Review of: "Competition and Post-Transplant Outcomes in Cadaveric Liver Transplantation under the MELD Scoring System" by H. Paarsch, A. Segre and J. Roberts

This paper presents an interesting empirical analysis of competition for liver transplants in the presence of an allocation rule known as MELD (Model for End-Stage Liver Disease) that allocates an available liver for transplant among competing transplant centers in a region known as a Donation Service Area based on a ranking of eligible patients' disease severity scores. Although the rule is somewhat complex as represented by figure 2 in the paper, "UNOS Rules for Allocating a Donor Liver by Status and Location", the essence of the scheme is to give the right of first refusal to the transplant center and patient in the Donor Service Area that is in the most serious need of a transplant in order to survive, as quantified by the patient's MELD score. As the authors note on page 7, "A MELD score is calculated using easily-measurable laboratory values; the MELD score predicts, with some accuracy, a particular potential liver recipients risk of dying without transplantation."

Though the MELD score is a strong guide to allocation of livers, there is discretion in whether a transplant is actually done since not every liver that may arrive may be a good match for a particular patient. Thus, a transplant center and patient face a difficult optimal stopping problem: should a patient eligible to receive a currently available liver for transplant do the transplant now if the "match quality" of the liver to the patient is not as high as would be desirable (perhaps leading to some risk of rejection or complications following the transplant), or should the liver be rejected and the patient wait for the arrival of another liver that provides a better match in the future, but at the risk that the patient's condition could serverely deteriorate if a suitable new liver did not come along after this one?

Previous studies by Howard (2002) and Alagoz *et. al.* (2007) studied the transplant from a "single agent" decision perspective, formulated as an optimal stopping problem. The contribution of this paper is to point out that when competition between transplant centers is taken into account, there is a possibility for suboptimal outcomes to occur under competition. Specifically, the authors hypothesize that competition for livers among transplant centers in the Donation Service Area interacts with the MELD rule in a way that produces a sub-optimal outcome that they call the *competitive impatience effect*, "competition makes patient/surgeon decision-makers more likely to accept a donor organ than when no competition exists, which means (all other things being equal) the matches made under competition are predicted to be of weakly lower quality." (p. 11-12).

The authors conduct an empirical analysis of liver transplant outcomes using a unique panel data set on liver transplants with several years of follow up observations that enabled them to gain informaton on the *ex post* outcomes. Specifically, their data contain "2,322 observations concerning 105 transplant centers in 53 Donation Service Areas in the eleven UNOS regions — 870 each concerning one-month and one-year durations and 582 concerning three-year durations." (p. 28). Their empirical analysis was conducted to address the primary question, "does the presence of local competition affect the post-transplantation outcomes?" (p. 29). The authors hypothesize that the liver transplant decision, particularly the quality of the match of the liver to the transplant patient, as well as the quality of the donated liver and the health status of the patient, "affects post-transplantation graft survival duration in a weakly-positive way according to the following function $T = \tau(Q^*(z, m))$, $\tau' > 0$" where $Q^*(z, m)$ denotes the threshold quality of a liver that leads to a transplant for a patient with observed characteristics z in a Donation Service Area with "market characteristics" (principally the number of competing transplant centers) m.

However the authors do not adopt a structural approach to estimation and thus do not attempt to actually solve for the threshold $Q^*(z, m)$. Instead they attempt to make inferences about how competition among transplant centers affect *ex post* outcomes by adopting a Cox proportional hazard approach (i.e. a reduced form approach to study the issue indirectly) though they acknowledge that "We cannot, of course, implement Cox's approach because we do not have micro-level data, only center-specific averages." (p. 31). However despite this, they state "We should note, however, that the center-specific, risk-adjusted average graft survival-rate and patient survival-rate data are generated using results from a Cox proportional hazard-rate model" (p. 31) which I found hard to interpret, since the actual empirical work amounts to running regressions where the dependent variables are the center-specific graft-survival rate (GSR) and patient survival rate (PSR). Since these regressions aggregate over patients, the patient specific variables z no longer enter these regressions, but the market specific variables m still do.

The authors estimate their regressions using both Ordinary Least Squares (OLS) and Least Absolute Deviations (LAD) using HAC corrections to produce robust standard error estimates and included transplant center specific fixed effects (dummy variables). They find that "Additional competition is predicted to decrease the average graft survival rate by between 2.3 and 9.3 percent, depending on the number of transplant centers in the Donation Service Area and the estimation method." (p.36). They also find that "The share of transplantations performed by a transplant center in a Donation Service Area is predicted to increase the average graft survival rate; its p-value is 0.001 under least squares." (p. 36). For example, "a Donation Service Area in which two transplant centers compete, where one transplant center performs two-thirds of the transplants, while the other performs one third: the larger transplant center has an average graft survival rate that is around 6.6 percent greater than the smaller one: bigger is better in this empirical specification."

Finally, the fixed effects that they estimate indicate large differences in post-transplant success rates (i.e. survival rates) across different transplant centers. However the authors are unable to address what the underlying determinants of this heterogeneity in success rates might be.

I found the empirical analysis to be well done and was convinced that there seems to be a strong effect of competition that tends to result in less successful liver transplants as reflected by a lower post-transplant survival rate. The same is true of the overall patient survival rates as their table 5 shows.

Thus, I feel that the authors have uncovered an important empirical result and thus they have demonstrated empirically that a "competitive impatience effect" does appear to exist. I cannot think of any explanations or reasons to suggest their results are spurious or misleading. Thus, I think there is promise for this paper to be published in a leading journal, but the question is, "is this empirical finding a sufficient contribution to justify publishing this paper in the JPE"?

In my opinion there needs to an additional theoretical contribution to the paper to justify publishing this paper in the JPE. I believe the authors are capable of making this contribution so I would suggest giving the authors a chance to revise and resubmit in view of the importance of the question and the novelty of their empirical finding. The main weakness of the paper is that the abstract and the way the paper is written makes it appear that the authors are finding empirical confirmation for a well known theoretical prediction. However as far as I can see, there is no theoretical prediction and no theory is offered about how competition between transplant centers might affect transplant decisions.

While the authors provide an intuitive story and conjecture that "Competitors are like having an higher discount rate in the problems investigated by Howard [2002] as well as Alagoz et al. [2004, 2007a,b]" their Section 3, "Theoretical Framework" really is short on theory and long on intuitive stories, but stories that I did not find completely convincing or compelling.

Though the authors do acknowledge on page 11 that "While we do not solve for the equilibrium of a (potentially asymmetric) game of incomplete information, we appeal to game-theoretic notions when interpreting empirically the effects of competition on the post-transplantation outcomes for cadaveric livers in adults in the United States." I did not find their intuitive reasoning to be compelling that theory necessarily predicts something like a competitive impatience effect for several reasons.

First, the authors have not convinced me that there is absence of competition even in Donation Service Centers where there is a single transplant center. I could imagine that there are competing doctors working at this center, each representing different patients. If this is the case, then the same story the authors offered for their competitive impatience effect could be told in microcosm as a suboptimal result of competition between different doctors and patients that occurs within a single transplant center. If there are now more transplant centers, then perhaps there could be more competition in the Donation Service Area in aggregate, though not necessarily. One could imagine one DSA with 2 medium sized transplant centers and another with a single big one and in both there are the same number of doctors/patients vying for liver transplants. If the loci of competition is primarily among different doctors then it is far from clear to me that there should be any competitive impatience effect indexed by the number of competing transplant centers in a DSA.

So the authors need to make a case that the doctors working in a given transplant centers are not really competing with each other, but instead we should think of a transplant center as operating more like a firm and the doctors are more like the employees of this firm. Then the intuitive story for the competitive impatience effect makes more sense, since more transplant centers are like a market with more firms, and we do have a strong intuition that competition should produce different outcomes relative to a “monopoly” solution.

So I suggest that the authors should appeal more to Industrial Organization theory or better, develop a new theory that is reasonably tailored to the type of competition that exists in this “market” and is able to a) justify why decision making on transplants by doctors in a given transplant center is better approximated as that of decisions of employees at a profit-maximizing firm and thus aligned with the financial interests of the transplant center and not on the financial interests of individual doctors associated with the transplant center who may have potentially competing practices and serve different patients, and b) to actually work out a theory in detail that at least illustrates that the competitive impatience effect really is implied by a well-defined theory (as opposed to relying on conjectures and a simple diagram that has not actually been derived by comparing how competitive outcomes are affected by the number of transplant centers).

I would not require the authors to have to build and *structurally estimate* a full blown competitive model of transplants. However I think they should be able to at least produce a model that shows that what they call the competitive impatience effect is a theoretical possibility. Otherwise, if they don’t or can’t do that, they should not write up the paper to create the impression that they are providing an empirical confirmation of a prediction of economic theory. Instead, they should write up the paper as an interesting and important empirical finding, and leave the question to future theorists to see if they can provide a theoretical explanation of the competitive impatience effect that is currently in the status of an empirical finding, but one for which there is no theoretical explanation that I am aware of.

I think a paper that only presents the empricial finding but does not also provide a well defined model that also shows that theory does predict it as well (and thus provides further insight into why it occurs and how policy varaibles such as MELD contribute to it or ameliorate it) is publishable in a leading journal, though perhaps not the JPE.

I think a paper that does both, and provides an innovative model that results in clear insight would be a contribution that would be worthy of publication in the JPE. I recommend that the authors be given that chance since I think they do have good intuition and the empirical support for the result is strong. Below I outline a simple model that may enable the authors to formalize and prove that the competitive impatience effect exists. It is not the only model that could be formulated, and the authors may have specific objections or reservations about the model I suggest below.

Thus I am not demanding that the authors formulate and solve a model exactly along the lines of the one I sketch below, but I provide a model just to show that I think it is possible to do this. I suggest that the simplest model would compare transplant outcomes under two different situations: 1) “the monopoly case” where there is single transplant center in the DSA, and 2) “the duopoly case” where there are two competing transplant centers in the DSA. This is a relatively clean way to illustrate the effect theoretically and the model I formulated below was done so in a way that I think would maximize the chance of actually finding the competitive impatience effect theoretically. There are cases known in Industrial organization theory where the monopoly solution is known to be “efficient” and the duopoly solution is known to be “inefficient”. An example is the recent paper by Iskahkov, Rust and Schjerning (2011) “A Dynamic Model of Leap-Frogging Investments and Bertrand Price Competition”. Though dynamic competitive models solved under the Markov Perfect Equilibrium solution concept can easily yield multiple equilibria, I think the nature of stochastic evolution of health status of patients and the MELD allocation rule can interact to result what in effect is a randomly alternating move game. These games have a greater likelihood of having unique equilibria, though the analysis of randomly alternating move game of price and investment competition cited above does not have a unique equilibrium, bur rather a continuum of equilibrium outcomes.

Thus there is some risk that the authors might try to formulate and solve a Markov Perfect equilibrium like the duopoly transplant center game suggested below and either find a unique equilibrium but one that does not imply a competitive impatience effect, or it might have multiple equilibria and some of these equilibria might be consistent with a competitive impatience effect and others do not exhibit a competitive impatience effect. Thus, doing a revision of this paper in the way I suggest is not entirely without risk.

1. The monopoly transplant center problem

I formulate the decision problem of a single transplant center as a regenerative stopping problem that differs from the non-regenerative formulations of Howard (2002) and Alagoz *et. al.* (2007). These latter stopping problems can be viewed as representing the perspective/problem of a single patient who is waiting to receive a liver and the decision problem ends when either a) the patient dies, or b) the patient receives a transplant live. The formulation below, on the other hand, treats the hospital/transplant center as the decision maker that faces an inflow of patients and livers over an infinite horizon. A transplant of a liver does not end the decision problem. Instead a new patient can arrive either after an existing patient dies or receives a transplant liver. The new patient can be considered to be the "regeneration" of a new stopping problem for this new patient and so on.

Actually, to make this model as comparable to a "duopoly problem" where there are two transplant centers competing for the available supply of livers for transplant, we will assume that at any point in time there are at most two sickest liver transplant patients that are eligible for transplants. Let (h_1, h_2). I adopt the normalization that higher values of h_i denote better health and lower values denoting worse health, with $h_i = 0$ denoting either a) the death of the patient, or b) no patient currently, for $i \in \{1, 2\}$. Thus, a state $(0, 0)$ means that there are no patients eligible for transplants, and $(0, h_2)$ and $(h_1, 0)$ denote situations where there is only one patient eligible (the other either died or received a transplant and so far no other eligible patient has arrived to "replace" this patient.

I assume that the health status of the two sickest eligible patients evolve as indpendent first order Markov processes, witth transition probabilities $H_1(h_1'|h_1)$ and $H_2(h_2'|h_2)$. I follow Howard (2002) and Alagoz *et. al.* (2007) by assuming that livers have a condition or "quality" q with higher values of q indicating a "better condition" liver that increases the chance of a successful transplant, all other things equal. I let $q = 0$ denote a state where no liver has "arrived" in the given period. The arrival of livers is assumed to be an *IID* process $\{q_t\}$ with CDF $F(q)$.

After a transplant or a death of a patient, a new patient arrives to replace the dead or successfully transplanted patient and I assume that the initial health of these "replacement" patients is drawn from a given distribution $H_0(h_i)$, $i \in \{1, 2\}$. If $H_0(0) > 0$, this means that there is a positive probability that no patient will arrive in any given period t to replace one of the transplant patients who has either died or had a transplant in period $t - 1$.

Unlike the Howard (2002) and Alagoz *et. al.* (2007) papers, I assume that the transplant center is a profit-maximizer and makes a rational, dynamic cost/benefit decision about whether to do a transplant on a given patient or not. I assume that the center follows the Meld scoring priority rule where it offers a liver to the patient in worse health (patient 1 if $h_1 < h_2$) but that the doctors working at the center (whose interests I assume to be perfectly aligned with the profit motives of the center not the welfare of a particular patient) have sufficient authority and credibility with their patients that if they recommend against doing a transplant for any reason, the patient will follow their advice. This means that, in effect, the Meld priority rule amounts to only a guideline to the transplant center and not a hard and fast allocation rule.

If a liver of quality q is available and transplanted in a patient with health status h, the center expects a payoff of $B(h, q)$ conditional on the transplant being deemed a success, which occurs with probability $p(h, q)$. However if the transplant fails, the center expects malpractice liability $L(h, q)$. Further, if the center ignores the Meld ordering and transplants a liver into a patient, say patient 2, with better health $h_2 > h_1$, then in addition to the net benefit $EB_2(h_2, q)$ the center expects from doing the transplant on patient 2, where

$$EB_2(h_2, q) = B(h_2, q)p(h_2, q) + (1 - p(h_2, q))L(h_2, q),$$

the center also expects a net cost from a potential malpractice lawsuit from patient 1 of $EC_1(h_1, q)$. Thus, the net expected "profit" (or loss) from ignoring the Meld ordering and doing a transplant on the healthier of the two patients, 2, is

$$R_2(h_1, h_2, q) = EB_2(h_2, q) - EC_1(h_1, q).$$

If the transplant center follows the Meld ordering and does the transplant on patient 1, I assume that it is immune from a lawsuit from patient 2 and thus its expected profit is

$$R_1(h_1, h_2, q) = EB_1(h_1, q).$$

If tghe center decides not to do transplants on either patient its cost is the expected liability from malpractice lawsuits from *both* patients, or

$$R_0(h_1, h_2, q) = -(EC_1(h_1, q) + EC_2(h_2, q)).$$

Of course, I am presuming that there are actually two patients here. If there are no patients, then there is no decision to be made and the expected payoff/loss is 0. If there is only one patient, say only $h_1 > 0$ and $h_2 = 0$, then we have

$$R_1(h_1, 0, q) = EB_1(h_1, q)$$

and

$$R_0(h_1, 0, q) = -EC_1(h_1, q)$$

I enforce these restrictions by imposing that $EC_1(0, q) = 0$ and $EC_2(0, q) = 0$, and similarly, $EB_1(0, q) = 0$ and $EB_2(0, q) = 0$.

Of course there can be no expected costs or benefits to the center in anyy period where no liver has arrived to allocate to either patient, $q = 0$. So we also have $R_0(h_1, h_2, 0) = 0$, and $R_1(h_1, h_2, 0) = 0$ and $R_2(h_1, h_2, 0) = 0$.

Let $V(h_1, h_2, q)$ be the present value of discounted profits that the transplant center expects when it adopts an optimal dynamic tansplant strategy for an infinite stream of transplant patients in a stationary, infinite horizon Markovian setting. I assume that benefits to the transplant center are sufficiently high relative to malpractice and other fixed operating and wage costs (that I assume are already implicitly embodied in the $B(h, q)$ and $L(h, q)$ and $EC_i(h, q)$ functions, $i \in \{1, 2\}$) that $V(h_1, h_2, q) > 0$, otherwise the center would exit the transplant business in any state where $V(h_1, h_2, q) < 0$.

The Bellman equation for V is given by

$$\begin{aligned} V(h_1, h_2, q) = max\, [&R_1(h_1, h_2, q) + \beta EV(0, h_2), \\ &R_2(h_1, h_2, q) + \beta EV(h_1, 0), \\ &R_0(h_1, h_2, q) + \beta EV(h_1, h_2)] \end{aligned}$$

where $\beta \in (0, 1)$ is the center's discount factor for future profits, and

$$EV(h_1, h_2) = \int_{q'} \int_{h_1'} \int_{h_2'} V(h_1', h_2', q') F(dq') H_1(dh_1'|h_1) H_2(dh_2'|h_2).$$

If patient 1 has either died or received a transplant, a new "replaccement" is drawn from the distribution H_0, so we have

$$EV(0, h_2) = \int_{q'} \int_{h_1'} \int_{h_2'} V(h_1', h_2', q') F(dq') H_0(dh_1') H_2(dh_2'|h_2).$$

Similarly, if patient 2 has died or received a transplant, we have

$$EV(h_1, 0) = \int_{q'} \int_{h_1'} \int_{h_2'} V(h_1', h_2', q') F(dq') H_1(dh_1'|h_1) H_0(dh_2').$$

If both patients has died or if one gets a transplant and the other dies, we have

$$EV(0,0) = \int_{q'} \int_{h_1'} \int_{h_2'} V(h_1', h_2', q') F(dq') H_0(dh_1') H_0(dh_2').$$

Assume that the support of h_1 and h_2 and q is the unit interval $[0, 1]$. Then the solution to the stopping problem partitions the unit cube into three regions

0 . A region where no transplant is done on either patient,

1 . A region where a transplant is done on patient 1,

2 . A region where a transplant is done on patient 2.

The duopoly transplant center problem

Now consider a situation where there are two transplant centers and at most two sickest patients eligible for transplants at any given time, but assume one patient, with health status h_1 is assigned to transplant center 1 and the other, with health state h_2, is assigned to transplant center 2. I assume that the MELD allocation rule is rigidly enforced when there are two competing transplant centers. That is, if a liver of quality q comes available in any period, it is directed with probability 1 to the transplant center with the patient that is in the worst health state. Thus if $0 < h_1 < h_2$, then a liver goes to transplant center 1. The other possible case where a liver goes to transplant center 1 with probability 1 is when $h_2 = 0$, i.e. where there is patient at transplant center 2 that is in need of or eligible for a liver transplant.

I assume there is complete information at the two transplant centers about the health states of each of their respective patients, as well as the quality q of the liver. Further I assume that the shelf life of a liver for transplant is sufficiently short that if it is delivered to the center that has the patient in the worst health but for some reason the liver is not used, then the liver is wasted and cannot be re-transported to the other transplant center in time to be of use to the other patient.

Let $V_1(h_1, h_2, q)$ be the expected value of profits to transplant center 1, and let $V_2(h_1, h_2, q)$ be the expected profits of transplant center 2 in a *Markov Perfect equilibrium* of the "transplant game". This means that the two transplant centers have adopted transplant strategies that are mutual best responses (i.e. each maximizes the respective center's expected profits taking the strategy of the opponent as given as in the usual notion of Nash equilibrium) and further, this equilibrium must hold in every possible state (h_1, h_2, q) and only be a function of this state, as opposed to other "past history" of the game.

Let $\rho_1(h_1, h_2, q)$ be the probability that transplant center 1 will do a transplant if it is allocated a liver (i.e. if $q > 0$ and $0 < h_1 < h_2$). Similarly let $\rho_2(h_1, h_2, q)$ be the probability that transplant center 2 will do a transplant when it is allocated a liver (i.e. when $q > 0$ and $0 < h_2 < h_1$). Although each center only has one patient, its decision depends on the health status of the patient at the other center since if one center does not do a transplant in a period where it gets a liver, its expectations on whether its patient will be eligible to get a new liver next period will depend on the health status of *both* patients due to the MELD allocation rule. I will write down the equations for ρ_1 and ρ_2 in a bit but first I write the Bellman equation for firm 1. First suppose that $h_1 < h_2$ so that transplant center one has the option to to a transplant on its patient in accordance with the MELD rule. Then we have

$$V_1(h_1, h_2, q) = max\left[R_1(h_1, q) + \beta EV_1(0, h_2), R_0(h_1, q) + \beta EV_1(h_1, h_2)\right]$$

where $R_1(h_1, q)$ is the expected return to doing a transplant and $R_0(h_1, q)$ is the expected cost of not doing a transplant (factoring in the expected cost of a lawsuit by the patient if the patient dies or his/her health deteriorates rapidly and the patient blames the transplant center for not doing a transplant when it had a liver on hand to transplant). Note that in this case R_1 and R_0 only depend on (h_1, q) and not (h_1, h_2, q) since I assume that the patient at the other transplant center has no claim against transplant center 1 (i.e. ability to file malpractice lawsuit) since the center is "protected" by the MELD rule. We have

$$R_1(h_1, q) = EB_1(h_1, q)p(h_1, q) + (1 - p(h_1, q))EL_1(h_1, q)$$

where as before $p(h_1, q)$ is the probability the transplant is a success, $EB_1(h_1, q)$ is the expected payoff to the transplant center in the event the transplant is deemed to be a success, and $EL_1(h_1, q)$ is the expected payoff (liability) to the transplant center in the event that the transplant is not a success. For R_0 we have

$$R_0(h_1, q) = -EC(h_1, q)$$

the expected cost of a malpractice suit if it were filed by the patient if the patient was not given the liver.

The Bellman equation will be the same in the case $h_2 = 0$ and $h_1 > 0$, i.e. where there is no eligible patient for a liver transplant at center 2 but one eligible patient at center 1. Then of course the liver will go to center 1 with probability 1, and this center will make a decision on whether to do the transplant using the same Bellman equation, except that $h_2 = 0$ in this case.

Now consider the other case, where either $h_1 > h_2 > 0$, or where $h_1 = 0$ and $h_2 > 0$. Then the liver goes to transplant center 2 with probability 1 and there is no decision for transplant center 1 to make. In this case its value function is given by

$$V_1(h_1, h_2, q) = \beta\left[\rho_2(h_1, h_2, q)EV_1(h_1, 0) + (1 - \rho_2(h_1, h_2, q))EV_1(h_1, h_2)\right].$$

AS in the monopoly case we have

$$EV_1(h_1, h_2) = \int_{q'}\int_{h_1'}\int_{h_2'} V_1(h_1', h_2', q')F(dq')H_1(dh_1'|h_1)H_2(dh_2'|h_2),$$

$$EV_1(h_1, 0) = \int_{q'}\int_{h_1'}\int_{h_2'} V_1(h_1', h_2', q')F(dq')H_1(dh_1'|h_1)H_0(dh_2'),$$

$$EV_1(0, h_2) = \int_{q'}\int_{h_1'}\int_{h_2'} V_1(h_1', h_2', q')F(dq')H_0(dh_1')H_2(dh_2'|h_2).$$

Given firm 1's values in the states $\{(h_1, h_2)|h_1 < h_2$ or $h_1 > h_2 = 0\}$ where center 1 does have the option to do a transplant, its probability of doing a transplant is given by

$$\rho_1(h_1, h_2, q) = I\left\{R_1(h_1, q) + \beta EV_1(0, h_2) \geq R_0(h_1, q) + \beta EV_1(h_1, h_2)\right\}.$$

The equations for firm 2's value functions and probability of doing a transplant are defined similarly, and a Markov perfect equilibrium is any solution to the pair of Bellman functional equations for firms 1 and 2.

Note that the game can be modified to include incomplete information by adding extreme value shocks $(\epsilon_0^1, \epsilon_1^1)$ associated with the decsion to do a transplant or not, respectively, that are observed only by transplant center 1 and not by transplant center 2. Then the value function for firm 1 has two extra arguments, $V_1(h_1, h_2, q, \epsilon_0^1, \epsilon_1^1)$, and is given by

$$V_1(h_1, h_2, q, \epsilon_0^1, \epsilon_1^1) = max\left[R_1(h_1, q) + \epsilon_1^1 + \beta EV_1(0, h_2), R_0(h_1, q) + \epsilon_0^1 + \beta EV_1(h_1, h_2)\right]$$

and if we assume the scale parameter of the normalized extreme value components $(\epsilon_0^1, \epsilon_1^1)$ of the return to doing a transplant or not is σ the probability $\rho_1(h_1, h_2, q)$ that center 1 will do a transplant is

$$\rho_1(h_1, h_2, q) = \frac{exp\{v_1(h_1, h_2, q)/\sigma\}}{exp\{v_0(h_1, h_2, q)/\sigma\} + exp\{v_1(h_1, h_2, q)\sigma\}}$$

where

$$v_0(h_1, h_2, q) = R_0(h_1, q) + \beta EV_1(h_1, h_2)$$

and

$$v_1(h_1, h_2, q) = R_1(h_1, q) + \beta EV_1(0, h_2),$$

where

$$EV_1(h_1,h_2) = \int_{q'}\int_{h_1'}\int_{h_2'} \phi_\sigma(v_0(h_1',h_2',q'),v_1(h_1',h_2',q'))F(dq')H_1(dh_1'|h_1)H_2(dh_2'|h_2),$$

where

$$\phi_\sigma(v_0,v_1) = \sigma\left[exp\{v_0/\sigma\} + exp\{v_1/\sigma\}\right],$$

and $EV_1(0,h_2)$ is defined similarly,

$$EV_1(h_1,h_2) = \int_{q'}\int_{h_1'}\int_{h_2'} \phi_\sigma(v_0(h_1',h_2',q'),v_1(h_1',h_2',q'))F(dq')H_0(dh_1')H_2(dh_2'|h_2).$$

Note that if we treat the state space of the game as the unit cube again (i.e. points (h_1,h_2,q) where $0 \le h_1 \le 1$, $0 \le h_2 \le 1$ and $0 \le q \le 1$), the state space divide along the line $h_1 = h_2$ into two separate regions. In the half-cube where $h_1 < h_2$ firm 1 can do a transplant and this region is further subdivided into a region where firm 1 does the transplant and a region where it decides not to do the transplant. In the $h_1 > h_2$ half-cube then firm 2 has the option to do the transplant and this region is also further divided into a region where firm 2 decides to do the transplant and the remaining region where it does not do the transplant.

As a result, just as in the monopoly case, the overall state space ends up partitioned into three regions

0 . A region where no transplant is done on either patient,

1 . A region where a transplant is done on patient 1,

2 . A region where a transplant is done on patient 2.

By solving both the monopoly and duopoly problems it should be possible to characterize the three regions in each case. The "competitive impatience" effect would then be evident if the region 0 in the duopoly problem is a strict subset of the corresponding region in the monopoly transplant problem.

Offhand I see no elmentary reasoning that would immediately lead to this conclusion. In fact the two region 0's may not necessarily be ordered by set inclusion and thus the determination of "competitive impatience" might be a bit more tricky to define in such a case. However I think this sort of model provides a start on simple theoretical framework where it might be possible to formally define the competitive impatience effect and prove that it exists under certain conditions/assumptions. Both problems should be do-able and the MPE should be relatively easy to compute and potentially even unique since the MELD allocation scheme and the randomly evolving health statuses of the two patients have the effects of making this a "randomly alternating move" game. Though this by itself is not guaranteed to result in a unique equilibrium, there is some chance that the solution could have nice properties and might be realatively easy to compute via simple successive approximations methods. The monopoly problem can be solved efficiently as a single agent dynamic programming problem using policy iteration.