Fall 2015 John Rust
Economics 611 Georgetown University

Problem Set 1, Due in class Tuesday September 15th

0. Equation solving Find all solutions to

$$\exp(x) + x + \sin(x) = 0 \tag{1}$$

1. Monopoly profit maximization with linear demand: Bertrand vs. Cournot approaches Consider a monopolist that has a constant returns to scale production function and can produce any (continuous) amount of a good $q \geq 0$ at a constant marginal $c \geq 0$. Suppose the monopolist faces a linear demand function for its product, $q_d = a - bp$, where q_d is the quantity of the monopolist's good that customers demand when the price is p and $a > 0$ and $b > 0$ are positive constants. However we can also compute the inverse demand function as $p = \frac{1}{b}(a - q)$ and interpret p as the per unit price the monopolist could receive if the monopolist produced an amount q and put the entire amount q up for auction in the market under the requirement that all consumers pay the same per unit price p (i.e. no price discrimination).

a. Compute the monopolist's optimal price and quantity under the assumption that the monopolist is a *price setter,* i.e. the monopolist chooses the price that maximizes its profits. We refer to this as the *Bertrand model* of the monopolist's behavior.

b. Compute the monopolist's optimal price and quantity under the assumption that the monopolist is a *quantity setter,* i.e. the monopolist chooses the quantity that maximizes its profits. We refer to this as the *Cournot model* of the monopolist's behavior.

c. Show that the Bertrand and Cournot solutions are the same in this case.

d. Now suppose that instead of a *linear* demand function the monopolist faces a general demand function $q = D(p)$ where $D(p)$ is a differentiable function of p satisfying $D'(p) < 0$. Will the Cournot and Bertrand solutions be the same in the case of a general demand function? (If you say yes, then provide a proof that they result in a same profits, price and quantity produced, otherwise if you say no, then provide an example of a demand function $q = D(p)$ where the Bertrand and Cournot solutions are different.)

e. How do your answers to parts a to d above change if the monopolist faces *a fixed capacity constraint* K? That is, the monopolist can produce at constant returns to scale with marginal cost of c for any $q \leq K$, but at least in the short run (i.e. assuming that we are considering a period of time too short for the monopolist to have time to invest and increase its production capacity K) the monopolist cannot produce any more than K.

2. Duopoly solution with linear demand: Bertrand vs. Cournot approaches Assume that instead of a monopoly, the market is served by 2 firms that both have constant returns to scale production functions with identical marginal costs of production of c for each firm. Assue that the market demand function is linear and the firms are producing *perfect substitutes* and consumers are perfectly informed and face no *switching or transportation costs* of choosing one firm's product versus the other's. This implies that under either the Cournot or Bertrand models, the price of the good must be the same for both firms, since no consumer would buy from the firm with the higher price. Thus, if the market price of both firms' products is p we assume the *total demand* is still linear, $q_d = a - bp$, where q_d is the total amount demanded. Since supply must equal demand, and if q_1 is the amount produced by firm 1 and q_2 is the amount produced by firm 2, then we have $q_1 + q_2 = q_d$.

a. Compute the *Cournot equilibrium quantities* where each firm sets quantity of production as its decision variable. Note that in a Cournot equilibrium each firm's optimal quantity q_i, $i \in \{1,2\}$ depends on its *expectation* of the profit maximizing quantity that its opponent will choose. In a Cournot equilibrium both firms must have *correct expectations*, that it, the amounts each firm expects that the other firm will produce must actually equal the amount that its opponent *actually* will produce and sell.

b. Compute the *Bertrand equilibrium prices* for each firm now under the assumption that instead of choosing quantities of production, the firms choose their respective prices for selling their goods. Just at in the Cournot equilibrium case, in equilibrium the two firms must have correct expectations about the price their opponent will charge its customers.

c. Repeat parts a and b above in the case where the two duopolists have different marginal costs of production, c_1 and c_2. Compute the prices, production/sales, and profits for the two firms in both the Bertrand and Cournot cases.

d. Now consider parts a and b in the case of a general demand function $q_d = D(p)$, where $D'(p) < 0$. Can you say anything about the Cournot and Betrand solutions in the general case when the firms have the same marginal costs of production c? In particular, are the Bertrand and Cournot prices and outputs the same or different from each other? If they are the same, provide a proof of this, if they are different provide an example or an argument as to why they are different.

e. Which of the two equilibrium concepts, Bertrand equilibrium or Cournot equilibrium, do you think is more realistic? That is, when you think about what happens in the real world, which of these models provides a better approximation to the way two firms actually make decisions and compete with each other in actual situations?

3. Cournot and Bertrand oligopoly Now suppose that there are N firms competing in the market. Initially assume that there is linear aggregate demand, $q_d = a - bp$, the firms produce a homogenous good, and that suppy equals demand so that if firms $i = 1,\dots,N$ produce amounts $q_1, q_2, \dots, q_N$, respectively, we have $q_d = q_1 + q_2 + \cdots + q_{N-1} + q_N$. Assume initially that all firms have constant returns to scale production functions with identical marginal costs of production c.

a. Compute the Bertrand and Cournot oligopoly solutions. Show that the Bertrand equilibrium is *competitive* for any finite value of $N > 1$, i.e. $p_B^* = c$ where p_B^* is the Bertrand equilibrium price. However show that under the Cournot equilibrium, show that the Cournot equilibrium price p_C^* satisfies $p_C^* > c$, but that as $N \to \infty$, we have $p_C^* \to c$.

b. Repeat part a except explain what happens if one half of the firms (say firms i where i is an even number) have a constant marginal cost of productio c_1 and the other half (say firms i where i is an odd number) have a marginal cost $c_2 > c_1$. Characterize what the Bertrand and Cournot equilibrium prices, and profits and output would be in this case, both for finite numbers of firms N and as $N \to \infty$.

4. Bertrand duopoly with capacity constraints Suppose there are two firms with identical marginal costs of production c and both can produce at constant returns to scale up to a capacity constraint K_1 for firm 1 and a capacity constraint K_2 for firm 2. What is the Bertrand equilibrium in this case? For simplicity, you can assume that the market demand function is linear.