

Putting theorems 1 and 2 together, there is a symmetric relationship between the two problems if both Z.1 and Z.2 hold. Note that if Z.1 fails, it is still true that a solution to (D) will be a solution to (P), but not conversely. If Z.2 fails but not Z.1, a solution to (D) will be a solution to (P), but not conversely.

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## UNCERTAIN LIFETIMES AND THE WELFARE ENHANCING PROPERTIES OF ANNUITY MARKETS AND SOCIAL SECURITY

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This paper explores the implications of social security programs and annuity markets through which agents, who are characterized by different distributions of length of lifetime, share death-related risks. When annuity markets operate, a non-discriminatory social security program affects only the intragenerational allocation of resources. In the absence of private information regarding individual survival probabilities, such a program will lead to a non-optimal intragenerational allocation of resources. However, the presence of adverse selection considerations gives rise to a Pareto improving role for a mandatory non-discriminatory social security program.

## 1. Introduction

In the absence of bequest motives, the existence of uncertainty with respect to the length of lifetime will lead individuals to save for their old age via annuities. This form of savings, which shares the risks related to old age, makes possible higher rates of return on forgone consumption by making claims to future payments conditional on an individual's continued life. The role of annuities as a device for sharing uncertainty about the length of one's life and observations on the absence of complete markets for such contracts constitute an important part of Diamond's (1977) suggestions for evaluating social security type programs.

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The same view underlies the recent work of Kotlikoff and Spivak (1981) who consider the family institution as a substitute, albeit imperfect, for complete annuity markets. Likewise, Sheshinski and Weiss (1981) examine alternative forms of financing a publicly provided, actuarially fair annuity program. Eckstein, Eichenbaum and Peled (1983) and Abel (1983) examine different aspects of wealth distribution and welfare gains from the introduction of a social security program to an economy where all private annuity markets are exogenously excluded. A different kind of risk that may be inefficiently allocated in a decentralized equilibrium is considered by Merton (1981), who examines the features of a Pareto improving social security program in an intertemporal model with non-tradeable, randomly productive human capital.

A potential limitation of the above analyses is the absence of an explicit specification of which features in the environment prevent decentralized equilibria from attaining optimal allocations. Thus, Kotlikoff and Spivak confine their analysis to the relative efficiency of equilibria when risk-sharing opportunities are restricted on a 'family' basis, while Merton's Pareto improving policy depends on unspecified reasons that restrict the tradeability of human capital in the first place. On the other hand, Sheshinski and Weiss acknowledge at the outset that their analysis pertains to the social security program only to the extent that privately issued annuities are ruled out. In contrast, we describe here an economy in which private information about individual specific survival probabilities may inhibit the efficient operation of annuity markets. We then compare the impact of a social security type program in such an informationally constrained environment with its impact on the same economy with full information. The private information turns out to have three important implications regarding both positive and normative aspects of social insurance: (a) it is shown that a mandatory social security type program may yield a Pareto superior equilibrium allocation; (b) the publicly provided annuity program is different from, and exists simultaneously with, privately supplied annuities which satisfy residual demands for annuities; and (c) because public and private annuities offer different rates of return they are not perceived of as perfect substitutes by individuals. Consequently, both aggregate private savings and observable rates of return in the economy are affected by the level of the public program. While we concentrate in this work on the uncertainty which is related to length of life, we envision extensions and refinements of our stylized analysis that reflect a variety of individual uncertainties.

Our analysis is conducted within a version of Samuelson's (1958) overlapping generations model which provides a convenient framework for studying the intergenerational allocation of resources in an intertemporal economy populated by finitely lived agents. In the specific model under consideration, agents live at most two periods. In order to introduce a

natural role for annuities, we assume that while life during the first period is certain, death can occur at the beginning of the second period with a positive probability. In addition, we assume that survival probabilities differ across cohorts. These features allow us to examine the role of an annuity-like social security program under a variety of assumptions concerning the risk-sharing contracts available in the economy.

When individuals' survival probabilities are commonly known, Pareto optimal allocations are characterized by equal consumption levels across both periods of life of agents that reach old age. Consequently, ex ante marginal rates of substitution of consumption across periods are *not* equal at those optimal allocations for agents with different survival rates. Rather, with no population growth, marginal rates of substitution associated with optimal allocations must equal group specific actuarially fair rates. A competitive equilibrium, under this public information assumption, will support a discriminatory annuity structure with precisely this return structure, in which each group of cohorts with common survival probability shares the death-related risks among its own members.

The optimal sharing of old age risks obtained by private markets is destroyed by considering individual survival probabilities as private information. As was shown by Rothschild and Stiglitz (1976), Wilson (1977), and Riley (1979a, 1979b) in their work on insurance equilibria with private information, decentralized equilibria may yield inefficient allocations. In our context the same problem arises because agents with high survival probabilities — a group which constitutes high risk for annuity issuers — impose an externality which harms other agents without necessarily gaining anything themselves. One way to improve that equilibrium allocation which is consistent with this information structure involves a government-imposed non-discriminatory annuity program which takes the following form: agents are forced to contribute a prespecified amount to the program when young, and are paid off at a rate of return which equals the economy-wide actuarially fair rate of return if they reach old age. Residual demands for annuities will be supplied in a competitive *separating* equilibrium, so that, ex post, agents from different groups will have purchased some annuities with group specific actuarially fair rates of return. The resulting allocation is Pareto optimal and for a large class of economies, Pareto *dominates* the non-intervention equilibrium. Hence, unlike the analysis of Sheshinski and Weiss, social security and private annuities are not perfect substitutes from the point of view of individual agents in environments where there is a clear welfare enhancing role for social insurance. Consequently, aggregate savings of young agents will depend, in a systematic way, upon the magnitude of required contributions to the social security program. It is shown that the imposition of a Pareto improving social security system can, in some cases, increase the level of aggregate savings by young agents.

## 2. Complete annuity markets

In this section we describe the basic version of the model to be used for examining the implications of uncertain lifetimes on optimal resource allocations and market structures. The main implication of this section regarding the nature of optimal allocations is that, unlike other forms of diversity in individual characteristics such as preferences or endowments, diversity in publicly known survival probabilities affects in a fundamental way the nature of the price system which can support such allocations as decentralized equilibria. Specifically, allocations associated with stationary competitive equilibria will be optimal only if cohorts with different survival probabilities each face actuarially fair intertemporal terms of trade.

### The model

The economy to be studied is a variant of Samuelson's (1958) pure-exchange overlapping generations model. At each period  $t$ ,  $t \geq 1$ , the population consists of old members of generation  $t-1$  who all die at the end of that period, and young members of generation  $t$ . Each generation  $t$  is partitioned into two distinct groups, A and B, whose relative size is fixed for all  $t$ , so that for each agent of type A there are  $\gamma$  agents of type B,  $\gamma > 0$ . Members of each group live at most two periods, the first of which they survive with certainty. Death can occur at the beginning of the second period with probability  $(1-\pi_i)$ ,  $0 < \pi_i < 1$ ,  $i=A, B$ . It is assumed that  $\pi_A < \pi_B$ . With a continuum of agents, each of whom correctly perceives his death to occur with probability  $1-\pi_i$ , where  $i$  indicates the agent's group, a proportion  $(1-\pi_i)$  of group  $i$ ,  $i=A, B$ , passes away after living only one period. There is no aggregate uncertainty implied by agent's random lifetimes. In this and the following section the survival probability of any given agent is assumed to be public information.

There is a single non-storable and non-producible consumption good in this economy. Each young agent is endowed at birth with  $w$  units of the good. Generations are of equal size so that each member of generation  $t$  is viewed as giving birth to one identical agent [member of generation  $(t+1)$ ] before the uncertainty about his continued life is resolved. Preferences over lifetime consumption  $(c_1^i, c_2^i)$  of a representative member of group  $i$  are given by the expected utility function

$$U^i[c_1^i, c_2^i] = u(c_1^i) + \pi_i u(c_2^i), \quad i = A, B, \quad (1)$$

with  $u' > 0$ ,  $u'' < 0$ ,  $u'(c) \rightarrow \infty$  as  $c \rightarrow 0$  and  $u'(c) \rightarrow 0$  as  $c \rightarrow \infty$ . Notice that our specification of  $U^i[c_1^i, c_2^i]$  embodies the assumption that preferences are separable over time. In this we follow Yaari (1965) and Barro and Friedman (1977) who utilize this specification of preferences to deal with the problem of

parameterizing utility over lifetime consumption bundles when the length of lifetime itself is uncertain. Similar assumptions are made throughout the literature.

Before we discuss specific market structures and the effects of various government interventions, we characterize the sets of feasible and optimal stationary allocations of this economy.

**Definition.** A stationary allocation  $\{(c_1^i, c_2^i) | i = A, B\}$  is feasible if it satisfies

$$c_1^A + \gamma c_1^B + \pi_A c_2^A + \gamma \pi_B c_2^B = w(1 + \gamma). \quad (2)$$

Notice that this definition reflects our assumption about the absence of aggregate uncertainty regarding the number of deaths in each group.

**Definition.** A feasible stationary allocation  $\{(c_1^i, c_2^i) | i = A, B\}$  is optimal if there does not exist another feasible stationary allocation  $\{(c_1^i, c_2^i) | i = A, B\}$  such that

$$U^i[c_1^i, c_2^i] \geq U^i[\bar{c}_1^i, \bar{c}_2^i], \quad i = A, B,$$

with strict inequality for some  $i$ .

It can be shown that an interior allocation,  $\{(c_1^i, c_2^i) | i = A, B\}$ , is optimal if for some  $\delta_i > 0$ ,  $i = A, B$ , it solves the problem:

$$\begin{aligned} &\text{maximize } \delta^A U^A(c_1^A, c_2^A) + \delta^B U^B(c_1^B, c_2^B) \\ &\text{subject to (2).} \end{aligned} \quad (3)$$

A necessary and sufficient condition for an interior allocation to be optimal is that it satisfies (2) and has the property that

$$\frac{u'(c_1^A)}{u'(c_2^A)} = \frac{u'(c_1^B)}{u'(c_2^B)}. \quad (4)$$

Given heterogeneity with respect to survival probabilities, (4) also implies that optimal allocations have the property that ex ante marginal rates of substitution are *not* equalized across members of different groups, i.e.

$$\frac{1}{\pi_A} \frac{u'(c_1^A)}{u'(c_2^A)} \neq \frac{1}{\pi_B} \frac{u'(c_1^B)}{u'(c_2^B)}. \quad (5)$$

In a competitive equilibrium agents equate their expected intertemporal marginal rate of substitution to the rate of return on savings that they face.

Consequently, the competitive equilibrium will be (full information) Pareto optimal if and only if all agents face actuarially fair rates of return,  $(1/\pi_i)$  for agents of type  $i$ ,  $i = A, B$ .

The above results should be contrasted with those that would arise under different sorts of diversity across agents such as heterogeneity with respect to the preferences and/or endowments. In this light, imagine that  $\pi_A$  and  $\pi_B$  represent different time discount parameters in (1) rather than survival probabilities. Under these circumstances,  $\pi_A$  and  $\pi_B$  would not enter the resource constraint (2), which would then be  $c_1^A + \gamma c_1^B + c_2^A + \gamma c_2^B = w(1 + \gamma)$ . The optimality condition would then equate each of the terms in (5) to unity so that a unitary intertemporal tradeoff between  $c_1$  and  $c_2$  for both groups would support an optimal allocation. Similarly, heterogeneity with respect to endowments does not affect the way in which consumption levels enter the feasibility constraint. A more general discussion of the incorporation of agent specific attributes in economy-wide resource constraints and the resulting implications for the existence and optimality of competitive equilibria is provided by Prescott and Townsend (1984).

In order to highlight potential inefficiencies associated with private information, rather than other kinds of (intergenerational) inefficiencies, we assume the existence of a safe asset whose (gross) real rate of return is unity. It is convenient to describe a competitive equilibrium in which agents allocate consumption intertemporally by purchasing annuities which are supplied by competitive firms that specialize in holding the safe asset.

We model an annuity bond at period  $t$  as a claim to a certain quantity of the consumption good at period  $t+1$  which is payable only if the original purchaser of the annuity is alive. Normalizing the purchasing price of a period  $t$  annuity to one unit of the good at  $t$ , the annuity's rate of return represents the intertemporal terms of trade faced by its buyer. Given the postulated heterogeneity of the population with respect to survival probabilities, we can potentially think of two kinds of annuity equilibria: a *pooling* equilibrium, in which the same annuity is purchased by members of both groups, and a *separating* equilibrium in which agents with different survival probabilities purchase annuities with different rates of return. Following Rothschild and Stiglitz (1976), we define an equilibrium in this market as a set of contracts such that when agents maximize expected utility: (i) no contract in the equilibrium set makes negative expected profits, and (ii) there is no contract outside the equilibrium set that, if offered, will make a non-negative profit. Clearly, the absence of aggregate uncertainty in this economy implies a similar absence of uncertainty regarding the profits of the annuity-supplying firms. Therefore, in either a pooling or separating equilibrium real profits must be equal to zero.

Let  $R_i(t)$  be the real payoff, at  $t+1$ , to an annuity purchased at time  $t$  by a member of group  $i$  contingent on his being alive at  $t+1$ , and let  $D_i(t)$  denote the utility maximizing purchase of such annuities by that agent,  $i \in \{A, B\}$ .

In the pooling equilibria,  $R_A(t) = R_B(t) = R(t)$  and profits at period  $t+1$  are given by

$$D_A(t+1) + \gamma D_B(t+1) - R(t)[\pi_A D_A(t) + \gamma \pi_B D_B(t)] = 0. \quad (6)$$

In a stationary equilibrium  $D_A(t) = D_A$ ,  $D_B(t) = D_B$  and  $R(t) = R$  for all  $t$ . Substituting these identities into (6), we obtain:

$$D_A + \gamma D_B - R[\pi_A D_A + \gamma \pi_B D_B] = 0,$$

or

$$R = \frac{D_A + \gamma D_B}{\pi_A D_A + \gamma \pi_B D_B}. \quad (7)$$

It is clear that this *economy-wide* rate of return lies between  $1/\pi_A$  and  $1/\pi_B$ . Notice, however, that the pooling contract can never be an equilibrium contract because at this rate, given by (7), positive profits can be made by restricting the sales of such annuities to only one of the groups. Hence, the equilibrium will necessarily be a separating one with each contract netting zero profits, so that the separating payoffs are given by  $R_A = 1/\pi_A$  and  $R_B = 1/\pi_B$ . But these are precisely the intertemporal rates of return which induce an equilibrium in which (4), the necessary and sufficient condition for optimality, is satisfied. To see this, notice that the problem of the representative young agent of group  $i$  of generation  $t$  is:

$$\begin{aligned} &\text{maximize } u(c_1^i) + \pi_i u(c_2^i) \\ &\text{s.t.} \end{aligned}$$

$$c_1^i = w - D_i(t),$$

$$c_2^i = R_i(t) D_i(t).$$

Given our assumptions on  $u(\cdot)$ , necessary and sufficient conditions for the solution of this problem are given by

$$\frac{u'(c_1^i)}{\pi_i u'(c_2^i)} = R_i(t), \quad i = A, B. \quad (8)$$

These conditions imply (4) when  $R_i(t) = 1/\pi_i$  for  $i = A$  and  $i = B$ . Notice that, given the strict concavity of  $u(\cdot)$ , (8) implies that

$$c_1^i = c_2^i = \frac{w}{1 + \pi_i}, \quad i = A, B, \quad (9)$$

for all agents who live for two periods. Finally, since allocation (9) satisfies the resource constraint (2), the essential features of the competitive annuity equilibrium have been completely described.

It is worthwhile noting that the above equilibrium is essentially one in which the old of group  $i$  share the estate of the deceased members of their own group. In effect, competitive annuity markets discriminate between groups in an actuarially fair way, and thereby induce risk-sharing *within* each specific group rather than across the entire generation. The result of this market structure is an optimal transfer of goods both between and *within* generations. Conversely, any annuity policy which does *not* discriminate between members of different groups would lead to an inefficient form of risk-sharing and a non-optimal equilibrium. These results are not inconsistent with the existing literature. For example, Barro and Friedman and Sheshinski and Weiss assume that agents face an economy-wide actuarially fair rate of return on annuities. To the extent that agents are homogeneous with respect to survival probabilities, the implicit market structures that were considered in these works will lead to optimal allocations. However, when there exists heterogeneity among agents with respect to this attribute, an economy-wide actuarially fair rate of return does not correspond to the decentralized equilibrium nor would it result in an optimal allocation if it were imposed. Finally, we note that in such environments optimally designed social security systems will have no effect on aggregate savings as long as required contributions do not exceed the amount that individuals would save in the absence of such government interventions. Essentially, this follows from the fact that there is no welfare-enhancing role for a social security program here. Hence, to achieve a Pareto optimal allocation, a social security program requires individuals to purchase publicly provided annuities which are perfect substitutes for private modes of saving. Because of this, increases in publicly mandated savings are simply offset, in a one-to-one way, by decreases in private savings. Aggregate savings will depend on the magnitude of a social security program only to the extent that the latter leads to a non-optimal equilibrium.

### 3. Annuity markets and social security in the presence of private information

We now turn our attention to the performance of the economy in the presence of private information regarding survival probabilities. Recent developments in the literature regarding the economics of information have pointed out the large differences between the properties of classical Walrasian equilibria and information equilibria, i.e. equilibria in which buyers and sellers have private information regarding the qualitative nature of the good which is being traded. It is well documented in the literature that these differences relate to both the existence and optimality of competitive

equilibria. Of particular interest for the problem at hand is the widely known result, obtained by Rothschild and Stiglitz (1976) and Wilson (1977) among others, that, even when competitive equilibria exist under such circumstances, the associated equilibrium allocations need not be Pareto optimal.

On the other hand, one of the prime justifications given in the literature for a government-run social security program is the alleged need to correct various sorts of market failures which give rise to inefficient forms of risk-sharing. Diamond (1977), for example, asserts that there are a number of market failures in the present U.S. economy which a social security system could help alleviate. While Diamond does not provide a model of the reasons for these alleged market failures, he does discuss in depth the problems of insuring the risks associated with a varying length of working life. Prominent among these are that attempts to insure such risks face severe *moral hazard* and *adverse selection* problems. Both problems arise due to the existence of private information in those markets. While this section does not attempt to model the specific phenomena alluded to by Diamond, we try to capture the essence of his arguments in favor of government intervention by considering the nature of competitive annuity markets in the presence of private information regarding survival probabilities. This is done by showing that the framework developed by Rothschild and Stiglitz and Wilson for dealing with the nature of competitive insurance markets in the presence of private information can be easily extended to deal with annuity markets. Once this is done, the resulting competitive equilibrium is shown to be generally non-optimal so that there is a potential Pareto improving role for the government in such economies. Moreover, it turns out that the set of optimally designed mandatory social security regimes which lead to an equilibrium which Pareto *dominates* the non-intervention equilibrium allows for the co-existence of private annuity markets and the government-run program. Furthermore, while the government annuities are actuarially fair in an economy-wide sense, the resulting equilibrium in the residual private annuity market is a separating one so that rates of return on privately issued annuities are actuarially fair in a group-specific sense. Hence, private and public modes of savings will not be perfect substitutes from the point of view of the individual agents in the system. While one group of agents would always like to invest more in the public program, another group of agents, who view private market rates of return parametrically, would like to opt out of the system. However, were they allowed to do so, the resulting equilibrium would be one in which they would be uniformly worse off.

#### 3.1. The competitive annuity market

Excepting our specification of the information sets of agents, the economy to be discussed is the same as that analyzed in section 2. We assume that



agents in our economy know their own survival probabilities as well as  $\gamma$ , the number of B type agents per agent of type A. Similarly, the government knows the values of  $\pi_A$ ,  $\pi_B$ , and  $\gamma$ . However, no agent, including the government, knows whether any other particular individual belongs to group A or B. Analogous to Rothschild and Stiglitz or Wilson, we define an annuity policy,  $\alpha$ , as a two-dimensional vector  $(s^*, R^*)$  so that if a young agent purchases the policy  $\alpha$  his consumption vector  $(c_1, c_2)$  becomes  $(w - s^*, R^* s^*)$  if he lives two periods and  $(w - s^*, 0)$  if he lives only one period.<sup>1</sup> The above specification implies that sellers specify both 'prices and quantities' ( $R$  and  $s$ ) in annuity contracts. Rothschild and Stiglitz argue at length that price and quantity competition coupled with free entry into insurance markets is the appropriate notion of competition in these sorts of markets. In particular, they point out that price competition is clearly a special case of price and quantity competition because nothing in the definition of the latter prevents firms from offering for sale a set of annuities which can be bought in different quantities, but which have the same rate of return if the purchaser survives. Hence, firms which adopt pure price strategies cannot hope to successfully compete with firms who specify both price and quantities. The interested reader is referred to Rothschild and Stiglitz for a more detailed discussion of this point.

Since the consumption vector of each young agent can be represented by the annuity policy that he purchases, the expected utility of agent  $i$  associated with an insurance policy  $(s, R)$  can be represented by an indirect utility function  $V^i(\cdot, \cdot)$ , given by

$$V^i(R, s) = u(w - s) + \pi_i u(Rs), \quad i = A, B. \quad (10)$$

The slope of the indifference curve of a young member of group  $i$ , through the allocation  $(w - s, Rs)$ , is

$$\frac{dc_1}{dc_2} = -\pi_i \frac{u'(Rs)}{u'(w - s)}, \quad i = A, B. \quad (11)$$

Given our assumptions on  $u(\cdot)$ , and the fact that  $\pi_B > \pi_A$ , the slope of a type B indifference curve will be greater in absolute value than the slope of a type A indifference curve for any given contract  $(s^*, R^*)$  (see fig. 1).

Put alternatively, for any given rate of return  $R^1$ , a type B young person would like to purchase a larger number of annuities than a type A young person ( $\alpha^A$  and  $\alpha^B$  in fig. 1).

Because of the serious and as yet unresolved controversies in the literature regarding the appropriate definition of equilibrium for markets such as these,

<sup>1</sup>Thus, an insurance contract specifies a particular lifetime consumption allocation rather than a budget line. See, for example, point  $\alpha$  in fig. 1.

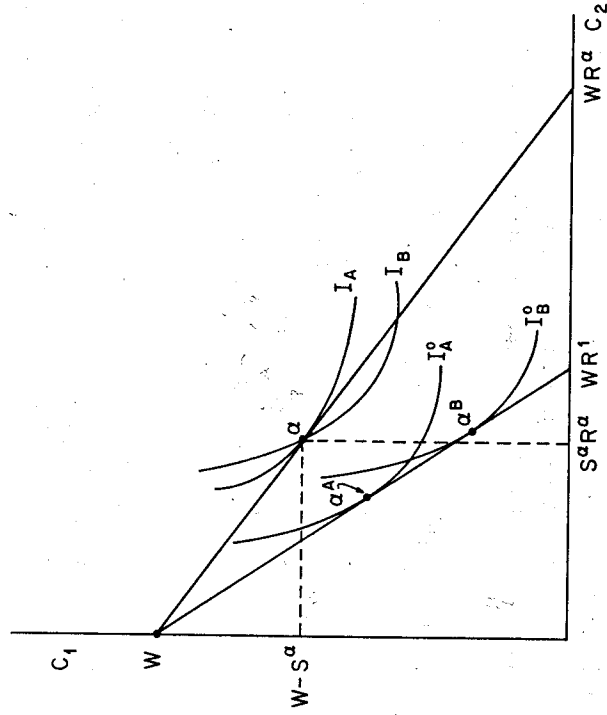


Fig. 1

we consider Wilson's two alternative definitions of equilibrium, both of which are motivated by the desire to describe an equilibrium set of policies for a situation in which firms can costlessly enter the market. We confine ourselves to characterizations of the stationary equilibria of the economy.

A Rothschild/Stiglitz (E1) equilibrium (which was used in section 2) is a set of contracts such that when agents choose contracts to maximize their expected utility, (i) no contract in the equilibrium set makes negative expected profits, and (ii) there is no contract outside the equilibrium set that, if offered, will make a non-negative profit. As Rothschild and Stiglitz point out, the E1 equilibrium is of the Nash-Cournot type in that each firm assumes that the contracts its competitors offer are independent of its own actions.

A Wilson (E2) equilibrium is the same as the E1 equilibrium except that firms' expectations are modified by assuming that each firm will correctly anticipate which of those policies that are offered by other firms will become unprofitable as a consequence of any changes in its own policies. The firm then offers a new policy only if it makes non-negative profits after all the other firms have made the expected adjustment in their policy offers. As Riley (1979a) points out, the nature of the decentralized equilibrium is quite sensitive to assumptions about rivals' reactions to contemplated moves. In fact, the aforementioned definitions of equilibrium are only two of the many which have been proposed.



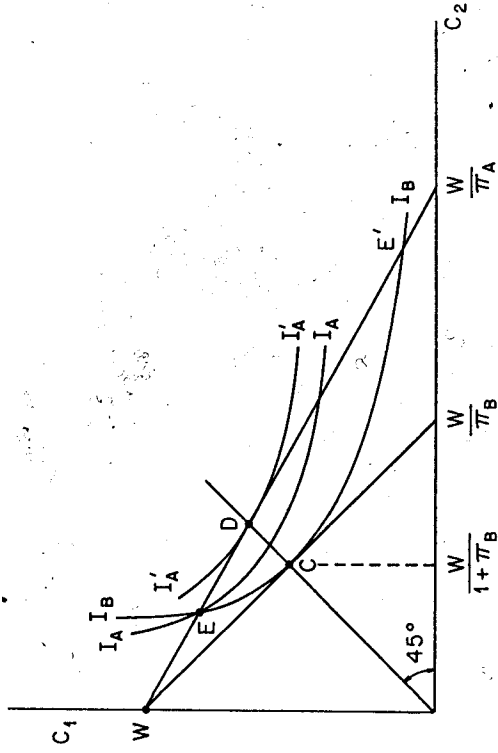


Fig. 3. The E1 separating equilibrium.

to the members of group B than contract C. Letting  $c_1^A(E)$  and  $c_2^A(E)$  denote first and second period consumption under some contract  $E$  which lies along  $(w, w/\pi_A)$  and recalling that  $c_1^B = c_2^B = w/(1 + \pi_B)$ , we require that

$$(1 + \pi_B)u\left(\frac{w}{1 + \pi_B}\right) \geq u(c_1^A(E)) + \pi_B u(c_2^A(E)) \quad (14)$$

for the contracts  $C$  and  $E$  to be a separating equilibrium. Hence, if an E1 equilibrium exists, the quantity of annuities that pay  $1/\pi_A$  purchased by a type A individual,  $s_1$ , solves the following problem:

$$\begin{aligned} \max_s & u(w - s) + \pi_A u(s/\pi_A) \\ \text{s.t.} & \end{aligned} \quad (15)$$

$$(1 + \pi_B)u\left(\frac{w}{1 + \pi_B}\right) \geq u(w - s) + \pi_B u\left(\frac{s}{\pi_A}\right). \quad (16)$$

However, because the constraint is clearly binding, we may replace the weak inequality in (16) with strict equality.<sup>2</sup> Hence, the set of potential E1

<sup>2</sup>Any  $s$  which satisfies (16) with strict inequality cannot be a solution to the maximization problem (15) since the derivative of the maximand with respect to  $s$  at any point that satisfies (16) is non-zero. The point at which the derivative becomes zero,

$$s = \frac{\pi_A}{1 + \pi_A} w,$$

violates (16).

equilibria can be found by examining the solutions of

$$(1 + \pi_B)u\left(\frac{w}{1 + \pi_B}\right) = u(w - s) + \pi_B u\left(\frac{s}{\pi_A}\right). \quad (17)$$

It is straightforward to verify that there always exist two solutions to (17) (corresponding to the points  $E$  and  $E'$  in fig. 3), where the indifference curve of a type B agent,  $I_B$ , intersects the  $(w, w/\pi_A)$  line. Since the indifference curve of the representative type A agent through point  $E'$  always lies below the one going through point  $E$ , the E1 equilibrium is given by  $(C, E)$ , if it exists. However, for the reasons pointed out by Rothschild and Stiglitz, and which are discussed below, even the contracts pair  $(C, E)$  may fail to be an equilibrium.

To establish (iii) we begin by considering the contract  $F$  in fig. 4 which lies above both  $I_A$  and  $I_B$ . If  $F$  is offered, members of both group B and A will purchase it in preference to contracts  $C$  and  $E$ , respectively. If it makes non-negative profit when both groups buy it,  $F$  clearly upsets the potential E1 separating equilibrium  $(C, E)$ . This is the case if the aggregate actuarially fair line is given by  $(w, \bar{R}w)$ , while this is not the case if that line is given by  $(w, \hat{R}w)$ . Hence, in the former case there does not exist an E1 equilibrium, while in the latter case there does. Notice then that, from eq. (13) the class of economies for which an E1 equilibrium exists is monotonically increasing in  $\gamma$ , since  $\bar{R}$  decreases monotonically in  $\gamma$ .

From the point of view of annuity supplier, members of group B are the 'high risk' agents, having a larger probability of claiming payments on their

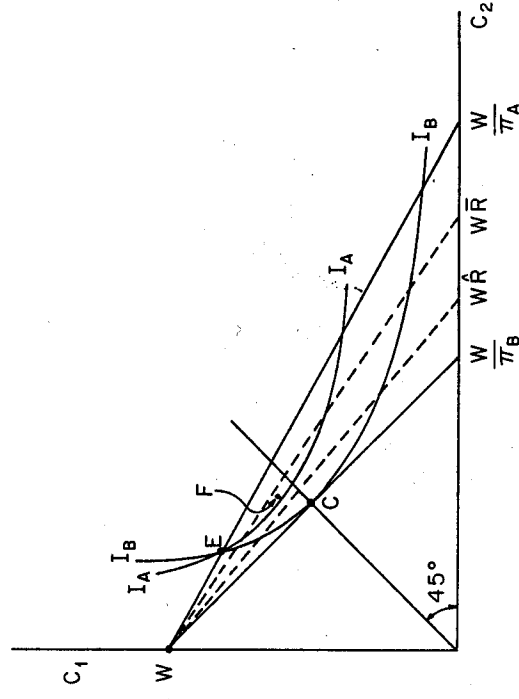


Fig. 4. The potential non-existence of an E1 equilibrium.



annuities. In order to sort them out, agents of type B are offered their most preferred actuarially fair contract. The externality imposed thereby on the 'low risk' agents of type A is reflected by the fact that they are *not* offered their most preferred actuarially fair contract, but rather a contract that is sufficiently worse to deter agents of type B from purchasing it. If this deterring contract is worse for agents of type A than a pooling contract, separation cannot occur in equilibrium, since agents of type B always prefer the pooling contract to their own best actuarially fair contract. Finally, note that the deterring contract of group A is determined independently of  $\gamma$ , while the pooling contract depends on that parameter. Consequently, for sufficiently small  $\gamma$ , the pooling contract will be close enough to the best actuarially fair contract for group A, thus rendering the deterring contract inferior to the pooling contract.

Before considering the optimality of the E1 equilibrium when it exists, we turn our attention to the class of E2 equilibria. Wilson demonstrates, in a more general context, that if an E1 equilibrium exists, it is also an E2 equilibrium. Given the above results, we need only illustrate that an E2 equilibrium exists when the E1 equilibrium does not. Such a case is displayed in fig. 5. Suppose that  $F$  is the proposed E2 equilibrium. Can it be broken by the contract  $A$  as was the case with the E1 equilibrium? To see that it cannot, notice that firms now take into account the fact that if contract  $A$  is offered only type B people will purchase contract  $F$  which will therefore be unprofitable and withdrawn. As such, if  $A$  is offered, it is expected that members of both groups will buy it. But under those conditions, the contract  $A$  will yield negative profits. as a result, no firm will offer contract  $A$ . In a similar way, it can be shown that there does not exist any contract which

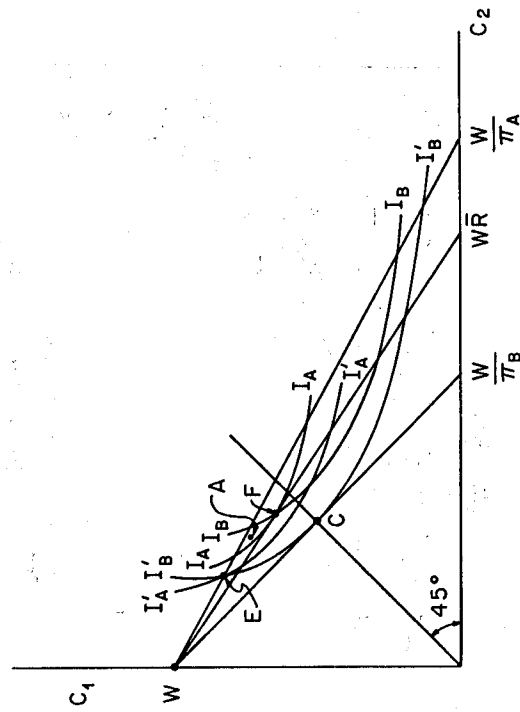


Fig. 5. The E2 pooling equilibrium.

will break the proposed equilibrium. Hence, the E2 equilibrium is given by contract  $F$ .

In general, then, one can derive the E2 equilibrium as follows: when the parameters of the problem are such that an E1 equilibrium exists, the E1 and E2 equilibria are the same and are given by the solution to (15); when the E1 equilibrium does not exist, the E2 equilibrium is a pooling equilibrium  $(x, \bar{R})$ , where  $\bar{R}$  is given by (13) and

$$x = \arg \max_s \{u(w-s) + \pi_A u(\bar{R}s)\}. \quad (18)$$

In summary, this subsection outlines two notions of competitive equilibria that can be shown to exist in our economy. Those two equilibria concepts, suggested by Wilson and Rothschild-Stiglitz, involve strategic considerations on the part of the competing firms. Prescott and Townsend (1984) provide compelling arguments for the non-existence of a non-strategic competitive equilibrium in environments of which ours is a special case. We should note, however, that our discussion of competitive equilibria is not meant to be exhaustive. Rather, we require some notion of a competitive equilibrium that can be shown to exist in our adverse selection economy in order to evaluate the desirability of government intervention.

### 3.2. The welfare improving role of mandatory social security

The above examples illustrate the fact that, with private information regarding survival probabilities, the presence of high risk individuals (type B agents) exerts a negative externality on other agents. In the case where an E1 equilibrium exists [contracts  $(C, E)$  in fig. 3], this externality is purely destructive in that while group A is worse off than it would be in the absence of private information ( $I_A$  versus  $I'_A$ ), group B is not better off. On the other hand, in a pooling equilibrium (fig. 5) at least the members of group B are better off than they would be in the absence of private information, while group A is still worse off.

Given these negative externalities, it should not come as a surprise that there exist Pareto improving policies which the government can undertake. Needless to say, in considering such policies, we restrict ourselves, a priori, to interventions which do not require that the government be able to distinguish between agents of different types. Notice that the definitions of the E1 and E2 equilibria impose that each contract which is purchased in equilibrium earn non-negative profit. But, as Wilson points out, this restriction arises because of the expectations that firms have regarding the effect of withdrawing an unprofitable policy on its aggregate profits. It does not arise as a consequence of the self-selection problem. Thus, the search for allocations



Notice that  $\bar{x}^P < s^P$ , since a member of group B who purchased actuarially fair annuities from the endowment  $(w - s^P, s^P \bar{R})$  could achieve a higher utility than  $u(c_1^P) + \pi_B u(c_2^P)$  (point K in fig. 6), thus violating (23).

Given these results we can prove the following theorem that demonstrates the Pareto improving role of the mandatory annuity program.

**Theorem.** For any mandatory contribution level  $\bar{x} \in [\bar{x}^P, s^P]$  to the public annuity program, the allocation  $(c_1^A(\bar{x}), c_2^A(\bar{x}))$  and  $(c_1^B(\bar{x}), c_2^B(\bar{x}))$  Pareto dominates the initial pooling equilibrium allocation, and can be supported as a separating equilibrium in private markets for residual demands for annuities.

*Proof.* By definition, the utility of a member of group B at  $(c_1^B(\bar{x}), c_2^B(\bar{x}))$  is

$$(1 + \pi_B) u \left[ \frac{w - \bar{x}(1 - \pi_B \bar{R})}{1 + \pi_B} \right].$$

Since  $1 - \pi_B \bar{R} < 0$ , this function is monotonically increasing in  $\bar{x}$ . By construction,  $u(c_1^B(\bar{x}^P)) + \pi_B u(c_2^B(\bar{x}^P)) = u(c_1^P) + \pi_B u(c_2^P)$ , so that for any  $\bar{x} \geq \bar{x}^P$  the members of group B prefer  $(c_1^B(\bar{x}), c_2^B(\bar{x}))$  to  $(c_1^P, c_2^P)$ . In order to establish that the members of group A prefer the allocation  $(c_1^A(\bar{x}), c_2^A(\bar{x}))$  to the allocation  $(c_1^P, c_2^P)$ , we first note that the constraint (22) is binding at the solution to the maximization problem (21). This is the case because the allocation associated with the unconstrained solution to (21) is on the 45° line from the origin, to the right of  $(c_1^B(\bar{x}), c_2^B(\bar{x}))$  which is also on that line. Consequently, the members of group B will always prefer the unconstrained allocation. Consider then  $s'$ , the smallest  $s$ , which solves (22) as an equality. Both  $(w - \bar{x} - s', \bar{R}\bar{x} + s'/\pi_A)$  and  $(c_1^B(\bar{x}), c_2^B(\bar{x}))$  lie on the same indifference curve of a type B person. Moreover, that indifference curve passes through or above the allocation  $(c_1^P, c_2^P)$ . In fact,  $(c_1^P, c_2^P)$  lies on that indifference curve only when  $\bar{x} = \bar{x}^P$ . The group A indifference curve through  $(w - \bar{x} - s', \bar{R}\bar{x} + s'/\pi_A)$  is flatter than the group B indifference curve, and hence passes above  $(c_1^P, c_2^P)$ . Thus, the pair  $\{(c_1^A(\bar{x}), c_2^A(\bar{x})), (c_1^B(\bar{x}), c_2^B(\bar{x}))\}$  Pareto dominates  $(c_1^P, c_2^P)$ . Finally, recall that the only case in which an E1 separating equilibrium fails to exist is the case in which some pooling allocation is preferred by a type A person. But since  $(c_1^A(\bar{x}), c_2^A(\bar{x}))$  is preferred by the members of group A to their best pooling allocation,  $(c_1^P, c_2^P)$ , the separating contracts constitute a separating equilibrium. Q.E.D.

The resulting equilibria with both public and private annuities for two alternative levels of mandatory contribution to the public program are described in fig. 6. The pair of final allocations for A and B, depicted by points E and C, respectively, correspond to the lowest level of contribution,  $x^1$ , that gives rise to a Pareto superior outcome. The pair of allocations

denoted by J and K, for A and B respectively, correspond to a mandatory contribution of  $s^P$ , which happens to equal the amount of annuities offered at the initial pooling equilibrium.

Several interesting implications emerge from examining the outcomes of these public interventions. First, the resulting equilibrium allocations are Pareto non-comparable. Group A is better off with a smaller social security contribution (point E) than with the larger contribution (point J), while the converse holds for group B. This follows from the fact that the public program subsidizes the members of group B. In fact, all of the equilibria corresponding to mandatory social security contributions between  $x^1$  and  $x$  are Pareto non-comparable, while each of them dominates the initial pooling equilibrium F. Secondly, there exist sufficiently large levels of the mandatory contribution to the social security program for which aggregate private savings by young agents increases. This is clearly the case for the mandatory contribution given by  $x$ . At the corresponding equilibrium allocation pair  $(J, K)$ , the members of both groups save more than they do at the initial allocation F. However, at the smaller mandatory contribution  $x^1$ , resulting in the equilibrium allocations  $(E, C)$ , the young members of group B increase their total savings relative to F while the converse hold for group A. Thus, the impact on aggregate private savings by young agents cannot be determined in the absence of further restrictions on the model.

**Case II. Separating equilibrium.** Unlike case I, if the decentralized equilibrium is a separating one, a mandatory annuity program of the kind considered here may not result in an allocation that Pareto dominates the initial equilibrium. Members of group B are made better off by being offered  $\bar{R}$  rather than their actuarially fair rate  $1/\pi_B$ . However, the members of group A can become worse off. What is perhaps surprising is that there do exist cases where the public program is Pareto improving. Since general results are not available in this case, we demonstrate the possibility of Pareto dominating an initial separating equilibrium by a public annuity program with an example given by fig. 7. We begin with a separating equilibrium with group A at-point E, and group B at point C. A mandatory contribution of size  $x$  to a social security program in which annuities have a real rate of return of  $\bar{R}$  gives rise to the group-specific actuarially fair (broken) budget lines in fig. 7. Private annuities purchased by group B then attain allocation G along indifference curve  $I_B^1$ . If that curve intersects group A's budget line at a point like H, which is preferred to point E by group A, we have a separating equilibrium that dominates the decentralized one.

In concluding this section we note that while there are still ongoing and unresolved controversies in the literature regarding the appropriate concept of equilibrium for private information economies such as those studied here, our results are quite encouraging in that they suggest an important motive for mandatory social security. Furthermore, the model develops a framework



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# INCOME TAX POLICY AND LIFETIME LABOR SUPPLY

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A progressive income tax structure provides incentives for individuals to alter their rate of work and their age of retirement. Compared to a zero tax or proportional tax equilibrium, progressive taxation induces individuals to take less leisure in the form of retirement in exchange for more leisure during the worklife, especially at high wage levels. The imposition of a special pension tax provision on top of a progressive tax structure offsets the distortion on leisure alternatives imposed by progressivity. Indeed, the pension tax deferral provision can neutralize the impact of tax progressivity on the work profile over life. The magnitude of these tax inducements in the U.S. tax structure are non-trivial and therefore are expected to alter labor supply decisions over the lifetime. The model finds empirical support using data from the Social Security Newly Entitled Beneficiaries Survey.

## 1. Introduction

A substantial literature has arisen to explain the precipitous decline in labor force participation rates of older workers in the United States since World War II.<sup>1</sup> These studies concentrate on the labor supply effects of social security or private pension rules that may involve actuarially unfair pension formulas, forced savings or restraints on minimum or maximum retirement ages. The presumably actuarially unfair social security formula, in particular, has been treated as an implicit tax on work at older ages.<sup>2</sup>

At the same time, another tax policy that may significantly affect the retirement age has been almost completely ignored. It is well known that by saving for retirement through pensions, individuals can reduce their lifetime tax liability.<sup>3</sup> Contributions to pension plans made directly by the firm are

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<sup>1</sup>For example, participation rates of males aged 65 or older are now less than half of their 1947 level.

<sup>2</sup>Most of these studies are reviewed in Mitchell and Fields (1982), but also see Gustman and Steinmeier (1983), Mitchell and Fields (1984), Parsons (1980), and Ward (1983).

<sup>3</sup>Since 1974, income saved through IRA and Keogh Plans is also subject to this tax policy, and hence, results developed below for pensions operated by firms can be extended to these new plans.