Bad Decisions

John Rust

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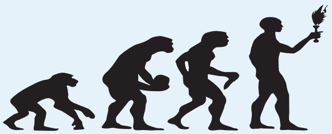
The Singularity is Near



A book by Ray Kurzweil (Viking Press, 2005)

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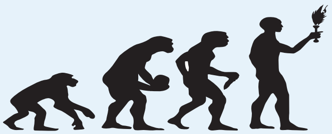

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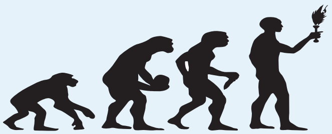
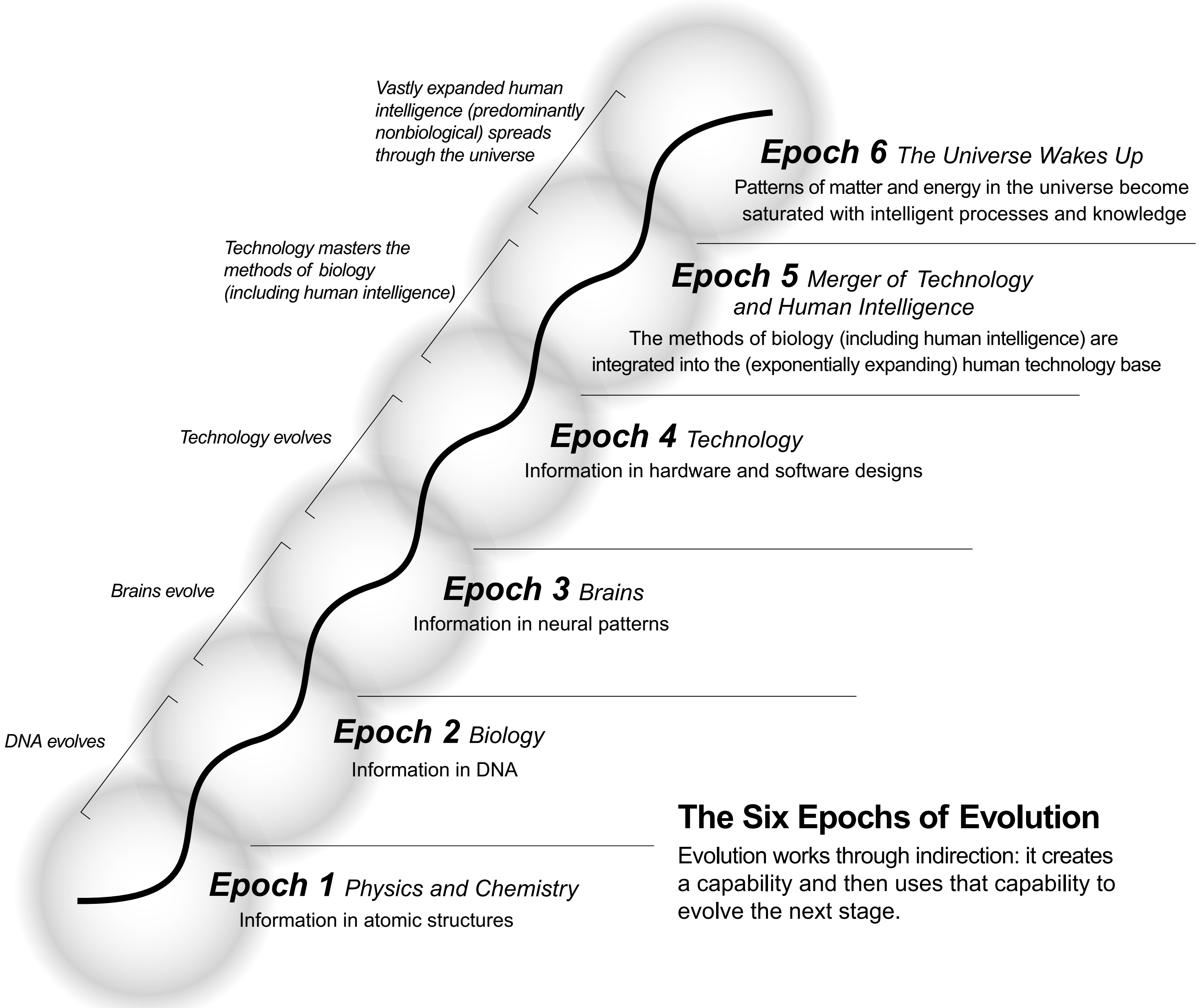
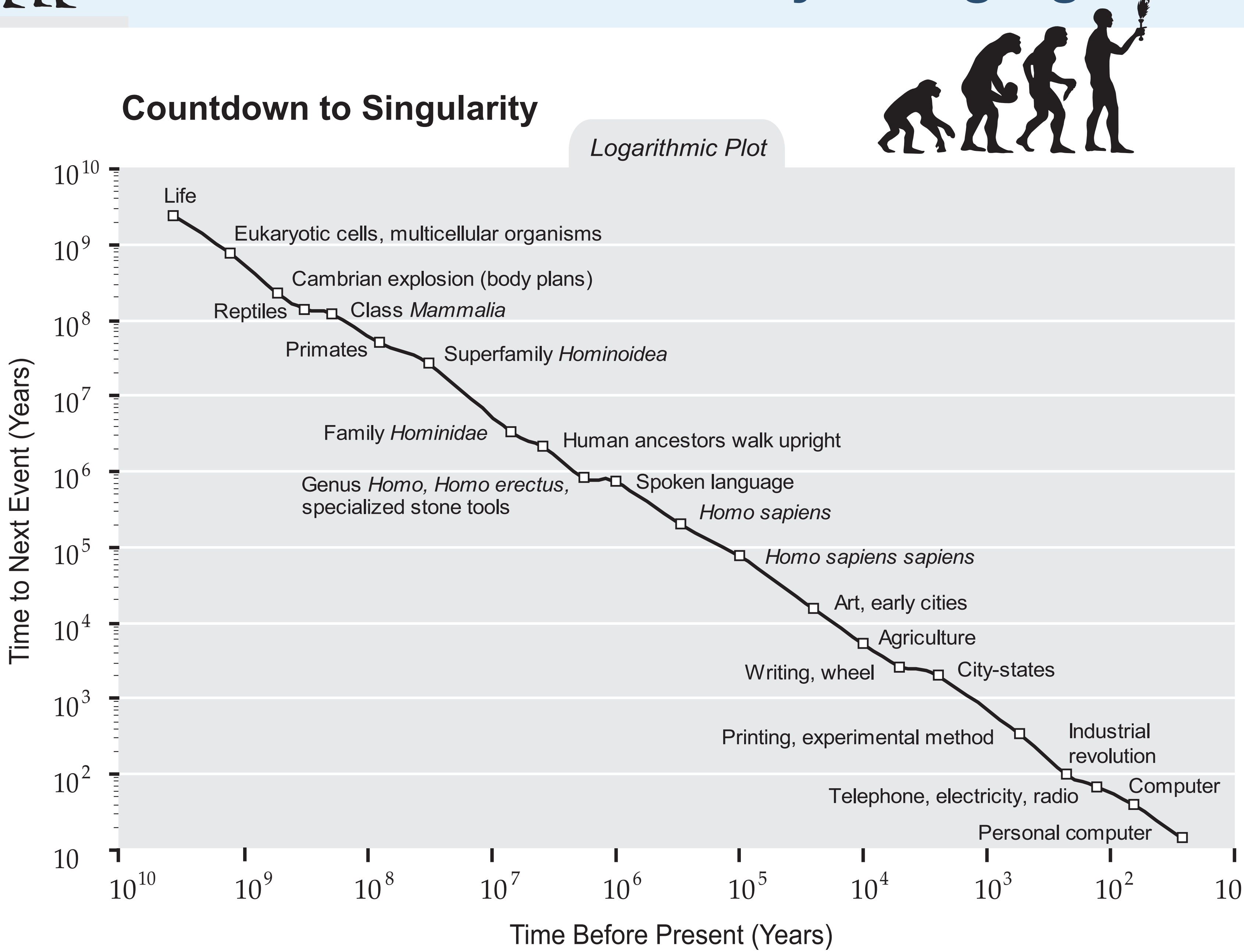
5. In particular humanity, as we now know it will be obsolete — superseded by a new generation of super intelligent quasi biological androids — *andro super sapiens.*

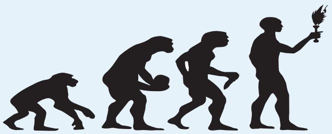


The Six Epochs



The New Growth Theory – Loglog scaling





The Accelerating Rate of Change

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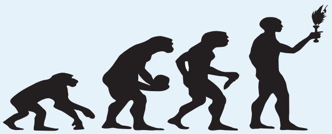

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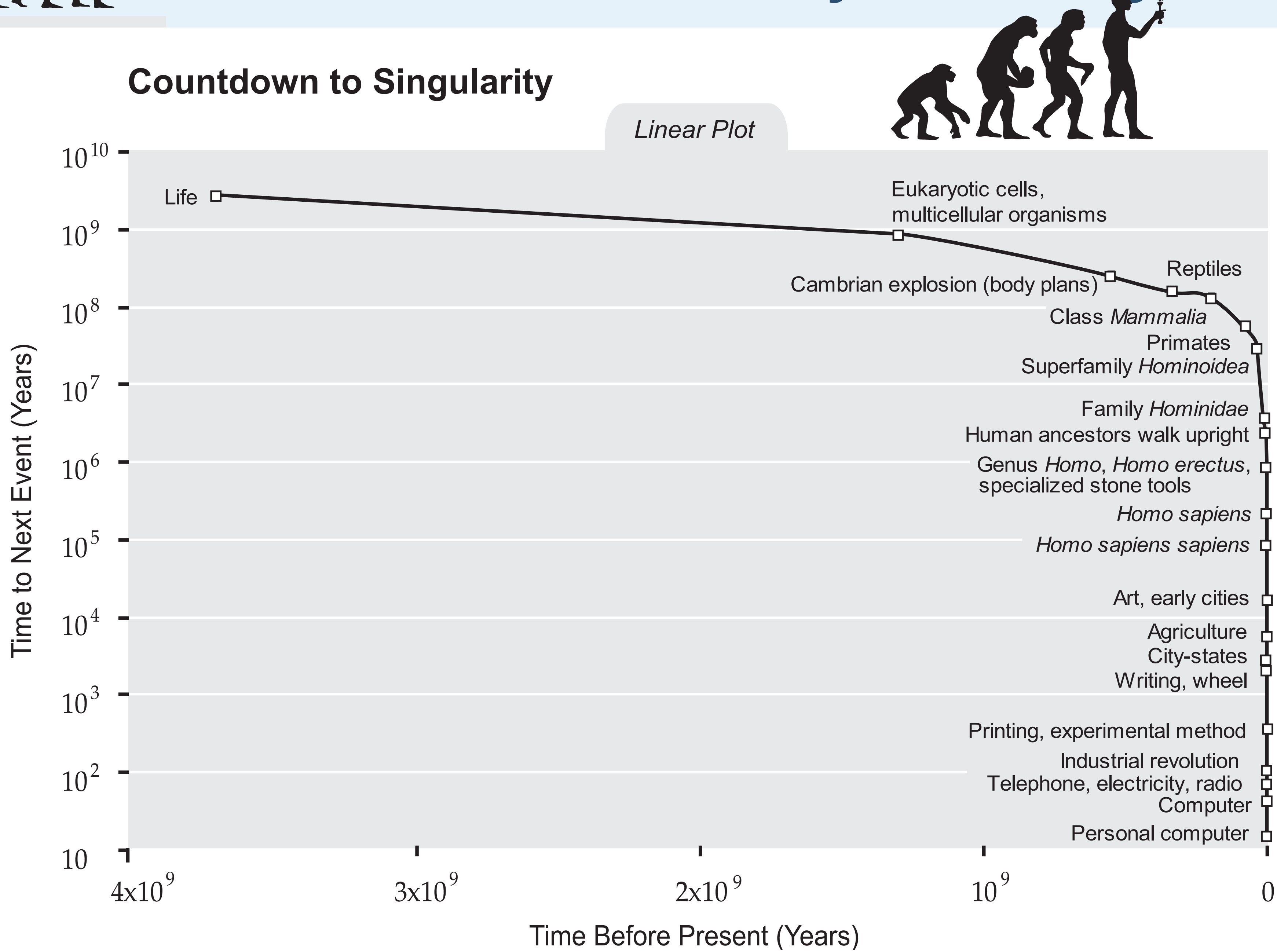

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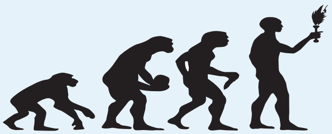

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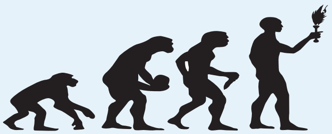

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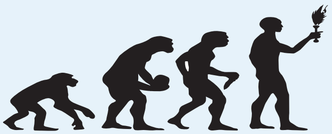

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7. 1997 is also significant: it marks the year when IBM's Deep Blue defeated Garry Kasparov. Ever since then the world's best chess players have been computers!

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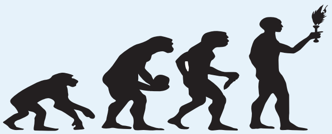

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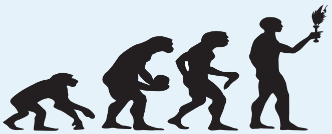
3. The result is a huge speedup in the rate of evolution: "These technologies — releasable parallel synthesis and error correction — permit us to assemble long, relatively error-free DNA constructs far more rapidly and inexpensively than has been possible to date. They can therefore constitute basis of a bio fab, and much like semiconductor chip lithography, these processes can be expected to keep steadily improving over time. That frees us to think about what we will build in the fab. (p. 48).



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- Ray Kurzweil, p. 487, concluding sentences of *The Singularity is Near*

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If we are so smart, why are we so dumb?

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4. Our increasingly sophisticated technologies are still affected by neanderthal instincts in our primitive mamallian brains, including primitive urges to rape, plunder, pillage, and kill. Our ability to make rational decisions is compromised by hormones such as testosterone, cortisone, adrenaline, and other sex and “flight or fight” hormones.

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5. I now present several examples of bad decisions and bad decision makers. Then I will define what I mean by “bad decision.”

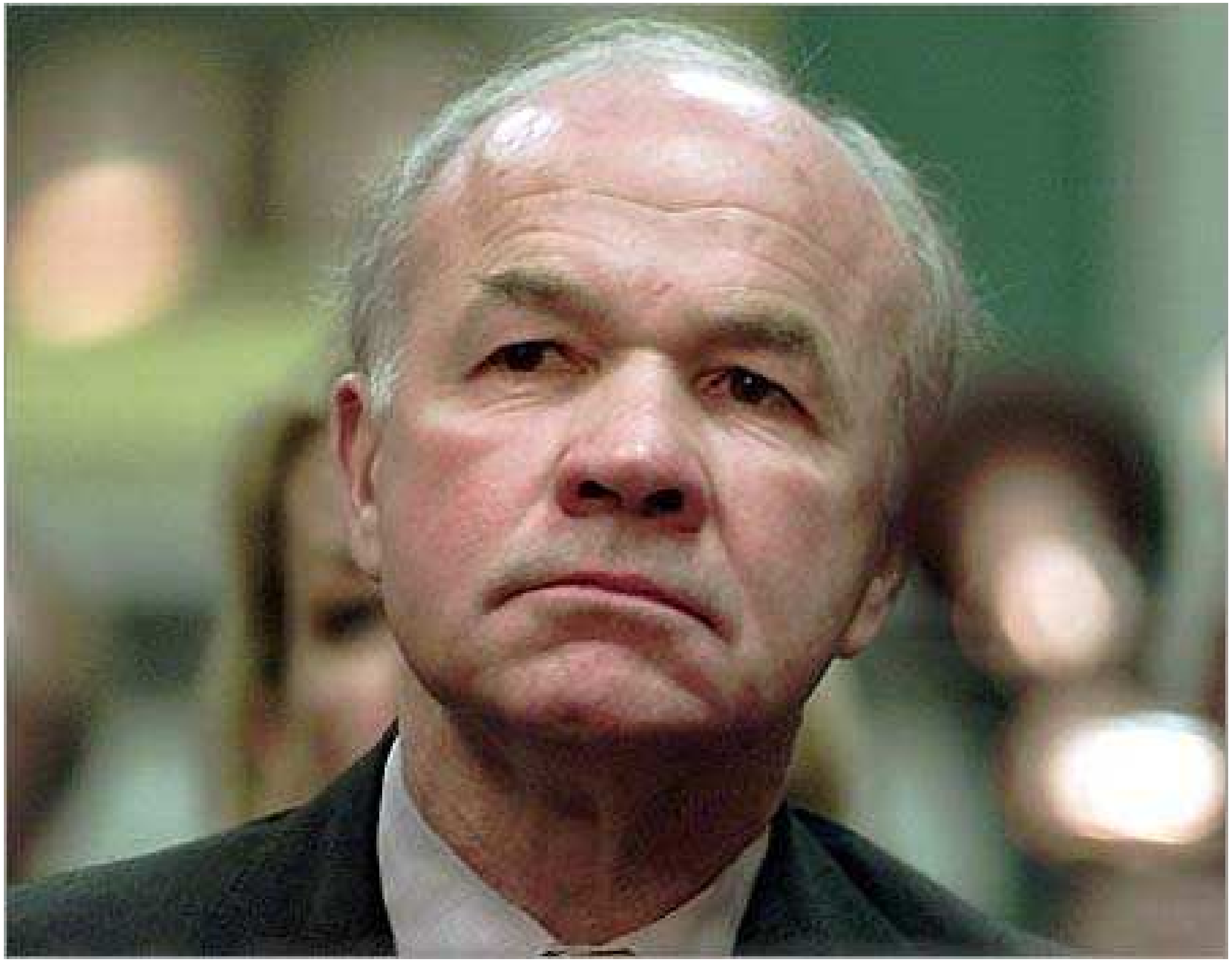
Floyd Landis

Bill Clinton

Click here to see Bill perform

Kim Jong-Il

Kenny Lay



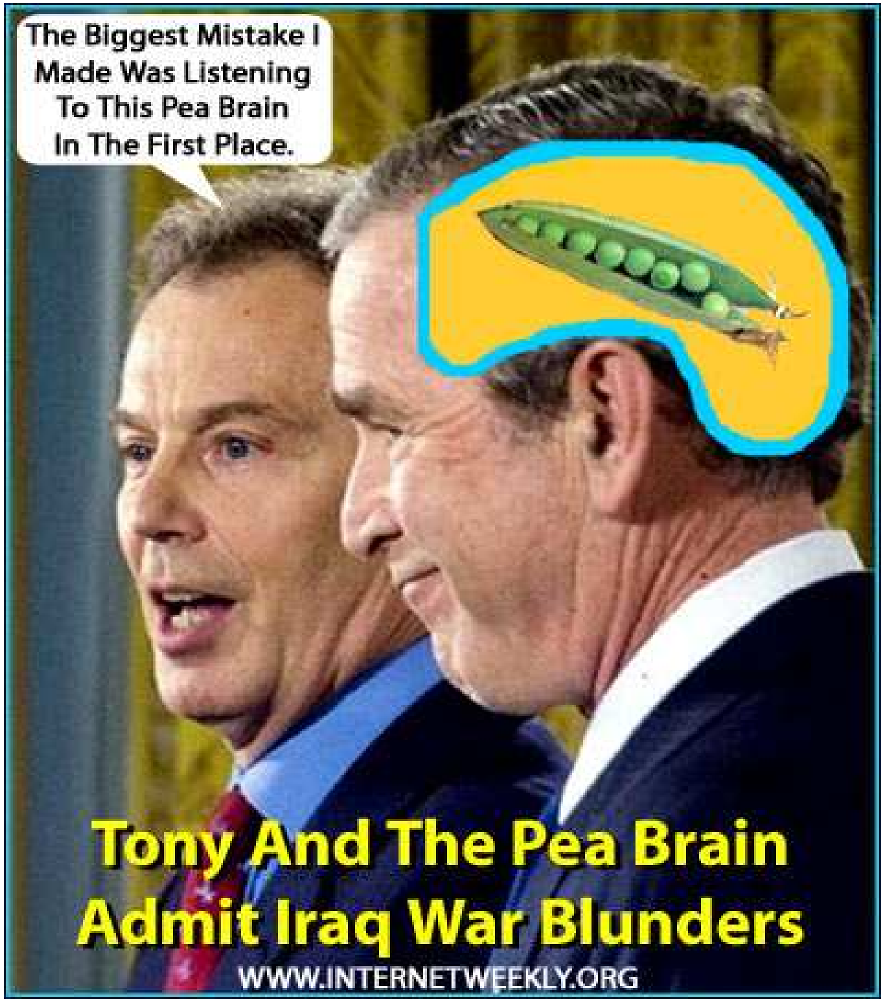
Saddam Hussein

Saddam in Happier Times

(shown after receiving billions in U.S. arms from Donald Rumsfeld)


BREAKING NEWS
BUSH: ONE OF THE WORST
DISASTERS TO HIT THE U.S.

Tony Blair


The Biggest Mistake I
Made Was Listening
To This Pea Brain
In The First Place.
Tony And The Pea Brain
Admit Iraq War Blunders
WWW.INTERNETWEEKLY.ORG

“Observing” Ex Ante Beliefs

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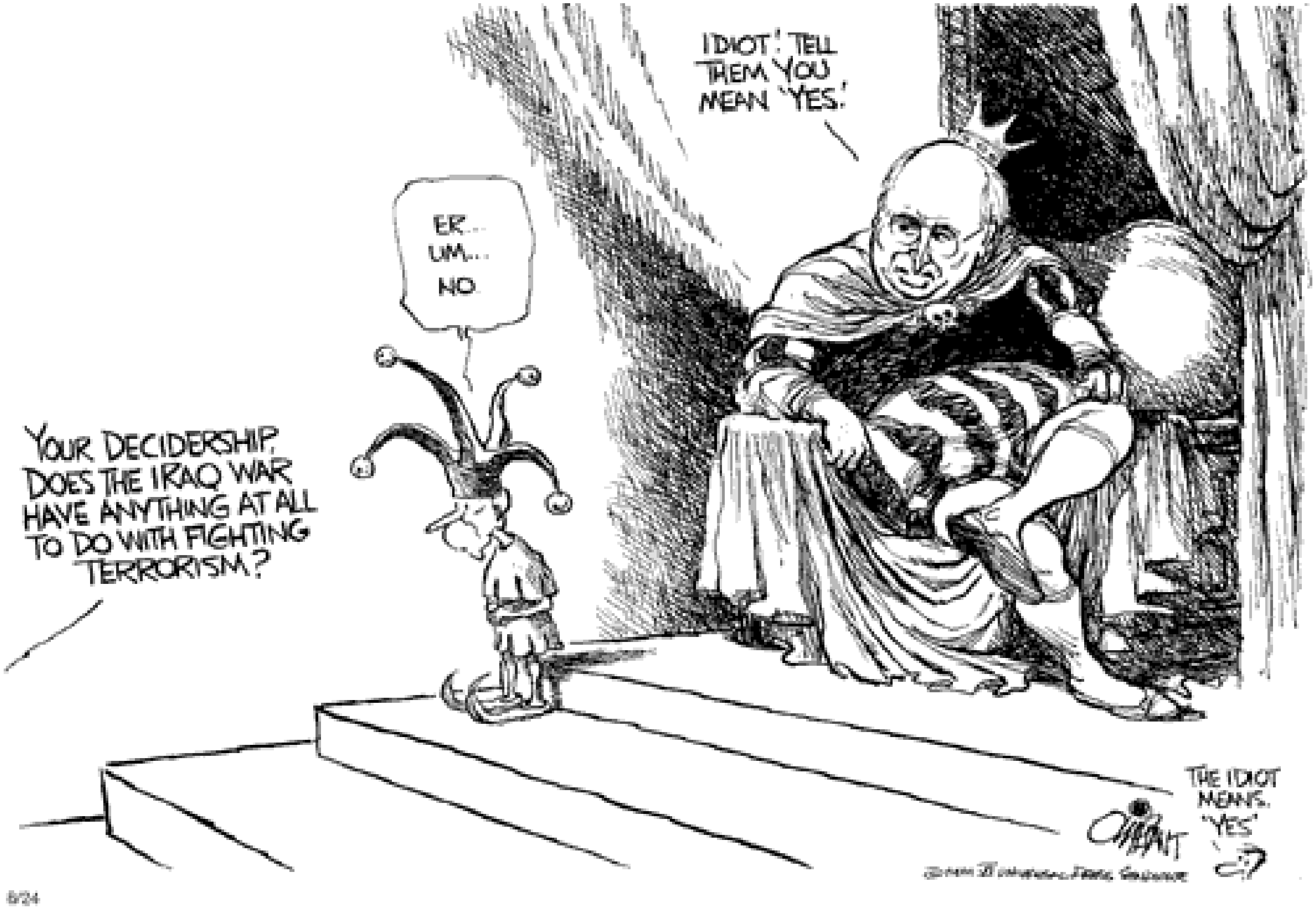
3. "We have to answer the big question — what will this action achieve? There seems to be a larger hole in this than on anything."
4. "Most of the assessments from the US have assumed regime change as a means of eliminating Iraq's WMD threat. But none has satisfactorily answered how that regime change is to be secured, and how there can be any certainty that the replacement regime will be any better."

Last but not least, the “king” of bad decision makers

King George II

"Oh shit, he is even dumber than I thought"

Who makes the decisions: Bush or Cheney?



George Bush, the Decider

The 2006 “Bush Prize” for Bad Decision Making

2006 Bush Prize for Bad Decision Making

A Joint Award to Hassan Nasrullan and Ehud Olmert

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6. The common feature of all the examples I presented are decision makers with *seriously distorted perceptions of reality.*

Definition of a bad decision

1. Definition: A bad decision is a decision under uncertainty that is made by a decision maker (DM) (according to either an expected utility or non-expected utility criterion) whose subjective probability distribution that is greatly at odds relative to the objective probability distribution governing the *ex post* payoff relevant states of nature in the sense that the loss (under the objective probability measure) from taking the decision is large.

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2. Consider the expected utility case. Let μ_s be the DM’s *subjective probability measure.* Then the decision (and decision rule) are defined by

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3. and the *value function* (indirect utility function) is

$$V_{\mu_s}(I, d_s) = E_{\mu_s}\{U(\tilde{X}, d_s(I)|I\}.$$

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3. where K is a sufficiently *LARGE* positive number representing the expected large loss that the DM would incur if he/she had rational beliefs.
4. In other words, if the DM had rational (or approximately rational) beliefs, he/she would never voluntarily choose to make the bad decision.

Definition of a crazy decision

1. Definition: A crazy decision is one that is taken even after a credible authority has informed the DM that his/her beliefs are greatly at odds with the objective probability distribution and will result in very large *ex ante* losses. Further, the DM knows the decision will result in a high probability of very bad *ex post* losses, even relative to the DM's distorted subjective beliefs, but the DM does it anyway.

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3. Example: A daughter of "Christian scientists" has a treatable cancer but will surely die if chemotheory is not given immediately. The church tells the parents that it is consistent with God's will to give their daughter chemotherapy. The parents still refuse, flee with their daughter to avoid arrest, and she soon dies of cancer.

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5. "Unfortunately, Washington — the political process and the media — judges decisions based solely on outcomes, not on the quality of the decision making." Robert Rubin, from 2003 memoire, *In an Uncertain World*

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4. If I am unwilling to question preferences, and if I concede that beliefs about most uncertain events in the real world are unavoidably subjective, then on what grounds can I justify questioning another person’s beliefs?

The Identification Problem

1. Rust (1994) and Magnac and Thesmar (1998) proved that the discrete dynamic choice model is nonparametrically unidentified. Roughly speaking, given any beliefs μ and any subjective discount factor β there is an equivalence class containing infinitely many different utility functions that rationalize any choice probability (decision rule).

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3. This means that we lack a strong scientific basis for looking back at individual decisions by individual decision makers and trying to determine what beliefs and preferences lead to their decisions.

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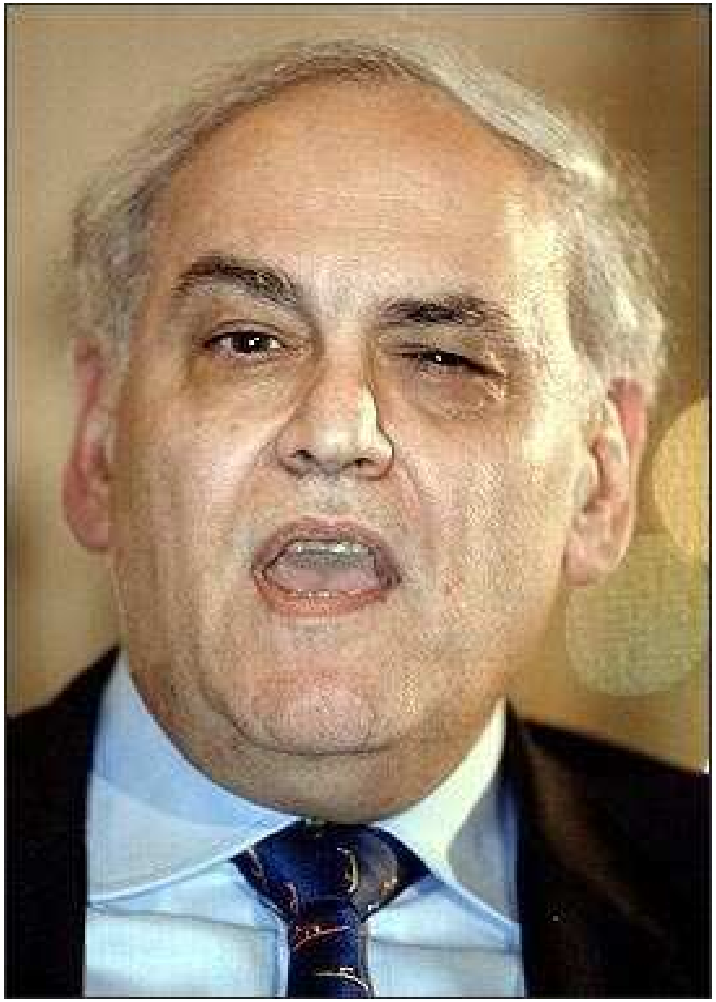
6. In addition, there is a real danger that advice to a powerful leader will be biased: advisors will seek the favor of the leader by providing the advice that they perceive the leader wants to hear.

The Role of Expert advisors

A Bush war advisor, now a Bush war critic



Richard Perle

Biased advice and the Iraq war decision

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3. Cheney received a $20 million departing bonus from Halliburton, the company he was President of before becoming President of the United States. Since the war started, Halliburton has received over $20 billion in Iraq reconstruction contracts, most of which were no bid, cost-plus contracts.

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5. How to do this?

Scientific Advice and George Bush

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Good Decisions

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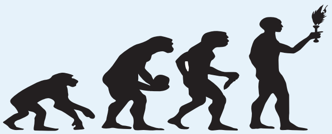

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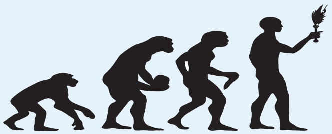

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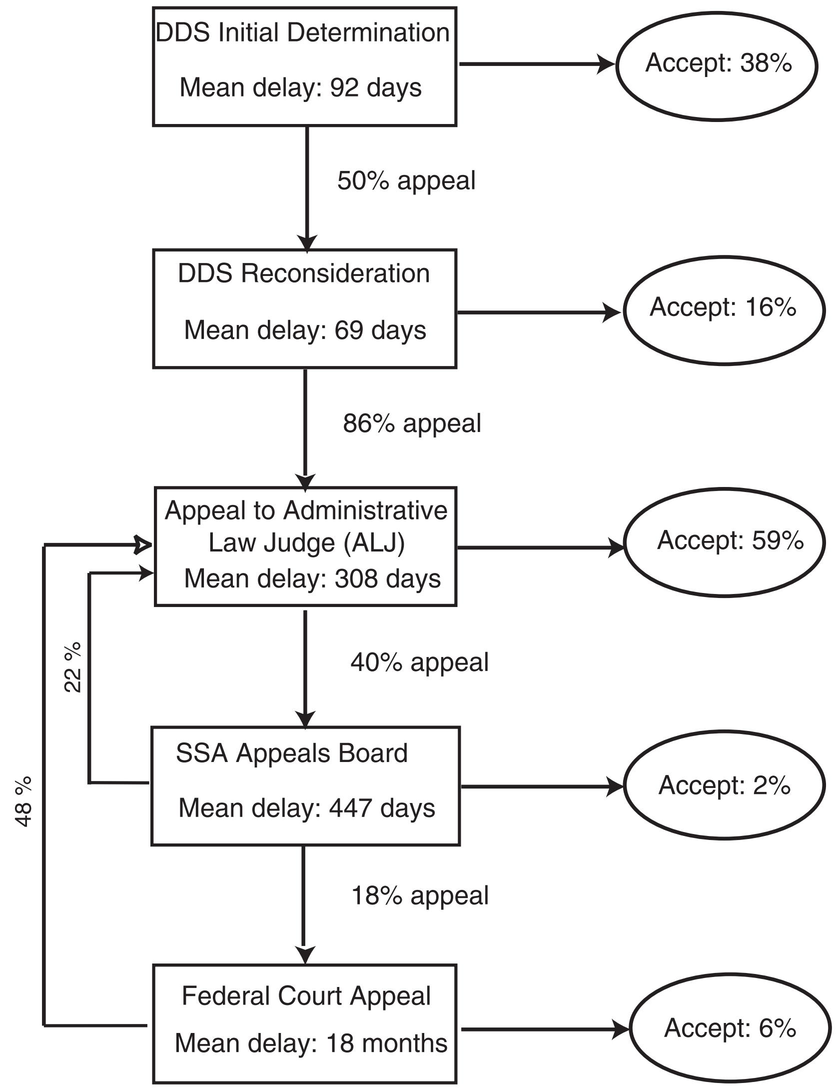
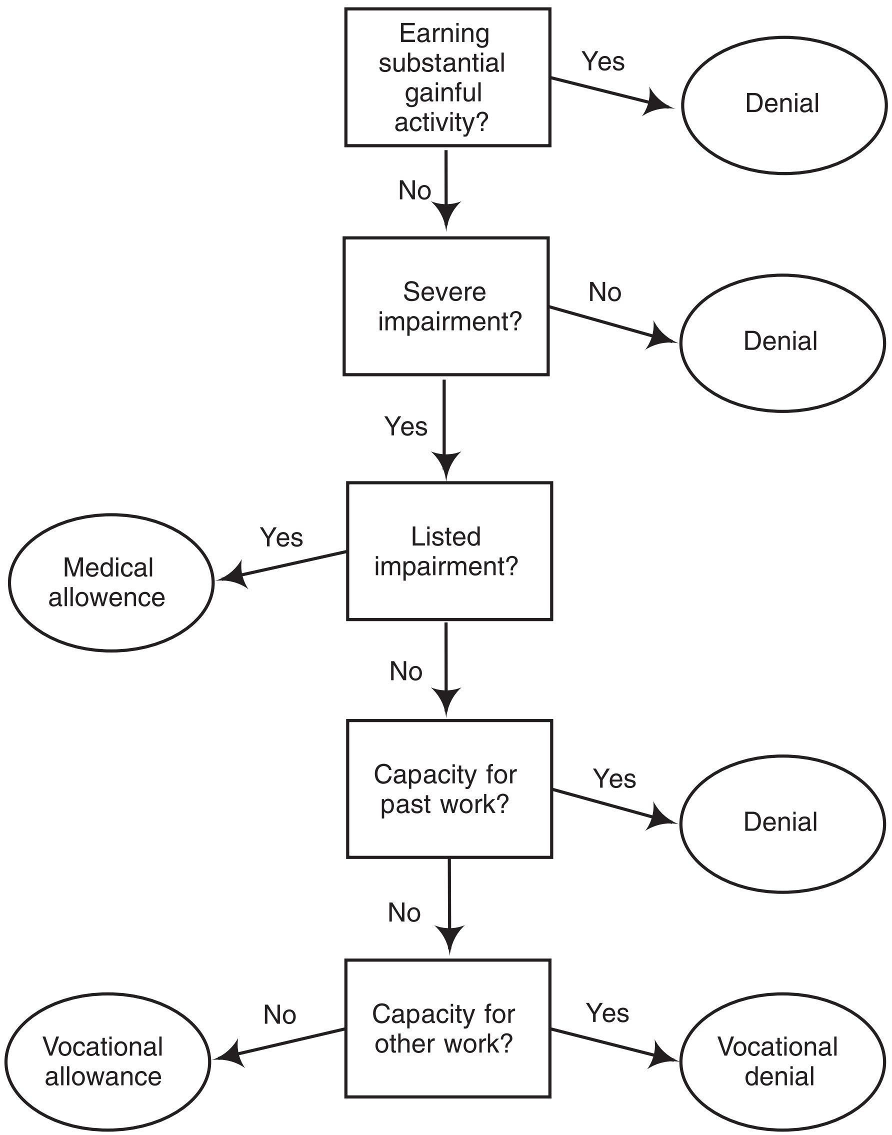
5. We use our empirical results and the *Neyman-Pearson Lemma* to design a more accurate disability screening process.
6. Our “computerized” DI screening process reduces award errors to **16%** and rejection error rates to **50%**

The DI Award Process



The 5 Stages



Earning substantial gainful activity?
Yes
Denial
No
Severe impairment?
No
Denial
Yes
Listed impairment?
Yes
Medical allowence
No
Capacity for past work?
Yes
Denial
No
Capacity for other work?
No
Vocational allowance
Yes
Vocational denial

How did we get these results?

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2. We compared their *self-reported disability status* $\tilde{d}$ to the SSA’s *ultimate award decision* $\tilde{a}$
3. We argued that individuals are truthful, accurate reporters of their “true” disability status $\tilde{\tau}$, so by comparing $\tilde{a}$ and $\tilde{d}$ we can infer error rates in the SSA’s bureaucratic award and appeal process.

Two Measures of “Disability”

1. **SSA** *The inability to engage in any substantial gainful activity (SGA) by reason of any medically determinable physical or mental impairment, which can be expected to result in death, or which has lasted, or can be expected to last, for a continuous period of at least 12 months.*

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2. **HRS** *Do you have a health condition that prevents you from working entirely?*
3. Define three binary random variables $\tilde{a}$, $\tilde{d}$ and $\tilde{\tau}$ as follows:

$$\tilde{a} = \begin{cases} 1 & \text{if person is ultimately awarded SSDI/SSI benefits} \\ 0 & \text{otherwise} \end{cases}$$

$$\tilde{d} = \begin{cases} 1 & \text{if person reports they are unableto work} \\ 0 & \text{otherwise} \end{cases}$$

$$\tilde{\tau} = \begin{cases} 1 & \text{if someone is “truly disabled”} \\ 0 & \text{otherwise} \end{cases}$$

Definitions of Classification Errors

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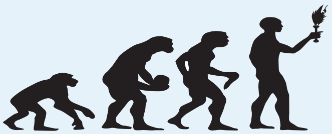

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4. Surprisingly, when we do these computations in this more realistic case, the rate of classification errors and the Type I and II error rates differ by only a small amount from the error rates we obtained in the “easy” case when we assumed that $\tilde{d} = \tilde{\tau}$.

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2. Nagi (1969) compared an “expert decision” (a moderated group decision of an examining team consisting of a physician, psychologist, social worker, occupational therapist, and a vocational rehabilitation expert) to SSA’s award decision

Expert	SSA Award Decision		Total
Team Decision	Awarded	Denied	
Can Work	291 (19.3%)	492 (52.1%)	783 (31.9%)
Cannot Work	1,219 (80.7%)	452 (47.9%)	1,671 (68.1%)
Total	1,510 (61.5%)	944 (38.5%)	2,454 (100.0%)

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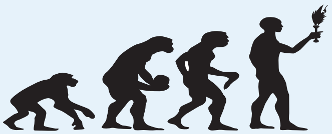

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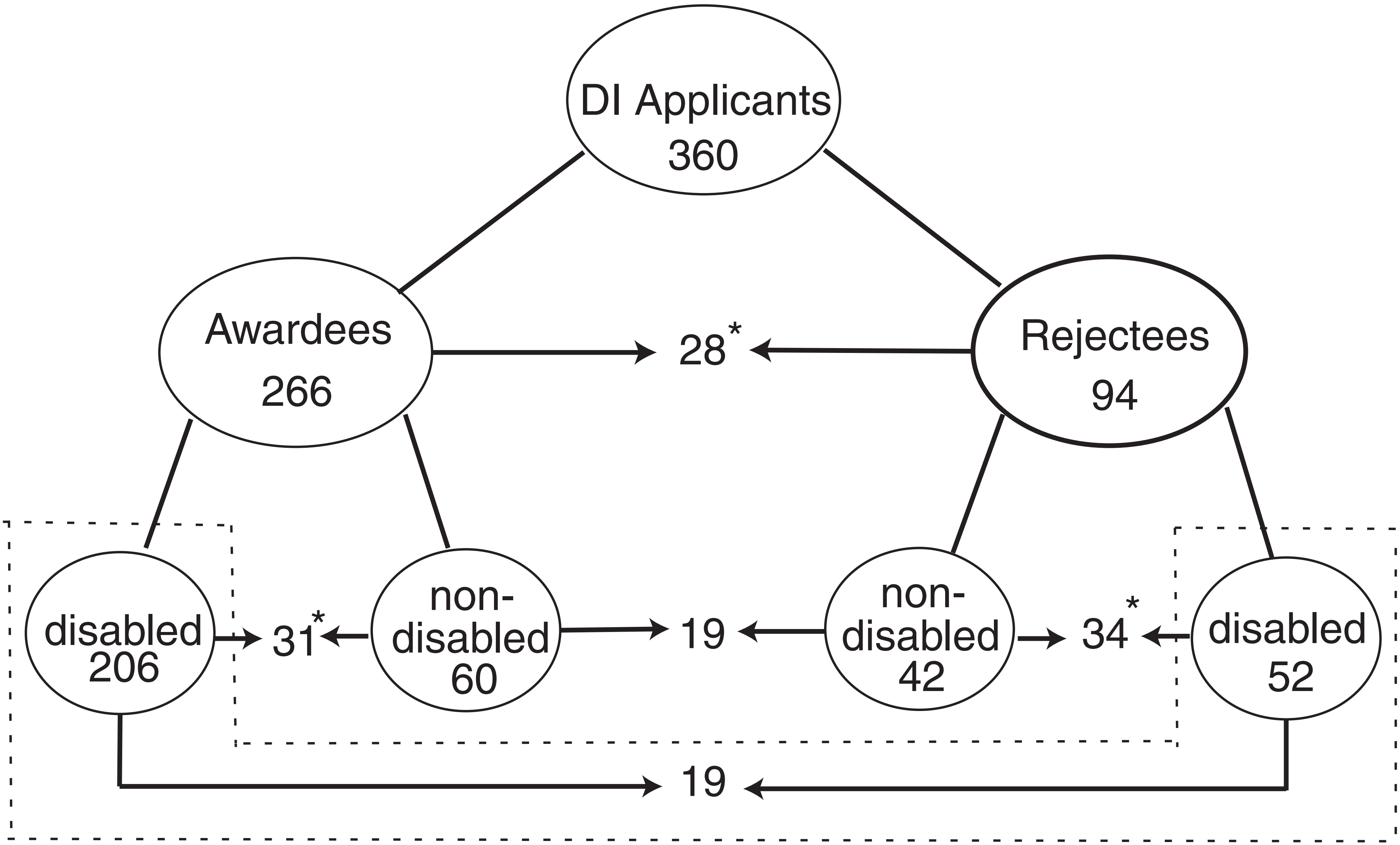
2. We don't have access to an independent expert, so we compare self-reported disability to SSA's award decision

Self-Reported	SSA Award Decision		Total
Disability Status	Awarded	Denied	
Not Disabled	60	42	102
	(22.6%)	(44.7%)	(28.3%)
Disabled	206	52	258
	(77.4%)	(55.3%)	(71.7%)
Total	266	94	360
	(73.9%)	(26.1%)	(100.0%)

Summary of Classification Errors



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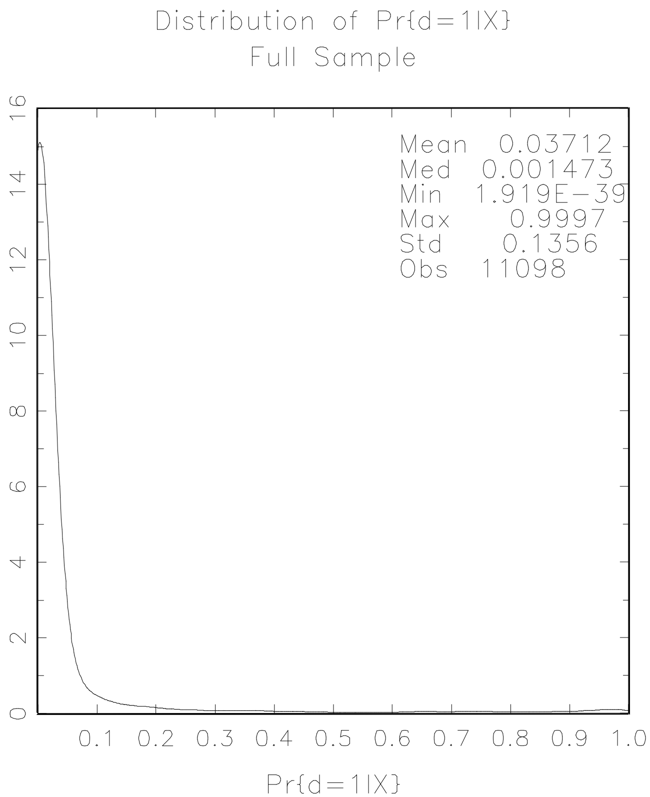
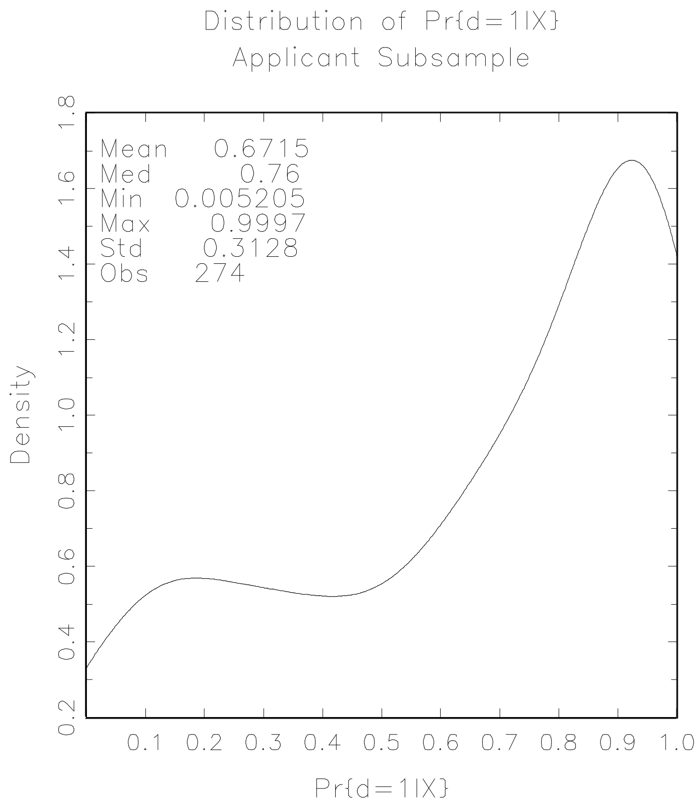
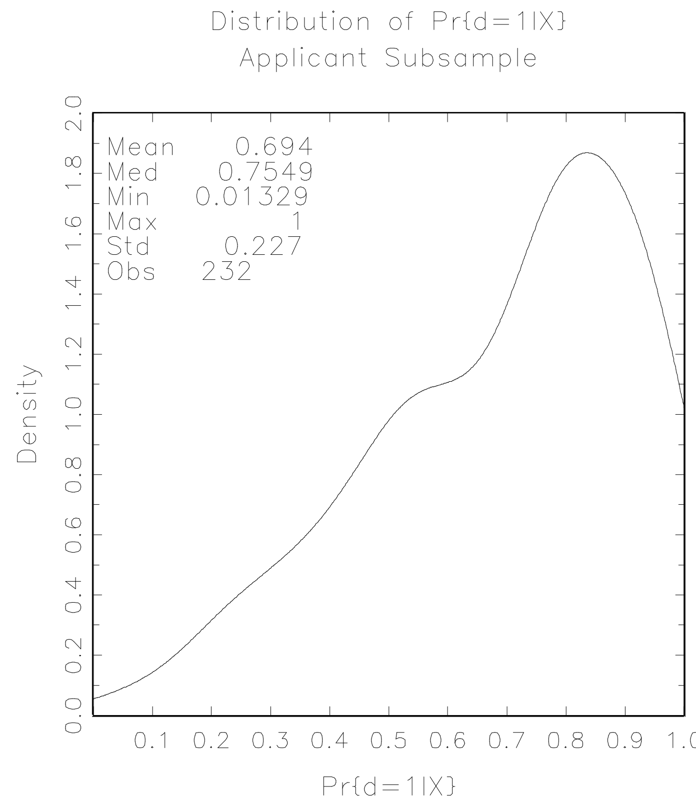
5. By varying the cutoff λ_c we can achieve any desired award rate.

Distributions of $\Pr\{\tilde{d} = 1|x\}$







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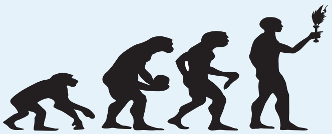
4. Contrary to the GAO analysis, we find that the appeal stage to the ALJs, and the high rate of reversals, substantially reduces the rate of rejection errors without increasing the rate of award errors.

Improving Return to Work Incentives



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4. Under the $1 for $2 offset, DI recipients can work and remain on DI, but they lose $1 of DI benefits for every $2 earned above the SGA threshold (about$9000 per year).
5. The 1999 law enabled SSA to conduct *randomized experiments* designed to measure the induced entry effect.

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4. Would the decreased costs due to “induced exit” from DI outweigh the increase in costs due to “induced entry”?
5. Congress mandated that a randomized experiment be conducted to answer this question and assess the cost effectiveness of the $1 for $2 offset.

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5. This means that potentionally millions of people would be living in counties that were randomly selected into the treatment group of counties whose citizens are eligible for the $1 for $2 offset.

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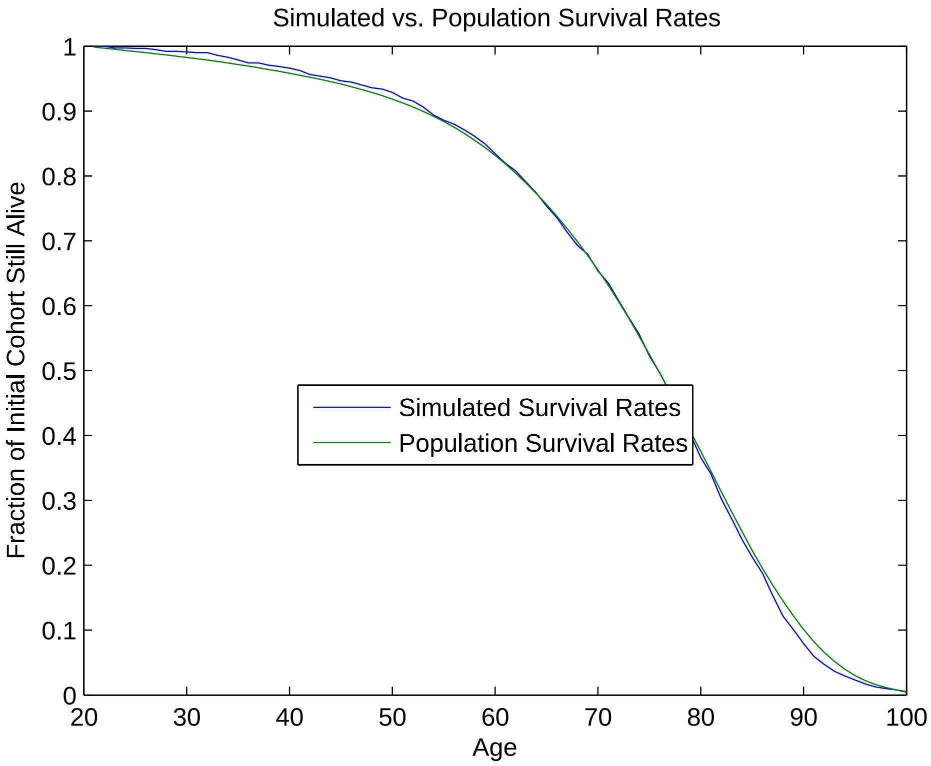
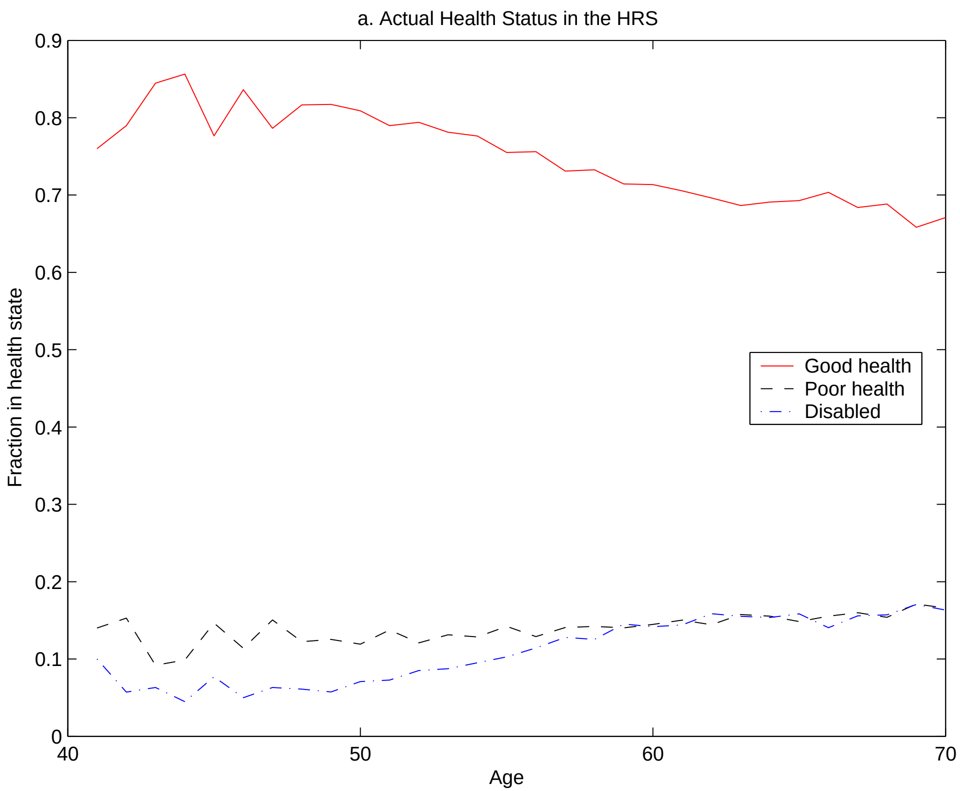
3. Other state variables include the individual’s social security status, and their *average wage* which updated recursively as

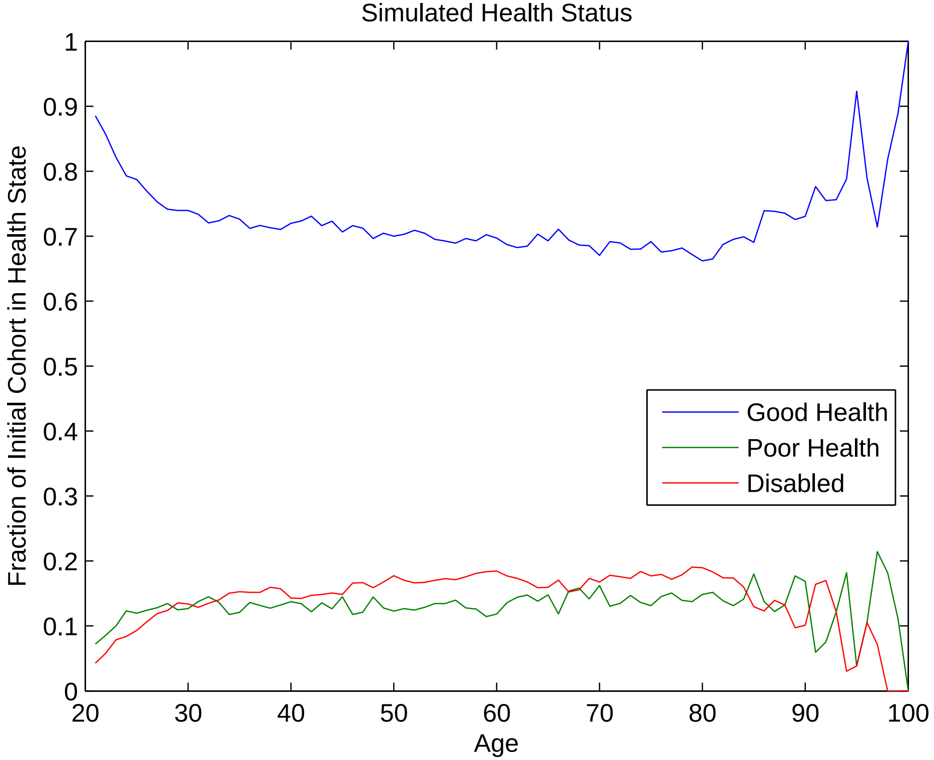
$$aw_{t+1} = \frac{t}{t+1} aw_t + \frac{1}{t} y_t.$$

Simulated vs. Population Survival Rates



Simulated vs. Actual Health Status





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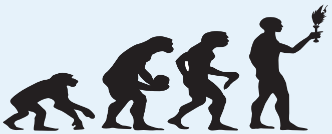

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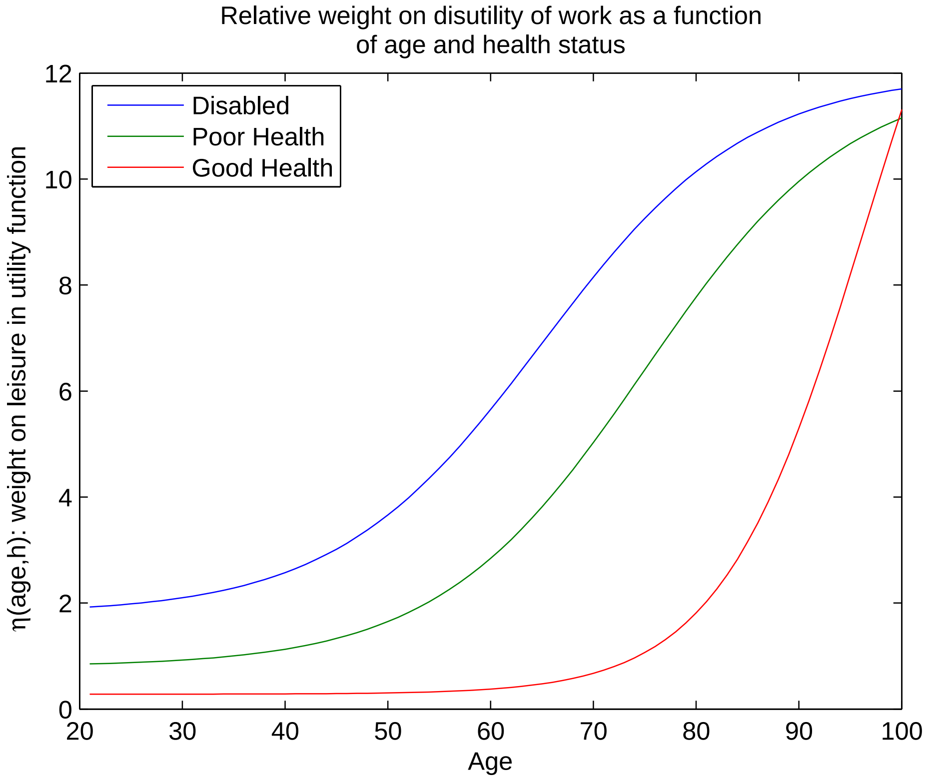
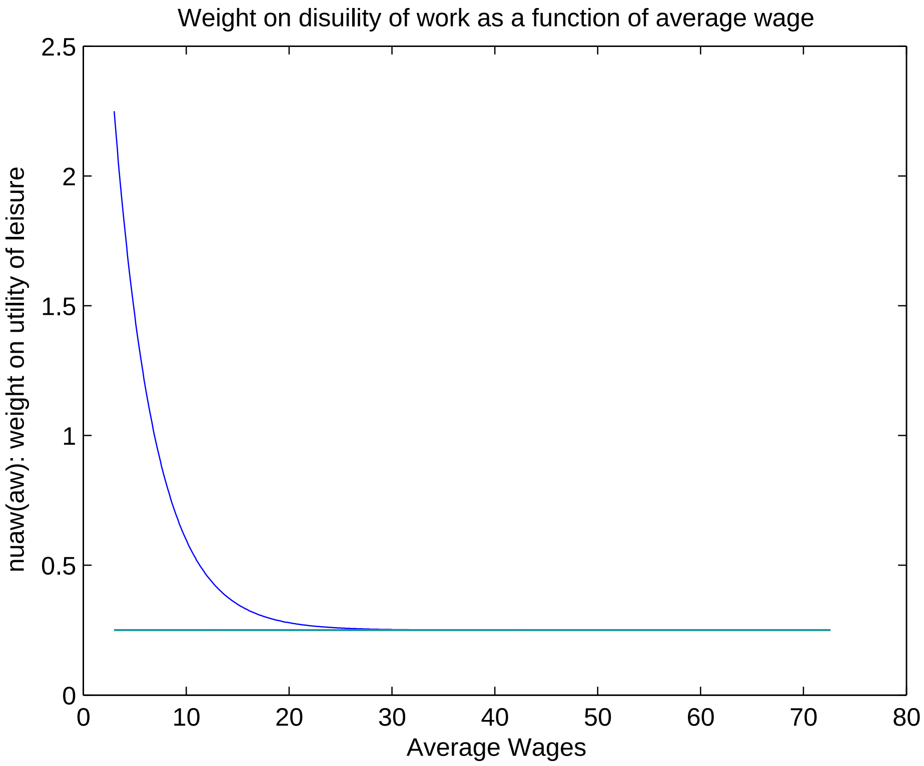
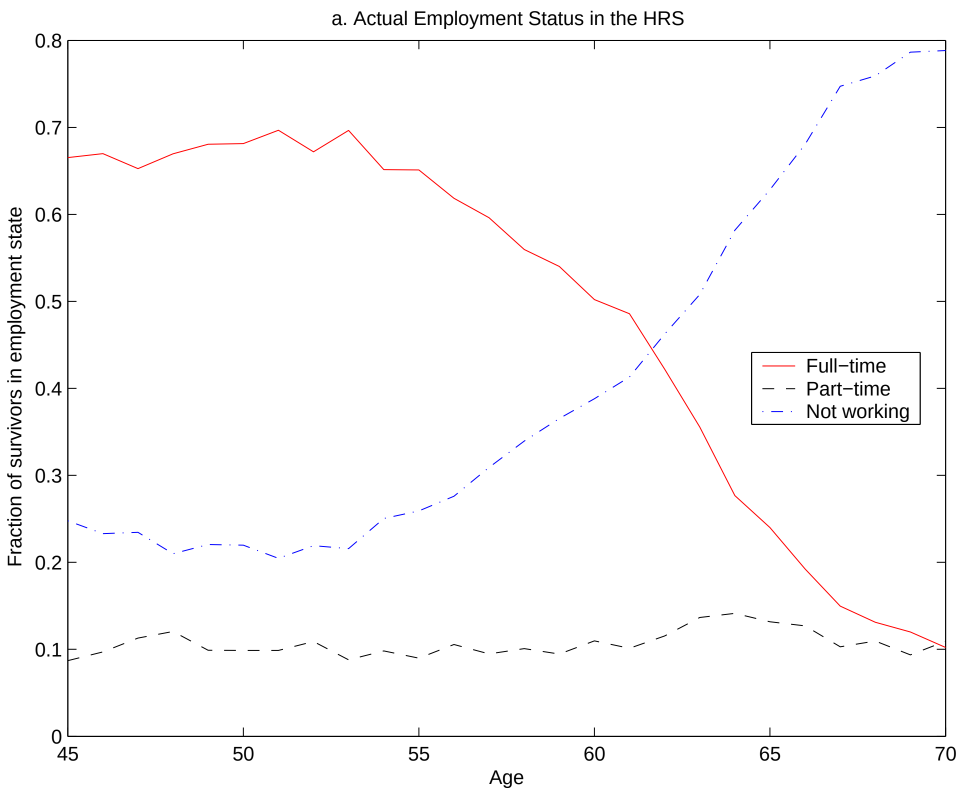
$$\log(y_{t+1}) = \alpha_1 + \alpha_2 \log(aw_t) + \alpha_3 t + \alpha_4 t^2 + \eta_t.$$

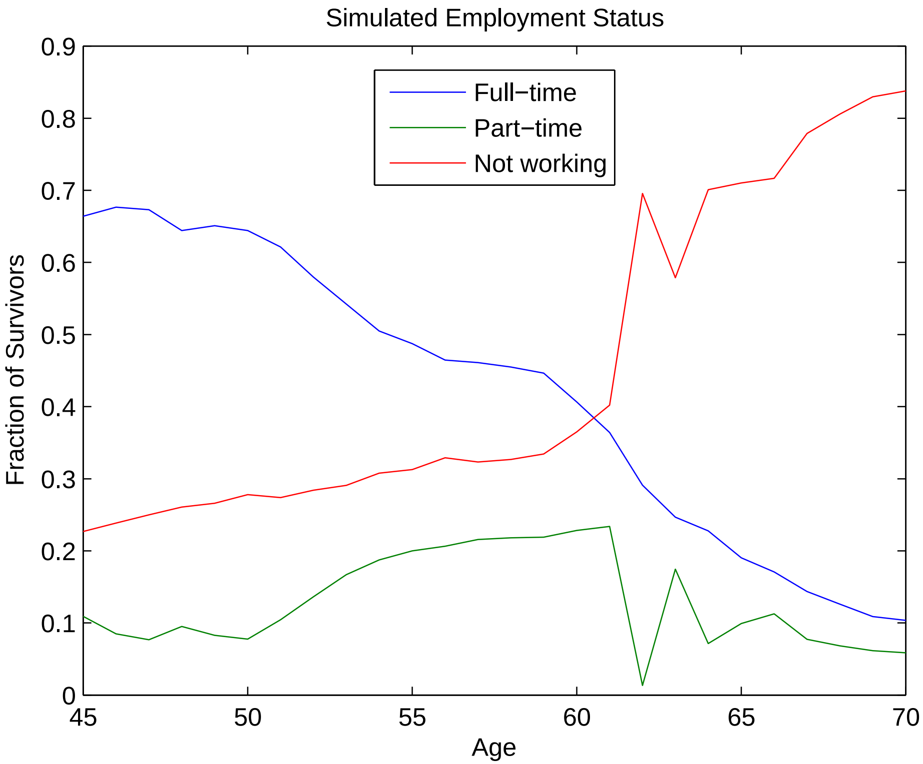
Weights on Leisure by Age and Average Wag



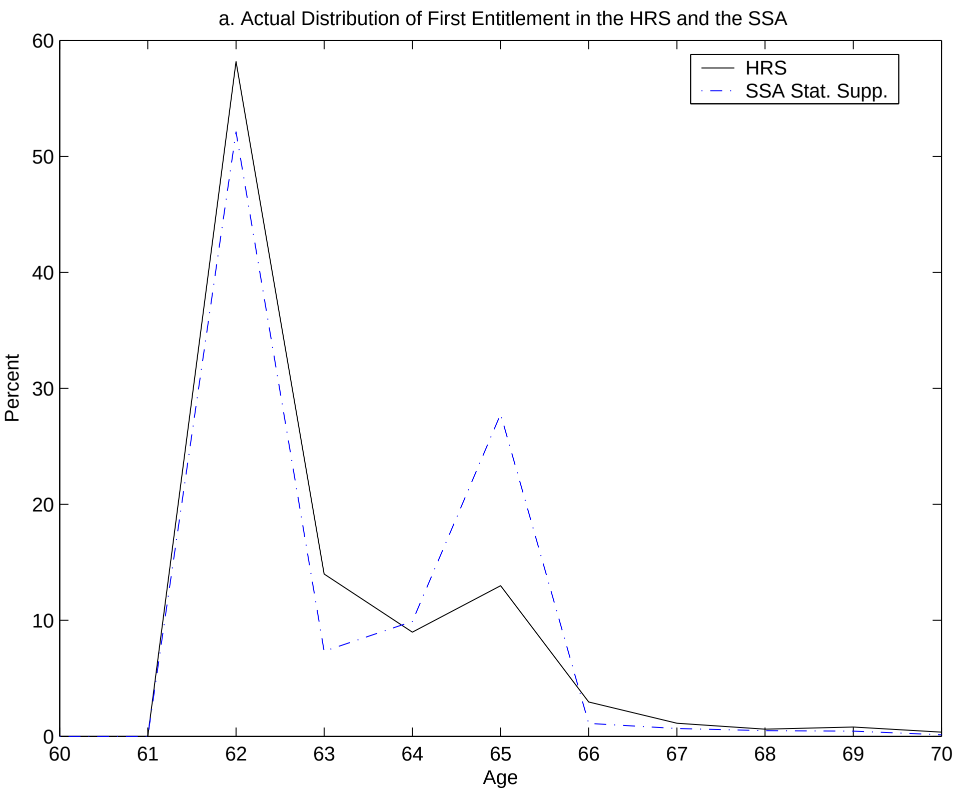


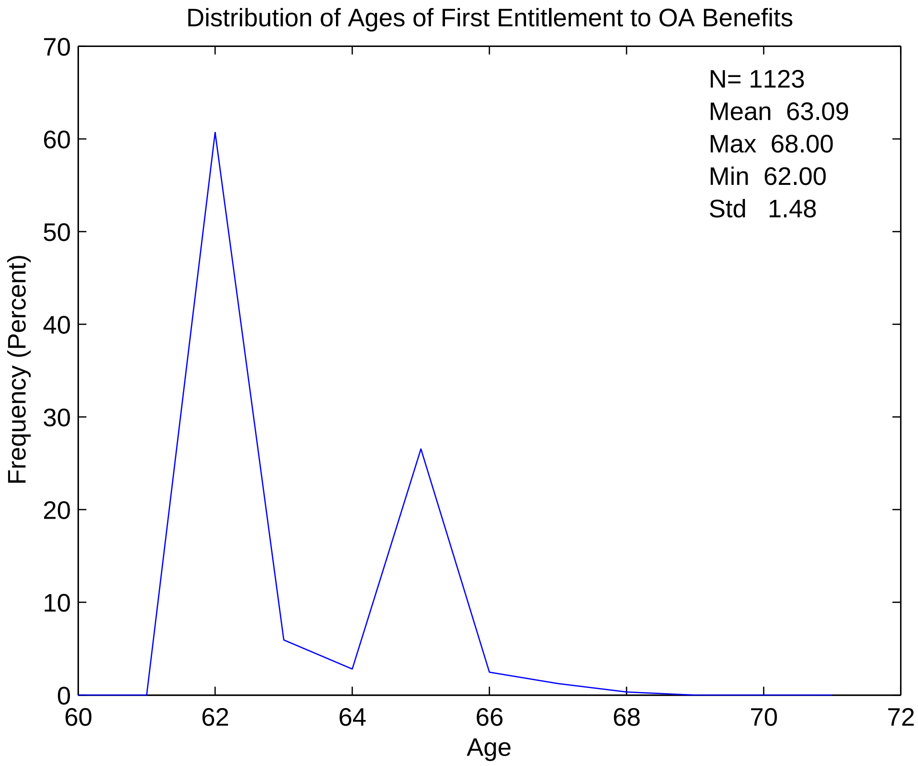
Simulated vs. Actual Labor Supply

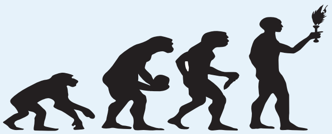




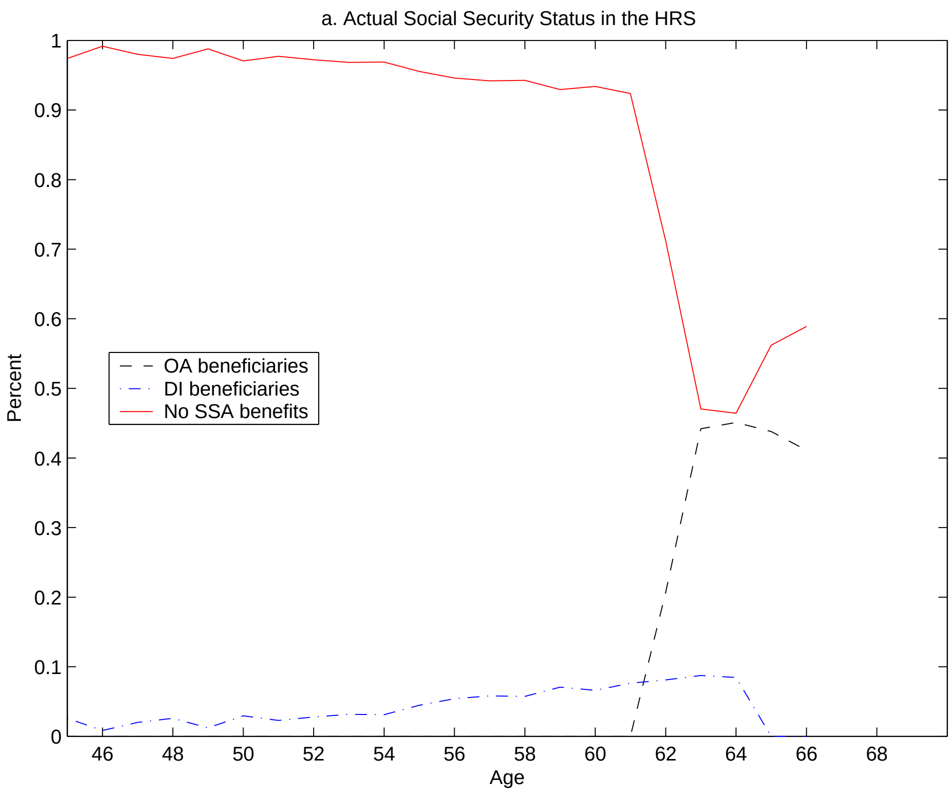
Simulated vs. Actual Social Security Receipt

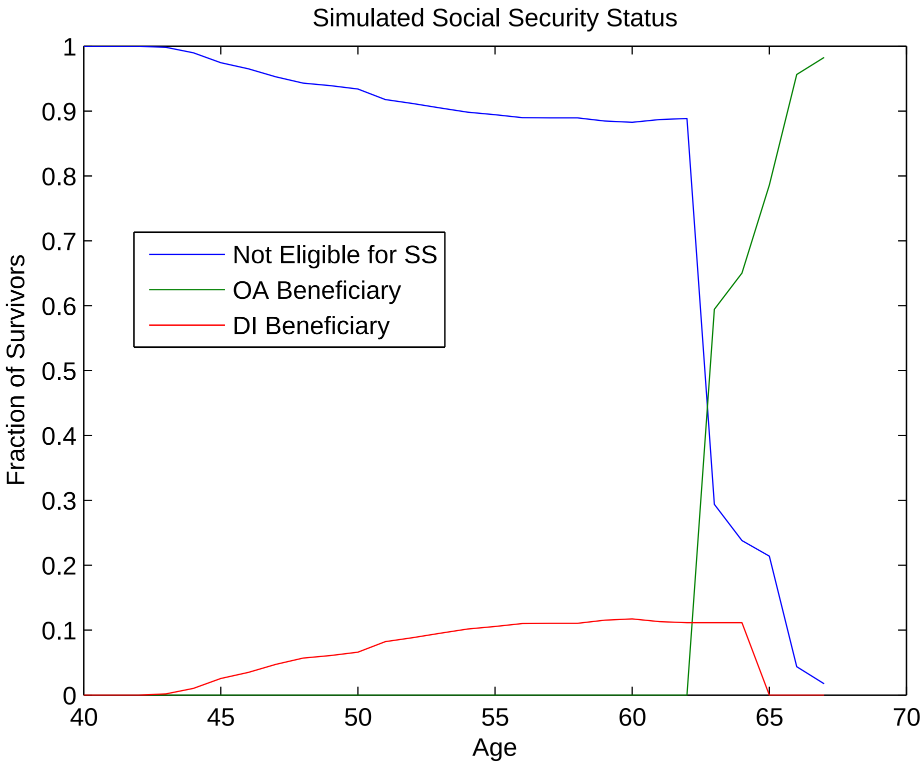




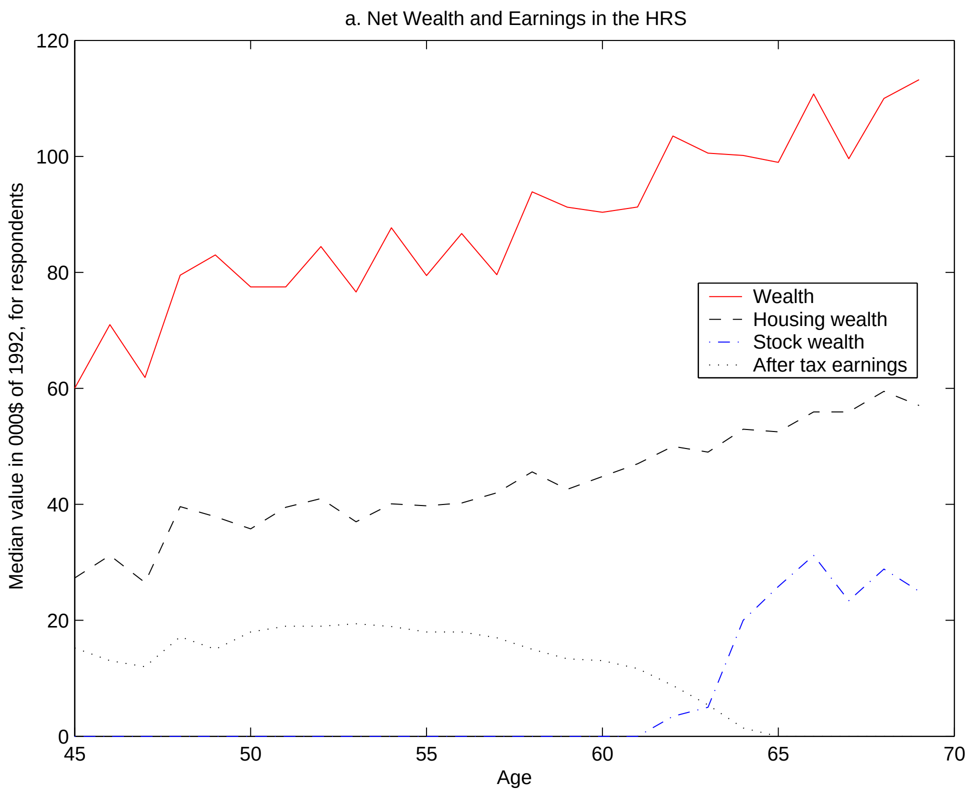


Simulated vs. Actual Social Security Status

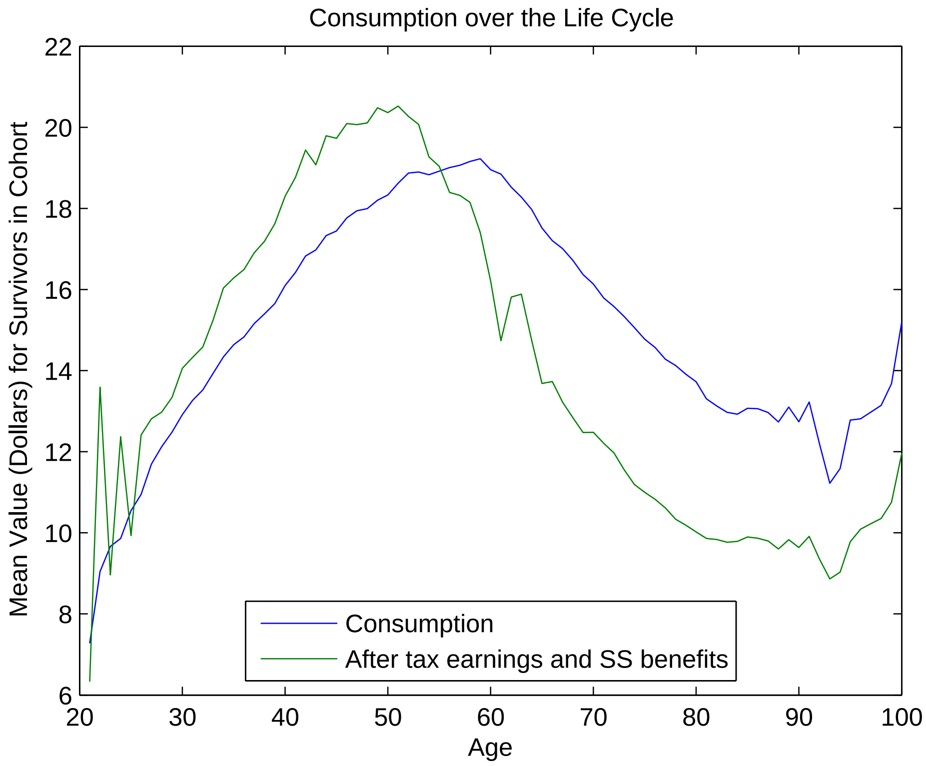


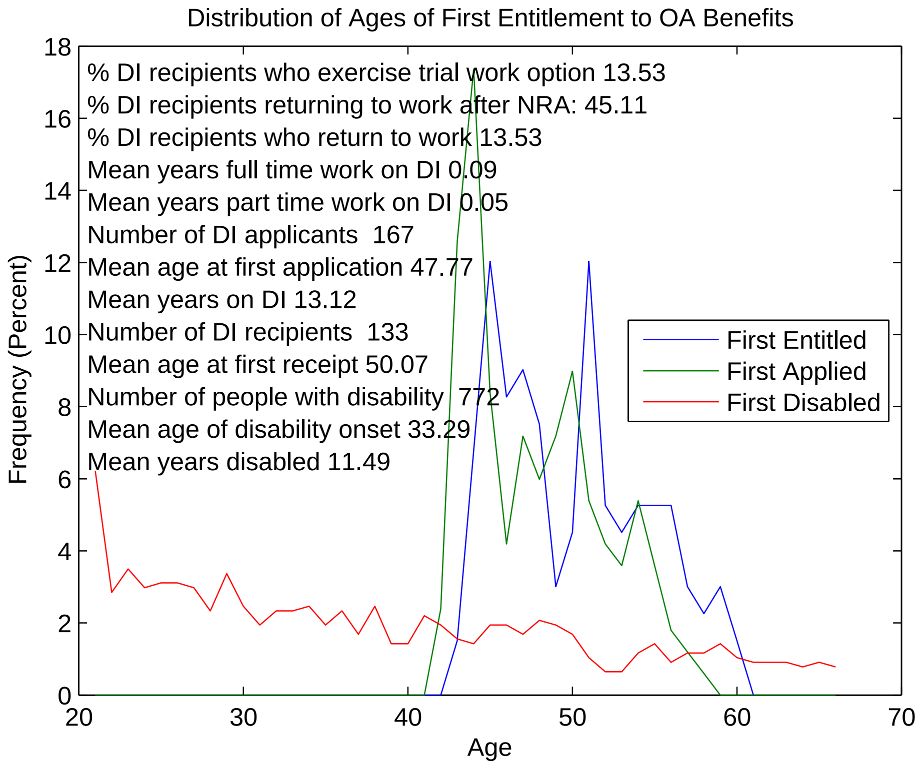


Simulated vs. Actual Net Worth

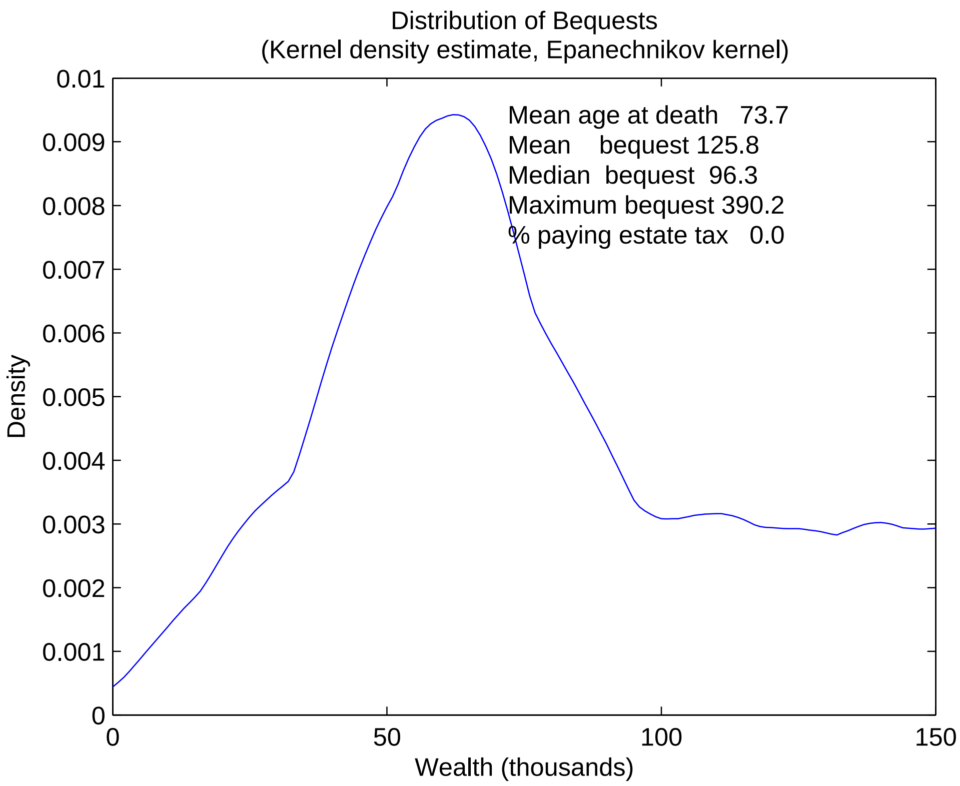


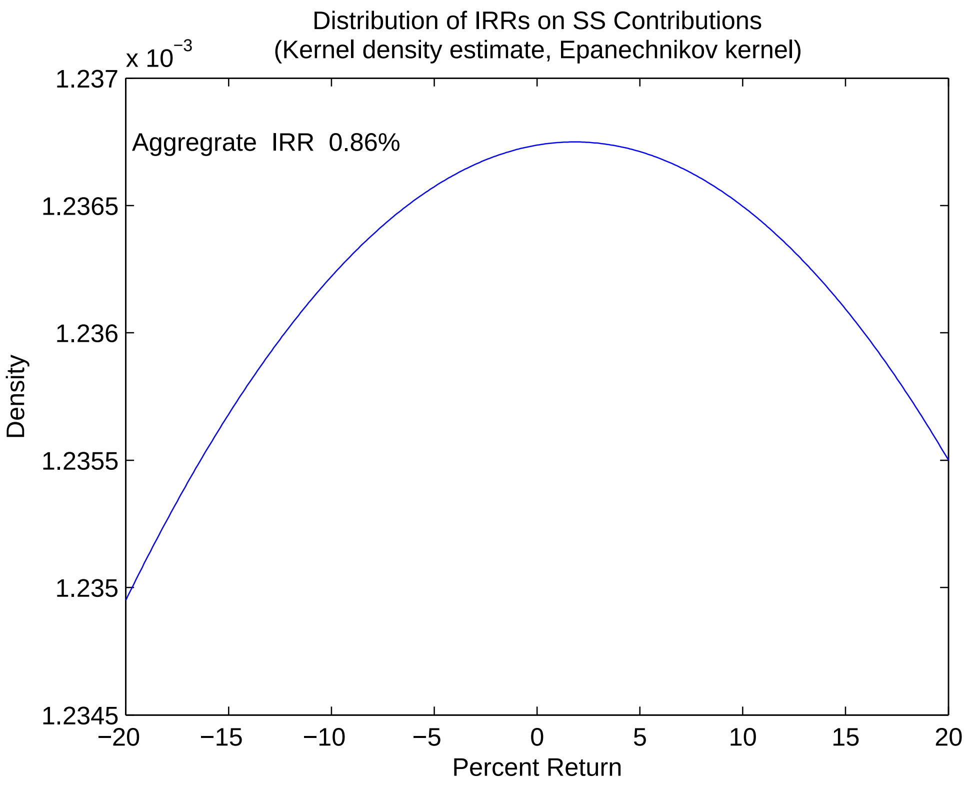
Consumption, Wages and DI Receipt



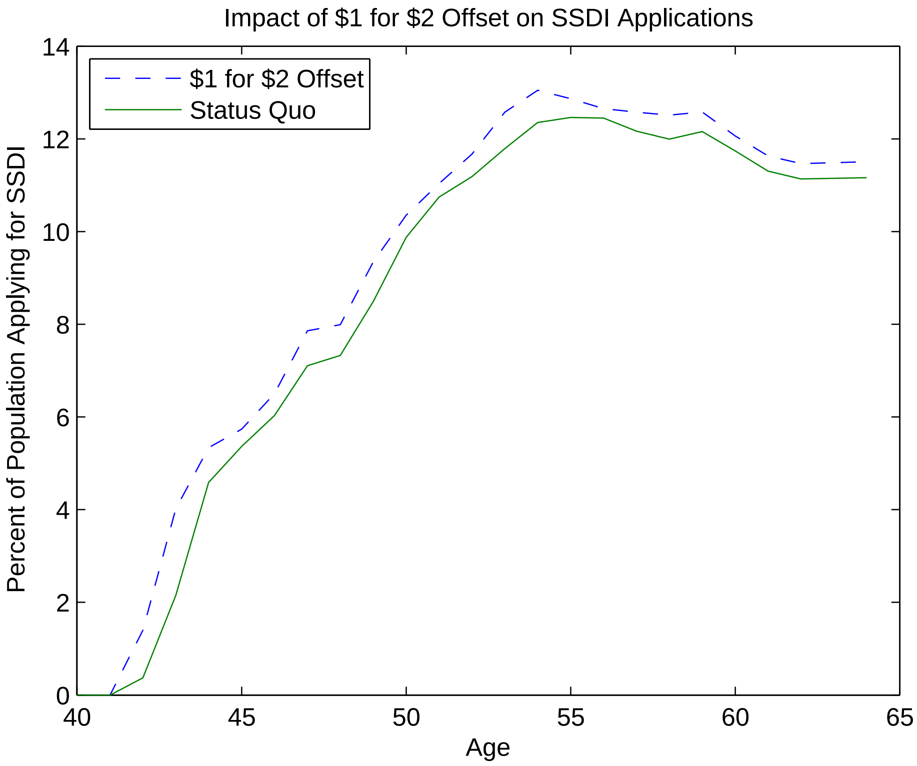


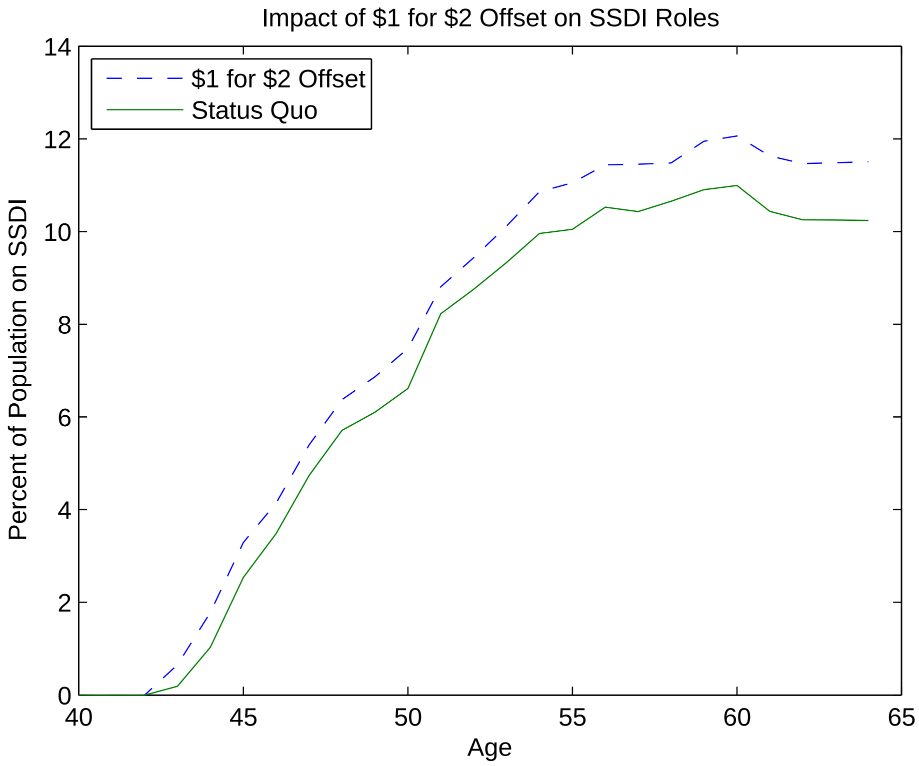
Bequests, IRR on Social Security



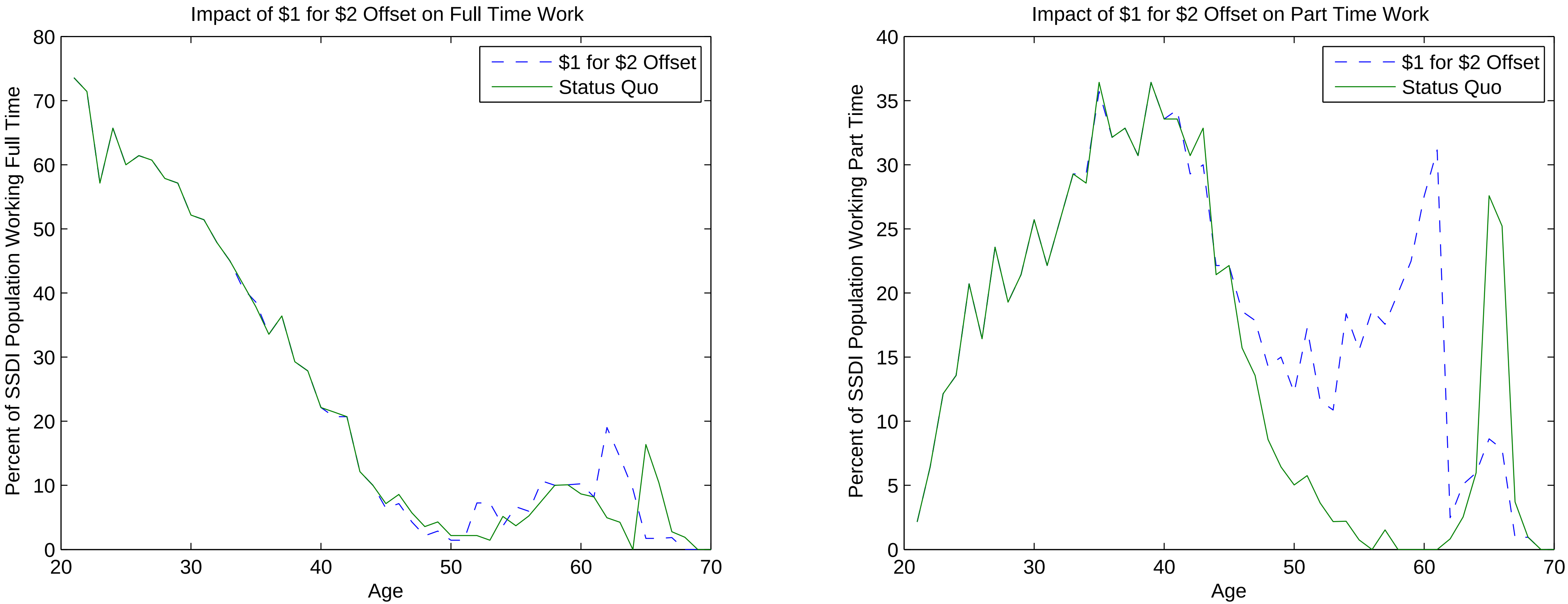


Impact on DI Applications and Rolls

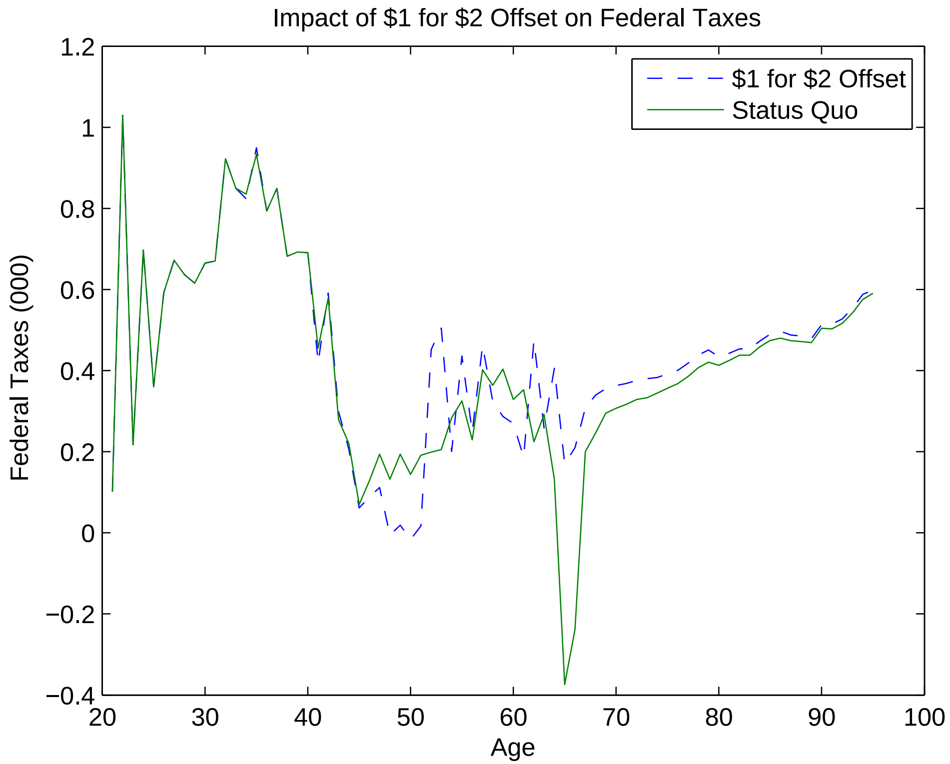




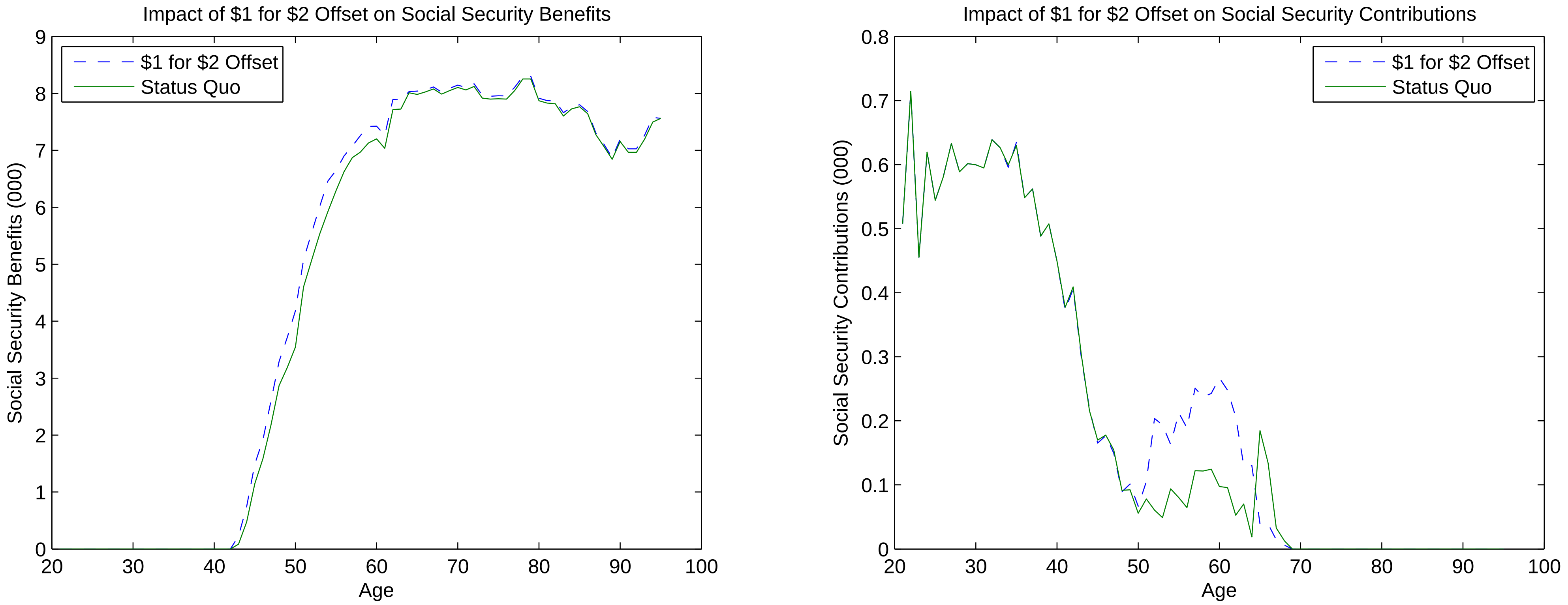
Impact on Labor Supply



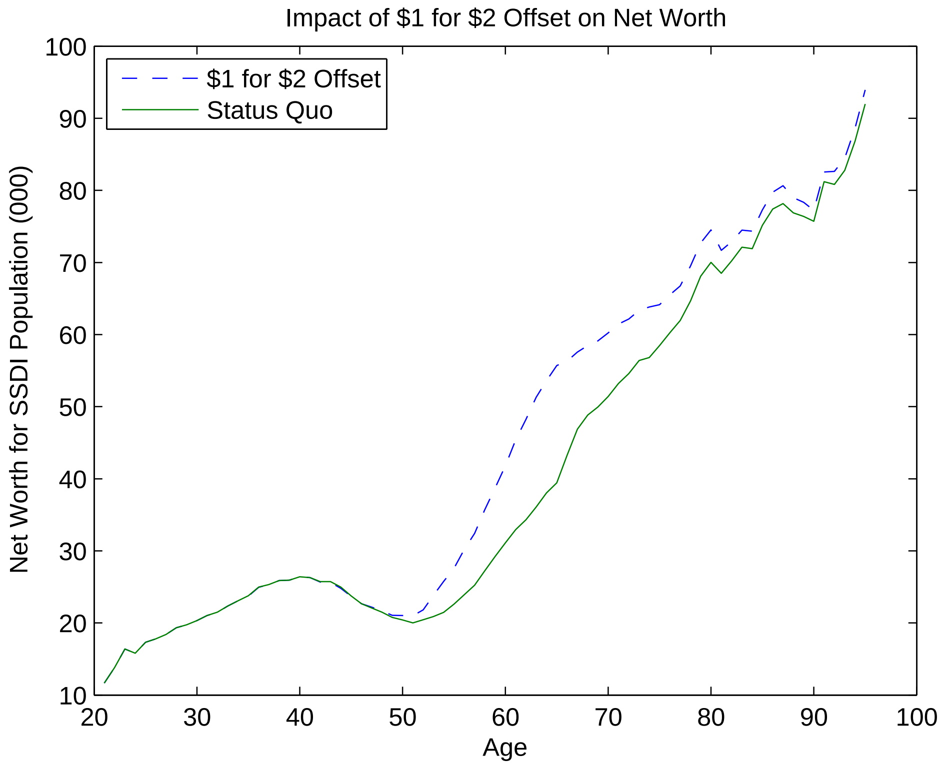
Impact on Wages and Taxes

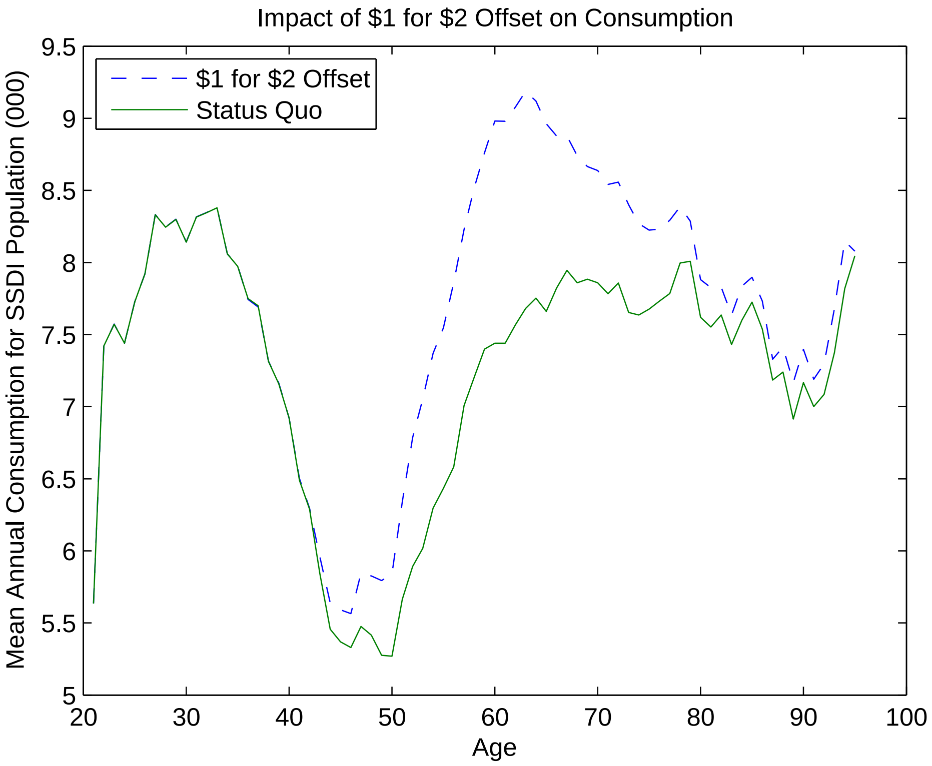


Impact on Social Security

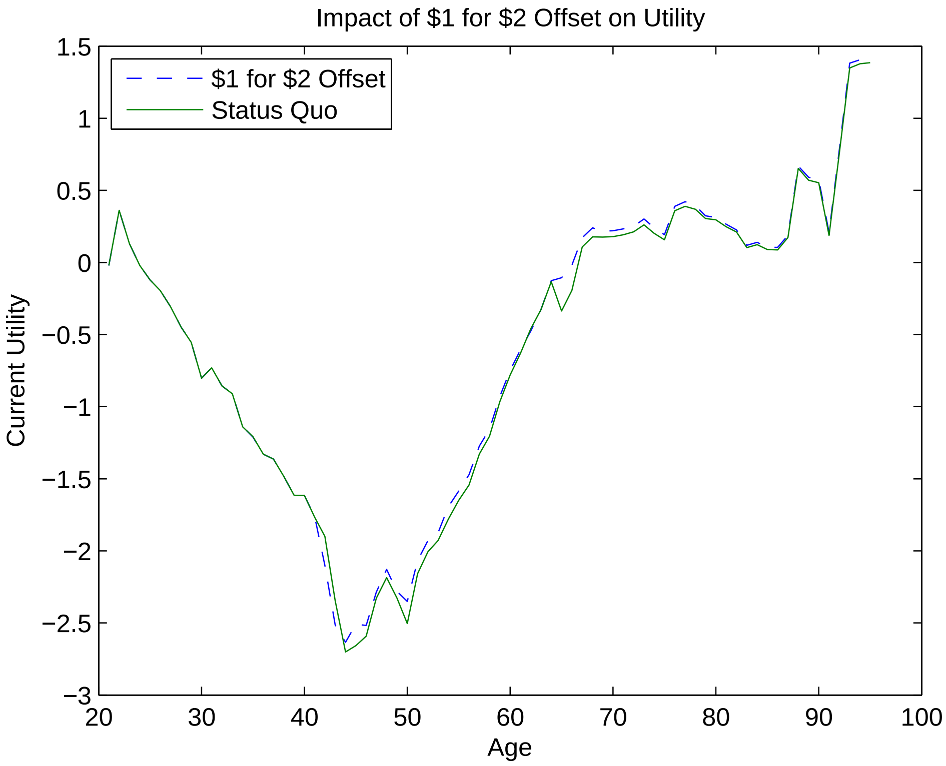


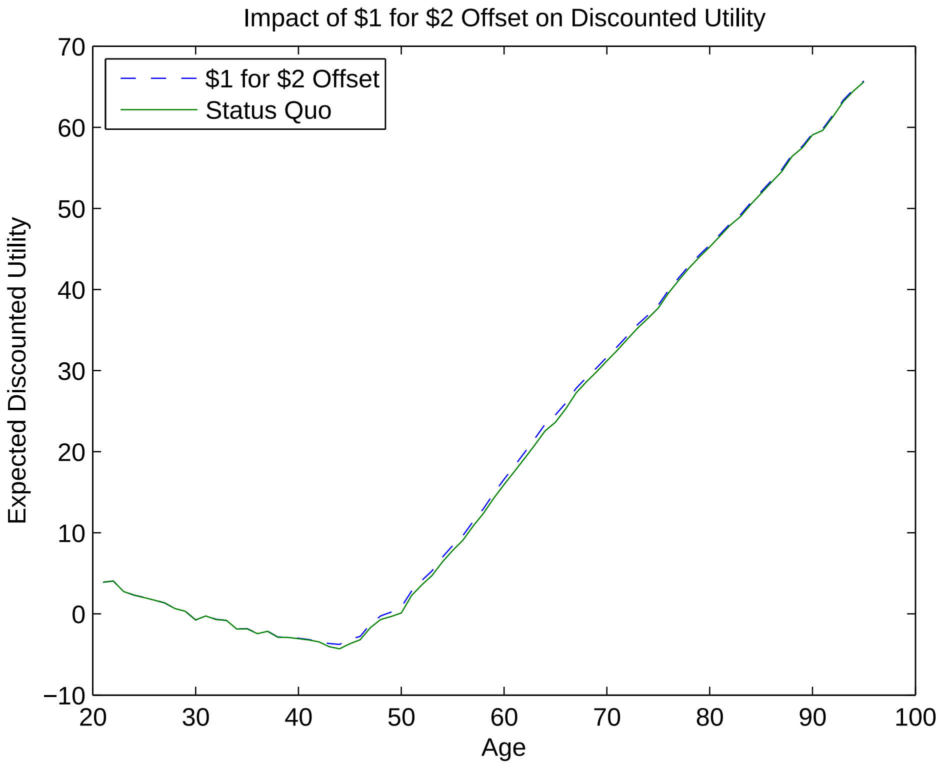
Impact on Wealth and Consumption



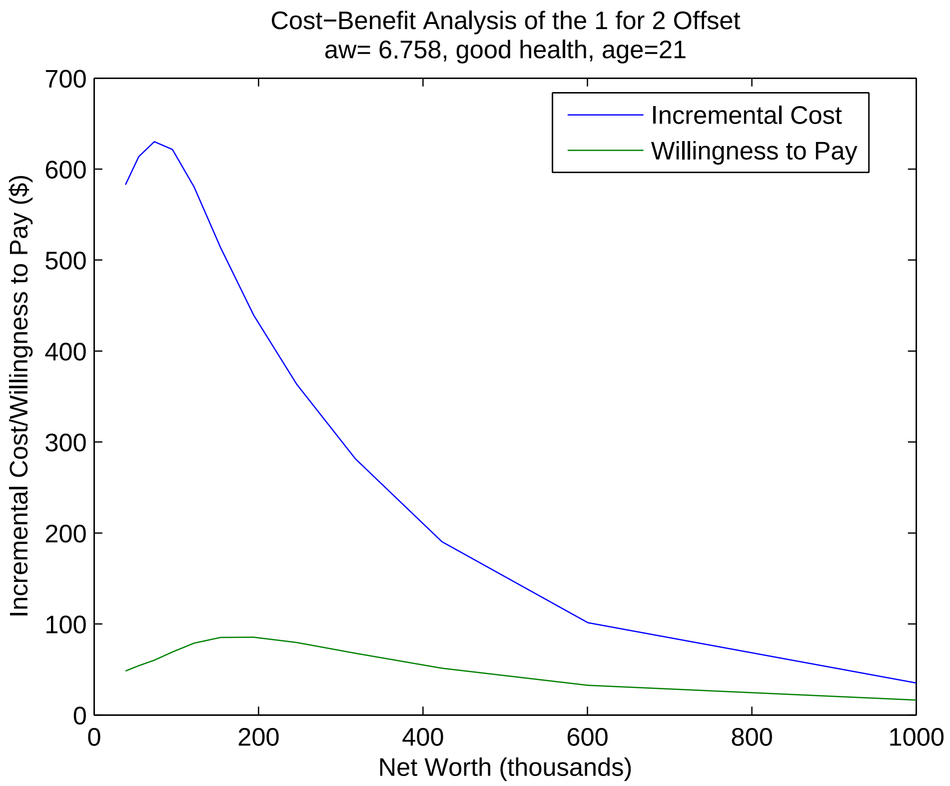


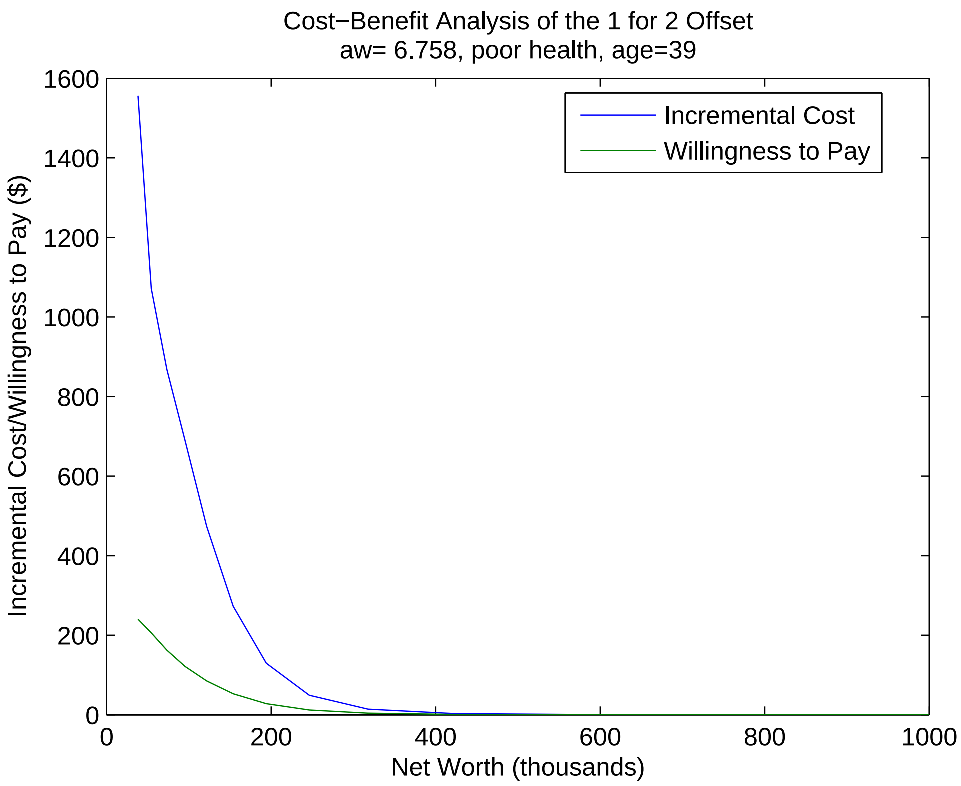
Impact on Welfare





Cost-Benefit Analysis





Improving Car Rental Profits

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4. Unfortunately, due to confidentiality restrictions on the data, we are not at liberty to disclose the name of the company and cannot provide much more detail on the exact makes/models of these cars or their locations.

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8. We then have all objects necessary to *simulate* the rental operations of this company

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5. We solve the stopping problem numerically and characterize compare the optimal replacement policy to the firm’s current replacement policy.

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5. The only aging effect that we can detect is a *rental contract composition effect.*

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5. The average age at sale is about 3 years, and the mean odometer at time of sale is about 70,000 kilometers.
6. *We observe only a very few cars that are over 5 years old or whose odometers have more than 140,000 kilometers.*

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4. Of course, it is unreasonable to suppose that rental rates would not decrease if the company kept its vehicle stock indefinitely
5. *Customers prefer new cars, all other things equal!*

The Extrapolation Problem

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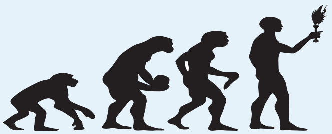

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5. We also assume that with appropriate *odometer-based discounts* on rental vehicles, customers can be induced to rent older vehicles.

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3. The *optimal replacement threshold* in the pessimistic scenario is 150,000 kilometers, about twice as large as under the *status quo.*
4. *Expected discounted profits increase significantly.* Depending on the type of car, we predict profits will be between *18-240 percent larger* than the *status quo* if it adopts the optimal replacement policy.

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3. Drawback of experiments: they are costly and time consuming, and may *contaminate* customers who receive discounts, leading them to expect similar discounts at other locations.
4. In the absence of experimental data, we believe *model-based predictions* such as ours, can be useful devices to help a company evaluate the profitability of its current operating strategy.

2.1 Analyzing Rentals

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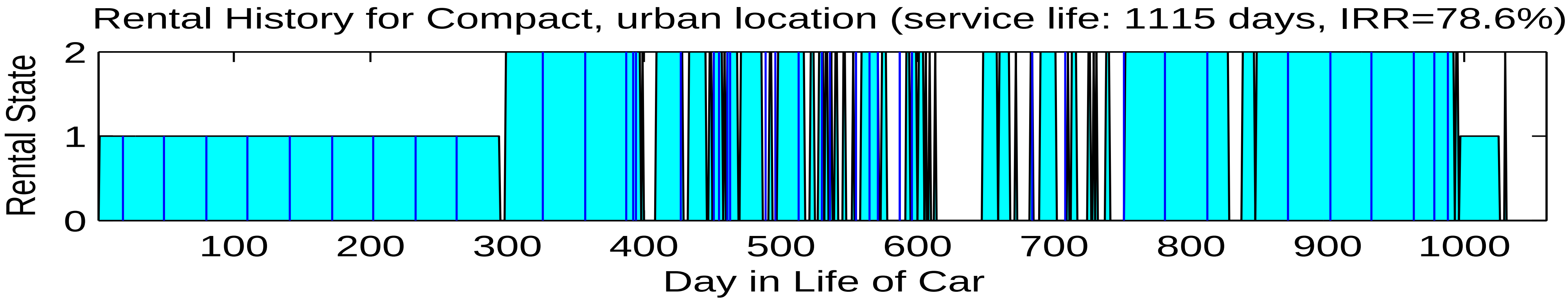
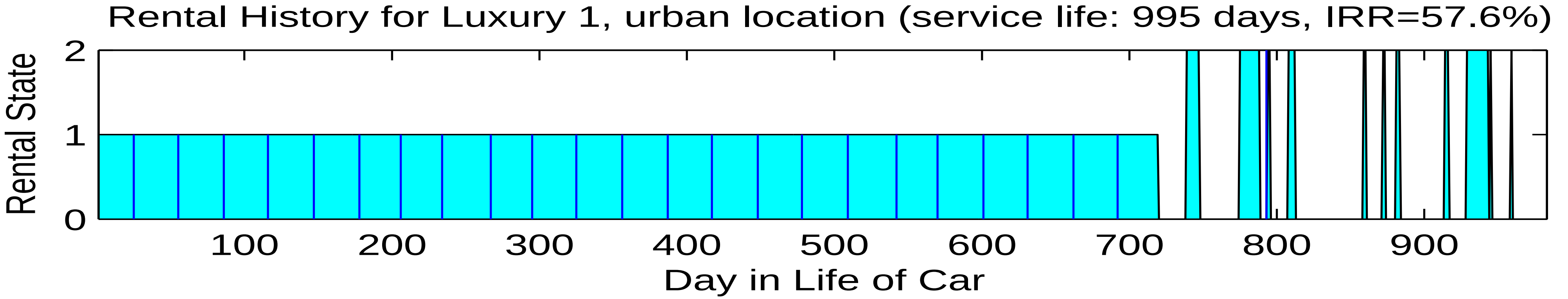
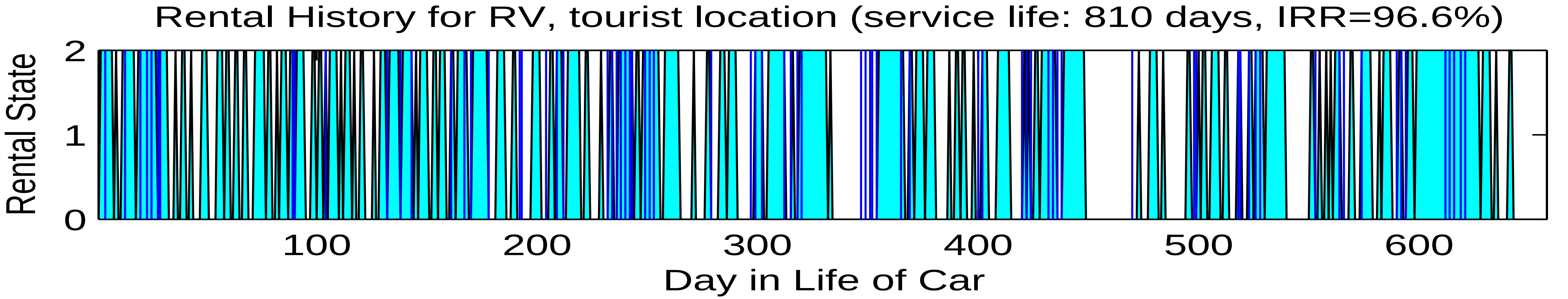
2. Customers are allowed to *roll over* a 30 day long term contract into a *defacto* equivalent of a long term lease.
3. There is a penalty for early returns of vehicles in long term contracts, generally equal to 20% of the lost rental revenue for the unfinished remaining days in the contract.

Typical Rental Histories







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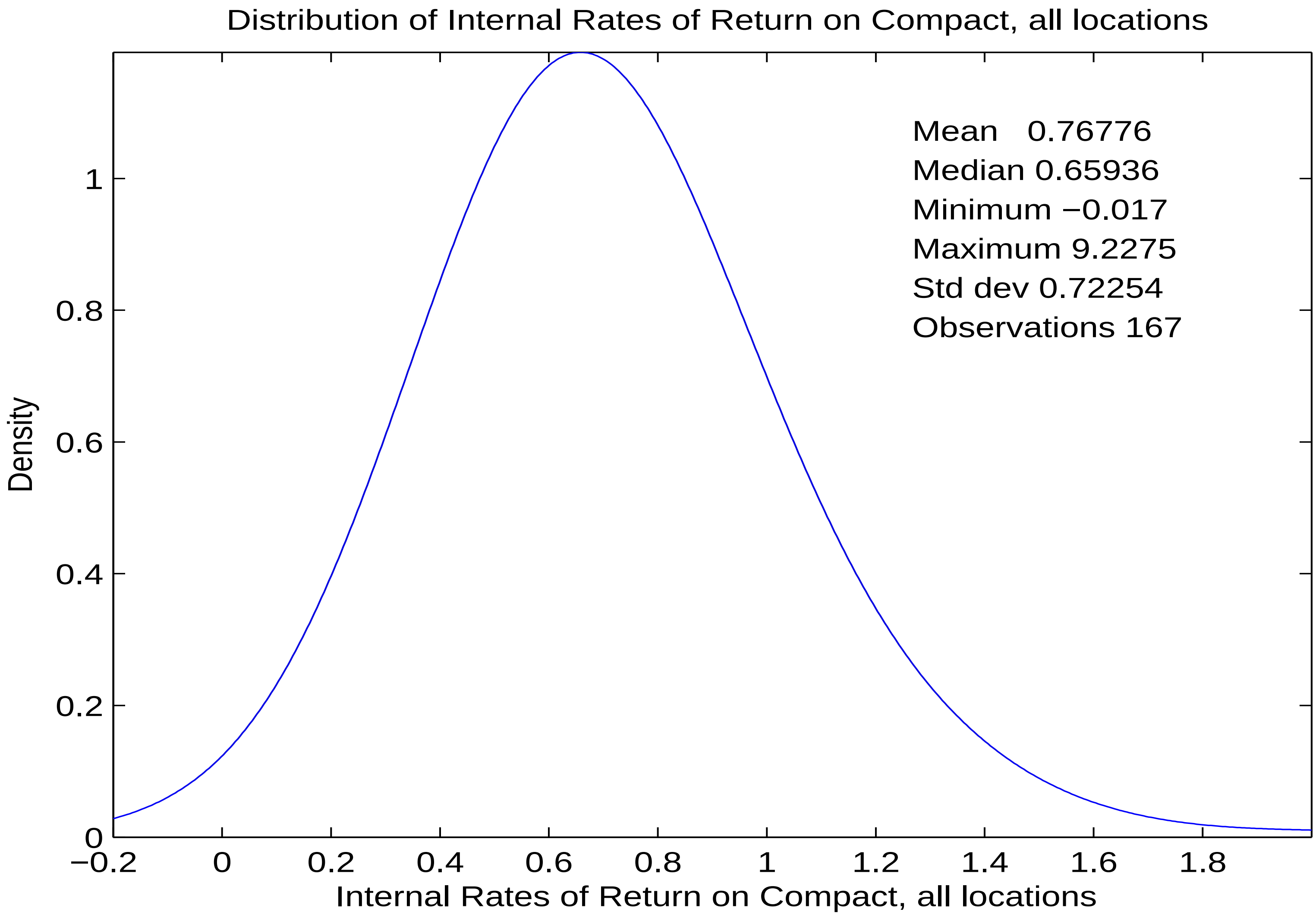
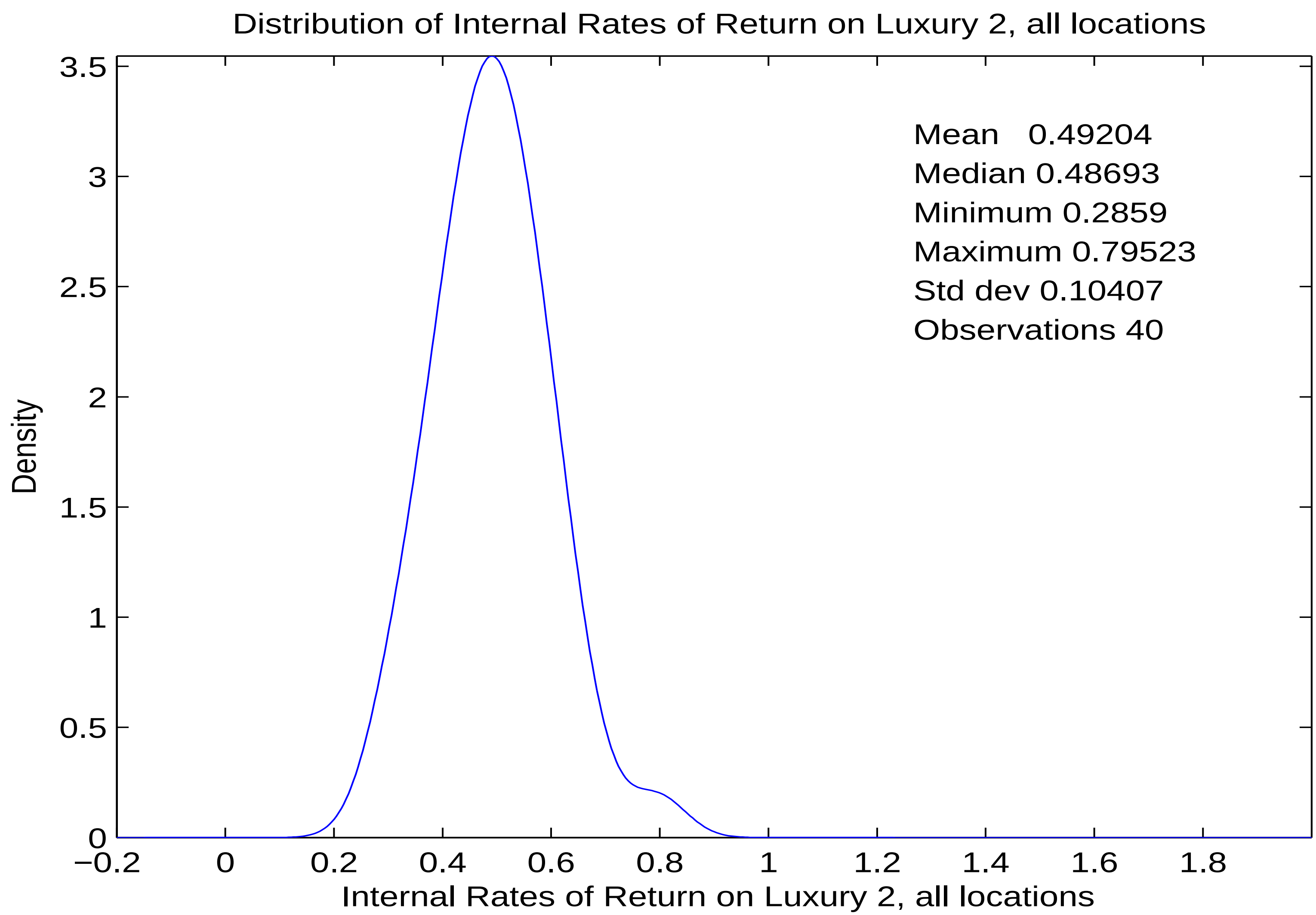
3. We see that for each of the cars illustrated in figure 1, the realized rates of return are extraordinarily high. *These high returns are not atypical.*

2.2 Analyzing Returns

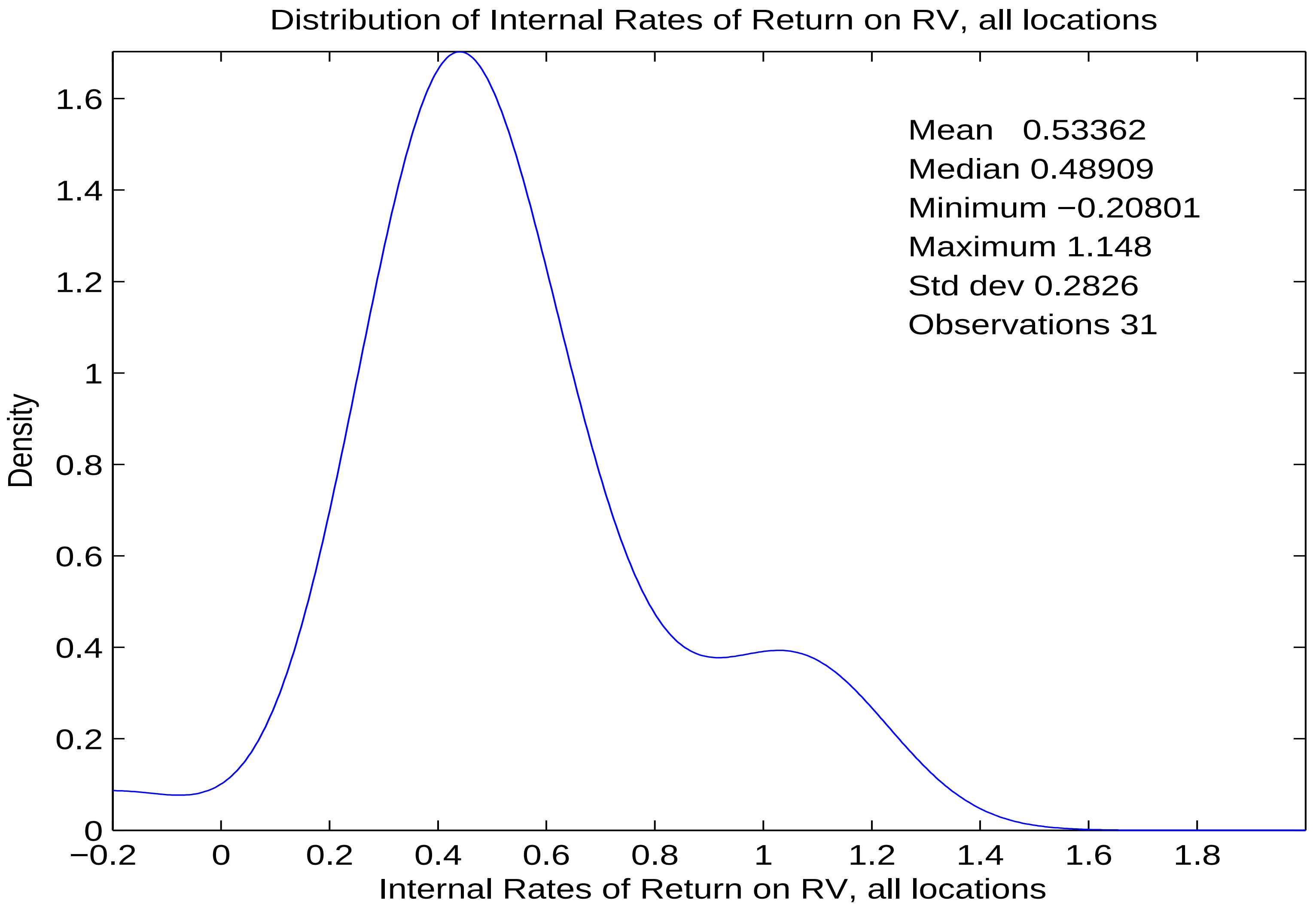
Return distribution: Compact



Return distributions: Luxury



Return distribution: RV



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3. Counterbalancing this, is that cars are more likely to be on the lot between successive short term rental spells.
4. *Which contract is more profitable: long or short term?*

Regression results: IRR

Variable	Compact	Luxury	RV
Constant	0.575^{*}	-0.006	0.999
Utilization Rate	0.003	0.522^{*}	1.366^{*}
Fraction Rented Long Term	-0.220^{*}	-0.076	-0.876^{*}
Total Maintenance costs ($000)	$-7.46e^{-5*}$	$-2.00e^{-5}$	$6.978e^{-6}$
Odometer (000 km)	0.0007	-0.0004	-0.001
Age at Sale (years)	0.151^{*}	0.072	-0.154
New Price ($000)	-0.104^{*}	-0.036^{*}	-0.082^{*}
Sale Price ($000)	0.008	-0.002	0.063
Short term rental rate	0.003^{*}	0.0006^{*}	0.004^{*}
Long term rental rate	0.037^{**}	0.020^{**}	0.009
Observations, R^2	167,81%	40,78%	31,86%

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3. and coefficients on *age* and *odometer* are insignificantly different from 0.

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5. In the luxury case, age has a positive coefficient and total maintenance cost has a negative coefficient.
6. But even in this case, the effect of age on IRR is small: the regression results predict that keeping a luxury car for 100 more days increases the IRR by 0.03.

2.3 Is regression enough?

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5. If so, we would expect a *positive correlation* between vehicle age and odometer values and the error term in the regression, so the estimates for these coefficients could be *upward biased.*

Results for simulated optimal replacements

Variable	Compact	Luxury	RV
Constant	0.875^*	0.619	-0.261
Utilization Rate	0.750^*	0.399^*	0.791^*
Fraction Rented Long Term	-0.535^*	-0.142^*	-0.597^*
Odometer (000 km)	0.002	-0.0003	0.002^*
Age at Sale (years)	0.122^*	-0.002	-0.040
Sale Price ($000)	0.009	0.002	0.003
Short term rental rate	0.0006	0.0003	0.003
Long term rental rate	0.008	-0.005	0.008
Observations, R^2	100,67%	100,56%	100,56%

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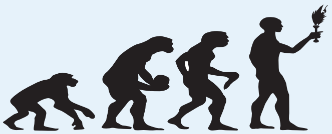

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6. Unfortunately, we do not have either of these instrumental variables in our data set.

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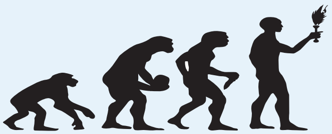

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5. We will simulate the model under the *status quo* and show it provides a good approximation to the data we observe.
6. Then we use the model to compute and simulate *alternative rental policies.* We show that certain alternative policies result in *significantly higher profits.*

3.1 Overview

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- We refer to the latter two states, 3 and 4, as *lot spells.*

Other State Variables

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- The other potential state variable of interest, the vehicle's *age* a_t creates complications due to *non-stationarity.*
- Since age and odometer are highly correlated, we feel that not much damage is done from excluding a_t as an explicit state variable, and *deriving* the implied distribution of vehicle ages from our model.

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- With this information, we can construct a complete econometric model of the company's rental operations, and conduct stochastic simulations of the model to see how accurately it can represent the company's actual operations.

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6. A model of the company's ***replacement decision,*** i.e. the factors that motivate it to sell a given car at a particular point in time.

3.2 Resale Price Model

The resale price model

- We have data on both the new price $\overline{P}(\tau)$ as well as the realized sales price $P_t(o_t, \tau)$ of each car, where τ denotes a particular make and model of vehicle, which we will also call a *car type.*

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- For each of the three car types $\tau \in \{\text{compact}, \text{luxury}, \text{RV}\}$, we estimated a simple linear regression model with the log depreciation rate, $\overline{P}(\tau)/P_t(o_t, \tau)$, as the dependent variable

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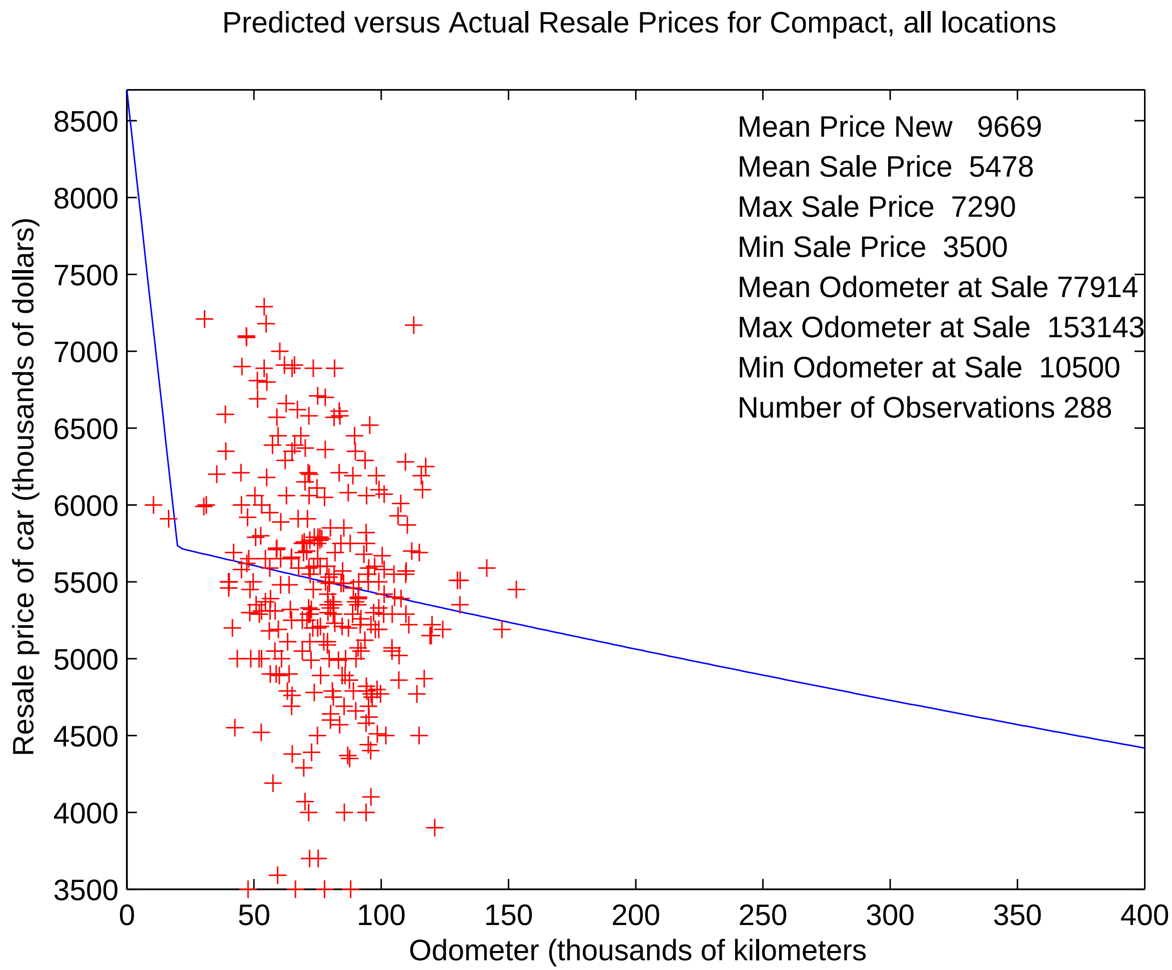
- We also estimated regressions that included vehicle age and other variables such as the number of accidents and the total accident repair cost.

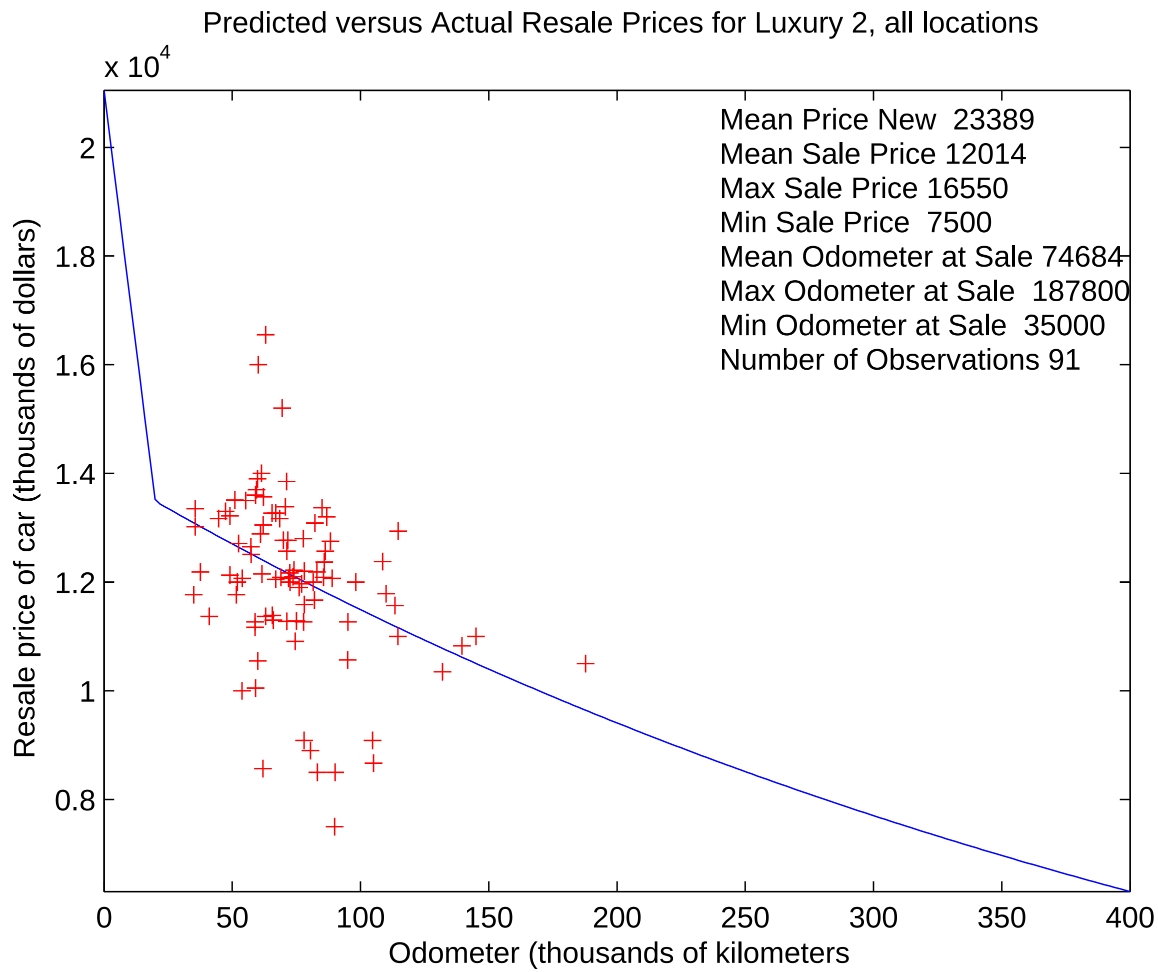
Resale price regression

Variable	Compact	Luxury	RV
Constant	-0.4789^{*}	-0.6201^{**}	-0.8521^{*}
Age (days)	-0.0001^{*}	-0.0004^{*}	-0.0004^{*}
Odometer (000 km)	-0.0007^{*}	-0.0011^{*}	0.0016
Number of Accidents	-0.0112	0.0006	0.0371
Accident Repair Costs	$-0.8.88e^{-6}$	$-4.672e^{-6}$	$-1.654e^{-6}$
Internal Rate of Return	0.1629^{**}	0.067	0.394^{*}
Maintenance Cost per Day	0.0092	-0.0039	-0.0053
N, R^2	288,38.9%	91,42.0%	41,48.1%

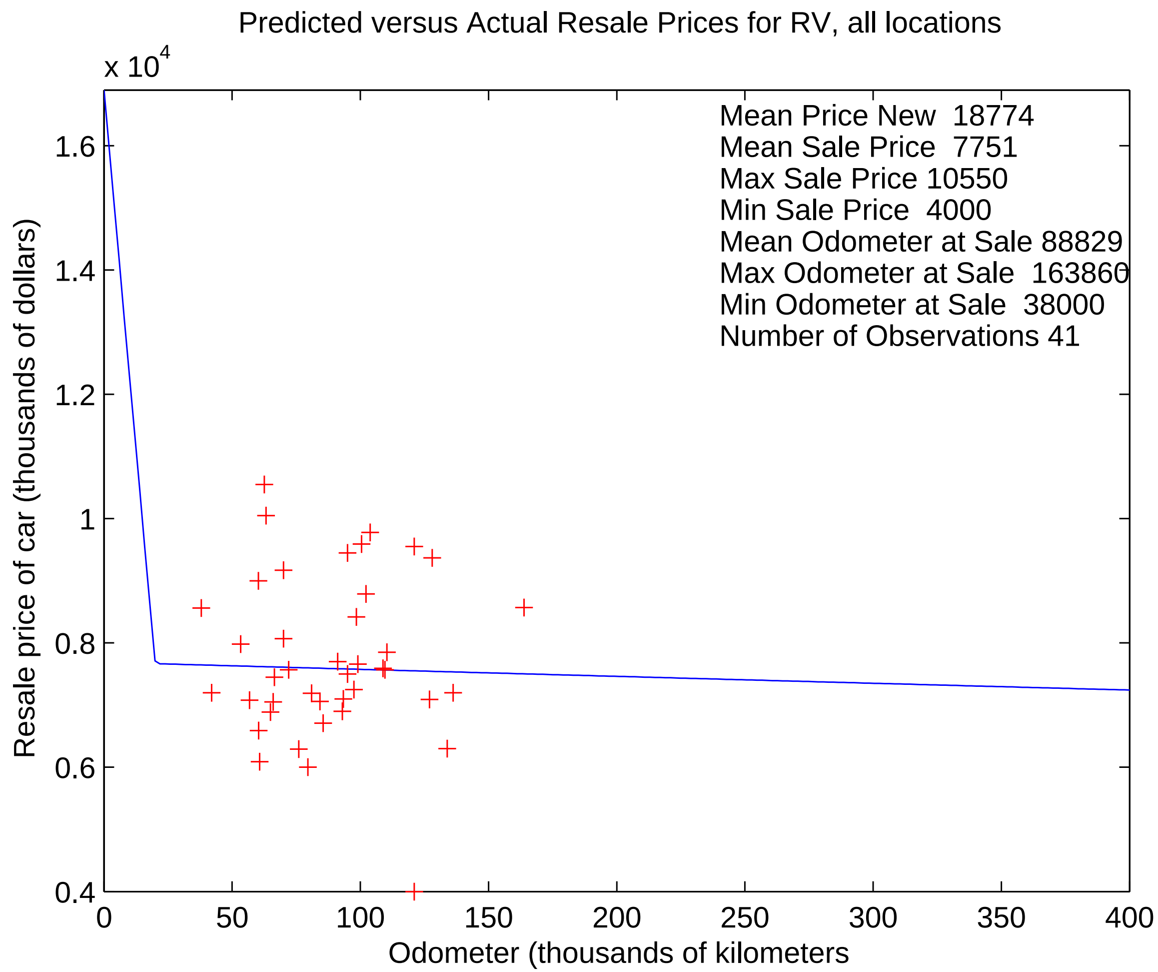
Resale prices: Compact



Resale prices: Luxury



Resale prices: RV



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Comments

Improving Disability Determinations

Improving Return to Work Incentives

Improving Car Rental Profits

3.2 Resale Price Model

- The resale price model
- Resale price regression
- Resale prices: Compact
- Resale prices: Luxury
- Resale prices: RV
- Comments
- Conclusions: resale prices

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4. However predictions of the optimal replacement policy are not sensitive to our assumptions about the precise shape of the depreciation curve for cars with odometer values of less than 20,000 kilometers.

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3. *Conclusion 2:* the rapid initial depreciation implies that vehicle replacement is a significant investment that can be *amortized* by keeping the vehicle sufficiently long before next replacement.

3.3 Vehicle Usage Model

Vehicle Usage Model

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4. We assume that the number of kilometers travelled each day by a rental customer are *IID* draws from an exponential distribution with parameter λ_r.
5. Conditional on spell length d, it follows that $F(o'|d, r)$ is a gamma distribution, since a sum of *IID* exponential random variables has a gamma distribution.

Lifetime Usage Identities

Improving Disability Determinations
Improving Return to Work Incentives
Improving Car Rental Profits
3.3 Vehicle Usage Model
- Vehicle Usage Model
- Lifetime Usage Identities
- Estimating the Usage Model

1. Suppose that at time of sale, a rental car had been rented for N^s days under short term rental contracts and N^l days under long term rental contracts. Then the odometer value on the car at time of sale, $\tilde{o}$, is given by

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2. Thus, we have

$$E\{\tilde{o}|N^l, N^s\} = \lambda_1 N^l + \lambda_2 N^s. \tag{5}$$

Estimating the Usage Model

1. Since we do accurately observe N^l and N^s for each rental car, we can estimate λ_1 and λ_2 as coefficients on a simple linear regression

   $$o_i = \lambda_1 N_i^l + \lambda_2 N_i^s + \varepsilon_i \tag{6}$$

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2. Estimation results

Variable	Compact	Luxury	RV
λ_1	78.7	86.6	95.4
λ_2	157.1	140.8	167.7

3.4 The Replacement Model

The Replacement Decision

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3. Among the variables in the vector x_t are the vehicle's age and predicted odometer value (based on the regression estimate $\hat{o}_t$ using the observed values of N_t^l and N_t^s from the rental contract data, as discussed above), duration in the lot, average daily maintenance costs, and utilization rate.

Logit Estimation Results

Variable	Compact	Luxury	RV
Constant	-13.06^{**}	-12.27^{*}	-14.67^{*}
Age (days)	0.0077^{*}	0.0011	0.0125^{*}
Odometer (km)	0.0050	0.0987^{*}	-0.038
Duration, Age < 500	0.0206^{**}	-11.99^{**}	-6.069^{**}
Duration, Age $\in [500, 1000)$	0.0867^{**}	0.0471^{*}	0.0399^{*}
Duration, Age > 1000	0.1362^{**}	0.1736^{*}	0.1744^{*}
Maintenance Cost	0.00003^{*}	0.2030	-0.0188
Utilization Rate	0.4049	-1.616	1.989
N, $\log(L)/N$	$36262, -0.017$	$6445, -0.022$	$7192, -0.017$

Replacement Conclusions

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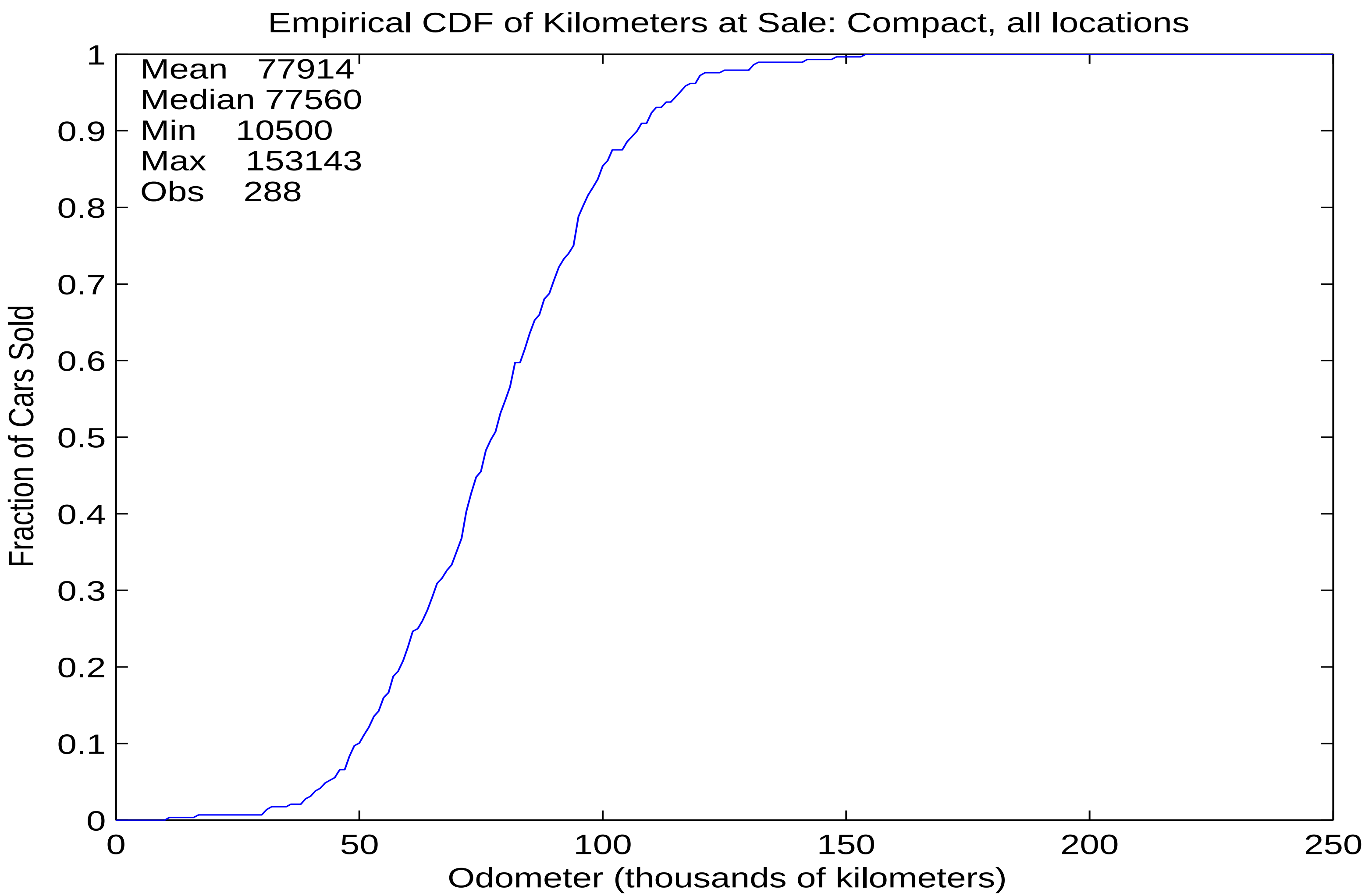
2. Model fits about as well if age is excluded from the logit.
3. Besides age and odometer, the only variable whose coefficient estimates are statistically significant and has signs that are (generally) consistent with our *a priori* expectations is duration.

Odometer at Sale: Compact



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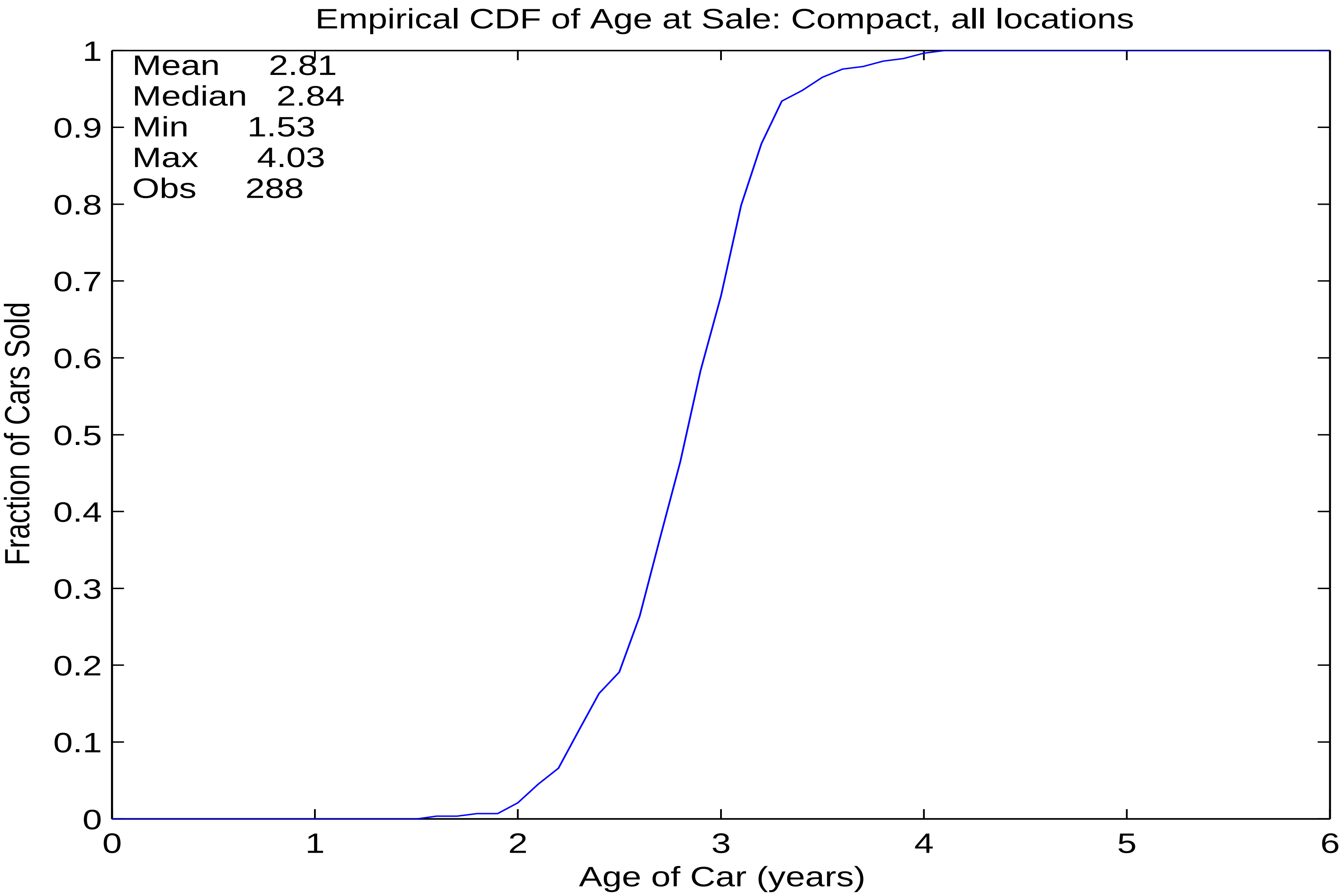
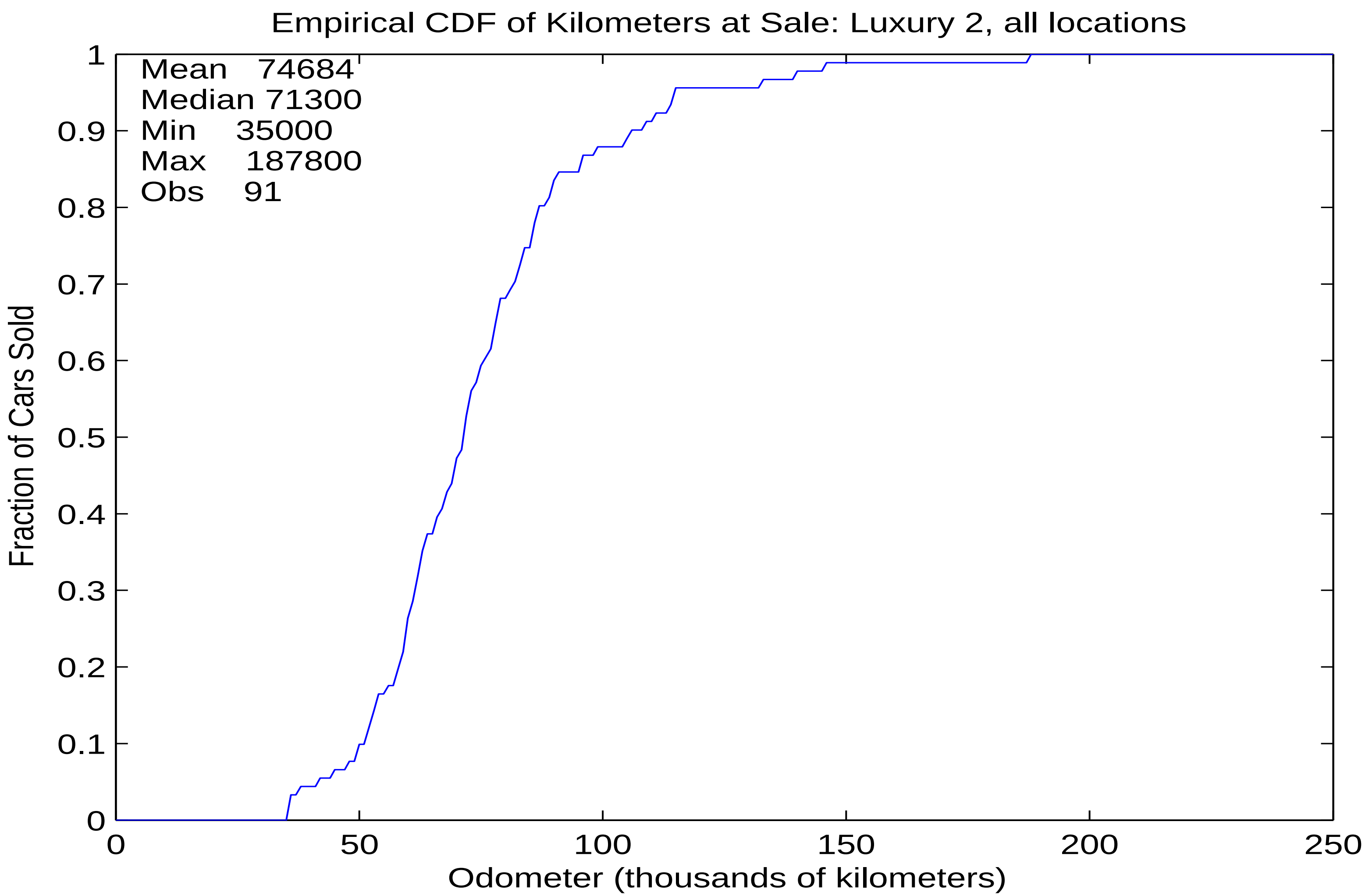


Odometer at Sale: Luxury



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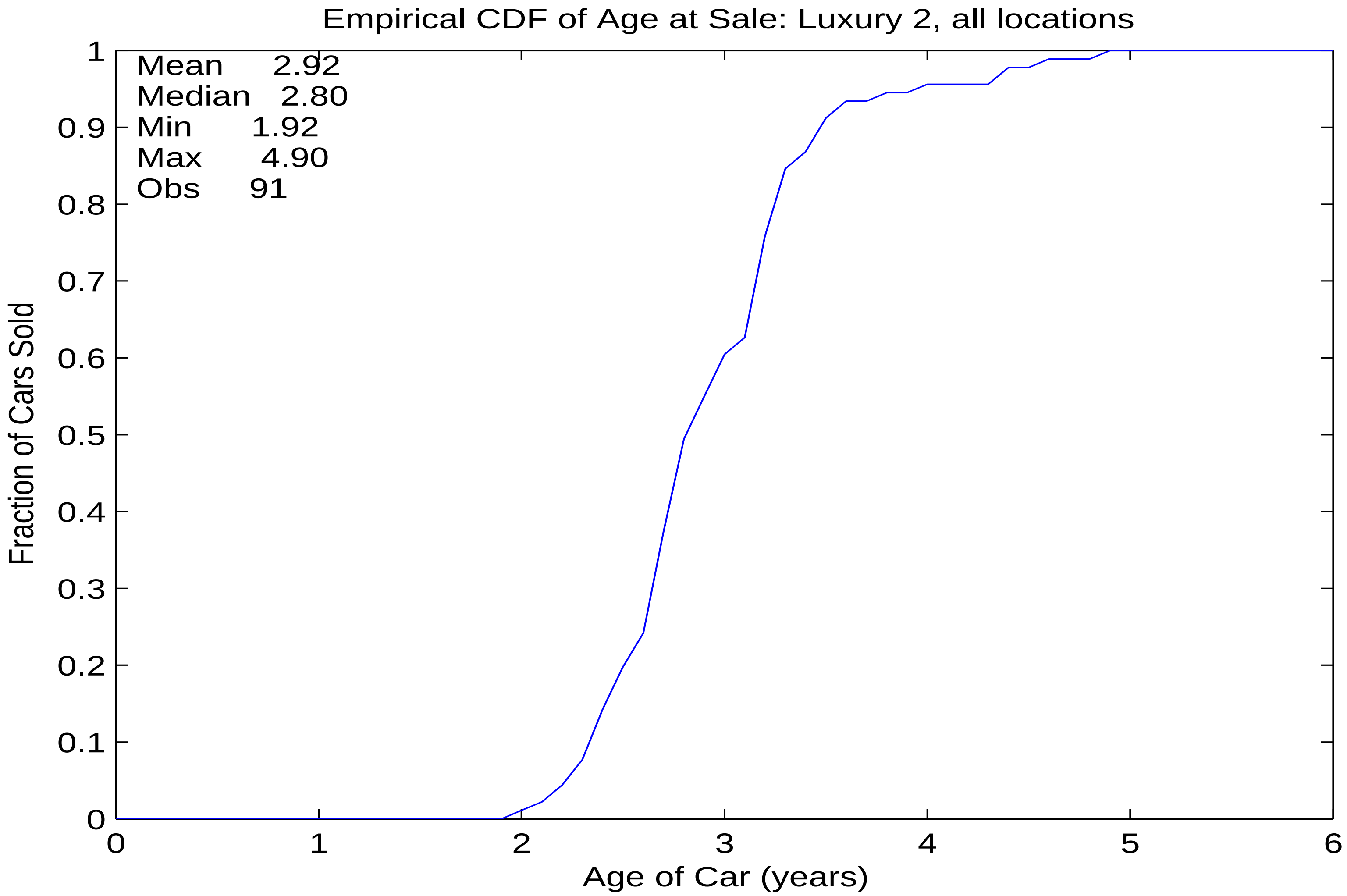
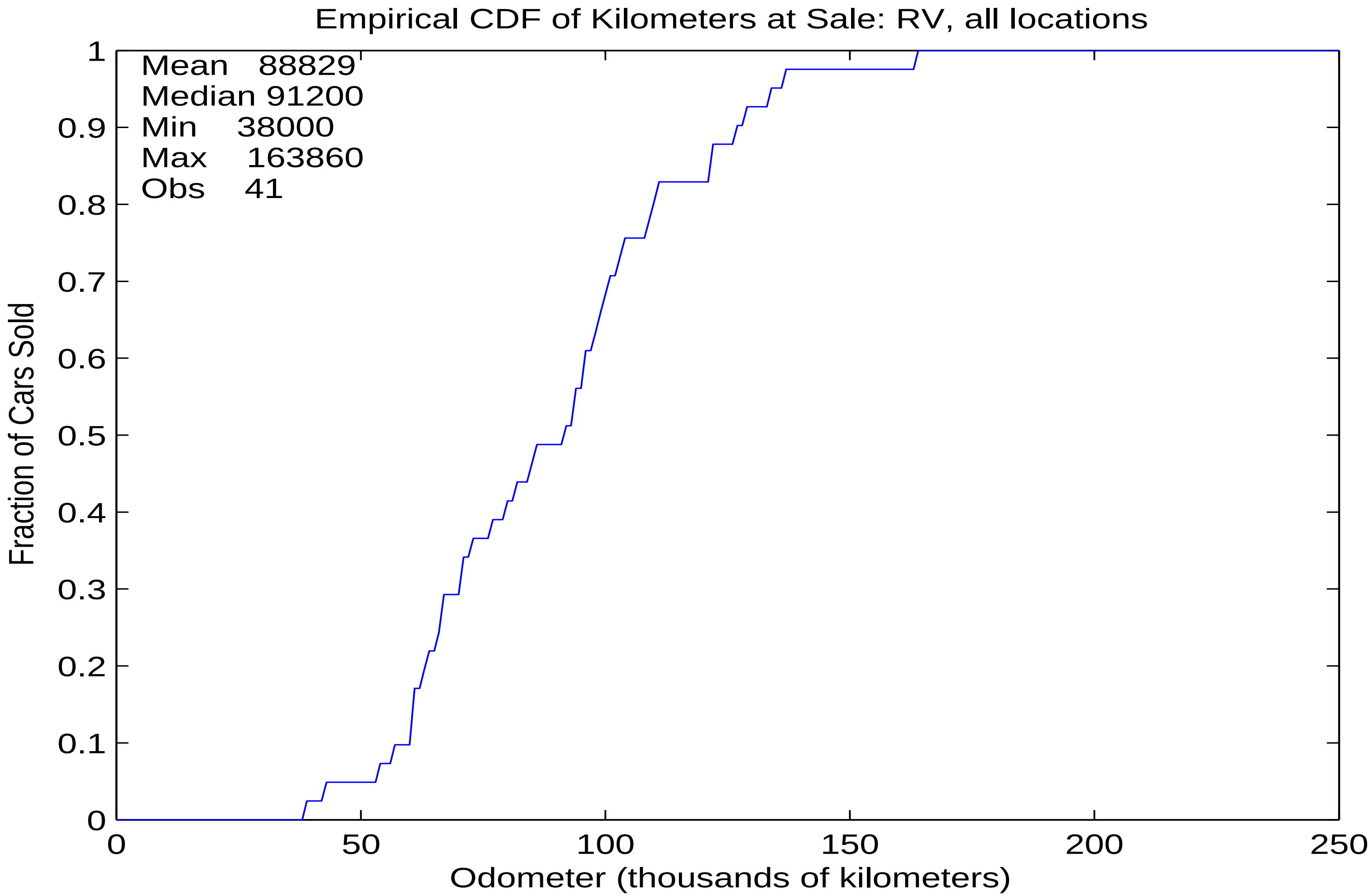


Odometer at Sale: RV



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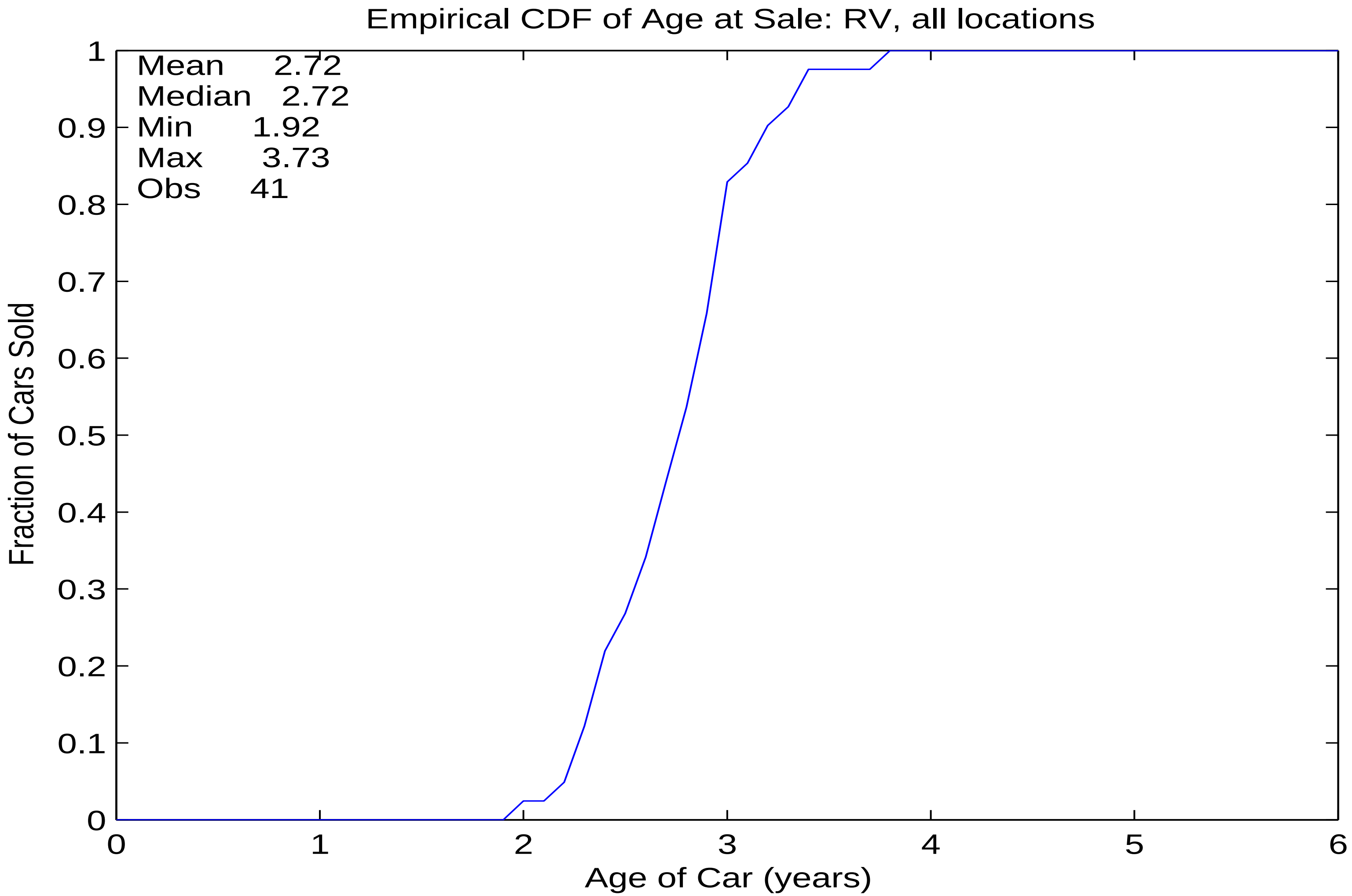

3.5 The Duration Model

Estimated Spell Durations

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$$f(d|r) = \begin{cases} f(1|r) = h(0, r) \\ f(d|r) = \prod_{j=0}^{d-2} [1 - h(j, r)] \, h(d-1, r) \quad d \geq 2 \end{cases}$$

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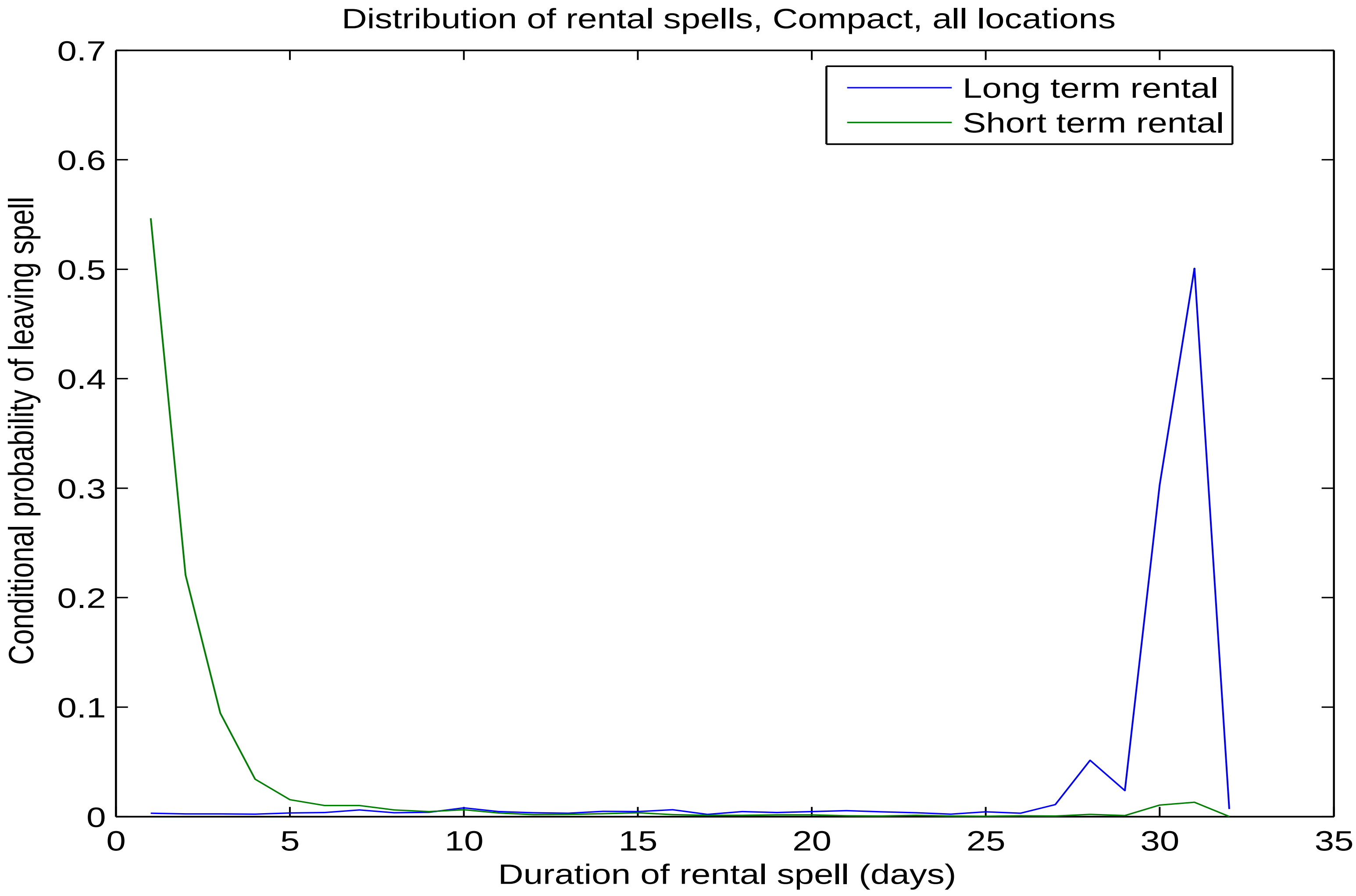
(8)

4. Since we have sufficiently many observations of rental spells, we were able to estimate the hazard functions for these spells *non-parametrically.*

Rental Durations: Compact



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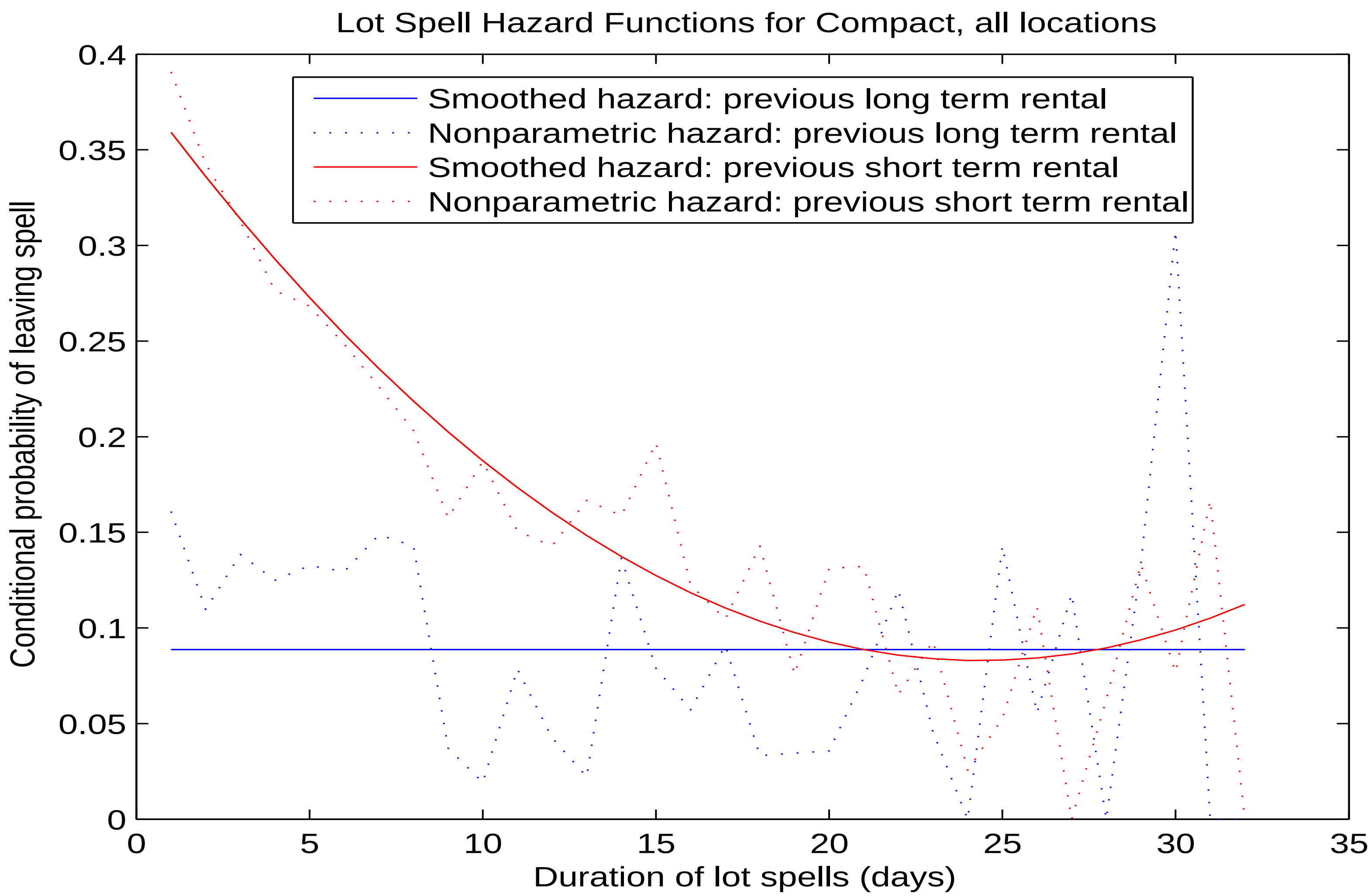
6. We also *regression smoothed* the non-parametric hazard estimates.

Lot Durations: Compact



3.6 Transition Models

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5. However for rental spells, there is a conceptual distinction between a rental spell that terminates with an immediate transition to a new rental spell versus the case where an existing rental contract continues for one more day.
6. The former case can be viewed as an immediate "roll over" of one rental contract to another one.

Functional Forms

1. Since there are three possible destination states for transitions out of rental spells (i.e. long term contract, short term contract, or lot spell), we used a *trinomial logit model* to estimate these probabilities.

$$\pi(r'|r,d,o) = \frac{\exp\{v(r,d,o)\theta_{r'}\}}{\sum_{\rho\in\{1,2,l(r)\}}\exp\{v(r,d,o)\theta_\rho\}}, \tag{9}$$

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2. $v(r,d,o)$ is a vector-valued function of the variables (r,d,o) and θ_ρ is an alternative-specific vector of parameters, for $\rho = \{1,2,l(r)\}$ (where $l(r)$ denotes a lot spell, either of type 3 if $r=1$ or type 4 if $r=2$) with the same dimension as v.

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3. As is well known, it is not possible to identify all three of the θ_ρ vectors. Therefore we make an identifying normalization that $\theta_1 = 0$, i.e. we normalized the parameters for transition to long term contract to zero.

Trinomial Logit Estimates

Variable	Compact	Luxury	RV
Estimates of θ_2 (from short term rental)			
Constant	4.60^{**}	3.01^{**}	4.14^{**}
Odometer, o (000 km)	0.011^{*}	0.002	0.001
Duration, d	-0.068^{*}	-0.039^{*}	-0.087^{*}
$I\{d >= 29\}$	-0.421	0.006	0.079
$I\{r = 1\}$	-6.65^{**}	-6.29^{**}	-6.33^{**}
Estimates of $\theta_{l(r)}$ (from lot spell)			
Constant	3.88^{**}	3.70^{**}	4.36^{**}
Odometer, o (000 km)	0.0204^{*}	0.007^{*}	0.010^{*}
Duration, d	-0.077^{**}	-0.082^{**}	-0.120^{**}
$I\{d >= 29\}$	-1.50^{**}	-1.07^{*}	-0.77
$I\{r = 1\}$	-4.49^{**}	-3.44^{**}	-3.77^{**}
N, $\log(L)/N$	$16246, -0.606$	$3617, -0.484$	$2142, -0.583$

Lot Spell Transitions

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2. We use a *binomial logit model* to estimate these probabilities

$$\pi(r' = 1|r, d, o) = \frac{\exp\{v(o, d)\theta_r\}}{1 + \exp\{v(o, d)\theta_r\}}, \quad r \in \{3, 4\} \tag{10}$$

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3. Note that there are far fewer observations on transitions out of type 3 lot spells due to the high frequency of roll over of long term rental contracts.

Binomial Logit Estimates

Variable	Compact	Luxury	RV
Estimates of θ_3 (previous rental long term)			
Constant	2.26^*	1.59^*	2.34^*
Odometer, o (000 km)	0.010	-0.002	-0.009
Duration, d	-0.05^*	-0.004	-0.038
N, $\log(L)/N$	$173, -0.326$	$181, -0.490$	$43, -0.511$
Estimates of θ_4 (previous rental short term)			
Constant	3.63^*	1.94^*	4.54^*
Odometer, o (000 km)	0.021^*	0.013^*	-0.009
Duration, d	-0.06^*	-0.003	-0.03^*
N, $\log(L)/N$	$5162, -0.077$	$961, -0.683$	$922, -0.090$

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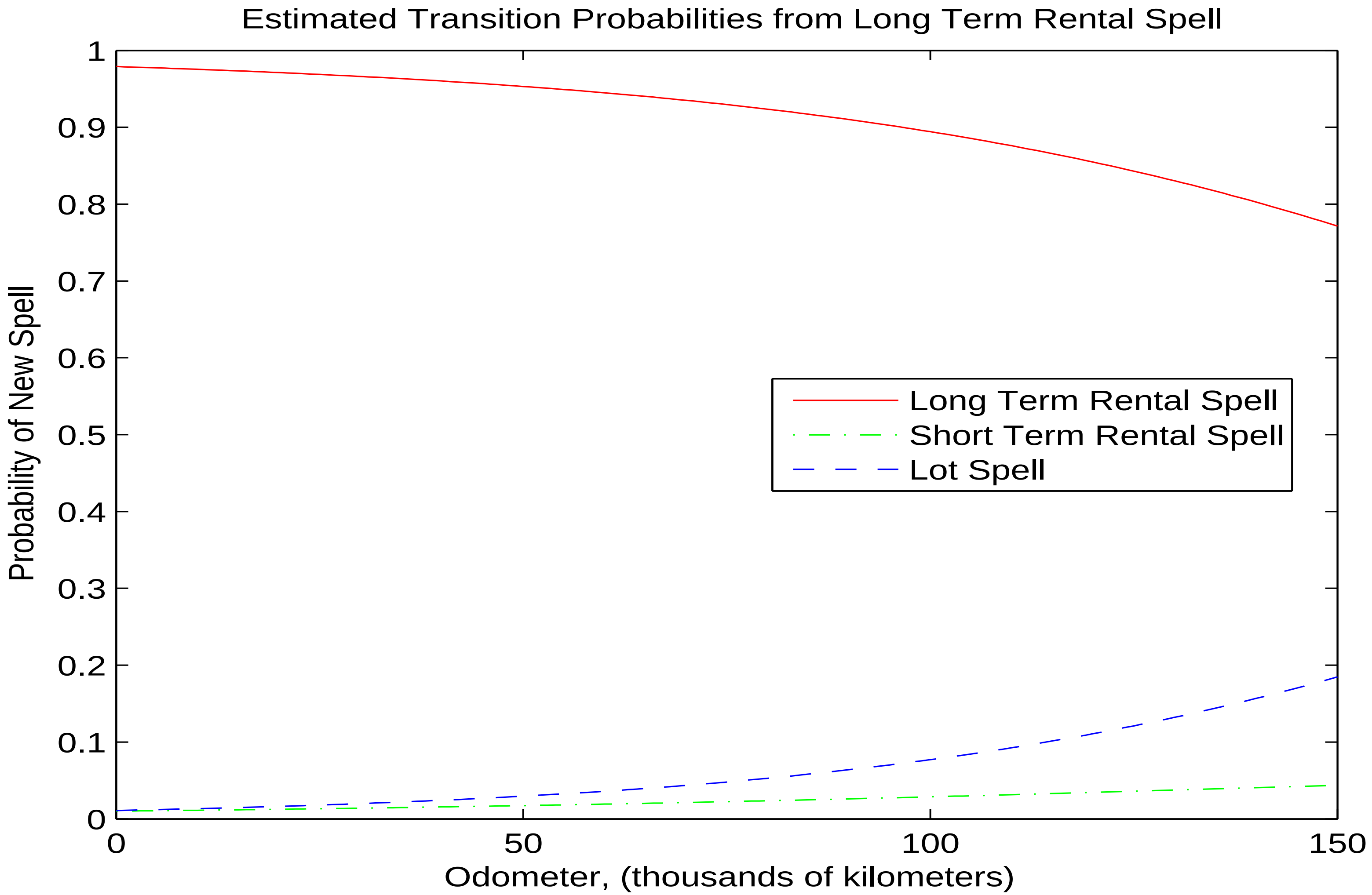
- That is, as the odometer value increases (i.e. the age of the car increases) the probability of transitions into long term rental contracts decreases and the probability of transitions into short term rental contracts increases.

Conclusions

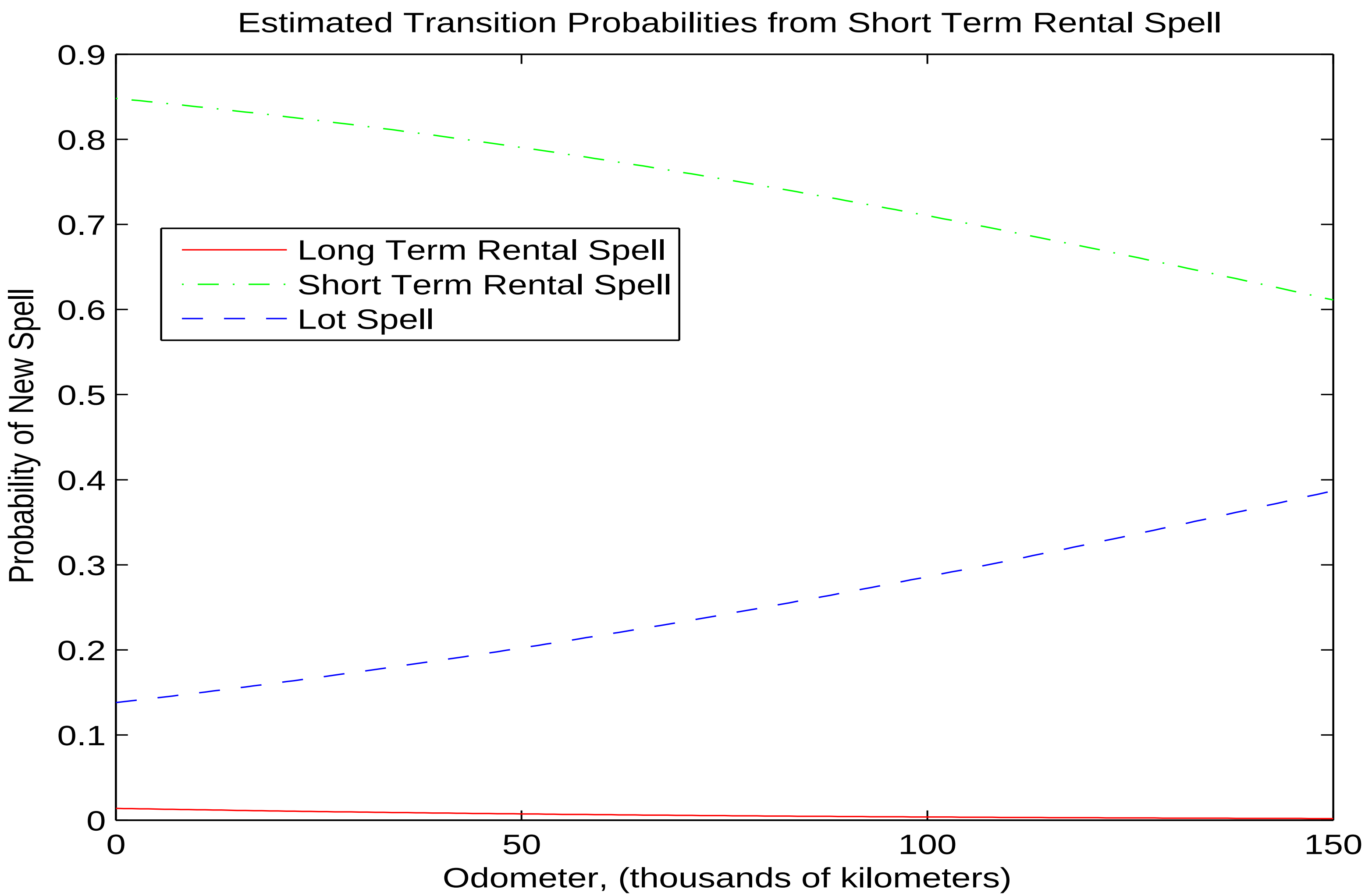
- There are two key points to take away from the transition probability estimates
  1. for all car types, there is a very high probability that cars will be initially rented in long term contracts,
  2. the results provide clear evidence of "contract age effects".
- That is, as the odometer value increases (i.e. the age of the car increases) the probability of transitions into long term rental contracts decreases and the probability of transitions into short term rental contracts increases.
- *However we find no other aging effects in spell durations or in maintenance costs.*

3.7 Probability Graphs

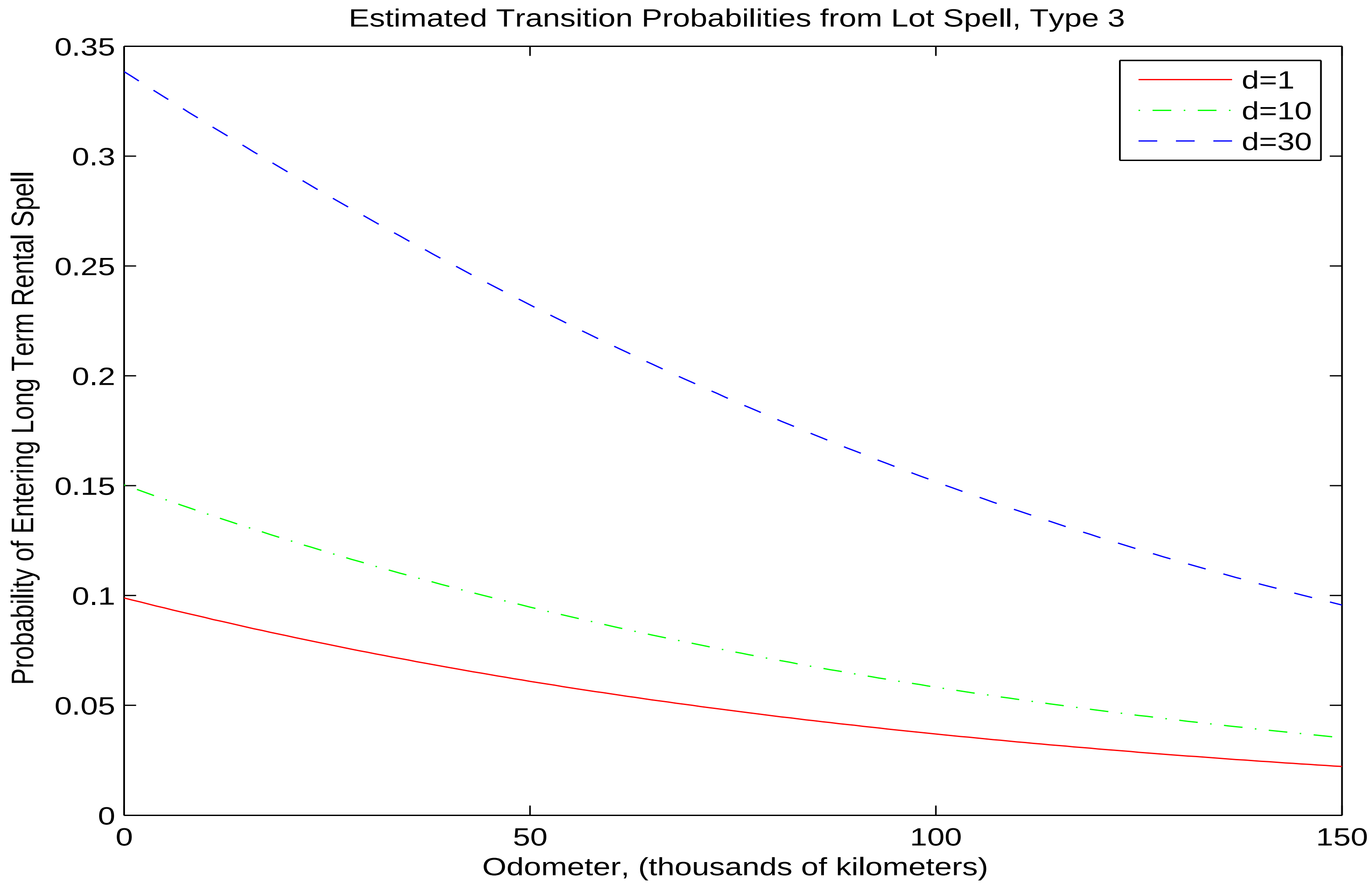
Transition from Long Term



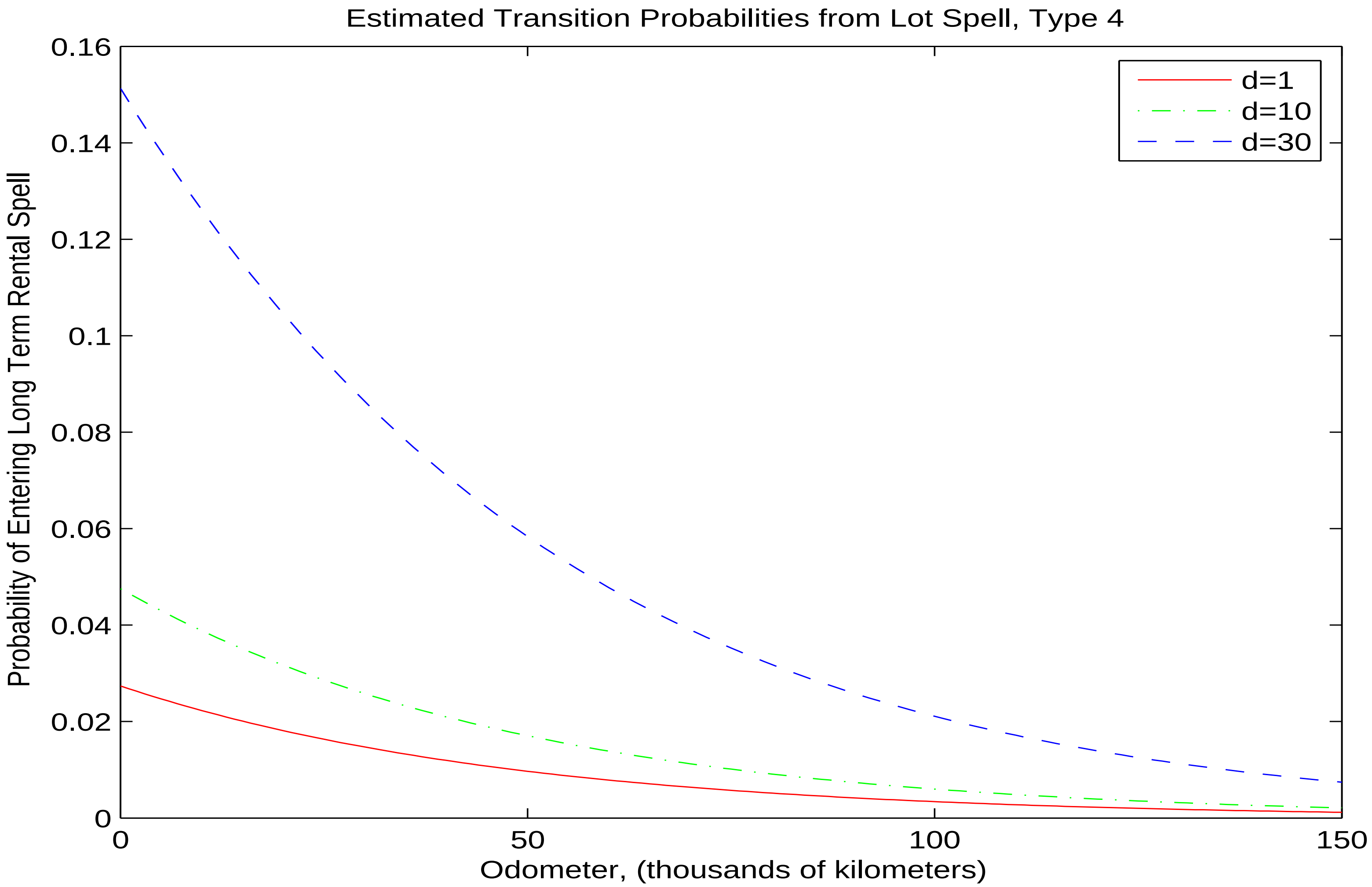
Transition from Short Term



Transition from Lot Type 3

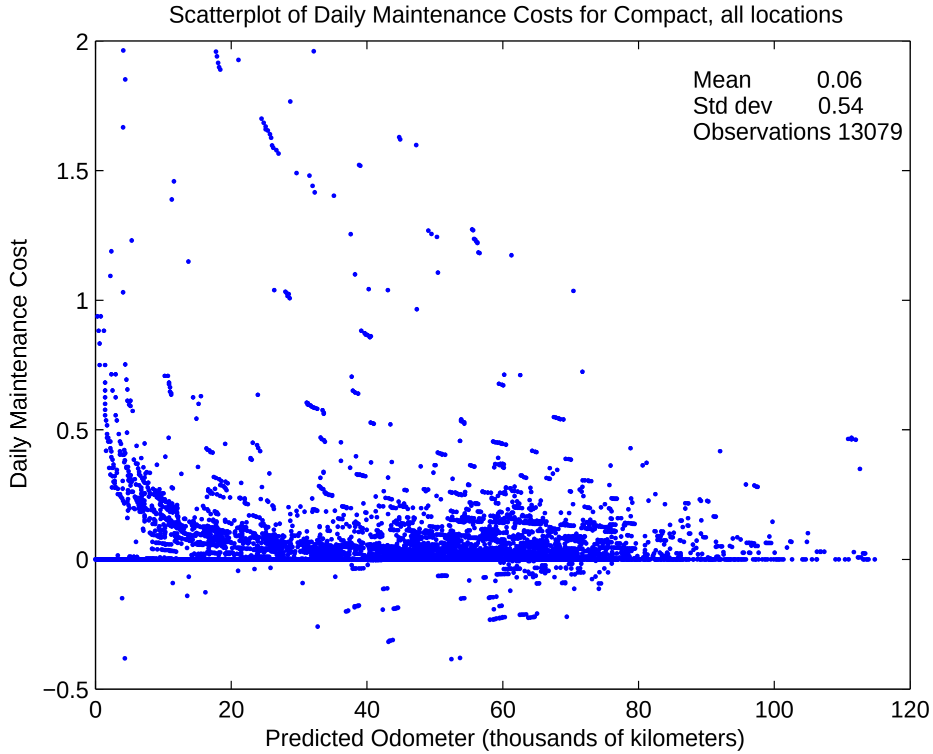


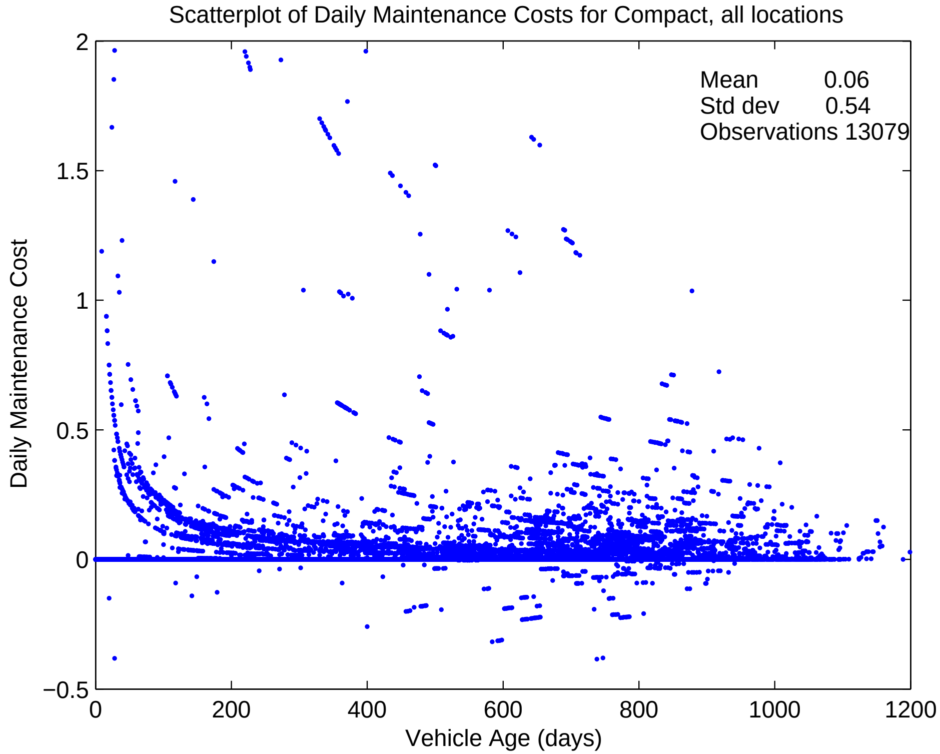
Transition from Lot Type 4



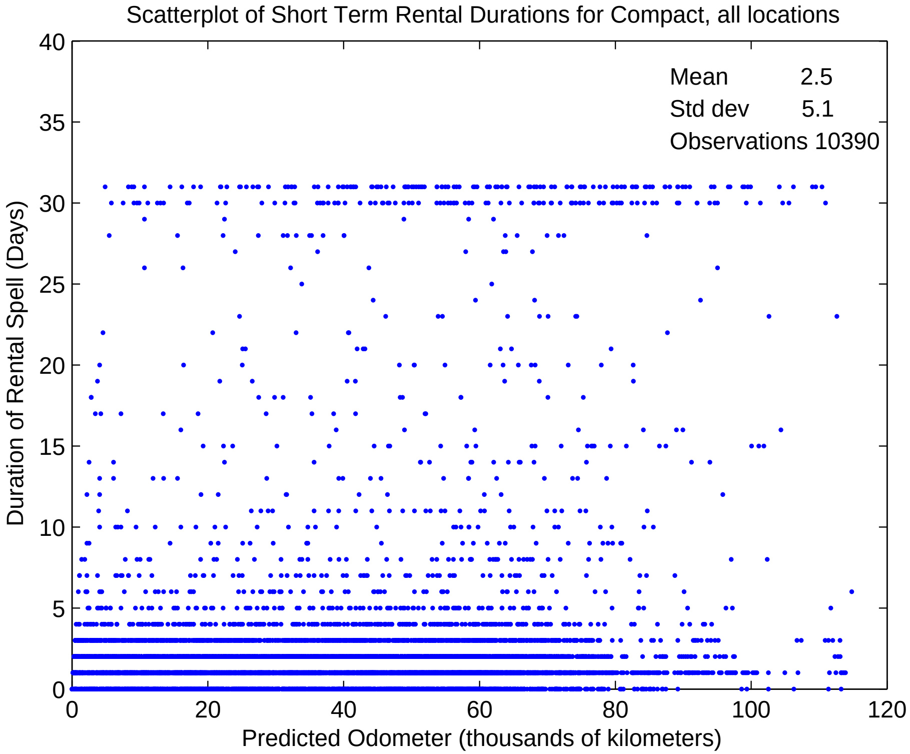
3.8 No Other Age Effects

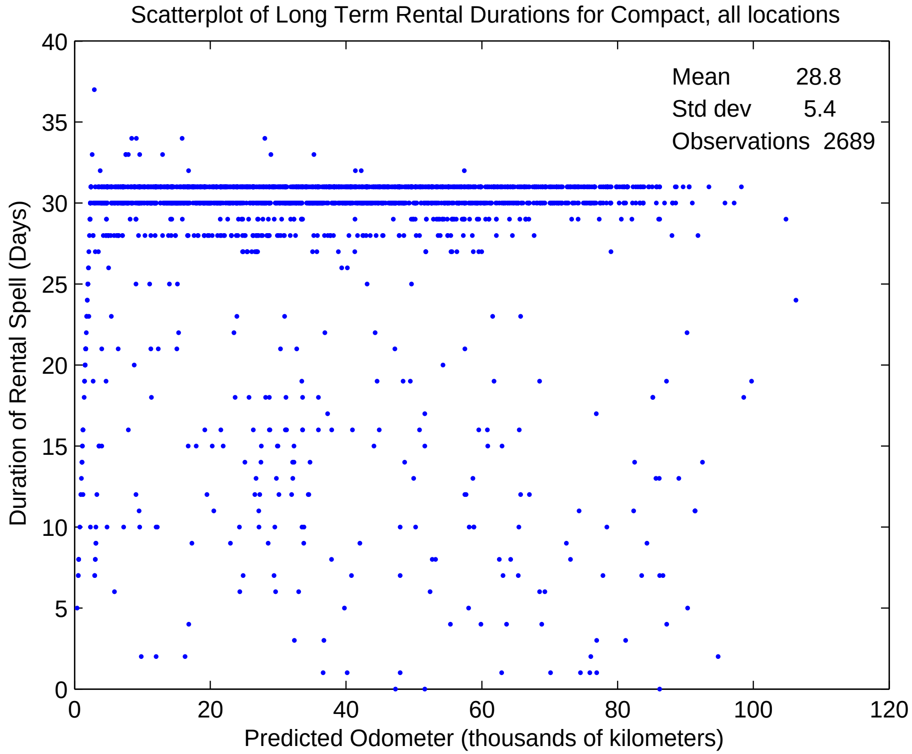
No Aging in Maintenance



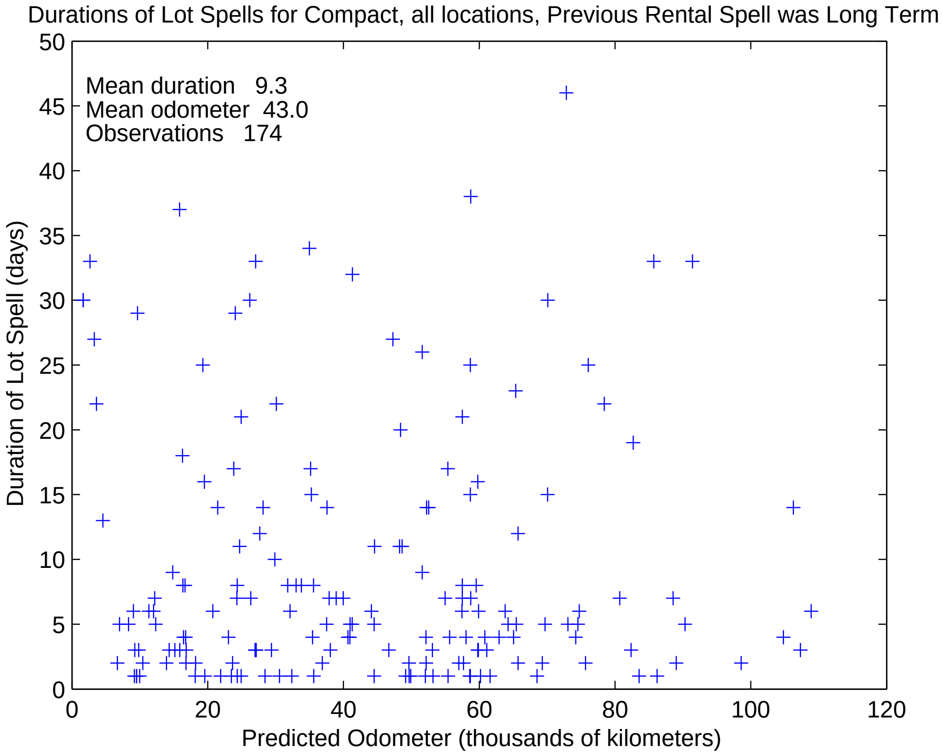


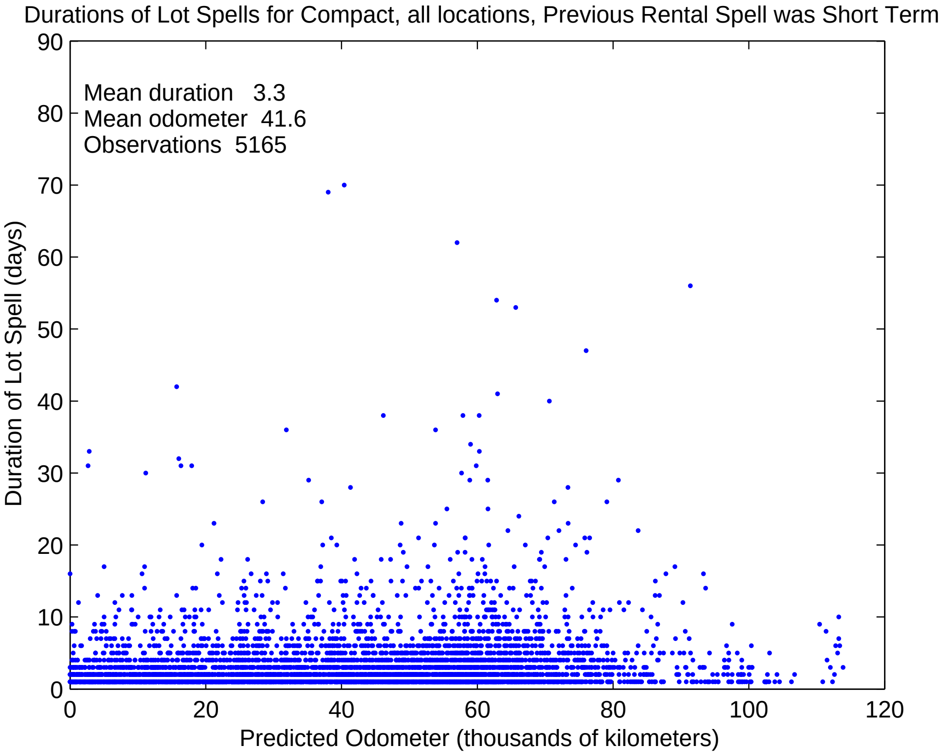
No Aging in Rental Durations





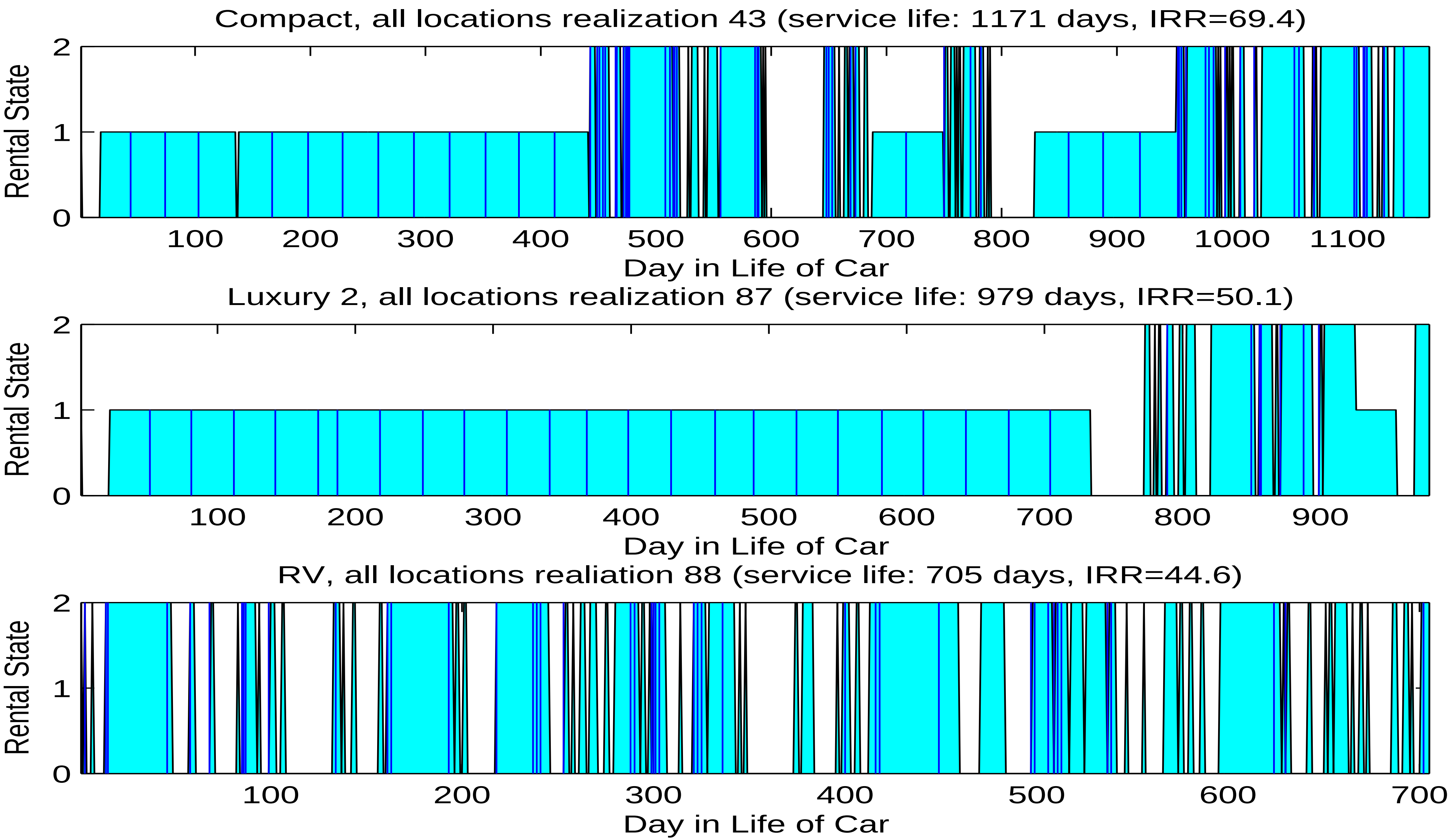
No Aging in Lot Durations





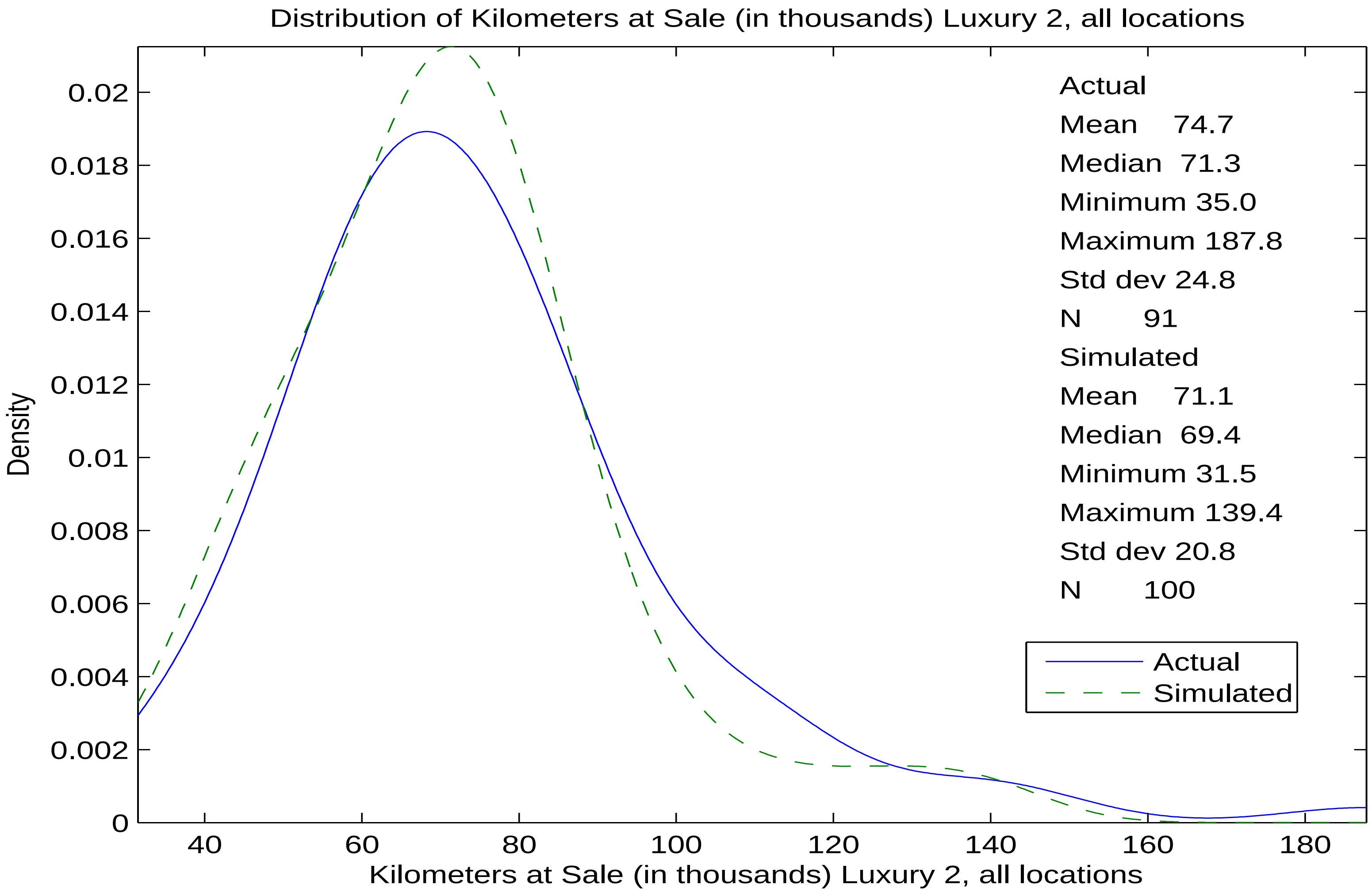
4 Simulating the Model

Simulated Histories

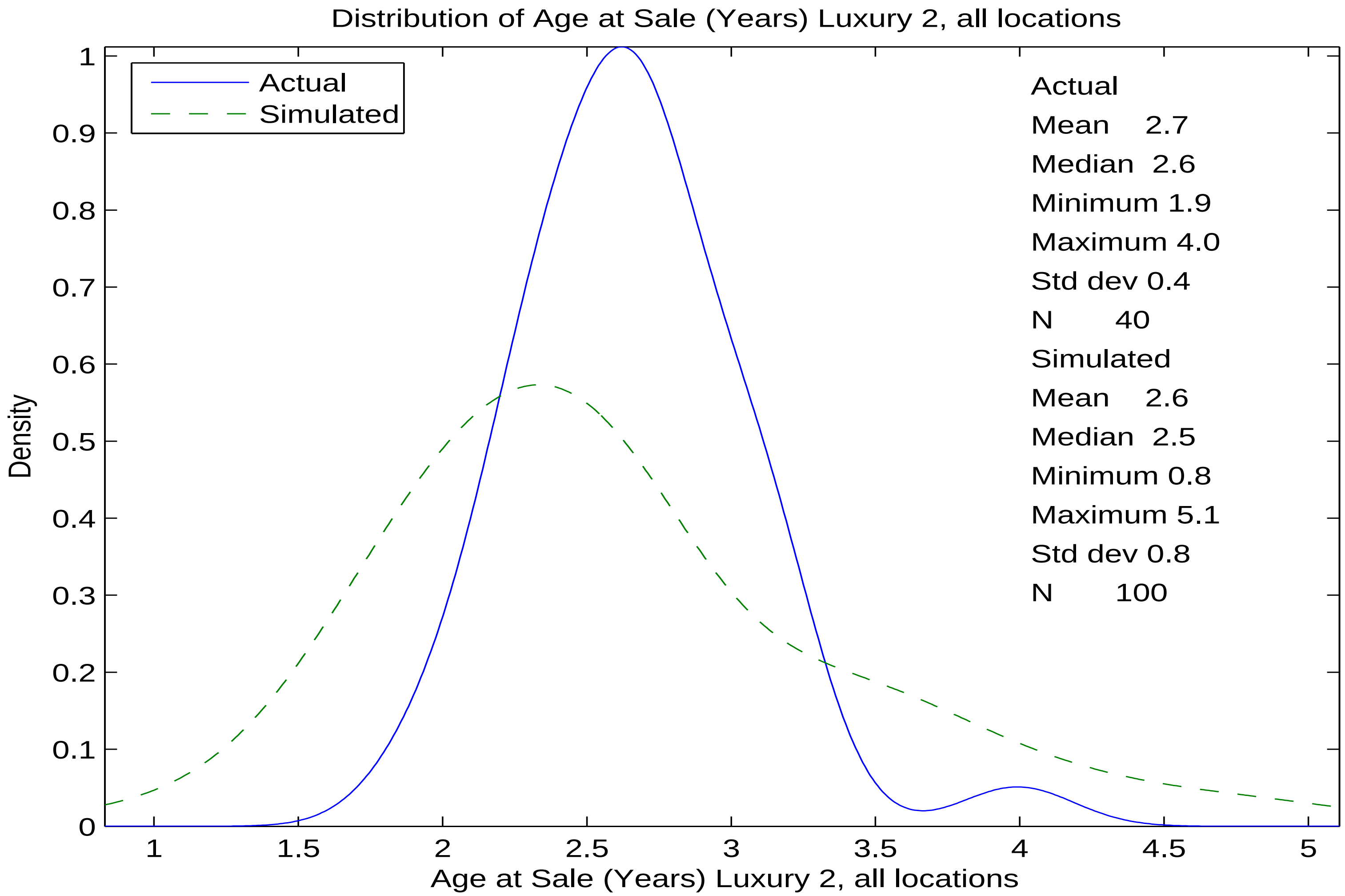


3.1 Odometer and Age

Odometer at Replacement

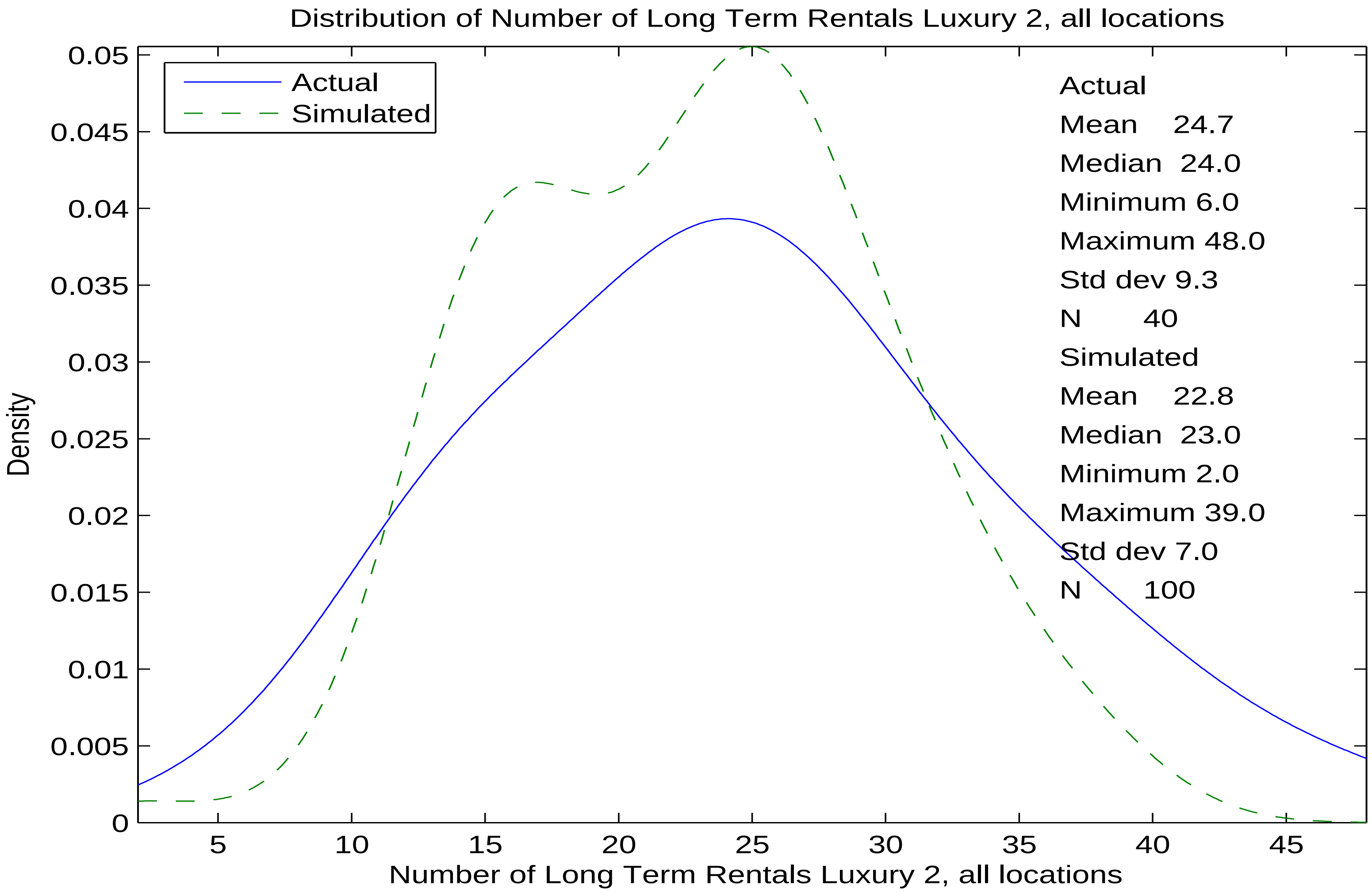


Age at Replacement

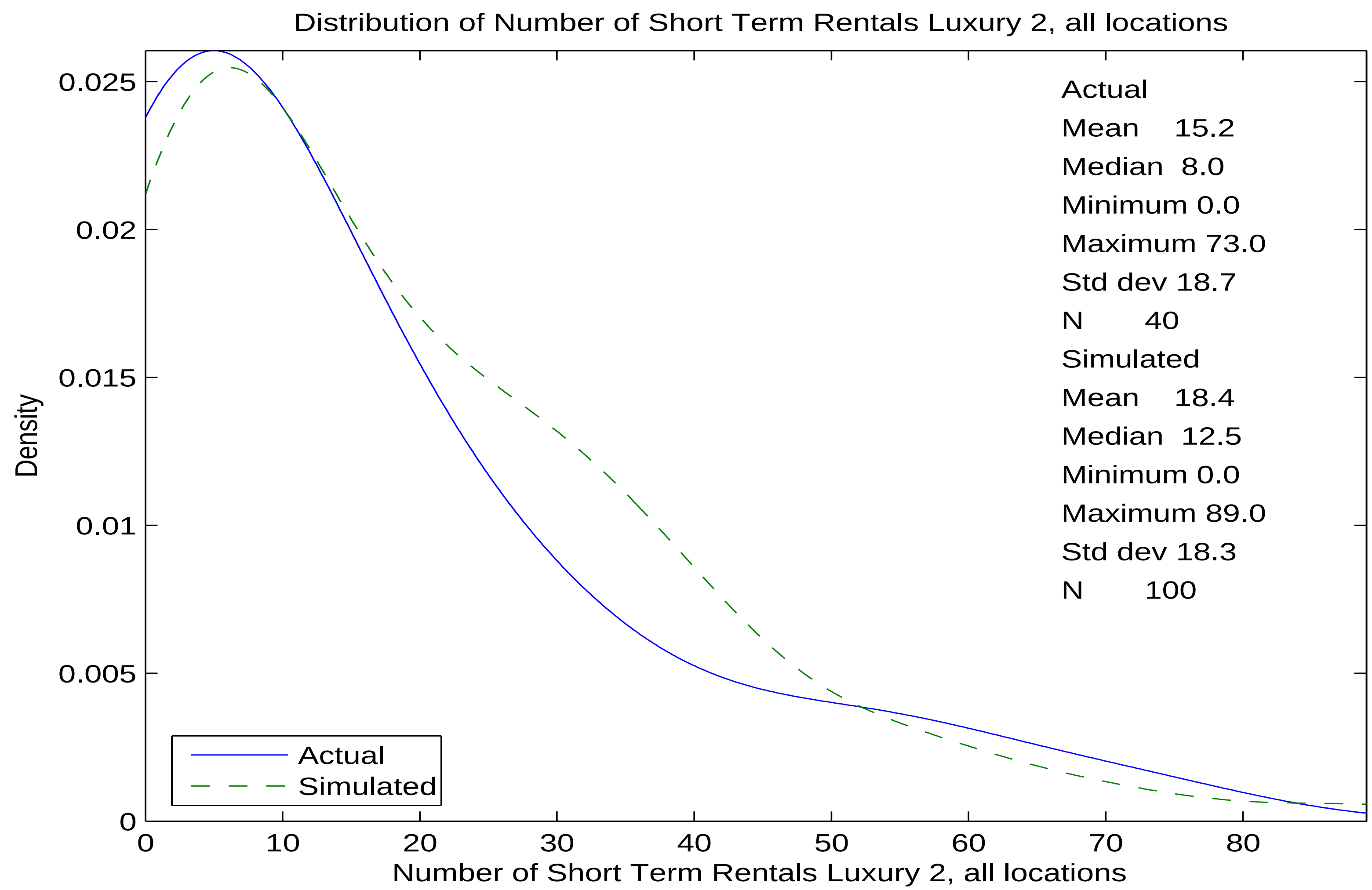


4.2 Number of Spells

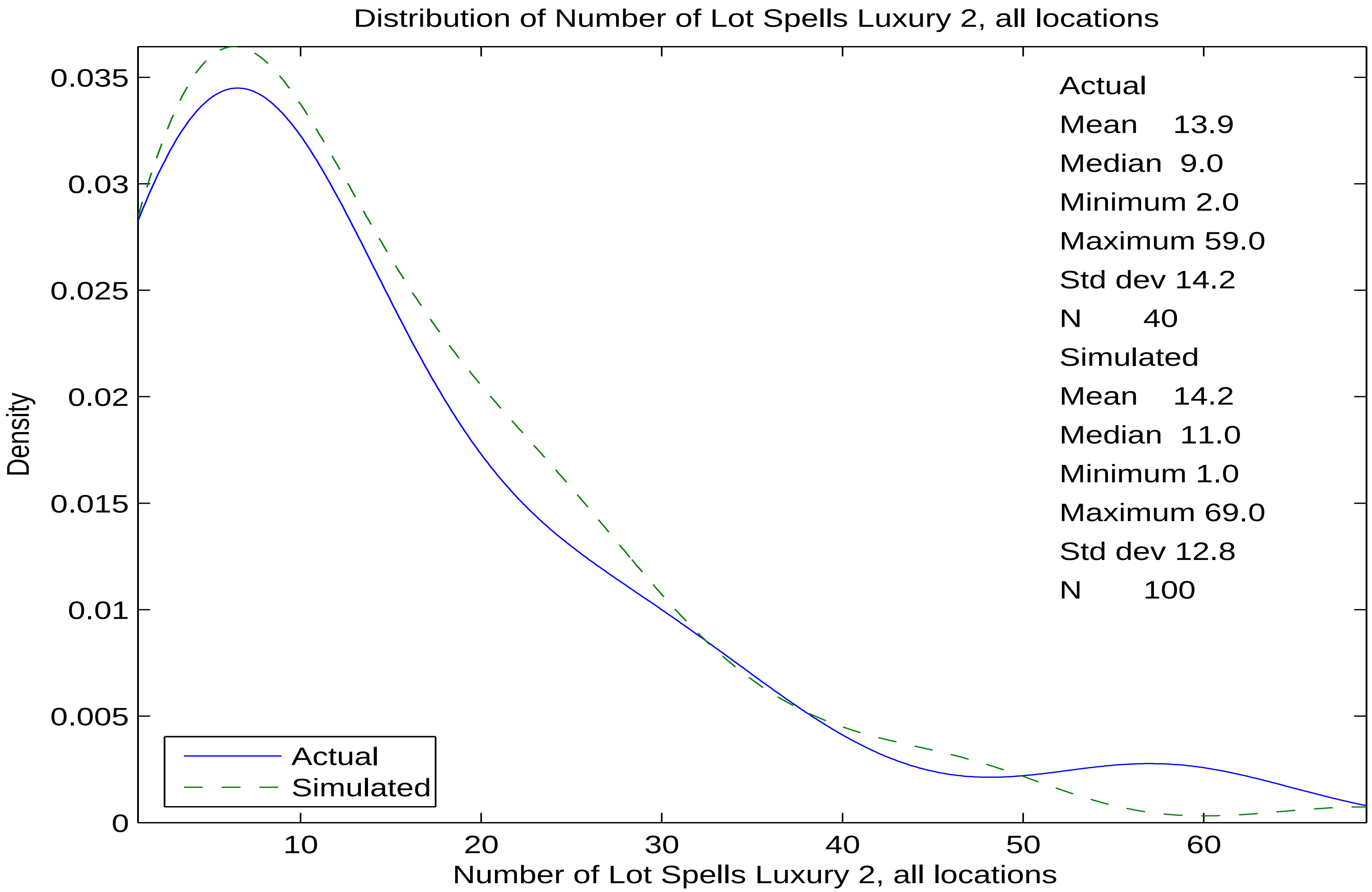
Number of Long Term Rentals



Number of Short Term Rentals

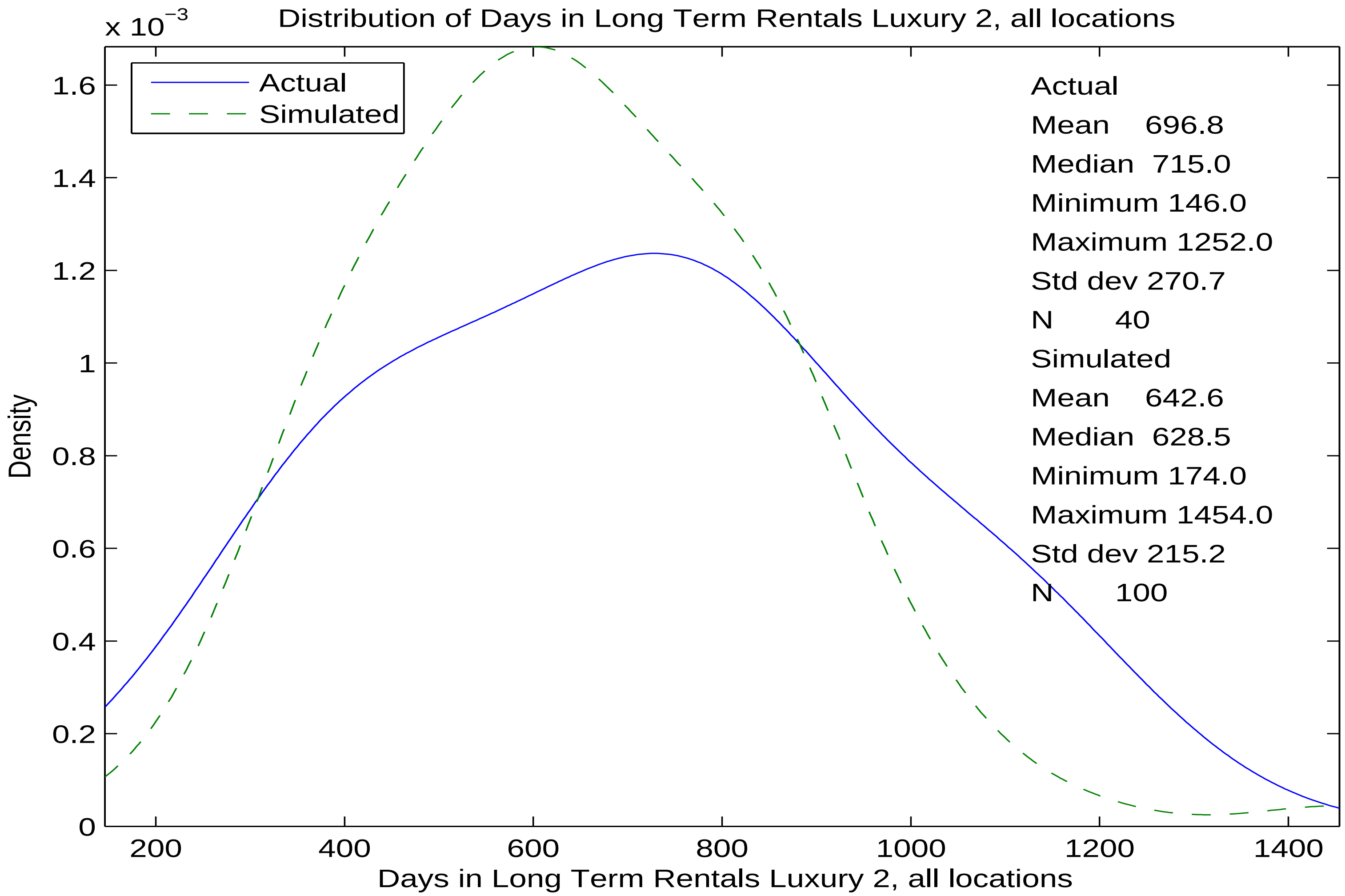


Number of Lot Spells

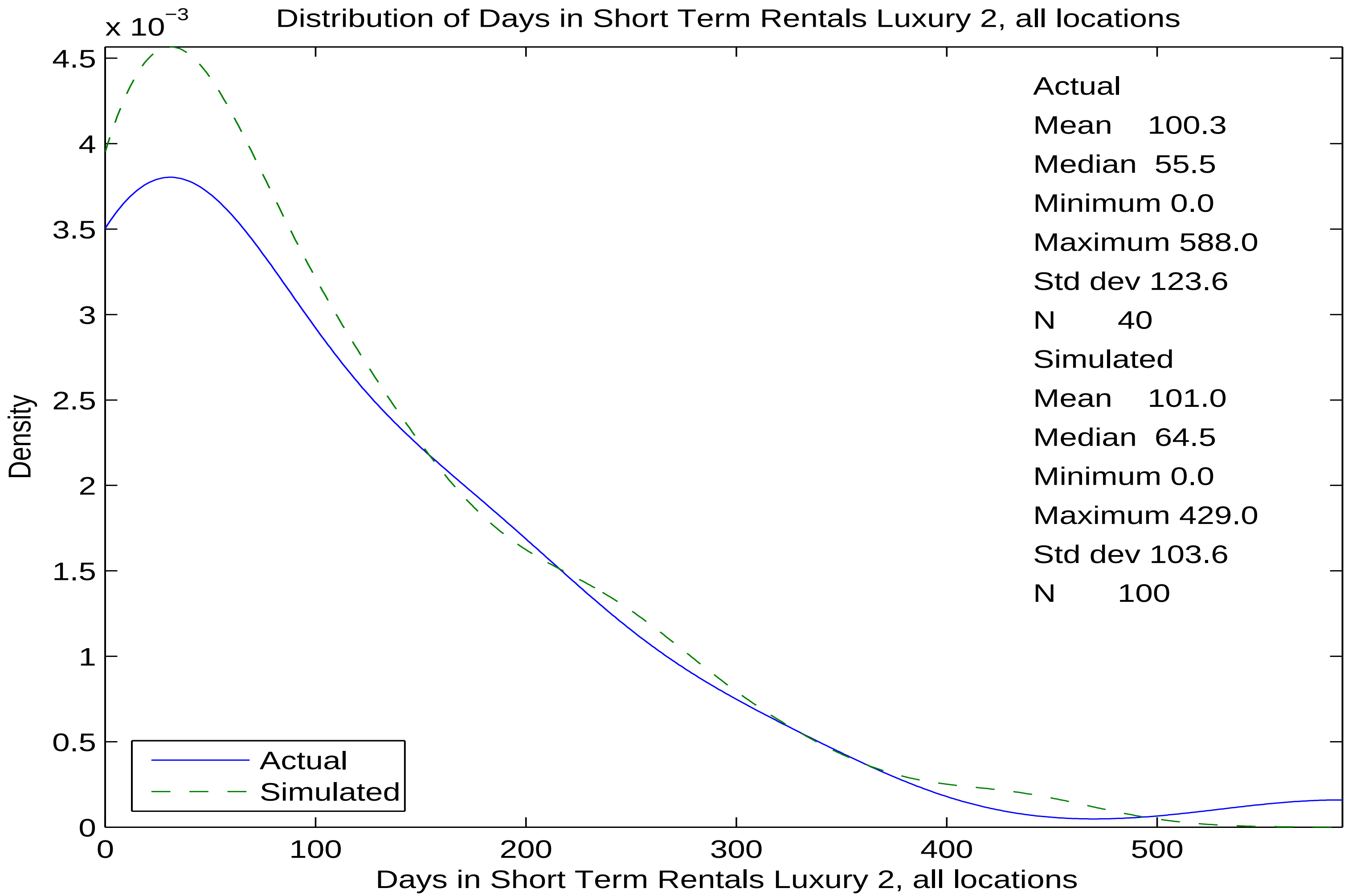


4.3 Days in Spells

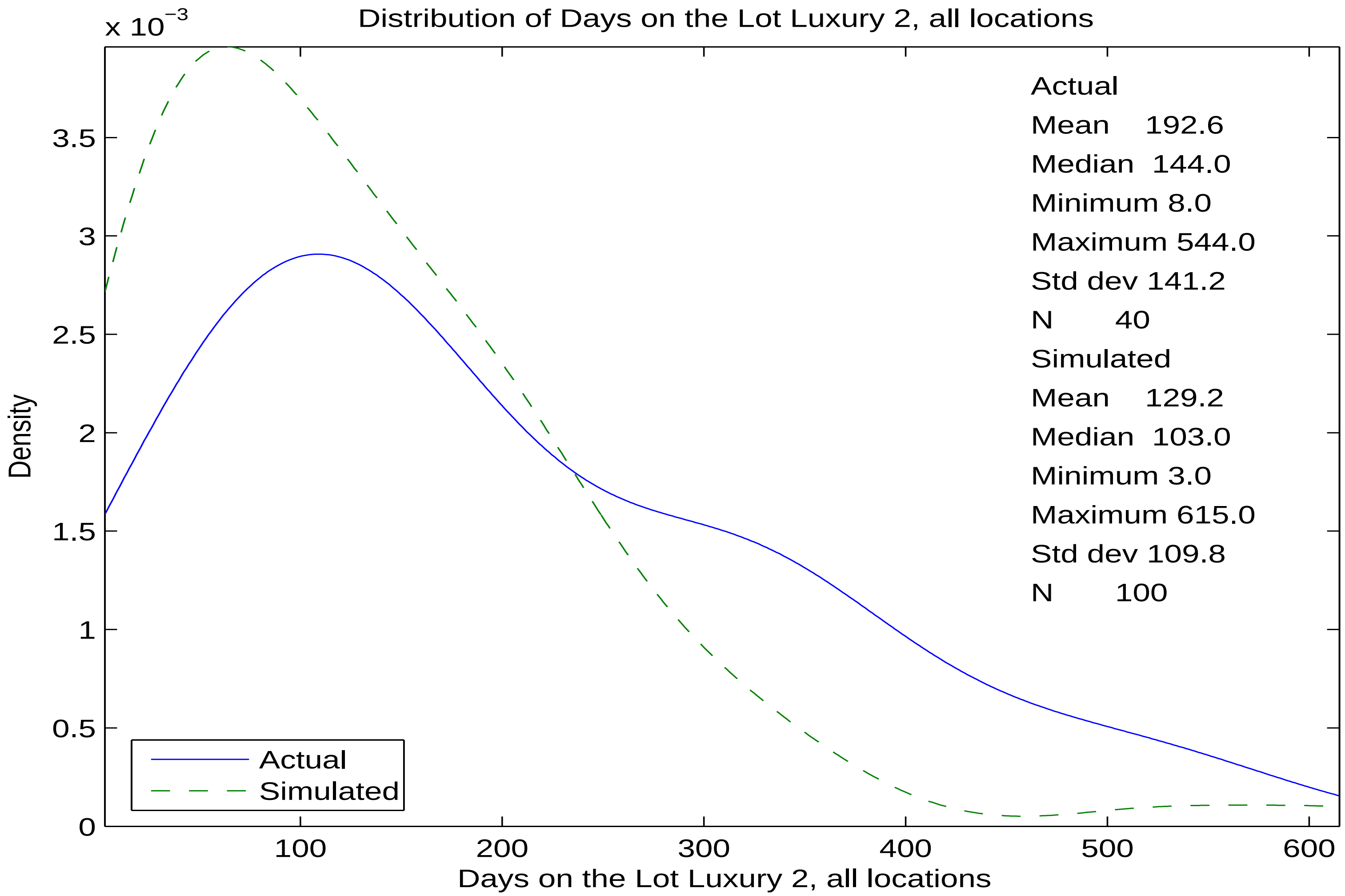
Days in Long Term Rentals



Days in Short Term Rentals

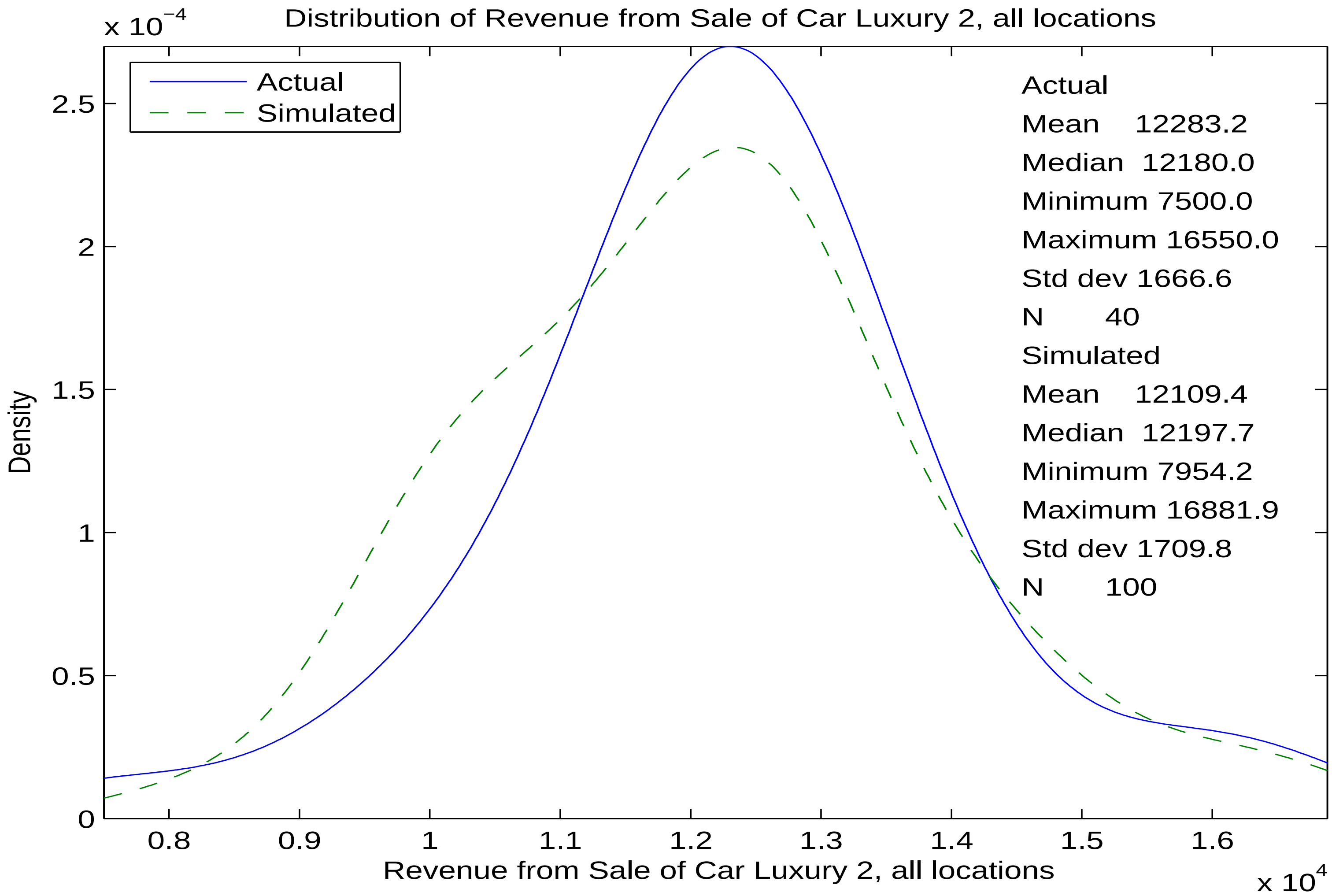


Days in Lot Spells

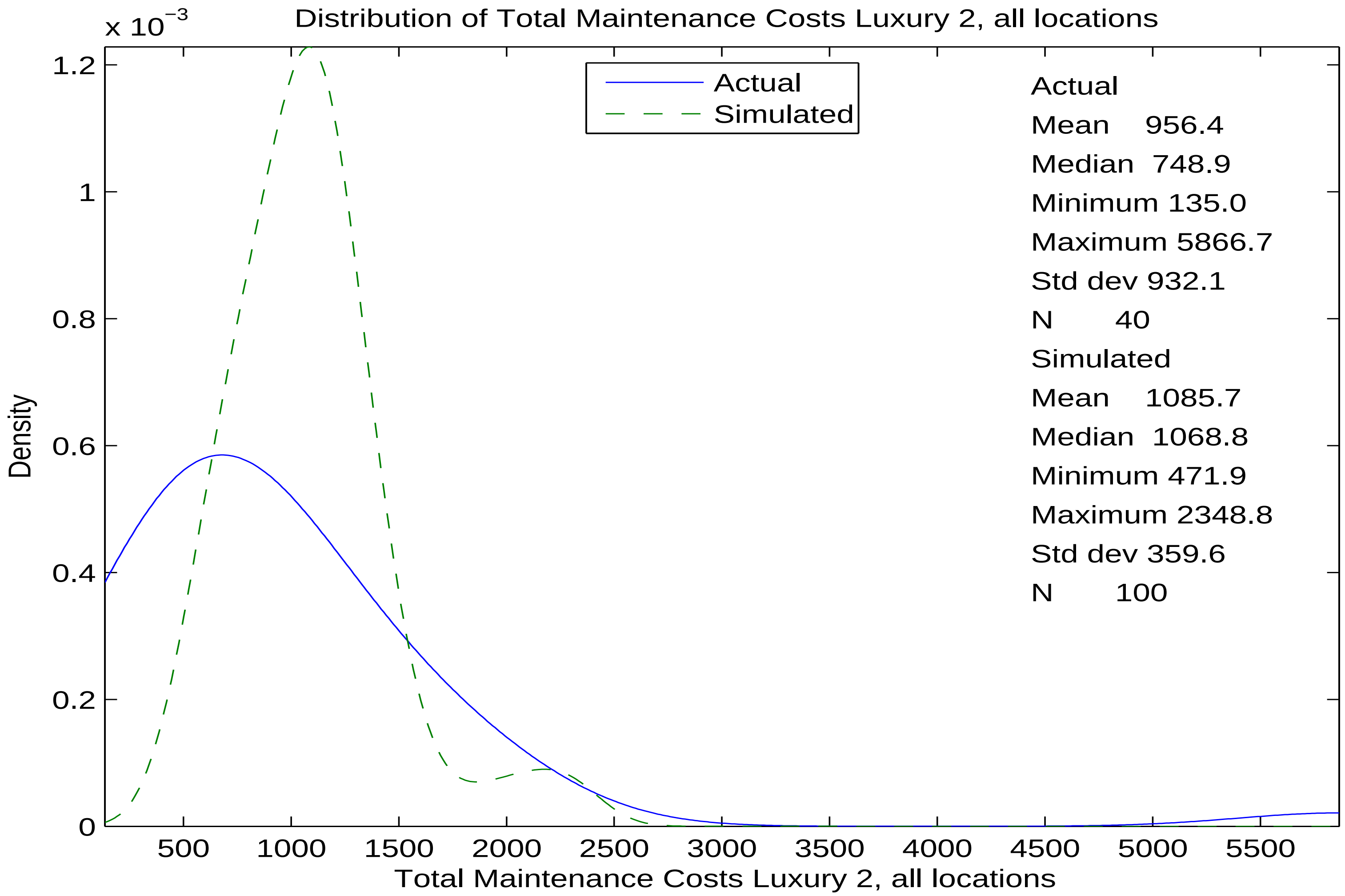


4.4 Financial Outcomes

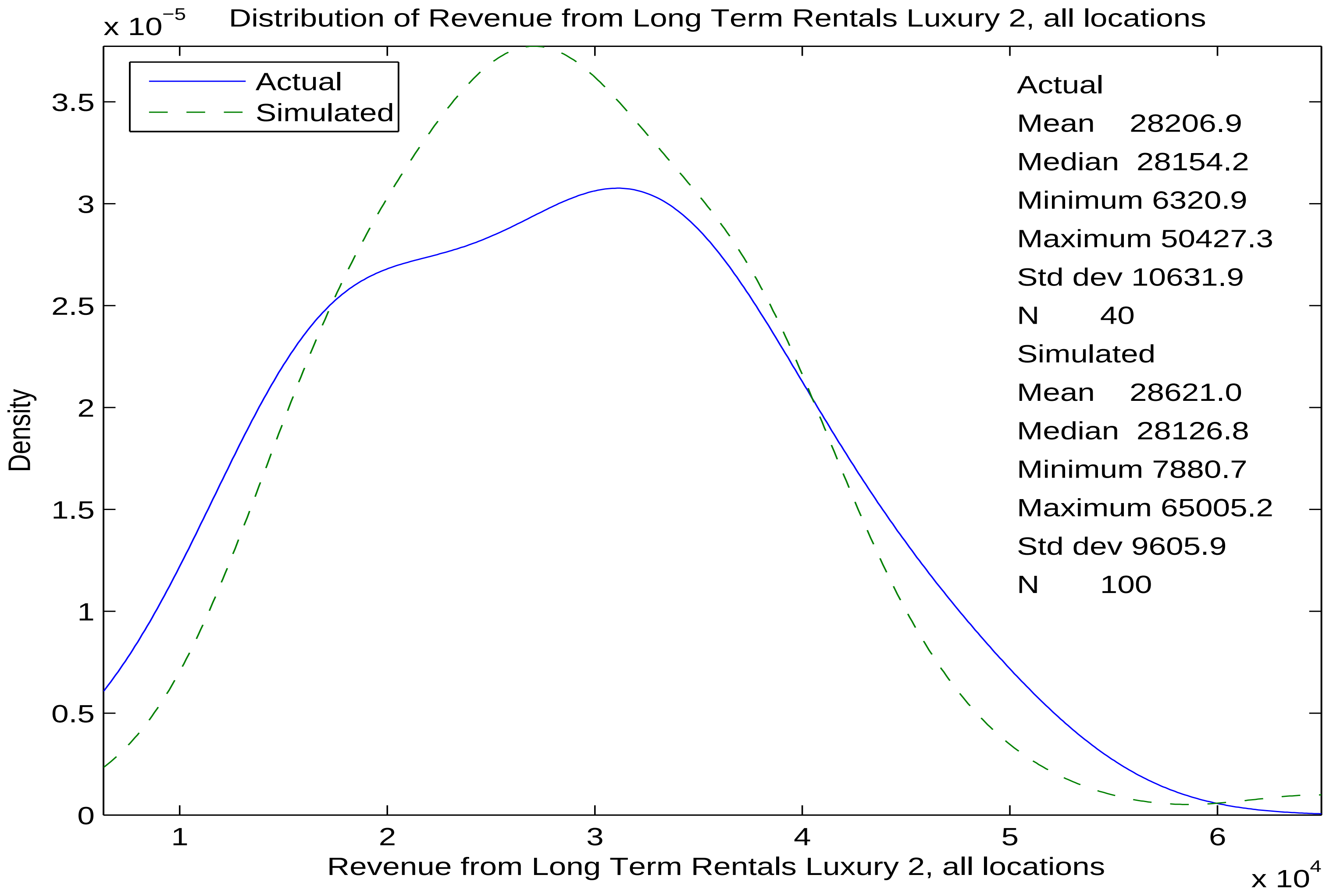
Vehicle Sales Proceeds



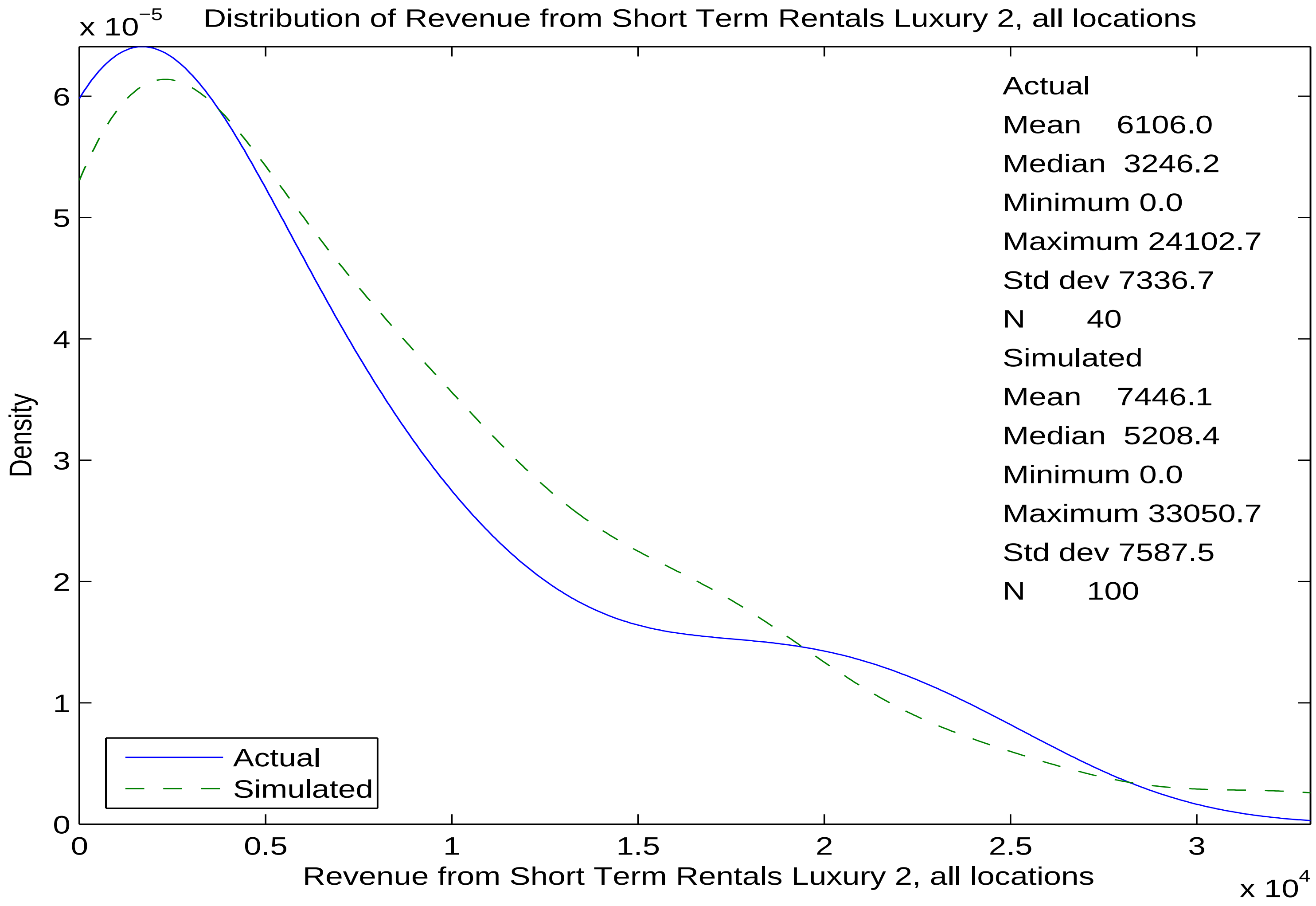
Maintenance Costs



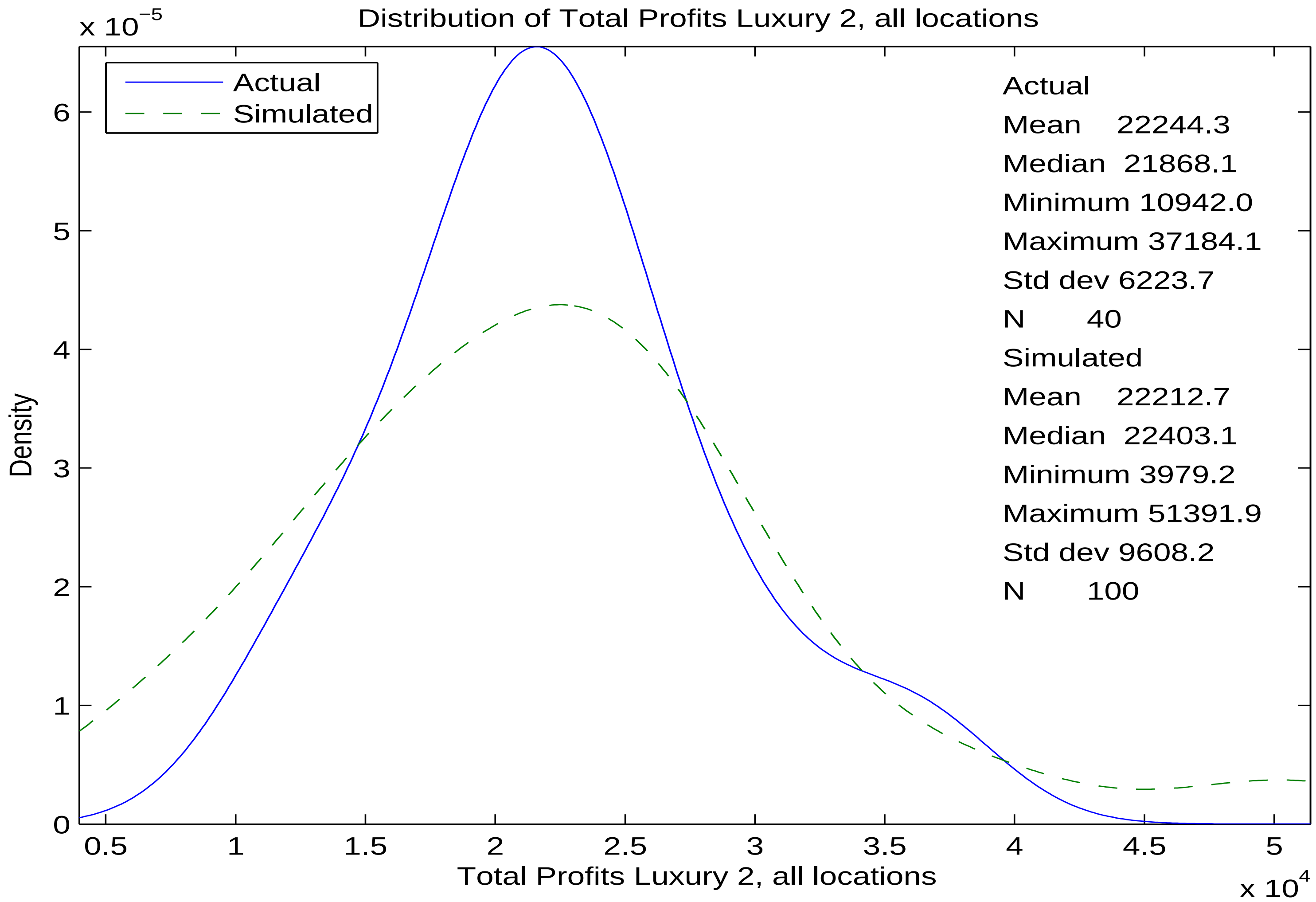
Long Term Rental Revenues



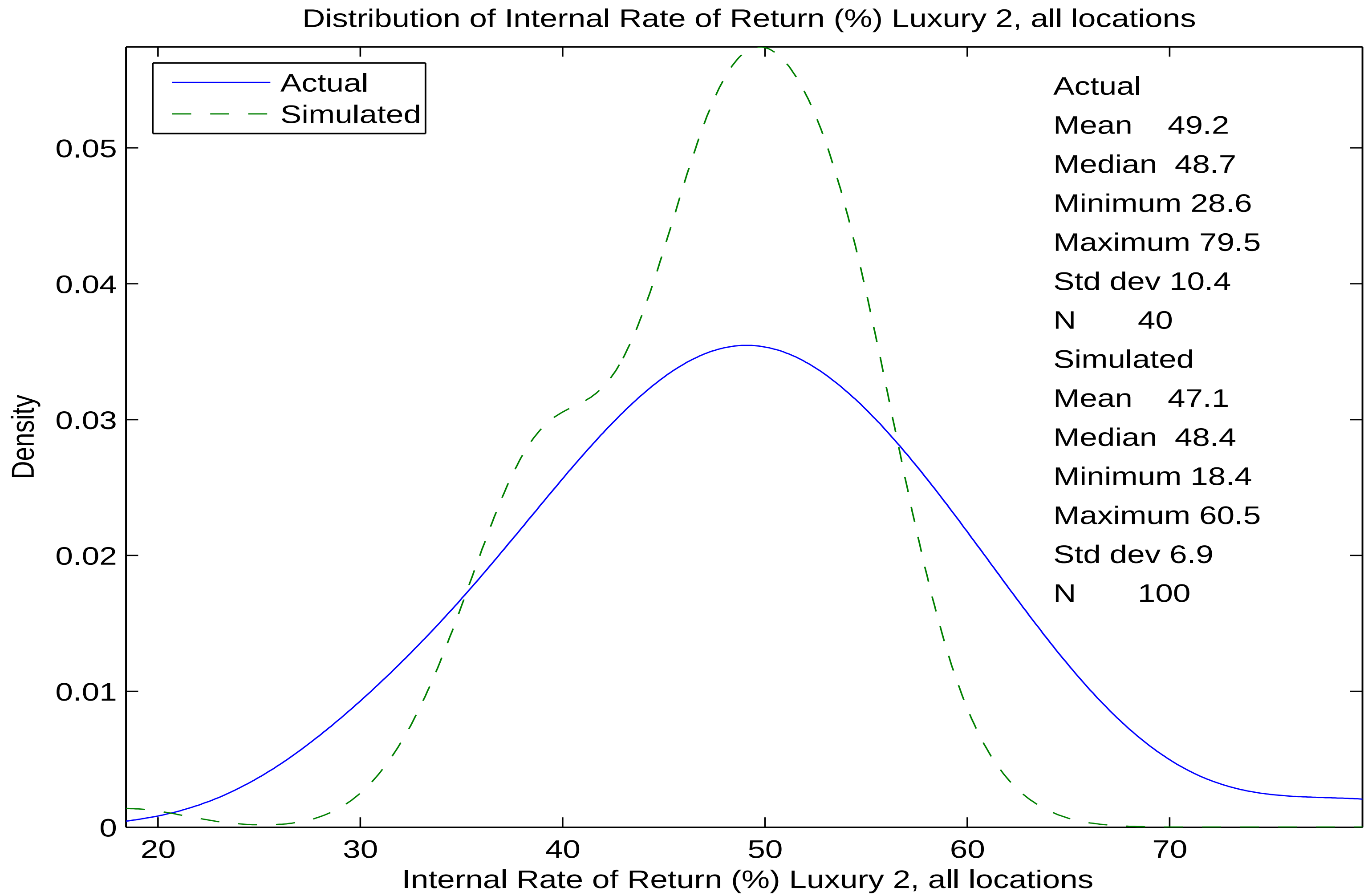
Short Term Rental Revenues



Total Profits



Internal Rates of Return



5 Optimal Replacement Theory

Optimal Stopping Theory

1. While it is possible to evaluate *specific hypothetical alternatives* to the company's *status quo* operating policy using simulation methods similar to the previous section, there are more efficient methods available for searching over *all possible policies* to find the *optimal replacement policy.*

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2. Mathematically, the optimal replacement problem is equivalent to a specific type of *optimal stopping problem* known as a *regenerative optimal stopping problem* (see Rust, 1987).
3. The term "regenerative" is used, since the decision to replace a vehicle does not stop or end the decision process, but rather results in a "regeneration" or "rebirth", i.e. a replacement of an old vehicle by a brand new one.

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Dynamic Programming

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2. We show that the optimal strategy takes the form of a *threshold rule,* i.e. the optimal time to replace a car occurs when its odometer value o exceeds a threshold value $\overline{o}(d, r, \tau)$ that depends on the current rental state r, the duration in that state d, and the car type τ.

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3. Using numerical methods, we solve the dynamic programming problem and calculate the optimal stopping thresholds $\overline{o}(d, r, \tau)$.
4. We also compute the optimal *value functions* $V(r, d, o, \tau)$. This function provides the expected discounted profits (over an infinite horizon) under the optimal replacement policy for a vehicle that is in state (r, d, o).

Valuing Alternative Policies

1. It is also possible to compute the value of any alternative operating strategy μ, which can include *mixed* or probabilistic operating strategies where the decision to replace a car is given by a conditional probability distribution $\mu(r, d, o, \tau)$.

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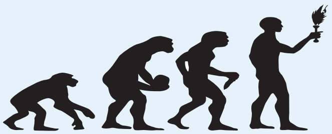

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5. *We will show that the optimal policy entails keeping cars significantly longer than the company currently keeps them.*

6 Numerical Results

No Extrapolation Case

1. As we noted above, if we solve the regenerative optimal stopping problem under the assumption that the only aging effects are 1) the depreciation in vehicle resale values, and 2) the "rental contract composition effect", then the optimal stopping thresholds is $\overline{o}(r,d) = \infty$, i.e. it is *never optimal to sell an existing vehicle.*

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2. This follows from the assumption that average daily maintenance costs EM do not increase as a function of odometer value, and that rental rates do not decrease as a function of odometer values.
3. While there is substantial empirical justification for these assumptions *over the range of our observations,* it is questionable that these assumptions will continue to be valid as a vehicle's odometer and age increases indefinitely, far beyond the range for which we have any observations.

The Pessimistic Case

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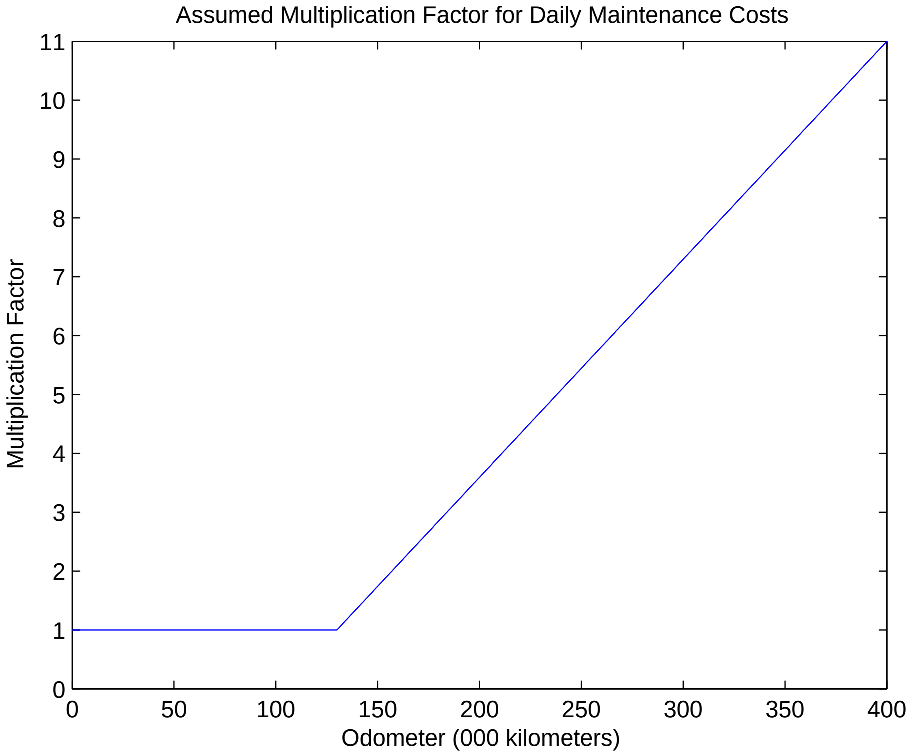
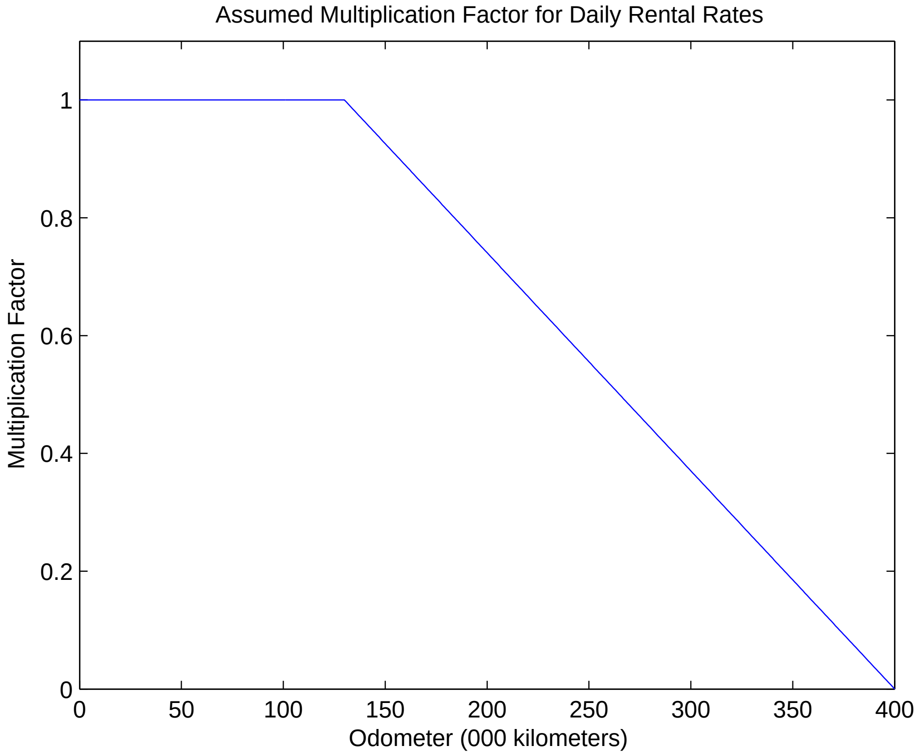
2. That is, we will assume that beyond the range of our observations, maintenance costs increase at a very rapid rate as odometer increases, and that to induce customers to rent older vehicles, daily rental rates must be steeply discounted.
3. Specifically, after a vehicle hits 130,000 kilometers, we assume that maintainence costs increase rapidly and rental rates must be decreased rapidly to induce customers to rent older cars.

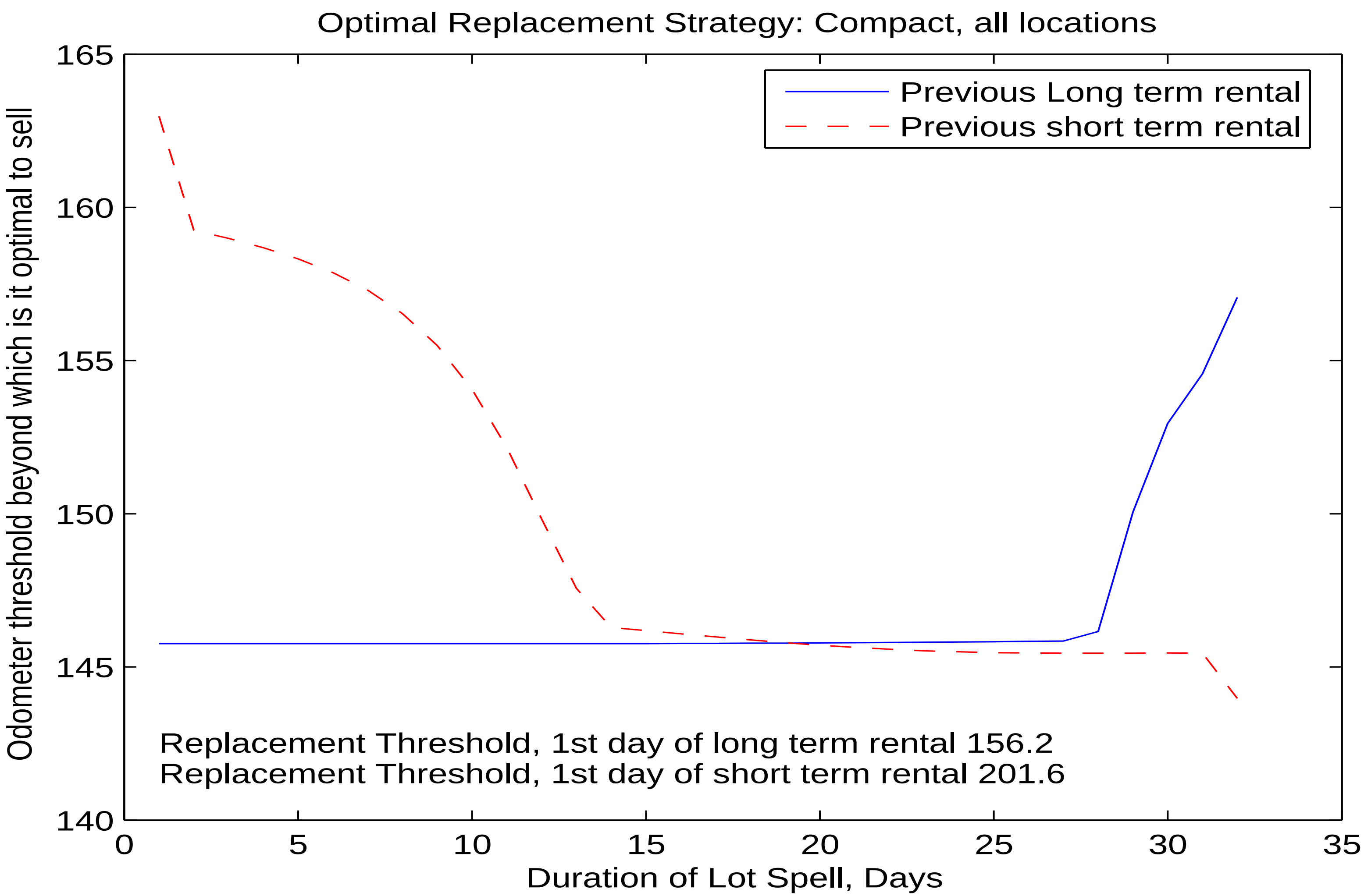
Multiplication Factors



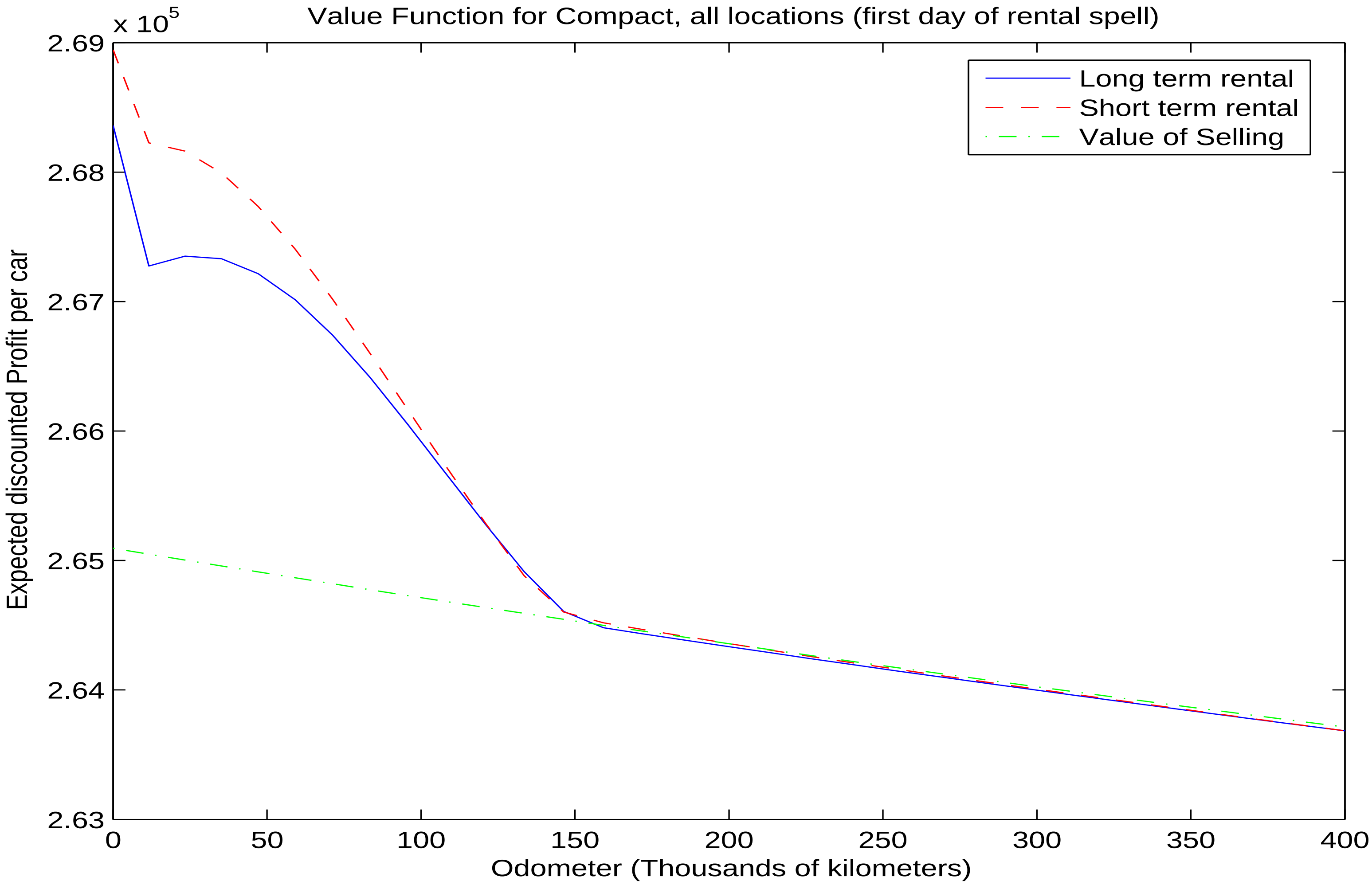


6.1 Results for Compact

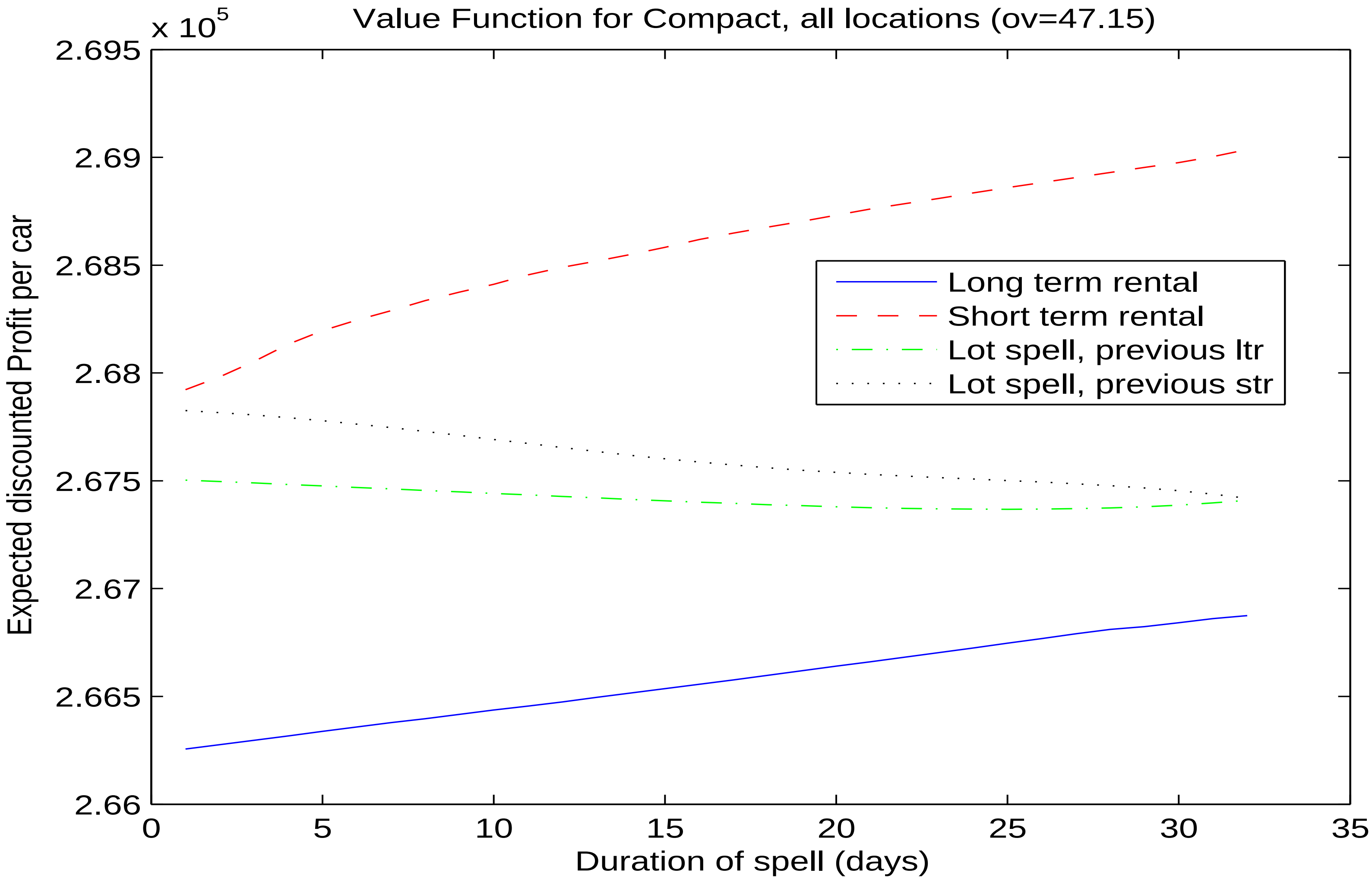
Optimal Thresholds: Compact



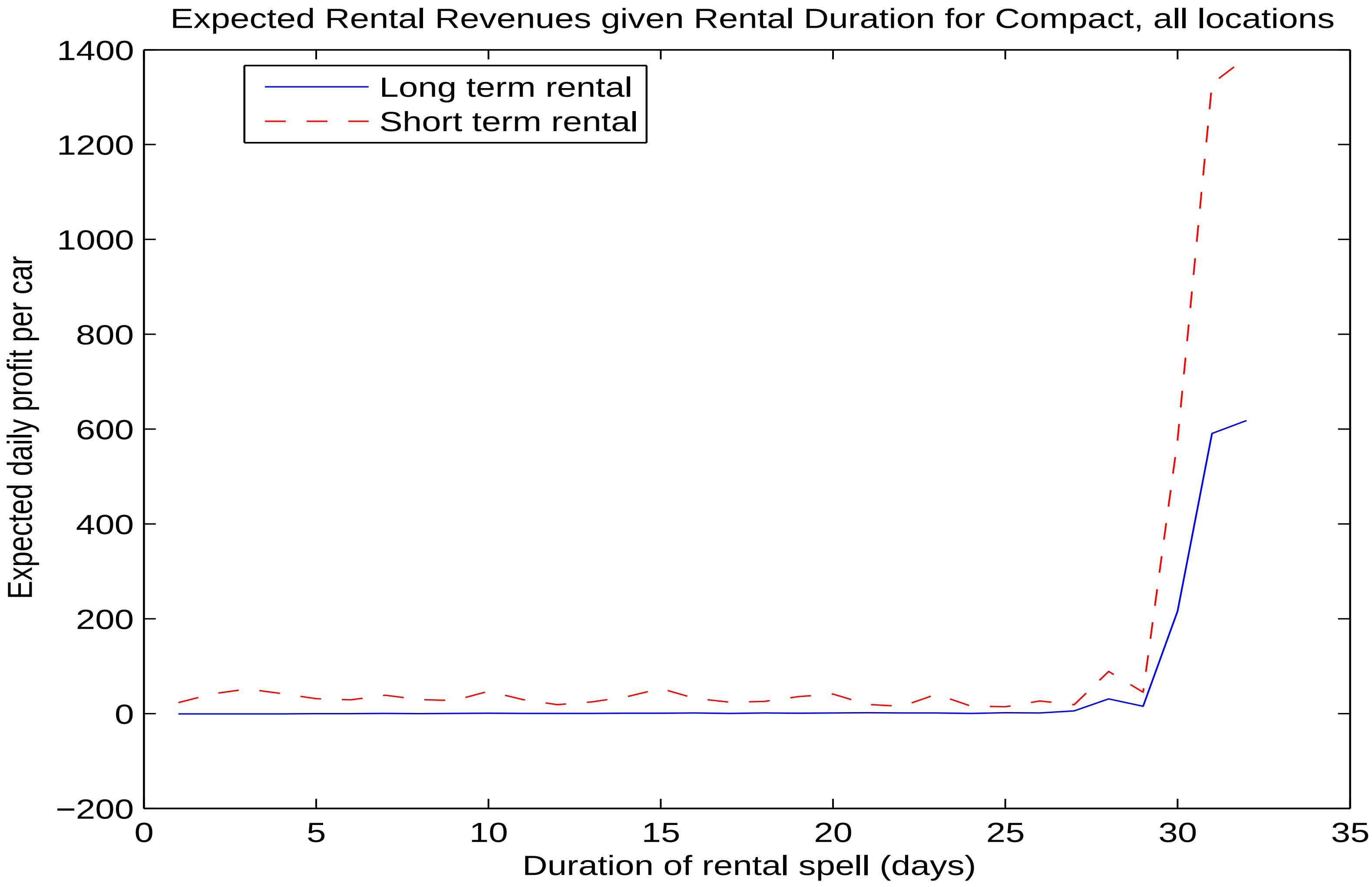
Optimal Values: Compact



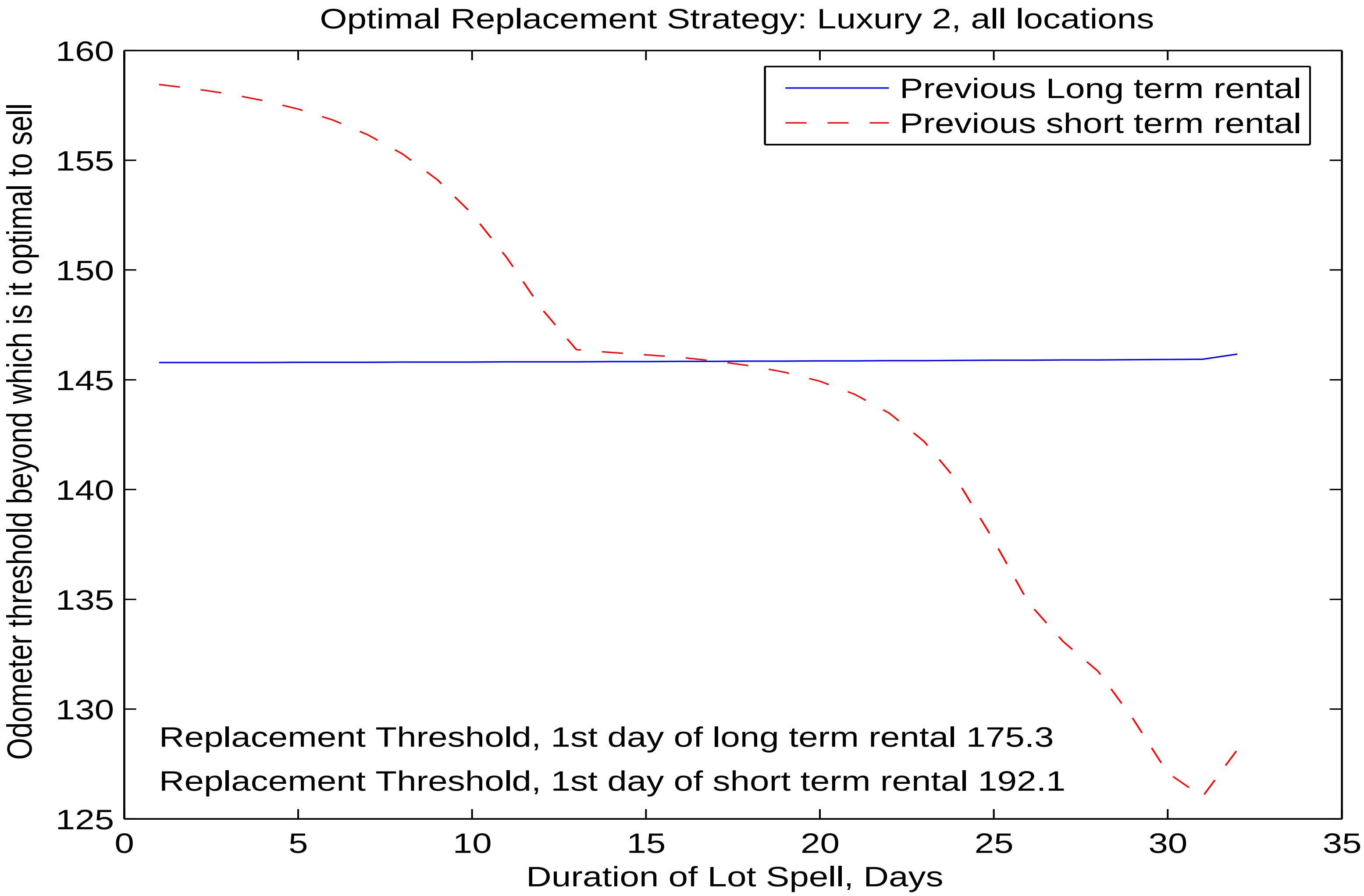
Optimal Values: Compact



Expected Revenue: Compact



Optimal Thresholds: Luxury



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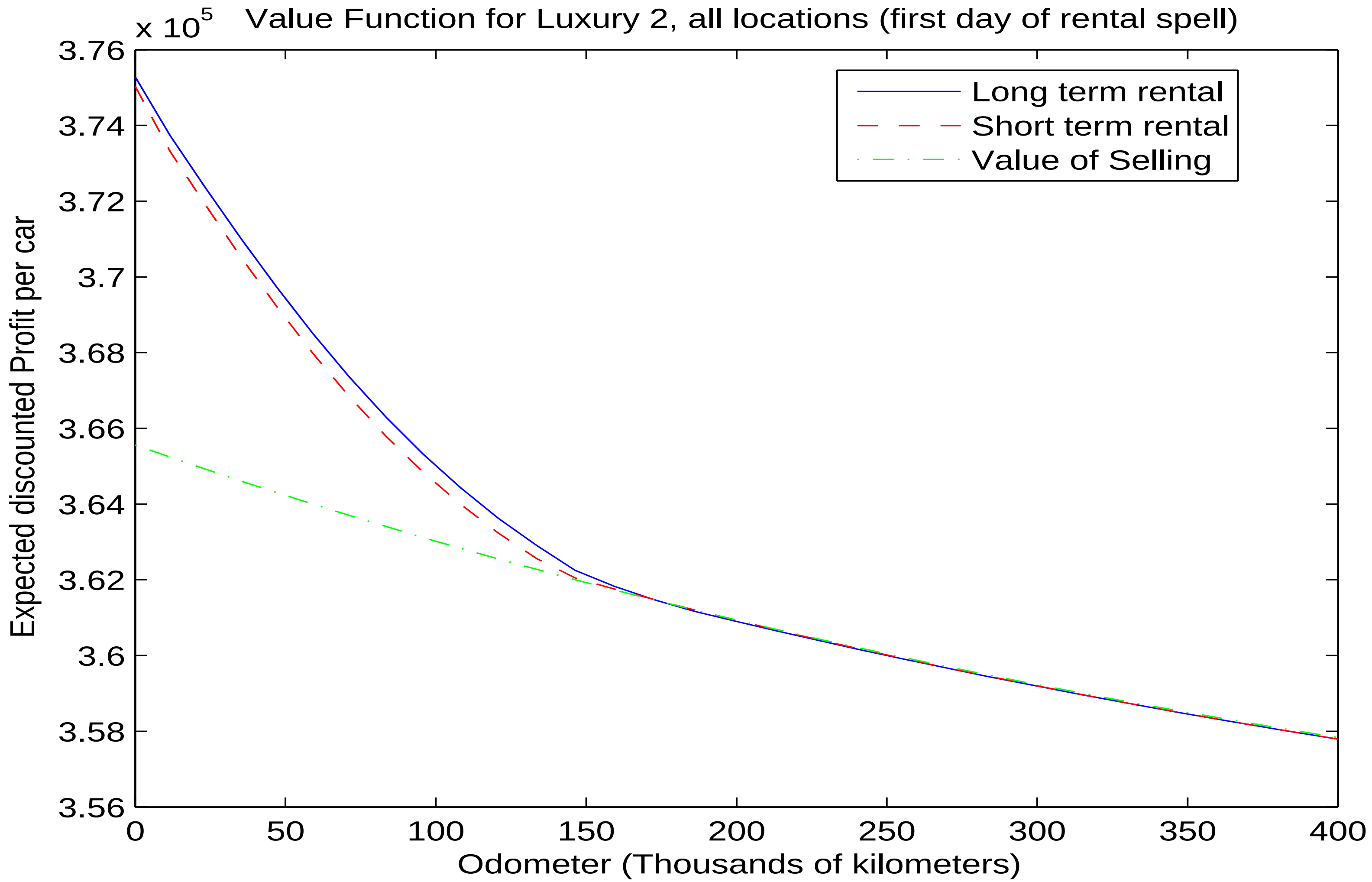
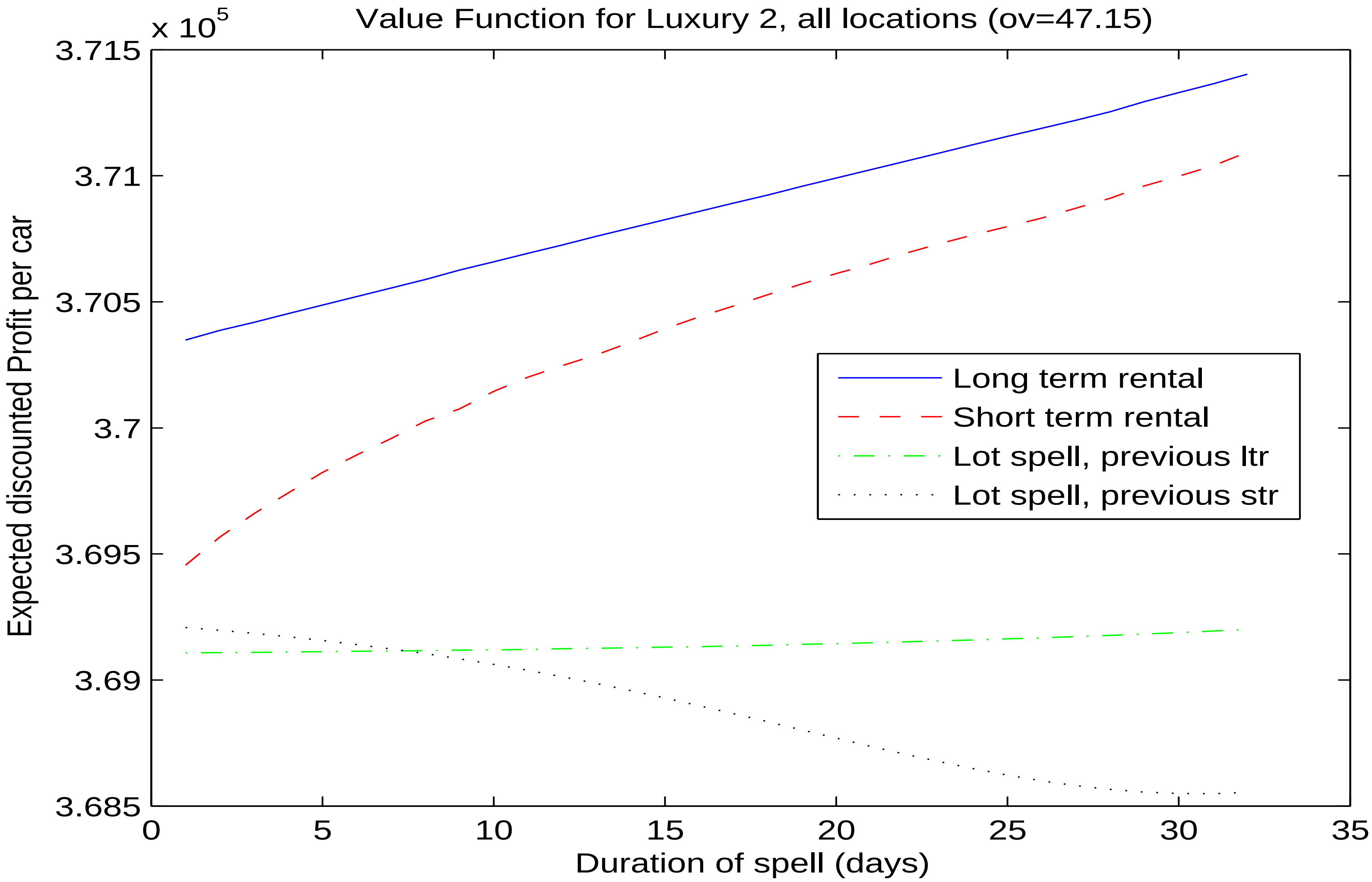
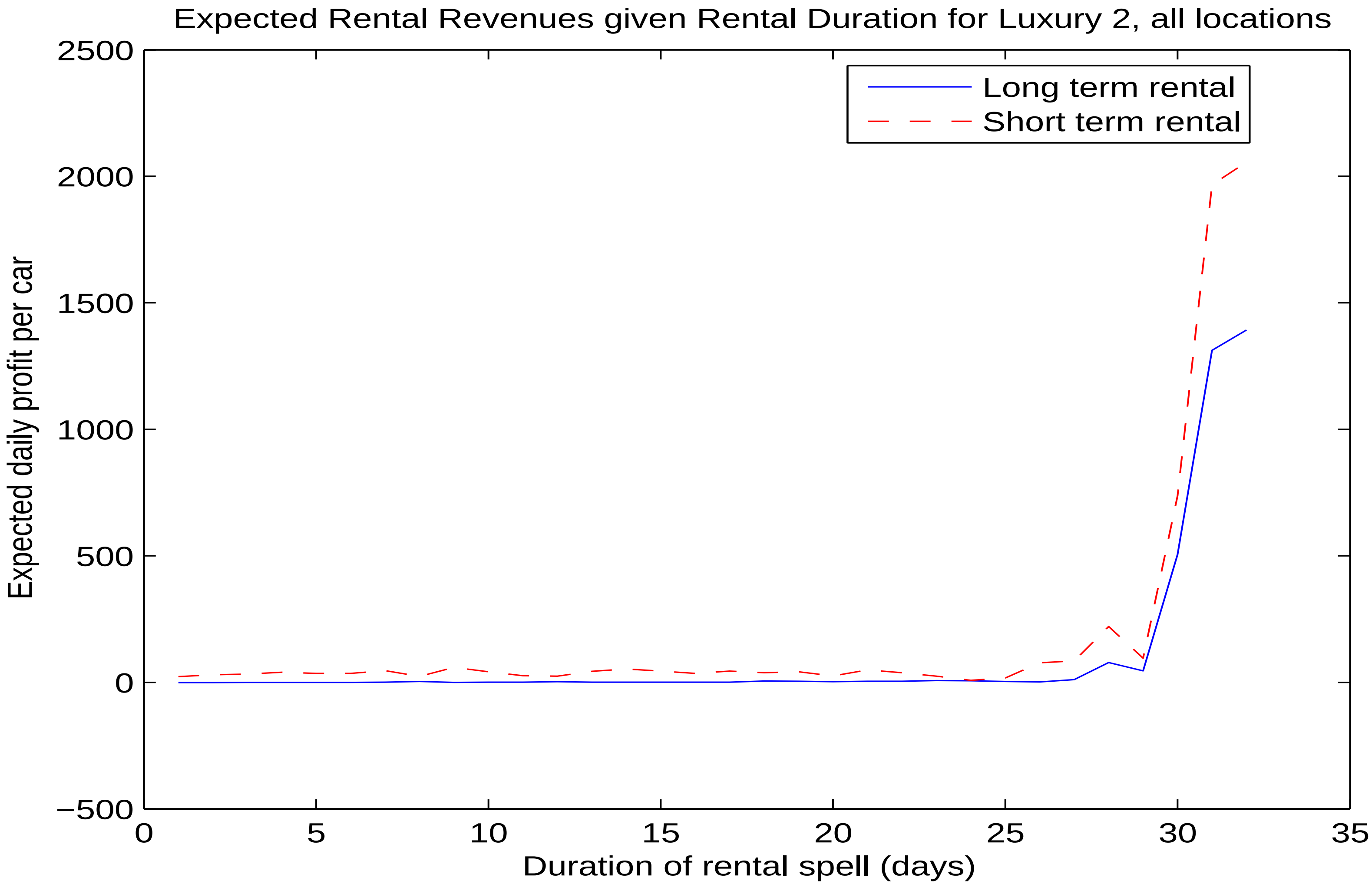


Optimal Values: Luxury

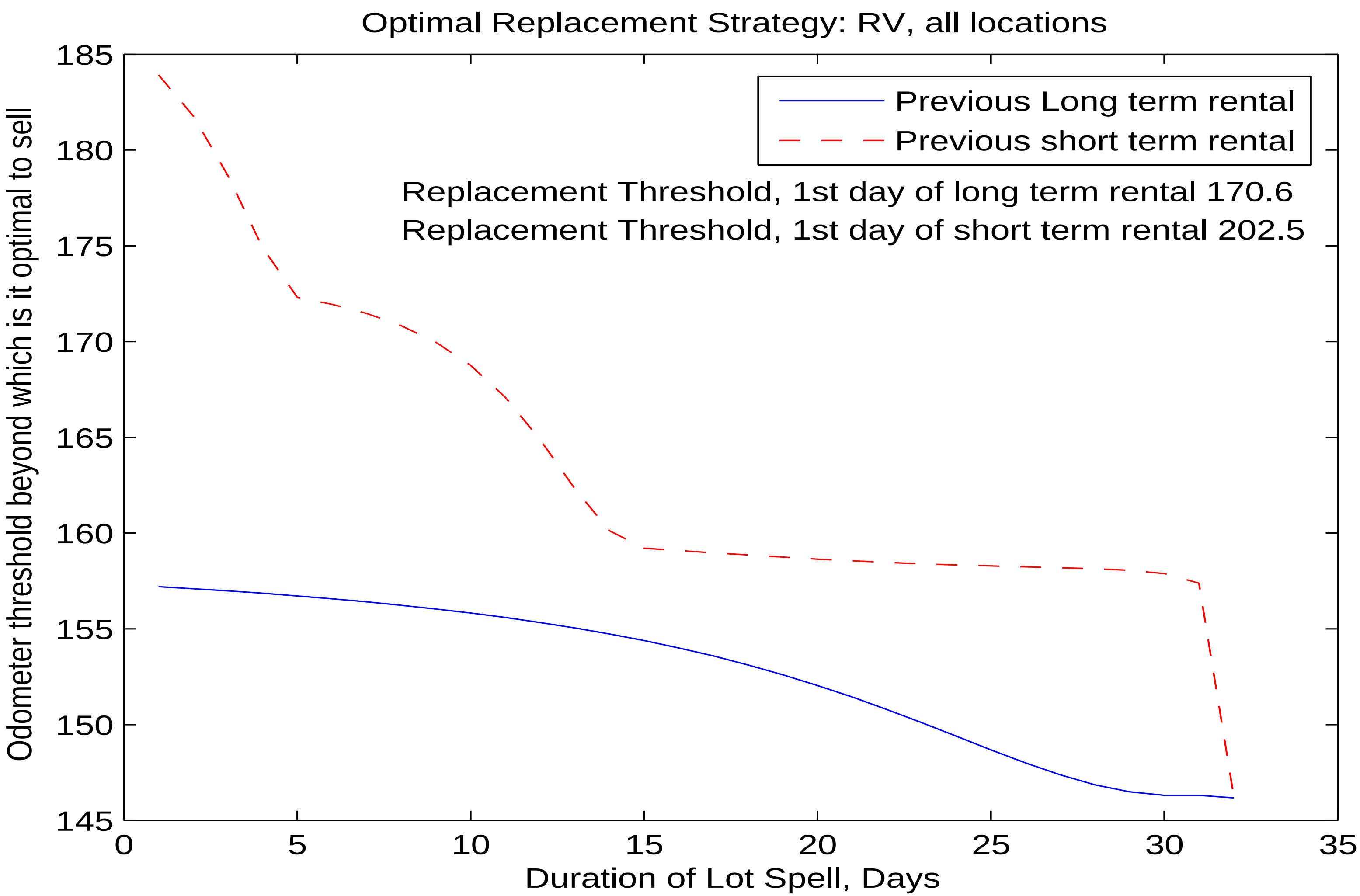


Expected Revenue: Luxury

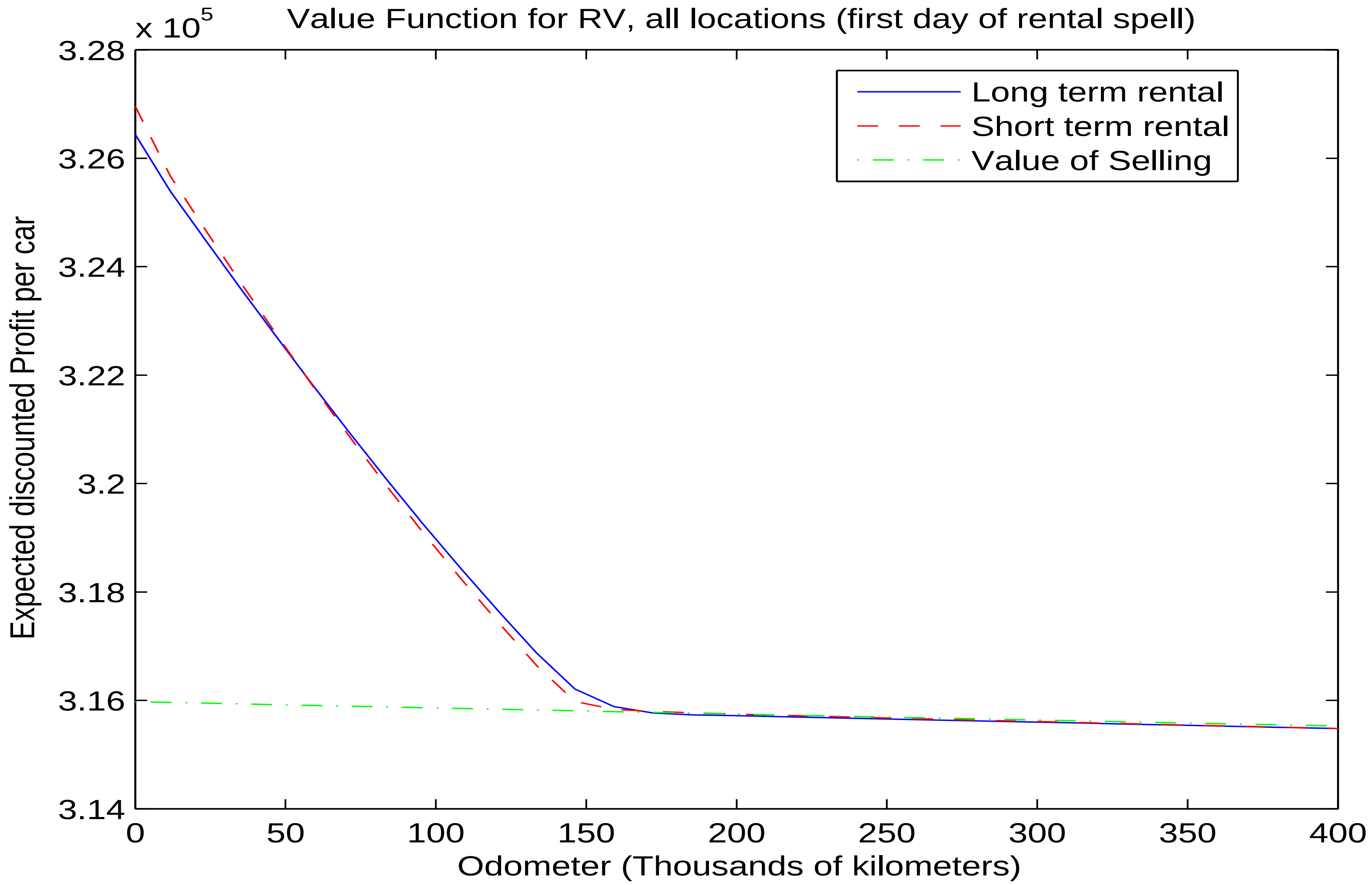


6.3 Results for RV

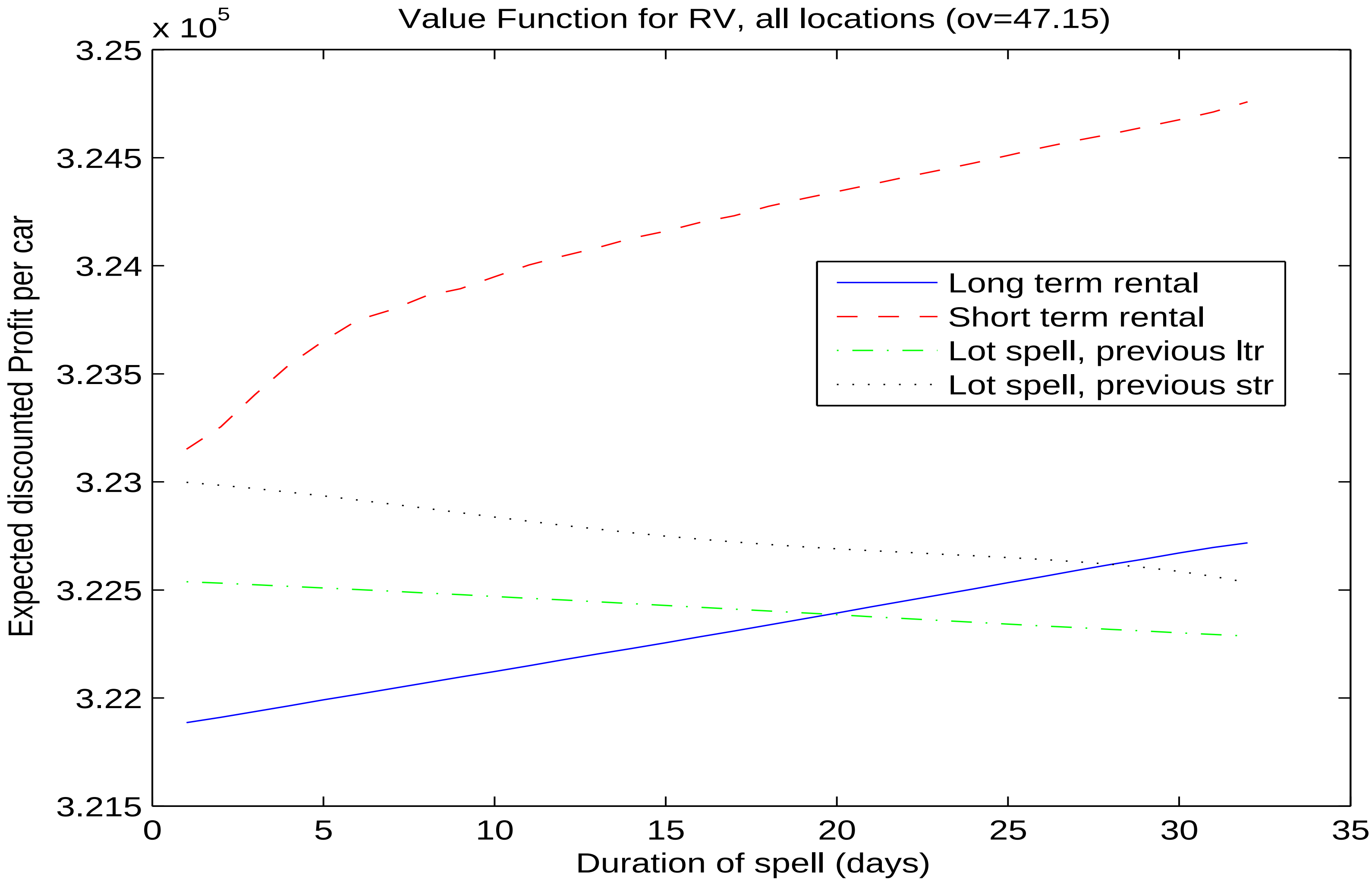
Optimal Thresholds: RV



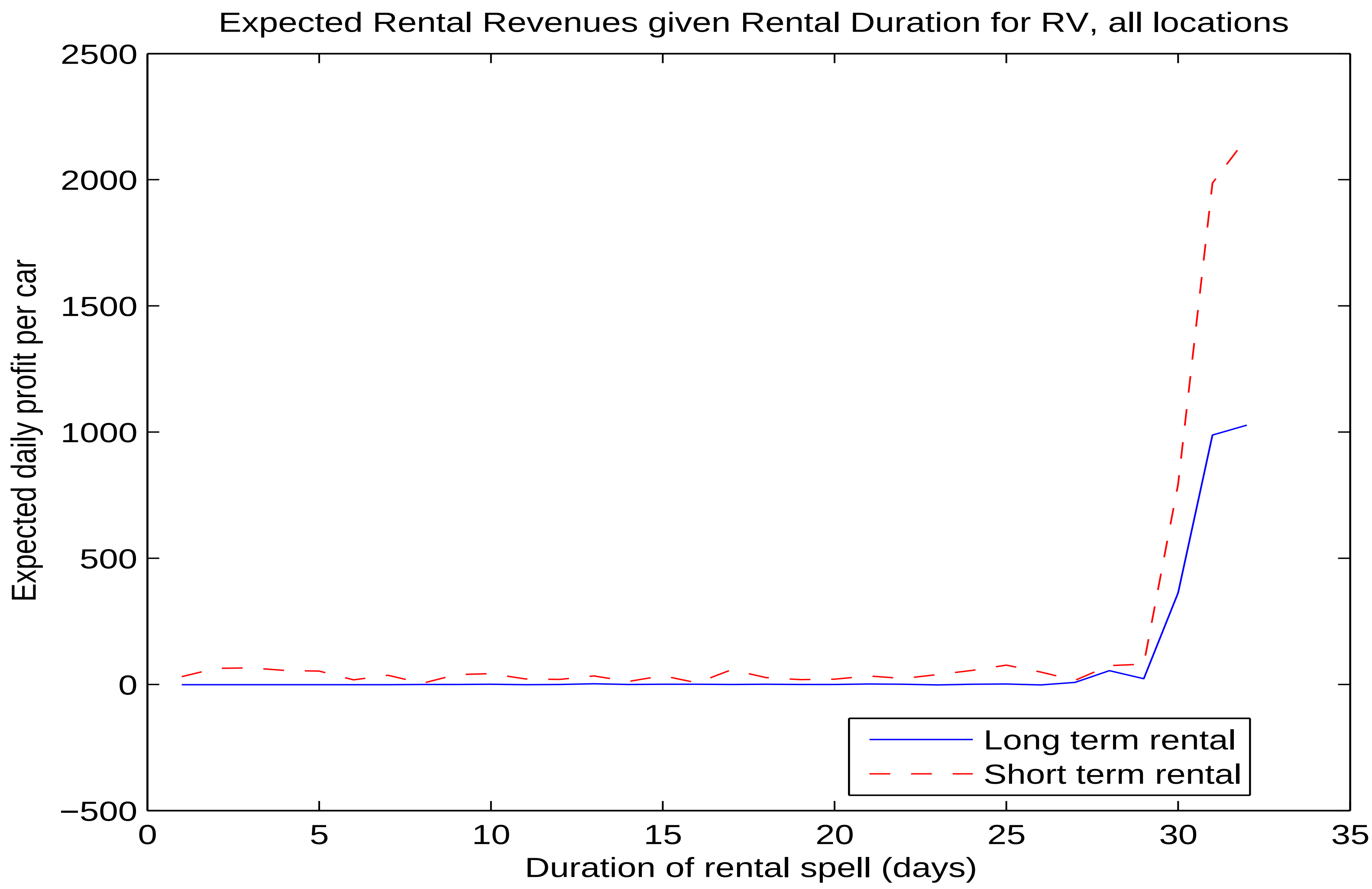
Optimal Values: RV



Optimal Values: RV



Expected Revenue: RV



6.4 Profit Comparisons

Profit Comparisons

Quantity	Compact	Luxury	RV
$\overline{P}$	9668	23389	18774
Expected Discounted Values Under Optimal Replacement Policy			
$V(0,0,r_0)$	268963	374913	327057
$(1-\beta)V(0,0,r_0)$	22.11	30.81	26.88
$V(0,0,r_0)/\overline{P}$	27.8	16.0	17.4
Expected Discounted Values Under *Status Quo* Replacement Policy			
$V_\mu(0,0,r_0)$	196589	318247	136792
$(1-\beta)V_\mu(0,0,r_0)$	16.16	26.16	11.24
$V(0,0,r_0)_\mu/\overline{P}$	20.3	13.6	7.3
Ratio of Expected Values: Optimal Policy versus *Status Quo*			
$V(0,0,r_0)/V_\mu(0,0,r_0)$	1.37	1.18	2.39

6.5 Assessing Robustness

Even More Pessimistic Case

1. To assess the robustness of our conclusions, we solved for the optimal replacement policy under even more pessimistic assumptions about maintenance costs and the rental discounts that would be required to induce customers to rent older cars.

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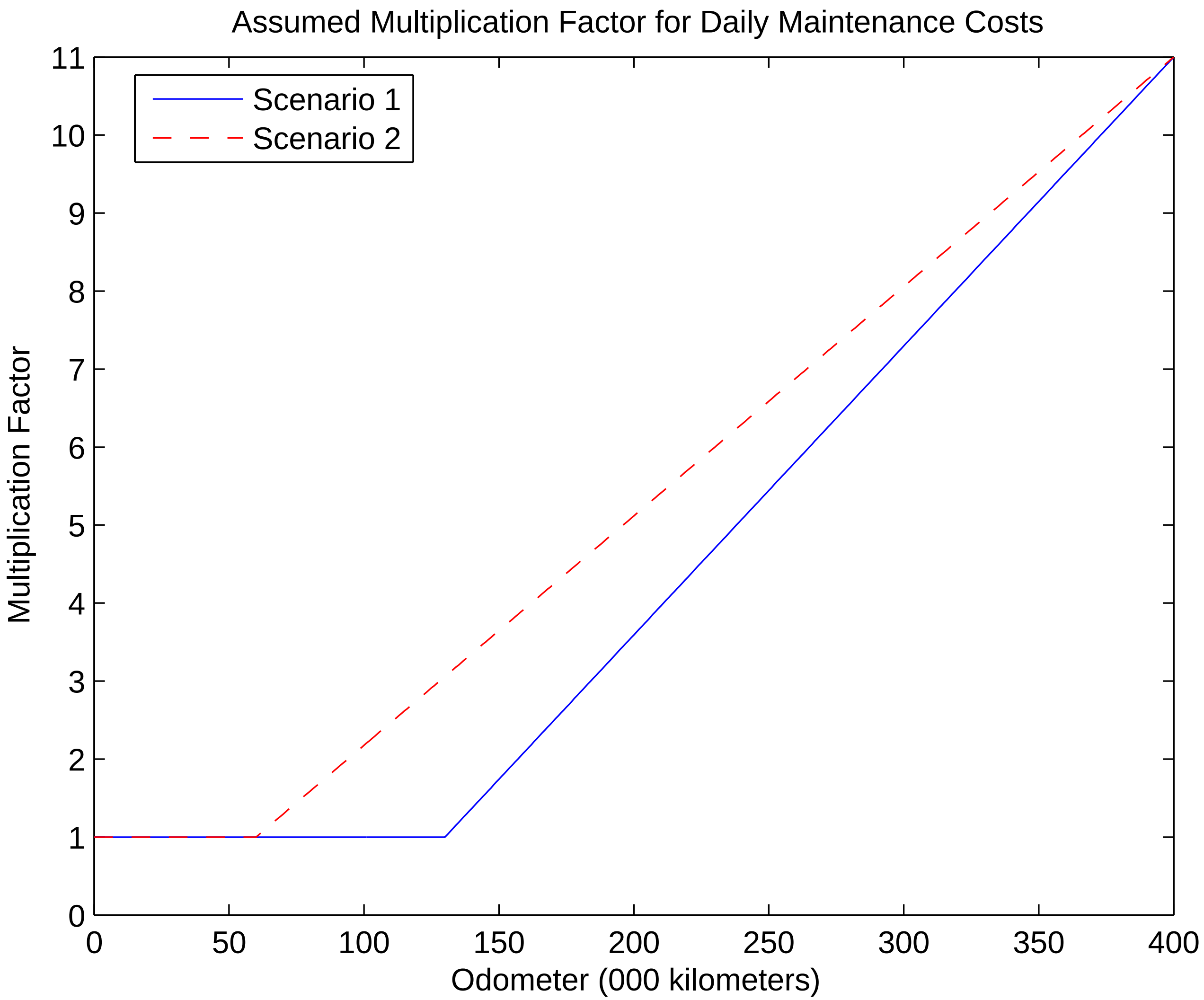
2. Under this more pessimistic scenario, maintenance costs start to accelerate far earlier, at *60,000 kilometers.*
3. We assume that rental rates start decreasing at a linear after 60,000 kilometers until they hit zero at 210,000 kilometers.

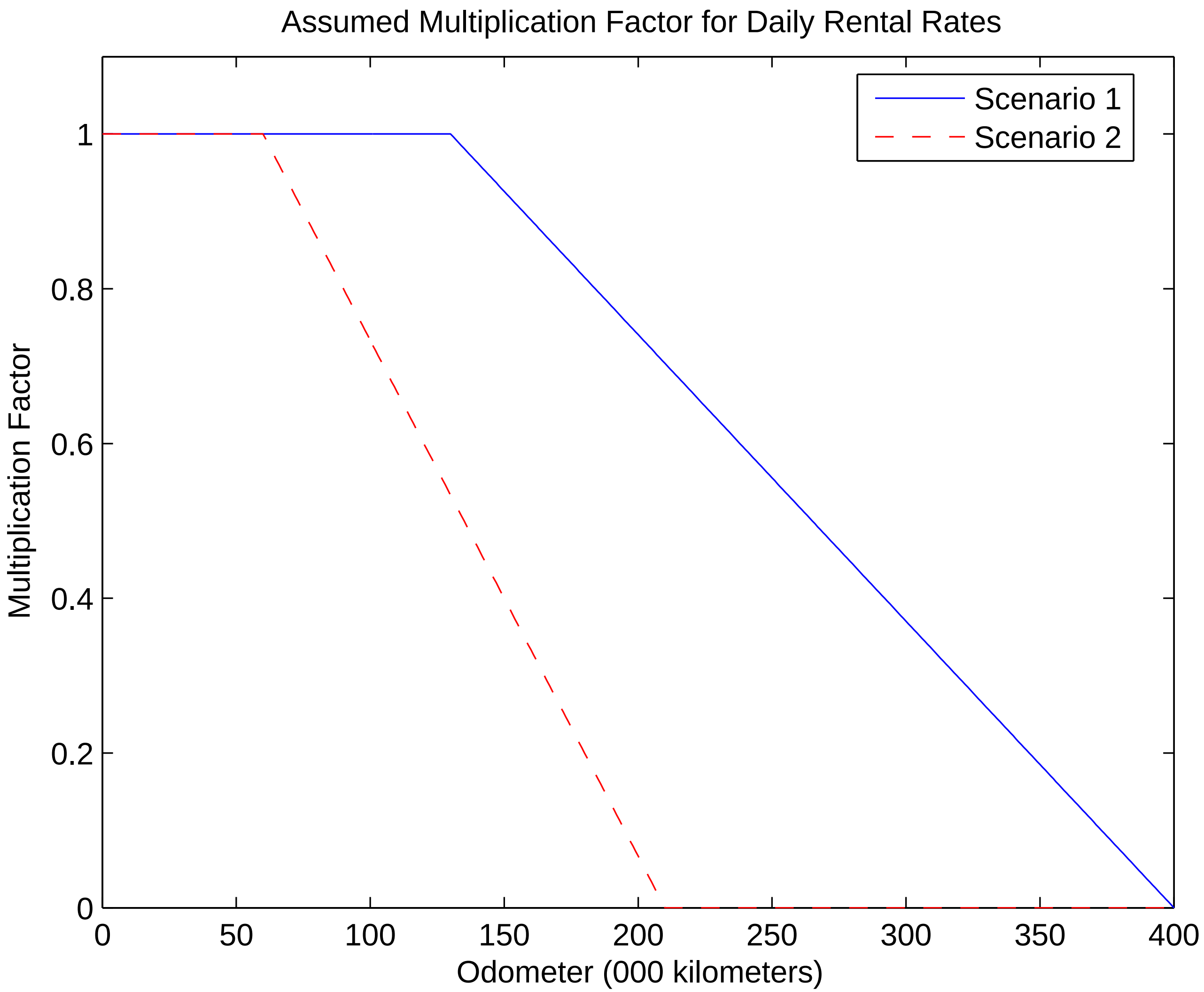
Even More Pessimistic Case

1. To assess the robustness of our conclusions, we solved for the optimal replacement policy under even more pessimistic assumptions about maintenance costs and the rental discounts that would be required to induce customers to rent older cars.
2. Under this more pessimistic scenario, maintenance costs start to accelerate far earlier, at *60,000 kilometers.*
3. We assume that rental rates start decreasing at a linear after 60,000 kilometers until they hit zero at 210,000 kilometers.
4. *Even under this even more pessimistic scenario, the optimal replacement policy still entails keeping cars about twice as long (in terms of age or odometer value) as the company currently keeps them.*

Maintenance Factors



Rental Factors



Profit Comparison

Quantity	Compact	Luxury	RV
Expected Discounted Values Under Optimal Replacement Policy			
$V(0,0,r_0)$	245680	337853	275614
$(1-\beta)V(0,0,r_0)$	20.19	27.77	22.65
$V(0,0,r_0)/\overline{P}$	25.4	14.4	14.7
Ratio of Expected Values: Optimal Policy versus *Status Quo*			
$V(0,0,r_0)/V_\mu(0,0,r_0)$	1.25	1.06	2.01

Unanswered Questions

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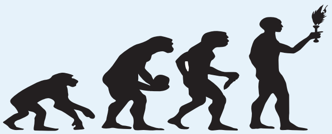
- We believe we have provided convincing evidence that via modest changes in the company's operating strategy, it can significantly increase discounted profits.
- However our analysis leaves a number of unanswered questions:
  1. Given how successful this company is at what it does, how could it fail to recognize the benefits from keeping its vehicles longer?

Unanswered Questions

- Our analysis has been focused mainly on the narrow question of the timing of replacement decisions,
- We believe we have provided convincing evidence that via modest changes in the company’s operating strategy, it can significantly increase discounted profits.
- However our analysis leaves a number of unanswered questions:
  1. Given how successful this company is at what it does, how could it fail to recognize the benefits from keeping its vehicles longer?
  2. Are there any overlooked considerations, constraints, or regulations that might explain why the company decides to replace its rental vehicles “too frequently”?



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  1. Why doesn't the company adjust the rental rates to equalize the relative profitability of long and short term contracts?
- Our analysis also revealed big differences in the overall profitability of different vehicles. In particular, the stream of discounted profits from rental of the RV or luxury car types are 20 and 40% higher, respectively.
  1. If these vehicles are so much more profitable, why not allocate more lot space on the margin to luxury and RVs, or alternatively, increase rental rates on compact cars to increase their relatively profitability?

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- Otherwise if there is one type of car that has a higher return per dollar invested, then the firm would be better off investing the marginal dollar in the car type that yields the highest possible returns.
- Our analysis has revealed that of the three car types we have analyzed, the compact has the *highest rate of return* even though it has the lowest *discounted value of profits per car.*

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- That is, if the company wants to get the highest return on its investment, it would appear it should allocate more of its vehicle “portfolio” to compacts and less to luxury or RVs.
- However if it is interested in maximizing the expected present value of profits, then it would appear that it should allocate more of its vehicle portfolio to the luxury and RV car types.

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- Clearly some customers will want to rent compacts, others will prefer RVs and others will prefer to have luxury vehicles.
- If the company happens to be "stocked out" of a particular customer's most preferred type of vehicle, having a portfolio with sufficiently close substitutes may enable the company to keep that customer, as opposed to the customer walking down to the next rental company window to see if a competitor has their preferred vehicle in stock and ready to rent.

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- However we believe the analysis we have conducted in this paper constitutes a fundamental "building block" toward a more complete analysis of this optimal (i.e. profit maximizing) operation of this company.
- Whatever portfolio allocation of rental vehicles, and rental rates the company chooses, it will want to adopt a vehicle replacement policy that is optimal conditional on its vehicle portfolio and rental rate structure.

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- Rental rate structures are more complicated if we allow contracts with odometer-based discounts, and usage-based rental schemes.

Toward a Complete Model

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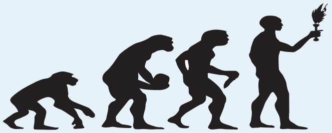
- Nested within this problem is the regnerative optimal stopping problem, that we have solved in this paper, that delivers the value function $V_{ij}(\mathcal{R})$ for all car types at all of the firm's rental locations.



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- In this larger competitive game, the firm's value and the optimal strategy for its vehicle portfolio and rental rate structure will clearly depend on the portfolios and rental rate structures chosen by its competitors.
- Solving for the overall competitive equilibrium problem in the rental market remains a challenging area for future research.

Recessional Hymns

Saddam in happier times

(shown after receiving billions in U.S. arms from Donald Rumsfeld)

George Bush, compassionate conservative