Creative Destruction Among Grocery Stores

Job Market Paper, Nan Yang*

November, 2011

Abstract

Technological innovations in inventory, logistics, and sales give grocery chain stores a profitability advantage over old-fashioned local stores. With chain stores advancing, local store incumbents gradually exit. Two questions concerning this creative destruction process are central to competition policy. How do chain stores make entry decisions? How does a chain store's entry impact incumbent stores' profitability and survival? In this paper, I develop a tractable dynamic oligopoly model to examine these two questions. The model's all Markov-perfect equilibria that survive natural refinements can be quickly computed by finding the fixed points of a sequence of low-dimensional contraction mappings. I estimate this model using observations of grocery stores' entry and exit in small Dutch municipalities. The average sunk cost of entry can be multiple times a store's expected discounted profit, possibly because Dutch zoning regulation greatly limits potential entrants' locations. The high average sunk cost considerably delays chain stores' expansion. An entering chain store reduces local incumbents' net present values by 29% to 66%. A policy experiment with the estimated model shows that cutting average sunk costs by 40% doubles chain store entry.

---

*Tilburg University, Department of Econometrics and OR. E-mail: N.Yang@tilburguniversity.edu

JEL Code: L13, L83

Keywords: Creative Destruction, Entry and exit, Dynamic oligopoly model, Markov-perfect equilibrium, Nested-Fixed-Point Algorithm, Retail grocery.

I wish to thank my advisors Jaap Abbring and Jeffrey Campbell for their constant encouragement and inspiring guidance. This paper benefits tremendously from the comments and discussions generously shared by Eric Bartelsman, Bart Bronnenberg, Lapo Filistrucchi, Pieter Gautier, Tobias Klein, Carlos Santos, Catherine Schaumans, and Benjamin Tanz. I am grateful to the 2009 Institute for Computational Economics at the University of Chicago and the Ziena Company for providing the KNITRO licence. Finally, I bid my compliments to Rob Grim and Jan Tilly for their help on the data. I am fully responsible for the remaining errors.

A replication package is available at the author's website `http://www.myyang.name/research.html`.

1 Introduction

In this paper, I develop and estimate a tractable dynamic oligopoly model for the retail grocery industry. The analysis quantifies the determinants of chain stores' entry and exit decisions and evaluates how these decisions influence incumbent stores' profitability and their survival. In the recent decades, technologies like barcode scanner, computerized inventory management, and modern logistic systems have continued chain grocery stores' century-long expansion, at the expense of old-fashioned independent local grocery stores exited. Understanding the determinants and consequences of chain stores' entry and exit in this creative destruction process is central to competition policy. For instance, policy makers might want to create favorable market conditions to encourage chain stores' entry. A policy experiment with the estimated model shows that reducing the entry cost by 40% effectively doubles chain store entry in the Dutch cities sampled in this paper. Regulators might also want to support local stores' participation in this industry. In the Netherlands, many local governments offer subsidy programs to small supermarkets.[1] The estimated model suggests that a subsidy package valued 150 million Euro is required in the next 10 years to maintain on average one operating local store per postcode area in the sampled cities in 2020.

In empirical industrial organization, there is a large literature studying firms' entry into oligopoly market. Despite the problem's inherently dynamic nature, a majority of the empirical studies assume that firms' one-shot choices settle the industry into its long-run steady state (e.g., Bresnahan and Reiss 1990, Berry 1992, Mazzeo 2002, and Seim 2006). Good departure points as they are, these models cannot incorporate firms' dynamic considerations into their entry and exit decisions. For instance, entering firms in these static analyses do not have the option to cease operation and avoid negative profit in future. When studying a dynamic industry with uncertainty, ignoring such valuable option often leads to biased estimates of important market determinants, as demonstrated by Abbring and Campbell (2010). By adding ongoing demand uncertainty and sunk costs in its dynamic analysis, this paper mitigates the aforementioned bias of firm values, and the bias of entry's impact on profits.

The simplicity and tractability of the paper's model differentiate it from a small but grow-

[1] For an overview of these programs, see the document "Subsidiemogelijkheden kleine tot middelgrote buurtsupermarkten" (In English, Subsidy opportunities for small and medium local supermarkets) published by the Dutch administrative authority National Board for the Retail Trade (Hoofdbedrijfschap Detailhandel, HBD) at http://www.hbd.nl/websites/hbd2009/files/Supermarkten/Subsidiemogelijkheden%20kleine%20tot%20middelgrote%20buurtsupermarkten.pdf.

ing literature that employs dynamic oligopoly models in the spirit of Ericson and Pakes (1995) to characterize market structure changes (e.g., Xu 2008, Collard-Wexler 2010, Gowrisankaran, Lucarelli, Schmidt-Dengler, and Town 2010, and Ryan forthcoming). Specifically, I keep the estimable game-theoretical model simple and deliver clear-cut results on computation and multiplicity of its Markov-perfect equilibria (MPE): (1) An MPE satisfying some natural refinements always exists, and all such equilibria can be quickly computed by finding the fixed points of a sequence of low-dimensional contraction mappings; (2) The driving force for any equilibrium multiplicity is identified. I then derive a fairly intuitive and computable condition for the estimated MPE to be unique. Since Ericson and Pakes's seminal work, it is well known that the lack of results on MPE computation and multiplicity poses serious challenges on this class of dynamic oligopoly models' estimation. Enormous efforts have been devoted to tackling these issues, with considerable success. Aguirregabiria and Mira (2007), Pakes, Ostrovsky, and Berry (2007), Bajari, Benkard, and Levin (2007), Pesendorfer and Schmidt-Dengler (2008) and Weintraub, Benkard, and Roy (2008) all provide various feasible computation or estimation methods. However, these papers and their applications are largely silent on the issue of multiple equilibria. Besanko, Doraszelski, Kryukov, and Satterthwaite (2010) convincingly demonstrate the severity of the multiplicity problem in the class of models considered by those authors. In general, even after reasonable refinement, the number of equilibria remains unclear, rendering the computation of all MPE nearly impossible. This poses a serious threat to the reliability of counterfactual policy experiments, because results from such experiments are generated only by the computed equilibrium. By contrast, the simplicity of my model allows me to verify the uniqueness of the estimated equilibrium. Additionally, the contraction-based algorithm allows me to quickly examine the effects of a large variety of policy changes by computing the equilibria for many sets of parameter values.

My modeling and estimation strategy builds on several previous papers. Firstly, the proposed model is rooted in Abbring, Campbell, and Yang (2010). However, I extend their framework in two important dimensions. First, their framework is statistically degenerate: for any set of parameter values, their model's predicted market outcome is unique. Therefore, it can not be directly applied to analyzing real market data. Following Rust (1987), I introduce various sources of unobservable transitory shocks into retailers' decision problems to rationalize the real data. The contraction-mapping property of the equilibrium computation scheme and the transitory shock structure motivate a direct application of the Nested-Fixed-Point

Algorithm developed by Rust to estimate the model. Second, I adapt Abbring, Campbell, and Yang's entry phase by allowing potential entry of low-cost retailers. This generalization is a crucial step towards accommodating creative destruction in the model.

In my model, retailers of two formats, chain stores and local stores, enter, compete, and exit in infinite-horizon oligopoly markets with stochastic demand. Entry requires paying a format-specific stochastic sunk costs of establishment. After entry, a chain store either becomes a high profitability retailer, or settles for the same low profitability as its local rivals. Each active retailer receives a profit every period, determined by market competition, its profitability type, realized demand, and a shock on profit. This shock can make continuing operation unprofitable. In the model's MPE, a retailer enters a market if the expected discounted profit from operation covers the sunk cost of entry, and exits if continuation gives negative expected discounted profit. By exploring the difference in retailers' equilibrium entry and exit choices under varying market conditions, I identify (the ratio of) sunk costs and store profit. This identification strategy is in the spirit of Bresnahan and Reiss (1990, 1993) and Berry (1992).

The model's simplicity allows me to make a first attempt at extending the estimation of dynamic oligopoly models in one dimension: recovering persistent unobservable heterogeneity. In the retail grocery industry, it is common to observe that a chain store exits while a nearby local store remains active. In light of this, I assume that whether a chain retailer has a superior profitability type than its local rivals is stochastically determined upon entry. This profitability type stays unchanged over time, but remains unobserved to the econometrician. The stores' optimal entry and exit choices are informative on the joint type distribution of all the incumbent chain stores, so I infer this joint distribution with Bayes' rule following every observed entry and exit choice. This approach turns out to be very tractable.

I apply the model to a panel dataset of grocery stores' entry and exit in small Dutch municipalities from 2002 to 2010. There are three reasons to champion the Dutch data. First, a clear pattern of creative destruction is visible. During the 9-year period, the number of chain stores increased almost 25% in the sampled cities, while around half of the local stores that were active in 2002 exited before 2010. Second, stores often cluster in residential areas because Dutch consumers highly value proximity when planning grocery shopping trips.[2] This feature suggests that I can use residential postcode to partition the country into isolated markets and obtain a large cross section of markets for estimation. Finally, after 2002, most Dutch

[2]See Van Lin and Gijsbrechts (2011).

supermarket chains had established their logistic network, and were active in franchising during the sample period. This allows me to treat continuation and entry decisions for individual chain stores independently from other outlets in the same chain.[3]

The refined MPE is verified to be unique given the estimated structural parameters. In this equilibrium, the chain stores, upon successfully establishing their advantageous profitability position, earn a flow profit 5.5 times that of their local rivals. When a chain store enters, it deteriorates a local store incumbent's expected discounted profit by 29%-66%. However, chain stores' expansion is considerably delayed by a very high average sunk cost of entry. A policy experiment shows that cutting the average sunk cost of entry by 40% will double the number of chain store entrants in the next 10 years. If the average sunk cost can be reduced by 90%, the number of incumbent chain stores in 2020 will double, and very few local stores will remain. To provide budgetary advice for the Dutch subsidy programs, I also compute the required costs for various policy targets. For instance, ensuring that on average one active local store per postcode area in 2020 requires in total 150 million Euro subsidy from 2010 to 2020. Almost 500 million Euro is needed to contain the percentage of postcode areas that only have chain stores to below 30%.

The remainder of this paper proceeds as follows. The next section provides background information on the Dutch retail grocery industry to guide the modeling choices. It also describes the sources and construction of my dataset. Section 3 presents the model's primitives. Section 4 establishes the results on equilibrium refinements, existence and uniqueness. The constructive proof of equilibrium existence also provides a procedure to compute the equilibrium values and market transition probabilities. Section 5 describes the likelihood construction and estimation procedure. The estimation results and the policy analysis are reported in Section 6. Further discussion and a conclusion appear in Section 7. Computational details and proofs are collected in the appendices.

[3] Jia (2008) and Holmes (2011) study the network effect in chain stores' expansions in the US. In their models, chain store headquarter incorporates the economies of scale from network expansion in their outlet entry decisions. Therefore, the decisions of entering different markets are associated. Using US data, both authors confirm network effect as a driving force behind the market leaders' vast expansion. Incorporating such network effects in the model will result in complications that are beyond the scope of this paper. Fortunately, the near-completed logistic network and the franchising activities of the Dutch chain stores reduced the concern of not modeling the network effect. This simplification gives me leverage to set up a model featuring both dynamics and oligopoly competition, from which Jia and Holmes respectively abstracted away.

2 Grocery Stores in Small Dutch Municipalities

2.1 The Dutch Grocery Stores: A Snapshot

The Netherlands is not famous for its dining culture. Nevertheless, according to Statistics Netherlands (Centraal Bureau voor de Statistiek, CBS), Dutch people spent almost 40 billion euro on food, beverages and tobacco in 2010. This is around 15% of the total domestic consumption. Grocery supermarkets enjoyed 86% of the market share from food and 62% from beverage and tobacco, boasting a turnover of 25 billion Euro in 2010. Even after adjusting for inflation, the supermarkets' total turnover increased over the past decade.

	2007	2008	2009	2010
Albert Heijn	29.5%	31.2%	32.8%	33.6%
C1000	14.3%	13.1%	11.7%	11.5%
Super de Boer	7.3%	6.7%	6.5%	5.5%
Jumbo	4.4%	4.7%	4.9%	5.5%
Superunie (total)	30.0%	30.7%	29.6%	29.6%
Aldi	8.9%	8.4%	8.3%	7.9%
Lidl	4.0%	4.7%	5.4%	5.6%
Other Chains	1.6%	0.5%	0.8%	0.8%
Total	100%	100%	100%	100%

Table 1: Market Shares (sales) of the Dutch Supermarket Chains.

Source: Original from AC Nielsen report, cited by "Dossier Supermarkten (feiten en cijfers)" (In English, Supermarket Profiles (facts and figures)) composed by HBD.

The major Dutch supermarket chains include four national ones (Albert Heijn (AH), C1000, Super de Boer, and Jumbo), two international hard discounters[4] (Aldi and Lidl, both German supermarket chains), and a dozen of smaller regional chains. Table 1 presents their market shares in recent years. The largest chain AH has a market share of one third, which is still growing. The other three national chains and the two hard discounters roughly take another one third. The regional chain stores contracted with the purchasing organization Superunie altogether take the rest.[5]

[4]The hard discounters are small to medium supermarkets that use aggressive pricing strategy, have limited assortment, and predominately focus on private labels.

[5]Superunie negotiates and buys products in the wholesale market on behalf of the regional chain stores. It

All the supermarket chains in Table 1 have been active since 2002, the beginning of the data period. No other large chain entered the Dutch market since then. During this period, Dutch chain stores' expansion had been much more modest than that of their US counterparts studied by Jia and Holmes. For instance, from 1962 to 2005 in the US, the number of Wal-Mart's general merchandise outlets increases from none to 3,176, and the number of the general distribution centers from none to 43 (Table 2 in Holmes 2011). During 2002-2010, the most aggressive expander Albert Heijn opened 154 outlets, which accounts for less than 20% of the currently active stores. Furthermore, no new distribution center was built by Albert Heijn during this period, suggesting that the logistic network had been close to completion. The second largest chain (C1000) even experienced contraction during this period.

Several mergers and acquisitions of supermarket chains occured during the observation period. In 2006, the holding company of supermarket chains Konmar and Edah, Laurus, sold all its stores to third parties, including AH and C1000. In 2009, the parent company of supermarket Jumbo bought another chain, Super de Boer, from its French holding company Groupe Casino. During these merger and acquisitions, many stores were taken over, but very few were forced to leave the market. In the second case, Super de Boer and Jumbo still operated as two distinct chains after the acquisition.[6]

Despite the presence of the hard discounters, vertical differentiation among the supermarket chains remains limited. Most of the Dutch supermarket chains have only one store format[7] and adopt a nation-wide uniform pricing policy. All of the national chains and most of the regional chains provide a full spectrum of products to capture the maximum scope of potential consumers. Even the quality-oriented AH provides, and aggressively advertises, its

also handles the lion's share of logistics for the members, delivering products from its own warehouses using its own container-bearing trucks. Its presence integrates the contracted regional chains in purchasing and logistics to a very large extent, creating economies of scale and enabling them to compete with the national and international chains. For this reason, I do not categorize the small regional chains affiliated to Superunie as local stores in my estimation.

[6]In 2011, Super de Boer stores started converting into Jumbo stores. The data used in this paper ends in 2010.

[7]AH is one exception. Besides the ordinary outlets, it has around 30 hypermarkets, "AH XL", and around 50 convenience stores, "AH To Go". The AH XL stores are larger in size than ordinary AH. They also have more departments in multimedia products, kitchen products, etc.. Nevertheless, its primary business activity remains retail grocery. AH To Go stores are mostly located in the train stations and airports, serving primarily commuters and tourists. In the empirical analysis, I treat AH XL as ordinary AH stores, and exclude AH To Go from the sample.

heavily discounted private label EuroShopper. As pointed out by Van Lin and Gijsbrechts (2011), distance to store is the first concern for Dutch grocery shoppers. This is arguably the most prominent characteristic that horizontally differentiates the supermarkets. To stay close to their potential customers, grocery stores usually locate in residential areas. The Dutch zoning regulation (bestemmingsplan) imposes strict restriction on using designated dwelling properties for business purposes, which limits the locations suitable for entry. Municipalities draft these plans in consultation with local residents, and they revise them only infrequently.[8]

As discussed earlier, many Dutch supermarket chains actively franchise their brands. For instance, Albert Heijn has around 200 franchisees among its 800 stores.[9] Almost 90% of the 400 outlets in the second largest chain C1000 are independent franchisees.[10]

2.2 The Data

The longitudinal dataset used in the model's estimation is uniquely constructed from two major sources. The annual establishment-level data of store entries and exits is extracted from the online version of the REview and Analysis of Companies in Holland (REACH) database. Small municipalities' isolated postcode areas are selected to form the cross-sections of independent markets for retail grocery. To measure the demand on these markets, annual populations on the postcode level are retrieved from the CBS database StatLine.

2.2.1 The REACH Data

The online version of the REACH database contains the establishment-level data for all Dutch business that have ever been active after January 1, 2002. It is created and maintained by the Dutch consultancy Bureau van Dijk (BvD), under the delegation and authorization of the Dutch Chamber of Commerce (Kamer van Koophandel, KvK). The business profiles come from the Dutch Trade Register (Het Handelsregister, DTR) archive provided by KvK. In the Netherlands, registration in DTR is compulsory for "every company and almost every legal

[8]Establishing or changing a zoning plan often requires approval from residents, municipal council, and existing entrepreneurs. This can be a long and painful process which takes months. A description on the procedure of revising a zoning plan is published (in Dutch) by the Dutch Chamber of Commerce at http://www.kvk.nl/ondernemen/huisvesting/bestemmingsplannen/wijziging-bestemmingsplan-door-de-gemeente/

[9]In Dutch at http://www.ah.nl/artikel?trg=albertheijn/article.feiten.

[10]In Dutch at http://www.denationalefranchisegids.nl/firstfranchise/KG_c1000.htm.

entity" (KvK website).[11] The registration also requires truthful provision of, among other things, the business' name, main activities, location, date of incorporation, ownership data, and form of legal organization. These pieces of information are all recorded by REACH. Additionally, businesses ceasing operation must deregister from the DTR. REACH includes the deregistration date.[12]

I focus on the period between January 1, 2002 and December 31, 2010 to construct an 9-year annual panel. If an establishment (1) has been active in this period, (2) has a four-digit SBI[13] code 4711 (supermarket and such with a general assortment of food and drink) listed among its main activities,[14] and (3) does not have a name either strongly suggesting that other business activity dominates grocery retail, or indicating that the store is specialized in a narrow category of groceries,[15] I classify it as a grocery store and include it in the sample. For each store, I treat its recorded "year of incorporation" as the entry time, and the year of deregistration as the exit time.[16] The store's format is primarily recovered from the ownership data; if it is labeled as a "branch" in REACH, then it is categorized as a chain store outlet. This criterion may overlook the franchisees, which are very often recorded as single location firms. Therefore, I supplement this criterion by (1) matching the store names

[11]For the details on the scope of the registration, see http://www.kvk.nl/english/traderegister/020_About_the_trade_register/registrationinthetraderegister/Whomustregister.asp. It is safe to assume that all the relevant stores are included in this database.

[12]REACH does provide concise financial data for medium companies with annual turnover between 1.5 and 50 million euro in several recent years. For the majority of the stores studied in this paper, the financial data are either not available for the full sample period, or of poor quality.

[13]SBI (De Standaard Bedrijfsindeling) is the Dutch standard industry classification code. For its details, please refer to the link (in Dutch) http://www.cbs.nl/nl-NL/menu/methoden/classificaties/overzicht/sbi/default.htm. Although REACH also provides BIC and SIC codes in the business profiles, the original registration at KvK only contains the SBI code.

[14]Most of the Dutch supermarkets specialize in food-items and household non-durable supplies, flirting little with other retail sectors. According to HBD, supermarket chains only have a 29% share in the drugstore products market, 8% in non prescription medicine, and 1% in clothing. Therefore, the presence of other retailers is not likely to influence grocery stores' entry and exit decisions. For this reason, I do not consider the competition between grocery stores and other retailers.

[15]To this end, I eliminate the stores whose names contain, "wijnen" (wine), "kaas" (cheese), "noten" (nuts), "eetcafe" (small restaurant), etc., and a range of terms suggesting that a store is specialized in exotic foreign products (immigrant stores). A complete list of the terms used for the elimination can be found in the documentation for the data processing, which is published on the author's website.

[16]In case of store ownership change due to the aforementioned mergers, I treat the records of outlet takeovers the same as store continuation if the store name remains after the takeover, and as a sequence of exit followed by entry if the store name changes.

with a list of major brands of supermarket chains, and the names of known franchisees; (2) matching the website of the store with a list of chain stores' websites; (3) categorizing stores with 40 or more employees as chain stores. If an establishment is not labeled as a chain store after applying all the above criterions, it is deemed a local store.

2.2.2 Markets Definition and the StatLine Data

To create a sample of markets with well-defined demand, shoppers in one sampled market should not buy groceries from stores in another sampled market. To ensure this, I adopt a similar strategy as in Bresnahan and Reiss (1991), and define markets as the major residential postcode areas in the small Dutch municipalities.

The demographic data are obtained from the publicly available dataset of StatLine. First, from the 2009 list of Dutch municipalities, I eliminate 22 metropolitan areas. These areas often have multiple densely populated postcode areas adjacent to each other. Shoppers are likely to traverse through the areas to buy groceries. In addition, many of these areas are either business districts, or packed with tourists throughout the year. For them, the number of residents is not a proper measure of local demand. This elimination leaves me with 375 small municipalities. I further drop the tourist town Noordwijkerhout and the beach resort Burgh-Haamstede from the sample. The remaining municipalities are often composed of isolated villages, each having a distinct postcode. Next, I choose the postcode areas with a number of inhabitants between 4,000 and 12,000 in 2009 in these municipalities as isolated markets for retail grocery. Eventually, I obtain 877 such markets. The numbers of inhabitants in these markets from 2002 to 2010 are extracted as the indicator for market demand. Stores in REACH are subsequently matched to these markets based on their location information. In total, 2,588 stores have been active during 2002 to 2010 in these markets. Among them, 1,559 are chain store outlets and 1,029 are local stores.

2.2.3 Summary Statistics

Table 2 presents some summary statistics for the selected markets. From 2002 to 2010, the number of residents grew in roughly half of the markets, and declined in the other half. Overall, the magnitude of population growth outmatched the magnitude of decline, and hence the markets' population distribution shifted to the right. The percentage of annual absolute population change has a median of 0.7%. The population change more often accumulated than canceled out over the 9-year period. As a result, the percentage of absolute population

	Mean	10%	25%	Median	75%	90%
Population in 2002	7103	4463	5251	6760	8719	10214
Population in 2010	7213	4585	5386	6885	8849	10310
% of Pop. change 2010 v.s. 2002	2.7	-5.8	-3.4	0.0	3.7	9.6
Absolute % of pop. change 2010 v.s. 2002	6.5	0.6	1.5	3.5	6.1	10.4
% of Annual pop. change 2002-2010	0.3	-1.2	-0.7	-0.1	0.7	1.7
Absolute % of annual pop. change 2002-2010	1.1	0.1	0.3	0.7	1.2	2.1
Disposable income per capita 2008 (k Euro)	14.73	13.30	13.88	14.50	15.30	16.40
% of Population w/ income 2008	71	69	70	72	73	74
Disp. inco. for pop. w/ inco. 2008 (k Euro)	20.33	18.28	19.10	19.90	21.10	22.62
No. of chain entrants, 2002-2010	0.57	0	0	0	1	2
No. of chain exited, 2002-2010	0.23	0	0	0	0	1
No. of local entrants, 2002-2010	0.55	0	0	0	1	2
No. of local exited, 2002-2010	0.59	0	0	0	1	2

Table 2: Summary Statistics.

The mean values are arithmatic average over all the markets.

The income data is from StatLine, and is at the municipality level. Income includes labor and capital income, as well as social benefits.

change from 2002 to 2009 has a median of 3.5%.

The income statistics are only available at the municipality level. Fortunately for the analysis, the Netherlands has a very equal income distribution, even among the small municipalities. For this reason, I am content with the postcode-level population data as a proxy for local demand.

During the sample period, chain store entry happened in less than half of the markets, and the same can be said for chain store exit, local store entry, and exit. On average, store entry or exit happened less than once per market during the 9-year period. This is perhaps not surprising given that these markets were mostly developed residential districts in a developed country. Nevertheless, the chain stores' advance and the local stores' consequent exits still changed the *market composition* of chain and local stores considerably, as summarized in Table 3. The number of markets populated only by chain stores rose from 340 in 2002 to 406 in 2010. In 2002, there were 234 markets without a single chain store, 62 of them with monopoly local store, and 26 of them with duopoly local stores. In 2010, the numbers had dropped to 154, 50, and 11 respectively. The numbers of markets with 2, 3, and 4 or more chain stores all increased over this period.

Market Composition in 2002						
No. chains\locals	0	1	2	3	4+	Subtotal
0	139	62	26	5	2	234
1	176	108	37	13	6	340
2	99	51	19	5	6	180
3	46	21	9	6	1	83
4+	19	16	3	0	2	40
Subtotal	479	258	94	29	17	877
Market Composition in 2010						
No. chains\locals	0	1	2	3	4+	Subtotal
0	86	50	11	5	2	154
1	197	94	27	10	2	330
2	121	59	22	8	4	214
3	65	44	9	2	5	125
4+	23	20	4	5	2	54
Subtotal	492	267	73	30	15	877

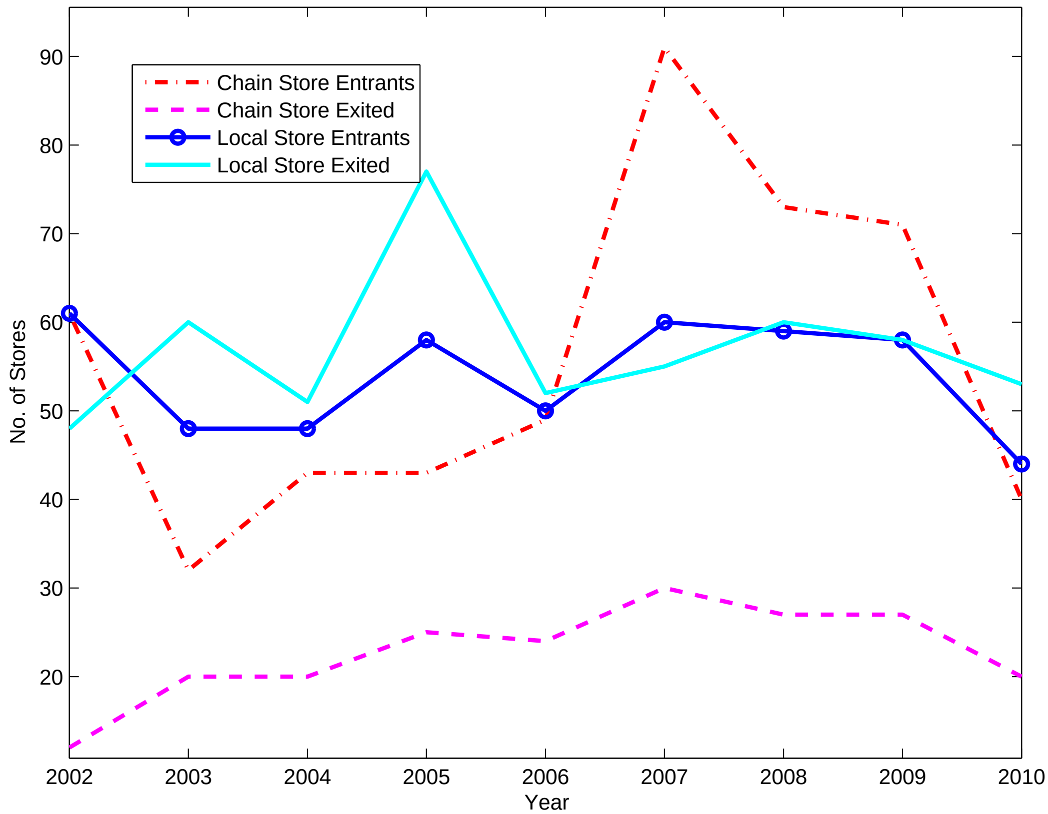
Table 3: Market Compositions

The total number of the active chain stores in the sample rose from 1105 in 2002 to 1354 in 2010, while the total number of the active local stores declined from 556 to 551. Behind the negligible contraction of the local stores is a high annual turnover: As shown in Figure 1, around 50-70 local stores exited each year, which was roughly 10% of all the active local stores. This high turnover rate suggests that the local stores are vulnerable to changes in market conditions.

3 The Model

With the nine-year panel data in hand, I introduce an oligopoly model of stores' entry and exit to rationalize observed market dynamics in terms of structural primitives, particularly, sunk costs of entry, the profitability advantage of chain stores, and uncertainty.

Figure 1: Numbers of Incumbents, Entrants, and Exited Stores in Each Year: 2002-2010

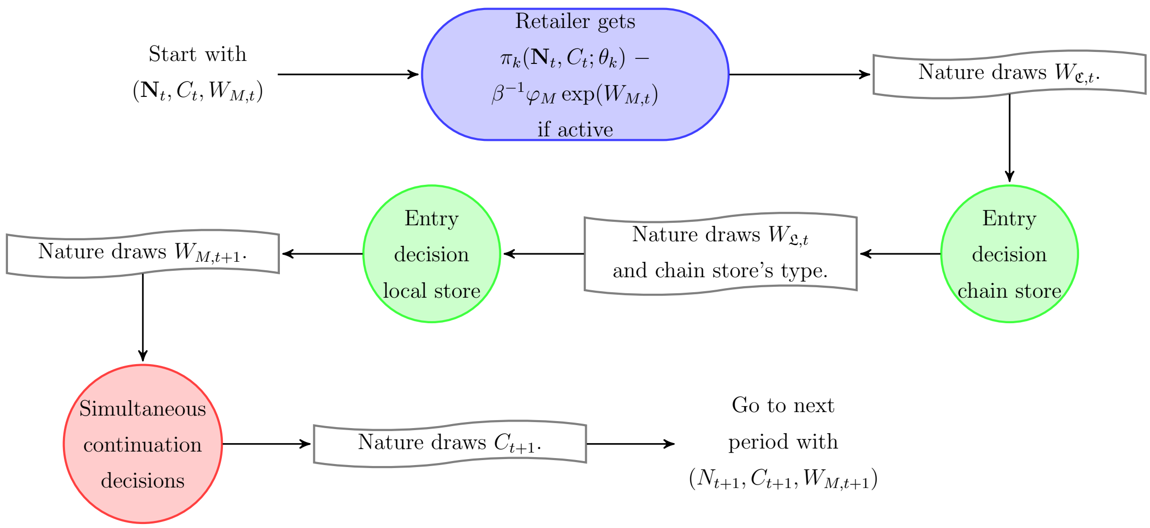


In the remainder of the paper, I denote a $n-$vector by $\mathbf{x} = (x_1, x_2, \ldots, x_n)$. A capital letter denotes a random variable (vector), the corresponding lower case is reserved for its realization. The random variable corresponding to Y in the next period is Y'. The expectation taken over the random variable X (or X', depending on the occasion) is denoted by $\mathbb{E}_X$. The conditional density function is written as $f(\cdot|X = x)$ when random variable X equals x, while $f(\cdot;\theta)$ means that θ is function f's parameter.

3.1 Primitives

The same set of primitive assumptions applies to all the markets. Time is discrete and the horizon is infinite, $t \in \mathbb{Z}_\star \equiv \{0, 1, \ldots\}$. Retailers are observed to differ in their formats: local store ($\mathfrak{L}$) or chain store ($\mathfrak{C}$). A countable number of chain store retailers and a countable number of local retailers potentially serve each market. At a given time t, some of the retailers are *active*, and the rest are *inactive*. Active retailers either have low ($\mathcal{L}$) or high

Figure 2: The Sequence of Events and Actions within a Period



($\mathcal{H}$) *profitability types* (or *types* in short). These types are not observed by econometrician, though they are common knowledge to the retailers once realized. A 2×1 vector $\mathbf{N}_t$, the variable for *market structure*, records the numbers of active type-$\mathcal{L}$ and type-$\mathcal{H}$ stores. The number of active local stores and the number of active chain stores are recorded in another 2×1 vector $\tilde{\mathbf{N}}_t$, the variable for *market composition*. Strategic entry and exit determine the market structure and market composition's evolutions.

Here and throughout the paper, I use ι_k to denote a 2×1 vector with a one in position k and zeros elsewhere, so $\iota_{\mathcal{H}} = \iota_{\mathfrak{C}} \equiv (0, 1)$ and $\iota_{\mathcal{L}} = \iota_{\mathfrak{L}} \equiv (1, 0)$. An empty market can be denoted with $\iota_0 \equiv (0, 0)$.

Figure 2 illustrates the sequencing of these variables' realizations and retailers' actions within period t. The stochastic number of consumers in period t is denoted by $C_t \in [\hat{c}, \check{c}]$, with $\check{c} < \infty$. The market-level cost shock on profit in period t is denoted by $W_{M,t}$. Given the inherited values of $\mathbf{N}_t, C_t$ and $W_{M,t}$, all active retailers begin the period by serving the market. The profits from this stage are $\pi_{\mathcal{H}}(\mathbf{N}_t, C_t; \theta_{\mathcal{H}}) - \beta^{-1}\varphi_M \exp(W_{M,t})$ for a type-$\mathcal{H}$ retailer and $\pi_{\mathcal{L}}(\mathbf{N}_t, C_t; \theta_{\mathcal{L}}) - \beta^{-1}\varphi_M \exp(W_{M,t})$ for a type-$\mathcal{L}$ retailer. The discount factor $\beta < 1$ is common to all retailers. The parameters $\theta_{\mathcal{H}}$ and $\theta_{\mathcal{L}}$ measure how the state variables affect the profits. A retailer's profit decreases with the number and profitability types of its competitors, and it improves with market demand. Also, the technological advantage of a type-$\mathcal{H}$ retailer over a type-$\mathcal{L}$ gives it greater profit. Formally, we have

Assumption 1 (Monotone Producer Profit)**.** For any profitability type $k \in \{\mathcal{L}, \mathcal{H}\}$, number of consumers $c \in [\hat{c}, \check{c}]$, and market structure $\mathbf{n} \in \mathbb{Z}^2_\star$ such that $\mathbf{n}$ at least includes one type-k store

1. $\pi_k(\mathbf{n}, c; \theta_k) \leq \check{\pi} < \infty$;
2. $\pi_k(\mathbf{n} + \iota_{\mathcal{H}}, c; \theta_k) < \pi_k (\mathbf{n} + \iota_{\mathcal{L}}, c; \theta_k)$;
3. there exists an $\overline{n} \in \mathbb{N}$ such that $\pi_k(\mathbf{n}, c; \theta_k) < 0$ if the number of stores in $\mathbf{n}$ is larger than $\overline{n}$; and
4. $\pi_{\mathcal{L}}(\mathbf{n}, c; \theta_{\mathcal{L}}) \leq \pi_{\mathcal{H}}(\mathbf{n}, c; \theta_{\mathcal{H}})$ for any $\mathbf{n}$ that includes at least one type-$\mathcal{L}$ and one type-$\mathcal{H}$ retailer.

After production, a shock on the chain store's sunk cost of entry $W_{\mathfrak{C},t}$ is realized. *One* chain store retailer that has never attempted to enter the market makes an entry decision after observing this shock. After this decision, a shock to the local store's sunk cost of entry $W_{\mathfrak{L},t}$ is realized. *One* local store that has never attempted to serve the market observes this shock, and subsequently decides on entry. Upon entry, the chain store retailer pays a sunk cost of $\varphi_{\mathfrak{C}} \exp(W_{\mathfrak{C},t}) \geq 0$. The local store pays $\varphi_{\mathfrak{L}} \exp(W_{\mathfrak{L},t}) \geq 0$ for entry. The restriction on the number of potential entrants each period fits the data: In the Dutch local markets considered in the estimation, entry by more than one chain stores or local stores in the same year is very rarely observed (less than 20 cases in the 877 markets over 9 years[17]). Generalizing the restricted entry phase to several more sophisticated specifications[18] will affect neither the model's equilibrium existence and uniqueness result, nor the computation and estimation strategy. Computational complication is the only substantial cost of considering those specifications.

All local stores have profitability type $\mathcal{L}$ after entry. chain stores do not learn their types until entering the market. After entry, a chain store has probability ω to become a type-$\mathcal{H}$ retailer, and with the complimentary probability it becomes a type-$\mathcal{L}$ retailer. Each store's profitability type is not only learned by its manager, but also observed by all retailers. I assume that the type realization of a chain store occurs right after its entry. Hence, the local

[17] In the estimation, I only consider the likelihood contribution from the first entry

[18] For instance, (1) simultaneous entry by a fixed or random number of chain stores, followed by a fixed/random number of local stores, with format-specific shocks on sunk costs; (2) sequential entry by an infinite number of chain stores, followed by an infinite number of local stores, with format-specific shocks on sunk costs.

store entrant following this chain store observes this type. This timing assumption is not essential to any of the main results.

A retailer with an entry opportunity cannot delay its choice[19], so the payoff to staying out of the industry is normalized to be zero. After the entry phase, the profitability shock in period $t+1$, $W_{M,t+1}$, is revealed to all active retailers. Then, all of them— including those that just entered the market— decide simultaneously between survival and exit. Exit is irreversible[20] but otherwise costless. It allows firms to avoid future periods' negative profits. All retailers' entry and exit decisions maximize their expected discounted profit.

In the period's final stage, the number of consumers C_t evolves exogenously following a first-order Markov process. This concludes the updating of all the exogenous random components in this market. With the updated values of $\mathbf{N}_{t+1}, C_{t+1}$ and $W_{M,t+1}$, the market moves on to next period.

I follow Rust and assume that the unobservable shocks $W_{\mathfrak{C},t}, W_{\mathfrak{L},t}$, and $W_{M,t}$ are conditionally independent in the following way.

Assumption 2 (Markov and Conditional Independence)**.** For any $t \in \{1, \ldots\}$, the transition density of the process $\{C_t, W_{M,t}, W_{\mathfrak{C},t}, W_{\mathfrak{L},t}\}$ factors as

$$f(C_{t+1}, W_{M,t+1}, W_{\mathfrak{C},t+1}, W_{\mathfrak{L},t+1}|C_t, W_{\mathfrak{C},t}, W_{\mathfrak{L},t}, W_{M,t}; \theta) = \\ f_C(C_{t+1}|C_t; \theta_1) f_{W_M}(W_{M,t}|C_t; \theta_2) f_{W_\mathfrak{C}}(W_{\mathfrak{C},t+1}|C_{t+1}; \theta_3) f_{W_\mathfrak{L}}(W_{\mathfrak{L},t+1}|C_{t+1}; \theta_4),$$

in which f_C is the conditional density for the Markov variable C_t, f_{W_M} is the conditional density for the profitability shock $W_{M,t}$, $f_{W_\mathfrak{C}}$ and $f_{W_\mathfrak{L}}$ are the densities of the shocks on sunk costs. $\theta_1, \theta_2, \theta_3$ and θ_4 are parameters of the density functions.

3.2 Markov-Perfect Equilibrium

I focus on *Markov-perfect equilibrium* (MPE) of the model. This is a subgame-perfect equilibrium in strategies that are only contingent on payoff-relevant variables. Assumption 2 ensures that conditional on C_t, the realized shocks do not help in predicting their future realizations. Therefore, conditional on C_t, $W_{\mathfrak{C},t}$ is payoff-relevant only to the chain store entrant,

[19] Given the franchise feature and the large number of chain brands, lots of people can be potential owners of chain stores. In this industry, opportunities to open a store come and go quickly. Hence, for a potential store owner, delaying entry to a later year often is not practical.

[20] Although a chain store outlet's closure does not mean the exit of the chain from the retail grocery market, it is rarely observed that a chain store opens another outlet in the same market that it has once withdrawn from.

$W_{\mathfrak{L},t}$ only to the local store entrant, and $W_{M,t+1}$ only to incumbents. More specifically, for a chain store retailer contemplating entry in period t, the payoff-relevant variables are $C_t, W_{\mathfrak{C},t}$, and the market structure $\mathbf{M}_{\mathfrak{C},t}$ just before this store's possible entry. For a local entrant, the payoff-relevant variables include $C_t, W_{\mathfrak{L},t}$ and $\mathbf{M}_{\mathfrak{L},t}$. Next, denote the market structure after the period's final entry with $\mathbf{M}_{E,t}$. For an active retailer contemplating survival, the payoff-relevant variables are this market structure, the current number of consumers C_t, the profitability shock $W_{M,t+1}$, and this retailer's profitability type. Throughout the paper, I focus on symmetric equilibria. Hence, retailers' brands are not payoff-relevant once conditioning on the variables mentioned here.

Suppose that $W_{\mathfrak{C},t}, W_{\mathfrak{L},t}$ and $W_{M,t}$ all have infinite support. A *Markov strategy* is a couple (a^E, a^S) of functions

$$\begin{aligned} a^E : \mathbb{Z}^2_\star \times [\hat{c}, \check{c}] \times \mathbb{R} \times \{\mathfrak{L}, \mathfrak{C}\} &\longrightarrow [0,1] \\ a^S : \mathbb{Z}^2_\star \times [\hat{c}, \check{c}] \times \mathbb{R} \times \{\mathcal{L}, \mathcal{H}\} &\longrightarrow [0,1]. \end{aligned}$$

This allows for mixed strategies. For each potential entrant with format $k \in \{\mathfrak{L}, \mathfrak{C}\}$, this strategy's *entry rule* a^E assigns a probability of becoming active to any $(\mathbf{M}_{k,t}, C_t, W_{k,t}, k)$. Similarly, its *survival rule* a^S assigns a probability of remaining active in the next period to each possible value of the payoff-relevant state $(\mathbf{M}_{E,t}, C_t, W_{M,t}, k)$ for all active retailers, where $k \in \{\mathcal{L}, \mathcal{H}\}$. Since calendar time is not payoff-relevant in MPE, I hereafter drop the t subscript from all variables.

To characterize equilibria, it is useful to define two value functions, each corresponding to a particular node of the game tree within each period. The *post-entry value* $v^E(\mathbf{m}_E, c, k)$ equals the expected discounted profits of a retailer that has profitability type k, faces market structure $\mathbf{m}_E$ and number of consumers c just after all entry decisions have been realized, and just before the profitability shock W_M is revealed. For a potential entrant, this value function gives the expected discounted profits from entry, and hence it determines optimal entry choices. The realized shocks on sunk costs $w_{\mathfrak{C}}, w_{\mathfrak{L}}$ do not enter this value, because they are "sunk" upon entry, and do not help predicting future shocks (Assumption 2). The *post-survival value* $v^S(\mathbf{m}_S, c, k)$ equals the expected discounted profits of a type-k retailer,*gross of next period's fixed costs*, facing market structure $\mathbf{m}_S$ and number of consumers c, just after all survival decisions have been realized. This value net of the (discounted) profitability shock, $v^S(\mathbf{m}_S, c, k) - \varphi_M \exp(w_M)$, equals the payoff to a surviving retailer following the simultaneous continuation decisions, so it is central to the analysis of exit.

The value functions v^E and v^S satisfy

$$v^E(\mathbf{m}_E, c, k) = \mathbb{E}_{\mathbf{M}_S, W_M}\left[a^S(\mathbf{m}_E, c, W'_M, k)(v^S(\mathbf{M}_S, c, k) - \varphi_M \exp(W'_M)) \,\middle|\, \mathbf{M}_E = \mathbf{m}_E, C = c\right], \tag{1}$$

and

$$v^S(\mathbf{m}_S, c, k) = \beta \mathbb{E}_{\mathbf{M}_E, C}\left[\pi_k(\mathbf{m}_S, C') + v^E(\mathbf{M}'_E, C', k) \,\middle|\, \mathbf{M}_S = \mathbf{m}_S, C = c\right]. \tag{2}$$

For (a^E, a^S) to form a symmetric Markov-perfect equilibrium, it is necessary and sufficient that no firm can gain from a one-shot deviation from (a^E, a^S) (e.g. Fudenberg and Tirole, 1991, Theorem 4.2):

$$\begin{aligned} a^E(\mathbf{m}_\mathfrak{C}, c, w_\mathfrak{C}, \mathfrak{C}) &\in \arg\max_{a \in [0,1]} a(\omega \mathbb{E}_{\mathbf{M}_E}[v^E(\mathbf{M}_E, c, \mathcal{H}) \,|\mathbf{M}_\mathfrak{L} = \mathbf{m}_\mathfrak{C} + \iota_\mathcal{H}] \\ &\quad +(1-\omega)\mathbb{E}_{\mathbf{M}_E}[v^E(\mathbf{M}_E, c, \mathcal{L}) \,|\mathbf{M}_\mathfrak{L} = \mathbf{m}_\mathfrak{C} + \iota_\mathcal{L}] - \varphi_\mathfrak{C} \exp(w_\mathfrak{C})), \quad (3) \\ a^E(\mathbf{m}_\mathfrak{L}, c, w_\mathfrak{L}, \mathfrak{L}) &\in \arg\max_{a \in [0,1]} a\left(v^E(\mathbf{m}_\mathfrak{L} + \iota_\mathcal{L}, c, \mathcal{L}) - \varphi_\mathfrak{L} \exp(w_\mathfrak{L})\right), \quad (4) \\ a^S(\mathbf{m}_E, c, w_M, k) &\in \arg\max_{a \in [0,1]} a\left(\mathbb{E}_{\mathbf{M}_S}\left[v^S(\mathbf{M}_S, c, k) \,\middle|\, \mathbf{M}_E = \mathbf{m}_E\right] - \varphi_M \exp(w_M)\right). \quad (5) \end{aligned}$$

The conditional expectations in (1), (2), (3), and (5) are computed given that other retailers follow (a^E, a^S), and the retailer of interest enters or remains active. Note that a chain store entrant's payoff depend on the post-entry values for both types. Together, conditions (1)–(5) are necessary and sufficient for a strategy (a^E, a^S) to form a symmetric Markov-perfect equilibrium with payoffs v^E and v^S.

4 Equilibrium Analysis

In Section 4.1, I begin the equilibrium analysis with markets that can accommodate at most two retailers simultaneously, regardless of their types. The restriction on the number of retailers can be rationalized by a formidable entry cost for a third entrant. Note that under this restriction, the number of simultaneously active retailers never exceeds two in equilibrium, if the market is initially empty. This is the simplest example where strategic interactions between retailers are retained. Using this example, I introduce several equilibrium refinements and illustrate how to compute the equilibrium value functions, the choice probability of retailers, and the market structure transition probabilities under the refined equilibria. I formally define the equilibrium refinements in Section 4.2. Finally, I generalize the technique and procedure used in the duopoly example, and establish the general results on equilibrium existence, uniqueness, and computation in Section 4.3.

Figure 3: Reduced-form Representation of the Duopoly Continuation Game

	Survive	Exit
Survive	$v^S(2\iota_{\mathcal{H}}, c, \mathcal{H}) - \varphi_M \exp(w_M)$ $v^S(2\iota_{\mathcal{H}}, c, \mathcal{H}) - \varphi_M \exp(w_M)$	$v^S(\iota_{\mathcal{H}}, c, \mathcal{H}) - \varphi_M \exp(w_M)$ 0
Exit	0 $v^S(\iota_{\mathcal{H}}, c, \mathcal{H}) - \varphi_M \exp(w_M)$	0 0

Note: In each cell, the upper-left expression gives the row player's payoff. Please see the text for further details.

4.1 A Duopoly Example

If at most two retailers can serve a market at the same time, the equilibrium computation consists of five steps. Under proper equilibrium refinements, in each of the steps, the post-entry value function restricted to part of the state space is determined by the unique fixed point of a contraction mapping. Then, the post-survival values, the strategy, the choice probabilities, and the transition probabilities are constructed. The results from the completed steps are used as inputs in the following steps. I sketch the procedure here, and collect the omitted details in Appendix A.

Step 1: Duopoly Market with Two Type-$\mathcal{H}$ Retailers The equilibrium computation begins with market structure of two active type-$\mathcal{H}$ retailers. In a Markov-perfect equilibrium, the survival rule $a^S(2\iota_{\mathcal{H}}, c, w_M, \mathcal{H})$ satisfies (5): It is a Nash equilibrium of the static simultaneous-move game with payoffs given by the expected value of continuation given c and w_M. Figure 3 gives the reduced-form representation of this static game with the two possible pure strategies "Survive" and "Exit". The upper-left expression in each cell is the row player's payoff. Both retailers receive the duopoly post-survival payoff $v^S(2\iota_{\mathcal{H}}, c, \mathcal{H}) - \varphi_M \exp(w_M)$ from joint continuation. Since no retailer will further enter this saturated market in the next period, the duopoly post-survival payoff satisfies a special case of Equation (2):

$$v^S(2\iota_{\mathcal{H}}, c, \mathcal{H}) = \beta \mathbb{E}_C \left[\pi_{\mathcal{H}}(2\iota_{\mathcal{H}}, C') + v^E(2\iota_{\mathcal{H}}, C', \mathcal{H}) \,\middle|\, C = c \right].$$

If a retailer survives alone, it earns the monopoly post-survival value $v^S(\iota_{\mathcal{H}}, c, \mathcal{H}) - \varphi_M \exp(w_M)$.

Now, suppose that $v^S(2\iota_{\mathcal{H}}, c, \mathcal{H}) > \varphi_M \exp(w_M)$. If the monopoly post-survival value $v^S(\iota_{\mathcal{H}}, c, \mathcal{H}) > \varphi_M \exp(w_M)$, "Survive" is a dominant strategy in this static game. If $v^S(\iota_{\mathcal{H}}, c, \mathcal{H}) \leq$

$\varphi_M \exp(w_M)$, any symmetric equilibrium strategy other than "Survive" is not *renegotiation-proof*: Such strategy either involves mixing, or is pure "Exit". In either case, retailers' expected payoff from such a strategy always equals zero. So, if retailers can renegotiate, they will both rationally choose joint continuation to get positive payoff. Therefore, if we require the MPE to be renegotiation-proof, both retailers will choose "Survive" with probability one if $v^S(2\iota_{\mathcal{H}}, c, \mathcal{H}) > \varphi_M \exp(w_M)$. If instead $v^S(2\iota_{\mathcal{H}}, c, \mathcal{H}) \leq \varphi_M \exp(w_M)$, then any symmetric equilibrium strategy, mixing or pure "Exit", gives retailers zero expected payoff. Since $v^E(2\iota_{\mathcal{H}}, c, \mathcal{H})$ is computed based on the symmetric equilibrium payoff to this static game, these facts together yield the following special case of Equation (1):

$$\begin{aligned} v^E(2\iota_{\mathcal{H}}, c, \mathcal{H}) &= \mathbb{E}_{W_M}\left[\max\{0, v^S(2\iota_{\mathcal{H}}, c, \mathcal{H}) - \exp(W'_M)\} \,\middle|\, C = c\right] \\ &= \mathbb{E}_{W_M}\left[\max\{0, \beta\mathbb{E}_C\left[\pi_{\mathcal{H}}(2\iota_{\mathcal{H}}, C') + v^E(2\iota_{\mathcal{H}}, C', \mathcal{H}) \,\middle|\, C = c\right] - \varphi_M \exp(W'_M)\} \,\middle|\, C = c\right]. \end{aligned} \tag{6}$$

The expectation in Equation (6) is taken over the exogenous variables C' and W'_M only, and does not depend on any survival rule. This ensures that the equation's right-hand side defines a contraction mapping, with its unique fixed point pinning down $v^E(2\iota_{\mathcal{H}}, \cdot, \mathcal{H})$. Then, it is straightforward to compute $v^S(2\iota_{\mathcal{H}}, \cdot, \mathcal{H})$. The reasoning leading to Equation (6) highlights the key technical insight that makes the equilibrium computation simple. The equilibrium duopoly payoffs can be computed without knowledge of the retailers' payoffs in other market structures, because retailers receive positive expected payoff in a symmetric equilibrium only when joint continuation is individually profitable.

For some distributions of W_M, the expectation over W_M in Equation (6) has a closed-form expression. In Appendix A, I work out an example under normally distributed W_M. By using closed-form expression to compute $v^E(2\iota_{\mathcal{H}}, \cdot, \mathcal{H})$, one can avoid the numerical integration over W_M. This is one of the major consequence and benefit from Assumption 2.

Step 2: Type-$\mathcal{L}$ Duopolist Facing a Type-$\mathcal{H}$ Rival. Next, consider a type-$\mathcal{L}$ retailer who faces a type-$\mathcal{H}$ competitor. Because a type-$\mathcal{H}$ retailer earns higher flow profit than a type-$\mathcal{L}$ retailer does in each period, I require the equilibrium to be *natural* in its survival rules: a type-$\mathcal{H}$ retailer never exits when a type-$\mathcal{L}$ competitor survives. In a natural MPE, the type-$\mathcal{L}$ retailer's survival implies the survival of the type-$\mathcal{H}$ rival. Moreover, no retailer will further enter this market following its survival, given the formidable sunk cost of entry for a third store. Therefore, this retailer's post-survival value does not depend on any unknown strategy of rival stores. Consequently, Equation (1) defines a contraction mapping with its unique fixed point determining $v^E(\iota_\mathcal{L}+\iota_\mathcal{H},\cdot,\mathcal{L})$. Then, the post-survival value $v^S(\iota_\mathcal{L}+\iota_\mathcal{H},\cdot,\mathcal{L})$ naturally follows.

The equilibrium refinements discussed above narrow the equilibrium set to the *renegotiation-proof natural Markov-perfect equilibria* (RNMPE). With the computed RNMPE value functions, I can further compute the equilibrium entry rules to a market occupied by a type-$\mathcal{H}$ monopolist for a chain store and a local store, and the survival rule for a type-$\mathcal{L}$ retailer facing a type-$\mathcal{H}$ rival.

By imposing distributional assumptions on $W_\mathfrak{C}, W_\mathfrak{L}$, I can compute the *choice probabilities* for a chain store and a local store to enter when the market is monopolized by a type-$\mathcal{H}$ retailer, $P^E(\iota_\mathfrak{C}|\iota_\mathcal{H},\cdot)$ and $P^E(\iota_\mathfrak{L}|\iota_\mathcal{H},\cdot)$, by integrating out the sunk cost shocks. With these probabilities in hand, I further compute $\mathbb{P}^E(\mathbf{m}_E|\iota_\mathcal{H},\cdot)$, the *transition probability* of entry for the post-entry market structure to become $\mathbf{m}_E$, when the pre-entry market structure is $\iota_\mathcal{H}$.

Similarly, the transition probability of survival, $\mathbb{P}^S(\iota_\mathcal{L}+\iota_\mathcal{H}|\iota_\mathcal{L}+\iota_\mathcal{H},\cdot)$, follows from the post-survival value $v^S(\iota_\mathcal{L}+\iota_\mathcal{H},\cdot,\mathcal{L})$ and some distributional assumption on W_M.

Step 3: Type-$\mathcal{H}$ Monopolist & Type-$\mathcal{H}$ Duopolist Facing One Type-$\mathcal{L}$ Rival. A type-$\mathcal{H}$ monopolist's post-survival value function, $v^S(\iota_\mathcal{H},\cdot,\mathcal{H})$, depends on whether entry would occur next period. The relevant entry rules have been calculated in Step 2. The post-survival payoff for a type-$\mathcal{H}$ duopolist who face a type-$\mathcal{L}$ rival, $v^S(\iota_\mathcal{L}+\iota_\mathcal{H},\cdot,\mathcal{H})$, depends on whether the type-$\mathcal{L}$ store exits. The relevant survival rule has been calculated in Step 2 as well. Thus, Equation (1) again defines a contraction mapping with its unique fixed point simultaneously determining the post-entry value functions $v^E(\iota_\mathcal{H},\cdot,\mathcal{H})$ and $v^E(\iota_\mathcal{L}+\iota_\mathcal{H},\cdot,\mathcal{H})$. The associated post-survival payoffs, the transition probabilities $\mathbb{P}^S(\iota_\mathcal{H}|\iota_\mathcal{L}+\iota_\mathcal{H},\cdot)$ and $\mathbb{P}^S(\iota_0|\iota_\mathcal{L}+\iota_\mathcal{H},\cdot)$ follow.

Step 4: Duopoly Market with Two Type-$\mathcal{L}$ Retailers. The survival problem on a duopoly market with two type-$\mathcal{L}$ retailers is a carbon copy of the static game presented in Figure 3. If simultaneous survival is individually profitable, then the renegotiation-proofness requires both retailers to choose survival in a symmetric equilibrium. Otherwise, the strategy in the static game assigns non-negative probability to "Exit", and results in zero expected payoff. Therefore, similarly to Equation (6), the necessary condition for $v^E(2\iota_\mathcal{L},\cdot,\mathcal{L})$ defines a contraction mapping, with its fixed point determining $v^E(2\iota_\mathcal{L},\cdot,\mathcal{L})$. Consequently, $v^S(2\iota_\mathcal{L},\cdot,\mathcal{L})$ follows. Then, the associated choice probabilities $P^E(\iota_\mathfrak{C}|\iota_\mathcal{L},\cdot)$ and $P^E(\iota_\mathfrak{L}|\iota_\mathcal{L},\cdot)$, and the transition probabilities of entry for market structures succeeding this monopoly market $\mathbb{P}^E(\iota_{\mathcal{L}} + \iota_{\mathcal{H}}|\iota_{\mathcal{L}}, \cdot)$, $\mathbb{P}^E(2\iota_{\mathcal{L}}|\iota_{\mathcal{L}}, \cdot)$, and $\mathbb{P}^E(\iota_{\mathcal{L}}|\iota_{\mathcal{L}}, \cdot)$ are computed analogously to their counterparts in Step 2.

Step 5: The Rest. First, I compute a type-$\mathcal{L}$ monopolist's value functions. The survival decision of this retailer depends on whether entry happens next period. The likelihood of entry is given by $P^E(\iota_{\mathfrak{C}}|\iota_{\mathcal{L}}, \cdot)$ and $P^E(\iota_{\mathfrak{L}}|\iota_{\mathcal{L}}, \cdot)$, the choice probabilities that have been calculated in Step 4. Therefore, a special case of Equation (1) defines a contraction mapping, with its unique fixed point determining the post-entry value $v^E(\iota_{\mathcal{L}}, \cdot, \mathcal{L})$.

Next, I compute the entry rule to an empty market by a chain store. This entry rule depends on whether the local store following this chain store in the entry sequence enters. Such entry choice is characterized by the entry rules for a local store to a monopoly market, which have been determined in Steps 2 and 4. With this entry rule in hand, I obtain the associated choice probabilities of entry by integrating out $W_{\mathfrak{C}}$.

Furthermore, I determine the entry rules to an empty market by a local store and the survival rule of a monopolist type-$\mathcal{L}$ retailer as the optimal choice rules in single-agent decision problems. The associated choice and transition probabilities have closed-form expressions under the normality assumption.

Finally, all that remain undetermined is the survival rule for duopoly retailers of identical type and the associated choice and transition probabilities. Reconsider the static game presented in Figure 3: in a RNMPE, if the post-survival value for a duopolist exceeds $\varphi_M \exp(w_M)$, then both retailers continue for sure. Otherwise, checking the post-survival value for a monopolist is essential.

If the monopoly post-survival value also exceeds $\varphi_M \exp(w_M)$, the reduced-form continuation game has no pure strategy equilibrium. Instead, it admits a unique mixed-strategy equilibrium, in which each retailer chooses a survival probability to leave its rival indifferent between exiting and surviving. The equilibrium thus has the following survival rules for $k \in \{\mathcal{H}, \mathcal{L}\}$.

$$a^S(2\iota_k, c, w_M, k) = \begin{cases} 1 & \text{if } v^S(2\iota_k, c, k) > \varphi_M \exp(w_M), \\ \frac{v^S(\iota_k, c, k) - \varphi_M \exp(w_M)}{v^S(\iota_k, c, k) - v^S(2\iota_k, c, k)} & \text{if } v^S(2\iota_k, c, k) \leq \varphi_M \exp(w_M), v^S(\iota_k, c, k) > \varphi_M \exp(w_M) \\ 0 & \text{otherwise.} \end{cases}$$

If the monopoly post-survival value is low than $\varphi_M \exp(w_M)$, pure exit is the only equilibrium strategy of the static game–no unilateral deviation not improve the payoff, and no mixing is possible. In this case, the survival rule in the RNMPE is

$$a^S(2\iota_k, c, w_M, k) = \begin{cases} 1 & \text{if } v^S(2\iota_k, c, k) > \varphi_M \exp(w_M), \\ 0 & \text{otherwise.} \end{cases}$$

In this duopoly example, the RNMPE is always unique. For the unique equilibrium, I can compute the transition probabilities for a duopoly market with two type-k retailers by integrating out W_M. With this part of the survival rule and transition probabilities determined, the equilibrium construction is concluded.

4.2 Equilibrium Refinements

The RNMPE are a subset of MPE which possesses theoretically and empirically plausible implications. First, in a natural MPE, a type-$\mathcal{H}$ retailer never exits when a type-$\mathcal{L}$ competitor survives. It is a natural selection of equilibria, because a type-$\mathcal{H}$ retailer earns higher flow profit in each period. The exit pattern in the data also supports such an equilibrium: The majority of the exited stores are local stores, which are type-$\mathcal{L}$ retailers by assumption. Formally, we have

Definition 1. *A* natural *Markov-perfect equilibrium is a symmetric Markov-perfect equilibrium in a strategy* (a^E, a^S) *such that for all* c, w_M *and all* $\mathbf{m}$ *that includes at least one type-*$\mathcal{H}$ *and one type-*$\mathcal{L}$ *retailer,* $a^S(\mathbf{m}, c, w_M, \mathcal{L}) > 0$ *implies that* $a^S(\mathbf{m}, c, w_M, \mathcal{H}) = 1$.

Cabral (1993) restricts attention to similar natural equilibria in a model with deterministic productivity progression.

Second, renegotiation-proofness requires all active retailers of the same type to continue for sure, if the joint continuation is profitable for every one of them. This requirement only has a bite when joint incumbency yields positive expected payoff, while any other market structure, including those with less retailers of this type, yields negative expected payoff.[21] In this case, I only consider the natural MPE in which all retailers of this type choose to continue, hence these retailers cannot further gain by a joint change of actions. Since these retailers repeatedly interact, it seems reasonable to assume that they are able to "renegotiate" onto this more profitable option. Formally, we have

[21] This seemingly odd situation only arises when $k = \mathcal{L}$. If (1) two type-$\mathcal{L}$ retailers can deter future entry of any chain store by joint continuation, while one type-$\mathcal{L}$ retailer's continuation cannot, and (2) the gain from not having a chain store in future may dominate the harm from a decrease in present flow profit, solo continuation is less profitable than joint continuation.For an example, please refer to Abbring, Campbell, and Yang (2010).

Definition 2. *A natural Markov-perfect equilibrium is renegotiation-proof if, for any* (m, c, w_M)*, no one-shot agreement satisfying the following properties can be negotiated:*

- *all retailers in the agreement change their survival actions once;*
- *the agreement is self-enforcing, so no retailer in the agreement has incentive to unilaterally change the agreed action;*
- *if one type-k firm is in the agreement, all type-k firms are,* $k \in \{\mathcal{L}, \mathcal{H}\}$*; and*
- *the payoffs to all retailers in the agreement are strictly improved.*

Finally, I need to make a technical remark on a retailer's equilibrium actions on "indifference" points. When no other active stores have the same state variables as this retailer, and different actions give this retailer the same payoff, one can generate multiple equilibria by varying this retailer's action. For instance, when the payoff to entry net of sunk cost exactly equals zero, both entry and staying out give an entrant the same payoff, and both are consistent with payoff maximization. Similarly, when the payoff to survival as the only retailer of a certain type subtracting profitability shock exactly equals zero, both exit and continuation give this retailer the same payoff, and both are consistent with payoff-maximization. I require that retailers choose inactivity in these situations to eliminate this unimportant equilibrium multiplicity. In my empirical implementation, the shocks on costs and profit are continuous random variables. Therefore, such equilibrium multiplicity occurs with measure zero and reduces to a technicality. Formally, we have

Definition 3. *A Markov strategy* (a^E, a^S) *with corresponding value functions* v^E, v^S defaults to inactivity *if*

- *for all* $\mathbf{m}$*,* $a^E(\mathbf{m}, c, w_{\mathfrak{C}}, \mathfrak{C}) = 0$ *whenever* $\mathbb{E}_{\mathbf{M}_E}[\omega v^E(\mathbf{M}_E, c, \mathcal{H}) + (1-\omega) v^E(\mathbf{M}_E, c, \mathcal{L}) \mid \mathbf{M}_{\mathfrak{C}} = \mathbf{m}] = \varphi_{\mathfrak{C}} \exp(w_C)$*;*
- *for all* $\mathbf{m}$*,* $a^E(\mathbf{m}, c, w_{\mathfrak{L}}, \mathfrak{L}) = 0$ *whenever* $\mathbb{E}_{\mathbf{M}_E}\left[v^E(\mathbf{M}_E, c, \mathcal{L}) \mid \mathbf{M}_{\mathfrak{L}} = \mathbf{m}\right] = \varphi_{\mathfrak{L}} \exp(w_L)$*;*
- $a^S(\mathbf{m}, c, w_M, k) = 0$ *whenever* $v^S(\mathbf{m}, c, k) = \varphi_M \exp(w_M)$ *and* $m_k = 1$*.*

for all $k \in \{\mathcal{L}, \mathcal{H}\}$ *and all* $c, w_{\mathfrak{C}}, w_{\mathfrak{L}}, w_M$*.*

Throughout the paper, I restrict attention to equilibria with strategies that default to inactivity.

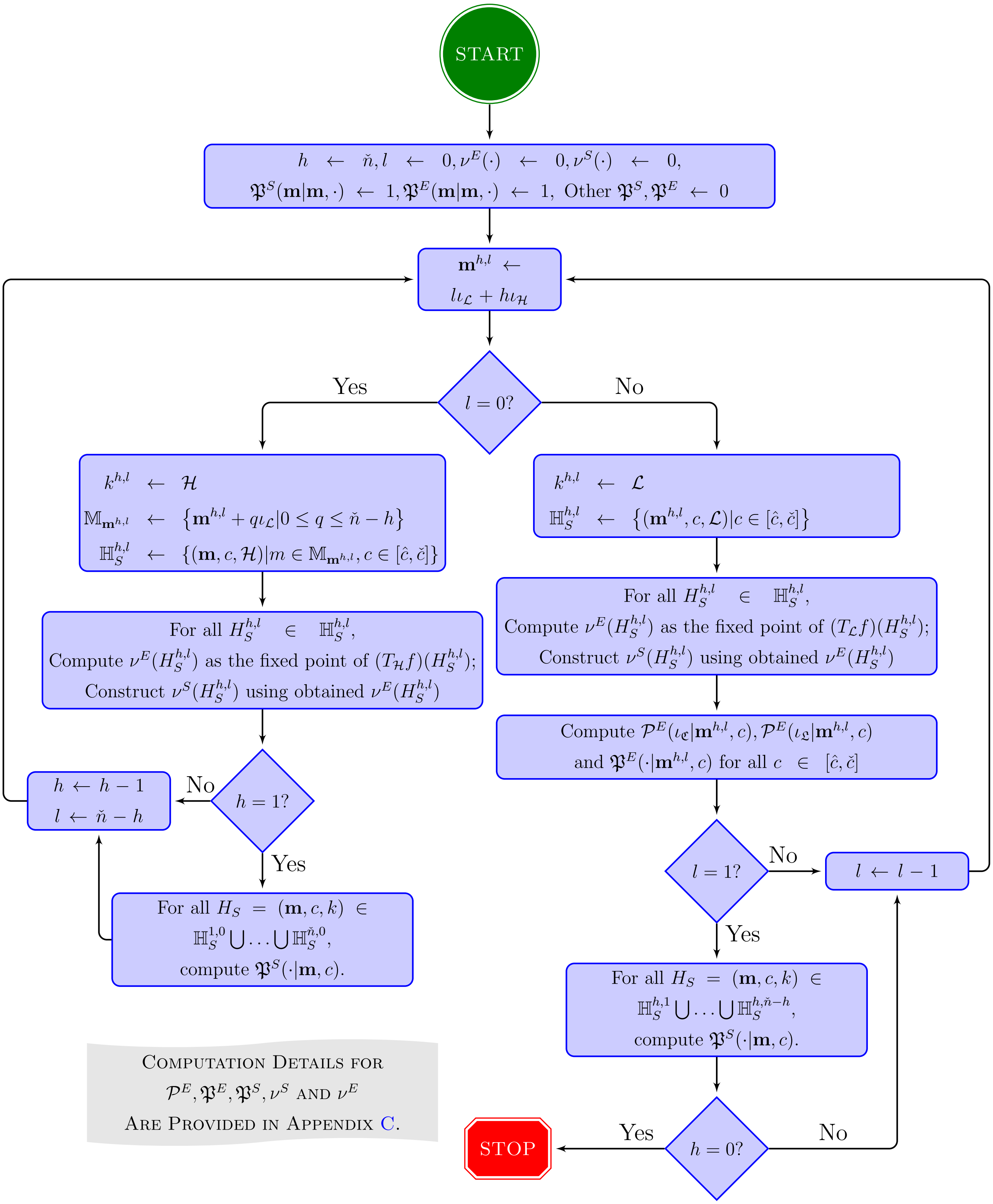
4.3 Equilibrium Existence, Uniqueness and Computation

The five-step approach discussed in Section 4.1 can be naturally extended to a general procedure that solves for a RNMPE by computed the fixed points of a sequence of contraction mappings. This procedure partitions the state space and traverses through the parts in steps. In each step, the post-entry value function restricted to the part of the state space is determined by the fixed point of a contraction mapping. I emphasize that the procedure calculates *candidate* value functions, choice probabilities and transition probabilities by denoting these with ν^E, ν^S, $\mathcal{P}^E$, $\mathfrak{P}^E$, and $\mathfrak{P}^S$ instead of v^E, v^S, P^E, $\mathbb{P}^E$ and $\mathbb{P}^S$.

Item 3 in Assumption 1 and non-negative sunk costs of entry imply that there exists a upper bound for the number of simultaneously active retailers in an initially empty market. Denote this bound by $\check{n}$. Consequently, no retailer will rationally enter the market $\check{n}\iota_{\mathcal{H}}$, and at most $\frac{(\check{n}+2)(\check{n}+1)}{2} - 1$ different non-empty markets structures can arise when market evolves. The state space is then partitioned into $\frac{(\check{n}+2)(\check{n}+1)}{2} - 1$ parts, with each step of the procedure computing the payoff function restricted to one of these parts. The computation procedure starts by considering the saturated market $\check{n}\iota_{\mathcal{H}}$. Procedure 1 presents this as a flow chart.

In this procedure, h indexes the number of type-$\mathcal{H}$ retailers, and l indexes the number of type-$\mathcal{L}$ retailers. In the course of the computation, h decreases from $\check{n}$ to 0. For each level of h, l decreases from $\check{n} - h$ to 0. For any pair of (h, l) such that $l > 0$, the post-entry value of type-$\mathcal{L}$ retailers facing h type-$\mathcal{H}$ rivals, $\nu^E(l\iota_{\mathcal{L}} + h\iota_{\mathcal{H}}, \cdot, \mathcal{L})$, is computed as the fixed point of a functional operator $T_{\mathcal{L}}$. This operator is defined by the appropriate generalization of Equation (9). The type-$\mathcal{L}$ retailer rationally expects all of its rivals to remain whenever it receives positive payoff, so this retailer's value only depend on future states in which all currently active retailers survive. Since any subsequent entry leads to a market structure with higher h or higher l, and the algorithm proceeds in descending order of (h, l), the payoff-relevant entry transition probabilities and the post-entry value functions have been computed before this step.

When l reaches 1, the choice probabilities of survival for $1, 2, \ldots, \check{n} - h$ type-$\mathcal{L}$ retailers facing h type-$\mathcal{L}$ retailers are computed. With these probabilities in place, the next step is to simultaneously compute the payoff for a type-$\mathcal{H}$ retailer in a market with $h - 1$ type-$\mathcal{H}$ and $0, 1, \ldots, \check{n} - h$ type-$\mathcal{L}$ rivals. This payoff is computed as the fixed point of a functional operator $T_{\mathcal{H}}$. Evaluating this operator requires choice probabilities of survival from the type-$\mathcal{L}$ competitors, the relevant entry transition probabilities, and the corresponding post-entry value functions. Again, the descending order of (h, l) guarantees that these values have been



Procedure 1: Calculation of a Candidate Equilibrium

computed before this step. Therefore, $T_{\mathcal{H}}$ and $T_{\mathcal{L}}$ are always contraction mappings with unique fixed points ν^E. Upon verifying that ν^E is indeed the post-entry value function for a RNMPE, this procedure serves as a constructive proof for the equilibrium existence.

Proposition 1 (Equilibrium Existence)**.** *Procedure 1 always computes a renegotiation-proof natural Markov-perfect equilibrium. The equilibrium value functions $v^S = \nu^S, v^E = \nu^E$, the choice probabilities $P^E = \mathcal{P}^E$ and the transition probabilities $\mathbb{P}^E = \mathfrak{P}^E$ and $\mathbb{P}^S = \mathfrak{P}^S$.*

Proof. See Appendix B. □

In Procedure 1, when equilibrium survival rule on $(\mathbf{m}, c, w_M, k)$ involves mixed actions, the mixing probability p is determined by the following polynomial equation.[22]

$$\sum_{i=0}^{m_k-1} (1-p)^{m_k-1-i} p^i \binom{m_k-1}{i} (\nu^S(\underline{\mathbf{m}} - (m_k-1-i)\iota_k, c, k) - \varphi_M \exp(w_M) = 0. \qquad (7)$$

where $\underline{\mathbf{m}} = \mathbf{m}$ if $k = \mathcal{L}$, and $\underline{\mathbf{m}} = m_k \iota_k$ if $k = \mathcal{H}$. The degree of the polynomial equals the number of mixing stores minus one. When the number of mixing stores exceeds two, multiple roots of the polynomial may exist between $[0,1)$, resulting multiple choice probabilities. Given each distinct set of choice probabilities, the contraction mappings $T_{\mathcal{H}}$ and $T_{\mathcal{L}}$ produce a unique equilibrium post-entry value function. Thus, the uniqueness of the equilibrium entirely rests on the uniqueness of the mixing probability.

If both C and W_M are discrete variables (or discretized for computational purposes), the state space for any retailer facing continuation decision has a finite number of points. This implies that one can only create a finite number of distinct sets of choice probabilities, and hence a finite number of RNMPE by combining different mixing probabilities. By recording all the admissible mixing probabilities and repeating the algorithm for every possible set of these probabilities, I can compute the payoffs and choice probabilities for all such RNMPE.

Nevertheless, equilibrium uniqueness is still an empirically desirable feature. It is also central to ensure the reliability of policy experiments. The following proposition and its corollary further address the uniqueness issue.

Proposition 2 (Equilibrium Uniqueness)**.** *The renegotiation-proof natural Markov-perfect equilibrium of the model is unique if for any $(\mathbf{m}, c, w_M, k)$ such that $v^S(\underline{\mathbf{m}}, c, k) \leq 0$, the polynomial equation (7) admits no more than one root in $[0,1)$.*

[22]In practice, I do not need to solve this polynomial equation to compute the model. Computational details are included in Procedure 3 in Appendix C

Proof. See Appendix B. □

A sufficient condition for the polynomial (7) to have a unique root is that $v^S(\mathbf{m}, c, k)$ is non-increasing in the number of type-k stores in $\mathbf{m}$.

Corollary 1. *If the value functions of a natural Markov-perfect equilibrium satisfy* $v^S(\mathbf{m}, c, k) \geq v^S(\mathbf{m}+\iota_k, c, k)$ *and* $v^E(\mathbf{m}, c, k) \geq v^E(\mathbf{m}+\iota_k, c, k)$ *for all* $(\mathbf{m}, c, k)$*, it is the unique renegotiation-proof natural Markov-perfect equilibrium.*

The natural MPE whose value functions satisfies the monotone conditions in Corollary 1 is named payoff-monotone natural MPE. Three important remarks supplement this corollary. First, a payoff-monotone natural MPE is always renegotiation-proof. If the equilibrium post-survival payoff is monotone, and joint continuation for retailers of a same type is profitable, then survival is the dominant strategy. If all such retailers survive, no coalition can be formed to further improve their payoffs. Therefore, payoff-monotonicity ensures that retailers always adopt the renegotiation-proof strategy. Second, because all the post-survival value functions relevant to any particular mixing probability have been determined before the probability is computed, the payoff-monotonicity is more easily testable than directly examining the number of admissible roots of the polynomial. Finally, the monotonicity should be checked in the same ordering as the equilibrium computation. If the monotonicity is only violated in the later steps, multiple equilibria, if any, still agree on the values, choice probabilities and transition probabilities computed in the earlier steps where monotonicity held.

5 Estimation

Throughout Section 4, the choice probabilities, the transition probabilities and the equilibrium value functions are independent of the shocks to profit and sunk costs. Nevertheless, they still depend on retailers' profitability types and the market structure. The data, on the other hand, only contain information on the store formats, which are noisy indicators of the active retailers' profitability types. Therefore, to construct a likelihood function using the choice and transition probabilities, I need to assess the underlying market structure through the joint distribution of active chain stores' types. Because all active retailers' profitability types are public information for the players of the game, retailers' equilibrium decisions are informative on the underlying market structure. Hence, I infer the joint distribution for all active retailers' types from their observed equilibrium actions. From there, the likelihood function is constructed by integrating over this joint distribution. In Section 5.1, I use a duopoly example to illustrate the construction of the likelihood function. In Section 5.2, I generalize this construction and introduce the Nested-Fixed-Point algorithm to estimate the model.

5.1 A Duopoly Example

Consider a market with two active chain stores, AH and C1000. Their joint type distribution has four points in the support. Suppose that the initial (post-entry) probabilities in period 1 for these four points are given. Denote them by $p_1^{E,\mathcal{HH}}$, $p_1^{E,\mathcal{HL}}$,$p_1^{E,\mathcal{LH}}$, and $p_1^{E,\mathcal{LL}}$ respectively (the types in the superscript are alphabetically ordered by the stores' names). Denote the post-entry probability conditioning on the observed exits and survivals by $\tilde{p}_1^{E,k_1k_2}$, for $k_1, k_2 \in \{\mathcal{L}, \mathcal{H}\}$. If, for instance, both stores are observed to continue in period 1 under the demand c_1, This observation is informative on the underlying market structure. According to Bayes' rule, the conditional post-entry probability that AH is type-k_1 and C1000 is type-k_2 is updated to

$$\tilde{p}_1^{E,k_1k_2} = \frac{p_1^{E,k_1k_2}\mathbb{P}^S(\iota_{k_1} + \iota_{k_2}|\iota_{k_1} + \iota_{k_2}, c_1)}{\sum_{i\in\{\mathcal{L},\mathcal{H}\}}\sum_{j\in\{\mathcal{L},\mathcal{H}\}} p_1^{E,ij}\mathbb{P}^S(\iota_i + \iota_j|\iota_i + \iota_j, c_1)},$$

in which $\mathbb{P}^S(\iota_i + \iota_j|\iota_i + \iota_j, c_1)$ is the equilibrium transition probability for market $\iota_i + \iota_j$ to remain under demand c_1. The numerator is the the probability that AH has type-k_1, C1000 has type-k_2, and both of them survive under c_1. The denominator is the sum of probabilities that both AH and C1000 survival taking all possible type combinations among them, and under c_1. When no store exits, the post-survival probabilities coincide with the conditional post-entry probabilities.

If C1000 exits from the market, while AH remains, then the conditional post-entry probability that AH is type-k_1 and C1000 is type-k_2 is

$$\tilde{p}_1^{E,k_1k_2} = \frac{p_1^{E,k_1k_2}\mathbb{P}^S(\iota_{k_1}|\iota_{k_1} + \iota_{k_2}, c_1)/\binom{n_{k_2}}{1}}{\sum_{i\in\{\mathcal{L},\mathcal{H}\}}\sum_{j\in\{\mathcal{L},\mathcal{H}\}} p_1^{E,ij}\mathbb{P}^S(\iota_i|\iota_i + \iota_j, c_1)/\binom{n_j}{1}},$$

The numerator is the the probability that AH has type-k_1, C1000 has type-k_2, and only the type-k_2 store exits under c_1. Here, n_j is the number of type-j stores in the pre-survival market. If $j = k_1$, i.e., C1000 and AH have the same type, because $\mathbb{P}^S(\iota_{k_1}|\iota_{k_1} + \iota_j, c_1)$ summarizes the transition probabilities for both C1000's exit coupled with AH's survival and C1000's survival coupled with AH's exit, it needs to be divided by $\binom{n_j}{1} = 2$ to represent the observed market transition.

After C1000's exit, the post-survival type distribution in period 1 is determined by AH's type. The probabilities for AH to be type $\mathcal{L}$ and type $\mathcal{H}$ are $p_2^{S,\mathcal{L}}$ and $p_2^{S,\mathcal{H}}$ respectively. For $k_1 \in \{\mathcal{L}, \mathcal{H}\}$, they are constructed by summing $\tilde{p}_1^{E,k_1k_2}$ over C1000's two possible types, $p_1^{S,k_1} = \sum_{j\in\{\mathcal{L},\mathcal{H}\}} \tilde{p}_1^{E,k_1j}$.

Now suppose that after C1000's exit in period 1, another chain store Dirk enters in period 2 in demand state c_2. This equilibrium action is informative on the type of AH, the only incumbent in the pre-entry market. Hence, the post-survival (or pre-entry) probability for AH to be type-k_1, $k_1 \in \{\mathcal{L}, \mathcal{H}\}$, conditioning on the observed entry, is

$$\tilde{p}_1^{S,k_1} \frac{p_1^{S,k_1} P^E(\iota_{\mathfrak{C}}|\iota_{k_1}, c_2)}{\sum_{i\in\{\mathcal{L},\mathcal{H}\}} p_1^{S,i} P^E(\iota_{\mathfrak{C}}|\iota_i, c_2)}.$$

Then, the post-entry probability for AH to be type-k_1 and Dirk to be type-k_2, $k_1, k_2 \in \{\mathcal{L}, \mathcal{H}\}$ is constructed from $\tilde{p}_1^{S,k_1}$ by incorporating Dirk's initial type,

$$p_2^{E,k_1k_2} = \begin{cases} \omega\tilde{p}_1^{S,k_1} & \text{if } k_2 = \mathcal{H}, \\ (1-\omega)\tilde{p}_1^{S,k_1} & \text{if } k_2 = \mathcal{L}. \end{cases}$$

Because the distribution of chain store types is updated in each period, the observations' likelihood contributions are also computed iteratively over time. In the first period of this duopoly example, if both retailers continue to the next period, collect the conditional post-entry probabilities, $\tilde{p}_1^{E,k_1k_2}$, for the four possible type combinations in the *type distribution vector* $\tilde{\mathbf{p}}_1^E$. The underlying post-entry market structure assumes three values. Denote its distribution function by $\mathbb{P}(\mathbf{N}|\tilde{\mathbf{p}}_1^E, 0)$, where 0 is the number of active local stores. Then,

$$\mathbb{P}(\mathbf{N}|\tilde{\mathbf{p}}_1^E, 0) = \begin{cases} p_1^{E,\mathcal{HH}} & \text{if } \mathbf{N} = 2\iota_{\mathcal{H}}, \\ p_1^{E,\mathcal{HL}} + p_1^{E,\mathcal{LH}} & \text{if } \mathbf{N} = \iota_{\mathcal{H}} + \iota_{\mathcal{L}}, \\ p_1^{E,\mathcal{LL}} & \text{if } \mathbf{N} = 2\iota_{\mathcal{L}}, \\ 0 & \text{otherwise.} \end{cases}$$

The observed survival's likelihood contribution is determined by summing the appropriate transition probabilities over the distribution of underlying market structures,

$$\mathbb{P}^S(2\iota_{\mathcal{L}}|2\iota_{\mathcal{L}}, c_1)\mathbb{P}(2\iota_{\mathcal{L}}|\tilde{\mathbf{p}}_1^E, 0) + \mathbb{P}^S(\iota_{\mathcal{L}}+\iota_{\mathcal{H}}|\iota_{\mathcal{L}}+\iota_{\mathcal{H}}, c_1)\mathbb{P}(\iota_{\mathcal{L}}+\iota_{\mathcal{H}}|\tilde{\mathbf{p}}_1^E, 0) + \mathbb{P}^S(2\iota_{\mathcal{H}}|2\iota_{\mathcal{H}}, c_1)\mathbb{P}(2\iota_{\mathcal{H}}|\tilde{\mathbf{p}}_1^E, 0).$$

If AH continues but C1000 exits, then natural equilibrium requires that AH does not have inferior profitability type than C1000. Hence, the market structure $\iota_{\mathcal{L}} + \iota_{\mathcal{H}}$ can only be attributed to a type-$\mathcal{H}$ AH and a type-$\mathcal{L}$ C1000. Such an observation contributes to the likelihood by

$$\mathbb{P}^S(\iota_{\mathcal{L}}|2\iota_{\mathcal{L}}, c_1)\mathbb{P}(2\iota_{\mathcal{L}}|\tilde{\mathbf{p}}_1^E, 0, C1000) + \mathbb{P}^S(\iota_{\mathcal{H}}|2\iota_{\mathcal{H}}, c_1)\mathbb{P}(2\iota_{\mathcal{H}}|\tilde{\mathbf{p}}_1^E, 0, C1000)$$
$$+ \mathbb{P}^S(\iota_{\mathcal{H}}|\iota_{\mathcal{L}} + \iota_{\mathcal{H}}, c_1)\mathbb{P}(\iota_{\mathcal{L}} + \iota_{\mathcal{H}}|\tilde{\mathbf{p}_1^E}, 0, C1000).$$

where $\mathbb{P}(\cdot|\tilde{\mathbf{p}}_1^E, 0, C1000)$ denotes the market structure distribution conditional on C1000's type being no superior to the type of the surviving rival AH. In this example, $\mathbb{P}(2\iota_k|\tilde{\mathbf{p}}_1^E, 0, C1000) = \mathbb{P}(2\iota_k|\tilde{\mathbf{p}}_1^E, 0)$ for $k \in \{\mathcal{L}, \mathcal{H}\}$, whereas $\mathbb{P}(\iota_{\mathcal{L}} + \iota_{\mathcal{H}}|\tilde{\mathbf{p}}_1^E, 0, C1000) = \tilde{p}_1^{E,\mathcal{HL}}$.

In period 2 of the duopoly example, chain store Dirk's entry contributes to the likelihood function by

$$P^E(\iota_{\mathfrak{C}}|\iota_{\mathcal{L}}, c_2)\mathbb{P}(\iota_{\mathcal{L}}|\tilde{\mathbf{p}}_1^S, 0) + P^E(\iota_{\mathfrak{C}}|\iota_{\mathcal{H}}, c_2)\mathbb{P}(\iota_{\mathcal{H}}|\tilde{\mathbf{p}}_1^S, 0).$$

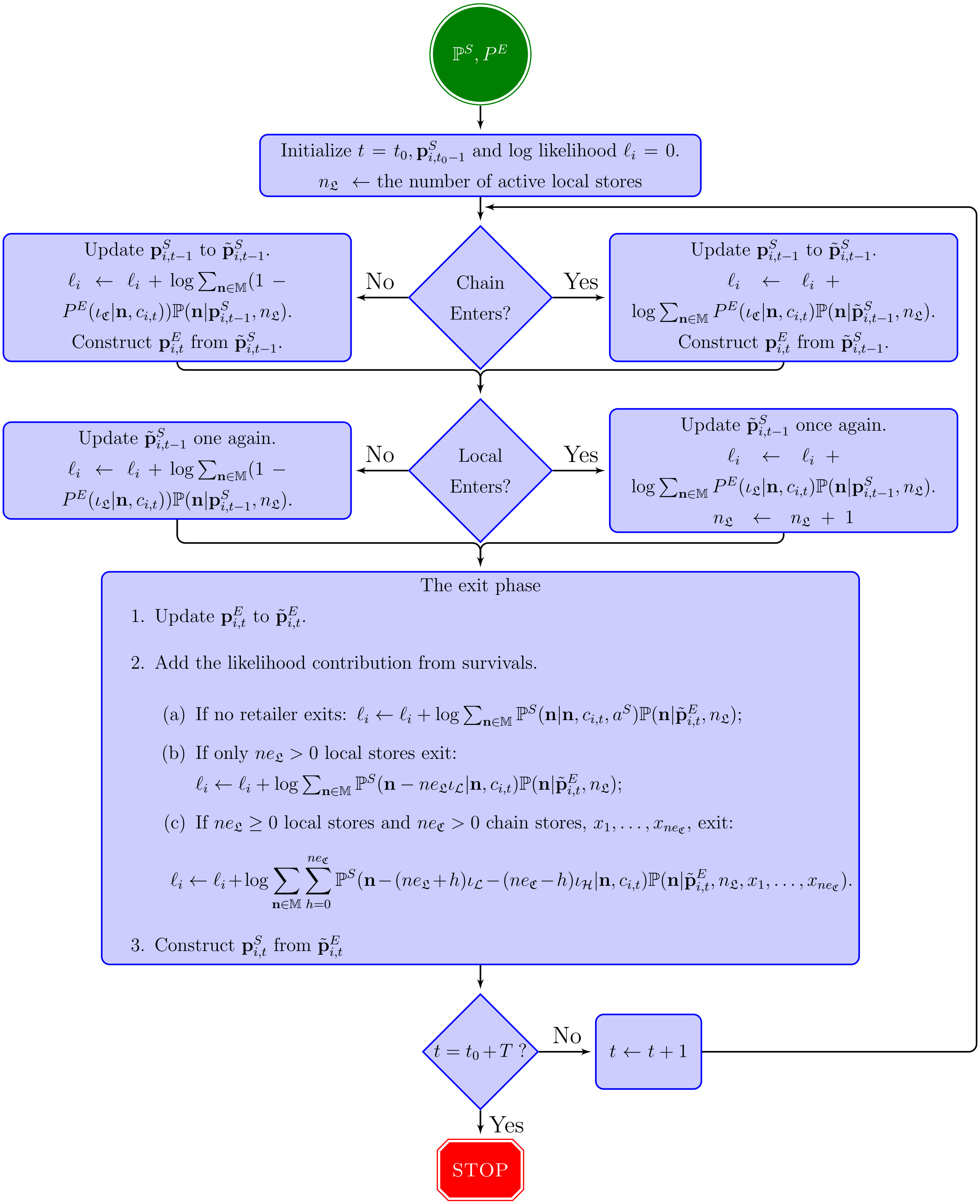
This is the sum of probabilities of entry under each possible pre-entry market structure. If instead a local store becomes active or no retailer enters in this period, the likelihood contribution is computed in a similar fashion by using the appropriate choice probabilities. Extending the market structure distribution function $\mathbb{P}(\mathbf{N}|\tilde{\mathbf{p}}, k)$ to the cases where $k > 0$ is straightforward.

Next, the likelihood construction moves on until the end of the sample. After each entry and exit, I update the type distribution vector using Bayes' rule, and compute the likelihood contribution for each observed action.

5.2 The Nested-Fixed-Point Algorithm

The parameter vector θ_1 governing C's evolution can be directly recovered from a partial likelihood of population dynamics. After θ_1 has been estimated, with the choice and transition probabilities computed by Procedure 1, the construction of the partial likelihood for the other parameters in Θ proceeds iteratively from the first period ($t = t_0$) of the data to the last period ($t = t_0 + T$). In any market $i \in \{1, \ldots, I\}$, this procedure starts with initializing $\mathbf{p}_0^S$, the type distribution vector for the post-survival market at the end of the pre-sample period. Then, using the observations on entries and exits, I update the type vector and compute the likelihood contributions, as illustrated in Section 5.1. The general procedure is depicted in following flow chart for Procedure 2.

In this procedure, $\mathbf{p}_{i,t}^E$ denotes the type distribution vector at time t in market i, with the retailer x's type fixed at k. The distribution function for underlying market structure $\mathbb{P}$



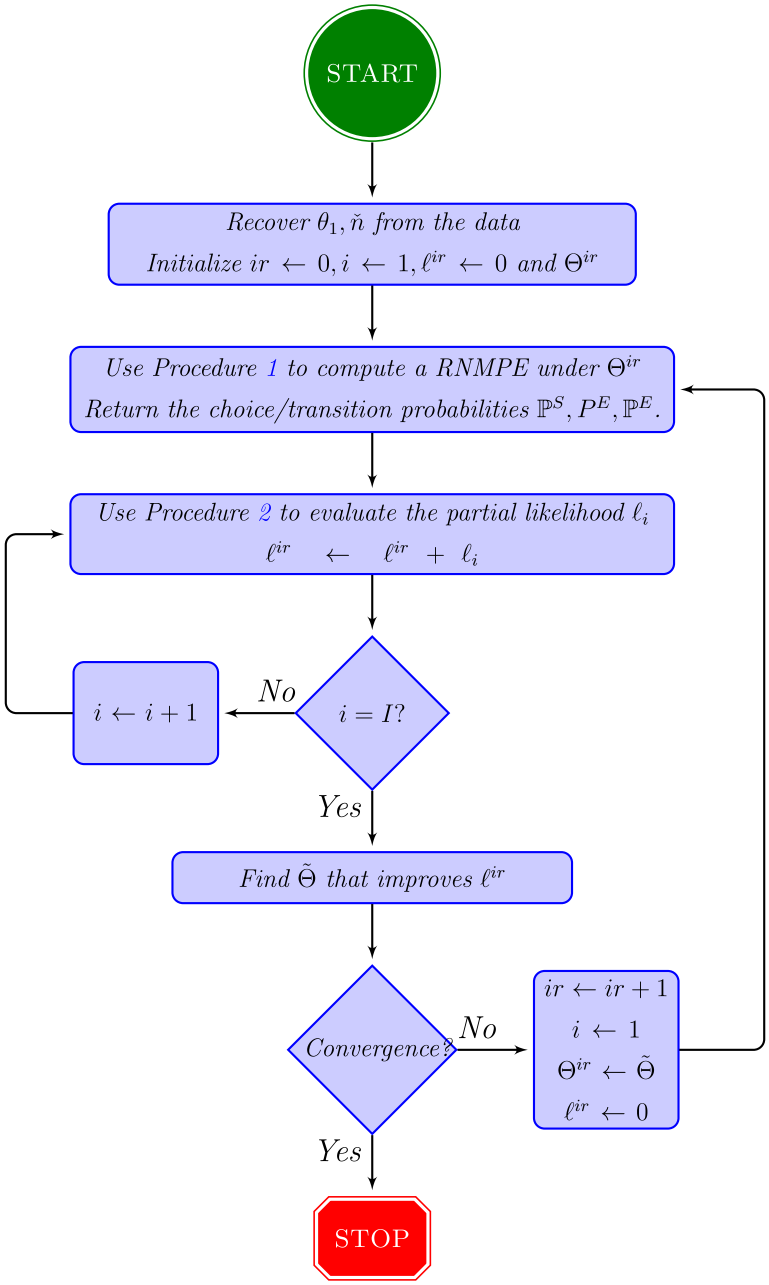
Procedure 2: Likelihood Construction for Market i

is computed by considering all active retailers' type distributions. The formula for $\mathbb{P}$ is in Appendix C.

To search for the optimal value of Θ, I use the Nested-Fixed-Point Algorithm (NFXP) proposed by Rust (1987). This algorithm iterates between an outer loop and an inner loop. It starts with estimating θ_1 directly from the demand data, and initialize values for the rest of Θ. The upper bound $\check{n}$ is recovered from the historical maximum number of simultaneously active retailers. Also, it can be set to any arbitrary finite number above the observed maximum. In the inner loop, it uses Procedure 1 to compute a RNMPE by finding the fixed points of a sequence of contraction mappings, and return the choice probabilities P^E and the transition probabilities $\mathbb{P}^E, \mathbb{P}^S$. Then, it uses Procedure 2 to evaluate the log likelihood contribution ℓ_i for every market $i \in \{1, \ldots, I\}$, and subsequently sums them up to form the partial likelihood for the structural parameters. In the outer loop, it searches for new parameter values of Θ to increase the likelihood value. When the parameter values are updated, they are passed to the inner loop to solve the model again and regenerate the likelihood value. The algorithm stops when no further improvement of the likelihood can be found.[23] The flow chart for Algorithm 1 presents the details.

[23] In practice, I use the optimization solver KNITRO's built-in stopping rules. For details, please refer to the KNITRO User's Manual http://www.ziena.com/docs/Knitro60_UserManual.pdf.

Algorithm 1 (Nested-Fixed-Point Algorithm).



6 Empirical Implementation and Results

This section discusses in order the details of the empirical implementation, the estimation results, and the policy experiments.

6.1 Empirical Implementation

The market demand $c_{i,t}$ is defined as the number of inhabitants at time t in postcode area i. Since all the postcode areas in the sample have a population between 4,000 and 12,000 in 2009, the demand indicator C is discretized on a 201-point grid with bounds $[3500, 12500]$. Each sampled value is located on its nearest grid point. The conditional distribution $f_C(C'|C;\theta_1)$ is assumed to be a mixture over 51 reflected random walks in C with uniformly distributed innovations. This mixture approximates a normally distributed innovation. The parameter θ_1 includes the mean and the standard deviation of the innovation targeted by the approximation. The mean is set to be 0, and the standard deviation equals 161.38, which is the sample standard deviation of $C_{i,t}$.

The upper bound on the number of simultaneously active stores $\check{n}$ is set to 11, the maximum number observed in the sample. Feasible alternatives $\check{n} = 12$ and $\check{n} = 13$ alter the estimation results negligibly. In total, 77 possible market structures can arise in market dynamics. Because not every possible market structure is well represented in the sample, nonparametric identification of the profit function $\pi_k(n, c; \theta_k)$ is nearly impossible. Therefore, I adopt a parametric approach to specify the profit function as

$$\pi_k(n, c; \theta_k) = \frac{\theta_k(c/500)}{\theta_{\mathcal{H}} n_{\mathcal{H}} + \theta_{\mathcal{L}} n_{\mathcal{L}} + 1}, \quad k \in \{\mathcal{L}, \mathcal{H}\}. \tag{8}$$

An intuitive interpretation for this parametrization is that the profit from every 500 inhabitants is divided into $\theta_{\mathcal{H}} n_{\mathcal{H}} + \theta_{\mathcal{L}} n_{\mathcal{L}} + 1$ shares, and a type-k retailer attracts θ_k shares of them. In this expression, the parameter θ_k measures the profitability of a type-k retailer. A type-$\mathcal{H}$ retailer earns a profit $\theta_{\mathcal{H}}/\theta_{\mathcal{L}}$ times that of a type-$\mathcal{L}$ retailer in the same market. Hence, this ratio captures the profitability advantage of a type-$\mathcal{H}$ relative to a type-$\mathcal{L}$ rival. Assumption 1 requires this ratio to be larger than 1.[24] In the estimation, I normalize $\theta_{\mathcal{L}}$ to 1. Under this normalization, the per period profit from every 500 inhabitants is $\theta_{\mathcal{H}}/(\theta_{\mathcal{H}} + 1)$ for a type-$\mathcal{H}$ monopolist, and 1/2 for a type-$\mathcal{L}$ monopolist.

I assume that the transitory shocks are normally distributed with mean 0 and variances $\sigma_M^2, \sigma_{\mathfrak{E}}^2$, and $\sigma_{\mathfrak{C}}^2$. For lack of historical observations, I assume that all the existing chain stores at the beginning of the sample period have an identical probability ω to be a type-$\mathcal{H}$ retailer. Finally, the annual discount rate is set to 5%.

With the above parameterizations, the variables to be estimated are down to the proba-

[24] When this ratio is larger than 1, $\pi_k(n, c; \theta_k) - \varepsilon$ for any $\varepsilon > 0$ honors all the requirements in Assumption 1. In practice, I discard the ε term.

bility for a chain store to become a type-$\mathcal{H}$ retailer after entry (ω), the profitability parameter for type-$\mathcal{H}$ retailers ($\theta_{\mathcal{H}}$), the profitability shock parameters φ_M, σ_M, and the sunk cost parameters $\varphi_{\mathfrak{C}}, \varphi_{\mathfrak{L}}, \sigma_{\mathfrak{C}}, \sigma_{\mathfrak{L}}$. For numerical stability, I fix $\sigma_{\mathfrak{L}}$ at 1.[25]

The NFXP estimation is coded in Matlab. The likelihood maximization invokes function `ktrlink`, which calls the KNITRO optimization libraries. With 201 grid points for the demand process, 201 grid points for the mixing probability, and $\check{n} = 11$, each likelihood evaluation typically takes less than 10 seconds on an off-the-shelf computer, and the entire estimation takes about 1-2 hours. The code and the companion documentations are downloadable from the author's website.

6.2 The Results

6.2.1 The Estimates

The estimates are reported in the first column of Table 4. The standard error and the confidence intervals are constructed by 100-time bootstrap. The bootstrap inference suggests that the estimates are reasonably accurate.

	Estimates	Standard Error	90% Confidence Interval	95% Confidence Interval
ω	65.44%	1.90%	[62.79%,67.93%]	[61.32%,68.45%]
$\theta_{\mathcal{H}}$	5.47	0.24	[5.17,5.90]	[5.07,5.97]
$\varphi_{\mathfrak{C}}$	221.22	19.31	[192.25,254.16]	[188.19,264.94]
$\varphi_{\mathfrak{L}}$	30.13	2.19	[27.24,33.49]	[26.20,34.45]
φ_M	1.58	0.16	[1.34,1.86]	[1.32,1.90]
$\sigma_{\mathfrak{C}}$	0.67	0.08	[0.54,0.80]	[0.53,0.83]
σ_M	1.27	0.08	[1.14,1.38]	[1.13,1.39]

Table 4: The Estimates and the Bootstrap Confidence Intervals

As expected, a large proportion, 65%, of the chain stores establish themselves as type-$\mathcal{H}$ retailers in the market after entry. The estimate of $\theta_{\mathcal{H}}$ is 5.5, confirming the advantageous position of the type-$\mathcal{H}$ retailers relative to the type-$\mathcal{L}$ competitors. This estimate helps to anchor the normalized equilibrium values to Euros. By matching the equilibrium implied

[25] In practice, separately identifying $\sigma_{\mathfrak{L}}$ and $\varphi_{\mathfrak{L}}$ turns out to be hard, because of limited variation in type-$\mathcal{L}$ stores' values. I also set $\sigma_{\mathfrak{L}}$ to 0.5 and 2 to check the robustness. The estimates, except $\varphi_{\mathfrak{L}}$, do not change much.

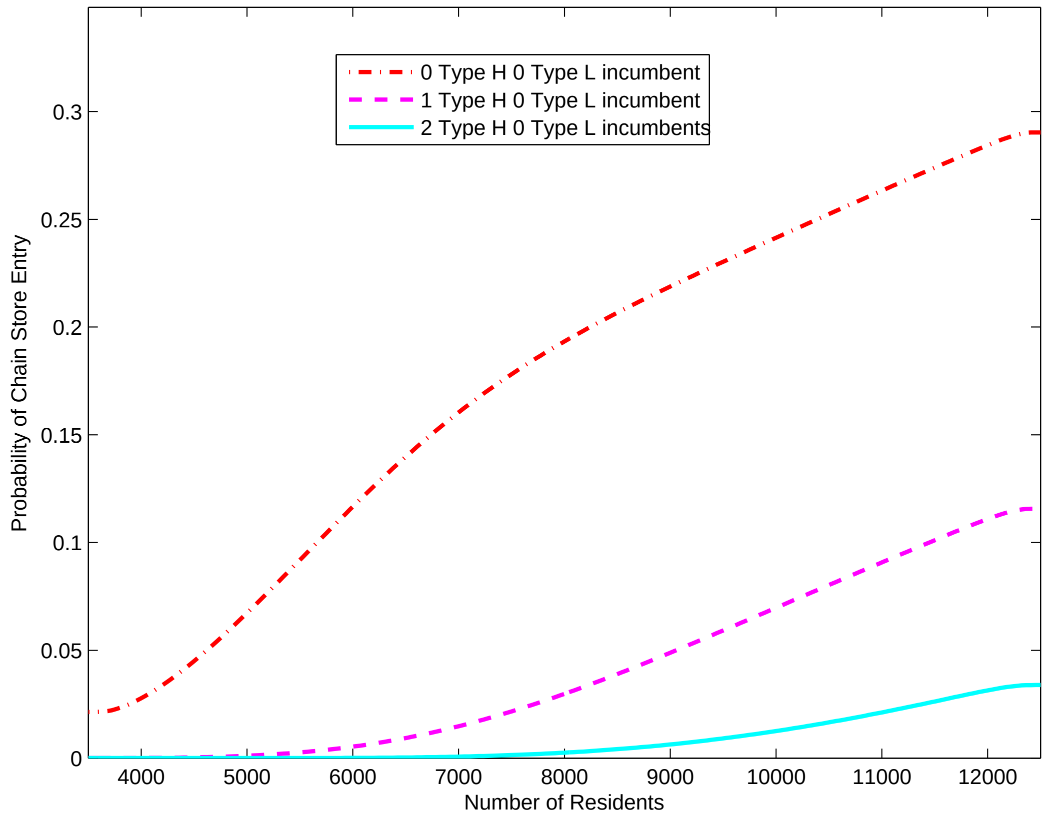
chain store profit in 2009 to the information published by the administrative authority HBD, I conclude that 1 unit of the cost/profit/payoff determined by the estimation is roughly worth €20,000.[26] Suppose that the market has one active type-$\mathcal{L}$ local store and one active type-$\mathcal{H}$ chain store. From serving every 500 inhabitants, the local store receives an annual net-of-shock profit of €2,700, and the chain store receives €14,700.

Conditional on entry, the average post-entry value expected by a chain store potential entrant is estimated to be around 1.8 million Euro, suggesting that the average sunk costs actually incurred are below this value. Unconditionally, the estimates of sunk cost parameters translate into setup costs of 5.5 million Euro for a chain store and 1 million Euro for a local store, averaged over the transitory shocks. Note that these values are average costs that potential entrants would have to face each period. Compared to stores' values, these average sunk costs turns out to be huge, implying a natural barrier to entry in this industry. Even for a would-be monopolist chain store potential entrant facing the largest market of 12.5 thousand inhabitants, the average sunk cost is almost 80% of its expected post-entry value. For a local store potential monopolist, the ratio is 1.4. A possible explanation for the natural barrier is the zoning regulation. As discussed before, the zoning regulation greatly limits the availability of business space in residential areas. When such space is unavailable in a certain year, entry is virtually impossible. The large one-off investment to open a chain store often yields a much higher expected rate of return than setting up a local store. For instance, on a market with 8,000 inhabitants, an active chain store is valued 40 to 100 times more than an local rival, depending on the market structure. This wedge is driven by the chain stores' superior profitability and longevity.

The probability of entry for a chain store is increasing in the number of consumers and decreasing in the number of incumbents, as depicted in Figure 4. Facing the high average sunk cost of entry, even a potential monopolist chain store only has a chance of 29% to enter the largest market with 12,500 residents in a year. For a potential duopolist chain store, this probability is merely 13%. According to the model's probabilistic prediction, over the sample period, the expected total numbers of chain and local entrants are 469 and 598 respectively, while the actual numbers are 503 and 486.

[26] In 2009, the estimated equilibrium implies a flow profit of 2.4 for a chain store, averaged over all the markets and the profitability shock. HBD reports that the net accounting profit for a supermarket chain outlet is around €102,920 in the same year. (The per-store sales is €5,146,000, and the net profit is 2%. See page 15 & 16 in "Dossier Supermarkten (feiten en cijfers)".) I reckon that the opportunity cost amounts to €50,000, which gives store owners roughly 1% economic return from investing in a supermarket.

Figure 4: Entry Probability for a Chain Store



The equilibrium value functions give the present value of stores' expected discounted profit. With the estimates in hand, I can evaluate a chain store entry's impact on such values. When an average chain store is certain to enter, Table 5 presents the expected percentage change of incumbent stores' values, with the percentage change of flow profit (excluding the shock) in the parenthesis. The change in values are computed using the estimated post-entry values averaged over the steady-state demand. Pre-entry markets in the same row (column) share the same number of type-$\mathcal{H}$ ($\mathcal{L}$) incumbent stores. Under the estimated parameter values of the model, the loss of store value inflicted by the new chain store is expected to range from 25% to 31% for a type-$\mathcal{H}$ incumbent retailer, and from 28% to 66% for a type-$\mathcal{L}$ retailer. The entry only results in a decline of 12%-60% in the flow profit for the incumbents, suggesting that a significant share of the damage in value is attribute to the reduced chance of survival.

Before proceeding to the policy experiments, I discuss the equilibrium multiplicity under the estimated parameters.

	0 Type-$\mathcal{L}$	1 Type-$\mathcal{L}$	2 Type-$\mathcal{L}$	3 Type-$\mathcal{L}$	4 Type-$\mathcal{L}$
0 Type-$\mathcal{H}$	-/-	-/-66.2%	-/-64.2%	-/-62.3%	-/-60.4%
	-	(-59.5%)	(-50.9%)	(-44.7%)	(-40.0%)
1 Type-$\mathcal{H}$	-31.3%/-	-30.1%/-59.2%	-29.2%/-57.0%	-28.6%/-54.6%	-28.3%/-52.1%
	(-34.6%)	(-31.7%)	(-29.3%)	(-27.3%)	(-25.5%)
2 Type-$\mathcal{H}$	-28.6%/-	-28.3%/-47.9%	-28.1%/-45.6%	-28.0%/-43.5%	-27.9%/-41.5%
	(-23.2%)	(-21.9%)	(-20.8%)	(-19.7%)	(-18.8%)
3 Type-$\mathcal{H}$	-26.9%/-	-26.8%/-37.7%	-26.7%/-36.3%	-26.7%/-34.9%	-26.7%/-33.7%
	(-17.5%)	(-16.8%)	(-16.1%)	(-15.4%)	(-14.9%)
4 Type-$\mathcal{H}$	-25.7%/-	-25.7%/-31.1%	-25.7%/-30.2%	-25.6%/-29.3%	-25.6%/-28.5%
	(-14.1%)	(-13.6%)	(-13.1%)	(-12.7%)	(-12.3%)

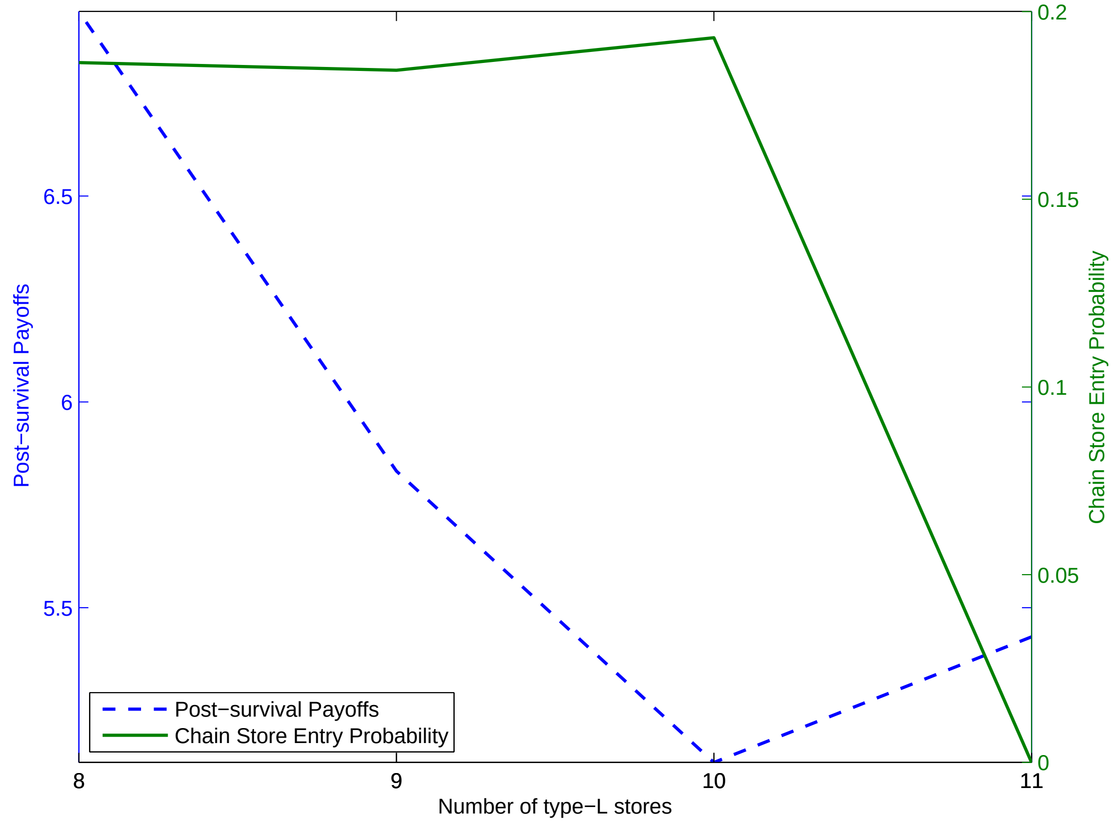
Table 5: The expected impact of one more chain on incumbents' values and flow profits

6.2.2 Equilibrium Multiplicity

First, I examine the equilibrium's uniqueness by checking the monotonicity of the equilibrium value function under the estimated parameter values, following Corollary 1. I find that when market structure includes at least one active type-$\mathcal{H}$ retailers, this condition is always satisfied for both type-$\mathcal{H}$ and type-$\mathcal{L}$ retailers. Given the ordering of Procedure 1, all the equilibrium payoffs and survival/entry rules computed prior to the step indexed by $(h = 0, l = 1)$ (which corresponds to the market structure $\iota_{\mathcal{L}}$) must be unique: If there were multiple RNMPE, they would have agreed on these payoffs and rules. This means that any equilibrium prediction of market transition involving type-$\mathcal{H}$ retailers is unique.

When the market is nearly saturated by only type-$\mathcal{L}$ stores, the entry deterrence effect leads to non-monotone values for the incumbents. Figure 5 visualizes such non-monotonicity for markets with no type-$\mathcal{H}$ chain store and 12,500 inhabitants ($C = \check{c}$). When there are no more than nine active type-$\mathcal{L}$ retailers, the probability for a chain store to enter next period declines with the number of type-$\mathcal{L}$ stores. When the number of post-survival type-$\mathcal{L}$ stores reaches ten, any chain store that enters in the coming period will saturate the market, and will not be followed by any other entrant. Therefore, its entry probability slightly increases. After the number of post-survival type-$\mathcal{L}$ stores reaches eleven, by the definition of $\check{n}$, no chain stores will further enter this market next period. Without the threat of entry, an active type-$\mathcal{L}$ retailer with ten type-$\mathcal{L}$ rivals enjoys a higher payoff from joint continuation than with nine rivals.

Figure 5: Non-Monotone Post-Survival Payoffs and Chain Entry Probabilities (No Type-$\mathcal{H}$ Rival & 12.5k inhabitants)



Such non-monotonicity does not induce multiple equilibria if the polynomial equation (7) never admits more than one root in $[0, 1)$. I further assess the possibility of multiple equilibria using simulation. To this end, I draw 5000 realizations of w_M, and compute the survival probability for each state $(\mathbf{m}, c, \mathcal{L}, w_M)$ where mixing takes place by finding all the roots to polynomial (7) in $[0, 1)$. In all cases, the survival probability is unique. Though not conclusive, this is one piece of strong evidence that the RNMPE under the estimated parameter values is unique.

6.3 Policy Experiments

I conduct two policy experiments to examine the effects of policy changes on market structure: cutting the sunk cost of entry and subsidizing type-$\mathcal{L}$ stores.

The estimated average sunk costs are high compared to store values, which suggests that reducing sunk cost of entry is an effective way to encourage store entry. In the Netherlands, policy makers can efficiently achieve such reduction by abolishing the zoning regulation. To examine the impact of this policy change, I simulate 50 times the market dynamics for 10 years, using equilibrium transition probabilities used for the simulation are computed under 46 equal-distance values of $\varphi_{\mathfrak{C}}, \varphi_{\mathfrak{L}}$, between the estimated values 221.22, 30.13 and their 90%-reduced values 22.12 and 3.01. The market structure at end of 2010 are used to initialize the simulations. The other primitive values are held constant in all simulations.

Table 6 presents some statistics on the market compositions for this experiment, averaged over all the simulations for three sets of different values of $\varphi_{\mathfrak{C}}$ and $\varphi_{\mathfrak{L}}$. The first column is the benchmark case under the estimated value $\varphi_{\mathfrak{C}} = 221.22$ and $\varphi_{\mathfrak{L}} = 30.13$. The second and third column corresponds to 50% and 10% of the estimated $\varphi_{\mathfrak{C}}, \varphi_{\mathfrak{L}}$. In general, creative destruction will continue to dictate the market dynamics: local store incumbents will be gradually replaced by chain stores. For lower values of the sunk costs, such process is considerably accelerated. According to the prediction, if abolishing zoning regulation can reduce the sunk costs to 60%, the number of chain store entrants in the next 10 years will double. If the reduction is 90%, the number of chain store entrants in the next 10 years will be seven times higher than in the benchmark case, and the number of chain store incumbents in 2020 will be twice as many as the benchmark value. With the competition intensified, the market selection on chain stores is more pronounced: the fraction of type-$\mathcal{L}$ chain stores in 2020 is only 2.6% under 10% sunk costs, almost ten times lower compared to the benchmark value of 23.8%. The local stores are suppressed harder under the lower sunk costs: very few local store will survive in 2020 if the sunk costs are cut to 10%. In fact, such creative destruction process will continue beyond 2020, resulting in markets populated only by type-$\mathcal{H}$ chain stores in the long run steady state.

Figure 6 depicts the stores' net present values in 2010 and 2020 against different levels of $\varphi_{\mathfrak{C}}$ and $\varphi_{\mathfrak{L}}$. One interesting finding is that for any given value of the sunk cost parameters, although the average number of type-$\mathcal{H}$ stores is predicted to be higher in 2020 than in 2010, the active type-$\mathcal{H}$ stores' value is also higher. This is because the entry of the chain stores are predicted to primarily happen in markets populated by type-$\mathcal{L}$. In 2010, around one third of the markets only had type-$\mathcal{L}$ stores. In 2020, this percentage will decrease to 24% even when the sunk costs are not cut, and to 2.5 when the sunk costs are cut to 10%. As a result, type-$\mathcal{L}$ stores absorbing a large part of the impact from the new chain entrants. However,

$\varphi_{\mathfrak{C}}$,$\varphi_{\mathfrak{L}}$ set at the level of	100%	60%	10%
10-Year average No. chain incumbents	1.45	1.56	2.38
10-Year average No. local incumbents	0.53	0.42	0.24
No. chains in 2020	1.39	1.59	3.00
No. locals in 2020	0.44	0.24	0.04
10-Year total No. chain entrants	0.48	0.96	3.55
10-Year total No. local entrants	0.39	0.38	0.63
10-Year total No. chain exited	0.62	0.90	1.92
10-Year total No. local exited	0.54	0.73	1.17
Average % of chains are type-$\mathcal{L}$ in 2020	23.99%	13.90%	2.62%
% of markets with only type-$\mathcal{L}$ stores in 2020	21.40%	9.45%	0.69%

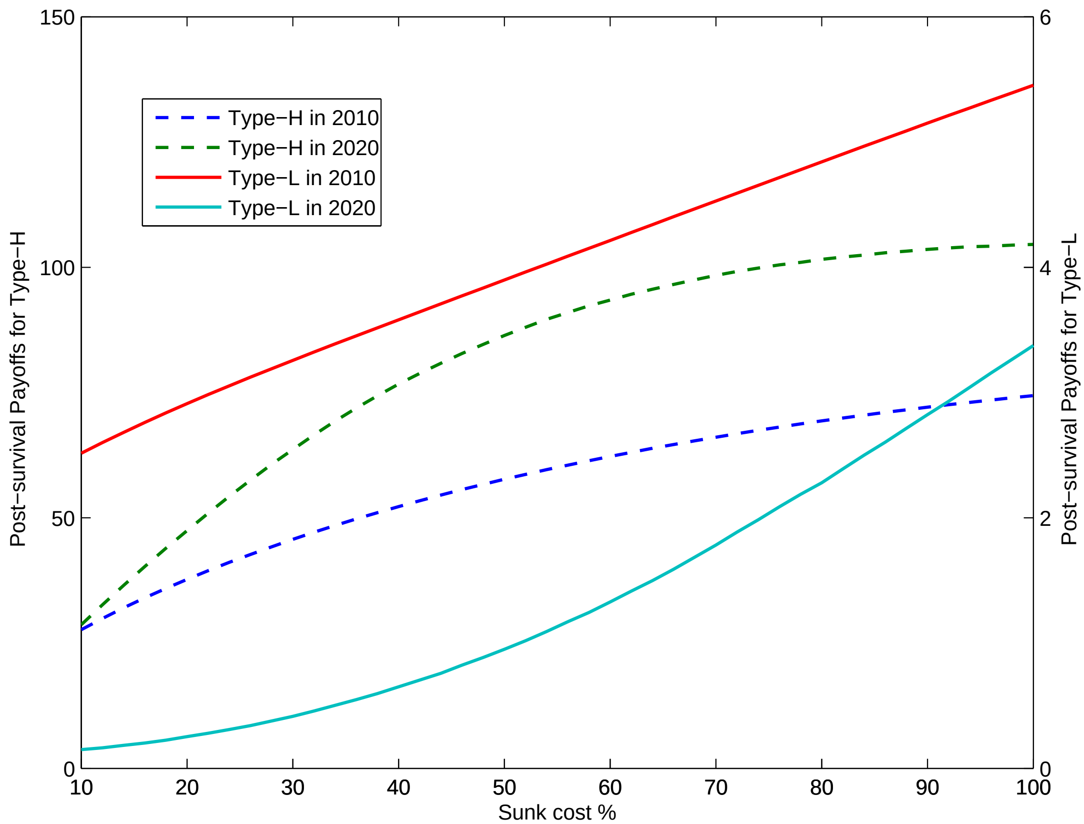
Table 6: Market Dynamics under Different $\varphi_{\mathfrak{C}}$ and $\varphi_{\mathfrak{L}}$

when $\varphi_{\mathfrak{C}}$ is cut to 10%, the fierce competition induced by chain stores' entry almost reverses this effect, and reduces the incumbent type-$\mathcal{H}$ stores' average value in 2020 to almost the same as that in 2010.

Anticipating the local stores' future, policy makers may want to provide subsidy to maintain the local stores' presence. In the Netherlands, various aid programs aiming at helping small supermarkets are available. In the second policy experiment, I consider a direct subsidy scheme–tax rebate for type-$\mathcal{L}$ stores. Like in the first experiment, I forward simulate 50 times the market evolution from 2010 to 2020, using the equilibrium transition probabilities computed under 51 equal-distance values of $\theta_{\mathcal{L}}$ between the normalized 1 (no subsidy) and 2. Recall the profit function specification in Equation (8), $\theta_{\mathcal{L}} = 2$ implies that the subsidy scheme matches every euro that a type-$\mathcal{L}$ store makes. The other primitive values are hold constant in all simulations.

Table 7 presents the market composition statistics for the second experiment, averaging over all the simulations for three different values of $\theta_{\mathcal{L}}$. The first column is the benchmark case under the normalized value $\theta_{\mathcal{L}} = 1$. The second and third column corresponds to 50% subsidy and the match-every-euro subsidy program respectively. Increased subsidy encourages local stores' participation. A 50% subsidy increases the number of local store entrants almost by three folds, and reduces their exits by 13%. In 2020, with 50% subsidy, the average number of incumbent local stores in each market will exceed 1. With 100% subsidy, this number will exceed 2.

Figure 6: Average Post-Survival Payoffs under Different $\varphi_{\mathfrak{C}}$ and $\varphi_{\mathfrak{L}}$

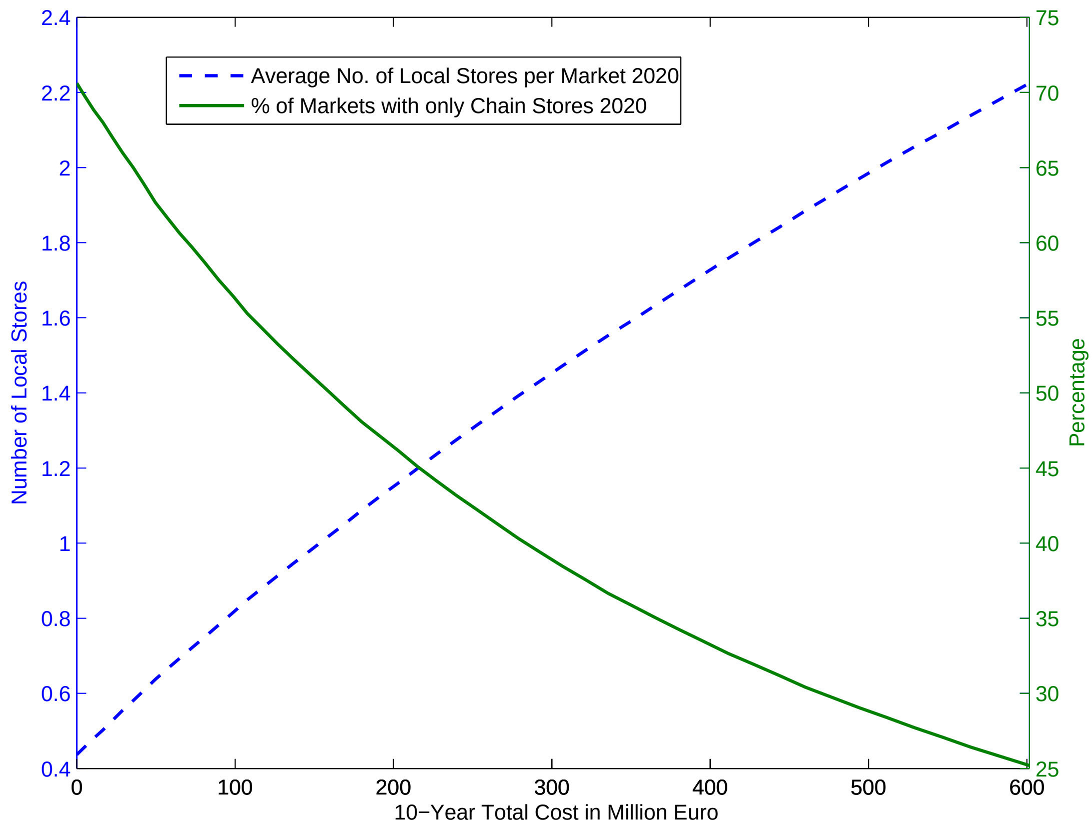


$\theta_{\mathcal{L}}$ set at the level of	100%	150%	200%
10-Year average No. chain incumbents	1.45	1.58	1.63
10-Year average No. local incumbents	0.52	0.99	1.56
No. chains in 2020	1.39	1.60	1.67
No. locals in 2020	0.44	1.20	2.22
10-Year total No. chain entrants	0.48	0.41	0.32
10-Year total No. local entrants	0.39	1.09	2.03
10-Year total No. chain exited	0.62	0.35	0.18
10-Year total No. local exited	0.54	0.47	0.39
% of markets with only locals in 2020	8.82%	9.52%	11.01%
% of markets with only chains in 2020	70.62%	45.08%	25.20%

Table 7: Market Dynamics under Different $\theta_{\mathcal{L}}$

Though such tax rebate program is effective in preserving the local stores, it can be costly to implement. Figure 7 is to help policy makers with the budgetary planning. To achieve the target of on average one local store per market in 2020, government will need to provide a 10-year aid package valued 150 million Euro. Limiting the percentage of markets without a single local store to around 30% in 2020 will require investing a sizable sum of almost 500 million Euro.

Figure 7: Policy Targets v.s. Budget



7 Discussion and Conclusion

In this paper, I develop and estimate a tractable oligopoly model for Dutch retail grocery industry. The chain stores' domination over their local rivals is estimated to be sizable. This quantitatively explains the creative destruction process at work in this industry. Indeed, in the long run steady state of the model, almost all local stores are predicted to exit the market.

The attractive features of this model are threefold. First, market-level data of local demand and store entry and exit are sufficient for the estimation. Store-level performance data, which are often hard to acquire due to stores' confidentiality policies, are not required. Second, clear-cut results on equilibrium existence, uniqueness, and computation ensure the reliability of the estimates and the counterfactual analysis. Third, the light computational burden allows counterfactual analysis of many policy alternatives at a very low cost.

Much of the model's simplicity arises from the contraction property of the Bellman equations for equilibrium payoffs. The proposed model can be extended in the following ways to accommodate more complex dynamics, while still retaining such a contraction property and leaving the central equilibrium existence, computation, and uniqueness results intact. First, more store-level heterogeneity can be accounted for by allowing more than two store formats and/or profitability types. More store formats can better capture, for instance, the size differences among stores. With more than two profitability types, the natural equilibrium requires higher type stores to continue for sure, if lower type stores survive with positive probability. Second, more market-level heterogeneity can be accounted for by allowing store profit to depend on first-order Markovian (income, household size, etc.) or time-invariant (region, ethic composition, etc.) market characteristics other than demand. The downside of such complications is the increased computational burden. With extra Markovian variables, the model has a larger state space. With market fixed-effects, distinct contraction mappings define the equilibrium payoffs for markets with different characteristics. Nevertheless, modern computer power and parallel computing techniques ensure that even with 877 distinct markets, the model can be estimated within reasonable time.

References

ABBRING, J., J. CAMPBELL, AND N. YANG (2010): "Simple Markov-perfect industry dynamics," *Federal Reserve Bank of Chicago Working Paper Series*. 2, 3, 22, 54, 55

ABBRING, J. H., AND J. R. CAMPBELL (2010): "Last-In First-Out Oligopoly Dynamics," *Econometrica*, 78(5), 1491–1527. 1

AGUIRREGABIRIA, V., AND P. MIRA (2007): "Sequential Estimation of Dynamic Discrete Games," *Econometrica*, 75(1), 1–53. 2

BAJARI, P., C. BENKARD, AND J. LEVIN (2007): "Estimating Dynamic Models of Imperfect Competition," *Econometrica*, 75(5), 1331–1370. 2

BERRY, S. (1992): "Estimation of a Model of Entry in the Airline Industry," *Econometrica*, 60(4), 889–917. 1, 3

BESANKO, D., U. DORASZELSKI, Y. KRYUKOV, AND M. SATTERTHWAITE (2010): "Learning-by-Doing, Organizational Forgetting, and Industry Dynamics," *Econometrica*, 78(2), 453–508. 2

BRESNAHAN, T. F., AND P. C. REISS (1990): "Entry in Monopoly Markets," *Review of Economic Studies*, 57(4), 531–553. 1, 3

——— (1991): "Entry and Competition in Concentrated Markets," *Journal of Political Economy*, 99(5), 977–1009. 9

——— (1993): "Measuring the Importance of Sunk Costs," *Annales d'Economie et de Statistique*, 31, 181–217. 3

CABRAL, L. M. (1993): "Experience Advantages and Entry Dynamics," *Journal of Economic Theory*, 59, 403–416. 22

COLLARD-WEXLER, A. (2010): "Demand Fluctuations in the Ready-Mix Concrete Industry," *Manuscript. New York University*. 2

ERICSON, R., AND A. PAKES (1995): "Markov-Perfect Industry Dynamics: A Framework for Empirical Work," *Review of Economic Studies*, 62, 53–82. 2

FUDENBERG, D., AND J. TIROLE (1991): *Game Theory*. MIT Press, Cambridge, MA. 17

Gowrisankaran, G., C. Lucarelli, P. Schmidt-Dengler, and R. Town (2010): "Government Policy and the Dynamics of Market Structure: Evidence from Critical Access Hospitals," *Manuscript. University of Arizona.* 2

Holmes, T. (2011): "The Diffusion of Wal-Mart and Economies of Density," *Econometrica*, 79(1), 253–302. 4, 6

Jia, P. (2008): "What Happens When Wal-Mart Comes to Town: An Empirical Analysis of the Discount Retailing Industry," *Econometrica*, 76(6), 1263–1316. 4, 6

Mazzeo, M. (2002): "Product Choice and Oligopoly Market Structure," *RAND Journal of Economics*, pp. 221–242. 1

Pakes, A., M. Ostrovsky, and S. Berry (2007): "Simple Estimators for the Parameters of Discrete Dynamic Games, with Entry/Exit Samples," *RAND Journal of Economics*, 28(3), 373–399. 2

Pesendorfer, M., and P. Schmidt-Dengler (2008): "Asymptotic Least Squares Estimators for Dynamic Games," *Review of Economic Studies*, 75(3), 901–928. 2

Rust, J. (1987): "Optimal Replacement of GMC Bus Engines: An Empirical Model of Harold Zurcher," *Econometrica*, 55, 999–1033. 2, 3, 15, 32

Ryan, S. (forthcoming): "The Costs of Environmental Regulation in a Concentrated Industry," *Econometrica.* 2

Seim, K. (2006): "An Empirical Model of Firm Entry with Endogenous Product-type Choices," *RAND Journal of Economics*, 37(3), 619–640. 1

Van Lin, A., and E. Gijsbrechts (2011): "Shopper Loyalty to Whom? On Store Acquisitions, Outlet Loyalty, and Store Performance," *Manuscript. Tilburg University.* 3, 7

Weintraub, G., C. Benkard, and B. Roy (2008): "Markov Perfect Industry Dynamics With Many Firms," *Econometrica*, 76(6), 1375–1411. 2

Xu, Y. (2008): "A Structural Empirical Model of R&D, Firm Heterogeneity and Industry Evolution," *Manuscript. New York University.* 2

Appendices

A Computational Details for the Duopoly Example in Section 4.1

In this appendix, I supplement Section 4.1 with more details on the five-step procedure computing the RNMPE.

Step 1: Duopoly Market with Two Type-$\mathcal{H}$ Retailers For some distributions of W_M, the expectation over W_M in Equation (6) has a closed-form expression. For instance, if W_M is assumed to be independent of C and normally distributed with mean 0 and variance $\sigma_{\mathfrak{C}}^2$, then,

$$
\begin{aligned}
& v^E(2\iota_{\mathcal{H}}, c, \mathcal{H}) \\
=& Prob\left(v^S\left(2\iota_{\mathcal{H}}, c, \mathcal{H}\right) > \varphi_M \exp(W_M')\right)\left(v^S\left(2\iota_{\mathcal{H}}, c, \mathcal{H}\right) - \int_{-\infty}^{\log\max\left\{0, v^S(2\iota_{\mathcal{H}}, c, \mathcal{H})\right\}} \varphi_M \exp(W_M')\phi(W_M')dW_M'\right) \\
=& \Phi\left(\frac{\log\max\left\{0, v^S\left(2\iota_{\mathcal{H}}, c, \mathcal{H}\right)\right\} - \log\varphi_M}{\sigma_M}\right) v^S\left(2\iota_{\mathcal{H}}, c, \mathcal{H}\right) \\
& - \exp(\sigma_M^2/2)\Phi\left(\frac{\log\max\left\{0, v^S\left(2\iota_{\mathcal{H}}, c, \mathcal{H}\right)\right\} - \log\varphi_M - \sigma_M^2}{\sigma_M}\right),
\end{aligned}
$$

where Φ is the c.d.f. for standard normal distribution. In this expression, $\log\max\left\{0, v^S\left(2\iota_{\mathcal{H}}, c, \mathcal{H}\right)\right\} - \log\varphi_M$ is the "ceiling" value of W_M to ensure profitable continuation for a type-$\mathcal{H}$ incumbent. The expectation over W_M is hence computed only on the interval $(-\infty, \log\max\left\{0, v^S\left(2\iota_{\mathcal{H}}, c, \mathcal{H}\right)\right\} - \log\varphi_M)$. By using this expression to compute $v^E(2\iota_{\mathcal{H}}, c, \mathcal{H})$, one can avoid the numerical integration over W_M. This is one of the major consequence and benefit of Assumption 2. In the remaining part of the duopoly example, as well as in the empirical implementation of the model, I maintain the normality assumption on W_M.

Step 2: Duopoly Market with Both Types of Retailers. Next, consider a type-$\mathcal{L}$ retailer who faces a type-$\mathcal{H}$ competitor. In a natural MPE, this retailer's survival implies the survival of the type-$\mathcal{H}$ rival. Following its survival, a chain store may further enter in next period, and regardless of this chain store's realized type, the type-$\mathcal{L}$ retailer receives zero continuation value in next period. Equation (1) defines $v^E(\iota_{\mathcal{L}}+\iota_{\mathcal{H}}, C, \mathcal{L})$ as

$$\begin{aligned} v^E(\iota_{\mathcal{L}}+\iota_{\mathcal{H}}, c, \mathcal{L}) =& \mathbb{E}_{W_M}[\max\{0, \beta\mathbb{E}_C[\pi_{\mathcal{L}}(\iota_{\mathcal{L}}+\iota_{\mathcal{H}}, C') - \varphi_M \exp(W'_M) \\ &+ (1-P^E(\iota_{\mathfrak{C}}+\iota_{\mathfrak{L}}|\iota_{\mathcal{L}}+\iota_{\mathcal{H}}, C'))v^E(\iota_{\mathcal{L}}+\iota_{\mathcal{H}}, C', \mathcal{L}) \,|\, C=c]\}]. \end{aligned}$$

Given the value of $P^E(\iota_{\mathfrak{C}}+\iota_{\mathfrak{L}}|\iota_{\mathcal{L}}+\iota_{\mathcal{H}}, C)$ for all C, the right-hand side of the equation defines a contraction mapping. Its unique fixed point determines $v^E(\iota_{\mathcal{L}}+\iota_{\mathcal{H}}, c, \mathcal{L})$. Using Equation (2), the associated post-survival payoff $v^S(\iota_{\mathcal{L}}+\iota_{\mathcal{H}}, c, \mathcal{L})$ can be quickly computed.

Next, consider a type-$\mathcal{L}$ retailer who faces a type-$\mathcal{H}$ competitor. In a natural MPE, this retailer's survival implies the survival of the type-$\mathcal{H}$ rival. Moreover, no retailer will further enter this market following its survival, given the formidable sunk cost of entry. Equation (1) defines $v^E(\iota_{\mathcal{L}}+\iota_{\mathcal{H}}, C, \mathcal{L})$ as

$$v^E(\iota_{\mathcal{L}}+\iota_{\mathcal{H}}, c, \mathcal{L}) = \mathbb{E}_{W_M}\left[\max\{0, v^S\left(\iota_{\mathcal{L}}+\iota_{\mathcal{H}}, c, \mathcal{L}\right) - \varphi_M \exp(W'_M)\}\right] \tag{9}$$

in which $v^S\left(\iota_{\mathcal{L}}+\iota_{\mathcal{H}}, c, \mathcal{L}\right) = \beta\mathbb{E}_C\left[\pi_{\mathcal{L}}(\iota_{\mathcal{L}}+\iota_{\mathcal{H}}, C') + v^E(\iota_{\mathcal{L}}+\iota_{\mathcal{H}}, C', \mathcal{L}) \,\middle|\, C=c\right]$. Again, the right-hand side of the equation defines a contraction mapping. Its unique fixed point determines $v^E(\iota_{\mathcal{L}}+\iota_{\mathcal{H}}, c, \mathcal{L})$, and subsequently $v^S(\iota_{\mathcal{L}}+\iota_{\mathcal{H}}, c, \mathcal{L})$. Again, the expectation over W_M on the interval $(-\infty, \log(v^S\left(\iota_{\mathcal{L}}+\iota_{\mathcal{H}}, c, \mathcal{L}\right)) - \log\varphi_M)$ has a closed-form expression.

Then, the entry rule to a market occupied by a type-$\mathcal{H}$ monopolist and the survival rule for a type-$\mathcal{L}$ retailer facing a type-$\mathcal{H}$ rival are determined as

$$\begin{aligned} a^E(\iota_{\mathcal{H}}, c, w_{\mathfrak{C}}, \mathfrak{C}) &= I\{\omega v^E\left(2\iota_{\mathcal{H}}, c, \mathcal{H}\right) + (1-\omega)v^E\left(\iota_{\mathcal{L}}+\iota_{\mathcal{H}}, c, \mathcal{L}\right) > \varphi_{\mathfrak{C}}\exp(w_{\mathfrak{C}})\}, \\ a^E(\iota_{\mathcal{H}}, c, w_{\mathfrak{L}}, \mathfrak{L}) &= I\{v^E(\iota_{\mathcal{L}}+\iota_{\mathcal{H}}, c, \mathcal{L}) > \varphi_{\mathfrak{L}}\exp(w_{\mathfrak{L}})\}, \\ a^S(\iota_{\mathcal{L}}+\iota_{\mathcal{H}}, c, w_M, \mathcal{L}) &= I\{v^S(\iota_{\mathcal{L}}+\iota_{\mathcal{H}}, c, \mathcal{L}) > \varphi_M\exp(w_M)\}. \end{aligned}$$

By imposing distributional assumptions on $w_{\mathfrak{C}}, w_{\mathfrak{L}}$, I can compute the *(joint) choice probability* for a chain store or a local store to enter when the market is monopolized by a type-$\mathcal{H}$ retailer, the demand is c, and the shocks on sunk costs are integrated out. Denote these probabilities by $P^E(\iota_{\mathfrak{C}}|\iota_{\mathcal{H}}, c)$ and $P^E(\iota_{\mathfrak{L}}|\iota_{\mathcal{H}}, c)$. In this duopoly example and in the empirical implementation, I assume that $w_{\mathfrak{C}}$ and $w_{\mathfrak{L}}$ independently and normally distributed, with mean 0 and standard deviations $\sigma_{\mathfrak{C}}, \sigma_{\mathfrak{L}}$ respectively. Then,

$$\begin{aligned} P^E(\iota_{\mathfrak{C}}|\iota_{\mathcal{H}}, c) &= Prob(a^E(\iota_{\mathcal{H}}, c, w_{\mathfrak{C}}, \mathfrak{C}) = 1) \\ &\quad = \Phi\left(\left(\log\left(\omega v^E\left(2\iota_{\mathcal{H}}, c, \mathcal{H}\right) + (1-\omega)v^E\left(\iota_{\mathcal{L}}+\iota_{\mathcal{H}}, c, \mathcal{L}\right)\right) - \log\varphi_{\mathfrak{C}}\right)/\sigma_{\mathfrak{C}}\right), \\ P^E(\iota_{\mathfrak{L}}|\iota_{\mathcal{H}}, c) &= Prob(a^E(\iota_{\mathcal{H}}, c, w_{\mathfrak{L}}, \mathfrak{L}) = 1) = \Phi\left(\left(\log\left(v^E\left(\iota_{\mathcal{L}}+\iota_{\mathcal{H}}, c, \mathcal{L}\right)\right) - \log\varphi_{\mathfrak{C}}\right)/\sigma_{\mathfrak{L}}\right) \end{aligned}$$

Because at most one extra retailer can rationally enter this market in a RNMPE, the entry and the subsequent type realization outcome leads to one out of three possible post-entry market structures with non-trivial probabilities. A function $\mathbb{P}^E(\mathbf{m}_E|\mathbf{m}_S, c, a^E)$ computes the *transition probability* of entry for the post-entry market structure to become $\mathbf{m}_E$, when the pre-entry market structure is $\mathbf{m}_S$, the demand is c, and the entry rule is given by a^E. With $P^E(\iota_{\mathfrak{C}}|\iota_{\mathcal{H}}, c)$ and $P^E(\iota_{\mathfrak{L}}|\iota_{\mathcal{H}}, c)$ computed as the functions of the entry rules, the transition probabilities for the pre-entry market with one type-$\mathcal{H}$ monopolist are compactly expressed as

$$\begin{aligned}
&\mathbb{P}^E(2\iota_{\mathcal{H}}|\iota_{\mathcal{H}}, c, a^E) = \omega P^E(\iota_{\mathfrak{C}}|\iota_{\mathcal{H}}, c),\\
&\mathbb{P}^E(\iota_{\mathcal{L}} + \iota_{\mathcal{H}}|\iota_{\mathcal{H}}, c, a^E) = (1-\omega)P^E(\iota_{\mathfrak{C}}|\iota_{\mathcal{H}}, c) + P^E(\iota_{\mathfrak{L}}|\iota_{\mathcal{H}}, c),\\
&\mathbb{P}^E(\iota_{\mathcal{H}}|\iota_{\mathcal{H}}, c, a^E) = (1 - P^E(\iota_{\mathfrak{C}}|\iota_{\mathcal{H}}, c))(1 - P^E(\iota_{\mathfrak{L}}|\iota_{\mathcal{H}}, c)).
\end{aligned}$$

Let function $\mathbb{P}^S(\mathbf{m}_S|\mathbf{m}_E, c, a^S)$ defines the transition probability of survival for the post-survival market to become $\mathbf{m}_S$, when the post-entry market is $\mathbf{m}_E$, the demand is c, and the survival rule is defined by a^S. Because the type-$\mathcal{H}$ retailer never exits before the type-$\mathcal{L}$ rival, the probability for market structure $\iota_{\mathcal{L}} + \iota_{\mathcal{H}}$ to remain solely relies on the type-$\mathcal{L}$ retailer's choice. Hence,

$$\mathbb{P}^S(\iota_{\mathcal{L}}+\iota_{\mathcal{H}}|\iota_{\mathcal{L}}+\iota_{\mathcal{H}}, c, a^S) = Prob(a^S(\iota_{\mathcal{L}}+\iota_{\mathcal{H}}, c, w_M, \mathcal{L}) = 1) = \Phi\left(\log\max\left\{0, v^S\left(\iota_{\mathcal{L}}+\iota_{\mathcal{H}}, c, \mathcal{L}\right)\right\}/\sigma_M\right).$$

For econometrician who does not observe the shocks $w_{\mathfrak{L}}, w_{\mathfrak{C}}$ and w_M, the transition probabilities and the choice probabilities are essential in forming the expectations in the equilibrium payoff function v_E's computation, and in building the likelihood function towards recovering Θ.

Step 3: Type-$\mathcal{H}$ Monopolist & Type-$\mathcal{H}$ Duopolist Facing One Type-$\mathcal{L}$ Rival. The post-entry payoff for a type-$\mathcal{H}$ monopolist, $v^E(\iota_{\mathcal{H}}, \cdot, \mathcal{H})$, is

$$\begin{aligned}
v^E(\iota_{\mathcal{H}}, c, \mathcal{H}) &= \mathbb{E}_{W_M}\left[\max\{0, v^S\left(\iota_{\mathcal{H}}, c, \mathcal{L}\right) - \varphi_M\exp(W'_M)\}\right],\\
&= \Phi\left(\frac{\log v^S\left(\iota_{\mathcal{H}}, c, \mathcal{H}\right) - \log\varphi_M}{\sigma_M}\right)v^S\left(\iota_{\mathcal{H}}, c, \mathcal{H}\right) - \exp(\sigma_M^2/2)\Phi\left(\frac{\log v^S\left(2\iota_{\mathcal{H}}, c, \mathcal{H}\right) - \log\varphi_M - \sigma_M^2}{\sigma_M}\right),
\end{aligned} \tag{10}$$

where $\log(v^S\left(\iota_{\mathcal{H}}, c, \mathcal{H}\right))$ is the ceiling value of W_M to ensure profitable continuation for the type-$\mathcal{H}$ incumbent.

If this incumbent faces a type-$\mathcal{L}$ rival in the post-entry market, the transition probability $\mathbb{P}^S(\iota_\mathcal{L}+\iota_\mathcal{H}|\iota_\mathcal{L}+\iota_\mathcal{H},c,a^S)$ gives the likelihood that this type-$\mathcal{L}$ retailer rationally chooses to leave the market. The post-entry payoff for the type-$\mathcal{H}$ retailer is

$$v^E(\iota_\mathcal{L}+\iota_\mathcal{H},c,\iota_\mathcal{H}) = (1-\mathbb{P}^S(\iota_\mathcal{L}+\iota_\mathcal{H}|\iota_\mathcal{L}+\iota_\mathcal{H},c,a^S))\mathbb{E}_{W_M}[\max\{0, v^S\left(\iota_\mathcal{H},c,\iota_\mathcal{H}\right) - \varphi_M \exp(W_M')\}] \\ + \mathbb{P}^S(\iota_\mathcal{L}+\iota_\mathcal{H}|\iota_\mathcal{L}+\iota_\mathcal{H},c,a^S)\left(v^S\left(\iota_\mathcal{L}+\iota_\mathcal{H},c,\iota_\mathcal{H}\right) - \mathbb{E}_{W_M}[\exp(W_M')|v^S(\iota_\mathcal{L}+\iota_\mathcal{H},c,\mathcal{L}) > \varphi_M\exp(W_M')]\right). \tag{11}$$

Because one more retailer may join the monopoly market, the post-survival payoffs in Equations (10) depends on the transition probabilities of entry defined by $\mathbb{P}^E$.

$$v^S\left(\iota_\mathcal{H},c,\mathcal{H}\right) = \beta\mathbb{E}_C[\pi_\mathcal{H}(\iota_\mathcal{H},C') + \mathbb{P}^E(2\iota_\mathcal{H}|\iota_\mathcal{H},C',a^E)v^E(2\iota_\mathcal{H},C',\mathcal{H}) \\ + \mathbb{P}^E(\iota_\mathcal{L}+\iota_\mathcal{H}|\iota_\mathcal{H},C',a^E)v^E(\iota_\mathcal{L}+\iota_\mathcal{H},C',\mathcal{H}) + \mathbb{P}^E(\iota_\mathcal{H}|\iota_\mathcal{H},C',a^E)v^E(\iota_\mathcal{H},C',\mathcal{H})|C=c].$$

Since entry is not possible in the duopoly market, the post-survival payoff in Equations (11) is

$$v^S\left(\iota_\mathcal{L}+\iota_\mathcal{H},c,\mathcal{H}\right) = \beta\mathbb{E}_C[\pi_\mathcal{H}(\iota_\mathcal{L}+\iota_\mathcal{H},C') + v^E(\iota_\mathcal{L}+\iota_\mathcal{H},C',\mathcal{H})|C=c].$$

Given that the relevant transition probabilities in Equations (10) and (11) have been determined, the expectations in the above post-survival payoffs are taken only over exogenously evolving variables C. In addition, the post-entry payoff $v^E(2\iota_\mathcal{H},\cdot,\mathcal{H})$ has been determined in Step 1. Therefore, Equations (10) and (11) together define a contraction mapping with its fixed point determining $v^E(\iota_\mathcal{H},\cdot,\mathcal{H})$ and $v^E(\iota_\mathcal{L}+\iota_\mathcal{H},\cdot,\mathcal{H})$. Obtaining $v^S(\iota_\mathcal{H},\cdot,\mathcal{H})$ and $v^S(\iota_\mathcal{L}+\iota_\mathcal{H},\cdot,\mathcal{H})$ from there is straightforward.

The continuation decision for the type-$\mathcal{H}$ monopolist is a single-agent problem. Hence, the survival rule for this retailer is $a^S(\iota_\mathcal{H},c,w_M,\mathcal{H}) = I\{v^S(\iota_\mathcal{H},c,\mathcal{H}) > \varphi_M\exp(w_M)\}$. Under the normality assumption, the associated choice probability $P^S(\iota_\mathcal{H}|\iota_\mathcal{H},c,\mathcal{H})$ and the transition probabilities $\mathbb{P}^S(\iota_\mathcal{H}|\iota_\mathcal{H},c,a^S)$ and $\mathbb{P}^S(\iota_0|\iota_\mathcal{H},c,a^S)$ follow immediately.

The continuation decision for the type-$\mathcal{H}$ duopolist may depend on the type-$\mathcal{L}$ rival's choice. If $v^S(\iota_\mathcal{H},c,\mathcal{H}) < v^S(\iota_\mathcal{H}+\iota_\mathcal{L},c,\mathcal{L})$, after the type-$\mathcal{L}$ rival's exit, the type-$\mathcal{H}$ retailer's solo continuation becomes unprofitable. This situation arises because the type-$\mathcal{L}$ retailer's continuation deters future entry by a chain store. Because the new chain store entrant is likely to become another type-$\mathcal{H}$ retailer, the incumbent type-$\mathcal{H}$ retailer is better off if continuing with the type-$\mathcal{L}$ rival than without. Hence, this type-$\mathcal{H}$ incumbent's continuation decision is dictated by the type-$\mathcal{L}$ rival's payoff $v^S(\iota_\mathcal{H}+\iota_\mathcal{L},c,\mathcal{L})$. If $v^S(\iota_\mathcal{H},c,\mathcal{H}) > v^S(\iota_\mathcal{H}+\iota_\mathcal{L},c,\mathcal{L})$, the type-$\mathcal{H}$'s continuation decision is unaffected by the type-$\mathcal{L}$ rival's choice. Combining these two cases, the survival rule for the type-$\mathcal{H}$ duopolist is determined as

$$a^S(\iota_{\mathcal{H}} + \iota_{\mathcal{L}}, c, w_M, \mathcal{H}) = I\{\max\{v^S(\iota_{\mathcal{H}}, c, \mathcal{H}), v^S(\iota_{\mathcal{H}}, c, \mathcal{L})\} > \varphi_M \exp(w_M)\}.$$

The associated choice probability $P^S(\iota_{\mathcal{H}}+\iota_{\mathcal{L}}, c, \mathcal{H})$ and the transition probabilities $\mathbb{P}^S(\iota_{\mathcal{H}}|\iota_{\mathcal{H}}+\iota_{\mathcal{L}}, c, a^S)$ and $\mathbb{P}^S(\iota_0|\iota_{\mathcal{H}} + \iota_{\mathcal{L}}, c, a^S)$ are easily obtained under the normality assumption. In light of how the type-$\mathcal{L}$'s presence affects the continuation of the type-$\mathcal{H}$ retailer, the ceiling value of W_M for the type-$\mathcal{H}$ incumbent's survival in Equation (11) is hence determined by $\log\max\{0, v^S(\iota_{\mathcal{H}}, c, \mathcal{H}), v^S(\iota_{\mathcal{L}} + \iota_{\mathcal{H}}, c, \mathcal{L})\}$. Under the normality specification, the expectation over W_M still has a closed-form expression.

Step 4: Duopoly Market with Two Type-$\mathcal{L}$ Retailers. The survival problem on a duopoly market with two type-$\mathcal{L}$ retailers is a carbon copy of the static game presented in Figure 3. In a RNMPE, if simultaneous survival is individually profitable, then retailers will both choose to survive. Otherwise, the strategy in the static game assign non-negative probability to "Exit", and results in zero expected payoff. Therefore, $v^E(2\iota_{\mathcal{L}}, \cdot, \mathcal{L})$ satisfies

$$\begin{aligned} v^E(2\iota_{\mathcal{L}}, c, \mathcal{L}) &= \mathbb{E}_{W_M}\left[\max\{0, v^S(2\iota_{\mathcal{L}}, c, \mathcal{L}) - \exp(W'_M)\} \,\middle|\, C = c\right] \\ &= \Phi\left(\frac{\log v^S(2\iota_{\mathcal{L}}, c, \mathcal{L}) - \log\varphi_M}{\sigma_M}\right) v^S(2\iota_{\mathcal{L}}, c, \mathcal{L}) - \exp(\sigma_M^2/2)\Phi\left(\frac{\log v^S(2\iota_{\mathcal{L}}, c, \mathcal{L}) - \log\varphi_M - \sigma_M^2}{\sigma_M}\right), \end{aligned}$$

in which $v^S(2\iota_{\mathcal{L}}, c, \mathcal{L}) = \beta\mathbb{E}_C\left[\pi_{\mathcal{L}}(2\iota_{\mathcal{L}}, C') + v^E(2\iota_{\mathcal{L}}, C', \mathcal{L}) \,\middle|\, C = c\right]$.

Similar to Equation (6), the necessary condition for $v^E(2\iota_{\mathcal{L}}, \cdot, \mathcal{L})$ defines a contraction mapping, with its fixed point determining $v^E(2\iota_{\mathcal{L}}, \cdot, \mathcal{L})$. Consequently, $v^S(2\iota_{\mathcal{L}}, c, \mathcal{L})$ can be computed.

Then, the entry rule to a market occupied by a type-$\mathcal{L}$ monopolist are determined as

$$\begin{aligned} a^E(\iota_{\mathcal{L}}, c, w_{\mathfrak{C}}, \mathfrak{C}) &= I\{\omega v^E(\iota_{\mathcal{L}} + \iota_{\mathcal{H}}, c, \mathcal{H}) + (1-\omega)v^E(2\iota_{\mathcal{L}}, c, \mathcal{L}) > \varphi_{\mathfrak{C}}\exp(w_{\mathfrak{C}})\}, \\ a^E(\iota_{\mathcal{L}}, c, w_{\mathfrak{L}}, \mathfrak{L}) &= I\{v^E(2\iota_{\mathcal{L}}, c, \mathcal{L}) > \varphi_{\mathfrak{L}}\exp(w_{\mathfrak{L}})\} \end{aligned}$$

The associated choice probabilities are

$$P^E(\iota_{\mathfrak{C}}|\iota_{\mathcal{L}}, c) = \Phi\left((\log\left(\omega v^E(\iota_{\mathcal{L}} + \iota_{\mathcal{H}}, c, \mathcal{H}) + (1-\omega)v^E(2\iota_{\mathcal{L}}, c, \mathcal{L})\right) - \log\varphi_{\mathfrak{C}})/\sigma_{\mathfrak{C}}\right),$$

$$P^E(\iota_{\mathfrak{L}}|\iota_{\mathcal{L}}, c) = \Phi\left(\log v^E(2\iota_{\mathcal{L}}, c, \mathcal{L} - \log\varphi_{\mathfrak{L}})/\sigma_{\mathfrak{L}}\right)$$

The transition probability for the post-entry market structure are

$$\mathbb{P}^E(\iota_{\mathcal{L}} + \iota_{\mathcal{H}}|\iota_{\mathcal{L}}, c, a^E) = \omega P^E(\iota_{\mathfrak{C}}|\iota_{\mathcal{H}}, c),$$

$$\mathbb{P}^E(2\iota_{\mathcal{L}}|\iota_{\mathcal{L}}, c, a^E) = (1-\omega)P^E(\iota_{\mathfrak{C}}|\iota_{\mathcal{L}}, c) + P^E(\iota_{\mathfrak{L}}|\iota_{\mathcal{L}}, c),$$

$$\mathbb{P}^E(\iota_{\mathcal{L}}|\iota_{\mathcal{L}}, c, a^E) = (1 - P^E(\iota_{\mathfrak{C}}|\iota_{\mathcal{L}}, c))(1 - P^E(\iota_{\mathfrak{L}}|\iota_{\mathcal{L}}, c)).$$

Step 5: The Rest. A type-$\mathcal{L}$ monopolist's survival decision depends on if entry happens next period. The likelihood of entry is given by $P^E(\iota_{\mathfrak{C}}|\iota_{\mathcal{L}}, c)$ and $P^E(\iota_{\mathfrak{L}}|\iota_{\mathcal{L}}, c)$. Therefore, the post-entry value satisfies

$$\begin{aligned}
&v^E(\iota_{\mathcal{L}}, c, \mathcal{L}) \\
&= \mathbb{E}_{W_M}\left[\max\{0, v^S(\iota_{\mathcal{L}}, c, \mathcal{L}) - \exp(W_M')\} \,\middle|\, C = c\right] \\
&= \Phi\left(\frac{\log v^S(\iota_{\mathcal{L}}, c, \mathcal{L}) - \log\varphi_M}{\sigma_M}\right) v^S(\iota_{\mathcal{L}}, c, \mathcal{L}) - \exp(\sigma_M^2/2)\Phi\left(\frac{\log v^S(\iota_{\mathcal{L}}, c, \mathcal{L}) - \log\varphi_M - \sigma_M^2}{\sigma_M}\right),
\end{aligned} \tag{12}$$

in which

$$\begin{aligned}
v^S(\iota_{\mathcal{L}}, c, \mathcal{L}) = \beta\mathbb{E}_C[&\pi_{\mathcal{L}}(\iota_{\mathcal{L}}, C') + \mathbb{P}^E(\iota_{\mathcal{L}} + \iota_{\mathcal{H}}|\iota_{\mathcal{L}}, C', a^E)v^E(\iota_{\mathcal{L}} + \iota_{\mathcal{H}}, C', \mathcal{L}) \\
&+ \mathbb{P}^E(2\iota_{\mathcal{L}}|\iota_{\mathcal{L}}, C', a^E)v^E(2\iota_{\mathcal{L}}, C', \mathcal{L}) + \mathbb{P}^E(\iota_{\mathcal{L}}|\iota_{\mathcal{L}}, C', a^E)v^E(\iota_{\mathcal{L}}, C', \mathcal{L})|C = c].
\end{aligned}$$

Given the quantities calculated in Steps 1–4, the right-hand side of (12) defines a contraction mapping with $v^E(\iota_{\mathcal{L}}, \cdot, \mathcal{L})$ as its fixed point. With this, $v^S(\iota_{\mathcal{L}}, \cdot, \mathcal{L})$ follows. The entry rule to an empty market by a chain store depends on whether the local store following this chain store in the entry sequence enters. The local store's entry choice is characterized by the entry rules to a monopoly market, which have been determined in Step 2 and 4. Hence,

$$\begin{aligned}
a^E(\iota_0, c, w_{\mathfrak{C}}, \mathfrak{C}) = I\{&\omega\mathbb{E}_{W_{\mathfrak{L}}}((a^E(\iota_{\mathcal{H}}, c, w_{\mathfrak{L}}, \mathfrak{L})v^E(\iota_{\mathcal{L}} + \iota_{\mathcal{H}}, c, \mathcal{H}) + (1 - a^E(\iota_{\mathcal{H}}, c, w_{\mathfrak{L}}, \mathfrak{L}))v^E(\iota_{\mathcal{H}}, c, \mathcal{H})) \\
&+ (1-\omega)(a^E(\iota_{\mathcal{H}}, c, w_{\mathfrak{L}}, \mathfrak{L}))v^E(2\iota_{\mathcal{L}}, c, \mathcal{L}) + (1 - a^E(\iota_{\mathcal{H}}, c, w_{\mathfrak{L}}, \mathfrak{L}))v^E(\iota_{\mathcal{L}}, c, \mathcal{L})) > \varphi_{\mathfrak{C}}\exp(w_{\mathfrak{C}}))\}.
\end{aligned}$$

Because $W_{\mathfrak{C}}$ and $W_{\mathfrak{L}}$ are independent, the associated choice probability of entry is

$$\begin{aligned}
P^E(\iota_{\mathfrak{C}}|\iota_0, c) = \Phi\Big(\Big(&\log\Big(\omega(P^E(\iota_{\mathfrak{L}}|\iota_{\mathcal{H}}, c)v^E(\iota_{\mathcal{L}} + \iota_{\mathcal{H}}, c, \mathcal{H}) + (1 - P^E(\iota_{\mathfrak{L}}|\iota_{\mathcal{H}}, c))v^E(\iota_{\mathcal{H}}, c, \mathcal{H})) \\
&+ (1-\omega)(P^E(\iota_{\mathfrak{L}}|\iota_{\mathcal{L}}, c)v^E(2\iota_{\mathcal{L}}, c, \mathcal{L}) + (1 - P^E(\iota_{\mathfrak{L}}|\iota_{\mathcal{L}}, c))v^E(\iota_{\mathcal{L}}, c, \mathcal{L}))\Big) - \log\varphi_{\mathfrak{C}}\Big)/\sigma_{\mathfrak{C}}\Big).
\end{aligned}$$

The entry rule and the choice probability of entry by a local store is similarly determined. The associated transition probabilities are

$$\begin{aligned}
&\mathbb{P}^E(\iota_{\mathcal{L}} + \iota_{\mathcal{H}}|\iota_0, c, a^E) = \omega P^E(\iota_{\mathfrak{C}}|\iota_0, c)P^E(\iota_{\mathfrak{L}}|\iota_{\mathcal{H}}, c), \\
&\mathbb{P}^E(2\iota_{\mathcal{L}}|\iota_0, c, a^E) = (1-\omega)P^E(\iota_{\mathfrak{C}}|\iota_0, c)P^E(\iota_{\mathfrak{L}}|\iota_{\mathcal{L}}, c), \\
&\mathbb{P}^E(\iota_{\mathcal{H}}|\iota_0, c, a^E) = \omega P^E(\iota_{\mathfrak{C}}|\iota_0, c)(1 - P^E(\iota_{\mathfrak{L}}|\iota_{\mathcal{H}}, c)), \\
&\mathbb{P}^E(\iota_{\mathcal{L}}|\iota_0, c, a^E) = (1-\omega)P^E(\iota_{\mathfrak{C}}|\iota_0, c)(1 - P^E(\iota_{\mathfrak{L}}|\iota_{\mathcal{L}}, c)) + (1 - P^E(\iota_{\mathfrak{C}}|\iota_0, c))P^E(\iota_{\mathfrak{L}}|\iota_0, c), \\
&\mathbb{P}^E(\iota_0|\iota_0, c, a^E) = (1 - P^E(\iota_{\mathfrak{C}}|\iota_0, c))(1 - P^E(\iota_{\mathfrak{L}}|\iota_0, c)), \\
&\mathbb{P}^S(\iota_0|\iota_{\mathcal{L}}, c, a^S) = \Phi\left(\log\left(v^S(\iota_{\mathcal{L}}, c, \mathcal{L}) - \log\varphi_M\right)/\sigma_M\right).
\end{aligned}$$

Finally, all that remain undetermined is the survival rule for duopoly retailers of identical type and the associated choice and transition probabilities. Reconsider the static game presented in Figure 3: in a RNMPE, if the post-survival value for a duopolist exceeds $\varphi_M \exp(w_M)$, then both retailers continue for sure. Otherwise, checking the post-survival value for a monopolist is essential.

If the monopoly post-survival value also exceeds $\varphi_M \exp(w_M)$, the reduced-form continuation game has no pure strategy equilibrium. Instead, it admits a unique mixed-strategy equilibrium, in which each retailer chooses a survival probability to leave its rival indifferent between exiting and surviving. The equilibrium thus has the following survival rules for $k \in \{\mathcal{H}, \mathcal{L}\}$.

$$a^S(2\iota_k, c, w_M, k) = \begin{cases} 1 & \text{if } v^S(2\iota_k, c, k) > \varphi_M \exp(w_M), \\ \frac{v^S(\iota_k, c, k) - \varphi_M \exp(w_M)}{v^S(\iota_k, c, k) - v^S(2\iota_k, c, k)} & \text{if } v^S(2\iota_k, c, k) \leq \varphi_M \exp(w_M), v^S(\iota_k, c, k) > \varphi_M \exp(w_M) \\ 0 & \text{otherwise.} \end{cases}$$

If the monopoly post-survival value is low than $\varphi_M \exp(w_M)$, pure exit is the only equilibrium strategy of the static game–no unilateral deviation not improve the payoff, and no mixing is possible. In this case, the survival rule in the RNMPE is

$$a^S(2\iota_k, c, w_M, k) = \begin{cases} 1 & \text{if } v^S(2\iota_k, c, k) > \varphi_M \exp(w_M), \\ 0 & \text{otherwise.} \end{cases}$$

Under either equilibrium, the transition probabilities for a duopoly market with two type-k retailers are

$$\begin{aligned}
&\mathbb{P}^S(2\iota_k | 2\iota_k, c, a^S) = Prob(a^S(2\iota_k, c, W'_M, k) = 1) + \mathbb{E}_{W_M}[a^S(2\iota_k, c, W'_M, k)^2 | 0 < a^S(2\iota_k, c, W'_M, k) < 1], \\
&\mathbb{P}^S(\iota_k | 2\iota_k, c, a^S) = \mathbb{E}_{W_M}[2a^S(2\iota_k, c, W'_M, k)(1 - a^S(2\iota_k, c, W'_M, k)) | 0 < a^S(2\iota_k, c, W'_M, k) < 1], \\
&\mathbb{P}^S(\iota_0 | 2\iota_k, c, a^S) = 1 - \mathbb{P}^S(2\iota_k | 2\iota_k, c, a^S) - \mathbb{P}^S(\iota_k | 2\iota_k, c, a^S) Prob(a^S(2\iota_k, c, W'_M, k) = 0) \\
&+ \mathbb{E}_{W_M}[(1 - a^S(2\iota_k, c, W'_M, k))^2 | 0 < a^S(2\iota_k, c, W'_M, k) < 1] \\
&= \Phi\left(\frac{\log v^S(2\iota_K, c, K) - \log \varphi_M}{\sigma_M}\right) + \max\left\{0, \Phi\left(\frac{\log v^S(\iota_K, c, K) - \log \varphi_M}{\sigma_M}\right)\right. \\
&\left. - \Phi\left(\frac{\log v^S(2\iota_K, c, K) - \log \varphi_M}{\sigma_M}\right)\right\} \times \frac{v^S(\iota_K, c, K) - \exp(\sigma_M^2/2)\Phi\left(\frac{\log v^S(\iota_K, c, K) - \log \varphi_M - \sigma^2}{\sigma}\right)}{v^S(\iota_K, c, K) - v^S(2\iota_K, c, K)}
\end{aligned}$$

With this part of the survival rule and choice and transition probabilities determined, the equilibrium construction is concluded. Finally, note that when computing $\mathbb{P}^S(\cdot | 2\iota_k, c, a^S)$, the expectation over W_M has a closed-form expression under normality specification in this duopoly case. However, when the number of retailers involved in mixing exceeds three, closed-form expression in general fails to exist. Therefore, I use important sampling to numerically integrate over W_M to compute the transition probabilities. Because the calculation of the transition probabilities for every state is only conducted once for every set of parameter values, this numerical integration does not pose formidable computational challenge.

B Proof for Section 5

B.1 Sketch Proof for Proposition 1

Proof. The proof for Proposition 1 requires straightforward extension to the equilibrium existence proof in Abbring, Campbell, and Yang (2010). Therefore, I only review the four key steps here, and refer interested readers to their paper.

1. Show that Procedure 1 covers ν^E, ν^S, and $\mathcal{P}^E$ for all $(\mathbf{m}, c, k)$. The descending order of (h, l) ensures such completeness.

2. Show that Procedure 1 always produces well-defined ν^E, ν^S for all $(\mathbf{m}, c, k)$. This is a nontrivial step. It is achieved by first proving that $T_{\mathcal{L}}$ and $T_{\mathcal{H}}$ are contraction mappings in Procedure 1. As discussed in Section 5, the descending order of (h, l) ensures it. Then, when equation (13) is invoked to compute the survival rules, it needs to have a root in $[0, 1)$ if the equilibrium survival rule cannot imply pure exit. Note that the right-hand side of equation (13) collapses to $\nu^S(\underline{\mathbf{m}} - (m_k - 1)\iota_k, c, k) - \varphi_M \exp(w_M)$ when $p = 0$ and to $\nu^S(\underline{\mathbf{m}}, c, k) - \varphi_M \exp(w_M)$ when $p = 1$. When mixing takes place, $\nu^S(\underline{\mathbf{m}}, c, k) \leq \varphi_M \exp(w_M^s)$; If the survival rule cannot imply pure exit, the monopoly post-survival payoff overcomes the profitability shock and $\nu^S(\underline{\mathbf{m}} - (m_k - 1)\iota_k, c, k) > \varphi_M \exp(w_M)$. Therefore, intermediate value theorem ensures that at least one root in $[0, 1)$ always exists.

3. Verify that the choice probabilities are generated by survival rules satisfying the requirement in Definition 1 and 2.

4. Verify that ν^E is constructed as an equilibrium post-entry payoff, and ν^S an equilibrium post-survival payoff. The computed $\mathcal{P}^E$ is consequently verified to be the choice probabilities under RNMPE, and $\mathfrak{P}^E$ and $\mathfrak{P}^E$ to be the transition probabilities.

□

B.2 Sketch Proof for Proposition 2

Proof. Proposition 2, as well as its proof, is based on the uniqueness proposition for the payoff-monotone equilibrium in Abbring, Campbell, and Yang (2010). I hence avoid reiterating on the details and only give the sketch here.

The contraction property of the functional operators $T_{\mathcal{L}}$ and $T_{\mathcal{H}}$ ensures that if fed with unique choice/transition probability, they always produce unique fixed points. Consequently, the uniqueness of the RNMPE solely rests on the uniqueness of the choice/transition probability under each state. It is rather simple to show that multiplicity of choice probability only arises when stores are mixing between continuation and exit, and polynomial equation 13 admits multiple roots between 0 and 1.

When the monotonicity condition in Corollary 1 is satisfied for some $(\mathbf{m}, c, k)$, the right-hand side of equation (13) changes *monotonically* from $\nu^S(\underline{\mathbf{m}} - (m_k - 1)\iota_k, c, k) - \varphi_M \exp(w_M) > 0$ when $p = 0$ to $\nu^S(\underline{\mathbf{m}}, c, k) - \varphi_M \exp(w_M) < 0$ when $p = 1$. Therefore, the polynomial only have one root between 0 and 1, if mixing takes place under $(\mathbf{m}, c, k)$. If this condition is satisfied for all $(\mathbf{m}, c, k)$, the RNMPE is unique. □

C Computation Details

C.1 Choice Probabilities & Transition Probabilities of Entry

As demonstrated in Section 4.1, the choice probabilities of entry for a local store depends on the post-entry payoffs. A chain store's choice probabilities depend on if the local store after it enters or not, and the post-entry payoffs generated from the sequence of entries.

$$\mathcal{P}^E(\iota_{\mathfrak{L}}|\mathbf{m}, c) = \Phi\left(\left(\log\left(\nu^E\left(\mathbf{m} + \iota_{\mathcal{L}}, c, \mathcal{L}\right) - \varphi_{\mathfrak{L}}\right) - \log\varphi_{\mathfrak{L}}\right)/\sigma_{\mathfrak{L}}\right),$$

$$\begin{aligned}
&\mathcal{P}^E(\iota_{\mathfrak{C}}|\mathbf{m}, c) \\
&= \Phi\Bigg(\bigg(\log\Big(\omega(\mathcal{P}^E(\iota_{\mathfrak{L}}|\mathbf{m} + \iota_{\mathcal{H}}, c)\nu^E\left(\mathbf{m} + \iota_{\mathcal{H}} + \iota_{\mathcal{L}}, c, \mathcal{H}\right) + (1 - \mathcal{P}^E(\iota_{\mathfrak{L}}|\mathbf{m} + \iota_{\mathcal{H}}, c))\nu^E\left(\mathbf{m} + \iota_{\mathcal{H}}, c, \mathcal{H}\right)). \\
&+ (1 - \omega)(\mathcal{P}^E(\iota_{\mathfrak{L}}|\mathbf{m} + \iota_{\mathcal{L}}, c)\nu^E\left(\mathbf{m} + 2\iota_{\mathcal{L}}, c, \mathcal{L}\right) + (1 - \mathcal{P}^E(\iota_{\mathfrak{L}}|\mathbf{m} + \iota_{\mathcal{L}}, c))\nu^E\left(\mathbf{m} + \iota_{\mathcal{L}}, c, \mathcal{L}\right))\Big) \\
&- \log\varphi_{\mathfrak{C}}\bigg)/\sigma_{\mathfrak{C}}\Bigg).
\end{aligned}$$

The candidate transition probability $\mathfrak{P}^E$ can be computed directly as the function of $\mathcal{P}^E$, instead of α_E,

$$\mathfrak{P}^E(\mathbf{m}_E|\mathbf{m}_S, c, \mathcal{P}^E) = \begin{cases} \omega \mathcal{P}^E(\iota_{\mathfrak{C}}|\mathbf{m}_S, c)\mathcal{P}^E(\iota_{\mathfrak{L}}|\mathbf{m}_S + \iota_{\mathcal{H}}, c) & \text{if } \mathbf{m}_E = \mathbf{m}_S + \iota_{\mathcal{H}} + \iota_{\mathcal{L}}, \\ (1-\omega)\mathcal{P}^E(\iota_{\mathfrak{C}}|\mathbf{m}_S, c)\mathcal{P}^E(\iota_{\mathfrak{L}}|\mathbf{m}_S + \iota_{\mathcal{L}}, c) & \text{if } \mathbf{m}_E = \mathbf{m}_S + 2\iota_{\mathcal{L}}, \\ \omega \mathcal{P}^E(\iota_{\mathfrak{C}}|\mathbf{m}_S, c)(1 - \mathcal{P}^E(\iota_{\mathfrak{L}}|\mathbf{m}_S + \iota_{\mathcal{H}}, c)) & \text{if } \mathbf{m}_E = \mathbf{m}_S + \iota_{\mathcal{H}}, \\ (1-\omega)\mathcal{P}^E(\iota_{\mathfrak{C}}|\mathbf{m}_S, c)(1 - \mathcal{P}^E(\iota_{\mathfrak{L}}|\mathbf{m}_S + \iota_{\mathcal{L}}, c)) & \\ +(1 - \mathcal{P}^E(\iota_{\mathfrak{C}}|\mathbf{m}_S, c))\mathcal{P}^E(\iota_{\mathfrak{L}}|\mathbf{m}_S, c) & \text{if } \mathbf{m}_E = \mathbf{m}_S + \iota_{\mathcal{L}}, \\ (1 - \mathcal{P}^E(\iota_{\mathfrak{C}}|\mathbf{m}_S, c))(1 - \mathcal{P}^E(\iota_{\mathfrak{L}}|\mathbf{m}_S + \iota_{\mathcal{H}}, c)) & \text{if } \mathbf{m}_E = \mathbf{m}_S, \\ 0 & \text{otherwise.} \end{cases}$$

C.2 Choice Probabilities & Transition Probabilities of Survival

When type-k stores are mixing between survival and exit under $\mathbf{m}, c, k, w_M)$, the following polynomial equation determines the equilibrium survival rules as its roots in $[0, 1)$, if ν^S gives the (candidate) post-survival equilibrium payoffs,

$$\sum_{i=0}^{m_k-1} (1-\alpha^S)^{m_k-1-i}(\alpha^S)^i \binom{m_k-1}{i} (\nu^S(\underline{\mathbf{m}} - (m_k - 1 - i)\iota_k, c, k) - \varphi_M \exp(w_M)) = 0. \quad (13)$$

where $\underline{\mathbf{m}} = \mathbf{m}$ if $k = \mathcal{L}$, and $\underline{\mathbf{m}} = \mathbf{m} - m_{\mathcal{L}}\iota_{\mathcal{L}}$ if $k = \mathcal{H}$. If there are more than one roots in $[0, 1)$, the equilibrium survival rules are multiple. To compute the choice probability under $(\mathbf{m}, c, k)$ and the transition probabilities conditional on $(\mathbf{m}, c)$, one would need to integrate the probabilistic survival rules over the random variable W_M. A blunt force implementation of such integration is to first discretize or draw W_M, then to solve (13) repeatedly and compute α^S under each grid point or realization of W_M. This approach is not only computationally expensive, owing to the task of finding high order polynomials' roots, but also inaccurate, due to the fact that the infinite support of W_M challenges the precision of the approximation. As a solution, I work with a "reversed approach" which first discretizes α^S on the interval $[0, 1]$, then finds w_M satisfying

$$\sum_{i=0}^{m_k-1} (1-\alpha^S)^{m_k-1-i}(\alpha^S)^i \binom{m_k-1}{i} \nu^S(\underline{\mathbf{m}} - (m_k - 1 - i)\iota_k, c, k) = \varphi_M \exp(w_M).$$

This equation makes use of the observation that $\varphi_M \exp(w_M)$ in all of the polynomial coefficients, except the scaler, cancels out. Note that computing w_M only requires summation and multiplication, operations that computers can execute with much higher speed and precision than solving polynomials. Once all w_M are nailed, its density function f_{W_M} defines the probability that each corresponding α^S has in forming the choice/transition probabilities. Suppose that there are S grids points for α^S between $[0,1]$. Denote these points as $p[1],\dots,p[S]$ with $p[1]=0$ and $p[S]=1$, and the associated market-level shocks as $w_M[1],\dots,w_M[S]$. When the payoff-monotone condition in Proposition 2 is violated, $w_M[1],\dots,w_M[S]$ might not be an increasing sequence. I then use only the $J \le S$ w_M which follow a descending order from $w_M[S]$. Relabel them as $x[1],\dots,x[J]$, and the associated survival probabilities as $q[1],\dots,q[J]$. The choice probability of survival under the normality assumption is[27]

$$\sum_{j=2}^{J} \left(\Phi(x_M[j]/\sigma_M) - \Phi(x_M[j-1]/\sigma_M)\right)\left(q[j]+q[j-1]\right)/2 + \Phi(x_M[J]/\sigma_M).$$

The transition probabilities are computed using the same approach. Again, the computation of α^S is not necessary. The computational details for the choice probabilities and the transition probabilities of survival are demonstrated in the flow chart below.

C.3 Functional Operator $T_{\mathcal{L}}$ and $T_{\mathcal{H}}$

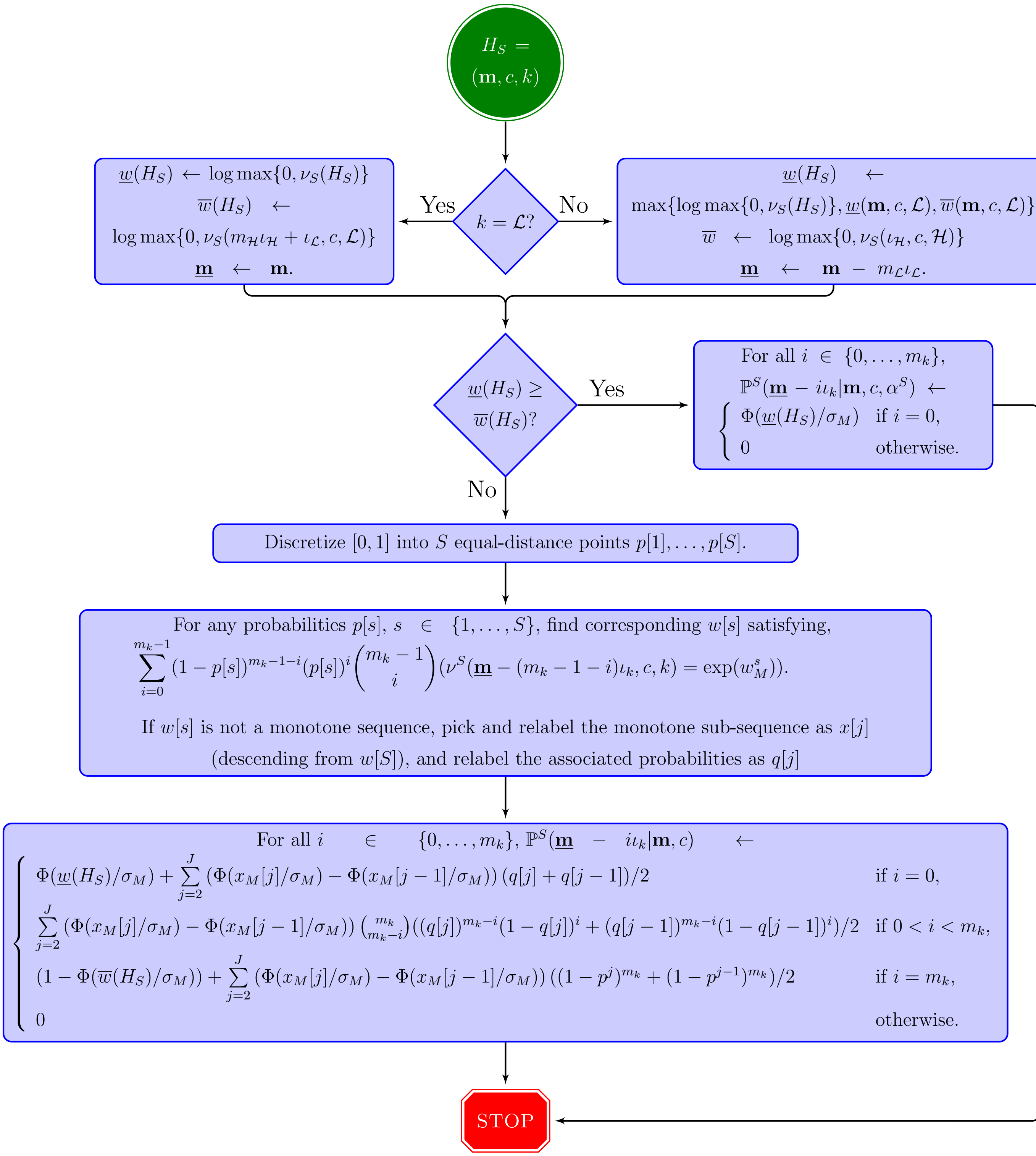
Finally, the functional operators $T_{\mathcal{L}}$ and $T_{\mathcal{H}}$ are defined as

$$\begin{aligned}(T_{\mathcal{L}}f)(H_S^{h,l}) =& \Phi\left(\frac{\log\max\left\{0, f^S\left(H_S^{h,l}\right)\right\} - \log\varphi_M}{\sigma_M}\right) f^S\left(H_S^{h,l}\right) \\ & - \exp(\sigma_M^2/2)\Phi\left(\frac{\log\max\left\{0, f^S\left(H_S^{h,l}\right)\right\} - \log\varphi_M - \sigma_M^2}{\sigma_M}\right)\end{aligned} \tag{14}$$

and

$$\begin{aligned}(T_{\mathcal{H}}f)(H_S^{h,l}) =& \sum_{j=0}^{m_{\mathcal{L}}} \mathbb{P}^S(m_{\mathcal{H}}\iota_{\mathcal{H}} + j\iota_{\mathcal{L}}|\mathbf{m},c) f^S(m_{\mathcal{H}}\iota_{\mathcal{H}} + j\iota_{\mathcal{L}}, c, \mathcal{H}) - \exp(\sigma_M^2/2) \\ & \times \Phi\left(\frac{\log\max\left\{0, f^S\left(H_S^{h,l}\right), \nu^S\left(m_{\mathcal{H}}\iota_{\mathcal{H}} + \iota_{\mathcal{L}}, c, \mathcal{L}\right), \nu^S\left(m_{\mathcal{H}}\iota_{\mathcal{H}} + m_{\mathcal{L}}\iota_{\mathcal{L}}, c, \mathcal{L}\right)\right\} - \log\varphi_M - \sigma_M^2}{\sigma_M}\right)\end{aligned} \tag{15}$$

[27]This is equivalent to picking the largest mixing probability when equilibrium multiplicity occurs. When equilibrium multiplicity occurs, ideally, the RNMPE that generates the highest likelihood value should be picked. However, the iterative nature of the NFXP algorithm forbids a straightforward selection of the equilibria.



Procedure 3: Compute the Candidate Choice/Transition Probabilities of Survival

in which

$$\mathbb{P}^S(m_{\mathcal{H}}\iota_{\mathcal{H}}|\mathbf{m},c) = \max\left\{0, \Phi\left(\frac{\log\max\left\{0, f^S\left(H_S^{h,l}\right)\right\} - \log\varphi_M}{\sigma_M}\right)\right.$$

$$- \max\left\{\Phi\left(\frac{\log\max\left\{0, \nu^S\left(m_{\mathcal{H}}\iota_{\mathcal{H}} + \iota_{\mathcal{L}}, c, \mathcal{L}\right)\right\} - \log\varphi_M}{\sigma_M}\right),\right.$$

$$\left.\left.\Phi\left(\frac{\log\max\left\{0, \nu^S\left(m_{\mathcal{H}}\iota_{\mathcal{H}} + m_{\mathcal{L}}\iota_{\mathcal{L}}, c, \mathcal{L}\right)\right\} - \log\varphi_M}{\sigma_M}\right)\right\}\right\},$$

and

$$f^S\left(\mathbf{m}, c, k\right) = \beta\mathbb{E}_C\left[\pi_k(\mathbf{m}, C') + \sum_{\mathbf{M}^E}\mathbb{P}^E(\mathbf{M}^E|\mathbf{m}, C', \mathcal{P}^E)g^E(\mathbf{M}^E, C', k)\,\middle|\, C = c\right].$$

with

$$g^E(\mathbf{M}^E, C', k) = \begin{cases} f(\mathbf{M}^E, C', k) & \text{if } (\mathbf{M}^E, C', k) \in \mathbb{H}_S^{h,l} \\ \nu^E(\mathbf{M}^E, C', k) & \text{if } (\mathbf{M}^E, C', k) \in \mathbb{H}_S^{h+,l+}, \text{ for } h+ \geq h, l+ > l \text{ or } h+ > h, l+ \geq l. \end{cases}$$

At last,

$$\nu^S\left(\mathbf{m}, c, k\right) = \beta\mathbb{E}_C\left[\pi_k(\mathbf{m}, C') + \sum_{\mathbf{M}^E}\mathbb{P}^E(\mathbf{M}^E|\mathbf{m}, C', \mathcal{P}^E)\nu^E(\mathbf{M}^E, C', k)\,\middle|\, C = c\right].$$

C.4 Market Structure Distribution Function $\mathbb{P}$

The distribution function for underlying market structure $\mathbb{P}$ is computed by considering all active retailers' type distributions. Formally, it is

$$\mathbb{P}(\mathbf{N}|\mathbf{p}, n_{\mathfrak{L}}) \equiv$$

$$\begin{cases} \displaystyle\sum_{j \text{ counts of } \mathcal{H}, |\mathbf{p}|-j \text{ counts of } \mathcal{L} \text{ among } k_1, k_2, \ldots, k_{n_{\mathfrak{C}}}} p^{k_1 k_2 \ldots k_{n_{\mathfrak{C}}}} & \text{if } N = j\iota_{\mathcal{H}} + (|\mathbf{p}| - j + n_{\mathfrak{L}})\iota_{\mathcal{L}}, j \in \{0, \ldots, |\mathbf{p}|\} \\ 0 & \text{otherwise.} \end{cases}$$

For a market structure with j type-$\mathcal{H}$ stores and $(|\mathbf{p}| - j + n_{\mathfrak{L}})$ type-$\mathcal{L}$ stores, its probability is the sum of all elements in the type distribution vector $\mathbf{p}$ whose sequences of types $\{k_1, k_2, \ldots, k_{n_{\mathfrak{C}}}\}$ have j counts of $\mathcal{H}$ and $|\mathbf{p}| - j$ counts of $\mathcal{L}$.