Econ 551b Econometrics II
Problem Set 2

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Question 1

Let $Y \sim N(0, \Sigma)$ where Y is $k \times 1$ random vector and Σ is $k \times k$ positive definite matrix. Prove $Y'\Sigma^{-1}Y \sim \chi^2(k)$. (*Hint*: Apply Jordan Decomposition on Σ.)

Question 2

The Legendre Polynomials, denoted by $P_0(x)$, $P_1(x)$, $P_2(x)$,..., can be obtained by finding the set of orthogonal vectors that span the same subspace in $L^2([-1,1])$ as 1, x, x^2, x^3,... Using the Gram-Schmidt Orthogonalization Process, find the first four Legendre Polynomials. Then normalize the Legendre Polynomials so that they are orthonormal.

Question 3

Let $\hat{\beta} = (\hat{\beta}_1, \hat{\beta}_2)'$ be the OLS estimate in the regression:

$$y = \beta_1 x_1 + \beta_2 x_2 + \epsilon$$

And $\overline{\beta} = (\overline{\beta}_1, \overline{\beta}_2)'$ be the estimate from stepwise regression. That is, first obtain $\overline{\beta}_1$ as the OLS estimate from the regression:

$$y = \beta_1 x_1 + \mu$$

Then obtain $\overline{\beta}_2$ as the OLS estimate from the following regression:

$$y - P(y|x_1) = (x_2 - P(x_2|x_1))\beta_2 + \eta$$

Where $P(y|x_1)$ and $P(x_2|x_1)$ are projections of y and x_2 on x_1 respectively, i.e., they are the predicted values in the regression of y and x_2 on x_1 respectively. Show that $\hat{y} = x_1\hat{\beta}_1 + x_2\hat{\beta}_2$ and $\overline{y} = x_1\overline{\beta}_1 + (x_2 - P(x_2|x_1)\overline{\beta}_2$ are identical. Suggest when stepwise regression can be useful.

Question 4

Suppose $(Y, X) \sim N(0, \Sigma)$, show that $E(Y|X) = X\beta^*$, where $\beta^* = (E(X'X))^{-1}E(X'Y)$.