# Dynamic Structural Models of Retirement and Disability

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#### Summary of Proposal:

We propose to use all available waves of the Health and Retirement Survey (HRS/AHEAD) to estimate a comprehensive dynamic programming (DP) model of behavior at the end of the life cycle that provides a detailed treatment of the Social Security Administration's (SSA) Old Age and Survivors (OASI), Supplemental Security Income (SSI) and Disability Insurance (DI) programs. Major changes to these programs are being contemplated. Yet, we currently lack a unified model of social insurance at the end of the life cycle that can help us evaluate the behavioral and distributional impacts of these policies. Particular attention is paid to developing, estimating, and testing a multi-stage dynamic programming (DP) model of the SSI and DI application, appeal, and award process, for (possibly) heterogeneous agents.

We are developing a tractable empirical model that captures an individual's decisions regarding (1) labor supply and retirement (3) application for OA, DI and SSI benefits, and (4) consumption and savings.

The resulting model will allow us to derive predictions of the behavioral and welfare implications of policy changes. While there is a large literature using reduced-form and static structural models that has investigated some of these issues, it suffers from two major shortcomings. First reduced-form model cannot be used for welfare analysis or to predict behavior responses to policy changes. Second, static structural models do not accurately reflect the level of complexity and uncertainty facing individual decision makers, nor do they capture the important dynamic elements of the decision processes. The DP model we will develop will circumvent these shortcomings, providing a tractable framework for analyzing individual behavior and well-being, and forecasting their response to a wide range of policy changes.

Our model could provide new insights into a number of puzzling aspects about disability in the U.S. One puzzle is to determine the factors responsible for the pronounced swings in DI incidence rates in recent years. Another puzzle is determine why the fraction of Americans receiving SSI and DI benefits continues to increase despite overwhelming epidemiological evidence of steady improvements in various objective indicators of health status. The SSA is currently contemplating significant changes to the disability award process, in order to reduce delays, and reduce large unexplained state-level differences in award rates.

We will use detailed health and functional status indicators from the HRS to evaluate whether or not there are alternative screening rules that can reduce the level of classification errors in the DI award process. Our estimated DP model will produce detailed predictions of the behavioral and welfare effects of changes in benefit levels, delays, the probability of being awarded benefits, and the probability that a DI beneficiary will be audited. This framework will allow us to develop methodologies for characterizing efficient policies, i.e., those that minimize the expected discounted cost of providing a stream of social insurance benefits subject to the constraint that individuals' expected discounted utilities are at least as high as under the status quo.

#### 1 Introduction

We propose to develop a unified empirical model of social insurance at the end of the life cycle using data from the Health and Retirement Survey (HRS). We will pay particularly close attention to accounting for the three key components of social insurance provided by the Social Security Administration (SSA): 1) Old Age and Survivors Insurance (OASI), 2) Medicare and Medicaid, and 3) Disability Insurance (DI) and Supplemental Security Income (SSI) benefits. Because of the rapid aging of the U.S. society, none of these programs are in long run actuarial balance, and sooner or later Congress will have to decide on whether significant tax increases or benefit cuts will be made to keep the program in the black. A number of important changes will begin to take effect in coming years as a result of the 1983 Social Security amendments, which was intended to create incentives for delayed retirement in order to restore the long run balance in the OASI program. In addition to increasing contribution rates, the 1983 amendments increased the normal retirement age (NRA) from 65 to 67, increased the delayed retirement credit (DRC) from 1% to 8%, and decreased the "retirement test" tax on post-retirement earnings from 50% to 33%. In recent years much more radical changes to the Social Security program have been proposed, including full or partial privatization, the introduction of individual accounts, and proposals that would invest a substantial share of the Trust Fund in equities. Another change that was enacted recently is the removal of the earnings test for those 65 and over years of age: the effects of this change can be easily examined in our framework.

Substantial changes to the SSI and DI programs are also being contemplated, resulting in part from the huge increases in processing delays and backlogs following the rapid growth in applications and appeals during the early 1990s. As part of its "Disability Process Redesign" (DPR) plan, the SSA is currently considering major changes in the multi-stage application and appeal process. The proposed changes include simplifications designed to reduce delays in initial award and appeal decisions, and the use of standardized functional impairment indices to reduce large unexplained state-level differences in award rates. All of these proposed policy changes will have significant effects on the structure of the Social Security program, that cannot be accurately predicted, and examined, using reduced-form methods that estimate behavioral relationships under the status quo. SSA currently does not have a unified behavioral model that would enable it to forecast the behavioral and welfare implications of any of these policy changes. The only comprehensive way to deal with these multitude of questions is by modeling explicitly the decision processes by both the individuals and the SSA, under realistic assumptions governing their behavior.

Fortunately, in recent years increasingly realistic dynamic structural models have been formulated and estimated in the academic literature. These models include Rust and Phelan (1997), which estimated a detailed dynamic programming model of the OASI and Medicare program. In contrast to reduced-form papers in the literature, the innovation of Rust and Phelan is that their paper showed that a number of previously puzzling aspects of retirement behavior are simply artifacts of particular details of the Social Security rules. In particular, it showed that OASI and Medicare benefits have complex interacting incentive effects, and must be modeled as such. For example, while many of the features of the OASI rules encourage early retirement, the fact that individuals do not qualify for Medicare benefits until the normal retirement age creates a substantial incentive for a significant fraction of "health insurance constrained" individuals to delay retirement. Their DP model accounts for wide variety of phenomena observed in the data, including the pronounced peaks in the distribution of retirement ages at 62 and 65. These results illustrate the importance of developing an integrated dynamic model of social insurance at the end of the life cycle. Such a model can help us to understand a number of other puzzling aspects about individuals' decisions regarding retirement, disability application, and health insurance, and could have substantial prac-

tical value as a tool to assist policy evaluation and forecasting by government organizations such as the SSA. We will use our model to analyze a number of important policy-related questions and issues including:

- 1. Why does the fraction of Americans on the DI and SSI roles continue to increase when epidemiological studies find that health of older Americans has improved over time?
- 2. What is the relative importance of changes in award rates, unemployment rates, and social factors in the large swings in DI incidence rates in recent years?
- **3.** What role do delays in the DI award process have on incentives to apply or appeal? Will proposals to speed up this process increase the number of applications and awards?
- 4. Will the 1983 Social Security Amendments, particularly the increase in the NRA and DRC, cause individuals, to significantly delay the age at which they apply for OA benefits? How would individuals' be affected if the Medicare eligibility age (MEA) were also changed?
- 5. Will the increase in the NRA increase the incentive to apply for SSI and DI benefits prior to the NRA? If so, to what extent will any reduction in the costs of the OA program due to the increased NRA be offset by an increase in the cost of the SSI/DI program?
- **6.** How would retirement incentives and individual welfare be affected by an introduction of "individual accounts" similar to the plan Governor Bush has proposed?

Although our model can address a wide range of policy issues, we will devote particular attention to modeling the dynamics of disability, mortality and health, and the factors influencing decisions to apply for SSI and DI benefits since these are relatively volatile programs that have grown at unsustainable rates in the past.

## 2 Overview of Issues and Trends in the DI Program

There is a large empirical literature studying the factors affecting DI applications and awards, and a somewhat smaller literature on the SSI program. This literature, (see, e.g. Rupp and Stapleton, 1996 or Stapleton et al. 1994 for an overview) has identified a number of important factors: 1) benefit levels, 2) program leniency as measured by award probabilities and audit rates, 3) strength of the demand for labor (as measured by unemployment rates), 4) the availability (or lack thereof) of alternative sources of support, and 5) social attitudes, particularly those affecting any possible stigma associated with receiving DI benefits. However, the relative importance of these factors is still not well understood, hampering SSA's ability to do policy analysis and short and long term forecasting. Figures 2.1 and 2.2 illustrate some of the key historical and forecasted trends in the DI program.<sup>1</sup>

Figure 2.1 summarizes the historical and projected trends in the size and cost of the DI program, measured by the DI prevalence rate and by the ratio of DI expenditures to GDP. The right hand side of Figure 2.1 shows a rapid rise in the cost of the DI program since its inception in 1956 until the mid 1970s, interrupted only by a decrease in the cost of the program during a period of retrenchment from 1977 to 1990, and a decrease during the economic boom of recent years. The Actuary forecasts continued growth in the program over the next 75 years, topping out at roughly 0.9% of GDP by 2075. The left hand panel of figure 2.1 plots historical and projected prevalence of DI over the period 1988 to 2075. We see that prevalence rates have increased steadily over the period 1988 to 1996, pausing briefly in 1997 and 1998. The Actuary forecasts a particularly rapid

<sup>&</sup>lt;sup>1</sup> We thank Eli Donkar and William Ritchie of the Social Security Office of the Actuary for providing forecasts of GDP and DI roles and expenditures. These forecasts were used in the 1999 Social Security Trustee's Report and are discussed in more detail in an appendix to the Final Report of the 1999 Technical Panel on Assumptions and Methods of the Social Security Advisory Board, of which Rust was co-author.

increase in DI prevalence until 2030, by which time most of the baby boom generation will have reached normal retirement age. Thereafter prevalence continues to grow at a more moderate rate reaching 7% of the insured population by 2075. The "adjusted" prevalence curve reflects a forecast based on a counterfactual assumption that the age distribution of the U.S. remains at its 1998 values. The unadjusted prevalence rates increase from 0.4 to 0.7 between 1999 and 2075, whereas the adjusted prevalence rate only increases to 0.5. Thus, in rough terms, population aging accounts for only one third of the projected increase in prevalence of DI in the next 75 years.

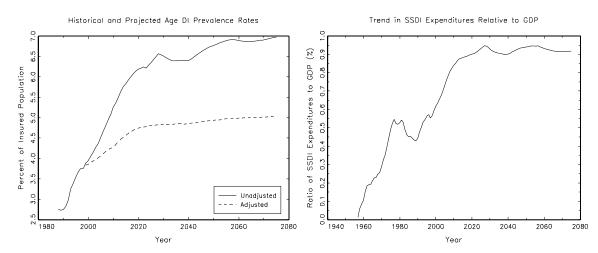


Figure 2.1: Historical and Forecasted Growth in SSDI Roles and Costs

Figure 2.2 illustrates some of the historical volatility in application and award rates. The left hand panel plots the trend in the "crude" acceptance rate—the ratio of the number of new DI awards to the total number of applications and appeals files in a given year. The right hand panel plots the ratio of DI applications and awards to the DI insured population. The award rate reached its lowest level in 1982 during the Reagan Administration, during a clamp-down on the DI and SSI programs. There was a large increase in audits, also known as "Continuing Disability Reviews" (CDR), during this period. The combined effect was to strongly discourage individuals from applying for DI benefits, as is evident in the right hand panel of figure 2.2. On the other hand DI applications and awards peaks in 1974. This peak in applications was due to several factors: 1) a recession in the early 1970s, 2) a rapid increase in benefit levels due to an error in the 1972 Social Security amendments which resulted in an inadvertent double indexing of Social Security benefits to inflation, and 3) a lenient policy towards DI applicants reflected in the high award rates in figure 2.2. SSI was also introduced in 1974, so the public may have actually perceived that SSA as encouraging applicants, reducing perceived "hassle costs" to applying for DI or the stigma associated with receiving benefits. DI application rates began growing rapidly again in the early 1990s following a sustained period of growth in award rates. The causes of this rapid burst of growth are not fully understood, but high unemployment rates in the early 1990s, and a cutback in state General Assistance (GA) programs are thought to important contributing factors. The passage of the Americans with Disabilities Act in 1990 was designed to force employers to accommodate workers with disabilities and thus reduce the incidence and prevalence of individuals receiving DI benefits, however, it may have had an opposite effect by helping to reduce the stigma of being disabled.

On the other hand, application rates declined equally quickly after 1993. This was also the peak year for enrollments in the AFDC program, and a period of high social stigma towards welfare

recipients may have been one of the most important factors motivating the tough 1996 Welfare Reform Act. While part of the decline in application rates might be ascribed to an increase in real or perceived stigma towards AFDC and SSI recipients, the years after 1993 have also constituted the longest peacetime economic boom in recorded history. Only within a structural model can one make an attempt to disentangle the relative contributions of these two possible explanations.

Trends in SSDI Acceptance Rate

Figure 2.2: History of SSDI Application and Award Rates, Roles, and Costs

Our own previous empirical work using the HRS data shows that self-reported health and disability status indicators are powerful predictors of which individuals choose to apply or appeal for DI benefits. Nevertheless, the patterns of fluctuation in aggregate DI award and application rates suggests that individual decisions are governed only partly by "objective" physical and health conditions. Social, political, and economic factors have just as significant impact on application and awards at the aggregate level.

The paradox that DI prevalence rates have grown while the objective health status of Americans has improved, suggests that the concept of "disability" used by the SSA is not based on an absolute objectively determinable measure of physical status, but is rather more akin to a socially defined concept whose absolute standards may change over time with changes in the political, social, and technological climate. Clearly, the nature of physical/mental conditions that are regarded as disabling is very different in today's "information economy" than they were in an industrial/agrarian economy in the 1800s. It is not surprising therefore that SSA's Actuarial Study 114 (1999) documents significant changes in the distribution of impairments that are listed as the primary reasons being awarded DI benefits. For example, mental conditions in recent years account for a much larger share of disabilities than in 1982; 20% compared with only 11% of all awards, respectively (See also Gruber and Kubik 1997 for further discussion).

Although the DI program was intended to be an insurance program, and not a means-tested transfer program, the DI program effectively has many of the characteristics of a welfare program. Our previous research indicates that most DI beneficiaries are in the lowest income brackets. A possible explanation for the upward trend in the prevalence of DI is the fact that a richer, more technically advanced society can afford the luxury of supporting an increasing fraction of its least healthy and productive citizens.

We believe that the way SSA administers the DI award process can help to create a "social standard" that has a powerful impact on the public's perception of the thresholds for mental and physical impairments that are sufficiently severe to constitute "disability". Indeed, we have shown

(see Benítez-Silva et al. 2000) that DI applicants satisfy an "unbiased reporting" hypothesis, i.e., their self-reported disability status is an unbiased indicator of SSA's ultimate award decision. This finding suggests the possibility that tightening or loosening of DI award rates may have a double effect. Its direct effect is on the individuals' incentives for applying for benefits since it affects their chances of success. The indirect effect is through the individuals' self-perceptions of whether or not they believe they are, in fact, disabled. Our model incorporates this fact and allows "disability" to be a social standard that evolves slowly over time. Whether the current social standard is just a "veil" that affects individuals' decisions (through calculations of costs and benefits to filing for DI or SSI benefits), or whether the social standard also has direct affects on their self-perceptions of their capacity to work, is an extremely important issue that we plan to analyze.

## 3 Reduced-form Models of the DI/SSI Award Process

The DI and SSI programs can be viewed as a game between applicants and the government. The outcome of this game depends on the objectives of the government (e.g. Social welfare maximization), and preferences of the individuals. There is, however, an additional complication: the "government" is not one decision maker, but rather a hierarchical bureaucracy with over 15,000 administrators. These are divided among the 54 state Disability Determination Services (DDS's) that process initial applications and reconsiderations, and the more than 1,000 Administrative Law Judges (ALJ's) and Appeals Board that process appeals. The complete DI application, award, and appeal process has been modeled econometrically, using the SIPP panel data, in Hu et al. (1997) and Lahiri et al. (1995), and Benítez-Silva et al. (1999) with the Health and Retirement Survey. The latter paper estimated a detailed reduced-form multi-stage model of an individual's decision to apply for SSDI and SSI benefits, the DDS "first stage" award decision, the decision by rejected applicants to appeal an initial rejection, and the "second stage" decision by SSA's Administrative Law Judges and Appeals Board whether to award or deny an appeal. The paper finds that there is a large apparent return to appealing an initial rejection by the DDS. The "ultimate award rate" rises from about 46%, at the first stage decision made by the DDS, to about 73% when the option to appeal was considered. However, this increased award rate comes at the cost of substantial delays, which appear to function as an implicit type-dependent "application fee" that dissuades opportunistic behavior. Indeed, the study not only shows that an individual's self-assessed health status is one of the most powerful predictors of application and appeal decisions, but also that it is a very powerful predictor of SSA's award decision.

The Benítez-Silva et al. (1999) study also revealed the importance of opportunistic economic factors and policy incentives affecting an individual's decision to apply for DI benefits. For example, only very few individuals who are over 62 apply for DI benefits, specifically because they can get early Social Security retirement benefits at this age. Even though DI pays a benefit equal to the full Primary Insurance Amount (PIA) payable at normal retirement age, in contrast to the early retirement benefits, the costs associated with long delays outweigh the difference in benefits.<sup>2</sup>

However, Benítez-Silva et al. (1999) is just an exploratory analysis into the overall disability award process, and their reduced-form results cannot be used to forecast the effects of policy changes, such as changes in award rates, audit rates, or benefit levels. The only way this can be done is via a structural econometric approach that models the individual's application decision.

<sup>&</sup>lt;sup>2</sup> Other econometric studies have revealed the importance of occupational and economic factors (e.g. the local unemployment rate) on disability application and award decisions. It is also well known (see Social Security Advisory Board, 1998), that there are large differences in the delays and award rates among the 54 state-run DDS's and the over 1,000 ALJ's who handle appeals of DI denials.

There are a number of static structural models of the DI application process, such as Halpern and Hausman (1978) and Kreider (1999). The problem with these models is that they are incapable of capturing the dynamic aspects of the application and appeal process. We use dynamic programming to model the sequential decision process of whether or not to apply for DI benefits, allowing us to explicitly account for the potential returns, uncertainties and "hassle costs" associated with submitting an initial application for benefits, and deciding whether to appeal or re-apply upon denial. The success of the DP approach in modeling and predicting retirement behavior (see, e.g. Rust and Phelan 1997) strongly suggests that it will yield accurate forecasts of changes in various aspects of the DI program and award process.

## 4 Dealing with the Endogeneity of Disability and Health Status

A pervasive concern with the use of self-reported health and disability measures in behavioral models is that they are biased and endogenous. A commonly suggested explanation is that survey respondents exaggerate the severity of health problems and incidence of disabilities in order to rationalize labor force non-participation, application for disability benefits and/or receipt of those benefits. We re-examined this issue (see Benítez-Silva et al. 2000) using a subsample of HRS respondents who applied for DI or SSI benefits, and for which the SSA's award/deny decision could be ascertained. For these individuals we have two independent indicators of the individuals disability status: the individual's self-reported disability status  $\tilde{d}$  and SSA's award decision  $\tilde{a}$ . The indicator  $\tilde{d}$  equals 1 if the respondent reports that they have a health problem that prevented them from working entirely, and 0 otherwise. The indicator  $\tilde{a}$  equals 1 if the respondent was ultimately awarded DI or SSI benefits (including those who were initially denied but were successful upon appeal), and 0 otherwise. We tested the hypothesis—which we term the unbiased reporting (UR) hypothesis—that DI and SSI applicants are aware of the criteria and decision rules that SSA uses in making awards, and individuals act as if they were applying these same criteria and rules when reporting their own disability status. The UR hypothesis amounts to moment restrictions given by

$$E\{\tilde{a} - \tilde{d}|x\} = 0 \iff Pr\{\tilde{a}|x\} = Pr\{\tilde{d}|x\},\tag{1}$$

where x denotes a vector of objectively measurable health and socioeconomic characteristics. Using a variety of parametric and non-parametric tests (e.g. Horowitz and Spokoiny (1999)), that have optimal rate of convergence against a broad class of non-parametric alternatives, we were unable to reject the UR hypothesis, even when we used a large number of objectively measurable health and socioeconomic characteristics similar those the SSA uses in making its award decisions, as instruments. Thus our results provide evidence that DI applicants do not exaggerate their disability status—at least in anonymous surveys such as the HRS.

Given the relatively low sample sizes and the absence of functional form restrictions, the power of these conditional moment tests can be low. Hence, we also performed Wald and Likelihood Ratio tests of a parametric version of the UR hypothesis where the conditional probabilities are derived from the bivariate probit function:

$$Pr\{\tilde{a}|x\} = E\{I[x\beta_a + \epsilon_a \ge 0]\}, \quad Pr\{\tilde{d}|x\} = E\{I[x\beta_d + \epsilon_d \ge 0]\}, \tag{2}$$

where  $(\epsilon_a, \epsilon_d)$  have a bivariate normal distribution with mean 0, unit variance, and correlation coefficient  $\rho$ . Here, the UR hypothesis amounts to the restriction  $\beta_a = \beta_d$ . We employ both a one-group model and two-group model. In the latter we allow for two types of individuals in the population. In this model there are two sets of parameter, one for each group,  $\beta_{dj}$ ,  $\beta_{aj}$ , and

corresponding  $\rho_j$ , j = 1, 2. The UR hypothesis imply then that  $\beta_{aj} = \beta_{dj}$  for j = 1, 2. Again, we are unable to reject the UR hypothesis at conventional significance levels.

The parametric model suggests the following interpretation of the UR hypothesis. Without loss of generality, SSA's ultimate award decision can be represented by an index rule depending on information x that is observed by the econometrician and other information  $\epsilon_a$  that is observed by SSA, but unobserved by the econometrician. The coefficient vector  $\beta_a$  represents the weights SSA assigns to various health conditions and socioeconomic characteristics in coming up with an overall "disability score"  $x\beta_a + \epsilon_a$ . Individuals with sufficiently high disability scores are awarded benefits (represented by the arbitrary cutoff  $x\beta_a + \epsilon_a \geq 0$ ) and the rest are denied benefits. Similarly, the individual's self-reported disability status can also be represented by an index rule with the corresponding vector of weights  $\beta_d$ , and private information  $\epsilon_d$ , that is observed by the individual but not by the econometrician. The UR hypothesis amounts to a rational expectations restriction that individuals use the same variables and weighting vector — i.e. the same criteria — as SSA uses to make DI determinations. However, individuals' self-reports also depend on private information  $\epsilon_d$ , that is unobserved by the SSA. Likewise, the SSA's award decision may be affected by "bureaucratic noise"  $\epsilon_a$  that the individual does not observe. This implies that the indicators  $\tilde{a}$  and d are not perfectly correlated, however, if UR restriction holds then they have identical conditional probability distributions. Thus, while we conclude that self-reported disability status appears to be "accurate" and "exogenous" in the sense of conforming to the same standards SSA uses, from an aggregate perspective disability status is endogenous, since the UR hypothesis implies that changes in SSA's criteria for determining disability status affects individuals' self-perceptions of disability status. An important aspect of our research will be to examine the implications of this hypothesis for policy making by SSA.

## 5 A DP Model of OASI, SSI, DI and Medicare

This section outlines our plans to estimate a dynamic programming (DP) model of male and female labor supply and social insurance application decisions at the end of the life cycle. The following key aspects of the U.S. Social Security program and private insurance and pensions will be modeled: (1) Old Age and Survivors (OASI); (2) Medicare/Medicaid (MC, MCA); (3) Disability, including Supplemental Security Income (SSDI, SSI); (4) private health insurance; (5) private pensions and annuities; (6) Unemployment and Worker's Compensation (UI, WC), and (7) joint decisions of couples in a household. Our initial work will focus on developing a model that incorporates items (1) to (4), and once this is successful we will go on to extend the model to incorporate items (5) to (8).

Our ultimate goal is to develop a DP model where decisions are made on a monthly basis. However, our initial work will focus on building a DP model where decisions are taken at annual intervals. A monthly model is necessary for modeling certain details of the DI application and appeal process, since a sequence of application, award/denial, and appeal decisions can often occur within a single year. We plan to use the value functions (to be described below) from our annual model as terminal value functions for a series of nested monthly level DP "sub-problems". This extended model will incorporate the month by month decisions on labor supply, pensions, disability unemployment, workmen's compensation application decisions, etc. The DP model will be programmed in C, using the Parallel Virtual Machine (PVM) library to distribute the computation over several networked Unix workstations located at Yale, Brown, and SUNY. All data and source code developed from NIH funding will be freely available to the research community over the web.

Perhaps the best way to describe the DP model and illustrate the feasibility of our approach

is to formulate, solve, and simulate a specific prototype of the DP model we are planning to extend, and empirically implement, over the course of this research project. We emphasize that the following model is preliminary and is presented to provide a concrete illustration of how a relatively simple and parsimoniously parametrized specification of a DP model can yield richly detailed, intuitively plausible solutions. We do not claim that this illustrative model is empirically "realistic" at this point. Indeed, the purpose of this proposal is to obtain the necessary funding to develop empirically realistic specifications whose unknown parameters will be econometrically estimated using the HRS/AHEAD data.

A key part of our research plan is to subject our models to rigorous specification and goodness of fit tests, as well as tests of the model's ability to provide out-of-sample predictions of individuals' responses to policy changes. One interesting set of out-of-sample predictive tests will be conducted by John Rust, who is an expert advisor to SSA's demonstration project for the 1999 "Ticket to Work and Work Incentives Improvement Act" (PL 106-70). This legislation is designed to create increased incentives for DI and SSI beneficiaries to return to work, and to ultimately leave the roles via a variety of policies, including: (1) altering the 24 month waiting period for Medicare benefits; (2) lengthening the trial work period; (3) gradually reducing benefits paid to workers similar to the OA "earnings test" rather than cutting off benefits entirely; and (4) expanding the use of job training, vocational rehabilitation, and placement services. The legislation requires the SSA to conduct controlled experiments with human subjects to gauge which of these policies (or combination thereof) would be most effective in encouraging DI beneficiaries to return the work, and ultimately leave the roles. This type of experiment provides an ideal opportunity for testing the predictions of our DP model. For each proposed policy change we can solve the DP model and generate predictions of how individuals will react to it. Then, we can compare these predictions to the outcome of the demonstration experiments to see how close the responses of the humans are to the predictions of the DP model.

If the DP model has a good in-sample fit (using the HRS/AHEAD data) and is able to accurately predict individuals' response to policy changes out-of-sample (and as discussed above, there is strong theoretical reason to believe that a structural DP is the most promising approach for predicting and analyzing policy changes), then it should be clear that our model could have a wide array of practical applications including evaluating the fiscal, welfare, and distributional implications of alternative proposals for reforming various parts of the U.S. Social Security system. We provide an example of how our model can be used to evaluate even fairly "radical" policy changes, including partial or complete "privatization" of the Social Security system, and recent proposals for changing the complex bureaucratic process for awarding disability benefits.

Although simplified in several respects, our illustrative model already constitutes one of the most ambitious and detailed computational models of life-cycle behavior that has ever been formulated and solved. We pay special attention to providing a fairly realistic treatment of the main features of the U.S. Social Security system including the Disability Insurance program. Except for the Ph.D. thesis of Benítez-Silva (2000), no other study has attempted to model both the consumption/savings and labor/leisure decisions jointly in a framework that includes a realistic treatment of the U.S. Social Security system.<sup>3</sup>

We solve an 80 period model by backward induction, starting from the terminal age of 100 and working backward until age 21, when we assume individuals enter the labor force. Agents in our model make three decisions at the start of each period. These choices are denoted by the three variables  $\{l_t, c_t, ssd_t\}$ . Here,  $l_t$  denotes leisure, that is, the amount of waking time devoted

<sup>&</sup>lt;sup>3</sup> Although French (2000) presents a similar model, there is no attempt made to analyze consumption and wealth accumulation.

to non-work activities, where the amount of waking time during a year (assumed to be 12 hours per day, or a total of  $12\cdot365$  hours over the year) is normalized to 1. Thus,  $l_t=1$  corresponds to not working at all,  $l_t=.543=(12\cdot365-2000)/(12\cdot365)$  corresponds to full time work (i.e., working 2000 hours per year), while  $l_t=.817=(12\cdot365-800)/(12\cdot365)$  corresponds to part time work (i.e., working 800 hours per year). The quantity  $c_t$  denotes consumption expenditures, which are, naturally, treated as a continuous variable. The quantity  $ssd_t$  denotes the individual's social security decision, where  $ssd_t=1$  denotes the decision to apply for Old Age benefits,  $ssd_t=2$  denotes the decision to apply for DI benefits, and  $ssd_t=0$  denotes the decision not to apply for benefits. Some of these choices may be infeasible under certain circumstances. For example, individuals who are under the early retirement age (denoted by ERA, currently set at 62) are not allowed to receive OA benefits. Hence, their choice set reduces to  $ssd_t \in \{0,2\}$ . After the normal retirement age the OA and DI programs are formally merged, thus the choice set reduces to  $ssd_t \in \{0,1\}$ . Also if a person is already receiving OA benefits they cannot re-apply for additional benefits, so they face no further choices unless their age t satisfies  $ERA \le t < NRA$ , in which case they still have the option to apply for DI benefits, even while receiving OA benefits.

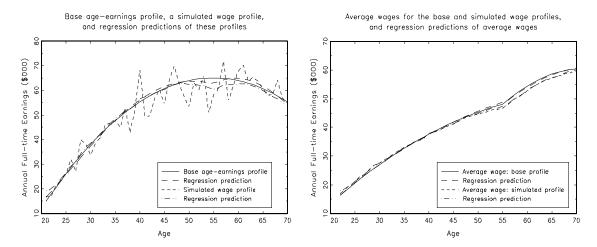
The state of an individual at any point in time can be summarized by four variables: Current age t, net (tangible) wealth  $w_t$ , the individual's Social Security state  $ss_t$ , and the individual's average wage,  $aw_t$ . The  $ss_t$  variable has three mutually exclusive values:  $ss_t = 0$  (not entitled to benefits),  $ss_t = 1$  (entitled to OA benefits), and  $ss_t = 2$  (entitled to DI benefits). As noted above, the OA and DI programs are merged after the NRA, so that any individual with  $ss_t = 2$  is automatically converted to  $ss_t = 1$  when  $t \geq NRA$ .

The average wage,  $aw_t$ , is a key variable in the DP model, serving two roles: (1) it acts as a measure of "permanent income" that serves as convenient "sufficient statistic" for predicting the evolution of annual wage earnings; and (2) it is key to accurately modeling the rules governing payment of Social Security benefits. The Social Security rules are fairly complicated, but can be summarized as follows. An individual's highest 35 years of earnings are averaged, and the resulting Average Indexed Earnings is denoted (on an annualized basis) as  $aw_t$ . The potential Social Security benefit rate for retiring at the normal retirement age (NRA), the so-called Primary Insurance Amount (PIA), is a piece-wise linear, concave function of  $aw_t$ , whose value (on an annual basis) is denoted by  $pia(aw_t)$ .

The fact that  $aw_t$  is an average of the 35 years of highest earnings suggests that to in order to accurately model the U.S. Social Security system we would need to carry as state variables an individual's entire past earnings history. However, due to the "curse of dimensionality" of solving high-dimensional DP problems, this approach is not feasible. Fortunately, it is possible to approximate the evolution of average wages in a Markovian fashion, i.e., to find a formula that allows us to make highly accurate predictions of next period average wage  $aw_{t+1}$  using only age, t, current average wage,  $aw_t$ , and current period earnings,  $y_t$ . Figure 5.1 illustrates this.

We assumed that the potential earnings an individual could make from working full time are given by IID deviations about a quadratic age-earnings profile such as is shown in the left panel of figure 5.1 below. Assuming for the moment that this is the "true model" for individual earnings, we generated an artificial data set of simulated earnings histories, assuming each individual works full time from age 21 until retirement at the ERA. For each simulated earnings history, we used the actual Social Security rules to compute the corresponding sequence of average wages. This is shown in the right panel of figure 5.1. Notice that since  $aw_t$  is a long moving average of past wages wages, the implied path for  $\{aw_t\}$  is far less variable than the path of earnings  $\{y_t\}$ .

Figure 5.1: Age-earnings profiles and the implied path of average wages



We used the observed sequence of average wages as regressors to estimate the following ("misspecified") log-normal regression model of an individual's annual earnings:

$$\log(y_t) = \alpha_1 + \alpha_2 \log(aw_t) + \alpha_3 t + \alpha_4 t^2 + \eta_t \tag{3}$$

Even though this regression does not correspond to the true process by which earnings are generated (i.e. as IID innovations around the quadratic "base case" age-earnings profile in the left panel of figure 5.1), the  $R^2$  of the above regression is .99940, and as we can see from the curves labelled "regression prediction" in the left panel of figure 5.1, the misspecified regression model in equation (3) tracks the true age-earnings profile—the conditional expectation of the wage distribution—very closely. We also estimated a log-normal regression model for average wages:

$$\log(aw_{t+1}) = \gamma_1 + \gamma_2 \log(y_t) + \gamma_3 \log(aw_t) + \gamma_4 t + \gamma_5 t^2 + \epsilon_t \tag{4}$$

We also found of very high  $R^2$  (.99998) for this specification, with an extremely small estimated standard error, resulting from the low variability of the  $\{aw_t\}$  sequences. As we can see in the right hand panel of figure 5.1, the misspecified regression predictions of average wages, using the Markovian approximation in equation (4), provides a highly accurate estimate of the "true" average wages, i.e. the values that come out of the actual formula used by Social Security. This finding is highly encouraging, since it is a key result for an important computational simplification that allows us to accurately model the Social Security rules in our DP model with far fewer variables than would otherwise be needed. In subsequent research we will re-estimate these regression models using the restricted HRS data on Social Security earnings histories and average wages. We use simulated data here to avoid any possible confidentiality issues.

Our DP model also accounts for the other key details of the Social Security rules. For example, there is a penalty for retiring prior to the normal retirement age. That is, an individual's PIA is permanently reduced by an actuarial reduction factor of  $\exp(-g_1k)$ , where k is the number of years prior to the NRA (to a maximum of NRA-ERA) that the individual first starts receiving OA benefits. Our DP model uses the actuarial reduction rate  $g_1 = .0713$  that is currently in effect in the U.S. If a person is accepted into the DI program, he/she receives the full PIA regardless of his/her age.

To increase the incentives to delay retirement, the 1983 Social Security reforms gradually increased the NRA from 65 to 67 and increased the delayed retirement credit. This is a permanent

increase in the PIA by a factor of  $\exp\{g_2k\}$ , where k denotes the number of years after the NRA that the individual delays receiving OA benefits. The rate  $g_2$  is being gradually increased over time. In the simulations below we use the current value of  $g_2 = 0.077$ . The maximum value of k is MRA-NRA, where MRA denotes a "maximum retirement age" (currently 70), beyond which further delays in retirement yield no further increases in PIA. For this reason, it is not optimal to delay applying for OA benefits beyond the MRA, because, due to mortality, further delays necessarily reduce the present value of OA benefits the person will collect over their remaining lifetime.<sup>4</sup>

The final aspect of the Social Security rules concerns taxation of benefits. Individuals whose combined income (including Social Security benefit) exceeds a given threshold must pay Federal Income taxes on a portion of their Social Security benefits. We incorporate these rules in our model as well as the 15.75% Social Security payroll tax, in addition to the Federal income tax, on wage earnings. In addition to these taxes, we account for the Social Security earnings test. If a person retires between the ERA and NRA, each dollar of earnings above a certain threshold (currently \$10,800) results in a 50 cent reduction in Social Security benefits. Between the NRA and MRA the implicit earnings test tax rate falls to 33% for earnings above a higher threshold (currently \$17,000). For individuals who are above the MRA, there is no earnings test.

We can summarize all of the Social Security rules via the Social Security benefit function  $ssb_t(aw, y, ss, ssd)$ . This function completely embodies the SSA's rules, including the determination PIA from average wages, the potential reduction in benefits due to earnings test, and the adjustment of the PIA for retirement before or after the NRA as discussed above. We have coded this function in C and Gauss, which are simplified versions of the SSA's own ANYPIA program, except that our version incorporates SSI and DI benefits as well as OA benefits.

Finally, our model provides a simplified account of the DI award/appeal process. We assume that if a person applies for DI benefits but continues to work, there is no chance of being awarded benefits due to the test for "substantial gainful activity". However, even if a person is not working, there is only a probability  $p_{award}(t, w_t)$  that a person of age t and wealth  $w_t$  will be awarded benefits. Even if benefits are awarded there is a six month waiting period before they can be paid. Adding on the typical delays in the application and appeal process, we assume that if a person applies for DI at the beginning of year t, then he/she will not receive any benefits during that year, but will be notified whether he/she is accepted into the program at the start of year t+1. At that time the individual can start receiving benefits. We do not model the trial work period at this stage, but rather assume that if a DI recipient works full or part time, he/she will be immediately detected and removed from the program. We assume that SSA also randomly audits DI recipients who are not working, and with probability  $p_{audit}(t, w_t)$  a DI recipient can be removed from the roles. We currently set the audit/removal probability to a very low value, about 1%, reflecting the fact that audits have not been extensively used since the Reagan/Bush administration, although the incidence of CDR's has been increasing in recent years.

To complete the specification of the DP model, we need to make some assumptions about individuals' health, mortality, and preferences for leisure and consumption. In this prototype version of the model we have not included an extra state variable to describe health status. Instead, we assume that all individuals of a given age are equally healthy, although we do implicitly account for

<sup>&</sup>lt;sup>4</sup> This is not quite true. It is possible that if wages continue to rise with age, then continued work could increase the individuals' average wage and thus the present value of Social Security benefits from delaying retirement past the MRA, even accounting for mortality. However, if wages do not increase sufficiently fast beyond the MRA, then it will still be true that it will not be optimal to delay retirement beyond the MRA, in the sense that further delays necessarily reduce the present value of Social Security benefits.

<sup>&</sup>lt;sup>5</sup> The earnings test provision has been recently eliminated for individuals 65 and over. However, it was still in place during the time the HRS data was collected and therefore we include it in our model. This policy change allows us to perform a policy experiment that we present in section 6.

a mild "aging effect" by additionally assuming that the disutility of labor effort gradually increases as the person ages. We further assume that the maximum possible age for any individual is 100, but individuals can also die prior to age 100 according to age specific death rates taken from the U.S. Decennial Life Tables (Table 2 in Vol 1., No. 1 of the 1997 Report of the U.S. Center for Disease Control).

If this proposal is funded, a major priority will be devoted to investigating the best way to incorporate health status into the DP model using the rich set of variables in the HRS/AHEAD data set. We will introduce one or more health state variables that can enable us to distinguish between temporary acute health problems and chronic health conditions, as well as measures that account for varying degrees of severity of health problems, with the most extreme of these (short of death) being interpreted as disabling health problems. We will also use HRS/AHEAD to estimate a richer model of mortality, including health, wealth, marital status, education, and other variables as covariates instead of just age and sex that standard life-tables provide.<sup>6</sup>

We assume that the individual's utility for consumption and leisure is given by

$$u_t(c,l) = \frac{[c^{\lambda_t} l^{1-\lambda_t}]^{1-\rho_t}}{(1-\rho_t)},$$
(5)

where we allow the parameters  $(\lambda_t, \rho_t)$  to depend on age, reflecting "aging effects". The parameter  $\rho_t$  is the coefficient of relative risk aversion, and it could potentially increase with age, although we fix  $\rho_t = 1.5$  for all t in the simulation results presented below. The parameter  $\lambda_t$  governs the relative weight the individual places on consumption versus leisure. Decreasing the value of  $\lambda_t$  corresponds to placing less weight on consumption and more weight on leisure, or, stated differently, decreasing the value of  $\lambda_t$  is equivalent to increasing the marginal disutility of labor supply. In the simulations below we assume that  $\lambda_t = .6/(1+t/50)$ , so that  $\lambda_t$  gradually decreases from  $\lambda_{21} = .563$  at age 21 to  $\lambda_{100} = .2666$  at age 100. This can be interpreted as a way of modeling the effect of gradually declining health with age. But, as noted above, there is no "disability" in our model: we do not allow the marginal disutility of effort to suddenly increase, or the wage rate to suddenly decrease. This will be incorporated into future extensions of the model that allow for a health state variable. We also assume that individuals have a bequest motive. This is captured by a utility of bequeathed wealth,  $w_t$  (assumed to occur at death), of  $B(w) = .2w^{(1-\rho_t)}/(1-\rho_t)$ , where  $\rho_t = 1.5$ .

Regarding wage earnings, we model the stochastic evolution of full-time wages, for full-time workers, via the regression model in equation (3). These wages are based on 2,000 hours of work per year. Part-time workers are assumed to work 800 hours per year, and at a wage rate that is only 83.75% of the full-time wage rate. Thus, the annual earnings paid to a part-time worker is 33.5% (.8375 · 800/2000) of the wages paid to a full-time worker, as generated by the model in equation (3).

We account for time and financial costs of applying for DI benefits as a reduction of consumption and leisure. Initially we assume zero financial costs to applying for DI, but assume that there is a small cost in leisure time equal to .005, or a total of 22 hours of waking time to gather the information necessary to pursue an application. We assume there are no time or financial costs to applying for OA benefits. In subsequent work we will treat these costs as unknown parameters to be estimated. We use the notation  $u_t(c, l, ssd, ss)$  to alert readers to the possibility of allowing for potential "stigma costs" to being on the DI roles. However, initially we assume there is no stigma.

Let  $V_t(w, aw, ss)$  denote the individual's value function, the expected present discounted value of utility from age t onward for an individual with current wealth w, average wage aw and in Social

<sup>&</sup>lt;sup>6</sup> We are well underway in pursuing this estimation using some parametric and semiparametric methods, which will enable us to predict the probability of death for an individuals with certain observed characteristics.

Security state ss. We solved the DP problem via numerical computation of the Bellman recursion for  $V_t$  given by

$$V_{t}(w, aw, ss) = \max \{u(c, l, ssd, ss) + \beta [(1 - d_{t})EV_{t+1}(w, aw, ss, c, l, ssd) + d_{t}EB(w, aw, ss, c, l, ssd)]\},$$

$$0 \le c \le w, l \in \{.54, .81, 1\}, ssd \in A_{t}(ss)$$
(6)

where  $A_t(ss)$  denotes the set of feasible Social Security choices for a person of age t in Social Security state ss (see discussion above) and  $d_t$  denotes the age-specific death rate for males, taken from the 1997 edition of the U.S. Decennial life tables as noted above. The functions  $EV_{t+1}$  and EB denote the conditional expectations of next period's value and bequest functions, respectively, given the individual's current state (w, aw, ss) and decision (c, l, ssd). Specifically, we have

$$EV_{t+1}(w, aw, ss, c, l, ssd) = \int_{y'} \sum_{ss'=0}^{2} V_{t+1}(wp_{t}(w, aw, y', ss, ssd), awp_{t}(aw, y'), ss') \times f_{t}(y'|aw)g_{t}(ss'|aw, w, ss, ssd)dy',$$
(7)

where  $awp_t(aw, y)$  is the Markovian updating rule for average wages that approximates Social Security's exact formula for updating an individual's average wage to reflect current earnings, and  $wp_t$  summarizes the law of motion for next period's wealth, that is,

$$wp_t(w, aw, y, ss, ssd) = R[w + ssb_t(aw, y', ss, ssd) + y' - \tau(y', w) - c],$$
 (8)

where R is the return on saving, and  $\tau(y, w)$  is the *tax function*, which includes income taxes such as Federal income taxes and Social Security taxes and potentially other types of state/local income and property/wealth taxes. The  $awp_t$ , derived from (4), is given by

$$awp_t(aw, y) = \exp\left\{\gamma_1 + \gamma_2 \log(y) + \gamma_3 \log(aw) + \gamma_4 t + \gamma_5 t^2 + \sigma^2/2\right\},$$
 (9)

where  $\sigma$  is the estimated standard error in the regression (4). Finally,  $f_t(y|aw)$  is a log-normal distribution of current earnings, given current age t and average wealth aw, that is implied by (3) under the additional assumption of normality of errors  $\eta_t$ . The discrete conditional probability distribution  $g_t(ss'|aw, w, ss, ssd)$  reflects uncertainty in whether a DI applicant will be awarded benefits, or a DI beneficiary will be audited and terminated from the roles. We allow this probability to depend on (w, aw) to reflect means-testing that occurs, for example, in the SSI program. Otherwise it reflects the deterministic transitions for OA benefits, e.g. if a person is over the ERA, is not receiving OA, and applies for OA benefits, then he/she becomes eligible with probability 1 (i.e., if  $ss_t = 0$ ,  $t \ge ERA$  and  $ssd_t = 1$ , then  $ss_{t+1} = 1$ ).

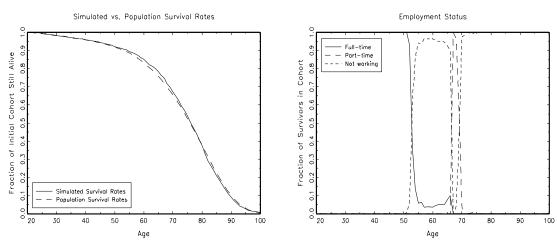
The DP algorithm in equation (??) was programmed in Gauss and C although. Nevertheless, some routines, such as the zbrent nonlinear root finding algorithm, were based on modified code from Press et al. (1992). The zbrent algorithm used was used to solve for optimal consumption (treated as zero to the first order condition, using numerical derivatives), and 15 points Gaussian quadrature was used to compute approximations to the integrals. At each time period the explicit optimizations in equation (??) were performed over a grid of 375 points in the (w, aw) state space (25 grid points for w and 15 grid points for aw), where w ranging from \$1 to \$1,000,000, and aw ranging from \$3,000 to \$72,600. Two-dimensional interpolation was used to compute approximate values for  $V_t(w, aw, ss)$  at (w, aw) points that are not on the predefined grid. The entire DP problem can be solved in less than 5 minutes on a laptop computer (400Mhz AMD K6 processor)Moreover, our software can generate 1,000 IID simulations of the optimal solution in under a second. We expect to obtain further speedups via use of more efficient numerical methods (such as polynomial

approximations to the value functions) and parallel processing techniques. It worth emphasizing that our initial experience make us confident that it is computationally feasible for us to estimate the types of models we are proposing.

Figures 5.2 to 5.4 illustrate the rich types of behavior that the DP model predicts. Each of the curves is an average of 1,000 *IID* simulations, with each simulation corresponding to a separate "person" followed from age 21 until their death. The left hand panel of Figure 5.2 compares the simulated versus the population survival rates, where the latter are those implied by the age-specific mortality rates from the U.S. Decennial life tables. We see that the average of the simulations is very close to the population values, which provides a useful check of the validity of the simulations. The remaining graphs in figures 5.2 through 5.4 present "mean paths" for various variables of interest The averages were computed at each age, for the subpopulation of survivors who lived until at least that age.

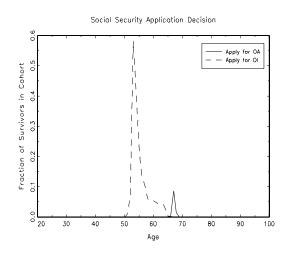
The right hand panel of figure 5.2 shows the employment status of the sample as a function of age. Between the ages of 21 and 50 virtually 100% of the sample works full time. Then, at age 51 there is a precipitous drop in labor force participation. By the age of 60 only about 5% of the surviving subpopulation is still working full-time. Finally, at age of 67 (the NRA), 99.7% of the simulation sample begins working part-time until age 73, when the remaining sample of survivors stops working for the rest of their lives.

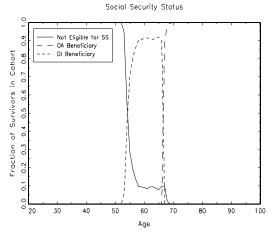
Figure 5.2: Mean Paths from 1,000 IID Simulations of the DP Model



The left hand panel of figure 5.3 provides some insight into what, at first glance, might appear to be a very puzzling pattern of employment behavior implied by the DP model. The graph shows the Social Security decisions made by the members of the simulation sample. Starting at age 50 there is a very rapid increase in the number of people applying for DI benefits, peaking at 54.65% of the surviving sample at age 53, and falling off sharply thereafter until reaching 0 at age 65. The right hand panel of figure 5.3 shows that as a result of the burst in DI applications between the age of 50 and 65, 91% of this sample succeeded in being awarded DI benefits by the time they reached the Social Security ERA. The remaining 9%, who did not apply for, or were not awarded DI benefits, applied for OA benefits at the NRA, 67. After that, with the automatic conversion of DI beneficiaries to the OA program, virtually all of the surviving sample were in the OA program for the remainder of their lives.

Figure 5.3: Predicted Paths for Social Security Decisions and States

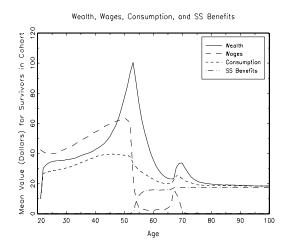


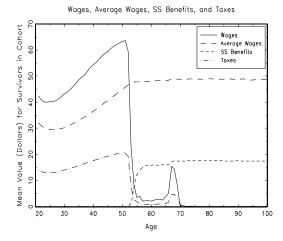


The fact that virtually all of the simulation sample decided to work part time at age 67 may also appear puzzling, but the left hand panel of figure 5.4 provides some insight into why this is happening. The figure shows trajectories for wages, Social Security benefits, consumption and wealth over the life-cycle. Starting with wages, we see that wages increased from their initial value of \$40,000 until peaking at \$63,258 at age 51. During this first 30 years, individuals consumed only 70% of their wage earnings, resulting in a rapid buildup of net worth that peaked at a value of nearly \$100,000 at age 53. At this point a large number of the sample were quitting their full-time jobs and applying for DI benefits. However, since the average Social Security benefit is only about \$15,000 and members of the sample kept consuming at a level considerably above \$15,000, even while they were receiving DI benefits, the members of this sample began rapidly de-accumulating their savings until the NRA when the average remaining wealth, \$23,239, was only slightly higher than average consumption expenditures, \$21,720. Thus, the explanation for the sudden rise in part-time employment can be explained by a need to replenish wealth stocks in order to have sufficient precautionary balances for the remainder of the life-cycle. The other explanation for the rise in part-time employment is that at age 67 the earnings test threshold increases from \$10,800 to \$17,000, but average part-time wage earnings are only \$15,000. Thus, these individuals were able to work part-time without subjecting themselves to the additional 33% earnings surtax. This brief stint of part-time post-retirement work enabled the sample to rebuild wealth stocks to nearly \$34,000 by the age of 70. Afterwards, consumption and wealth both declined to a level equal to the average Social Security OA benefit, which is slightly higher than \$17,000 for the surviving members in this age range.

The right hand panel of figure 5.4 shows the evolution of average wages and tax payments. Average wages start out at \$32,000 and steadily climb to \$48,000 by age 60, reflecting gradual human capital accumulation over the life-cycle. Tax payments increase from \$13,792 at age 21 to a maximum of \$20,540 at age 51, the point at which most of the members of this population begin applying for DI benefits. We see that about this same time, payment of Social Security benefits starts increasing rapidly, as more and more of the simulation sample succeeds in getting awarded DI benefits. It is straightforward to compare the expected discounted sum of tax payments to expected discounted sum of benefits. For this sample the internal rate of return on Social Security contributions is 4.2% if only the individual's tax contributions are counted, and 2.2% if we include the employer's matching contribution.

Figure 5.4: Earnings, Taxes, Social Security Benefits, Consumption, and Wealth





We conclude with some observations about this example. We have simulated a population of 1,000 individuals who are essentially identical, except for stochastic variations in realized wages and Social Security DI awards and audits. Thus, although there is still a great deal of heterogeneity in individuals' simulated labor supply, wage, consumption, and wealth paths, it would be easy for us to add additional sources of heterogeneity into the DP model. This would allow us to simulate a wide range of alternative types of behavior, such as individuals who choose to work full time well into their 70s and 80s before retiring, as well as individuals who never work a significant amount over their entire life-cycle. Our results were "concocted" to emphasize opportunistic behavior in applying for DI benefits. Here we have a sample of individuals, none of whom is ever truly disabled. Instead, their disutility for working is increasing very slowly with age. With full knowledge of the Social Security rules, and betting on the odds of getting accepted into the DI program, this sample of individuals makes unabashed use of the DI program as a type of special, stochastically awarded, "early retirement" benefits. The fact that these individuals are not "truly disabled" is especially transparent from the fact that once they reach NRA and their DI benefits are automatically converted to OA benefits, nearly all of these individuals voluntarily choose to return to work on a part-time basis. They do not choose to do this prior to the NRA (even though they have only \$23,000 of their \$100,000 accumulation of wealth at age 53) because they know that if they start working before the NRA they will fail the DI test for substantial gainful activity and consequently their benefits will be terminated. Note also, that their health, as proxied by their disutility for work, is worse in their 60s than in their 50s, yet they were receiving DI benefits during their 50s and able to return to work in their 60s.

We do not present this example to suggest that all, or even a majority of real DI beneficiaries are behaving opportunistically as the agents in our simulated sample are behaving. However, there are a number of features of disability that do suggest the possibility of opportunistic behavior on the part of a significant fraction of DI applicants and beneficiaries, including the fact noted above that the propensity to apply for DI benefits declines rapidly as individuals approach the ERA. This suggests to us that rational maximizing behavior with full knowledge of the Social Security benefit rules may, in fact, provide a good description of the behavior of many of the people who apply for DI benefits. Our purpose is not to pass judgement on whether opportunistic behavior is necessarily good or bad, but rather to determine to what extent it occurs and to consider whether there are alternative rules for paying out Social Security benefits that might lead to higher welfare, but at lower cost, than under the current system. In particular, our results suggest that it might be more

fruitful to view DI and SSI as types of welfare or early retirement programs. Furthermore, welfare gains provided by operating an expensive "monitoring technology" (i.e., the huge bureaucracy devoted to judging DI and SSI applications and handling appeals, audits and so forth), designed to screen out the "truly disabled", may not generate sufficiently accurate signals to be worth the cost.

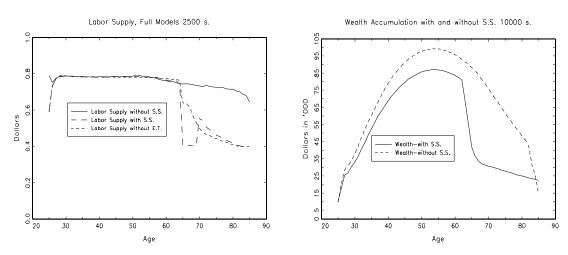
Before we can start to answer these important policy questions we need to find a specification of the DP model, including potential unobserved heterogeneity in various parameters, such as parameters governing earnings potential, preferences for leisure vs. work, risk aversion, subjective discount factors and mortality rates, etc., that enable us to provide a realistic model of individual behavior that pass the "Turing test". Formally this would be a model that would not be rejected by standard specification and goodness of fit tests, but informally, it would be a model for which stochastic simulations would appear statistically indistinguishable from real data such as we have in the HRS/AHEAD. We have considerable room for experimentation to provide empirically realistic DP models. We are very optimistic about the prospects of developing a satisfactory model. However, with this degree of freedom there is always a concern about "over-fitting". That is, there is always a concern that selecting particular functional forms, via "specification searching", that enable us to fit the data well in-sample, might not result in a model that forecasts well out-ofsample. As we noted above we plan to subject our model to one of the most demanding tests by evaluating the accuracy of predictions in out-of-sample predictive tests using participants in the SSA's Ticket to Work and Work Incentives Act demonstration projects. This is a demanding task not only because we are working with a completely different sample of individuals than those in the HRS/AHEAD, but also because the subjects in the demonstration projects will be facing a completely different set of policies than the HRS/AHEAD subjects (whom we use in estimating the unknown parameters of our model) were facing. Our predictions of the effect of policy changes would be made prior to the experimental results being known and would thus represent an honest and demanding test of the accuracy of the DP approach to policy forecasting.

### 6 Policy Analysis and Forecasting using DP Models

In this section we provide several concrete examples of how DP models can be used for policy forecasting and analysis. They also illustrate how it is possible to incorporate private pensions and annuities into the DP model. Similar to the previous section, these results are intended only as illustration purposes, and are not meant to be empirically realistic. Both of the examples are taken from Benítez-Silva's (2000) Ph.D. thesis.

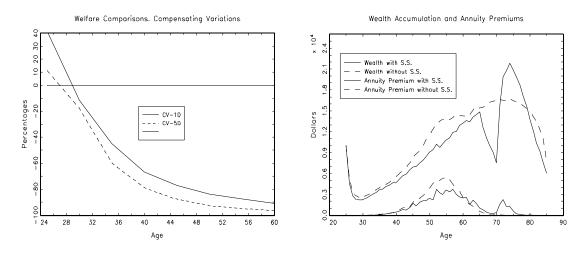
In the left panel of figure 6.1 we present the labor supply effects of Social Security, and the effects of removal of the earnings test provision for individuals 65 and over. For this we use similar model to the one presented in the previous section, but without disability and early retirement. We see that individuals decrease their labor supply at the time that they start receiving retirement benefits, as we saw in the previous section. More importantly, we also observe that the elimination of the earnings test provision has a sizable effect on labor supply, leading to an increase in participation among those of age 65 to 69, once the provision is eliminated. In the right panel of the figure 6.1 we observe the effect of the introduction of Social Security on wealth accumulation, namely, wealth accumulation is much lower, especially after the age of 65.

Figure 6.1: Labor Supply and Wealth Effects of Social Security



However, lower wealth accumulation does not necessarily mean that the individuals are worse off. In the left panel of figure 6.2 we present welfare comparisons (we show compensated variations, in terms of wealth to be compensated with, or wealth willing to pay) in a model with and without Social Security. The welfare implications are striking and rational individuals are well aware of these implications. We observe that young individuals have to be compensated because of the introduction of Social Security, that is, they are worse off under the Social Security regime. In contrast, those older than 40 years of age are willing to pay in order to have the Social Security system in place. This leads to a clear prediction of a "status quo" bias toward maintaining the Social Security system in its current form. This latter result is both novel and important since it provides with a rational explanation as to why Social Security reform seems to be such a delicate and complicated issue.

Figure 6.2: Welfare Comparisons and the introduction of Annuities



The right panel of figure 6.2 shows the wealth accumulation profiles of individuals in a model that allows for the existence of private annuities. Our DP model outline above can also be extended to include private pensions, these can be modeled in similar way to annuities. The figure shows

the paths of wealth and annuity purchases, with and without Social Security in the model. We clearly see the lower accumulation profile when Social Security is operation. Nevertheless, we also observe, similarly to figure 5.4, that after the NRA is reached there is a de-accumulation period, because of the increase in labor supply after the NRA is reached and because the earnings test provision does not apply. In this case the increase in wealth accumulation is more sizable because of the presence of private annuities; individuals want to accumulate enough resources so they can buy the annuity and smooth their consumption for the remainder of their lives. Note that the right panel of figure 6.2 indicates that annuity purchases are mainly late in the individual's life-cycle, and they are relatively in small amounts, on average. This is a possible resolution for the, so called, "annuity puzzle", i.e., the fact the annuity market is relatively small. We find that this situation is the optimal outcome in a DP model that accounts for the endogeneity of labor supply, annuities, and incorporates the Social Security rules.

We conclude this section with a brief discussion of "systematic" approaches to policy forecasting and analysis. We can view policy analysis conceptually as a "Stackelberg game" between the government and individuals. We assume that the government is interested in maximizing welfare of the citizens, but it may weight the welfare of different citizens differently, i.e. it may be interested in redistribution of income or wealth. We can represent a policy of the government abstractly by the symbol  $\pi$ . That is,  $\pi$  should be viewed as a vector of parameters governing details of things like tax rates, ages of early and normal retirement, disability award rates, audit rates, benefit levels and so forth. Given any policy  $\pi$ , individuals form their "best-replies," i.e., they maximize their expected discounted utility subject to the assumption that the government will commit to implementing the given policy  $\pi$ . Let  $V_{\pi}(s)$  denote the expected discounted utility of an individual in state s under policy  $\pi$ . If we identify the states s with individuals (i.e., each individual is in a given state s at the time that the government is determining its policy) we can let  $\mu(s)$  denote the population distribution of the different states (individuals). Finally, let  $w(s) \geq 0$  denote the weight the government places on an individual in state s. For example, if a component of s is income or wealth, this could reflect the government's relative concerns towards rich vs. poor people. Then a "utilitarian" government would choose a policy  $\pi^*$  that maximizes social welfare, i.e.  $\pi^*$  is the solution to

$$\pi^* = \underset{\pi}{\operatorname{argmax}} \int V_{\pi}(s)w(s)\mu(ds). \tag{10}$$

It is computationally feasible to solve "social planning problems" such as in equation (10). The main difficulty, however, is in specifying a weighting function w(s), which represents the government's "preferences" over individuals (states). An alternative, more ethically neutral approach that does not require the specification of the subjective weighting function w(s) is the dynamic mechanism designapproach. We discuss a simplified version of this approach here, which we refer to as computational mechanism design, because we intend to actually implement it in order to characterize efficient Social Security policies. The idea governing this approach is that corresponding to any government policy  $\pi$ , there is a cost  $c_{\pi}(s)$ , of delivering the utility level  $V_{\pi}(s)$  to an individual in state s. The cost  $c_{\pi}(s)$  is the expected present discounted value of benefits, less tax receipts, for a person in state s. If this cost is negative, it means that for person s the present value of their taxes exceeds the present value of the benefits they will receive. Let  $\pi_{sq}$  denote the policy of the government under the status quo. Then an efficient policy  $\pi^e$  is the solution to

$$\pi^e = \underset{\pi}{argmin} \int c_{\pi}(s)\mu(ds)$$
 subject to:  $V_{\pi}(s) \ge V_{\pi_{sq}}(s) \quad \forall s.$  (11)

Thus, an efficient policy  $\pi^e$  minimizes the net costs of providing government benefits to individuals, subject to the constraint that no individual is worse off under the efficient policy than they are

under the status quo. We can compute the expected cost of a government program under the status quo and compare it with the cost of an efficient policy. The difference represents a dollar valued estimate of the inefficiency under the status quo. The current aggregate cost of the U.S. Social Security system is in the range of \$10 trillion dollars, that is, this is the expected discounted value of Social Security's unfunded benefit obligations.

The most ambitious part of our research is to attempt to solve the mechanism design problem (11) numerically and provide a dollar estimate of the inefficiency in the current Social Security system. To make this computationally feasible, we will use a "parametric" approach to the mechanism design problem, whereby we search over a parametrized subclass of policies  $\pi$ , rather than the space of all possible policies  $\Pi$ . The latter approach requires much more sophisticated methods including recursive linear programming and the use of the revelation principle, and imposition of individual rationality and incentive-compatibility constraints. By looking at a subclass of parametrized policies, and explicitly solving for best-replies by dynamic programming, we avoid having to impose the incentive-compatibility constraints, and the individual rationality constraints are satisfied by virtue of the fact that an efficient policy guarantees that nobody is worse off under an efficient policy  $\pi^e$  than under the status quo. The main drawback of our approach is that by restricting our search to a parametric subspace, of the space II of all policies, we will only be able to provide a lower bound on the inefficiency of the current Social Security system. Stated differently, there may be alternative policies that we have not considered in our parametric sub-family of policies that could result in even higher cost savings than the policy we determine. Nevertheless we believe that a sufficiently flexibly parametrized set of policies will provide a good approximation for the "non parametric" solution to the mechanism design problem that does not place an a priori restriction on the space of possible policies. We believe our research in this area could lead to important theoretical as well as practical insights.

# 7 Econometric Implementation

We conclude this proposal by briefly outlining how the DP models will be estimated econometrically. The key is to partition the state vector for an individual at time t, which we denote as  $s_t$ , into observed and unobserved components. We assume the individual knows the full state  $s_t$ , but the econometrician only observes a subset of this information, which we denote by  $x_t$ . The remaining components of  $s_t$ , denoted by  $\epsilon_t$ , are observed by the individual but not by the econometrician. We account for unobserved states not only because it is more realistic to do so, but also because a model that has no unobserved components to the state variable would be statistically degenerate. This means that if we observe the individual's decision  $d_t$  and a full state  $s_t$ , the DP model predicts that there is a deterministic function, the optimal decision rule, linking these two variables:  $d_t = f_t(s_t)$ , where  $f_t$  is a function that specifies the individual's optimal decision at time t and in state  $s_t$ . This decision rule is computed by dynamic programming, as described in the previous section. However, we do not believe any deterministic functional relationship linking  $s_t$  and  $d_t$  would fit the data. This means that without the device of unobserved state variables, certain observed  $(d_t, s_t)$  could not be predicted to have occurred by the DP model, for any possible value of the unknown parameters of the model. As a result, such an observation would contradict the model and result in a zero likelihood of observation. To solve the "zero likelihood" problem, we allow for unobserved states, so that the resulting decision rule has the form  $d_t = f_t(x_t, \epsilon_t)$ , where only  $(d_t, x_t)$  are observed by the econometrician, while  $\epsilon_t$  is unobserved. For certain specifications of how unobservables enter the DP model, it can be arranged that any possible  $(d_t, x_t)$  combination could be "rationalized" for one or more values of the unobserved state variable  $\epsilon_t$ . This solves the zero likelihood problem and enables us to estimate the unknown parameters of the DP model by maximum likelihood.

We now briefly review results of due to Rust (summarized in Rust, 1994) that show how unobserved state variables can be incorporated into the DP model in a tractable fashion, resulting in a likelihood function that enables estimation of the unknown parameters of the DP model.

Let  $s_t = (x_t, \epsilon_t)$  and assume that the choices of interest are discrete, i.e., the individual chooses a decision  $d_t$  from a finite set of alternatives  $C_t(x_t)$ . Assume that  $\epsilon_t$  is a vector with the same dimension as the number of choices in the choice set  $C(x_t)$ . Then  $\epsilon_t(d)$  can be interpreted as a component of utility that depends on the decision d and other unobserved states of the agent. If we further assume that  $x_t$  and  $\epsilon_t$  are conditionally independent, that is,

$$p(x_{t+1}, \epsilon_{t+1} | x_t, \epsilon_t, d_t) = p(x_{t+1} | x_t, d_t) q(\epsilon_{t+1}), \tag{12}$$

and that  $\{\epsilon_t\}$  is an IID Type III extreme value process, then Rust (1994) showed that the DP recursions take the following form:

$$V_{t}(s) = \max_{d \in C(s)} \left[ u(s,d) + \beta \int V_{t+1}(s') p(ds'|s,d) \right],$$

$$= V_{t}(x,\epsilon),$$

$$= \max_{d \in C(x)} \left[ u(x,d) + \epsilon(d) + \beta \int_{\epsilon'} \int_{x'} V_{t+1}(x',\epsilon') p(dx'|x,d) q(d\epsilon') \right]. \tag{13}$$

We can rewrite equation (13) as follows:

$$V_t(x,\epsilon) = \max_{d \in C(x)} \left[ V_t(x,d) + \epsilon(d) \right], \tag{14}$$

where

$$V_t(x,d) = u(x,d) + \beta E V_{t+1}(x,d), \tag{15}$$

$$EV_{t+1}(x,d) = E_{x',\epsilon'} \left\{ V_{t+1}(x',\epsilon') | x, d \right\},$$

$$= E_{x',\epsilon'} \left\{ \max_{d' \in C(x')} \left[ V_{t+1}(x',d') + \epsilon'(d') \right] | x, d \right\},$$

$$= \int_{x'} \int_{\epsilon'} \max_{d' \in C(x')} \left[ V_{t+1}(x',d') + \epsilon'(d') \right] q(d\epsilon') p(dx'|x,d),$$

$$= \sigma \int_{x'} \log \left[ \sum_{d' \in C(x')} \exp \left\{ \frac{V_{t+1}(x',d')}{\sigma} \right\} \right] p(dx'|x,d), \tag{16}$$

and  $\sigma > 0$  is the scale parameter of a Type III extreme value distribution, the marginal distribution of  $\epsilon(d')$ .

Combining (16), (15), and (14) gives the following recursion for  $V_t(x,d)$ :

$$V_t(x,d) = u(x,d) + \beta \int_{x'} \sigma \log \left[ \sum_{d' \in C(x')} \exp \left\{ \frac{V_{t+1}(x',d')}{\sigma} \right\} \right] p(dx'|x,d), \tag{17}$$

with the terminal condition  $V_T(x,d) = u(x,d)$ , where T is the last year of life.

Equation (17) constitutes the basic recursion equation that we will be calculating in order to solve the DP problem with unobserved state variables. The value functions  $\{V_t(x,d)\}$  resulting

from these recursions enable us to derive the conditional choice probabilities  $P_t(d|x)$  that are, in turn, used for maximum likelihood estimation of the DP model:

$$P_{t}(d|x) = Pr\left\{d = \operatorname{argmax}_{d' \in C(x)} \left[V_{t}(x, d') + \epsilon(d')\right]\right\}$$

$$= \frac{\exp\{V_{t}(x, d)/\sigma\}}{\sum_{d' \in C(x)} \exp\{V_{t}(x, d')/\sigma\}}.$$
(18)

The (full) likelihood function for an (unbalanced) panel where individual i is followed for periods  $t = 1, ..., T_i$  is given by

$$L(\lbrace x_{it}, d_{it} \rbrace_{t=1}^{T_i}) = \prod_{i=1}^{I} \prod_{t=1}^{T_i} P_t(d_{it}|x_{it}) p(x_{it}|x_{it-1}, d_{it-1}).$$
(19)

In our model we also have continuous control variables such as consumption, which is not directly observed in the HRS/AHEAD data. Our solution to this problem is that in the value functions that determine the likelihood function (19) consumption is "substituted out" and the value function is only a function of wealth, average wages, Social Security status, health status, age and other variables we can observe. This allows us to estimate the DP model even though we do not observe all of the individual's decisions (such as consumption). We believe this approach to estimation of the DP model is computationally feasible and has been proven in many previous empirical applications to result in reasonable estimates of the model's unknown parameters. That is, the maximum likelihood estimation algorithm allows us to find a "best fitting" DP model, and it is typically very difficult to distinguish stochastic simulations of the best fitting DP model from the real data.

The main disadvantage of the above approach is that the unobserved component  $\epsilon_t$  is uncorrelated with the observed component  $x_t$ . To circumvent this problem we will adopt a strategy previously developed by Heckman and Singer (1984), Keane and Wolpin (1997) and Benítez-Silva et al. (2000a). To briefly explain our approach, assume that there are M "types" of individuals in the population of (unknown) proportions  $\phi_1, ..., \phi_M$ . Further assume that each individual's type has a (possibly) different set of parameters. Our objective then is to estimate the set of M parameter vectors and  $\phi_1, ..., \phi_M$ , corresponding to the M types of individuals. The main problem is that we do not know a priori what is the type of each individual in the sample. Nevertheless, one can estimate the proportion of each type in the population along with the rest of the parameters of the model by maximizing the likelihood function given by

$$L^{M}(\{x_{it}, d_{it}\}_{t=1}^{T_{i}}) = \sum_{m=1}^{M} \phi_{m} \left[ \prod_{i=1}^{I} \prod_{t=1}^{T_{i}} P_{t}(d_{it}|x_{it}) p(x_{it}|x_{it-1}, d_{it-1}) \right].$$
 (20)

Conceptually, the model in (20) is no more difficult to estimate than the model in (19). However, the problem is computational problem, namely one cannot use too many type. The exact number of types will be determined by computational considerations.

This is the first step in estimating and testing a candidate specification of the DP model. The more demanding next step is to see if the estimated DP model yields accurate predictions of how individuals respond to changes in policy. As discussed above, we will use out-of-sample predictive tests, including predictions of individual's responses to SSA's demonstration experiments for the Work Incentives Act to provide a rigorous test of the validity of our DP model.

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