

The U.S. Social Security System: What Does Political Sustainability Imply?*

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This paper examines how political constraints can shape the social security system under different demographics. A steady-state mapping between relevant economic and demographic variables and the social security tax rate resulting from a majority voting is provided. I calibrate an OLG model to the U.S. economy. Calculations using census population and survival probabilities projections and 1961–96 labor productivity growth deliver a social security tax rate of 13.3% (currently 11.2%) and a 54% replacement ratio (51.7%). This result reflects the median voter's aging, from 44 to 46 years, which dominates the decrease in the dependency ratio, from 5.45 to 4.72. *Journal of Economic Literature* Classification Numbers: H55, E17, D72. © 1999 Academic Press

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1. INTRODUCTION

The U.S. social security system has recently received enormous attention from both economists and policymakers. The discussion has focused on the demographic dynamics and its repercussion for the system's fiscal soundness and for the political representation of the different generations'

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opposing interests. In fact, several studies have argued that a graying America will not be able to honor its commitment to pay social security benefits to its retirees, unless a bigger financial burden is put on its working generations. The political process will thus have to reconcile the opposite interests of different generations of Americans.

This paper proposes a framework that can assess some demographic, economic, and political aspects of the social security debate. For different demographic and economic scenarios I attempt to determine the social security system that would receive the political support of a majority of the voters in the economy. In particular, this paper provides a steady-state mapping between probabilities of survival, population growth, and productivity growth rates, and the social security tax rate that would arise as an equilibrium outcome of a majoritarian voting game. In this context, for given realizations of these demographic and economic variables, a social security system is politically sustainable if it implements the equilibrium tax rate, and therefore it is supported by a majority of the voters.

I calibrate a large overlapping generation model with production to U.S. data. The economy is populated by several overlapping generations of workers and retirees. Agents are identical within a cohort, but differ across cohorts in working ability. They supply labor inelastically and retire at a mandatory age. The demographic aspect is summarized by the population growth rate and by the age-specific survival probabilities. The exogenous economic process is given by the labor productivity growth. These variables are parameterized using U.S. Census data and estimates. The usual calibration of an overlapping generation model is expanded to take into account the political process. In addition to the usual calibration targets,¹ the baseline economy is calibrated to obtain the current level of the U.S. social security tax rate as an equilibrium outcome of the political process. This allows us to extend the calibration to an additional parameter, the coefficient of relative risk aversion.

In this model, the social security system is unfunded, and its budget is balanced every year. Workers pay a proportional tax on their labor income; these contributions are entirely transferred to the retirees in equal shares. The U.S. social security system differs from this stylized version, mainly because of its annual unbalances. Indeed, the U.S. system was introduced in 1935 as a fully funded system. In the initial project, a trust fund had to be accumulated over the years through a slow but steady increase in the payroll tax rate in order to fund future benefits. However,

¹ The measured long-run values of the capital output and investment output ratios can be used to pin down two parameters of the model, typically the individual discount factor and the depreciation rate.

since the beginning the tax increases needed to fuel the fund lacked the necessary political support; the accumulation lagged behind schedule,² and the system has worked as an unfunded system.

In the context of an unfunded, balanced-budget social security system, survival probabilities, population growth, and productivity growth rates represent the relevant demographic and economic variables in the social security bill. A decrease in the population growth rate, associated with an increase in the survival probability for the elderly, leads to an older population. As the dependency ratio drops, either the retirement benefits decrease or the financial burden on the working population has to be increased. Increases in labor productivity growth, on the other hand, increase wages and thus increase total contributions to the system.

The steady-state mapping between the demographic and economic variables and the political equilibrium social security tax rate hinges crucially on the specification of the political system. In this paper I adopt a simple majoritarian political structure. Elections take place every period. Players in the voting game are all agents alive at every election. The equilibrium outcome of the voting game, i.e., the equilibrium tax rate, is determined at simple majority.

Since Hammond's (1975) contribution, this type of intergenerational game has been known to sustain (nonzero) social security systems through an implicit social contract that achieves cooperation among successive generations of egoistic players.³ Indeed, this approach typically generates a high degree of indeterminacy, as many tax rate sequences can be sustained as an equilibrium outcome of the voting game. The goal of this analysis is to provide a steady-state correspondence between some relevant demographic and economic variables and an equilibrium social security tax rate. Among the many possible equilibria, I thus choose to concentrate on steady-state equilibria induced by stationary strategy profile, i.e., by stationary implicit contracts. In particular, I focus on the steady-state subgame perfect equilibrium of the political game that maximizes the median age voter's remaining utility. This equilibrium has the feature that the initial median voter shares part of the gain from introducing the system

² In the initial project the trust fund was expected to finance one-third of the benefits through interest payments. The actual financing was 27% in 1950, 4.8% in 1960, 5.3% in 1970, 1.8% in 1980, and 7.3% in 1990. See Miron and Weil (1997).

³ This social contract has later been reinterpreted as an equilibrium outcome of a political process. See, among the others, Browning (1975) and Sjoblom (1985) as initial contributors to this literature, and Esteban and Sakovics (1993), Boldrin and Rustichini (1995), Cooley and Soares (to appear) and Azariadis and Galasso (1996, 1997) for later work.

with future median voters.⁴ Notice also that in adopting this equilibrium concept I abstract from transitional aspects. In fact, if the economy is outside the equilibrium steady state, the stationary strategy profile need not be an equilibrium along the transition path.

In this political environment, forward-looking, nonaltruistic agents may choose to support an unfunded system for two reasons. First, for some generations of voters the returns from the existing unfunded system may compare favorably to those that can be obtained from alternative investments.⁵ Second, the existence of an unfunded social security system tends to reduce the aggregate savings and thus the capital level. The resulting increase in the rate of return may increase the utility of a majority of the voters.⁶

Calculations performed using the U.S. Census population and survival probability projections deliver an equilibrium tax rate of 13.3%. This is obtained as a steady-state mapping from the projected annual population growth for the next 40 years of 0.78% and the 1961–96 average value of the productivity growth, into the corresponding equilibrium tax rate. The change in the tax rate, from the current 11.2% to 13.3%, is mainly due to the increase in the median voter's age, from 44 to 46 years, which dominates the negative effect of the decrease in the dependency ratio, from 5.45 to 4.72.

The next two sections describe, respectively, the economic environment and the political system. Section 4 presents some data on U.S. population, labor force, and labor productivity and the calibration. Section 5 shows the main findings and the sensitivity analysis. Concluding remarks and extensions are presented in Section 6.

2. THE ECONOMIC ENVIRONMENT

The economy consists of an overlapping generations model with production. Individuals are identical within cohorts. They face an age-specific

⁴ This is equivalent to the formulation by Cooley and Soares (to appear). Boldrin and Rustichini (1995), on the other hand, propose a "cookie monster" equilibrium, in which the first median voter runs away with the entire gain from introducing the system (the cookies) and leaves the future median voters indifferent between staying in the system or abandoning it.

⁵ Browning (1975) and Sjoblom (1985) provided a similar intuition. In a three-generation model they show that, if the social security decision were to be taken by the intermediate generation, the system would be pushed beyond its optimal level. See also Galasso (1998) for calculations on the profitability of the U.S. social security system for the median voter.

⁶ Cuckierman and Meltzer (1989) showed in the context of public debt decisions how this crowding out effect might play a crucial role in determining the median voter's decision. Boldrin and Rustichini (1995) and Cooley and Soares (to appear) applied this mechanism to social security decisions.

probability of surviving until the next period $(\pi_{t,i})_{i=0}^G$. Agents who reach the G th period of their life face certain death, $\pi_{t,G} = 0$. The demographic structure of the model can be synthesized by the population profile, which is obtained by combining the population growth rate, n_t , and these survival probabilities. The profile summarizes the fraction of population in each cohort: $\mu_{t,i}$, with $\sum_{i=1}^G \mu_{t,i} = 1$ for all t .

Agents work during the first J periods of their life and then retire. Labor is supplied inelastically.⁷ Retirement is mandatory at age 65. This assumption is not innocuous. In fact, several studies appearing in a book edited by Gruber and Wise (1998) show that many workers have a strong incentive to retire early, around age 62 in the United States. This early retirement decision clearly affects the social security system by reducing the dependency ratio. Here, I choose to abstract from early retirement for two reasons. First, as the agents are homogeneous within cohorts, at steady state they would all retire at the same age, either the mandatory age or earlier. Second, allowing for early retirement would introduce additional features into the social security system that, in the spirit of the model, would have to be agreed upon by a majority of the voters: the minimum retirement age and the degree of reduction in the benefits to be paid to early retirees. This would further complicate the political game, without adding much to the analysis, because of the agents' homogeneity.

2.1. Preferences

Agents value their lifetime consumption through a discounted lifetime utility function:

$$\sum_{j=0}^G \beta^j \left[\prod_{i=0}^j \pi_{t,i} \right] U(c_{t+j}^t) \quad \forall t, \quad (2.1)$$

where the c is the consumption, subscripts indicate the calendar time and superscripts the agent's period of birth, β is the individual discount factor, $\pi_{t,i}$ is the probability that an individual aged i at time t will survive until the next period $i + 1$, and $\pi_{t,0} = 1$.

⁷ Auerbach and Kotlikoff (1987) show that the existence of a social security system may have a negative impact on labor supply. Moreover, in the context of a life cycle model with altruism, Fuster (1997) suggests that the social security system tends to crowd out capital through labor supply distortions. By abstracting from endogenous labor decisions, I disregard the impact that these effects may have on the political decisions.

The utility function is assumed to have constant relative risk aversion:

$$U(c_{t+j}^t) = \frac{(c_{t+j}^t)^{1-\rho} - 1}{1-\rho}, \quad (2.2)$$

where ρ is the coefficient of risk aversion.

Every period agents decide how much of their resources to consume and how much to invest in claims to capital. Their resources are composed of returns from previous investment, labor income, and possible inheritance. For an agent born at time t , the sequence of budget constraints, from the first to the latest possible period of her life, can be written as follows:

$$c_{t+j}^t + a_{t+j+1}^t = a_{t+j}^t R_{t+j} + y_{t+j}^t + H_{t+j}^t \quad \forall j = 0, \dots, G, \quad (2.3)$$

where a_{t+j+1}^t and y_{t+j}^t represent the end-of-period assets holding and the disposable income at time $t+j$, and R_{t+j} is the interest factor on the assets purchased at time $t+j-1$.

Agents who do not survive until the last period, G , leave their savings as an unplanned or involuntary bequest. These asset holdings are assumed to be redistributed among agents of the same cohort in a lump-sum fashion.⁸ Therefore, those who survive until the successive period obtain an additional share of assets, $H_{t+j}^t = (1 - \pi_{t,j-1})a_{t+j}^t R_{t+j} / \pi_{t,j-1}$. Agents are born with no assets and leave no intentional bequest, i.e., $a_t^t = a_{t+G+1}^t = 0 \quad \forall t$.

Agents differ among generations in their working ability. Young and old generations are typically endowed with less human capital than middle-aged generations and thus earn lower wages. Retirees have no labor income, but they might be entitled to transfers, TR , from the social security system. The sequence of disposable incomes, y_{t+j}^t , for an agent born at time t is the following:

$$\begin{aligned} y_{t+j}^t &= \epsilon_{t+j,j} \cdot h \cdot w_{t+j} (1 - \tau_{t+j}) & \forall j = 0, \dots, J-1, \\ y_{t+j}^t &= TR_{t+j} & \forall j = J, \dots, G, \end{aligned} \quad (2.4)$$

where w_{t+j} is the wage rate per efficiency unit at time $t+j$, $\epsilon_{t+j,j}$ is a measure of the human capital or labor efficiency for the generation j at time $t+j$, h represents the number of hours worked, and τ_{t+j} and TR_{t+j} are, respectively, the social security payroll tax rate on wage income and the transfer to the old, at time $t+j$. In other words, the upper expression

⁸ Alternative redistribution schemes are examined in the sensitivity analysis.

in Eq. (2.4) represents the net wage income during the working period, while the lower one shows the income from old age transfers during retirement. In this model, the age-specific labor productivity ($\epsilon_{t+j,j}$) is exogenous, since there is no technology which allows agents to accumulate human capital.

2.2. Technology

There is a standard Cobb–Douglas production function with labor productivity growth:

$$Q_t = f[l_t \cdot (1 + \lambda)^t, k_t] = b \cdot k_t^\theta \cdot [l_t \cdot (1 + \lambda)^t]^{1-\theta}, \quad (2.5)$$

where λ is the labor productivity growth rate, l is labor per capita in efficiency units, k is capital per capita, b is the total factor productivity, and θ is the capital share of income.

The labor supply in efficiency units is equal to the fraction of workers in each cohort, multiplied by their specific human capital and by the number of hours supplied:

$$l_t = h \sum_{i=1}^J \epsilon_{t,i} \mu_{t,i}. \quad (2.6)$$

The total capital per capita in the economy is obtained by aggregating the net savings, or end-of-period asset holdings among generations:

$$k_t = \sum_{i=1}^G \frac{\mu_{t-i,i} a_t^{t-i}}{1+n}. \quad (2.7)$$

The profit maximization problem of the firm and equilibrium conditions yield the usual results in terms of competitive wage, w_t , and real return to capital, r_t :

$$\begin{aligned} w_t &= f_1[l_t \cdot (1 + \lambda)^t, k_t] \quad \text{and} \\ R_t &= 1 + r_t = f_2[l_t \cdot (1 + \lambda)^t, k_t] + 1 - \delta. \end{aligned} \quad (2.8)$$

Capital is assumed to depreciate at a rate δ .

Notice that this economy is not stationary. In fact, because of the labor productivity growth, all aggregate variables grow at a rate λ . However,

thanks to the CRRA utility function, standard transformations can be applied to make the economy stationary.⁹

2.3. The Social Security System

I consider a pure, self-financing, unfunded or pay-as-you-go (PAYG) social security system. The system consists of a sequence of transfers from the workers to the retirees. Each worker contributes to the system a percentage of her wage income, $\epsilon_{t,j}hw_t\tau_t$, and every retiree receives a transfer, TR_t . The budget is balanced every year so that the total amount of transfers made to the retirees is equal to the total amount of taxes collected:

$$\underbrace{hw_t\tau_t \sum_{i=1}^J \epsilon_{t,i} \mu_{t,i}}_{\text{contributions}} = \underbrace{TR_t \sum_{i=J+1}^G \mu_{t,i}}_{\text{benefits}} \quad \forall t. \quad (2.9)$$

This expression clarifies the importance of our key demographic and economic variables for evaluating the profitability of unfunded systems. In fact, for a given tax rate τ , the amount of benefits depends positively on the wages, and thus on the labor productivity growth, and on the ratio ϕ , which represents the number of contributors, weighted by their labor efficiency, divided by the number of the recipients:

$$\phi_t = \frac{\sum_{i=1}^J \epsilon_{t,i} \cdot \mu_{t,i}}{\sum_{i=J+1}^G \mu_{t,i}}. \quad (2.10)$$

The ratio ϕ depends positively on the population growth and negatively on increases in the survival probabilities for the elderly.

At its introduction in 1935, the U.S. social security system was created as a fully funded system. Tax rates were low at the beginning, but they were scheduled to be periodically increased. In particular, a trust fund was created and expected to be accumulated over time to provide the necessary funding for future benefits payments. However, as Miron and Weil (1997) recognize, the expected tax rate increases did not find the necessary political support, and the accumulation of the trust fund did not take place as planned. The extent to which the U.S. social security system has been run as an unfunded system is reflected in Fig. 1, which shows the annual balance of the system in proportion to the total benefits paid out (or contribution received) from 1955 to the estimates for 2002.

⁹ All individual variables and wages are divided by $(1 + \lambda)^t$.

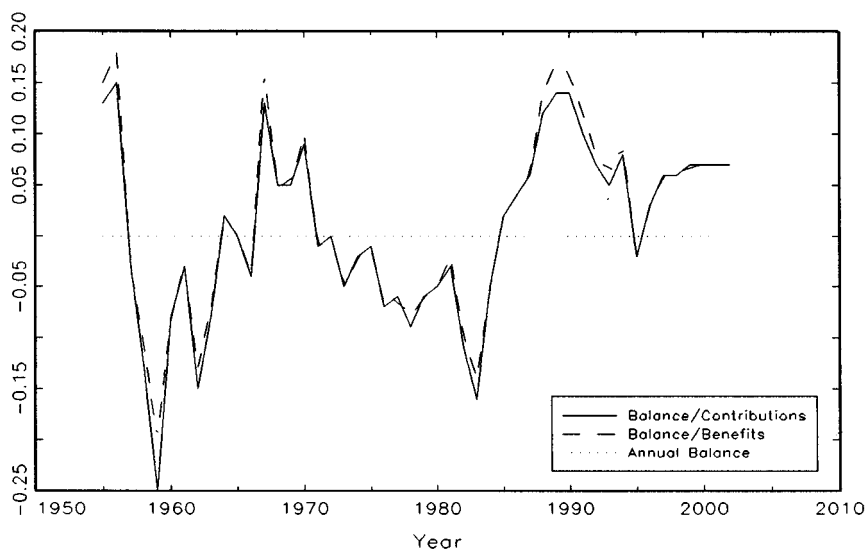


FIG. 1. U.S. Old Age and Survival Insurance Trust Fund: ratio of the trust fund annual balance to annual contributions (or benefits) from 1955 to 2002 (estimates).

The adoption in this model of a balanced, unfunded system greatly simplifies the analysis, since at each point in time there is only one policy decision to be taken. Clearly, for a given demographic structure of the population and wage level, the choice of the payroll tax rate (contributions) pins down the amount of transfer (benefits) as well. Moreover, this modeling choice is consistent with the objective of the paper, which is to examine how political constraints can shape the social security system under different demographic dynamics. In particular, for different realizations of the relevant economic and demographic variables, voters are required to determine the size of the system (i.e., the tax rate), provided that the system is balanced every year, and therefore fiscally sound.

Other studies have focused on unbalanced systems to analyze the fiscal sustainability of social security in response to demographic changes, when tax rate and individual benefits do not change. Auerbach and Kotlikoff (1987), for example, provide a welfare analysis of different possible policies for adjusting the social security system to an aging population.

2.4. *The Economic Equilibrium*

We can now define an economic equilibrium given the sequence of social security tax rates.

DEFINITION 2.1. For a given sequence of social security tax rates, labor productivity and population growth, $\{\tau_t, \lambda_t, n_t\}_{t=0}^\infty$, a competitive *economic equilibrium* is a sequence of allocations and prices, $\{c_t^{t-i}, w_t, R_t\}_{t=0, \dots, \infty}^{G-1}$, such that in every period,

- The consumer problem is solved for each generation i , i.e., agents maximize

$$\sum_{j=0}^{G-i} \beta^j \left[\prod_{x=i-1}^{j+i-1} \frac{\pi_x}{\pi_{i-1}} \right] U(c_{t+j}^{t-i})$$

with respect to $\{a_{t+j}^{t-i}\}_{j=1}^{G-i}$, given the budget constraints

$$c_{t+j}^{t-i} + a_{t+j+1}^{t-i} = a_{t+j}^{t-i} R_{t+j} + y_{t+j}^{t-i} + H_{t+j}^{t-i},$$

$\forall i = 0, \dots, G-1$ and $\forall j = 0, \dots, G-i$.

- The firms maximize their profits, and Eq. 2.8 is satisfied.
- Labor, capital, and goods markets clear, and thus, respectively, Eq. (2.6), Eq. (2.7), and the following expression are satisfied:

$$\sum_{i=1}^G (c_t^{t-i+1} + a_t^{t-i+1}) \mu_{t,i} = f(l_t, k_t) + (1 - \delta) \sum_{i=1}^G a_{t-1}^{t-i} \cdot \mu_{t-1,i}. \quad (2.11)$$

It is convenient at this point to define the expected utility for an agent aged i at time t in a competitive equilibrium.

DEFINITION 2.2. For a given sequence of social security tax rates, labor productivity, and population growth, $\{\tau_t, \lambda_t, n_t\}_{t=0}^\infty$, the remaining expected utility in a competitive equilibrium for an agent aged i at time t , $v_t^{t-i}(\{\tau_t, \lambda_t, n_t\}_{t=0}^\infty)$, is

$$v_t^{t-i}(\{\tau_t, \lambda_t, n_t\}_{t=0}^\infty) = \sum_{j=0}^{G-i} \beta^j \left[\prod_{x=i-1}^{j+i-1} \frac{\pi_x}{\pi_{i-1}} \right] \frac{(c_{t+j}^{t-i})^{1-\rho} - 1}{1 - \rho}, \quad (2.12)$$

where $\{c_t^{t-i}, w_{t+j}, R_{t+j}\}_{t=0, \dots, \infty}^{G-1}$ is a competitive equilibrium for a given $\{\tau_t, \lambda_t, n_t\}_{t=0}^\infty$.

3. THE POLITICAL SYSTEM

In this paper social security decisions are the equilibrium outcome of a voting game played by successive generations of voters, with elections taking place every period. Agents are forward looking in their voting

behavior, and they take fully into account the consequences of their decisions on future elections.

Players in this voting game are all agents alive at every election. For each generation i at time t , I consider a representative player, whom I refer to as generational player i . At every election there are G generational players, one for each generation alive. Notice that the mass of voters differs across cohorts because of the cohort size and because of the different participation rates at elections. It follows that the vote of each generational player i at time t has a different relative weight, $\psi_{t,i}$, depending on the mass of voters she represents, where $\sum_{i=1}^G \psi_{t,i} = 1 \ \forall t$. The introduction of generational players is consistent with the agent's homogeneity within cohort and allows for players' strategic voting. Notice, in fact, that with an infinite number of agents, each one having zero mass, no agent can be pivotal in the voting. Individual agents' deviations do not matter, since they cannot affect the outcome of the game. It follows that any strategy profile is an equilibrium. This consideration is typically used in this literature to support the assumption of sincere voting. The aggregation through generational players restores the strategic aspects of this intergenerational game.

Individual action spaces are the set of social security tax rates, $[0, 1]$. An action for a generational player i at time t is a tax rate: $q_t^i \in [0, 1]$. The action profile at time t is the vector of actions played at time t by all generational players alive: $q_t = \{q_t^1, q_t^2, \dots, q_t^G\}$.

I consider a majoritarian political system in which the political outcome is preferred to any other outcome by a majority of voters, and in which individuals have single peaked preferences over political outcome, i.e., tax rate. The realized tax rate at time t , τ_t , is thus assumed to be the median of the distribution of actions played by players alive at time t .

For a given sequence of profiles of actions, $(q_0, \dots, q_t, q_{t+1}, \dots)$, and corresponding outcomes, $(\tau_0, \dots, \tau_t, \tau_{t+1}, \dots)$, and given a pair of sequences $\{\lambda_t, n_t\}_{t=0}^\infty$, the expected payoff for a generational player i at time t is given by her expected utility, $v_t^{t-i}(\{\tau_t, \lambda_t, n_t\}_{t=0}^\infty)$, as in Definition 2.2.

The history of the game at time t describes the sequence of social security tax rates up to time $t - 1$. For simplicity we have let time begin at 0. The history at time t is

$$h_t = (\tau_0, \tau_1, \dots, \tau_{t-1}) \in [0, 1]^t. \quad (3.1)$$

A time t strategy for a player of age i alive at time t will therefore be a mapping

$$s_t^i: h_t \rightarrow [0, 1]. \quad (3.2)$$

Let s_t represent the collection of strategies adopted at time t by all generational players alive, $s_t = \{s_t^1, s_t^2, \dots, s_t^G\}$, and let s_t^{-i} be the collection s_t without the element s_t^i . For a given pair of sequences $\{\lambda_t, n_t\}_{t=0}^\infty$, let $c_t^{t-i}(s_0, \dots, s_t, \dots, s_{t+G-i+1}, \{\lambda_t, n_t\}_{t=0}^\infty)$ be the consumption in a competitive equilibrium for a generational player i at time t , given the realized sequence of social security tax rates resulting from the sequence of strategies $(s_0, \dots, s_t, \dots, s_{t+G-i+1})$.

A generational player i at time t maximizes

$$\begin{aligned} & V_t^{t-i}(s_0, \dots, s_t^i, s_t^{-i}, \dots, s_{t+G-i+1}, \{\lambda_t, n_t\}_{t=0}^\infty) \\ &= \max \sum_{j=0}^{G-i} \beta^j \left[\prod_{x=i-1}^{j+i-1} \frac{\pi_x}{\pi_{i-1}} \right] \\ & \quad \times \frac{c_{t+j}^{t-i}(s_0, \dots, s_t^i, s_t^{-i}, \dots, s_{t+G-i+1}, \{\lambda_t, n_t\}_{t=0}^\infty)^{1-\rho} - 1}{1 - \rho} \end{aligned} \quad (3.3)$$

The political process embedded in this game is known to sustain nonzero intergenerational transfer systems through an implicit social contract that achieves cooperation among successive generations of voters. Today's voters agree to transfer resources to current retirees because they expect to be rewarded for their actions with a corresponding transfer in their old age. If the current voters fail to comply with the contract, and thus vote not to pay social security to current retirees, they would receive no transfers in their old age. Indeed, this approach typically generates a high degree of indeterminacy, as many transfer sequences can be sustained as a political equilibrium of the voting game.¹⁰

The aim of this paper is to map out a steady-state correspondence between the relevant demographic and economic parameters and a social security tax rate that arises as an equilibrium outcome of this voting game. Therefore, among the many possible equilibria, I choose to concentrate on steady-state equilibria induced by stationary strategy profiles. The use of stationary strategy profiles seems particularly appropriate for examining steady states. The steady-state equilibria they induce have the feature that successive voters of the same age receive the same remaining lifetime expected utility. Moreover, I focus on the equilibrium that for given steady-state values of labor productivity and population growth rates (λ, n) maximizes the remaining lifetime expected utility of the median age (aged m) generational voter. The outcome associated with this equilibrium is equivalent to the equilibrium outcome obtained by Cooley and Soares (to

¹⁰ See, among others, Boldrin and Rustichini (1995), and Azariadis and Galasso (1996, 1997).

appear). In this equilibrium, the gain from introducing a social security system is shared among successive voters. In fact, successive median voters vote for their most preferred social security tax rate and obtain the same remaining lifetime expected utility. Boldrin and Rustichini (1995), on the other hand, focus on an equilibrium in which the first median voter extracts all of the gain, leaving future median voters indifferent between supporting or dismantling the system.

Let $\hat{\tau} > 0$ be given. I will say that condition (A) is satisfied at time t when $\exists t' \in \{0, 1, \dots, t-1\}$ s.t. $\tau_j = 0 \forall j \leq t'$ and $\tau_j = \hat{\tau} \forall j > t'$.

In words, condition (A) is satisfied at time t when either the social security system has never been introduced ($\tau_{t'} = 0$) or it has been introduced with a tax rate level $\tau = \hat{\tau}$, and this tax rate has prevailed since then.

Consider now a strategy profile $(\hat{s}_t^i)_{i=1, \dots, G}^{i=1, \dots, G}$, such that

- $\hat{s}_t^i(h_t) = \hat{\tau}$ for $i \in [l, u]$, if condition (A) holds, and
- $\hat{s}_t^i(h_t) = 0$ for $i \in [1, J]$, if condition (A) does not hold.

This strategy profile requires generational players with intermediate age (from l to u) to vote an initial tax rate $\tau = \hat{\tau}$ in the first election in which social security is introduced, and to vote for this initial tax rate ($\tau = \hat{\tau}$) provided that $\hat{\tau}$ has always prevailed since its first introduction. Generational players at their working age are required to dismantle the system, $\tau = 0$, if the initial tax rate has previously been changed. Notice that the conditions above constrain the strategy profile of some generational players only, and only after certain histories.

For a given sequence of population and labor productivity growth rates, $\{\lambda_t, n_t\}_{t=0}^{\infty}$, and for a given capital stock, k , this strategy profile is a subgame perfect Nash equilibrium of the voting game if individual optimality conditions are satisfied $\forall t$ and $\forall i$:

$$V_t^{t-i}(\hat{s}_0, \dots, \hat{s}_t^i, \hat{s}_t^{-i}, \dots, \hat{s}_{t+G-i+1}) \geq V_t^{t-i}(\hat{s}_0, \dots, s_t^i, \hat{s}_t^{-i}, \dots, \hat{s}_{t+G-i+1}),$$

if $\sum_{i=l}^u \psi_i \geq 1/2$, and $\sum_{i=1}^J \psi_i \geq 1/2$, and if the capital associated with the outcome (tax rate) is equal to the given initial capital stock: $k = k(\tau)$.

In other words, all generational players, each one taking individual optimal decisions, have to be induced to vote according to the strategy profile at every election; and the votes of specific voters, intermediate or working generational voters, depending on the history of the game, have to constitute a majority of the votes at every election. Moreover, since I look at steady-state equilibria induced by a stationary profile, the initial level of capital when the outcome tax rate is implemented, k , needs to be exactly equal to the steady-state capital stock associated with the realized tax rate,

$k(\tau)$, so that no transition is required.¹¹ When the above strategy profile induces a positive tax rate $\hat{\tau} > 0$ as an equilibrium outcome, I call this profile a Reversible Social Security Strategy Profile. In fact, this strategy profile supports positive social security tax rates with a reversion to zero social security whenever a deviation takes place.

Notice that many positive tax rates $\hat{\tau} > 0$ could be sustained by this strategy profile. As discussed earlier, I choose to concentrate on the subgame perfect equilibrium that maximizes the remaining lifetime utility of the voter with median age for a given pair (λ, n) . If voters' preferences about tax rates can be ordered by age (see next section), this selection criterion amounts to requiring that the tax rate has to be determined by a majority of the voters, and therefore by the median aged voter. Notice that successive median aged voters obtain the same remaining lifetime utility.

DEFINITION 3.1. For given steady-state values of population and labor productivity growth rates (λ, n) , the *politically sustainable social security tax rate*, $\bar{\tau}(\lambda, n)$, is the outcome of a Reversible Social Security Strategy Profile $(\hat{s}_t^i)_{t=0, \dots, \infty}^{i=1, \dots, G}$ that maximizes the median age voter's indirect utility (generational player m at time t):

$$\begin{aligned} \bar{\tau} = \arg \max \sum_{j=0}^{J-m} \beta^j & \left[\prod_{i=m-1}^{j+m-1} \frac{\pi_i}{\pi_{m-1}} \right] \\ & \times U(\hat{a}_{t+j}^{t-m}(\tau) R_{t+j} - \hat{a}_{t+j+1}^{t-m}(\tau) \\ & + H_{t+j}^{t-m} + \epsilon_{t+j, m+j} h w_{t+j} (1 - \tau)) \\ & + \sum_{j=J-m+1}^{G-m} \beta^j \left[\prod_{i=J}^{j+m-1} \frac{\pi_i}{\pi_{J-1}} \right] \\ & \times U(\hat{a}_{t+j}^{t-m}(\tau) R_{t+j} - \hat{a}_{t+j+1}^{t-m}(\tau) + \phi_{t+j} h w_{t+j} \tau). \end{aligned} \quad (3.4)$$

Definition 3.1 associates with every pair of labor productivity and population growth rates a steady-state politically sustainable social security tax rate, $\bar{\tau}(\lambda, n)$.

In the next sections I map different pairs of labor productivity and population growth rates into the corresponding politically sustainable tax rates. In particular, I calibrate a baseline economy to map the observed average values of the demographic and economic variables (λ, n) into an

¹¹ Notice that this strategy profile may support equilibria in which the initial level of capital, k , is different from $k(\tau)$. In this case the strategy profile needs to be an equilibrium along the transition from the initial k to the stationary $k(\tau)$.

equilibrium tax rate of 11.2%. Then, I repeat the computation for different pairs of parameter values.

4. DATA AND CALIBRATION

The ultimate goal of this paper is to map out a steady-state correspondence between some relevant economic and demographic variables and the equilibrium social security tax rate. In particular, population growth, labor productivity growth, and changes in life expectancy have important implications for the level of retirement benefits. In fact, for a given payroll tax level, an aging population reduces the social security benefits by decreasing the ratio of contributors to beneficiaries, ϕ . Labor productivity growth, on the other hand, translates into higher wages and thus boosts benefits.

The U.S. demographic dynamics of the past 40 years has been characterized by an aging population, because of the combined effects of a large downward swing in fertility and the constant reduction in mortality rates. Fertility rates increased after the Second World War, generating a long baby boom; a long-lasting reduction followed 20 years later.¹² Improvements in health care and standard of living, on the other hand, have constantly reduced the mortality rate.

The combined effect of those two elements is captured in Fig. 2, which represents the rates of growth of the U.S. population and labor force from 1953 to 1996. This figure clearly shows that the population growth rate decreased until the 1970s and then fluctuated around a value of about 1% a year. The labor force, on the other hand, has experienced a higher average growth rate, especially in the late 1960s and the 1970s.

Figure 3 displays the low, middle, and high series of population projections for 1997–2035, as predicted by the U.S. Census. These data are used to simulate the baseline economy.

The effect of these demographic changes on the social security system is reflected in the dependency ratio, i.e., the ratio of covered workers to retirees (old age and survivor insurance beneficiaries). This was 16.4 in 1950 and decreased to 3.8 in 1990. Moreover, the Social Security Administration's projections predict that in 60 years there will be only 2.2 covered workers per retiree, as Table I shows.

¹² Fertility rates reached a peak value of 3.68 in 1959 and dropped to a mere 1.74 in 1976. Since then they have recovered slightly just above 2.00. The Social Security Administration's (SSA) actuarial projections suggest that it will remain close to this level, converging to 1.9 around the year 2020.

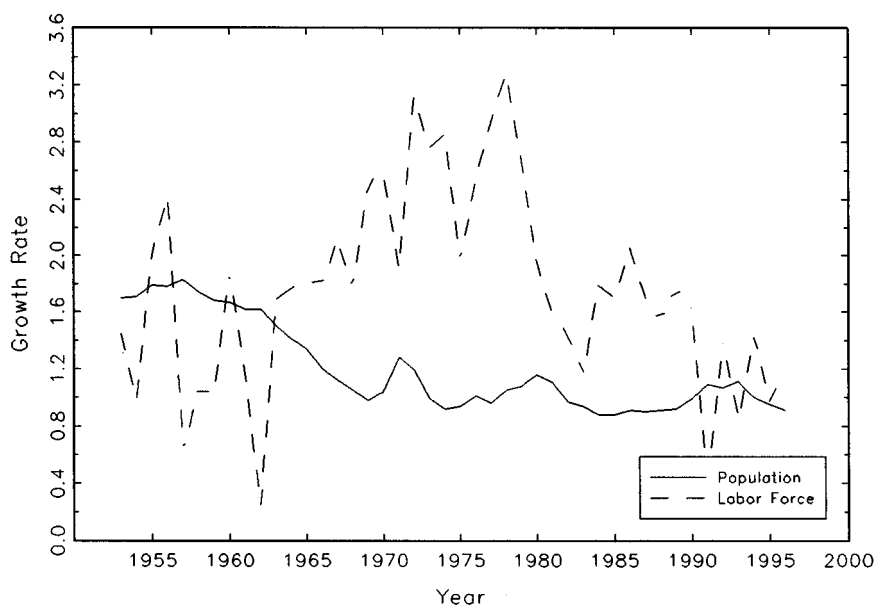


FIG. 2. U.S. population and labor force from 1953 to 1996.

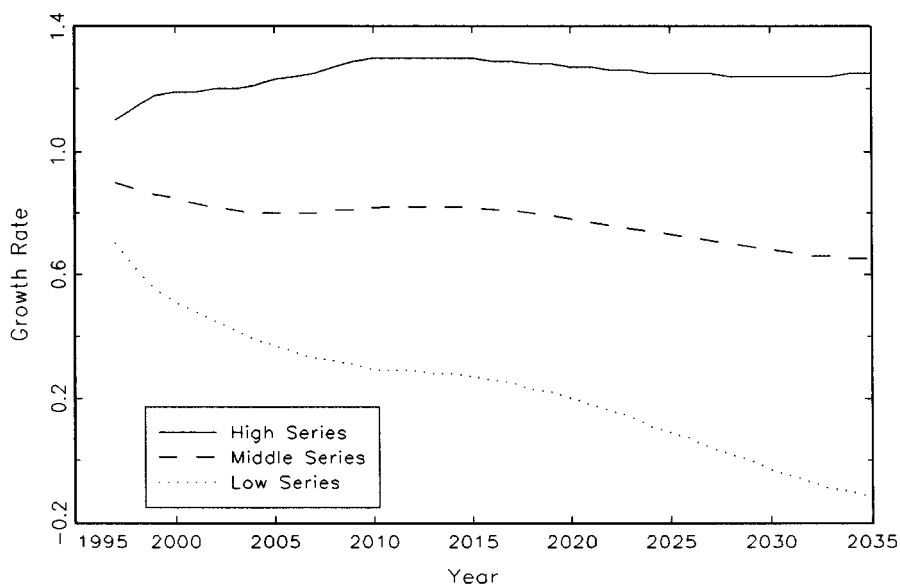


FIG. 3. U.S. Census projection for U.S. population from 1997 to 2035.

TABLE I
Elderly as Share of Total Population and Dependency Ratio

Year	65 and older /Total population ^a	Covered workers/OASI beneficiaries
1950	8.0%	16.39
1970	9.7%	4.11
1990	12.3%	3.76
2010	12.9%	3.55
2030	19.9%	2.34
2050	20.8%	2.2

^a Source: Steuerle and Bakija (p. 49, 1994) on Board of Trustees, OASDI historical data, and intermediate projections.

Data on the growth rates of U.S. labor productivity and wages are presented for the 1961–1996 period in Fig. 4. Labor productivity growth is measured as the growth rate of output per hours worked. The average rate of growth over this period has been around 2%. Wage growth rates are measured as hourly earnings growth rates. As the dashed line in Fig. 4 shows, wage growth has been lower than labor productivity, with an average rate of growth around 0.3%.

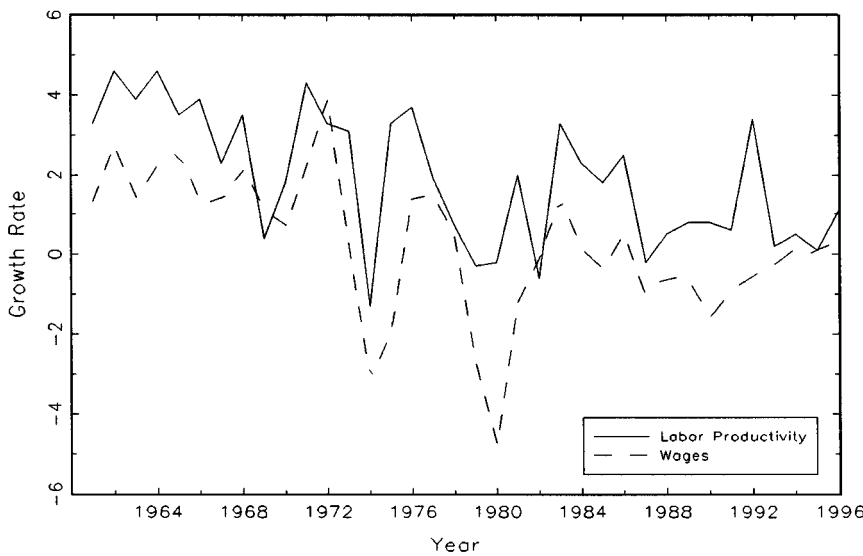


FIG. 4. U.S. labor productivity and wages from 1961 to 1996.

The data presented here are used to construct two alternative specifications of the model. In the main specification (M1), the relevant demographic and economic variables are, respectively, population and labor productivity growth. In the second specification of the model (M2), I use instead labor force and weekly earnings data.

4.1. Calibration Issues

The goal of the calibration exercise is to pin down some parameter values of the model economy by matching some measured data of the U.S. economy. Usually, in this type of life cycle model, only two measured stationary ratios, the capital output and the investment output ratio, can be matched.¹³ This allows the calibration of two parameters. All other values have to be obtained from empirical estimates.

In this paper, however, the calibration also aims at constructing a baseline economy in which the equilibrium social security tax rate equals the current U.S. OASI (Old Age Survival Insurance) payroll tax rate of 11.2%. These additional measured data allow the calibration of one more parameter.

Each period in the model corresponds to 1 year. Agents are born at age 18 and can live up to age 84. Every period they face an age-specific probability of death. The probabilities of survival used in the calibration of the baseline economy are obtained from *Vital Statistics of the U.S.* for 1992. Estimates of the survival probability for the years 2000 and 2050 are taken from Bureau of the Census Population Division Data. These are estimated using the middle series population projection. I use them to simulate changes in the baseline economy.

In parameterizing the technology, the productivity factor (b) is normalized to 1. The capital share of income (θ) is set equal to 0.36, as in Cooley and Prescott's (1995) calculation for the case with no home production, no government investment, and no explicit treatment of the inventories. The number of hours dedicated to work (h) is taken to be 0.423. This results from assuming 45 hours worked in a week out of a total of 98 hours, and an employment rate of 94%. The human capital or labor efficiency units profile, $(\epsilon_i)_{i=1}^J$, is calculated using the 1991 mean earnings for male individuals as published by the Bureau of the Census in *Current Population Reports*. The U.S. Census reports data for 18–64-year-old male individuals in nine age groups. The age-specific labor efficiency is then calculated by fitting these data with an exponential function as shown in Fig 5.

¹³ For a discussion of the calibration of overlapping generation models, see Auerbach and Kotlikoff (1987) and Rios Rull (1996).

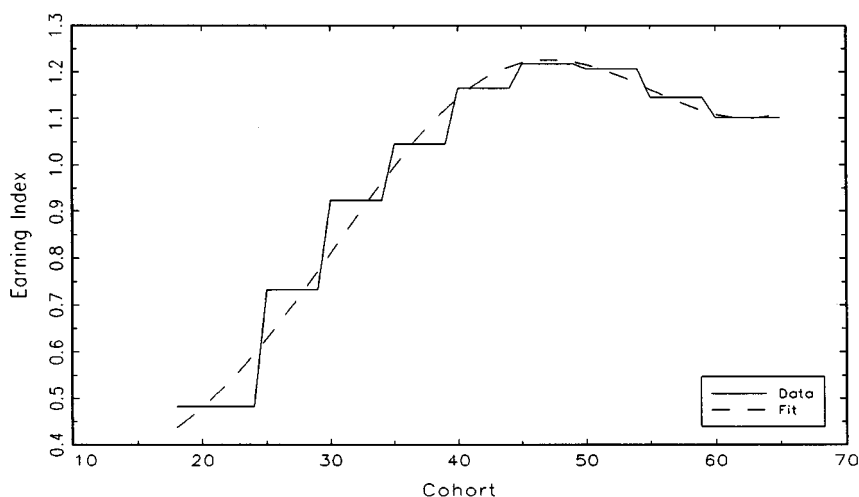


FIG. 5. Earning profile by age: constructed earning index and actual U.S. earning data by cohorts.

All of the above parameters are common to the baseline economy of both specifications of the model, M1 and M2.

M1, the primary specification, is further calibrated using growth rates of population and labor productivity. In its baseline economy, the annual labor productivity growth rate, λ , is taken to be equal to 1.94%, which is the rate of growth of output per hours worked in the 1961–96 period. The population growth rate, n , is set equal to 1.2%, the average rate of growth over the last 40 years.

M2 (the other specification) is parameterized to labor force and hourly earnings growth rate data. In its baseline economy, the annual rate of growth of the labor force, n , is equal to 1.76%, the average growth rate in the 1953–96 period, whereas the hourly earnings growth rate is set equal to 0.27%, the average over the last 35 years.

The parameterization of the political system requires the measured social security tax rate (11.2%) to be an equilibrium of the majoritarian voting game. Figure 6 shows that in the baseline economy generational players have single peaked preferences over tax rates, and that the preferred tax rate is increasing with the voter's age. Moreover, once the economy is calibrated to yield the tax rate of 11.2% as the median voter's preferred level, it can be shown that the (Reversible Social Security)

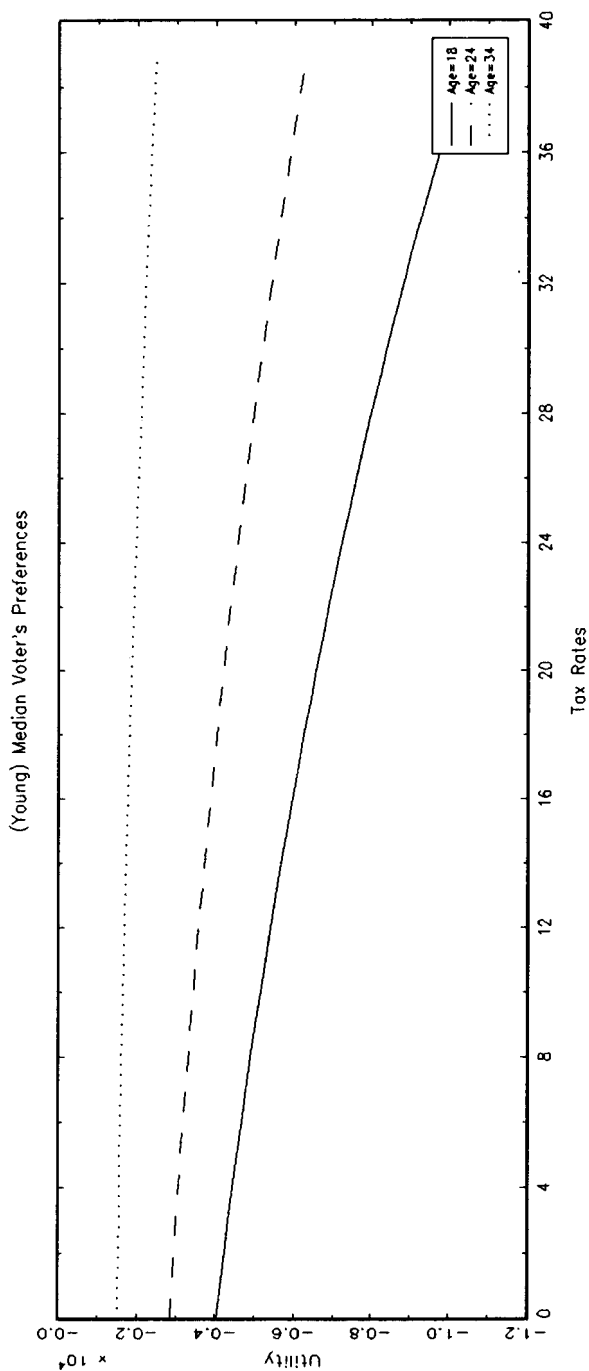


FIG. 6. Voter's preferences about social security tax rates by age: 18, 24, 34, 44, 54, and 64 years.

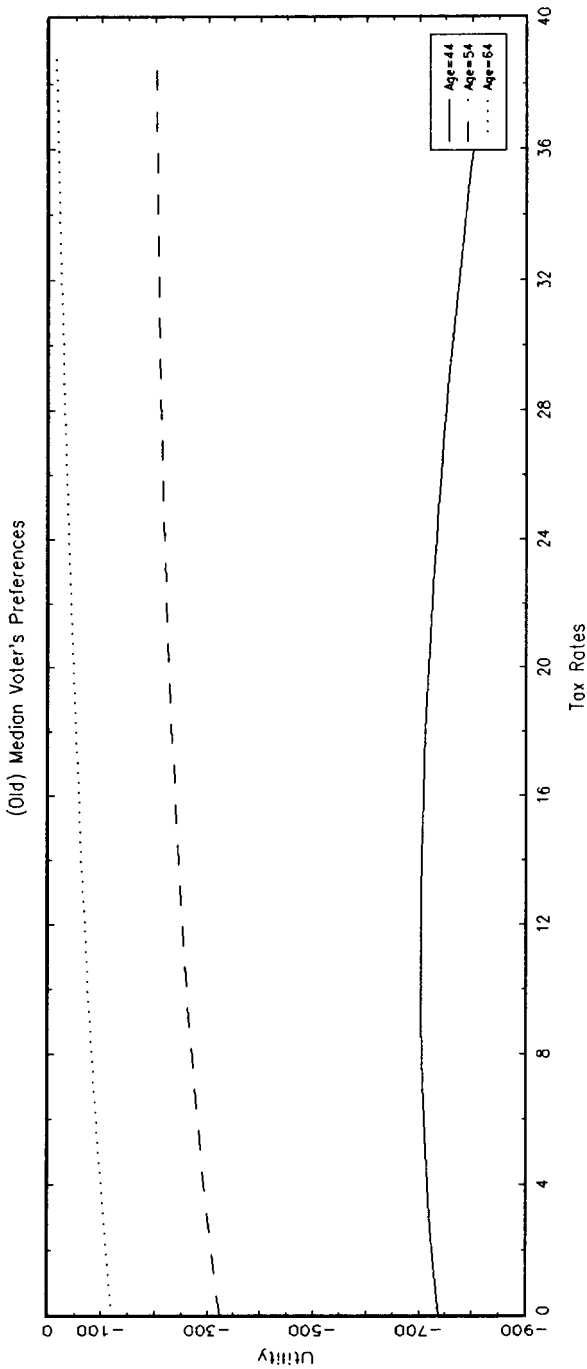


FIGURE 6—Continued

TABLE II
Population Growth Rate–Median Vote Age Correspondence

Population growth rates	Median voter age ^a
[0.25, 0.40]	48
[0.45, 0.65]	47
[0.70, 0.90]	46
[0.95, 1.15]	45
[1.20, 1.45]	44
[1.50, 1.70]	43
1.75	42

^a Using 1992 U.S. Census Probability of Survival and 1992 Presidential Elections Participation Rates. See Footnote 16.

strategy profile supporting this equilibrium tax rate is played by a majority of the voters.¹⁴

A crucial element in this calibration exercise is represented by the median voter's age. Using the voter's participation rate for the 1984, 1988, and 1992 presidential elections, as reported by the U.S. Census, I calculate the median voter's age at these elections to be 44 years.¹⁵ Probabilities of survival, population growth rate, and election participation rates are then used to mimic the age profile of the electors. Using the 1992 survival probabilities and the participation rates at the 1992 presidential elections, it is possible to find a correspondence between the population growth rate and the median voter's age.¹⁶ This correspondence is reported in Table II.

Finally, the steady-state measures of the capital–income ratio and of the investment–income ratio, together with the current value of the U.S. OASI social security tax rate, can be used to calibrate the depreciation

¹⁴ Recall that for a Reversible Social Security Strategy Profile to be an equilibrium, a majority of intermediate-age players have to support the tax rate, and a majority of working-age players have to punish any deviation. In this calculation, the intermediate-age generational players are between 38 and 70 years old and constitute 54% of the electors, whereas the working generations (from 18 to 64 years old) represent more than 81% of the voters.

¹⁵ The median age of the electorate in presidential elections is sensibly lower: 41 year in 1992, 40 in 1988 and 1980, and 39 in 1984. However, participation rates are much higher among older generations. See Galasso (1998).

¹⁶ In the baseline version of the model, a population growth rate of 1.2% would yield a median voter age equal to 45 years, instead of the 44 years computed from the data. To adjust for this discrepancy I redefine the median voter age to be the age i such that 48.6% of the voters are younger than or as old as i . In this case the baseline model yields a 44-year-old “adjusted” median voter for $n = 1.2\%$. Table II takes into account this adjustment. The sensitivity analysis provides some results for the unadjusted model.

TABLE III
Parameterization of the Baseline Economy: Specifications M1 and M2

Specification	<i>n</i>	<i>m</i>	<i>b</i>	θ	<i>h</i>	δ	λ	ρ	β
M1	1.2	44	1	0.36	0.423	5.408	1.94	2.073	1.0307
M2	1.76	44	1	0.36	0.423	6.537	0.27	2.2	1.0163

rate, δ , the individual discount factor, β , and the coefficient of relative risk aversion, ρ , in both models' baseline economies.

Since I exclude the government from the analysis, the calibration targets are¹⁷

$$\frac{K}{Y} = 2.94, \quad \frac{I}{Y} = 0.252, \quad \tau = 11.2\%.$$

In model 1, the depreciation rate, δ , turns out to be equal to 5.408%, the individual discount factor, β , is 1.0307, and the coefficient of relative risk aversion,¹⁸ ρ , is 2.073. In model 2, the depreciation rate is equal to 6.537%, the individual discount factor is 1.0163, and the coefficient of relative risk aversion is 2.2 (see Table III).

5. FINDINGS

This paper provides a mapping between relevant economic and demographic variables and the politically sustainable level of social security. In particular, I concentrate on the correspondence between population growth rate, labor productivity growth rate, age-specific survival probabilities (and the induced median voter's age), and the equilibrium social security tax rate.

The theoretical model suggests that a reduction in the population growth rate or an increase in the survival probabilities for the elderly has the opposite effect on the equilibrium tax rate. On one hand, it decreases

¹⁷ See Rios Rull (1996).

¹⁸ As terms of comparison, in utility functions with consumption and leisure, estimations for the coefficient of relative risk aversion range from 1.33 to 7.7. Auerbach and Kotlikoff (1987) use a value of 4. For a leisure preference parameter of 2/3, this value translates into $\rho = 2$ for a model without leisure. This parameter is crucial to the analysis, as it largely affects the savings decision. A low value of the coefficient of relative risk aversion leads to a lesser degree of consumption smoothing. In a model where social security can be seen as an alternative and better performing investment for the decision maker, a low value of ρ implies a large size of the system.

TABLE IV
Simulation Results

	Baseline	Simul.	"Control A"	Simul.	Simul.	Simul.
Population growth	1.2	0.78	0.78	0.78	0.78	0.78
Median voter age	44	46	44	46	47	47
Survival probability	1992	1992	1992	2000	2050	2000-50
Productivity growth	1.94	1.94	1.94	1.94	1.94	1.94
Dependency ratio	5.45	4.72	4.72	4.68	4.02	4.32
Equm tax rate	11.2%	13.3%	8.7%	13.2%	11.9%	13.9%
Replacement rate	51.7%	54%	35%	53%	41.2%	51.3%
K/Y	2.94	2.96	3.14	2.97	3.11	2.99
I/Y	0.252	0.241	0.256	0.241	0.253	0.244
Rate of return	6.84%	6.77%	6.06%	6.72%	6.18%	6.63%
Wages	1.174	1.177	1.218	1.180	1.21	1.185

the profitability of the social security system by reducing the dependency ratio; on the other hand, it raises the median voter's age. An increase in labor productivity has a positive impact on the size of the system.

The first part of this analysis focuses on changes in the demographics for the main specification of the model (M1). Results for the model parameterized to wage growth and labor force growth rates (M2) are reported in the sensitivity analysis. For a constant labor productivity growth of 1.94%, I simulate the baseline economy using the average of U.S. Census middle series estimates of the population growth rate from 1997 to 2035, $n = 0.78\%$, and the U.S. Census middle series estimates of the survival probability for the years 2000 and 2050. The results are reported in Table IV. The first column of this table describes the baseline economy, which was calibrated to have an equilibrium tax rate of 11.2%. Given the dependency ratio of 5.45, this yields a replacement ratio¹⁹ of 51.7%.

Column 2 shows the simulation of the baseline economy with a population growth rate of 0.78% and with the corresponding median voter's age, 46 years (see also Table II). This combination of values maps into an equilibrium tax rate of 13.3%, which yields a replacement rate of 54%. The aging in the population is reflected in the reduction of the dependency ratio to 4.72. The changes in the other variables, capital output ratio, rate of return, etc., are marginal.

To isolate the importance of changes in the dependency ratio from variations in the median voter's age to the determination of the equilibrium level of social security, I simulate the baseline economy for the

¹⁹ The average replacement ratio in the first year of retirement was around 61% in 1995. See Miron and Weil 1994 (1997).

average estimated population growth ($n = 0.78\%$), but keeping the median voter's age constant at 44 years, I call this economy "control A." As reported in column 3, the equilibrium tax rate drops to 8.7% and the replacement rate to 35%. The decrease in the dependency ratio (from 5.45 to 4.72) makes the system less profitable and induces the 44-year-old median voter to prefer alternative forms of saving. The capital-output ratio increases to 3.14, driving the interest rate down to around 6%.

The differences between the baseline economy and the economy simulated with the average estimated population growth and the corresponding median voter's age (column 2) can be examined by observing Figs. 7, 8, and 9, which show, respectively, consumption, assets, and income profiles for these two economies. A third, dashed line in the graphs describes a second control economy, "control B," which combines the new demographics, $n = 0.78\%$ and a 46-year-old median voter, with the previous social security system, $\tau = 11.2\%$.

The comparison between the baseline (*dotted line*) and this "control B" economy shows the impact of a reduction of the dependency ratio on the initial social security system, $\tau = 11.2\%$. The consumption profile (Fig. 7) tilts around the middle age and provokes a strong reduction in the old age consumption; the assets profile (Fig. 8) is almost identical. The income profile (Fig. 9) changes considerably: the higher capital level increases

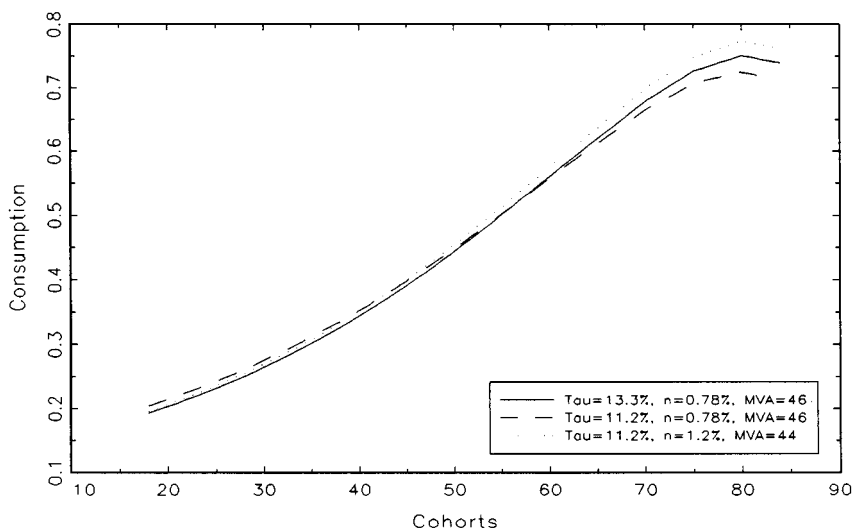


FIG. 7. Consumption profile over lifetime, for baseline, control, and simulated economy.

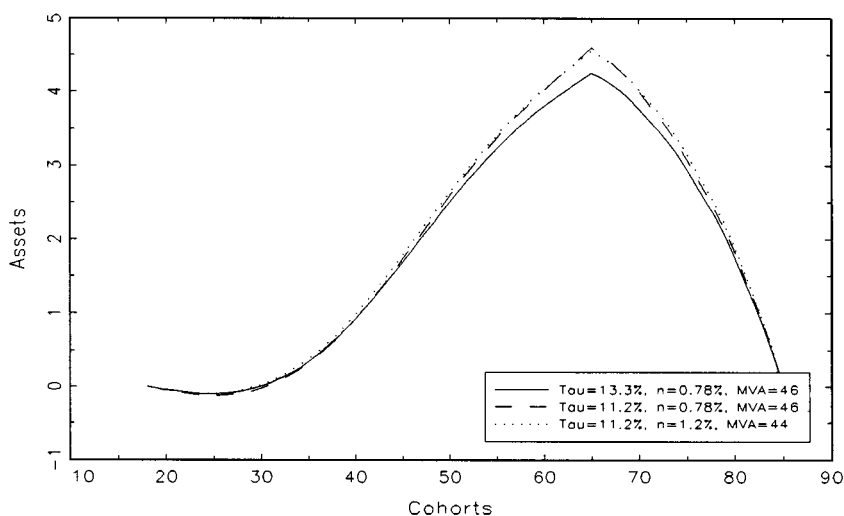


FIG. 8. Assets profile over lifetime, for baseline, control, and simulated economy.

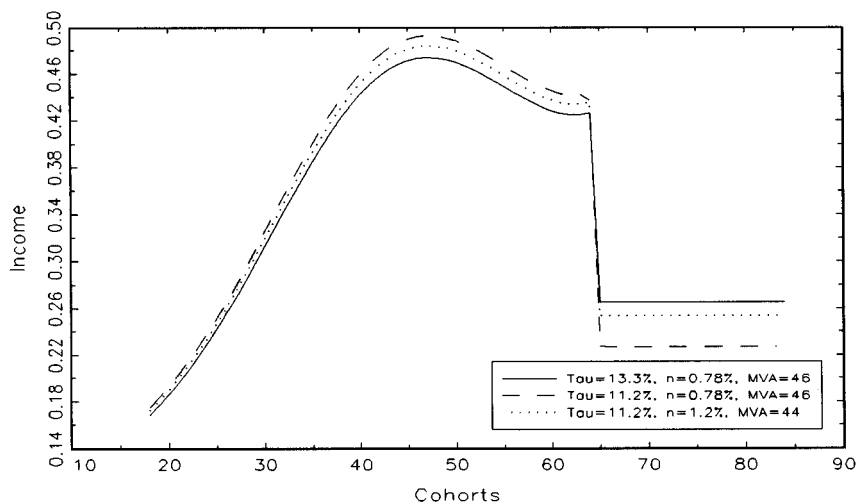


FIG. 9. Income profile over lifetime, for baseline, control, and simulated economy.

wages and therefore the labor income, whereas pensions are greatly reduced because of the decrease in the dependency ratio.

If the new (older) median voter is allowed to modify the system after the demographics have changed, the situation differs. The higher equilibrium tax rate, $\tau = 13.3\%$, associated with the new demographics modifies income and assets profiles. It decreases the disposable labor income and raises pensions, thus reducing assets accumulation and almost restoring the previous replacement rate. With respect to the "control B" economy, the consumption profile tilts strongly toward more old age consumption, responding to the incentives of an older median voter. Notice that the entire consumption profile lies below the baseline economy's.

The simulation performed using the estimated survival probability for the year 2000 yields almost identical results (column 4). The median voter is still 46 years old, while the dependency ratio decreases slightly to 4.68. This leads to an equilibrium tax rate of 13.2% and a replacement rate of 53%.

The adoption of the 2050 estimate probabilities of survival changes the picture quite dramatically (column 5). The estimated reduction in the mortality rate for the elderly translates into an older median voter, 47 years old, and a much lower dependency ratio, 4. With these demographics the model associates an equilibrium tax rate of 11.9% and a replacement rate of only 41%. Moreover, the capital output ratio is strongly affected. Column 6 reports the results obtained for an average between the survival probabilities of the years 2000 and 2050.

The second part of this analysis examines the combined effects of economic and demographic variables by calculating the equilibrium tax rate associated with different pairs of population and labor productivity growth rates.

Because of the uncertainty about the long-run pattern of the population, as shown in the large differences between the high and low series (see Fig. 3), I choose to simulate the baseline economy for a large interval of population growth rates, from 0.25% to 1.75%, and for the corresponding median voter's age, respectively from 48 to 42 years (see Table II). The survival probabilities are taken to be the 1992 value. Analogously, because of the lack of better estimates, labor productivity growth is allowed to vary from 1% to 3%.

The results are summarized in Fig. 10, which maps population and labor productivity growth rates into equilibrium tax rates. This graph helps to identify the different forces at work: a decrease in the growth rate of population reduces the profitability of the system and thus the equilibrium tax rate, because of a lower dependency ratio. However, it also implies an older population, and thus an older median voter. The resulting reduction in the number of contribution years for the median voter increases her

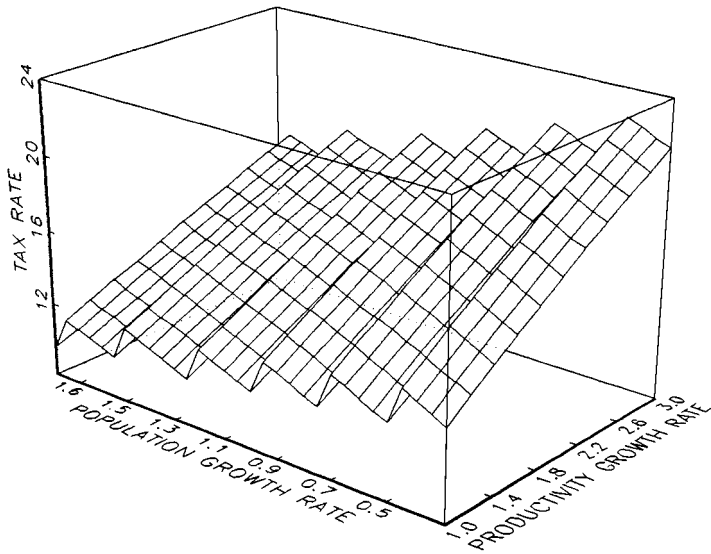


FIG. 10. Steady-state correspondence between equilibrium social security tax rate and pairs of population growth rate and labor productivity growth rate as obtained by the simulation.

return from investing in social security and thus the payroll tax rate. Increases in labor productivity univocally increase the tax rate by increasing the wage growth rate.

This figure suggests that the effect of the change in the median voter's age is predominant. In fact, low equilibrium levels of social security are typically obtained only for low productivity and for high population growth rates and therefore when the median voter is young.

5.1. Sensitivity Analysis: Model's Specification M2

This version of the model is parameterized to labor force and wage growth rates. Column 1 of Table V reports the calibrated baseline economy. The values of the variables are in line with the main model. However,

TABLE V
Sensitivity Analysis: Model Specification M2

	N	MVA	λ	β	ρ	τ	RR	K/Y
Baseline	1.76%	44	0.27%	1.0163	2.2	11.2%	61.6%	2.94
Simulation	1.10%	46	0.27%	1.0163	2.2	12.0%	53.7%	3.07

this economy features a lower rate of return (5.7% vs. 6.8% in the main model) and higher values of the replacement rate, 61.6%, and the dependency ratio, 6.62. Column 2 shows the results of a simulation performed using the U.S. Census labor force growth projections for 1996–2005 and a corresponding median voter’s age of 46 years. The equilibrium tax rate is higher than in the baseline economy, 12%, which confirms that the increase in the median voter’s age has a stronger impact than the drop in the dependency ratio. However, in this alternative model the magnitude of the change is much lower, and the replacement rate decreases.

5.2. Sensitivity Analysis: Involuntary Bequest

In the baseline model, unwilling bequests are redistributed among agents of the same cohort in a lump-sum fashion. Here, I examine three alternative ways of dealing with involuntary bequest.

First, I consider that the assets of the deceased are divided up among agents of the same cohort in proportion to their holdings. This amounts to assuming that agents enter a 1-year annuity contract with agents of the same generation to distribute the assets of the deceased. This redistribution scheme directly affects the realized rate of return. For example, survivors in the older generations receive higher returns on assets. The expected rate of return is, however, equal across cohorts. Column 1 of Table VI shows the calibration for a baseline economy with a proportional redistribution scheme. The calibration parameters are in line with the main model. Column 2 reports the results obtained by simulation of this economy with a population growth rate of 0.78% and a corresponding 46-year-old median voter. They are virtually identical to the main model: the equilibrium tax rate goes from 11.2% to 13.2%, and the replacement rate from 51.7% to 53.6%.

TABLE VI
Sensitivity Analysis: Involuntary Bequest

	Case I		Case II		Case III	
	Baseline	Simul.	Baseline	Simul.	Baseline	Simul.
Population growth	1.2%	0.78%	1.2%	0.78%	1.2%	0.78%
Median voter age	44	46	44	46	44	46
Productivity growth	1.94%	1.94%	1.94%	1.94%	1.94%	1.94%
IDF (β)	1.0212	1.0212	1.0396	1.0396	1.0529	1.0529
CRRA (ρ)	2.045	2.045	2.51	2.51	3.27	3.27
Equm tax rate	11.2%	13.2%	11.2%	13.2%	11.2%	13.7%
Replacement rate	51.7%	53.6%	51.7%	53.6%	51.7%	55.6%
K/Y	2.94	2.96	2.94	2.96	2.94	2.98

Second, I analyze a scheme which redistributes the total amount of unwilling bequest in a lump-sum fashion among survivors of all ages. Every agent alive obtains the same transfer. Columns 3 and 4 of Table VI report the calibration of this economy and the corresponding simulation. Again, the simulation results are not affected. In this case, however, the calibrated value of the coefficient of relative risk aversion is higher than in the baseline main model (2.51 vs. 2.073). This suggests that a constant, lump-sum bequest to every agent increases the incentive to have a social security system, because of the higher level of asset holdings.

Finally, in the third case all unwilling bequests are transferred as a lump sum to the youngest generation, the 18-year-olds. This redistribution scheme amplifies the effect detected in the previous case. As displayed in columns 5 and 6 of Table VI, the calibration of this economy requires a very high value of the coefficient of relative risk aversion for the equilibrium tax rate to be 11.2%. The large amount of assets transferred to the young tends to increase the capital level in the economy and thus to reduce the rate of return. This makes the social security system more profitable, both as an alternative form of saving and because of its role in crowding out capital. The simulated economy maps into an equilibrium tax rate of 13.7% and into a replacement rate of 55%.

5.3. *Sensitivity Analysis: Median Voter's Age*

In the baseline model, the computation of the median is adjusted to obtain a 44-year-old median voter as calculated from the data (see footnote 16). Here, I examine the results of the unadjusted model in which the median voter's age corresponding to a population growth rate of 1.2% is 45 years. The calibration of the model is reported in Table VII. Notice that the coefficient of relative risk aversion is higher than in the main model ($\rho = 2.436$), as an older median voter would prefer a larger system. However, the results of the simulation do not change. A population growth rate of 0.78% is now associated with a 47-year-old median voter, and it yields an equilibrium tax rate of 13.3% and a replacement ratio of 54.7%.

TABLE VII
Sensitivity Analysis: Median Voter Age

	n	MVA	λ	β	ρ	τ	RR	K/Y
Baseline	1.2%	45	1.94%	1.0425	2.436	11.2%	52.6%	2.94
Simulation	0.78%	47	1.94%	1.0425	2.436	13.3%	54.7%	2.96

6. CONCLUSIONS

Projections of future fertility and mortality rates show that the U.S. population will continue to age. Many studies have conjectured that additional social security reforms will be needed in order to ensure the fiscal sustainability of the system.

This paper concentrates on how political constraints can shape the social security system under different demographics. In particular, I provide a steady-state mapping between some relevant economic and demographic variables and the social security tax rate obtained as an equilibrium outcome of the majoritarian voting game.

This model suggests that, although current demographic trends decrease the dependency ratio and thus reduce the profitability of the social security system, they also raise the median age of the voters. These two effects have opposite impacts on the equilibrium tax rate.

Calculations performed using the U.S. Census population and survival probability projections and the 1961–96 average value of the productivity growth rate deliver a corresponding equilibrium social security tax rate of 13.3%. This increase reflects the expected aging of the median voter, from 44 to 46 years, which dominates the decrease in the dependency ratio, from 5.45 to 4.72. The new tax rate yields an almost unchanged replacement ratio, 54% versus the previous 51.7%.

The negative impact of the population aging becomes more relevant when the survival probability projection for the year 2050 is considered. In this case, the median voter is 47 years old, the dependency ratio drops to 4, and the corresponding tax rate is 11.9% (and the replacement ratio drops to 41%).

Simulations of the baseline economy for different values of the productivity growth rate quantify the impact of this economic variable on the social security system. In particular, using the 1997–2035 U.S. Census population projections, $n = 0.78\%$, the calculations suggest that a 25% drop in the current productivity growth rate, from 1.94% to 1.5%, is associated with an equilibrium tax rate of 11.8%, whereas a 25% increase to 2.5% maps into a 15.2% social security tax rate.

The notion of political sustainability presented here should be interpreted as a measure of *minimum* sustainability. It hinges crucially on the idea that workers who previously contributed to the system would not be even partially reimbursed if the system is dismantled. In fact, allowing for partial reimbursement of past contributions would decrease the profitability of the social security system, since the workers, and in particular the median voter, would not need to stay in the system until retirement to obtain some payments. An interesting, complementary exercise would be to calculate the amount of money that the median voter would need to

receive in order to be indifferent between continuing with the unfunded system and switching to a funded system or simply to private saving. This exercise would help define an alternative, stricter measure of political sustainability.

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