Spring 1997 | John Rust
Economics 551b | 37 Hillhouse, Rm. 27

HINTS TO MIDTERM EXAM

(Due: March 24, 1997)

Hint for Problem 1: Use the TCP/IP `ftp` command to get the data set `stockdat.dat` and its Gauss header file `stockdat.dht` by anonymous ftp from `gemini.econ.yale.edu` as follows:

1. `ftp gemini.econ.yale.edu`
2. `login:  anonymous`
3. `password:  econ551`
4. `ftp>cd /pub/John_Rust/courses/econ161/stats/stocks/dat`
5. `ftp>bin` (to change to binary transfer mode)
6. `ftp>get stockdat.dat`
7. `ftp>get stockdat.dht`
8. `ftp>quit`

If you are using Gauss under a Unix system you must do `transdat stockdat.dat` to translate the data set into Gauss's Unix system format. Otherwise you are ready to use the data in the Dos/Windows version of Gauss. Get into Gauss and open up the data set by typing `open f1=stockdat` and then load the data set into memory by typing `dat=readr(f1,rowsf(f1))`. To find out the names of the columns of the data set, do `nms=getname("stockdat")` and the 66×1 column vector `nms` will contain the names of each of the 66 columns in the 754×66 data matrix `dat`. You can use the `loc` procedure (available on the Econ 551 web page along with the other programs) to extract specific columns of the `dat` matrix by name, e.g. `dat[.,loc("VALAV",nms)]` will return the column of the market portfolio returns, where the portfolio is a value-weighted portfolio suggested by the CAPM model as a preferable means of forming an index of returns on the market portfolio.

Hints on Problem 2: In part 6 of problem 2 you are required to compute the posterior probability of being within a ball of radius .01 about θ^*. I suggest computing this probability by monte carlo simulations, drawing say 10,000 draws from the Dirichlet posterior and computing the fraction that lie in the ball of radius .01 about θ^*. How do you draw from a Dirichlet distribution? Use the following result:

Lemma: *Let $(\tilde{G}_1,\ldots,\tilde{G}_K)$ be independent random variables, where each $\tilde{G}_i$ has a gamma distribution with parameter (α_i,β). Then the random vector $\tilde{\boldsymbol{D}} = (\tilde{D}_1,\ldots,\tilde{D}_K)$ has a Dirichlet distribution with parameter $\boldsymbol{\alpha} = (\alpha_1,\ldots,\alpha_K)$ where each $\tilde{D}_i$ is defined by:*

$$\tilde{D}_i = \frac{\tilde{G}_i}{(\tilde{G}_1 + \cdots + \tilde{G}_K)}$$

Hints on Problem 4: Load the data sets `data1.asc` and `data2.asc` by anonymous ftp following the same instructions as above, but `cd /pub/John_Rust/courses/econ551/dat` and `get data1.asc` and `get data2.asc` (there is no need to switch to binary mode since these two data sets are in ASCII format). You will use `data1.asc` to estimate a trinomial logit model with 6 unknown alternative-specific coefficients by maximum likelihood.

To compute the maximum likelihood estimates you first need to write down the likelihood function and derivatives. Only first derivatives are required if you use BHHH but second derivatives are required if you use Newton's method. There are two ways to write down these formulae: 1) write down formulae that are specific to the case at hand, 2) write down formulae that will be useful for estimating any type of multinomial logit model with any number of alternatives.

1. **Alternative-specific trinomial code:** You will need to write a procedure `EVALMNL1.G` that codes the likelihood and first and second derivatives of the trinomial logit model with alternative-specific coefficients. This procedure will then be passed in the setup program that reads in the data and runs the main maximization procedure, `MAX.G`. Partition the unknown 6×1 parameter vector θ into two components, $\theta = (q_2, q_3)$ where q_2 is the 3×1 parameter vector for alternative 2 and q_3 is the 3×1 parameter vector for alternative 3 with q_1 normalized to 0 since it is unidentified. Next write down the log-likelihood function for this model as follows:

$$\begin{aligned}
\log\{P(1|x)\} &= -\log\{1 + \exp(xq_2) + \exp(xq_3)\} \\
\log\{P(2|x)\} &= xq_2 - \log\{1 + \exp(xq_2) + \exp(xq_3)\} \\
\log\{P(3|x)\} &= xq_3 - \log\{1 + \exp(xq_2) + \exp(xq_3)\}.
\end{aligned}$$

2. **General multinomial logit code:** you can write a general multinomial logit procedure `EVALMNL.G` that codes the likelihood and first and second derivatives for a general multinomial logit model with fixed coefficients and alternative-specific covariates. This procedure is then passed in the setup program that reads in the data and runs the main maximization procedure, `MAX.G`. This code will estimate an arbitrary MNL model with an arbitrary number of alternatives. The general model has a covariate vector $x = (x_1, \ldots, x_J)$ which is alternative-specific (with x_j being the covariate vector for alternative j) and the parameter vector θ is not alternative-specific. The likelihood function for the general MNL model has the following form:

$$\log\{P(d|x)\} = x_d\theta - \log\left\{\sum_{j=1}^{J} \exp\{x_j\theta\}\right\}.$$

You can "trick" the general MNL code into estimating the alternative-specific trinomial model of this assignment by defining an 18×1 covariate vector $x = (x_1, x_2, x_3)$ as follows:

$$\begin{aligned}
x_1 &= (0_{1\times 3}, 0_{1\times 3}) \\
x_2 &= (x_{1\times 3}, 0_{1\times 3}) \\
x_3 &= (0_{1\times 3}, x_{1\times 3})
\end{aligned}$$

where $x_{1\times 3}$ is the original non-alternative-specific covariate vector (dependence on observation i, $i = 1, \ldots, 2000$ is supressed for clarity). It is straightforward to verify that when the alternative-specific covariate vector (x_1, x_2, x_3) defined above is substituted into the general formula for the MNL log-likelihood we obtain the special case for the trinomial logit model with alternative-specific coefficients. We now complete the assignment by specifying the first and second derivatives of the log-likelihood function. We will do this for both the alternative-specific and general forms of the MNL model.

First derivatives for trinomial logit model with alternative-specific θ's

$$\begin{aligned}
\partial \log\left\{P(1|x)\right\}/\partial\theta &= (-xP(2|x), -xP(3|x)) \\
\partial \log\left\{P(2|x)\right\}/\partial\theta &= (x[1-P(2|x)], -xP(3|x)) \\
\partial \log\left\{P(3|x)\right\}/\partial\theta &= (-xP(2|x), x[1-P(3|x)])
\end{aligned}$$

First derivatives for general MNL model

$$\partial \log\left\{P(d|x)\right\}/\partial\theta = \sum_{j=1}^{J} \left[I\{j=d\} - P(j|x)\right] x_j$$

Second derivatives for trinomial logit model with alternative-specific θ's

$$\begin{aligned}
\partial^2 \log\left\{P(d|x)\right\}/\partial q_2 \partial q_2' &= -x'xP(2|x)[1-P(2|x)] \\
\partial^2 \log\left\{P(d|x)\right\}/\partial q_2 \partial q_2' &= -x'xP(3|x)[1-P(3|x)] \\
\partial^2 \log\left\{P(d|x)\right\}/\partial q_2 \partial q_3' &= x'xP(2|x)P(3|x)
\end{aligned}$$

Second derivatives for general MNL model

$$\partial^2 \log\left\{P(d|x)\right\}/\partial\theta\partial\theta' = -\sum_{j=1}^{J} x_j x_j' P(j|x) + \left[\sum_{j=1}^{J} x_j P(j|x)\right]\left[\sum_{j=1}^{J} x_j P(j|x)\right]'$$