Spring 1997 | John Rust
Economics 551b | 37 Hillhouse, Rm. 27

PROBLEM SET 1

Bayesian Basics

QUESTION 1

Derive optimal estimator with absolute error loss function:

$$l(\theta^*, \widehat{\theta}) = \left|\widehat{\theta} - \theta^*\right|.$$

QUESTION 2

Derive optimal estimator with asymmetric loss function:

$$l(\theta^*, \widehat{\theta}) = \begin{cases} a\left(\widehat{\theta} - \theta^*\right) & \text{if } \widehat{\theta} > \theta^* \\ b\left(\theta^* - \widehat{\theta}\right) & \text{if } \widehat{\theta} \leq \theta^* \end{cases}.$$

This loss function penalizes underestimation more than overestimation when $b > a$.

QUESTION 3

Suppose Beta distribution as a prior distribution of θ:

$$p(\theta) = \frac{1}{B(\alpha, \beta)} \theta^{\alpha-1} (1-\theta)^{\beta-1}$$

where $B(\alpha, \beta)$ is Beta function and $0 < \theta < 1$.

Derive the posterior distribution of θ when X_i $(i = 1, \ldots, N)$ follows Bernoulli distibution

$$f(X_i|\theta) = \theta^{X_i} (1-\theta)^{1-X_i}.$$

QUESTION 4

Show how the posterior distribution in Question 3 changes different value of N given an initial Beta prior which you can choose. You do this by choosing values for the hyperparameters of the Beta prior distribution in question 3. You must also choose the "true value" of θ^* to generate the $(x_1, \ldots, x_N)$ data. For example, choose $\theta^* = 0.3$ and generate Bernoulli random variables for $N = 10, 50, 100, 500$.