Government Reputation and Debt Repayment in Emerging Economies

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Abstract

Evidence shows that emerging economies that have repeatedly defaulted on their external obligations are still able to accumulate considerable amounts of debt. On average, an emerging economy defaulted 3 times every 100 years and the ratio of government debt to output is 40%. Existing models of sovereign debt are unable to jointly explain the debt to output ratios and the default frequency in these countries. To resolve this puzzle, I propose a standard small open economy model, with the addition that the government transits through different political states and these transitions cannot be directly observed by lenders. Moreover, after a default, the government and lenders bargain over the recovery rate. I argue that the combination of government reputation with endogenous periods of exclusion and debt renegotiation successfully accounts for the debt to output ratio in emerging economies. The model is consistent with the historical data that defaulting countries return to international loan markets only after settlement with creditors. In this framework, the terms of credit are determined by the lender's assessment of the government type: the sovereign rating. The model also generates the main empirical regularities of emerging economies.
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1 Introduction

Evidence shows that emerging economies that repeatedly defaulted on their external obligations are still able to accumulate considerable amounts of debt. Table 1 displays the average government

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debt to output ratio for countries that defaulted in the last decade. The average country, among emerging economies, experienced 3 defaults every 100 years and sustains an average external debt to output ratio around 40%.[1]

Table 1: Government Debt to Output Ratios and Default Episodes

Country	Gov.Debt/GNP 1980-2005 (%)	# of defaults	last default year t	Gov.Debt/GNP (%) year $t-1$
Russia	30	3	1998	64
Pakistan	36	2	1999	45
Ecuador	67	7	1999	88
Ukraine	16	2	2000	31
Argentina	37	5	2001	37
Indonesia	38	3	2002	43
Paraguay	31	7	2003	48
Uruguay	36	6	2003	66
Grenada	49	1	2004	87
Venezuela	35	8	2005	21
Average	39	4		53

Sources: World Bank, *Global Development Finance*;
Sturzenegger and Zettelmeyer (2005); and Standard & Poor's (2007).

One of the primary goals of a quantitative model of emerging market debt should be to generate the observed frequency of default in an equilibrium that sustains a level of external obligations similar to those displayed in Table 1. However, this remains a puzzle. Important recent contributions in the sovereign debt literature, such as Arellano (2007), Aguiar and Gopinath (2006) and Yue (2006), are unable to account for both debt to output ratios and recurrent default events.[2,3] This paper solves the debt to output puzzle by developing a model of government reputation where the risk of default and debt renegotiation are endogenous in an otherwise standard open economy environment.

During the last two decades the sovereign debt market has become increasingly competitive and it can be characterized as follows.[4] First, defaults are associated with "bad times" of the economy.

[1]Cases considered are defaults and/or debt restructurings (including reschedulings) in distressed circumstances involving external creditors. Defaults triggered by wars, revolutions, occupations and state disintegrations are excluded. All restructurings and defaults refer to federal or central government distress. Previous defaults since 1824 or year of independence (Pakistan 1947, Indonesia 1949, Ukraine 1991, Uruguay 1830, Grenada 1974).

[2]Arellano (2007), Aguiar and Gopinath (2006) and Yue (2006) are calibrated to Argentina and predict that, at the observed default frequency, the debt to output ratio should be 7.3% in the work of Arellano (2006); 18% in the case of Aguiar and Gopinath (2006); and 9.7% for Yue (2006).

[3]This was also a challenge in recent studies of unsecured consumer debt such as Chatterjee, Corbae, Nakajima and Rios Rull (2007, Econometrica) because high frequency of default makes unsecured debt very expensive which deters consumer borrowing. In addition to earnings shocks, and in order to jointly match the level of consumer debt and the default frequency, they consider unexpected medical expenses and private life-events (such as divorce) as a possible trigger for default.

[4]See Kletzer (1994), Eaton and Fernandez (1995), Sturzenegger and Zettelmeyer (2006) and Section 1.2 below.

Second, defaults are always followed by some form of repayment. Third, sovereign ratings, viewed as a measure of the willingness to pay of the sovereign, play an important role in determining countries access to international credit markets and are highly correlated with countries' interest rates. Fourth, countries are excluded from capital markets while the default situation lasts.

Based on the previous facts, I develop a model of sovereign debt, default and government reputation where the country is subject to income fluctuations. In this economy, an altruistic government makes the borrowing decisions and there is no commitment technology to repay the debt. Following Cole, Dow and English (1995), the government transits through two political states that affect how the government values the future. The state evolves as a Markov process rather than being permanent. According to these states we can classified the government as "aligned", if the government discounts the future at the same rate as consumers, or "misaligned", if the government discounts the future at a higher rate. The sovereign and risk neutral competitive financial intermediaries trade one period non-contingent bonds. A government default leads to financial autarky. However, the lenders and the government can renegotiate over debt a reduction. Only after repayment of the renegotiated debt can a government regain access to capital markets. Debt recovery rates are determined in a Nash bargaining mechanism consistent with the information structure.

The parameters of the model are estimated via simulated method of moments. In order to make a fair comparison of the results in this paper to those in previous studies in the sovereign debt literature, the moments chosen for the estimation are similar to the moments used in the calibration of Arellano (2007), Aguiar and Gopinath (2006) and Yue (2006).[5] It is important to note that the set of targeted moments does not include the debt to output ratio. After the estimation is done, and as a test of the model, I ask whether the mean debt to output ratio is consistent with the values presented in Table 1. I argue that the model is able to jointly account for the default frequency and the debt to output ratio in emerging economies. In particular, the combination of government reputation and endogenous recovery rates generates an average government debt to output ratio which is 92% of the ratio observed in the data, when the default probability is 3% per year.

One of the contributions of the paper is to develop a framework that is consistent with how the sovereign debt market for emerging economies works. International lenders learn about the political state from government borrowing and repayment behavior and summarize its reputation in the probability of a government being of the aligned type. I show that we can identify this probability with the sovereign rating. In equilibrium, the aligned government (hereafter the $a\!-\!type$), has a lower probability of default and borrow less than the misaligned government (hereafter the $m - type$). Thus, a default decision or taking more debt lowers the sovereign rating. Moreover, the $a - type$ government has a higher probability of entering the renegotiation stage than the $m - type$ government. This implies that after repayment the model generates an upgrade in the sovereign rating. Reputation becomes valuable because the terms of international loans depend not only on

[5]As in previous papers, I also base my analysis on the sovereign debt of Argentina.

economic fundamentals, but also on the sovereign rating.

Another important contribution of this paper is that incorporates both sovereign default and debt renegotiation into a dynamic general equilibrium model with private information. Post-default renegotiation and the endogenous determination of exclusion periods have important effects over the incentives to default. However, most of the recent papers in the literature, except Yue (2006), have considered zero recovery rates and exogenous exclusion periods after a default. Compared to a world with zero recovery rates (the environment of Arellano (2007) and Aguiar and Gopinath (2006)), interest rates at a particular debt level are lower because lenders have an expected recovery value that might be different from zero. Second, endogenous recovery rates introduce a level of contingency to debt contracts (they are function of the state of the economy in the repayment period) and make borrowing more appealing. As opposed to Yue (2006), countries are not forced to renegotiate in the default period and recovery rates are determined in the repayment period. Consistent with the data, countries have stronger incentives to repay in good times (see Kovrijnykh and Szentes (2007)).

The information structure and the endogenous determination of default penalties (recovery rates and periods of exclusion) are essential ingredients to obtain the main result. In section 5, I show that a model with full information and zero recovery rates generates an equilibrium debt to output ratio that is only 33% of that observed in the data. In addition, a model with full information and endogenous recovery generates a debt to output ratio that is only approximately 40% of the ratio in Argentina. In these models, reputation has no value (political states are observable), thus a repayment decision per se has no effect on future prices.

Emerging economies are characterized by volatile business cycles. Interest rates are countercyclical and highly volatile. Moreover, the current account is countercyclical and positively correlated with interest rates. At the estimated parameters, the benchmark model also accounts well for the business cycle behavior in these countries.

1.1 Related Literature

The pioneering work on sovereign debt and reputation is Eaton and Gersovitz (1981). In their model, countries were permanently excluded from credit markets after a default, so country's incentive to make repayments is to preserve its future access to foreign lenders. Bulow and Rogoff (1989) state that if markets are complete, sovereign debt can be sustained in equilibrium, only if it is possible to impose sanctions over a country after a default. I choose to model debt as a one period uncontingent bonds and countries are not allowed to borrow or save during default periods so their argument does not apply in this context. Moreover, the empirical evidence rejects the imposition of sanctions as a possible explanation of the existence of debt with no commitment.

This paper is closely related with Cole, Dow and English (1995) and Phelan (2006). They also study models with heterogenous agents where a players' type changes over time and is private information. Cole, et. al. (1995) focused on nineteenth-century bond defaults and subsequent resumptions. The level of debt and the repayment amount after a default are fixed. Phelan (2006) studies the effects of government reputation and shows that the unique equilibrium has the opportunistic government following a mixed strategy. Chatterjee, Corbae and Rios Rull (2007) also consider an environment with heterogenous borrowers and private information. They focus on unsecured consumer debt and the welfare consequences of imposing legal restrictions on the length of time that adverse events can remain on individual's credit record. The credit scoring technology implemented in this paper is similar to theirs. Another related paper is Alfaro and Kanczuk (2005). They study a model of sovereign debt with adverse selection where governments differ in their patience level but the borrowing level is exogenously given and countries have continuous access to credit markets. Moreover, they study equilibria where the government with a lower discount rate always default.

Recent quantitative models have related business cycles and sovereign debt in environments where the government cannot commit to pay back. Besides the models of Arellano (2006), Aguiar and Gopinath (2006) and Yue (2006) referenced before, other papers employed similar environments. In particular, Hatchondo, Martinez and Sapriza (2007) consider an environment where the government type can change but they consider zero debt recoveries and no exclusion from credit markets after a default. In their paper, government types are public information and debt contracts are contingent on types. Amador (2003) and Cuadra and Sapriza (2006) study sovereign default in a setup in which different government types alternate in power. The types disagree on the optimal allocation of resources within each period but do not differ in their willingness to pay, and therefore they receive the same treatment from lenders. Political instability affects the equilibrium spread through its effect on the weight on future utility flows. Another related work is the paper by Chang (2007). He studies the simultaneous determination of financial default and political crises in an open economy model. Political crises accompany default in equilibrium because of an information transmission conflict between the government and the public.

As in Kovrijnykh and Szentes (2007), the benchmark model predicts that there is some delay in reaching an agreement. In particular, the government chooses when to renegotiate. Consistent with the evidence they show, the bargaining mechanism used in this paper generates that incentives to repay are stranger in good times. In Kovrijnykh and Szentes (2007) the time elapsed from the default period to a return to markets is also endogenous. However, their model predicts periods of exclusion potentially long. Pitchford and Wright (2007) analyze what is the optimal tradeoff between efficient borrowing ex ante and the cost of default ex post. They present a model of sovereign borrowing default coupled with an explicit model of debt restructuring process in which delay arises due to both creditor holdout and free-riding on negotiation effort.

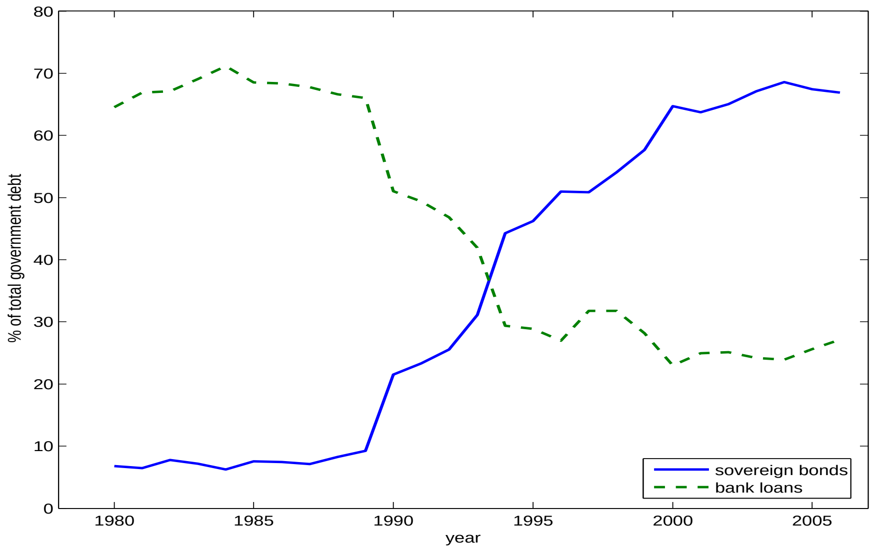
Default does not arise in equilibrium and the incentive to default is higher during good times in papers such as Kehoe and Levine (1993), Kocherlakota (1996), Alvarez and Jermann (2000) and Kehoe and Perri (2002). This paper differs from the literature on endogenous incomplete markets because in the model presented here default occurs in equilibrium and incentives to default are higher during low income realizations.

The rest of the paper is organized as follows. In Section 1.2 I present a description of the sovereign debt market in emerging economies. In section 2, I describe the environment. In section 3, I describe and define the equilibrium. Section 4, presents the computation algorithm and the estimation procedure. Finally, section 5 explains the main results of the paper.

1.2 The Emerging Economies Bond Market

During the past decades the market for sovereign lending has shift from syndicated bank loans towards bonds and has opened up the market to larger number of investors. Figure 1 shows the evolution of government debt by type (bonds vs bank loans) as a fraction of total government debt in emerging economies as a fraction of total debt. We observe a sharp decrease of the bank debt towards sovereign bonds specially towards the end of the 1980's.

Figure 1: Sovereign Debt in Emerging Economies by Type



Source: World Bank (2007).

Unlike loan restructurings, no formal mechanisms for sovereign bond renegotiation, or workouts, have been established. Markets have addressed the issues of bond workouts on a case-by-case basis, and essentially without intervention by creditor countries or multilateral institutions (see Chuhan and Sturzenegger (2003)). Two approaches to sovereign bond workouts have been followed: voluntary and involuntary (or concerted). Voluntary exchanges between a government and lenders typically re-profile debt service, but do not lower the nominal value of debt and impose small net present value reductions. A concerted renegotiation will involve a reduction in the net present value of the investment for investors. Table 2 displays the recovery rates and periods of exclusion during the most recent sovereign defaults. The sovereign ratings of these countries were always downgraded to the state of Selective Default (SD) or Default (D) until the repayment of the renegotiated debt. The recovery rates correspond to the difference between the present value of the new instruments versus the present value of the old instruments (see Sturzenegger and Zettelmeyer (2005) and Moody's (2006) for a comprehensive explanation).[6]

Table 2: Recovery Rates and Default Episodes

Country	Date of default	Time in default (months)	Recovery rate (%)
Russia	09/98	26	45
Pakistan	10/98	14	69
Ecuador	10/99	12	73
Ukraine	01/00	2	72
Argentina	11/01	42	27
Indonesia	04/02	5	na
Paraguay	02/03	14	na
Uruguay	04/03	2	87
Grenada	12/04	10	65
Venezuela	01/05	2	na
Average		13	62

Sources: Sturzenegger and Zettelmeyer (2005); Moody's (2006); and Standard & Poor's (2007).

In response to the increased demand for the evaluation of creditworthiness, several agencies such as Moodys and Standard & Poor's have developed expertise in estimating country risk.[7] Sovereign ratings have become an important component of the sovereign debt market. Standard & Poor's (2006) reports that a sovereign rating aggregates information from two main sources: economic and political risk. Both factors determine the sovereign's willingness to repay. The evidence shows that ratings directly affect the extension to government credit. Table (3) displays the correlation of sovereign ratings and country's spread over time observed for emerging economies[8]. Ratings have been good predictors of defaults. Countries in the lowest categories account for more than 70 percent of the total number of default episodes in a one year horizon (See Standard & Poor's and Moody's).

[6]While the renegotiation mechanism is different, recovery rates for rated corporate defaults are similar. Moody's (2006) documents that the average senior unsecured bond recovery rate is 58.3% in 2006.

[7]By 1970 S&P's rated only two countries, U.S. and Canada. The number of rated sovereigns rose to 12 in 1980, all rated 'AAA'. From that point on, there was a marked increase in the number of ratings and an expansion into lower rating categories.

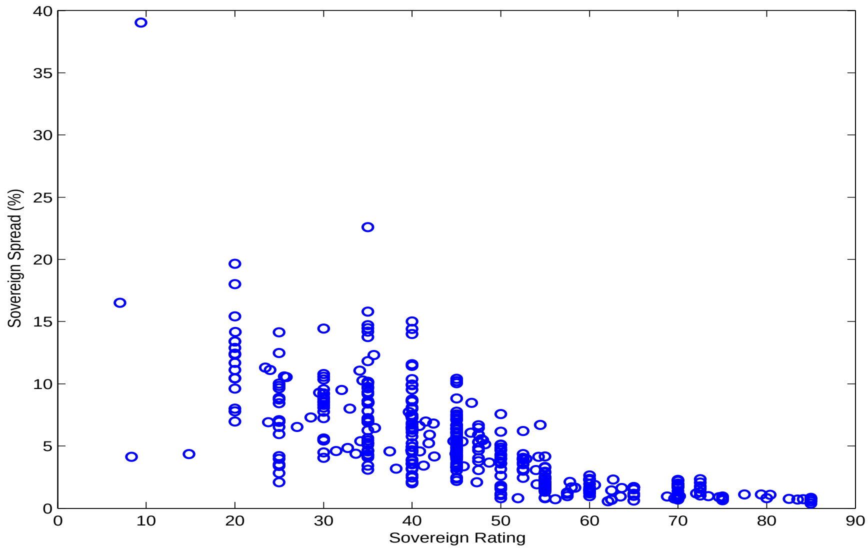
[8]A country's spread is the difference between a bond of the country and that of similar maturity in the U.S. For the table, the an index generated by JP Morgan, EMBI, is used to calculate the correlation with the ratings. Ratings are sovereign ratings reported by Standard & Poor's and transformed into numbers using a linear scale.

Table 3: Sovereign Ratings and Spreads

Country	Correlation Spreads and Ratings over time
Argentina	-0.86
Brazil	-0.32
Chile	-0.77
Ecuador	-0.75
Korea	-0.80
Malaysia	-0.85
Mexico	-0.82
Turkey	-0.82
Venezuela	-0.35

Sources: JP Morgan (2007); and Standard & Poor's (2007).

Figure 2: Sovereign Ratings and Interest Rates



Ratings affect the extension of government credit in four ways. First, a higher rating implies lower interest rates. Second, the presence of a "selective default" or "default" flag, constrains a country from market access. Third, after a default, countries emerge with lower ratings than before the episode. Fourth, countries that borrow more receive lower ratings. Higher credit ratings translate into lower country spreads (Cantor and Packer (1996) and Eichengreen and Mody (2000)). For these reasons, upgrades or downgrades in sovereign ratings have considerable impact in countries access to credit. Figure 2 shows the relation between rating and spreads for the group of countries in the previous table during the last decade. Each point corresponds to a year-country pair. One can observe a clear negative relation.

2 Environment

I study sovereign default and debt renegotiation in a dynamic small open economy with private information. The model consists of a large number of competing lenders and a sovereign that borrows from them. The country receives a stochastic stream of income and debt contracts are restricted to one-period bonds. Debt contracts are not enforceable.

2.1 Preferences and Endowment Process

Preferences of the representative consumer in period τ are given by

$$\mathbb{E}_\tau \sum_{t=\tau}^{\infty} \beta^t u(c_t) \tag{1}$$

where $u(c)$ is strictly concave and differentiable, $\beta \in (0,1)$ is the discount factor and c_t is aggregate consumption in period t. The endowment of the consumption good that the agents receive in each period is denoted by $y_t \in Y \subseteq \mathbf{R}_{++}$. I assume that y_t follows a Markov process with conditional distribution function $F(y_{t+1}|y_t)$.

International lenders are risk neutral. They can borrow or lend as much as necessary at the risk free rate r in international markets.

2.2 Government

The government chooses consumption to maximize the expected discounted utility of the representative consumer. As in Cole, Dow and English (1995) and Phelan (2006), I assume that the government transitions between two states. Political uncertainty, changes in the composition of the leading coalition, or changes in the distribution of power within the government generate that government's valuation of the future varies over time. The political state of the government is private information and changes over time. According to these states we can classify the government as "aligned" (the government discounts the future at the same rate as consumers, i.e. $\delta_a = 1$) or "misaligned" (the government discounts the future at a lower rate, i.e. $\delta_m < 1$). No restrictions are imposed on the value of δ_m besides $\delta_m \in [0,1)$. The political state evolves according to a first order markov process with the transition matrix given by:

$$\Pi = \begin{bmatrix} \pi_a & 1-\pi_a \\ 1-\pi_m & \pi_m \end{bmatrix}, \tag{2}$$

where π_i is the probability that at state i the government will stay at the same state in the following period. The transitions between government states are not observed by the lenders. For future reference, I will denote the unconditional probability of a government being an $a-type$ by

$$\pi^* = \frac{(1-\pi_m)}{(1-\pi_a)+(1-\pi_m)}. \tag{3}$$

2.3 Information Structure and Strategies

The credit industry accepts deposits and makes loans to the sovereign country. Asset markets are not complete. In each period, the government borrows or saves using only one period non-contingent bonds denoted by b. Let x and x' be any variable x at period t and $t+1$ respectively. The set of possible values of b is $B \equiv [\underline{b}, \overline{b}] \subset \mathbf{R}$, with $\underline{b} < 0 < \overline{b}$. When the government borrows from international markets b' will take negative values, $b' < 0$.

The level of debt, output and the actions of the government are observable. However, the political state is private information and lenders need to infer the sovereign's state based on its actions. I will denote by s the lender's posterior probability that the government is of the $a-type$ at the given observable state (b, y). In a markov perfect bayesian equilibrium, the actions of the players will be specified as function of s.

The government borrows at price $q(b', s', y)$ where b' is the borrowing level, s' is the market assessment of the government type, i.e. the probability that the government is of the $a-type$ and y is the current output realization. If the government borrows from foreign lenders, it receives $-q(b', s', y)b'$ units of current period goods and promises to deliver b' units of the following period good. The government is not committed to repay its debts. In equilibrium, the price function will reflect the risk of default and the recovery value of the defaulted debt. A government that defaults will remain in financial autarky until it repays an endogenously determined fraction of the defaulted debt. The variable h will denote the government status in international credit markets, $h \in \{In, Out\}$. If $h = In$, the government does not have an unresolved default and it is free to borrow or save in credit markets. On the other hand, if $h = Out$ the government still has to resolve an old default and it is not allowed to borrow or save.

In the default period, total consumption equals the endowment. If a country has an unresolved default, $h = Out$, and unpaid debt $b < 0$, the country is subject to exclusion from credit markets and the default situation entitles a proportional output loss $\lambda \in (0, 1)$. If the government decides to repay, the present value of the debt is reduced to a fraction $\phi(b, y, s; \sigma)$ of its original size. This fraction is determined at the renegotiation stage between the government and the lender and depends on the observable state in the repayment period (b, y, s) and on the probability that the lenders assign to the government they are facing in the renegotiation stage is of the $a-type$, σ.

The government and the lenders bargain over the debt recovery rate $\phi(b, y, s; \sigma)$. At the renegotiation stage, a Nash bargaining solution is defined in this environment with incomplete information. The properties derived from this mechanism are consistent with what we observe in the data and are explained in more detail in the next section. After the agreement, the present value of the debt is reduced by a fraction $\phi(b, y, s; \sigma)$ of its original size b.

2.4 Time Line

The timing of events can be summarized as follows: at the beginning of each period, the government inherited a level of debt b and credit history (s, h). The government's type and the country's endowment are realized. Only the endowment is revealed to the lenders. When $h = In$ the government decides to default or not. If it decides not to default, it chooses how much to borrow or lend b' at price $q(b', s', y)$ for the following period. If it chooses to default, consumption equals the endowment and $h' = Out$. If $h = Out$ the government has an unresolved default and has to decide to renegotiate or not. If the government repays the reduced debt, $\phi(b, y, s; \sigma)b$, it regains access to the credit market in the following period $(h' = In)$. In the case the government does not repay the old debt it will stay out of credit markets, $h' = Out$. Based on government actions the lenders will update their information about the sovereign's type, i.e. when $h = In$, s' is a function of the current observable government state (b, y, s), the default decision $d \in \{0, 1\}$ and the level of borrowing b'. When $h = Out$ the posterior s' is a function of (b, y, s), the repayment decision $z \in \{0, 1\}$ and the recovery rate $\phi(b, y, s; \sigma)$.

Period t			Period $t+1$
(b, s, h) given (i, y) are realized		Only the endowment y is revealed to lenders	(b', s', h')
If $h = In$	$b < 0$	government chooses to default or not - If $d = 0$ (no-default), gov. chooses b' at price $q(b', s', y)$ $s' = \Psi^{In}(b', 0, b, y, s)$ and $h' = In$ - If $d = 1$ (default) gov. cannot borrow or save $s' = \Psi^{In}(b, 1, b, y, s)$ and $h' = Out$	
	$b \geq 0$	government chooses b' at price $q(b', s', y)$ $s' = \Psi^{In}(b', 0, b, y, s)$ and $h' = In$	
If $h = Out$		government chooses to repay or not - If $z = 1$ (repay) bargain and pays back $\phi(b, y, s; \sigma)b$ $s' = \Psi^{Out}(1, b, y, s)$ and $h' = In$ - If $z = 0$ (no-repay) country suffers output loss λy $s' = \Psi^{Out}(0, b, y, s)$ and $h' = Out$	

3 Equilibrium

In this paper, I focus on Markov Perfect Bayesian Equilibria, and strategies are restricted to be stationary functions from beliefs (or more precisely posterior probabilities) to actions. Beliefs and strategies need to be specified for all possible states, even those that never happen in equilibrium. Markov strategies are defined relative to the state variable s, the sovereign rating.

Definition 1 *A* ***Stationary Markov Strategy*** *for a government in political state i is a set of stationary functions: consumption, $c_i(b, y, s, h)$; debt accumulation, $b'_i(b, y, s)$; default, $d_i(b, y, s)$ and debt repayment $z_i(b, y, s)$.*

At observable state $(b, y, s, h = In)$, the probability that lenders assign to a government of being of the $a - type$ after observing his debt, $b' \in B$, and default decisions, $d \in \{0, 1\}$, is denoted by $\sigma^{In}(b', d, b, y, s)$, that is

$$\sigma^{In}(b', d, b, y, s) \equiv \Pr(a|b', d, b, y, s; h = In).$$

If the government is out of the credit market, i.e. at state $(b, y, s, h = Out)$, the probability that it is of type a, conditional on the repayment decision $z \in \{0, 1\}$, is denoted by $\sigma^{Out}(z, b, y, s)$:

$$\sigma^{Out}(z, b, y, s) \equiv \Pr(a|z, b, y, s; h = Out).$$

This probabilities are defined for allocations on and off-the equilibrium path. The possibility of a type change implies that the sovereign rating (the posterior) at the beginning of the following period is:

$$\Psi^{In}(b', d, b, y, s) = \pi_a \sigma^{In}(b', d, b, y, s) + (1 - \pi_b)(1 - \sigma^{In}(b', d, b, y, s)), \tag{4}$$

and

$$\Psi^{Out}(z, b, y, s) = \pi_a \sigma^{Out}(z, b, y, s) + (1 - \pi_b)(1 - \sigma^{Out}(z, b, y, s)). \tag{5}$$

3.1 Government's Problem

The objective of the government is to maximize the expected discounted utility of the representative consumer. The government solves the following problem: If it is active in credit markets and has debt, it chooses to default or not and in the case of no-default how much to borrow or lend. If it is active but it does not have debt, it only chooses how much to borrow or save. If it is not active it chooses to renegotiate or not. The state variables for the government are its political state $i \in \{a, m\}$, the level of assets inherited from the previous period b, the endowment y, the probability the lender assigns to the government of being of the good type s and credit history

h. Let the value function of a government of type i at state (b, y, s, h) be denoted by $V_i(b, y, s, h)$. $V_{-i}(b, y, s, h)$ denotes the value function of the political state different than i.

If $b < 0$ and $h = In$, the government can choose between repaying its outstanding debt or defaulting. If the government settles the debt, it chooses its next period debt level b' and consumes. If it defaults, it cannot borrow or save and consumes its endowment. Based on its actions, the sovereign rating is updated to $s' = \Psi^{In}(b', d, b, y, s)$ and h' is set to In or Out if $d = 0$ or $d = 1$ respectively. The value function is

$$V_i(b, y, s, In) = \max\left\{v_i^{nd}(b, y, s), v_i^{d}(b, y, s)\right\}. \tag{6}$$

$v_i^{nd}(b, y, s)$ denotes the value of "no-default":

$$v_i^{nd}(b, y, s) = \max_{c \geq 0, b' \in B} \left\{u(c) + \beta\delta_i E_{y'|y}\left[\pi_i V_i(b', y', s', In) + (1 - \pi_i)V_{-i}(b', y', s', In)\right]\right\}, \tag{7}$$

$$\text{s.t. } c + q(b', s', y)b' \leq y + b,$$
$$s' = \Psi^{In}(b', d = 0, b, y, s).$$

$v_i^{d}(b, y, s)$ is the value of "default":

$$v_i^{d}(b, y, s) = u(y) + \beta\delta_i E_{y'|y}\left[\pi_i V_i(b, y', s', Out) + (1 - \pi_i)V_{-i}(b, y', s', Out)\right], \tag{8}$$

$$\text{s.t. } s' = \Psi^{In}(b, d = 1, b, y, s).$$

If $b \geq 0$ and $h = In$, the government does not have debt, so there is no default decision to make. In this case, the value for a government $i - type$ is given by

$$V_i(b, y, s, In) = v_i^{nd}(b, y, s). \tag{9}$$

If $h = Out$ the country is out of the credit market and has some unpaid debt $b < 0$. While the country is in this state, it suffers a proportional output loss of size λy. Only if the government repays the old debt, reduced to $\phi(b, y, s; \sigma)b$, it will regain access to credit markets. The recovery rate $\phi(b, y, s; \sigma)$ is endogenously determined in the bargaining process and can take any value in $[0, 1]$. The problem that the government solves is:

$$V_i(b, y, s, Out) = \max\left\{v_i^{r}(b, y, s), v_i^{nr}(b, y, s)\right\}. \tag{10}$$

$v_i^r(a, y, s)$ denotes the value of "repay":

$$v_i^r(b, y, s) = u(y + \phi(b, y, s; \sigma)b) + \beta\delta_i E_{y'|y}\Big[\pi_i V_i(0, y', s', In) + (1 - \pi_i)V_{-i}(0, y', s', In)\Big] \tag{11}$$

$$\text{s.t. } \; s' \; = \; \Psi^{h=Out}(z = 1, b, y, s).$$

$v_i^{nr}(b, y, s)$ denotes the value of "no-repay":

$$v_i^{nr}(b, y, s) = u(y(1 - \lambda)) + \beta\delta_i E_{y'|y}\Big[\pi_i V_i(b, y', s', Out) + (1 - \pi_i)V_{-i}(b, y', s', Out)\Big] \tag{12}$$

$$\text{s.t. } \; s' \; = \; \Psi^{h=Out}(z = 0, b, y, s).$$

The solution to this problem provides the value function $V_i(a, y, s, h)$ for $i = a, m$ and $\forall$ (b, y, s, h) and the optimal choices, $b' = g_i(b, y, s)$, $d_i(b, y, s)$, and $z_i(b, y, s)$ of assets, default and repayment respectively. These optimal choices allow me to characterize the default set $D_i(b, s) \subseteq Y$ and the renegotiation set $Z_i(b, s) \subseteq Y$ for a government in political state i as follows:

$$D_i(b, s) = \{y \in Y : d_i(b, y, s) = 1\}, \tag{13}$$

and

$$Z_i(b, s) = \{y \in Y : z_i(b, y, s) = 1\}. \tag{14}$$

For future reference, I can also define the set of endowments for which a government in political state i with posterior s chooses b':

$$E_i(b', b, s) = \{y \in Y : d_i(b, y, s) = 0 \;\&\; g_i(b, y, s) = b'\}. \tag{15}$$

3.2 Foreign Lender's Problem

Foreign creditors can borrow or lend at the risk free-rate $r \geq 0$. The market for sovereign bonds is competitive so they take the price schedule $q(b', s', y)$ as given. Recall from equation (4) that after choosing b' the government is thought to be of the $a - type$ at the beginning of the following period with probability $s' = \Psi^{In}(b', d = 0, b, y, s)$. From the default set defined in (13) it is possible to derive the equilibrium probability of default on a loan b' when the government is at state (b, y, s):

$$p(b', s', y) = \int_{D_a(b', s')} F(dy'|y)s' + \int_{D_m(b', s')} F(dy'|y)(1 - s').$$

The default probability takes into account the possible type change at the end of the period as well as the transition to different endowments levels. With probability $s' = \Psi^{In}(b', d = 0, b, y, s)$

the government is of the $a-type$ in the following period and will default in endowments states $y' \in D_a(b', s')$. Similarly, with probability $(1-s') = (1-\Psi^{In}(b', d=0, b, y, s))$ the government is of the $m-type$ in the following period and will default in endowments states $y' \in D_m(b', s')$.

The profit on a loan of size b' made to a government with assets b, endowment y and current rating s, denoted by $\Omega(b', s', y; p, \rho)$, equals the expected present discounted value of inflows less the current value of outflows. It depends on the price $q(b', s', y)$, on the probability of the government defaulting on it, $p(b', s', y)$, and on the expected present value of loan repayments after a default, $\rho(b, s, y)$. In equilibrium, the function $\rho(b, s, y)$ is derived from the decision rules of the government and the posterior function. Consider a situation where a government with prior s is in financial autarky. At the beginning of a period, before any action is taken, lenders expects to receive:

$$[sz_a(b, y, s) + (1-s)z_m(b, y, s)]\phi(b, y, s; \sigma)(-b),$$

that is, with probability s, lenders are facing an $a-type$ government and they will receive $\phi(b, y, s; \sigma)(-b)$ if the government chooses to repay, i.e. if $z_a(b, y, s) = 1$. Similarly, with probability $(1-s)$ they are facing a $m-type$ government and they will receive $\phi(b, y, s; \sigma)(-b)$ if the government chooses to repay, i.e. if $z_m(b, y, s) = 1$. Recursively, the expected recovery value on a defaulted loan is

$$\begin{aligned} \rho(b, s, y) &= [sz_a(b, y, s) + (1-s)z_m(b, y, s)]\phi(b, y, s; \sigma)(-b) \\ &\quad + \frac{1}{(1+r)}[s(1-z_a(b, y, s)) + (1-s)(1-z_m(b, y, s))]E_{y'|y}[\rho(b, s', y')], \end{aligned} \tag{16}$$

where the posterior s' corresponds to a government that chooses not to repay, i.e. $s' = \Psi^{Out}(z = 0, b, y, s)$. Hence, the profit on a loan of size b' made to a government with assets b, endowment y and current rating s, $\Omega(b', s', y; p, \rho)$ is

$$\Omega(b', s', y; p, \rho) = -q(b', s', y)(-b') + \frac{(1-p(b', s', y))(-b')}{(1+r)} + \frac{p(b', s', y)}{(1+r)^2}E\rho(b', s'', y'') \tag{17}$$

where $s'' = \Psi^{In}(b', d' = 1, b', y', s')$ is the posterior for the government after a default.

In most of the recent models of sovereign debt, it is assumed that countries obtain full discharge of debt after a default (see for example Arellano (2006) and Aguiar and Gopinath (2006)). This implies that, in these papers, the last term in expression (17) always equals zero. The exception is the paper by Yue (2006) that considers endogenous debt renegotiation in an environment with full information. However, in that paper, there is no delay in reaching an agreement and debt reduction is immediate because countries are forced to renegotiate in the default period.

Perfect competition implies that $\Omega(b', s', y; p, \rho) = 0$ and the equilibrium price function is:

$$q(b', s', y; p, \rho) = \begin{cases} (1+r)^{-1} & \text{if } b' \geq 0, \\ \frac{(1-p(b', s', y))}{(1+r)} + \frac{p(b', s', y)E_{y''|y}\rho(b', s'', y'')}{(-b')(1+r)^2} & \text{if } b' < 0. \end{cases}$$

This expression shows why the probability of default plays an important role in determining the equilibrium price. As expected, a higher probability implies a higher interest rate (a lower price). However, the value of the expected recovery value $\rho(b', s'', y'')$ sets a limit to the lowest value of the contract. At a given default probability, higher recovery rates will imply lower interest rates. Since $p(b', s', y) \in [0, 1]$ and $0 \leq E_{y'|y}[\rho(b', s'', y'')/(-b')] \leq 1$ the set of feasible prices is $Q = [0, (1 + r)^{-1}]$.

3.3 Debt Renegotiation: A Nash Mechanism

In an environment with full information, a "reasonable" outcome is the Nash bargaining solution because it is pairwise Pareto-efficient. In contrast, for meetings with incomplete information very few guidelines exist for the choice of the mechanism. Following Berentsen and Rocheteau (2004), I choose a particular mechanism, which maximizes the product of the expected surplus of the government and the lenders, given their beliefs, that coincides with the Nash bargaining solution when there is complete information. This mechanism has many implications consistent with what we observe in the data. These are explained in more detail in the section with the estimation results.

Upon the bargaining agreement, the present value of defaulted debt is reduced to a fraction φ of the unpaid debt b. The value of an agreement of size φ to a government in political state i is

$$v_i^r(b, y, s; \varphi) \quad = \quad u(y + \varphi b) + \beta\delta_i E_{y'|y}\Big[\pi_i V_i(0, y', s', In) + (1 - \pi_i)V_{-i}(0, y', s', In)\Big], \qquad (18)$$

that is the expected life time utility of repayment at state (b, y, s) when the debt recovery is φ. The value of the posterior is $s' = \pi_a\sigma + (1 - \pi_m)(1 - \sigma)$ where σ denotes the prior probability that lenders' assign to the government they are facing at the bargaining stage of being of the $a - type$.

In order to keep the model tractable, I assume that the threat point of the government in political state i is permanent autarky with output cost λy. Recursively, the autarky value $v_i^{aut}(y)$ for government of type i at income level y is

$$v_i^{aut}(y) = u(y(1 - \lambda)) + \beta\delta_i E_{y'|y}\Big[\pi_i v_i^{aut}(y') + (1 - \pi_i)v_{-i}^{aut}(y')\Big].$$

Thus, for a government of the $i - type$, the surplus of an agreement is

$$\Delta_i^B(\varphi; b, y, s) = v_i^r(b, y, s; \varphi) - v_i^{aut}(y). \qquad (19)$$

The surplus of the agreement for the lenders is the present value of the recovered debt:

$$\Delta^L(\varphi; b, y, s) = -\varphi b. \qquad (20)$$

If lenders have all the bargaining power, then they could extract all the government's surplus and viceversa. To analyze a general case, I assume that the government has bargaining power θ and the lender has bargaining power $(1 - \theta)$. The bargaining power is independent of σ and thus is not affected by the timing of the repayment decision. The optimal debt recovery rate $\phi(b, y, s; \sigma)$ satisfies the following program:

$$\phi(b, y, s; \sigma) \equiv \arg \max_{\varphi \in [0,1]} [\sigma \Delta_a^B(\varphi; b, y, s) + (1-\sigma)\Delta_b^B(\varphi; b, y, s)]^{\theta} [\Delta^L(\varphi; b, y, s)]^{1-\theta} \tag{21}$$

$$\begin{aligned} \text{s.t. } \Delta_a^B(\varphi; b, y, s) &\geq 0, \\ \Delta_m^B(\varphi; b, y, s) &\geq 0, \\ \Delta^L(\varphi; b, y, s) &\geq 0. \end{aligned}$$

Under this mechanism, the resulting offer maximizes the product of the expected surpluses of the players given their prior beliefs subject to the corresponding participation constraints of the government of type a, the government of type m and the lenders. The solution to this mechanism is a single offer. Hence, by construction, the value of φ that maximizes (21) is also incentive compatible.

Note that, in equilibrium, agreements are reached in one period. However, the period of financial exclusion is still endogenous in this model. At a given recovery schedule $\phi(b, y, s; \sigma)$, the government will optimally choose when to start the renegotiation process. The expected duration of financial exclusion increases with the equilibrium repayment fraction.

3.4 Definition of Equilibrium

A Markov Perfect Bayesian Equilibrium (MPBE) requires that, at every possible state (including those that only occur in off-the-equilibrium-path), agents' beliefs over the types and strategies of the other agents must be specified. Given these beliefs, each agent must choose actions that are the best responses to the strategies of the other agents. The government and the lenders only use stationary Markov strategies.

Definition 2 *A MPBE is a set of functions $V_i^*, \sigma^*, q^*, p^*, \rho^*, \phi^*$ and sets D_i^*, E_i^* and Z_i^* such that:*

1. *Given posterior functions, prices and the recovery rate, the value functions V_i^* and the sets D_i^*, E_i^* and Z_i^* for $i = a, m$, are consistent with the government's optimization problem.*

2. *Given posterior functions, prices and value functions V_i^* for $i = a, m$, the recovery rate $\phi(b, y, s; \sigma^*)$ is derived from the bargaining mechanism.*

3. *The equilibrium default probability $p^*(b', s', y)$ and expected recovery $\rho^*(b', s', y)$ are consistent with the government decision rules and the bargaining solution.*

4. *The equilibrium price is such that foreign creditors earn zero profits in expected value, that is at $q^*(b', s', y)$*
$$\Omega^*(b', s', y; p^*, \rho^*) = 0, \quad \textit{for all } (b', s', y),$$

5. *The function σ^* must be consistent with Bayes' rule (whenever possible) and posteriors are defined as in equations (4) and (5).*

The definition of equilibrium is standard. Condition 5 defines the function σ^* and it deserves a detailed explanation. This function must be consistent with Bayes' rule *whenever* applicable. When $h = In$, the probability that a government is type a conditional on their asset market behavior was denoted by $\sigma^{h=In}(b', d, b, y, s)$, and can be written as

$$\begin{aligned} \sigma^{In}(b', d, b, y, s) &= \frac{\Pr(b', d, b, y, s|a)\Pr(a)}{\Pr(b', d, b, y, s)} \\ &= \frac{\Pr(b', d|a, b, y, s)\Pr(a|b, y, s)}{\sum_i \Pr(b', d|i, b, y, s)\Pr(i|b, y, s)}, \end{aligned} \tag{22}$$

provided $\sum_i \Pr(b', d|i, b, y, s)\Pr(i|b, y, s) > 0$. When the conditioning set is empty, the definition of equilibrium does not impose any restriction. In the computation and estimation of the model, I assume that the off-the-equilibrium path posteriors σ are set equal to the priors s.[9]

Prior to the government's type realization, the posterior probability that a government with observable state $(b, y, s, h = In)$ who also defaults on his loan, is of type a is given by

$$\sigma^{In}(b, d = 1, b, y, s) = \begin{cases} 1 & \text{if } y \in D_a(b, s) \text{ and } y \notin D_m(b, s), \\ 0 & \text{if } y \notin D_a(b, s) \text{ and } y \in D_m(b, s), \\ s & \text{otherwise.} \end{cases}$$

The posterior $\sigma^{h=In}(b, d = 1, b, y, s)$ equals s in two circumstances. First, *on-the-equilibrium* path, when $y \in D_a(b, s)$ and $y \in D_m(b, s)$, $\sigma^{h=In}(b, d = 1, b, y, s) = s$ can be derived from equation (22). Second, *off-the-equilibrium* path, when $y \notin D_a(b, s)$ and $y \notin D_m(b, s)$, $\sigma^{h=In}(b, d = 1, b, y, s) = s$ by assumption because Bayes' rule is not applicable. One of the main differences between the environment in this paper and that in Chatterjee et. al. (2007) is that income is observable. This translates into a simpler posterior function, i.e. at a given state (b, y, s), $\sigma^h(\cdot)$ can take only three values: 1, 0 or s.[10]

The probability that a government with observable state $(b, y, s, h = In)$ who does not default on his loan and chooses b' is of type a is given by

$$\sigma^{In}(b', d = 0, b, y, s) = \begin{cases} 1 & \text{if } y \in E_a(b', b, s) \text{ and } y \notin E_m(b', b, s), \\ 0 & \text{if } y \notin E_a(b', b, s) \text{ and } y \in E_m(b', b, s), \\ s & \text{otherwise.} \end{cases}$$

[9] The estimation results that I present later are robust to different assumptions about the off-the-equilibrium path beliefs.

[10] Multiple computations show that existence of an equilibrium is still an issue but the space of parameters for which an equilibrium does not exists is considerably reduced.

Similarly, when $h = Out$, after observing the decision to repay or not, the probability that the government is of the $a - type$ is:

$$\begin{aligned}
\sigma^{Out}(z,b,y,s) &= \frac{\Pr(z,b,y,s|a)\Pr(a)}{\Pr(z,b,y,s)} \\
&= \frac{\Pr(z|a,b,y,s)\Pr(a|b,y,s)}{\sum_i \Pr(z|i,b,y,s)\Pr(i|b,y,s)},
\end{aligned}$$

provided $\sum_i \Pr(z|i,b,y,s)\Pr(i|b,y,s) > 0$. Hence, at $h = Out$, when the government repays the probability that it is of the $a - type$ is:

$$\sigma^{Out}(z=1,b,y,s) = \begin{cases} 1 & \text{if } y \in Z_a^*(b,s) \text{ and } y \notin Z_m^*(b,s), \\ 0 & \text{if } y \notin Z_a^*(b,s) \text{ and } y \in Z_m^*(b,s), \\ s & \text{otherwise.} \end{cases}$$

and when it does not repay

$$\sigma^{Out}(z=0,b,y,s) = \begin{cases} 1 & \text{if } y \notin Z_a^*(b,s) \text{ and } y \in Z_m^*(b,s), \\ 0 & \text{if } y \in Z_a^*(b,s) \text{ and } y \notin Z_m^*(b,s), \\ s & \text{otherwise.} \end{cases}$$

Once $\sigma^{In}(b',d,b,y,s)$ and $\sigma^{Out}(\phi,z,b,y,s)$ have been defined for each possible history and state, the new posterior, s', can be obtained from equations (4) and (5).

4 Computation and Estimation

The model is solved numerically. In this section, I describe the computation and estimation procedures. No restrictions are imposed on parameter values or equilibrium behavior. Off the equilibrium beliefs were defined in the previous section.

4.1 Model Specification

One period is a quarter. The utility function is assumed to be CRRA:

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma},$$

where γ is the coefficient of relative risk aversion. The endowment is modeled as a first order autoregressive process:

$$\ln(y_t) = \rho_y \ln(y_{t-1}) + \eta_t, \text{ with } \eta \sim N(0, \sigma_\eta).$$

The full set of 10 parameters in the benchmark model is:

$$\{r, \beta, \gamma, \delta_m, \pi_a, \pi_m, \lambda, \rho_y, \sigma_\eta, \theta\},$$

where r is the risk free rate; β is the discount factor of the representative consumer; γ is the coefficient of relative risk aversion; δ_m is parameter affecting the discount factor of the $m-type$ government; π_a and π_m are the transition probabilities; λ is the output cost during financial autarky; ρ_y and σ_η control the endowment process; and θ is the bargaining power of the government (probability that it makes the offer at the bargaining stage).

4.2 Parameter Values and Estimation

I classify the parameters into two groups. The first group consists of 6 parameters, $\{r, \beta, \gamma, \rho_y, \sigma_\eta, \lambda\}$, each of which can be pinned down independently of all other parameters by one target. The risk free rate r is set to 1.5% to match the average quarterly real return on 5 year U.S. T-bills in the last 35 years. The parameter γ is set equal to 2, a common value in the open economy literature. The discount factor $\beta = (1+r)^{-1}$. The endowment process is estimated using data from the Ministry of Finance of Argentina from 1980Q1 to 2006Q4 which are log seasonally adjusted real GDP. The data is detrended using the HP-filter with parameter 1600. The estimated values are $\rho_y = 0.86$ and $\sigma_\eta = 0.024$. The estimated values are used to derive a finite First Order Markov process by the Tauchen-Hussey method. The number of grid points for the endowment is set equal to 21. The additional loss of output in autarky, λ is set at 2% consistent with the finding of Sturzenegger (2002) that estimates the percentage of output contraction after default using a panel of 100 countries. This value of output loss during period of exclusion is also used in the papers of Aguiar and Gopinath (2006) and Yue (2006).

A second group, $\{\delta_m, \pi_a, \pi_m, \theta\}$ is jointly estimated to match target statistics of the Argentinean economy and recent default episodes. This is done by the Simulated Method of Moments. This procedure consists of minimizing the distance between data moments and moments extracted from the simulated model. That is, the parameters are chosen to minimize

$$L(\Theta) = [M^d - M^s(\Theta)]W^*[M^d - M^s(\Theta)]'$$

with respect to parameters Θ, where M^d are the moments from the data, $M^s(\Theta)$ are the moments from the simulated model at parameters Θ. W^* is positive definite and optimally derived weighting matrix. Given the potential for discontinuities in the model and the discretization of the state space, I used a simulated annealing algorithm to perform the optimization. Standard errors are computed from the derivative of the objective function with respect to the structural parameters evaluated at the point estimates. [11]

[11] See Gourieroux and Monfort (1996) and Ingram and Lee (1991). The appendix provides an explanation of the

The moments used in the estimation are the default frequency, the standard deviation of the current account to output ratio, the average period in the state of default and the ratio of the standard deviation of consumption to the standard deviation of output. Reinhart, Rogoff and Savastano (2003) report four episodes of sovereign defaults in Argentina's external debt from 1824 to 1999. In 2001, Argentina defaulted a fifth time on its external debts, making its average default frequency 2.78% or 0.68% quarterly. This value is similar to the average for emerging economies reported in Table 1. The current account to output ratio is computed for the period 1980q1 to 2006q4. The standard deviation is equal to 1.38%. The average period in the state of default is computed using data from Standard and Poor's (2007). This value is found to be equal to 4.33 quarters and it is also consistent with the estimates of Gelos et al. (2002), who use a large sample of emerging economies to find that during default episodes in the 1990's, economies were excluded from the credit markets for about a year. Many have documented that consumption is more volatile than output in emerging economies (see for example Neuymeyer and Perri (2005)). The relative volatility of consumption $\sigma(c)/\sigma(y)$ in Argentina for the period 1980Q1-2006Q4 is equal to 1.12.

These moments were chosen mainly for two reasons. First, it is important that the moments are informative. For instance if one of the moments is independent of the parameterization of the model, we are left to estimate an under-identified model. The size of the standard errors show that the model is identified. Second, and in order to make a fair comparison of the results in this paper to those in previous papers in sovereign debt literature the moments chosen are similar to the moments used in the calibration of Arellano (2007), Aguiar and Gopinath (2006) and Yue (2006).

4.3 Computation

Computation of the equilibrium requires three steps: an inner loop, where the decision problem of the government given parameter values, prices, the recovery schedule and posteriors is solved; a middle loop, where prices, the recovery schedule and posteriors are obtained; and an outside loop or estimation loop where parameter values that yield equilibrium allocations with the desired (target) properties are determined (the appendix contains a detailed explanation of each portion of this algorithm). To solve the model I use the discrete state space method. Posterior functions need to be defined for every element of the debt state space. The computational task is extremely burdensome, each equilibrium requires computing thousands of equilibrium loan prices and posteriors. Moreover, the existence of an equilibrium is not always guaranteed and this makes the estimation even more difficult.

To resemble what we observe in international credit markets, sovereign ratings are restricted to take a finite number of values, i.e. $s \in S \equiv \{s_1, s_2, \ldots, s_N\}$. Recall that the future score was specified in equations (4) and (5) and given by functions $s' = \Psi^{In}(b', d, b, y, s)$ and $s' = \Psi^{Out}(z, b, y, s)$. The

estimation procedure and how to obtain the optimal weighting matrix and standard errors.

technological assumption I make here is that the beginning of next period sovereign rating is

$$\hat{\Psi}^{In}(b', d, b, y, s) = \arg\min_{s_i \in S} |\Psi^{In}(b', d, b, y, s) - s_i|, \text{ when } h = In,$$

and

$$\hat{\Psi}^{Out}(z, b, y, s) = \arg\min_{s_i \in S} |\Psi^{Out}(z, b, y, s) - s_i|, \text{ when } h = Out.$$

In words, this is just saying that future scores are assigned to the grid point that is closest in absolute value. Given this technological assumption, in the computation of the model, $\hat{\Psi}^h$ is substituted everywhere I have Ψ^h. The value of N is set to 20 as in the available scales for sovereign ratings by Standard & Poor's and Moody's. Note that $s_1 = (1 - \pi_m)$ and $s_N = \pi_a$. Moreover, the Markov process for political states implies that the average value of s in the long-run is equal to π^*.

4.4 Estimated Parameters and Model Moments

Table 4 displays the parameters values that can be pinned down independently of all other parameters by one target. The estimated parameters as well as their standard errors and the model moments are reported in Table 5. The optimal weighting matrix in the SMM procedure is derived from the variance-covariance matrix of the moments.[12]

Table 4: Model Parameters: Independent Targets

Parameter		Value	Moment
Risk free interest rate	r	0.015	Q. real return US T-bill
Discount Factor	β	0.9852	$(1+r)^{-1}$
Risk aversion	γ	2	
Autocorrelation Endowment	ρ	0.86	Argentina Output 1980-2006
Std. Dev. Error	σ_η	0.024	Argentina Output 1980-2006
Output Loss	λ	0.02	Observed output loss

The discount factor of the misaligned government is found to be only 16% lower than that of the aligned government. The degree of impatience reflects some political instability in Argentina during this period. The values of π_a and π_m imply that in the stationary equilibrium a government will be of the aligned type approximately 76% of the time. Finally, the bargaining power, θ, is estimated to be 0.58 which shows that Argentina has a more favorable position in debt renegotiation than international investors.

[12]See the appendix for a comprehensive explanation.

Table 5: Estimated Parameters and Moments

Parameter		Value	Std. Error
Discount Factor $m-type$	δ_m	0.842	0.007
Probability $a-type$	π_a	0.931	$3.02e-05$
Probability $m-type$	π_m	0.776	0.019
Bargaining Power	θ	0.588	$8.23e-04$

Moment	Data (%)	Model (%)
Default Probability	0.68	0.65
Std. Dev. Current Account/Output	1.38	1.46
Average exclusion period (quarters)	4.33	4.07
Relative volatility of consumption $\sigma(c)/\sigma(y)$	1.12	1.16

The model is able to generate the default frequency observed in the data, as well as the standard deviation of the current account to output ratio. In equilibrium, the length of the period of exclusion and the relative volatility of consumption to output are consistent with what we observe in emerging economies. In the following section I discuss what these parameters imply for the debt to output ratio and how the differ from previous models of sovereign debt.

5 Resolution of the Debt to Output Puzzle

Recent models of sovereign debt explained how the risk of default interacts with business cycles in emerging economies (see Aguiar and Gopinath (2006), Arellano (2006) and Yue(2006) for example). However, at the observed default frequency, the level of debt to output ratio generated by these models was much lower than what we observe in the data. In particular, the debt to output ratio[13] was found to be 18% in Aguiar and Gopinath (2006), 7.33% in Arellano (2006) and 9.69% in Yue (2006). When computed from the the Global Development Finance *II* data set (compiled by the World Bank) the average ratio of Government Debt to GNP in Argentina since 1980 is 36.7%. The aim of this paper was to close this gap by introducing political uncertainty in a standard small open economy model. Effectively, the debt to output ratio computed at the estimated equilibrium is 33.8%, that represents 92% of that observed in the data. Note that the debt to output ratio was not part of the set of moments used in the estimation.

[13] These models were also calibrated to the Argentinean economy.

Table 6: Main Result

	Data	Model	Aguiar and Gopinath	Arellano	Yue
Debt to Output Ratio (%)	36.7	33.8	18.0	7.3	9.7

The combination of private information and the endogenous repayment schedule are able to account for the high debt to output ratio observed in the data. The information structure and the endogenous determination of default penalties (recovery rates and periods of exclusion) are the keys to understanding the intuition behind the main result. In the next section, I will show how the decisions of the government affect its reputation in credit markets and hence the price of new loans. In summary, a patient government may be able to get lower interest rates by choosing not to default and signal its type during bad times of the economy. Similarly, if the government has an unresolved default the aligned type is willing to repay sooner than the misaligned type to go back to financial markets with a higher reputation. Moreover, at some rating levels, a less patient government can mimic the behavior of the patient government and borrow at low interest rates. All these factors combine to generate the necessary incentives to make borrowing more attractive than defaulting. Furthermore, the introduction of endogenous recovery rates and periods of exclusion makes transiting through a default state much more costly for any government type and it has two effects over loan prices. First, at a given debt level, interest rates decrease because default probabilities are lower than in a world with costless return to markets. Second, the introduction of endogenous recovery rates makes borrowing more appealing because governments are able to obtain contingent repayments after a default.

The estimated parameters are in line with those used in previous studies of sovereign debt. In this paper, the aligned government discounts the future at a rate equal to 0.985 and the misaligned government discounts the future at a rate equal to 0.801 ($= .842\beta$). The discount factors used in the studies of Arellano (2007), Aguiar and Gopinath (2006) and Yue (2006) are 0.953, 0.80 and 0.74 respectively. Arellano (2007) and Aguiar and Gopinath (2006) do not consider the possibility of renegotiation and periods of exclusion are exogenous. The calibrated probability of returning to capital markets after a default implies that countries spend approximately 3.84 quarters in autarky for the case of Arellano (2006) and 2.5 years in the case of Aguiar and Gopinath (2006). Yue (2006) considers debt renegotiation and also finds that Argentina has a higher market power (equal to 0.83) than lenders at the renegotiation stage. However, that model implies that countries stay in financial autarky on average only one quarter, a period much shorter than that observed for emerging economies. The estimation of the benchmark model developed in this paper implies that the country stays in financial autarky for about 4 quarters and the bargaining power of Argentina is 0.588.

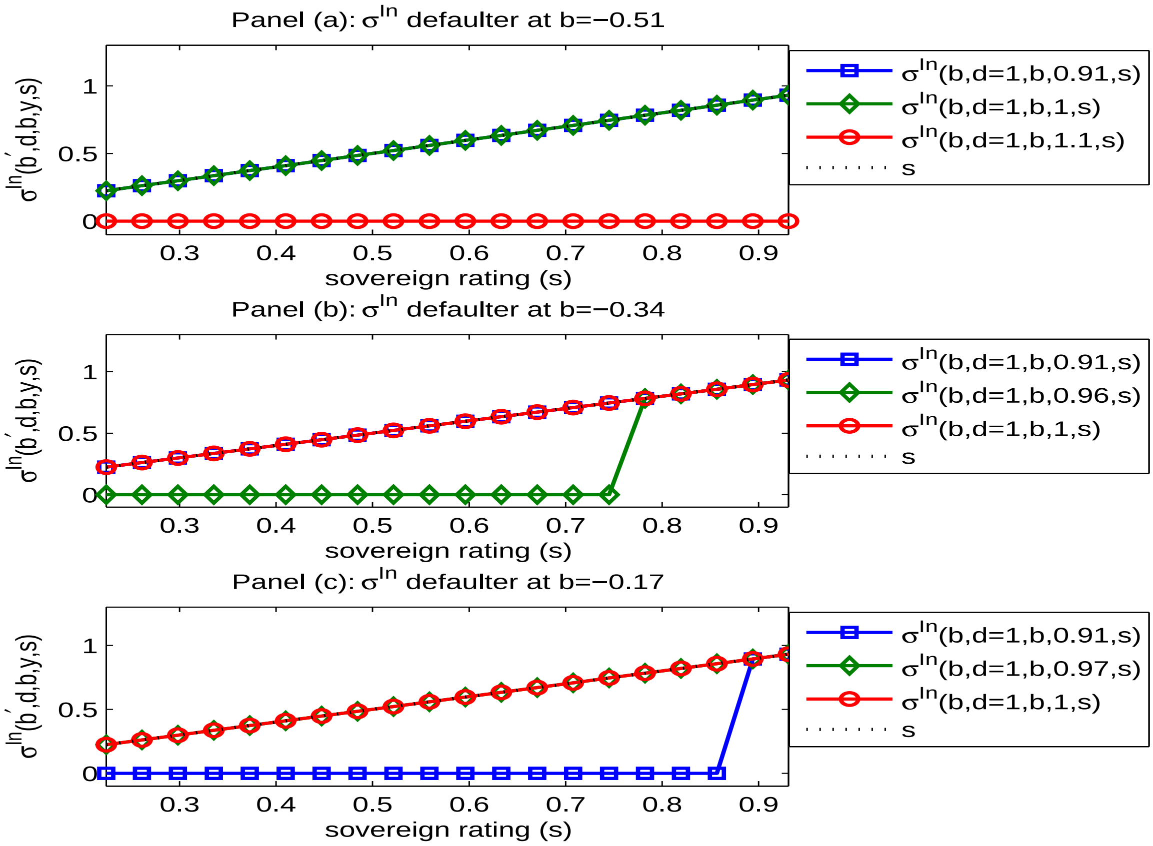
In Section 7, I show that the combination of private information and endogenous recovery rates are essential ingredients to sustain the observed debt to output ratios. A model with full information and zero recovery rates generates an equilibrium debt to output ratio that is only 11% (which is 33% of the value in the data). In addition, a model with full information and endogenous recovery generates a debt to output ratio that is still only 15% (approximately 40% of the value for Argentina).

6 Type Probabilities as Sovereign Ratings

Sovereign ratings are a key determinant of the interest rate a country faces. In the introduction, I described the relevance of sovereign ratings in international markets and in particular how they affect the loan prices for emerging economies. If the type probabilities generated by the model are consistent with the sovereign ratings observed in the data the following need to be true in equilibrium: (i) The type probability s drops after a default. (ii) The government increases its reputation after renegotiating and repaying and old debt. (iii) If the government takes on more debt s decreases. This amounts to asking, for example, if in equilibrium it is true that $D_a(b,s) \subseteq D_m(b,s)$ and $Z_a(b,s) \supseteq Z_m(b,s)$. Then, from the perspective of lenders, the type probability s (the probability that the government is of the $a-type$) can be identified with a sovereign rating.

I will start by describing the function $\sigma^{In}(b',d,b,y,s)$ in the case of a default $d=1$. It is important to start here because what a government reveals by its decisions will be the key determinant of the interest rate it faces. Figure 3 displays the posterior function $\sigma^{In}(b,d=1,b,y,s)$ as a function of the sovereign rating for different bond levels and evaluated at different income levels. From this figure we note that $\sigma^{In}(b,d=1,b,y,s) \leq s$ for all (b,y,s). Thus, a government contemplating to default recognizes that this will lower its rating and presumably raise its future interest rates. Panels (a), (b) and (c) of Figure 3 show the function σ^{In} after a default for different debt and income levels. In Panel (a) of Figure 3, we observe that for debt levels $b=-0.51$ (a debt level 50% higher than average debt) because both type of governments choose to default when income is equal to 0.91 or 1 (lower or equal to average income), the posterior probability that the government is of the $a-type$ equals the prior s. When income is higher than its average, $y=1.1$, only the misaligned government defaults and the rating drops to its lowest value after breaking the debt contract. In particular, $\sigma^{In}(b,d=1,b,1.1,s)=0$ for all s. The decision of no-default allows the aligned government to perfectly signal its type and increase its reputation.

Figure 3: Sovereign Ratings and Default

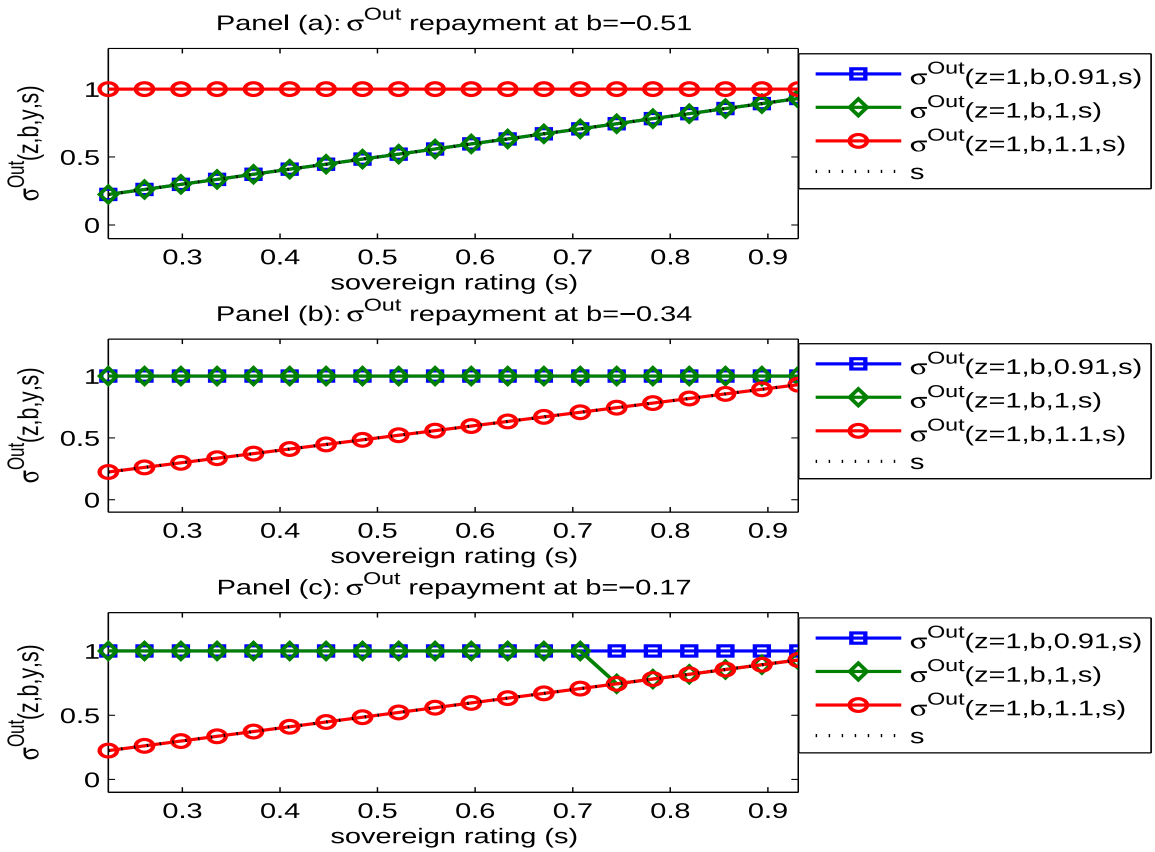


Panel (b) of Figure 3 shows the posterior probability σ^{In} at average debt $b = -0.34$. When income is at its lowest possible value $y = 0.91$ both government types choose to default and $\sigma^{In}(b, d = 1, b, y, s) = s$. As income rises, reputation provides the incentives for the aligned type government to choose not to default. The $a - type$ government chooses not to default for any s and the $m - type$ government chooses to default for $s \in [1 - \pi_m, 0.74]$. In this case $\sigma^{In}(b, d = 1, b, y, s \in [1 - \pi_m, 0.74]) = 0$ and the government that defaults is believed to be of the $m - type$. When $s > 0.74$ as well as when $y = 1$ for every s, none of the government types choose to default, so posteriors are defined off-the-equilibrium path and set equal to s, i.e. $\sigma^{In}(b, d = 1, b, y, s) = s$. Finally, Panel (c) of Figure 3 shows the posterior probability σ^{In} at $b = -0.17$ (50% of average debt). When income is low, $y = 0.91$, only the $m - type$ government defaults for $s \leq 0.85$ and $\sigma^{In}(b, d = 1, b, y, s \in [1 - \pi_m, 0.85]) = 0$. For higher sovereign rating levels none of the government types choose to default and $\sigma^{In}(b, d = 1, b, y, s \in (0.85.\pi_a]) = s$ as defined by the off-the-equilibrium assumption on posteriors. Similarly, when $b = -0.17$ and income is higher, $y = 1$ and $y = 1.1$, the posterior is defined off-the-equilibrium path.

Data on sovereign ratings show that countries increase their ratings after repayment. Also, if a government chooses not to repay, the rating is kept as its lower level. From Figure 4 we can infer that this is also an equilibrium outcome. In particular, $Z_m(b, s) \subseteq Z_a(b, s)$, i.e. the $a - type$

government is prone to repay more often than the $m-type$. Figure 4 displays the posterior function $\sigma^{Out}(z=1,b,y,s)$, for a government that chooses to renegotiate and repay a previously defaulted debt. As it is clear from Panels (a), (b) and (c) $\sigma^{Out}(z=1,b,y,s) \geq s$ for all (b,y,s) and in most cases, an align government will signal its type by resolving an old default.

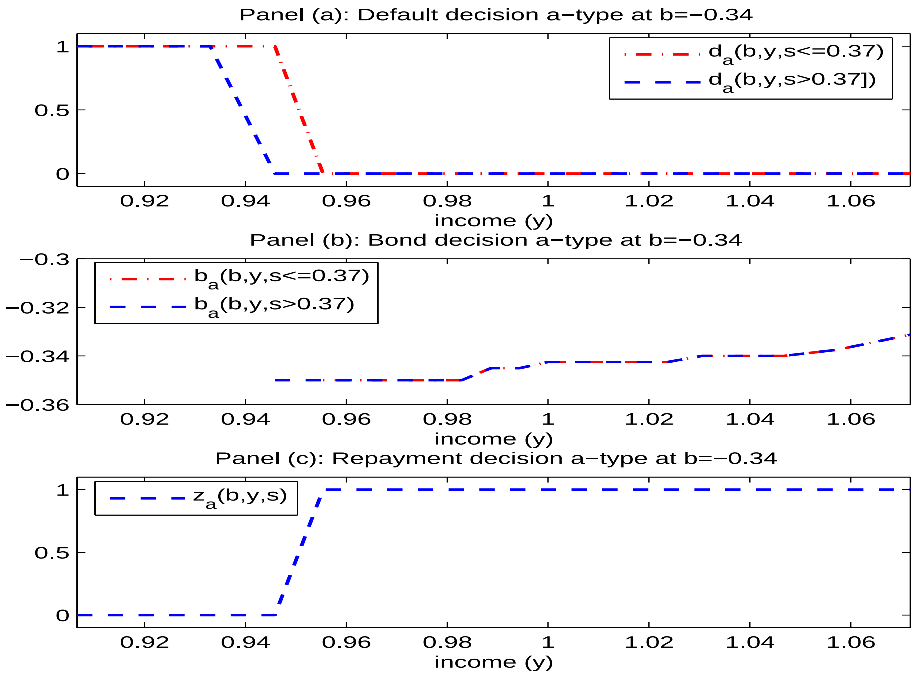
Figure 4: Sovereign Ratings and Renegotiation



Panel (a) of Figure 4 shows that when the debt level is 50% higher than average debt, $b = -0.51$, only the aligned type government chooses to repay. However, repaying is its optimal choice only for $y = 1.1$. In this case the repayment decision signals the government type perfectly and $\sigma^{Out}(z = 1, -0.53, 1.1, s) = 1$ for all s. From Panel (a) it is also possible to see that the posterior function is defined off-the-equilibrium for $y = 0.91$ and average income $y = 1$ for every s, i.e. $\sigma^{Out}(z = 1, -0.51, y \in \{0.91, 1\}, s) = 1$ because neither the $a - type$ nor the $m - type$ government chooses to repay. From Panel (b) of Figure 4 we observe that, when debt is equal to average debt, $b = -0.34$, at low and average income levels ($y = 0.91$ and $y = 1$ respectively) only the aligned type government repays and $\sigma^{Out}(z = 1, -0.34, y \in \{0.91, 1\}, s) = 1$. When income is higher, both government types would prefer to repay, so, in equilibrium, the posterior function is $\sigma^{Out}(z = 1, -0.34, 1.1, s) = s$. Panel (c) shows that when $b = -0.17$ (50% of average debt) only the aligned government renegotiates and repays when $y = 0.91$ for all s and $(y = 1, s \leq 0.71)$ implying that $\sigma^{Out}(z = 1, -0.17, y, s) = 1$. For other combinations of income and ratings, both government types choose to renegotiate and repay and hence $\sigma^{Out}(z = 1, -0.17, y, s) = s$.

The interaction between default, borrowing and repayment is crucial to determine the value of sovereign ratings. How much debt the government takes and when it decides to borrow affects the rating function and the prices the government face. In Figures 5 and 6 I plot the default, borrowing and repayment decision rules for both government types at average debt, $b = -0.34$, as a function of income and rating. Figure 5 corresponds to the $a - type$ type government and Figure 6 to the $m - type$.

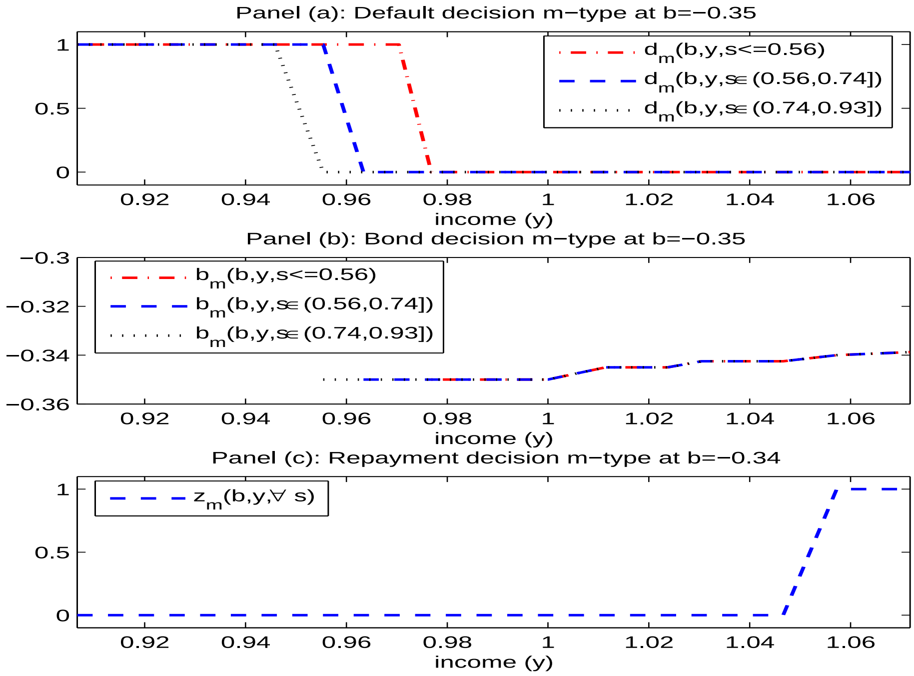
Figure 5: Decision Rule $a - type$



Panel (a) of Figures 5 and 6 shows that for low levels of income both government types decide to default when $b = -0.34$ and this decision depends on the level of sovereign rating. First, when the sovereign rating is less than 0.37 the $a - type$ government defaults for $y \in 0.91, 0.95$. For $s \in (0.37, 0.93]$ it chooses to default only for income levels below 0.94. The $m - type$ decision rule shows a similar dependence on sovereign rating. For $s \leq 0.56$ the misaligned government chooses to default for $y \leq 0.97$. When $s \in (0.56, 0.74]$ the $m - type$ government defaults for $s \leq 0.96$ and for higher ratings ($s > 0.74$) it defaults for $y \leq 0.95$.

Panel (b) of Figures 5 and 6 shows the borrowing decision rule for the $a - type$ and the $m - type$ government respectively. The dependence of the borrowing decision on the sovereign rating reflects the fact that the default decision also depends on it because the government is able to borrow only in periods that it chooses not to default. We observe that $b_a(b, y, s) leb_m(b, y, s)$ for all (y, s) and this will have important implications for sovereign ratings (see Figure 7 below).

Figure 6: Decision Rule $m - type$

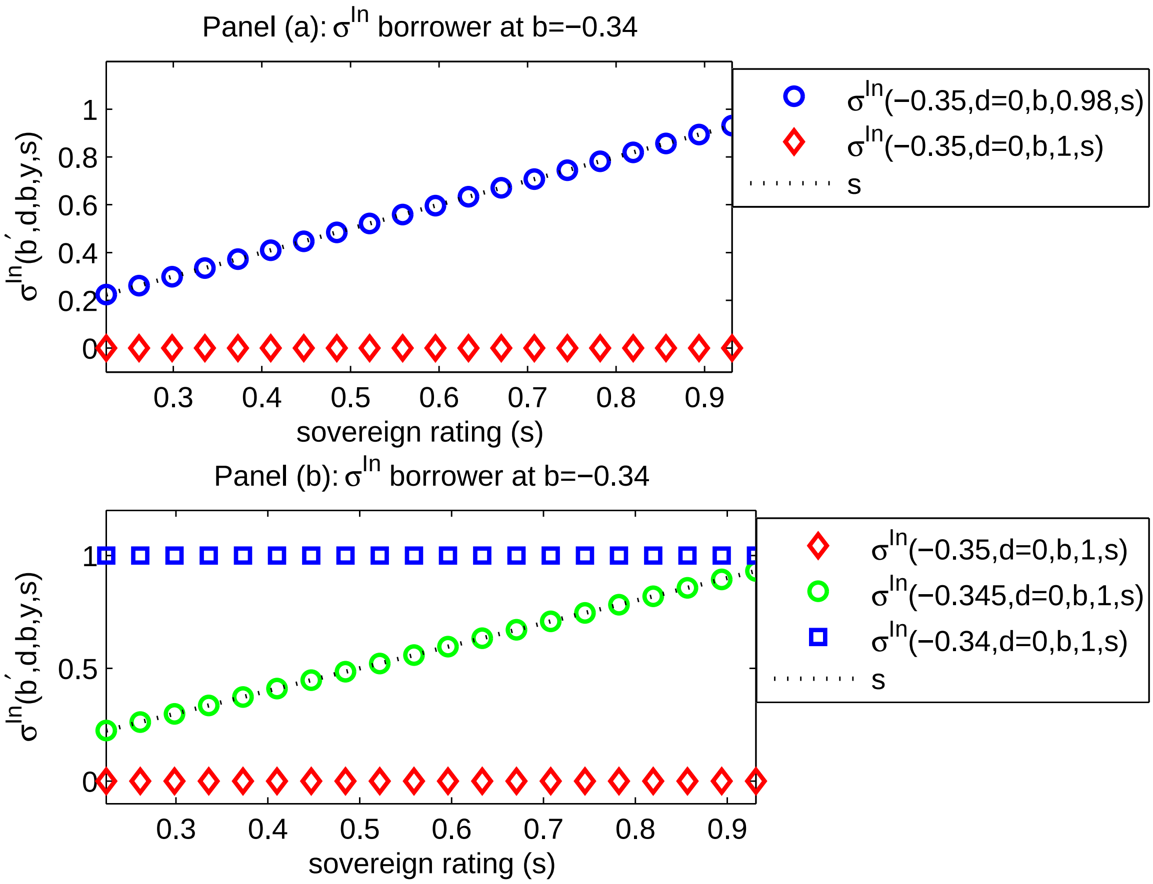


Panel (c) of Figures 5 and 6 displays the repayment decision rules for the $a - type$ and the $m - type$ government over income. As it was evident previously in Figure 4, the repayment set of the $a - type$ government includes the repayment set of the $m - type$, i.e. $Z_m(b, s) \subseteq Z_a(b, s)$. Moreover, incentives for repayment are higher in good times. This is consistent with the data presented in Kovrijnykh and Szentes (2007) on the timing of debt repayment. When $b = -0.34$ an $a - type$ government will choose to repay for income higher than 0.95. The $m - type$ government prefers to repay when $y > 1.045$.

Evidence on merging economies shows that the borrowing decision also affects the value of the future rating. In particular, taking on more debt negatively impacts the sovereign rating. Figure 7 shows that the model is also consistent with this fact. Panel (a) shows how the posterior function responds to the same level of borrowing but at different income levels. When income is $y = 0.98$ (below average) both government types choose the same level of borrowing. In this case, the $m - type$ government is able to mimic the $a - type$ government and maintain its reputation $\sigma^{In}(-0.35, d = 0, b = -0.34, y = 0.98, s) = s$ for all s. When income equals its average $y = 1$, only the $m - type$ government chooses $b' = -0.35$ and $\sigma^{In}(-0.35, d = 0, b = -0.34, y = 1, s) = 0$ for all s.

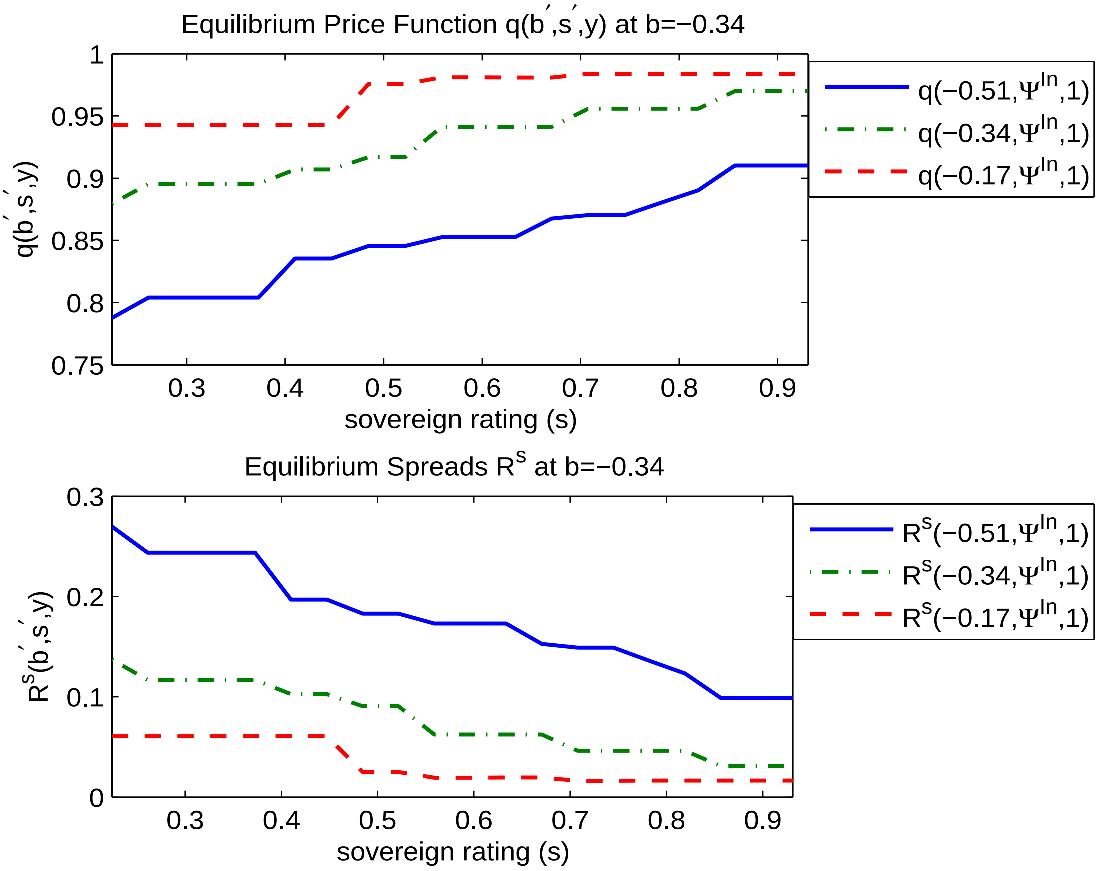
In Panel (b) of Figure 7 we observe how the posterior is affected by different borrowing levels evaluated at average income ($y = 1$). When $b' = -0.35$, the posterior takes value $\sigma^{In}(-0.35, d = 0, b = -0.34, y = 1, s) = 0$ because only the $m - type$ chooses this action. This government obtains a higher consumption level today at the cost of loosing its reputation. If $b' = -0.34$, $\sigma^{In}(-0.34, d = 0, b = -0.35, y = 1, s) = 1$ and the $a - type$ government perfectly signals its type. In this case, a lower amount of debt allows the government to build a reputation. When $b' = -0.345$ the posterior equals the prior because this is an off-the-equilibrium path outcome. In the data we observe that ratings are usually negatively correlated with the level of borrowing.

Figure 7: Posterior Function for Non-Defaulter



Next I plot the equilibrium bond price as a function of current rating. Recall that in equilibrium the current debt level and sovereign rating affect loan prices through the posterior function, i.e. $q(b', s' = \Psi^{In}(b', d = 0, b, y, s), y)$. Panel (a) of Figure 8 shows the bond price as a function of the sovereign rating s for different borrowing levels, b', evaluated at mean income and when current debt equals mean debt ($b = -0.34$). Panel (b) of Figure 8 shows how different bond prices over sovereign ratings translate into sovereign spreads $R^s \equiv 1/q - (1 + r)$, defined as the difference between the country interest rate and the risk free rate.

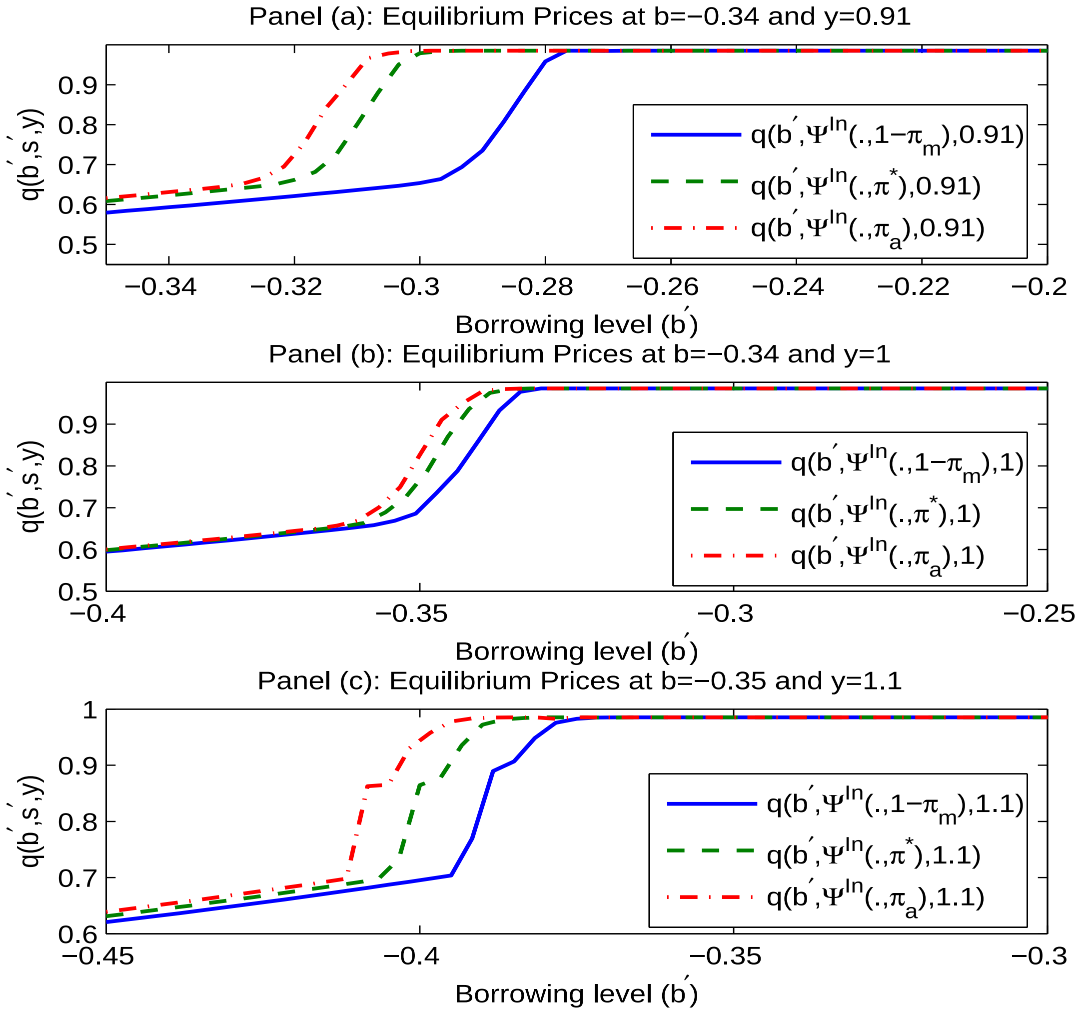
Figure 8: Sovereign Ratings and Loan Prices



The model generates a relation between sovereign ratings and spreads that is consistent with the data. Higher ratings imply lower spreads and viceversa. This relation between sovereign ratings and bond prices is important in creating the incentives to sustain higher debt levels in equilibrium because $a{-}type$ governments are able to obtain a cheaper interest rate by signaling its type. Another important property of the equilibrium is that at a given s, if default is optimal for a government of type i at b^1 with $b^1 > b^0$, then default is also optimal for b^0. That is, $D_i(b^1, s) \subseteq D_i(b^0, s)$. This implies that $p(b^0, s, y) \geq p(b^1, s, y)$ and $q(b^0, s, y) \leq q(b^1, s, y)$.

For completeness, in Figure 9, I show the price schedule as a function of future debt levels when $b = -0.34$ for different income levels and sovereign ratings. Panel (a), (b) and (c) correspond to low ($y = 0.91$), average ($y = 1$) and high income ($y = 1.1$) respectively.

Figure 9: Price Function for different income and rating levels

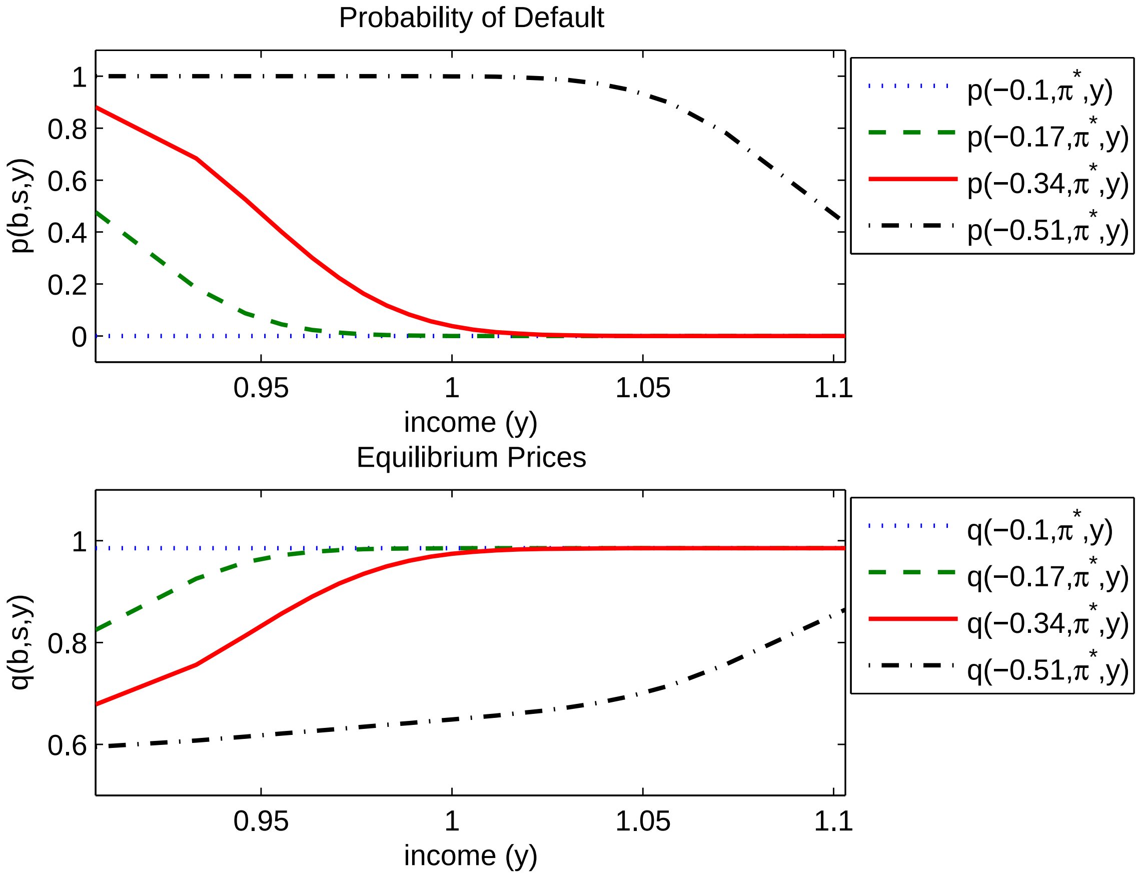


Higher ratings have value because they affect the price at which the governments borrow. Within each panel, the price function is plotted for $s \in \{1 - \pi_m, \pi^*, \pi_a\}$ that correspond to the lowest, the average and the highest sovereign rating. The relation between sovereign ratings and interest rates is also evident from this picture. A higher rating implies a lower interest (whenever the interest rate is not equal to $(1 + r)^{-1}$). From this figure we can see that as income increases the price for new loans also increases (interest rates decrease).

Default episodes observed during the last decade were associated with severe economic crisis. Figure 10 shows that in the model, default incentives are stronger during low income realizations. Panel (a) of this figure displays the default probability over income levels for different combinations of debt, evaluated at average score, π^*. When debt is $b = -0.51$, the probability of default is equal to one for most income levels. This impacts the bond price (see Panel (b) in Figure 10). At the average debt level $b = -0.34$ the default probability is strictly decreasing in y for $y \leq 1 = E[y]$. After that, the probability of default is equal to zero. This is consistent with actual debt levels in

Argentina. The government debt to output ratio in Argentina was 34% at the end of 2001 when output collapsed, interest rates went up and the government defaulted. After restructuring the debt, the government debt to output ratio was again 35% at the end of 2005 but output was above trend and bond prices were low (sovereign spreads were around 3.5%).

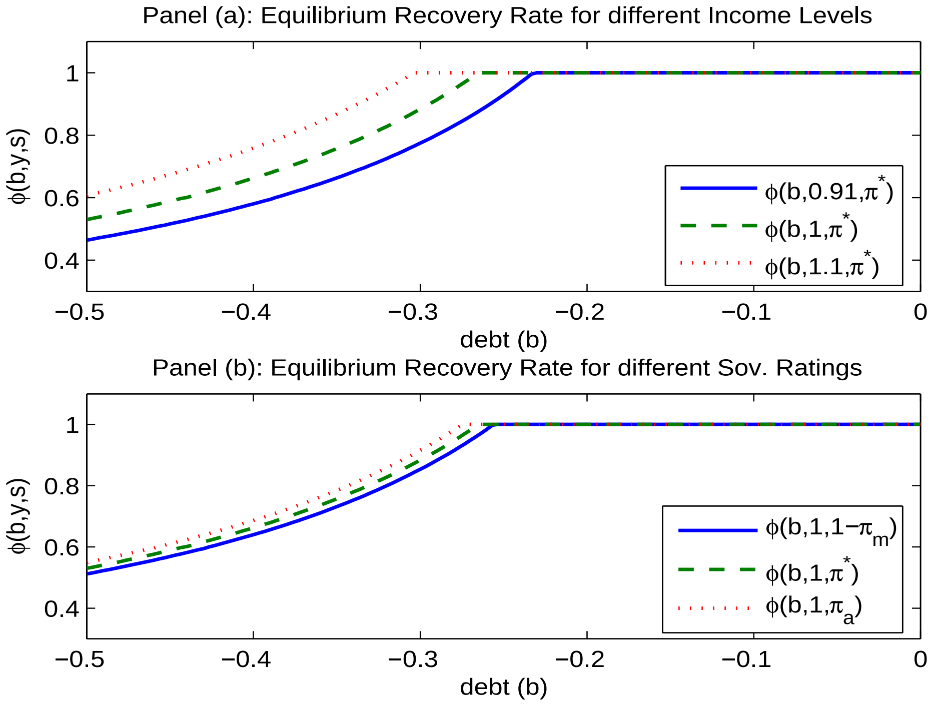
Figure 10: Default Incentives and Income



The introduction of endogenous recovery has important implications for resolving the sovereign debt puzzle. The country incentive to default depends on the renegotiation agreement on debt reduction. As in Yue (2006), it can be shown that for a given bargaining power θ, there exists a threshold $\bar{b}(y,s)$ such that the equilibrium recovery function ϕ satisfies

$$\phi(b,y,s) = \begin{cases} \frac{\bar{b}(y,s)}{b} & \text{if } b \leq \bar{b}(y,s), \\ 1 & \text{if } b > \bar{b}(y,s). \end{cases}$$

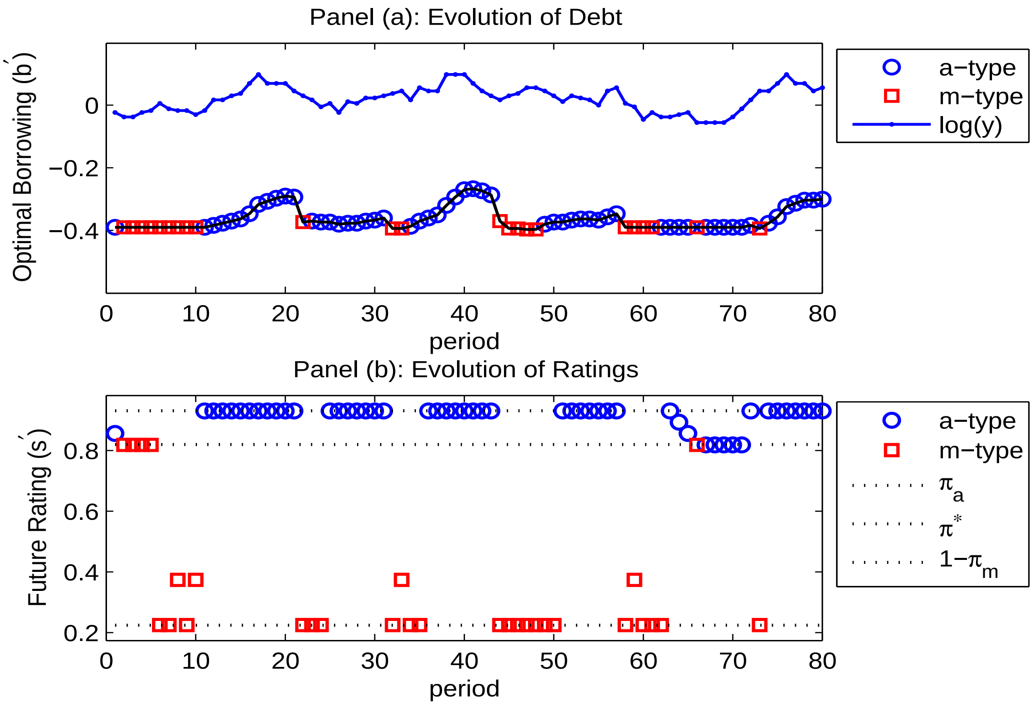
Figure 11: Recovery Schedule



Hence, debt recovery rates decrease with the amount of defaulted debt, and there is no debt reduction for debt levels smaller than the threshold. Observations from recent sovereign defaults are consistent with the equilibrium recovery schedule. Panel (a) of Figure 11 shows the equilibrium recovery schedule as a function of debt and income evaluated at π^*. The resulting recovery rate is increasing in the level of income. A novelty of this model is the relation between recovery rates and the sovereign rating. Panel (b) of Figure 11 displays the recovery rate evaluated at average income for different values of s. We can note that the recovery rate is increasing in s. In summary, the bargaining mechanism generates that repayment incentives are stronger in good times, debt reduction is lower as debt levels decrease and that countries that defaulted on a bigger debt stay longer in financial autarky. The average recovery rate is equal to 59%. The average recovery rate for emerging economies computed from Chuhan and Sturzenegger (2005) and Moody's (2006) report on recovery rates for defaults in the last decade is found to be 59%.

To conclude this section, in Figure 12, I show the evolution of debt and ratings in a simulation of 100 periods with no defaults. In Panel (a), circles correspond to the borrowing decision $b'_a(b, y, s)$ of the $a-type$ government. In Panel (b), circles correspond to the sovereign rating obtained by an $a-type$ government after the borrowing decision $s' = \Psi^{In}(b', d = 0, b, y, s)$. Similarly, in Panels (a) and (b), squares correspond to debt levels $b'_m(b, y, s)$ chosen by an $m-type$ government and ratings $s' = \Psi^{In}(b', d = 0, b, y, s)$ obtained by an $m - type$ government respectively. From Panel (a), we observe that the $m-type$ government generally borrows more than the $a-type$ government. Panel (b) show how fast lenders learn about the government type. The process for types implies that the score will tend to return to the mean π^* if no information can be extracted from the government actions, i.e. $\sigma^{In}(b', d = 0, b, y, s) = s$. Cycles of signaling and confusion alternate over time.

Figure 12: Evolution of Debt and Ratings



7 Why Private Information is Important

To be able to disentangle the effects of private information and endogenous recovery rates, in this section, I explore how the main results would change if two extensions of the benchmark model were considered. Both extensions share the characteristic that government types are public information. That is, in this environment, foreign lenders are able to observe the government type and the type changes over time. The first case corresponds to a model were recovery rates are zero and return to markets is exogenously given (as in Arellano (2006) and Aguiar and Gopinath (2006)). After a default countries are able to borrow again in the following period with probability α and with probability $(1 - \alpha)$ they stay one more period in financial autarky. In the second case, I consider a model also with full information but with endogenous recovery rates (as in Yue (2006)). In both cases I estimate the parameters of the models to match the moments used in the benchmark case. I develop the models with observable types and the equilibrium notion in the appendix. The equilibrium price function and the renegotiated repayment fraction will depend on the government type and not in the sovereign rating. In fact, the sovereign rating does not have any meaning in a context with full information.

7.1 Full Information Models: Estimation and Results

The risk free rate and the parameters of the endowment process are the same of the benchmark model. The output cost λ is set equal to 2% as in the benchmark model. The parameters left are estimated through Simulated Method of Moments as described before. The set of moments used is similar to that used for the benchmark model and depicted in Table 5. For the case of zero recovery rates the set of parameters to define are $\{\delta_m, \pi_a, \pi_m, \alpha\}$. These parameters are set in order to match the default probability, the standard deviation and the mean of the current account to output ratio. The value of α is chosen in order to obtain an average period of financial exclusion consistent with the data. For the case of endogenous recovery rates the set of parameters to define are $\{\delta_m, \pi_a, \pi_m, \theta\}$. These parameters are chosen in order to match the default probability, the standard deviation of the current account to output ratio, the mean recovery rate and the average period of financial exclusion.

The discount factor of the misaligned government is found to be similar in the three models. The model with full information and endogenous recovery rates predicts that the country will have an aligned government around 80%. For the case of the model with zero recovery rates this value is approximately 74%. The bargaining power of the government do not present major differences across the models either.

Table 7 displays the parameters values and the moments generated by the models.

Table 7: Effects of Private Information and Recovery Rates

		Benchmark	Full Info Zero Recovery	Full Info Endog. Recovery
Parameters				
δ_m		0.842	0.811	0.857
π_a		0.931	0.880	0.894
π_m		0.776	0.655	0.592
θ		0.588	–	0.694
α		–	0.231	–
Moments	**Data**			
Default probability	0.68	0.65	0.51	0.56
Std Dev CA/Y	1.38	1.46	1.25	1.52
Average exclusion period	4.33	4.07	4.33	2.8
$\sigma(c)/\sigma(y)$	1.12	1.16	1.02	1.13
Debt to Output ratio	36.7	33.8	11.5	15.1

As it is clear from the last row of Table 7, the debt to output ratios that models with full information predict are much lower than in the data. The model with zero recovery rates generates a debt to output ratio that is 1/3 of that in the data. Including endogenous recovery rates and periods of exclusion generates a higher debt to output ratio but still only 40% of the equilibrium ratio for Argentina.

When the political states are observable, reputation has no value. The main motivation for the borrowing and default decisions is consumption smoothing. In that case, the misaligned government will tend to default more often and because government types are known it will be charged a much higher interest rate than the aligned government. Moreover, for the aligned government it is not necessary to change its behavior in order to signal its type. This lowers the benefits from not defaulting in bad times.

8 Comparison to the Data: Business Cycle moments

The business cycle moments for emerging economies are well documented and the results obtained in this paper are consistent with previous studies.[14]

Using data from Argentina, I found that consumption and output are highly correlated ($corr(y, c) = 0.97$) and the correlation between the current account to output ratio, ca/y and output is -0.82. The data on output, consumption and current account is taken from the Ministry of Economics in Argentina for the period 1980q1 to 2006q4. Variables are real and seasonally adjusted. Output and consumption are detrended using the HP filter with parameter 1600. The data on spreads is computed from J.P. Morgan's Emerging Markets Bond Index (EMBI+) for Argentina from 1993 to 2001. The sovereign spread is the difference between the Argentinean interest rate and the real return on a U.S. Treasury bill of similar maturity. The standard deviation of spreads is 3.07% and the average spread is 7.75%. If the statistics on spreads are derived from 3-year foreign currency denominated bonds from 1993 to 2001 taken from Broner, Lorenzoni and Schmukler (2004) the value of the standard deviation is 1.69% and the average spread is 4.08%. The rating data is computed using the Standard & Poor's rating for Argentina since its first date available, 1993q3, to 2006q4. The correlation of ratings and output is 47% and the correlation with spreads is -81%.

Table 8 displays the moments from the data and those from the simulated model. The model generates business cycle statistics that are in line with the data. In particular, consumption is highly correlated with output and the current account is countercyclical.

[14] See Neumeyer and Perri (2005)

Table 8: Non-Target Moments

Moment	Data	Model	Arellano (2007)	Aguiar & Gopinath (2006)	Yue (2006)
Corr(y, c) (%)	97	92	91	97	n.a.
Corr($ca/y, y$) (%)	-82	-17	-15	-12	-14
EMBI Std. Dev. (%)	3.07	1.85	10.6	0.12	1.32
EMBI Mean Spread (%)	7.75	2.09	10.4	0.57*	1.84
Std. Dev. short-term spread(%)	1.68	1.85	10.6	0.12	1.32
Mean short-term spread (%)	4.08	2.09	10.4	0.57*	1.84
Corr($y, spread$) (%)	-76	-20	-22	-2	-18
Corr($ca/y, spread$) (%)	75	21	17	38	54
Corr($y, rating$) (%)	47	19	n.a.	n.a.	n.a.
Corr($spread, rating$) (%)	-81	-62	n.a.	n.a.	n.a.

*Max value is reported only

The model delivers a standard deviation of spreads that is consistent with short-term bond data but a lower mean spread (even when compared with short-term bonds). However, this anomaly of the model has been present in previous studies as well. Arellano (2006) obtains a much higher mean spread but at the cost of a standard deviation that is more than 3 times of that observed in the data. Aguiar and Gopinath (2006) only report the maximum value of spread recorded in their simulation and even so this value is only 13% of the mean of short term spreads and 7% of the average EMBI+. Yue (2006) reports mean spreads than are also lower than in the data.

The benchmark model is able to capture the countercyclical behavior of interest rates and the positive correlation between the current account and spreads. In particular, the correlation of income and spreads is -20 percent and the correlation between the spreads and the current account is 21 percent. A new dimension considered in this paper is the relation between interest rates, sovereign ratings and output. Consistent with the observations of the Argentinean economy in the past decade, ratings are positively correlated with output (corr($y, rating$) = 19 percent) and present a high negative correlation with spreads (corr($spread, rating$) $= -62$ percent).

In summary, the main regularities of emerging economies are that interest rates are countercyclical, the current account is countercyclical and interest rates and the current account are positively correlated. The model in this paper matches the data in that it simultaneously delivers a higher volatility of consumption relative to output, countercyclical interest rates and a countercyclical current account. It also displays a positive correlation between interest rates and the current account.

9 Conclusion

The average country, among emerging economies has experienced 3 defaults every 100 years, and sustains an average external debt to output ratio around 40%. Previous models of sovereign debt are unable to account jointly for the debt to output ratio and the observed default frequency. This paper solves the debt to output puzzle by developing a model of government reputation where the risk of default and debt renegotiation are endogenous in an otherwise standard open economy environment.

The parameters of the model are estimated via simulated method of moments to show that in equilibrium the average debt to output ratio predicted for Argentina is 34% (which is 92% of the ratio observed in the data). That is, the model is able to account jointly for the default frequency and the debt to output ratios observed in emerging economies. The information structure and the endogenous determination of default penalties, including recovery rates and periods of exclusion, are essential ingredients to obtain the main result. In section 5, I show that a model with full information and zero recovery rates generates an equilibrium debt to output ratio that is only 33% of that observed in the data. In addition, a model with full information and endogenous recovery generates a debt to output ratio that is 40% of the average debt to output ratio in Argentina. In these models, reputation has no value because political states are observable, thus a repayment decision per se has no effect on future prices.

In section 8, I show that the model account for the main regularities of emerging economies such as a higher volatility of consumption relative to output, countercyclical interest rates and a countercyclical current account. It also displays a positive correlation between interest rates and the current account.

In this paper, I show that a model of reputation and endogenous recovery rates accounts for the debt to output ratio and the default frequency. However, other puzzles are still present. The main anomaly of the paper is the low average interest rate (and spreads) generated in equilibrium. From Table 8 it is evident that previous studies are also unable to generate the observed mean spread. Other channels beyond the one identified here could affect the borrowing decisions of the government and lenders' expectations of the risk of default. The exploration of currency risk or debt with different maturities in a reputational model are left for future research.

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Appendix

A-1 Full Information Models

A-1.1 Zero Recovery Rates

The timing of events is as before. The government borrows at price $q(b', i, y)$ where i denotes the government type. The government problem can be stated as follows: At $b < 0$ and $h = In$ the government solves

$$V_i(b, y, In) = \max\Big\{v_i^{nd}(b, y), v_i^{d}(b, y)\Big\}.$$

$v_i^{nd}(b, y, s)$ denotes the value of "no-default":

$$v_i^{nd}(b, y) = \max_{c,b'}\Big\{u(c) + \beta\delta_i E_{y'|y}\Big[\pi_i V_i(b', y', In) + (1-\pi_i)V_{-i}(b', y', In)\Big]\Big\},$$

$$\text{s.t. } c + q(b', i, y)b' \leq y + b.$$

$v_i^{d}(b, y, s)$ is the value of "default":

$$v_i^{d}(b, y, s) = u(y) + \beta\delta_i E_{y'|y}\Big[\pi_i V_i(y', Out) + (1-\pi_i)V_{-i}(y', Out)\Big].$$

If $b \geq 0$ and $h = In$, the value for a government $i - type$ is given by

$$v_i(b, y, In) = v_i^{nd}(b, y).$$

If $h = Out$ the country is out of the credit market. In this case, recovery rates are zero, and the country is able to return to markets with probability α. While the country is in this state, it suffers a proportional output loss of size λy. The value of the government is:

$$\begin{aligned} V_i(y, Out) = u(y(1-\lambda)) + \beta\delta_i\alpha E_{y'|y}\Big[\pi_i V_i(0, y', In) + (1-\pi_i)V_{-i}(0, y', In)\Big] \\ \beta_i(1-\alpha)E_{y'|y}\Big[\pi_i V_i(y', Out) + (1-\pi_i)V_{-i}(y', Out)\Big]. \end{aligned}$$

For a government of type i, a loan b' generates profits given by:

$$\Omega(b', i, y) \quad = \quad -q(b', i, y)(-b') + \frac{(1 - p(b', i, y))(-b')}{(1+r)}, \tag{A.1.1}$$

so in equilibrium the price function is

$$q(b', i, y) = \begin{cases} (1+r)^{-1} & \text{if } b' \geq 0, \\ \frac{(1-p(b',i,y))}{(1+r)} & \text{if } b' < 0. \end{cases}$$

The equilibrium default probability $p^*(b', i, y)$ is:

$$p^*(b', i, y) = \int_{D_i(b')} F(dy'|y)\pi_i + \int_{D_{-i}(b')} F(dy'|y)(1-\pi_i).$$

The equilibrium notion is the standard Recursive Competitive Equilibrium, so given prices the value function and decision rules solve the government problem and the price function is consistent with the equilibrium decision rules.

A-1.2 Endogenous Recovery Rates

In this case recovery rates are determined in equilibrium and the value of a government while in financial autarky is not trivial. The government borrows at price $q(b', i, y)$ where i denotes the government type. The government problem can be stated as follows: At $b < 0$ and $h = In$ the government solves

$$V_i(b, y, In) = \max \left\{ v_i^{nd}(b, y), v_i^d(b, y) \right\}.$$

$v_i^{nd}(b, y, s)$ denotes the value of "no-default":

$$v_i^{nd}(b, y) = \max_{c, b'} \left\{ u(c) + \beta\delta_i E_{y'|y} \Big[\pi_i V_i(b', y', In) + (1-\pi_i)V_{-i}(b', y', In) \Big] \right\},$$

$$\text{s.t. } c + q(b', i, y)b' \leq y + b.$$

$v_i^d(b, y, s)$ is the value of "default":

$$v_i^d(b, y, s) = u(y) + \beta\delta_i E_{y'|y} \Big[\pi_i V_i(b, y', Out) + (1-\pi_i)V_{-i}(b, y', Out) \Big].$$

If $b \geq 0$ and $h = In$, the value for a government $i - type$ is given by

$$v_i(b, y, In) = v_i^{nd}(b, y).$$

If $h = Out$ the country is out of the credit market and has some unpaid debt $b < 0$. While the country is in this state, it suffers a proportional output loss of size λy. Only if the government repays the old debt, reduced to $\phi(b, y, i)b$, it will regain access to credit markets. The recovery rate $\phi(b, y, i)$ is endogenously determined and in principle can take any value in $[0, 1]$. The problem that the government solves is:

$$V_i(b, y, Out) = \max\Big\{v_i^r(b, y), v_i^{nr}(b, y)\Big\}.$$

$v_i^r(a, y)$ denotes the value of "repayment":

$$v_i^r(b, y) = \Big\{u(y + \phi(b, y, i)b) + \beta\delta_i E_{y'|y}\Big[\pi_i V_i(0, y', In) + (1 - \pi_i)V_{-i}(0, y', In)\Big]\Big\}$$

$v_i^{nr}(b, y, s)$ denotes the value of "no-repayment":

$$v_i^{nr}(b, y) = u(y(1 - \lambda)) + \beta\delta_i E_{y'|y}\Big[\pi_i V_i(b, y', Out) + (1 - \pi_i)V_{-i}(b, y', Out)\Big]$$

The repayment fraction is derived from solving the following problem.

$$\phi(b, y, i) \equiv \arg\max_{\varphi\in[0,1]}[\Delta_i^B(\varphi; b, y)]^\theta[\Delta^L(\varphi; b, y)]^{1-\theta}$$

$$\begin{aligned} \text{s.t. } \Delta_i^B(\varphi; b, y) &\geq 0, \\ \Delta^L(\varphi; b, y) &\geq 0. \end{aligned}$$

where Δ_i^B and Δ^L are defined as before.

For a government of type i, a loan b' generates profits given by:

$$\Omega(b', i, y) = -q(b', i, y)(-b') + \frac{(1 - p(b', i, y))(-b')}{(1 + r)} + \frac{p(b', i, y)}{(1 + r)^2}E_{y'',i''|y,i}\gamma(b', i'', y''), \quad \text{(A.1.2)}$$

From the zero profit condition, $\Omega(b', i, y) = 0$, the equilibrium price function can derived:

$$q(b', i, y) = \begin{cases} (1 + r)^{-1} & \text{if } b' \geq 0, \\ \frac{(1-p(b',i,y))}{(1+r)} + \frac{p(b',i,y)E_{y'',i''|y,i}\gamma(b',i'',y'')}{(-b')(1+r)^2} & \text{if } b' < 0. \end{cases}$$

The equilibrium default probability $p^*(b', i, y)$ and expected recovery $\gamma^*(b, i, y)$ are:

$$p^*(b', i, y) = \int_{D_i(b')} F(dy'|y)\pi_i + \int_{D_{-i}(b')} F(dy'|y)(1 - \pi_i)$$

and

$$\begin{aligned} \gamma^*(b, i, y) &= z_i(b, y)\phi(b, y, i)(-b') + \frac{\pi_i}{(1 + r)}(1 - z_i(b, y))E_{y'|y}[\gamma^*(b, i, y')] \\ &\quad + \frac{(1 - \pi_i)}{(1 + r)}(1 - z_i(b, y))E_{y'|y}[\gamma^*(b, -i, y')]. \end{aligned}$$

Again the equilibrium notion used in this case is the Recursive Competitive Equilibrium.

A-2 Computational Algorithm

Computation and estimation of the equilibrium requires three steps: an inner loop, where the decision problem of the government given parameter values, prices, the recovery schedule and posteriors is solved; a middle loop, where prices, the recovery schedule and posteriors are obtained; and an outside loop or estimation loop where parameter values that yield equilibrium allocations with the desired (target) properties are determined. In this section, I outline the algorithm to numerically solve for an equilibrium at given parameter values.

The first step is to set the grids over bonds, endowment and sovereign ratings. Recall the $s_1 = 1 - \pi_m$ and $s_N = \pi_a$. The endowment grid and the corresponding transition matrix is derived through the Tauchen-Hussey method. It is important to set a fine grid over bonds. Let k be the iteration number on the recovery rate function and j denote the iteration number on prices and posterior functions. Set $k = 0$ and $j = 0$. Let ϵ_ϕ and ϵ_p be small numbers that correspond to the tolerance value for the recovery rate and prices and posteriors respectively. Make an initial guess for the recovery rate schedule $\phi^{k=0}(b, y, s)$. The following procedure is used to obtain one equilibrium:

1. At a given recovery rate $\phi^k(b, y, s)$ solve for prices and posteriors $q^k(b', s', y)$ and $\Psi^k(\cdot, b, y, s)$ respectively as follows:
   (a) Make an initial guess the for price schedule $q^{k,j=0}(b', s', y)$ and posteriors $\Psi^{k,j=0}(\cdot, b, y, s)$
   (b) At given $q^{k,j}(b', s', y)$ and posteriors $\Psi^{k,j}(\cdot, b, y, s)$ solve the government problem and obtain the sets D_i, E_i and Z_i for $i = a, m$.
   (c) From the sets D_i, E_i and Z_i obtain new prices and posteriors $q^{k,j+1}(b', s', y)$ and $\Psi^{k,j+1}(\cdot, b, y, s)$.
      i. If $||q^{k,j+1} - q^{k,j}|| < \epsilon_p$ and $||\Psi^{k,j+1} - \Psi^{k,j}|| < \epsilon_p$ set $q^{k+1} = q^{k,j+1}$ and $\Psi^{k+1} = \Psi^{k,j+1}$. Moreover set $j = 0$ and go to 2.
      ii. Otherwise, set $q^{k,j+1} = q^{k,j}$ and $\Psi^{k,j+1} = \Psi^{k,j}$. Moreover, set $j = j + 1$ and go to 1b.
2. At given $q^{k+1}(b', s', y)$ and $\Psi^{k+1}(\cdot, b, y, s)$ solve the bargaining problem to obtain $\phi^{k+1}(b, y, s)$.
   (a) If $||\phi^{k+1} - \phi^k|| < \epsilon_\phi$ set $\phi^{k+1} = \phi^k$. Stop iterating. The functions $\phi^{k+1}(b, y, s)$, $q^{k+1}(b', s', y)$ and $\Psi^{k+1}(\cdot, b, y, s)$ are the equilibrium functions.
   (b) Otherwise, set $\phi^{k+1} = \phi^k$ and $k = k + 1$ and go to 1.

A-3 Estimation Procedure

The parameters $\{\delta_m, \pi_a, \pi_m, \theta\}$ are jointly estimated to match target statistics of the Argentinean economy and recent default episodes. This is done by the Simulated Method of Moments. This procedure consists of minimizing the distance between data moments and moments extracted from the simulated model. That is, the parameters are chosen to minimize

$$L(\Theta) = [M^d - M^s(\Theta)]W^*[M^d - M^s(\Theta)]' \tag{A.3.3}$$

with respect to parameters Θ, where M^d are the moments from the data, $M^s(\Theta)$ are the moments from the simulated model at parameters Θ. The estimator is consistent for any positive definite weighting matrix W. However, there is exists a matrix W^* that gives also efficient estimators. Thus, we say that W^* is an optimally derived weighting matrix. The weighting matrix is calculated via the bootstrap method. After an initial set of estimates are obtained using the identity matrix as an initial weight matrix, the model is simulated 3000 times using different draws for the innovations to the exogenous processes. After each simulation, the auxiliary moments are calculated and stored. The weighting matrix is then calculated as the inverse of the covariance matrix of the 3000 sets of auxiliary moments. A second estimation is then computed using this weighting matrix.

Given the potential for discontinuities in the model and the discretization of the state space, I used a simulated annealing algorithm to perform the optimization. The selection of auxiliary moments is crucial in order to obtain meaningful estimates of the structural parameters. If the moments from the simulated data are not sensitive to changes in the structural parameters, then little information can be gained via estimation. It is through the selection of good moments that identification is achieved. One statistic that is a useful indicator concerning the selection of moments is the standard error of the estimated parameter since it is a function of the derivative of the objective function (A.3.3) with respect to the structural parameter. In particular, the estimator $\widehat{\Theta}$ is asymptotically normal for fixed number of simulations S:

$$\sqrt{T}(\widehat{\Theta} - \Theta_0) \to N(0, Q_S(W^*))$$

where

$$Q_S(W^*) = \left(1 + \frac{1}{S}\right)\left[E_0 \frac{\partial L'}{\partial \Theta} W^{*-1} \frac{\partial L}{\partial \Theta}\right]^{-1}.$$

is the asymptotic variance covariance matrix. The covariance matrix of the estimator converges to the covariance matrix of the standard GMM estimator when $S \to \infty$.

The power of this procedure lies in the specification of the criterion function and the auxiliary moments to be estimated, similar to the importance of selecting moments in GMM. The precision of the estimates, measured through the asymptotic variance above, is related to the sensitivity of the auxiliary parameters to movements in the structural parameters through $\frac{\partial L'}{\partial \Theta}$. If the sensitivity is low, the derivative will be near zero, indicating a high variance for the structural estimates.