

# Asset Prices and Business Cycles with Financial Shocks

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**ABSTRACT.** Existing dynamic general equilibrium models have not been fully successful at explaining the observed high volatility of asset prices. We construct a general equilibrium model with heterogeneous firms, financial frictions, and financial shocks that addresses this issue. In our model only a fraction of firms can start new projects in a given period. Firms face a financial constraint that restricts external financing of new projects. There are two sources of aggregate uncertainty: productivity shocks and financial shocks that affect the tightness of the financial constraint. We show theoretically that both shocks result in asset price volatility. We then calibrate the model to the U.S. data to assess their quantitative importance. We find that productivity shocks generate only modest asset price volatility. However, our model with both productivity shocks and financial shocks generates a volatility in the price of equity comparable to the observed aggregate stock market volatility while also fitting key aspects of the behavior of aggregate quantities. In particular, the model is able to generate the aggregate investment volatility observed in the data.

**JEL Classification:** E20, E32, G12

**Keywords:** General Equilibrium, Business Cycles, Asset Pricing, Excess Volatility Puzzle

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## 1. INTRODUCTION

The excess volatility puzzle (Shiller (1981), and LeRoy and Porter (1981)) and the equity premium puzzle (Mehra and Prescott (1985)) are two fundamental challenges to theoretical models that have been developed in the finance and macroeconomics literature. Building a production economy model that would satisfactorily account for both high aggregate stock market volatility and business cycle fluctuations has proven to be rather difficult.

Jermann and Quadrini (2012) present a model with financial shocks and show that these shocks play an important role in explaining the observed business cycle fluctuations. In this paper we build a model with financial frictions and financial shocks and show that financial shocks play an important role in explaining not only business cycle fluctuations but also the high asset price volatility observed in the data. In particular, calibrating the model to the U.S. data, we find that it generates about 80% of the observed aggregate stock market volatility. In addition, the model generates a quarterly equity premium of 0.85%, i.e. over 70% of the observed equity premium. At the same time, the model generates time-series properties of aggregate quantities that match the macroeconomic data.

Our model resembles the model described in Kiyotaki and Moore (2011).<sup>1</sup> It is a dynamic stochastic general equilibrium model with heterogeneous entrepreneurs, financial frictions, and financial shocks. In every period only a fraction of entrepreneurs find new investment projects. Entrepreneurs who cannot find a new investment project are willing to buy claims to returns of other entrepreneurs' projects to replace their depreciated capital. We call these

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<sup>1</sup>We provide a detailed comparison of our model and Kiyotaki and Moore's model in Section 3.6. Other papers that analyze versions of the Kiyotaki and Moore (2011) model are Ajello (2012), Bigio (2012), Del Negro, Eggertsson, Ferrero, and Kiyotaki (2011), Driffill and Miller (2010), Salas (2010) and Shi (2011). These papers focus mostly on monetary policy and liquidity shocks, whereas we focus on asset prices and financial shocks, i.e., shocks to the access to outside financing.

claims *equity*. Markets are incomplete, and equity is the only financial asset that is traded in the economy.

Entrepreneurs face a financial constraint. We assume that they can pledge only a fraction of returns of the newly produced capital, i.e., sell only a fraction of the new project as equity. On its own, this friction is standard in the literature. The novel feature of our model is that we introduce financial shocks in that we assume that the tightness of the financial constraint changes over time.

We first show theoretically that as long as the financial constraint binds, productivity shocks and financial shocks result in changes in the price of equity. The intuition is as follows. An exogenous shock changes the supply of equity by entrepreneurs with an investment opportunity because it changes their wealth (productivity shock) or outside financing possibilities (financial shock). A productivity shock also changes the demand for equity by entrepreneurs without an investment opportunity by affecting their wealth. As a result of each of these two shocks, the amount of equity traded and the price of equity change. Based on these results, we argue that financial and productivity shocks are candidate sources of asset price fluctuations.

To assess the quantitative significance of these shocks, we calibrate the model to the U.S. data and solve it numerically. We find that a version of the model with productivity shocks but without financial shocks generates only modest asset price volatility. Once we add financial shocks, the model generates about 80% of the quarterly volatility in asset prices compared with the Wilshire 5000 Total Market Index. Moreover, the model generates over 70% of the observed equity premium. At the same time, the model generates the investment volatility observed in the data, unlike previous papers with convex adjustment costs or factor immobility (see, e.g., Guvenen (2009) and Boldrin, Christiano, and Fisher (2001)). These models generate less investment volatility than in the data.

It is important to emphasize that the high asset price volatility generated by the model comes from the interactions between the random project arrival, the financial friction itself, and the financial shocks, rather than solely from the financial shocks. Assuming for example that all entrepreneurs in the economy could find new investment projects in every period would imply that the financial constraint does not bind and the price of equity is constant. In that case, financial shocks would play no role for asset prices.

The rest of this paper is organized as follows. In the next section, we provide a more detailed discussion of the literature and discuss the place of this paper in the literature. Section 3 presents the model, and Section 4 characterizes its solution. In Section 5 we describe our calibration procedure and discuss the quantitative implications of the model. Section 6 concludes.

## 2. RELATED LITERATURE

Theoretically, it has been shown that financial frictions are important in explaining the fluctuations of aggregate macroeconomic quantities (see, for instance, Bernanke and Gertler (1989), Bernanke and Gertler (1990), Kiyotaki and Moore (1997), and Bernanke, Gertler, and Gilchrist (1999)). More recent papers have considered the effects of exogenous financial shocks (see, for example, Benk, Gillman, and Kejak (2005), Christiano, Motto, and Rostagno (2010), and Jermann and Quadrini (2012)). Their results suggest that financial shocks play an important role for macroeconomic fluctuations. In this paper we show that financial shocks are important for understanding not only the time-series properties of aggregate macroeconomic quantities but also asset price volatility.

By considering financial shocks, our paper distinguishes itself from Gomes, Yaron, and Zhang (2003). They analyze a model with financial frictions and productivity shocks, but

without financial shocks. They find that their model generates only modest asset return volatility.<sup>2</sup> Therefore, a departing assumption of our model is that fluctuations in productivity are not the only source of uncertainty in the economy. The second source of uncertainty in our model is financial shocks, namely, time variation in the fraction of a project an entrepreneur can finance with outside capital.<sup>3</sup>

Various approaches other than introducing financial frictions and financial shocks have been taken to explain the observed high asset price volatility in production economy models. A popular approach in the literature has been to assume convex capital adjustment costs along with various changes in the preference structure. In this vein, Jermann (1998) develops a model with habit persistence and convex capital adjustment costs; Guvenen (2009) builds a model with convex adjustment costs, Epstein-Zin preferences,<sup>4</sup> and limited stock market participation; and Campanale, Castro, and Clementi (2010) build a model with adjustment costs and disappointment averse agents. Boldrin, Christiano, and Fisher (2001) develop a model with habit persistence and labor and capital immobility across the consumption good sector and the investment good sector.

A shortcoming of models with convex capital adjustment costs (one can think of the factor immobility assumption in Boldrin, Christiano, and Fisher (2001) as an extreme version of adjustment costs) is the behavior of investment. First, convex adjustment costs imply that most firms invest (a little) in each time period, which is not in line with the data. Second, to generate high asset price volatility, the adjustment costs need to be quite severe, which in turn implies a counterfactually low aggregate investment volatility. Our model with idiosyncratic

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<sup>2</sup>Other attempts to build models with financial frictions that would generate a strong propagation of productivity shocks into the real economy and asset prices have not been very successful either (see, e.g., Kocherlakota (2000), Arias (2003), and Cordoba and Ripoll (2004)).

<sup>3</sup>We find that our model generates an asset return volatility very similar to Gomes, Yaron, and Zhang (2003) if we assume that this fraction is constant and productivity shock is the only source of uncertainty.

<sup>4</sup>Other papers with Epstein-Zin preferences and various departures from the standard neoclassical growth model that have studied asset price behavior include Tallarini (2000), Kuehn (2007), Croce (2012), and Kaltenbrunner and Lochstoer (2010).

investment opportunities improves upon the adjustment cost models by generating both the investment spikes at the firm level and the aggregate investment volatility observed in the data. At the same time, our model generates large asset price volatility.

A few papers have been more successful than convex adjustment cost models in explaining the joint behavior of investment and asset prices. For example, Gourio (2012) presents a model with a time-varying disaster probability. In his model, uncertainty in disaster probability plays a significant role in explaining the joint behavior of asset prices and aggregate quantities.<sup>5</sup> Favilukis and Lin (2011) build a model with firm-specific non-convex adjustment costs, Epstein-Zin preferences, and long-run growth. In their model, productivity shocks imply high asset price volatility, and the behavior of investment is in line with the data both at the firm level and at the aggregate level. Unlike these papers, we focus on financial frictions and show how financial shocks can generate high asset price volatility.

Our paper distinguishes itself along an important dimension from most of the papers discussed above. In those papers, asset price volatility is a feature of the first best allocation coming from preferences and technological constraints. Therefore, there is no role for the government as long as one assumes that the government is not able to change agents' preferences or overcome the technological constraints. In our model, on the other hand, asset price volatility originates from financial constraints and shocks. The government could try to relax the financial constraints by, for instance, lending to entrepreneurs with investment opportunities. Analyzing the optimal government policy in models of this kind is of interest, but in this paper, we focus on asset price volatility.<sup>6</sup>

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<sup>5</sup>In our opinion, the plausibility of the parameters in Gourio (2012) needs further investigation. He assumes that with an average probability of 1.7%, the U.S. economy loses 43% of its capital in one year.

<sup>6</sup>Del Negro, Eggertsson, Ferrero, and Kiyotaki (2011) perform an analysis of the Federal Reserve's policies applied during the recent recession in a model similar to ours.

### 3. THE MODEL

Time is discrete and infinite. There are two types of agents: a unit measure of ex ante identical entrepreneurs who consume, produce, and hold financial assets, but do not work, and a unit measure of identical hand-to-mouth workers who work and consume, but do not hold assets. There are two types of goods and two production technologies: a consumption good and a capital good, and a technology to produce the consumption good and a technology to produce the capital good. There is one type of financial asset traded: claims to returns of capital. Each period is divided into two subperiods. In the first subperiod, the consumption good is produced. In the second subperiod, the capital good is produced and consumption and asset trading take place.

Next we describe the details of the two production technologies, the asset trading structure, and the financial friction that the entrepreneurs face. We then present the entrepreneurs' and workers' optimization problems and define the competitive equilibrium in our model.

#### 3.1. Consumption Good and Investment Good Production Technologies.

In the first subperiod of each time period  $t$ , the consumption good production takes place. All entrepreneurs have access to the consumption good production technology. Entrepreneurs face a stochastic productivity shock, denoted by  $A_t$ , which is common to all of them. An entrepreneur  $T$  enters period  $t$  with capital  $k_t^T$ , hires labor  $l_t^T$ , and produces the consumption good  $y_t^T$  with the following technology ( $\gamma$  is the capital share parameter):

$$y_t^T = A_t(k_t^T)^\gamma(l_t^T)^{1-\gamma}.$$

Capital depreciates at rate  $\delta$  during the consumption good production, i.e., entrepreneur  $T$  enters the second subperiod with capital holdings  $(1 - \delta)k_t^T$ .

In the second subperiod, only a fraction  $\pi$  of entrepreneurs have the opportunity to start new projects. We model this “investment opportunity” as the entrepreneurs’ ability to access the capital good production technology. This technology enables them to produce new capital one-to-one from the consumption good, which is standard in the real business cycle literature. In practice apart from investment in the depreciated capital, firms adjust their capital stock by taking new projects. However, new projects are not always available. This technological constraint implies that investment responds to the entrepreneurs’ specific real opportunities rather than to the aggregate productivity shocks only. Our assumption is further motivated by the empirical observation that only a small fraction of firms invest a lot in a given year. In our quantitative analysis we use firm level data from the COMPUSTAT database to estimate  $\pi$ .

We assume that the arrival of the opportunity to access the capital good production technology is i.i.d. over time and over entrepreneurs.<sup>7</sup> We call entrepreneurs with access to the capital good production technology *investing entrepreneurs* and entrepreneurs without this access *noninvesting entrepreneurs*.

### 3.2. Trading and Financial Frictions.

In the second subperiod, consumption, capital good production, and asset trading take place. There is one type of financial asset traded: claims to capital returns (we refer to these claims simply as assets or equities).

Before we proceed with the discussion of the asset trading structure, we want to emphasize that the return per unit of capital is equal across entrepreneurs, independent of their capital holdings and independent of their opportunity to access the capital good production technology. Therefore, entrepreneurs are indifferent as to whose equity they hold.

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<sup>7</sup>The i.i.d. assumption is made for simplicity and is quite common in the literature. See, for instance, Angeletos (2007), Kocherlakota (2009), Kiyotaki and Moore (2011), and papers cited in footnote 1.

To see the above claim, consider entrepreneur  $T$  with capital  $k_t^T$ . In the first subperiod, he hires labor on a competitive labor market at wage  $w_t$  to maximize his profit, which can be written as

$$Profit(k_t^T; A_t, w_t) := A_t (k_t^T)^\gamma (l_t^T)^{1-\gamma} - w_t l_t^T.$$

The optimal behavior of entrepreneur  $T$  implies that he hires labor  $l_t^T = \left[ \frac{(1-\gamma)A_t}{w_t} \right]^{\frac{1}{\gamma}} k_t^T$ . This amount of labor equalizes the wage rate with the marginal product of labor:

$$w_t = MPL_t = (1-\gamma)A_t (k_t^T)^\gamma (l_t^T)^{-\gamma}.$$

Therefore,

$$Profit(k_t^T; A_t, w_t) = \gamma A_t \left[ \frac{(1-\gamma)A_t}{w_t} \right]^{\frac{1-\gamma}{\gamma}} \cdot k_t^T = r_t k_t^T,$$

where  $r_t := \gamma A_t \left[ \frac{(1-\gamma)A_t}{w_t} \right]^{\frac{1-\gamma}{\gamma}}$  denotes the return per unit of capital. Since all entrepreneurs face the same stochastic productivity shock,  $A_t$ , and hire labor at the same wage,  $w_t$  (determined by aggregate market clearing), the return on capital,  $r_t$ , is the same for all entrepreneurs.

To understand the trading structure in our economy, we first describe the capital and asset holdings of the entrepreneurs. Entrepreneurs can hold physical capital and equity to other entrepreneurs' capital returns. We define the individual state of entrepreneur  $T$  by  $(k_t^T, e_t^T, s_t^T)$ , where  $k_t^T$  is the physical capital held by the entrepreneur,  $e_t^T$  is the equity to other entrepreneurs' capital, and  $s_t^T$  is equity to entrepreneur  $T$ 's own capital sold to other entrepreneurs.

Physical capital  $k_t^T$  is used by entrepreneur  $T$  in the consumption good production and depreciates at rate  $\delta$ . We assume that physical capital is not traded in the economy. Equity

$e_t^T$  entitles entrepreneur  $T$  to the stream of returns of  $e_t^T$  units of other entrepreneurs' capital. Since the underlying capital depreciates at rate  $\delta$ , we can think of  $e_t^T$  as depreciating at rate  $\delta$ . Finally,  $s_t^T$  denotes claims to own capital returns sold by entrepreneur  $T$ , and we can think of these claims as depreciating at rate  $\delta$  as well. Therefore, an entrepreneur with an individual state  $(k_t^T, e_t^T, s_t^T)$  is entitled to returns from  $k_t^T - s_t^T + e_t^T$  units of capital.

In the second subperiod, entrepreneurs face a financial constraint, which restricts the amount of external financing. An investing entrepreneur that produces  $i_t^T$  units of new capital can sell at most a fraction  $\theta_t$  of returns from  $i_t^T$ . This means that an entrepreneur is able to finance only a fraction of his investment externally. This assumption is motivated by the empirical observation that firms do not fully finance their investments externally. We do not take a stand on the underlying reasons although problems of asymmetric information have a long-standing tradition in the theory of capital structure in corporate finance.

We further assume that  $\theta_t$  is a stochastic process common to all entrepreneurs. As before, the assumption that  $\theta_t$  is changing over time is motivated by an empirical observation. In the data, the amount of external financing that firms raise is changing over time. In our quantitative analysis we use the Flow of Funds data to put discipline on the evolution of the variable  $\theta_t$ . Although a micro-founded theory that endogenizes the time variation in  $\theta_t$  is of interest, we do not attempt to incorporate such a theory in the current paper. We provide empirical evidence supporting the stochasticity of  $\theta_t$  in Section 5 and incorporate it in our model to study its quantitative implications.<sup>8</sup>

We assume that claims to already installed capital can be traded without restrictions. This is in line with the notion that the problem of asymmetric information is less severe for already existing projects. These assumptions imply that the total amount of equity sold up

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<sup>8</sup>Empirical studies have documented that equity issuances cluster after information releases (see, e.g., Korajczyk, Lucas, and McDonald (1991), and Dierkens (1991)), and during business cycle expansions (see, e.g., Choe, Masulis, and Nanda (1993)). Therefore, theories that build on time-varying asymmetric information (see, e.g., Korajczyk, Lucas, and McDonald (1992)) make up one venue for endogenizing  $\theta_t$ .

until period  $t$  (denoted as  $s_{t+1}^T$ ) can be at most the sum of a fraction  $\theta_t$  of period  $t$  investment  $i_t^T$  and the depreciated period  $t$  capital holdings  $(1 - \delta)k_t^T$ :

$$(3.1) \quad s_{t+1}^T \leq \theta_t i_t^T + (1 - \delta)k_t^T.$$

To understand this constraint, we define  $k_{t+1}^T = (1 - \delta)k_t^T + i_t^T$  and rewrite (3.1) as

$$(3.2) \quad k_{t+1}^T - s_{t+1}^T \geq (1 - \theta_t)i_t^T$$

The left-hand side of inequality (3.2) captures the net amount of returns to entrepreneur  $T$ 's own capital that he must carry into period  $t + 1$ . Since he can sell at most  $\theta_t i_t^T$  of “new” equity, he must keep at least  $(1 - \theta_t)i_t^T$  of the newly produced capital unsold, which is captured in the right-hand side of inequality (3.2).

### 3.3. Entrepreneurs' Maximization Problem.

There is a unit measure of ex ante identical entrepreneurs, who hold capital, trade assets, and consume, but do not work. Ex post, entrepreneurs will differ in their capital and asset holdings. The budget constraint of an entrepreneur with capital and asset holdings  $(k_t^T, e_t^T, s_t^T)$  can be written as

$$c_t^T + i_t^T + q_t[k_{t+1}^T - s_{t+1}^T + e_{t+1}^T] \leq r_t[k_t^T - s_t^T + e_t^T] + (1 - \delta)q_t[k_t^T - s_t^T + e_t^T] + q_t i_t^T,$$

where  $r_t$  is the return on capital. The first term on the right-hand side is the return to which entrepreneur  $T$  is entitled. The second term is the market value of his depreciated unsold capital and asset holdings. The third term is the market value of equity to his newly installed capital at the market price  $q_t$ . The left-hand side sums up his expenditure. He can consume  $c_t^T$ , invest  $i_t^T$  with investment being generated one-to-one from the consumption good, and

carry unsold equity to his own capital  $k_{t+1}^T - s_{t+1}^T$  and outside equity  $e_{t+1}^T$  into period  $t + 1$ . These are traded at market price  $q_t$ .

The maximization problem of this entrepreneur can be written as follows (we drop the  $T$  superscripts for simplicity):

$$\begin{aligned} \max_{\{c_t, i_t, k_{t+1}, s_{t+1}, e_{t+1}\}_{t=0}^{\infty}} \quad & E_0 \sum_{t=0}^{\infty} \beta^t \log c_t \quad \text{s.t.} \\ (BC) \quad & c_t + i_t + q_t[k_{t+1} - s_{t+1} + e_{t+1}] \leq [k_t - s_t + e_t][r_t + (1 - \delta)q_t] + q_t i_t \\ (FC1) \quad & k_{t+1} - s_{t+1} \geq (1 - \theta_t)i_t \\ (FC2) \quad & e_{t+1} \geq 0. \end{aligned}$$

In this problem, expectations are taken over the stochastic processes for  $\theta_t$  and  $A_t$ , equilibrium processes for prices (taken as given and correctly forecasted by the entrepreneur), and the arrival of the investment opportunity (not explicitly included in the notation above). If the entrepreneur does not have an investment opportunity in period  $t$ , he must set  $i_t$  to zero.

Note that the returns of the unsold capital  $k_{t+1} - s_{t+1}$  and the returns from claims to other entrepreneurs' capital  $e_{t+1}$  are the same given the state of the economy. Moreover, trades in these assets in period  $t + 1$  are not subject to any restrictions. Therefore, inside equity  $k_{t+1} - s_{t+1}$  and outside equity  $e_{t+1}$  are perfect substitutes, and (FC1) binding is equivalent to the no-short-sales (FC2) binding, and we can sum them up without loss of generality.

The intuition for the equivalence of (FC1) and (FC2) is as follows. An entrepreneur who has the investment opportunity and whose (FC1) is binding will sell all his other assets  $e_t$  to take advantage of this profitable opportunity. Therefore, we can simplify the maximization

problem by defining net asset holdings  $n_t := k_t - s_t + e_t$  and write

$$\begin{aligned} \max_{\{c_t, i_t, n_{t+1}\}_{t=0}^{\infty}} \quad & E_0 \sum_{t=0}^{\infty} \beta^t \log c_t \quad \text{s.t.} \\ (BC) \quad & c_t + i_t + q_t n_{t+1} \leq n_t [r_t + (1 - \delta)q_t] + q_t i_t \\ (FC) \quad & n_{t+1} \geq (1 - \theta_t) i_t. \end{aligned}$$

### 3.4. Workers' Maximization Problem.

There is a unit measure of identical infinitely lived workers, i.e., agents who do not have access to consumption good and capital good production technologies. In each period, a worker decides how much to consume and how much labor to provide. For simplicity we assume that workers do not participate in asset trading. We discuss this assumption in more detail in section 3.6. A worker's maximization problem is thus static and can be written as

$$\max_{c'_t, l'_t} U \left( c'_t - \frac{\omega}{1 + \eta} (l'_t)^{1+\eta} \right) \quad \text{s.t.} \quad c'_t \leq w_t l'_t,$$

where  $c'_t$  is the consumption of the worker in period  $t$ ,  $l'_t$  is the labor provided by the worker in period  $t$ , the function  $U(\cdot)$  is increasing and strictly concave,  $\omega > 0$  and  $\eta > 0$ .<sup>9</sup>

### 3.5. Equilibrium.

A competitive equilibrium is quantities for entrepreneurs  $[\{c_t^j, i_t^j, n_{t+1}^j\}_{t=0}^{\infty}]_{j \in [0,1]}$ , quantities for workers  $[\{c_t^j, l_t^j\}_{t=0}^{\infty}]_{j \in [0,1]}$ , and prices  $(\{q_t, r_t, w_t\}_{t=0}^{\infty})$ , such that quantities solve workers' and entrepreneurs' problems given prices, input prices  $w_t, r_t$  are determined competitively, and markets clear.

### 3.6. Comparison with Kiyotaki and Moore (2011).

In Kiyotaki and Moore's model, entrepreneurs can hold equity  $n_t$  and fiat money  $m_t$ . The

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<sup>9</sup>This utility function is not standard in the business cycle literature. We analyze the sensitivity of our results to the choice of the utility function in Appendix C.

price of money in terms of the general consumption good is  $p_t$ . They assume that the financial shock variable  $\theta_t$  is constant over time. An entrepreneur can sell all of his money holdings, but he can sell only a fraction  $\phi_t$  of his equity holdings. The variable  $\phi_t$  is a stochastic process common to all entrepreneurs. The maximization problem of an entrepreneur in Kiyotaki and Moore's model is

$$\begin{aligned} \max_{\{c_t, i_t, n_{t+1}, m_{t+1}\}_{t=0}^{\infty}} \quad & E_0 \sum_{t=0}^{\infty} \beta^t \log c_t \quad \text{s.t.} \\ (BC) \quad & c_t + i_t + q_t n_{t+1} + p_t m_{t+1} \leq n_t [r_t + (1 - \delta)q_t] + q_t i_t + p_t m_t \\ (FC) \quad & n_{t+1} \geq (1 - \theta) i_t + (1 - \phi_t)(1 - \delta) n_t. \end{aligned}$$

In practice it is hard to document a restriction that limits the amount of equity an entrepreneur can sell in a given time period (in our model, a time period is a quarter). Therefore, in our model entrepreneurs are able to sell all of their equity holdings, i.e.,  $\phi_t = 1$  in every period. This effectively loosens the financial constraint. We also assume that  $\theta_t$  is a stochastic process. Since we do not focus on monetary policy, we abstract from fiat money.<sup>10</sup>

In terms of the workers, Kiyotaki and Moore assume infinitely lived workers who are borrowing constrained. They show that the borrowing constraint binds in steady state (because the rate of return on capital is lower than  $\frac{1}{\beta}$ ). This implies that workers do not participate in asset markets. We prove that the same result holds in our environment in Appendix A. By continuity, one can extend the argument to the neighborhood of a steady state. However, in our quantitative analysis shocks are large and in some states workers might want to save. To keep the analysis tractable we therefore simply assume that workers do not participate in asset markets.

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<sup>10</sup>We solve a version of our model with money to see how our quantitative results are affected. These results are discussed in section 5.4.

Abstracting from money and resaleability shocks,  $\phi_t$ , enables us to solve our model in closed form. Therefore, we can perform a detailed analysis of the dynamics of the model independent of the distance from steady state. We show theoretically that shocks to  $A_t$  and  $\theta_t$  imply movements in the price of equity and in the real variables. In contrast, Kiyotaki and Moore focus on steady state properties of their model and establish several properties in the neighborhood of the steady state. They restrict attention to the neighborhood of the steady state since it is unclear whether the steady state conditions that they derive are also valid further away from steady state.<sup>11</sup> As for a quantitative analysis of the model, Kiyotaki and Moore (2011) provide illustrative examples of impulse responses constructed using log-linear approximations around the deterministic steady state. We perform a detailed analysis of the business cycle properties of our model without relying on log-linear approximations in Section 5.

#### 4. CHARACTERIZATION OF THE MODEL

In this section we characterize the solutions to the workers' and entrepreneurs' problems. We show that the equilibrium is determined by a single equation in the price of equity. This enables us to do a comparative statics exercise in the exogenous shocks  $A_t$  and  $\theta_t$ . We show that both productivity shocks and financial shocks result in asset price volatility.

##### 4.1. Solving the Workers' Problem.

We start this subsection by simplify the workers' problem. We show that current output does not depend on the current realization of  $\theta_t$  and derive the relationships between labor, consumption, and aggregate output.

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<sup>11</sup>Kiyotaki and Moore show that in steady state workers do not save and investing entrepreneurs use all their money to invest in new projects.

A worker solves

$$\max_{c'_t, l'_t} U \left( c'_t - \frac{\omega}{1+\eta} (l'_t)^{1+\eta} \right) \text{ s.t. } c'_t \leq w_t l'_t,$$

where  $c'_t$  denotes the consumption of the worker,  $l'_t$  denotes the labor provided by the worker, and  $w_t$  denotes the wage rate.

Therefore,

$$(4.1) \quad l'_t = \left( \frac{w_t}{\omega} \right)^{1/\eta}.$$

Equation (4.1) holds for each worker. Therefore, the aggregate labor supply, denoted by  $L'_t$ , can be written as

$$(4.2) \quad L'_t = \left( \frac{w_t}{\omega} \right)^{1/\eta}.$$

The aggregate labor demand by the entrepreneurs, denoted by  $L_t$ , is determined by

$$w_t = A_t(1-\gamma)K_t^\gamma L_t^{-\gamma}.$$

In equilibrium, supply of labor by the workers is equal to demand of labor by the entrepreneurs, i.e.,  $L'_t = L_t$ . Therefore,

$$\begin{aligned} w_t &= \omega^{\frac{\gamma}{\eta+\gamma}} [(1-\gamma)A_t]^{\frac{\eta}{\eta+\gamma}} K_t^{\frac{\eta\gamma}{\eta+\gamma}} \\ L_t &= \left[ \frac{A_t(1-\gamma)}{\omega} \right]^{\frac{1}{\gamma+\eta}} K_t^{\frac{\gamma}{\gamma+\eta}}. \end{aligned}$$

Thus, we can express  $L_t$  as a function of parameters and aggregate states  $K_t, A_t$ . Note that  $L_t$  does not depend on the financial shock  $\theta_t$ . Therefore, in period  $t$ , aggregate output  $Y_t = A_t K_t^\gamma L_t^{1-\gamma}$  is not a function of  $\theta_t$ .

The joint dynamics of output and labor is determined by equation (4.2). Using equilibrium conditions, this equation can be rewritten as

$$L_t = \left(\frac{w_t}{\omega}\right)^{1/\eta} = \left(\frac{MPL_t}{\omega}\right)^{1/\eta} = \left(\frac{(1-\gamma)Y_t}{\omega L_t}\right)^{1/\eta}.$$

This implies

$$\begin{aligned} L_t^{1+\eta} &= \frac{(1-\gamma)Y_t}{\omega} \\ \implies (1+\eta) \log L_t &= \log Y_t + \log\left(\frac{1-\gamma}{\omega}\right) \\ \implies \text{corr}(\log L_t, \log Y_t) &= 1 \\ \implies (1+\eta)^2 \text{var}(\log L_t) &= \text{var}(\log Y_t). \end{aligned}$$

As we can see from the previous equation, the relative variance of labor and output is pinned down by the labor supply elasticity parameter  $\eta$ .

As for workers' consumption, since they cannot save, their aggregate consumption, denoted by  $C'_t$ , equals labor's share in output, i.e.,  $C'_t = (1-\gamma)Y_t$ . Therefore,  $\text{corr}(\log C'_t, \log Y_t) = 1$  and  $\text{var}(\log C'_t) = \text{var}(\log Y_t)$ . Since workers account for a large fraction of total consumption in the economy (the combined workers' and entrepreneurs' consumption), the dynamics of workers' consumption will significantly affect the dynamics of total consumption relative to output.

## 4.2. Solving the Entrepreneurs' Problem.

We begin this subsection by clarifying the role of the technological constraint and the financial friction. The maximization problem of an entrepreneur is

$$\begin{aligned} \max_{\{c_t, i_t, n_{t+1}\}_{t=0}^{\infty}} \quad & E_0 \sum_{t=0}^{\infty} \beta^t \log c_t \quad \text{s.t.} \\ (BC) \quad & c_t + i_t + q_t n_{t+1} \leq n_t [r_t + (1 - \delta)q_t] + q_t i_t \\ (FC) \quad & n_{t+1} \geq (1 - \theta_t) i_t. \end{aligned}$$

If all entrepreneurs in the economy had the ability to invest in every period, i.e.,  $\pi = 1$ , then  $q_t = 1$ . This is because no entrepreneur would be willing to pay more, given that he can produce new capital at price one.

If the investing entrepreneurs could finance all their new investment externally, i.e.,  $\theta_t = 1$ , then  $q_t = 1$  as well. If  $q_t$  was larger than one, then an investing entrepreneur would be able to decrease his consumption by one unit, increase investment by one unit, and sell claims to the newly produced capital at  $q_t > 1$ . He then could increase his consumption by one unit back to the original level and would end up with a net profit of  $q_t - 1 > 0$ . Therefore, this cannot be an equilibrium and  $q_t = 1$  at all times.

By this reasoning, both the technological constraint and the financial shocks are essential for generating asset price volatility in our model. In fact, we need the financial constraint (FC) to bind, otherwise  $q_t = 1$  as summarized in the following lemma.

**Lemma 4.1.** *The financial constraint binds for all investing entrepreneurs if and only if  $q_t > 1$ .*

**Proof:** If an entrepreneur has an investment opportunity, we can take the first order condition with respect to  $i_t$ . We denote the Lagrange multiplier on the budget constraint by  $\lambda_t$ , and the Lagrange multiplier on the financial constraint by  $\mu_t$ . The budget constraint

always binds, and therefore  $\lambda_t > 0$ . The necessary first order condition with respect to  $i_t$  is

$$(4.3) \quad (q_t - 1)\lambda_t = (1 - \theta_t)\mu_t$$

If  $\theta_t < 1$  then by equation (4.3)  $q_t > 1 \implies \mu_t > 0$  and vice versa.<sup>12</sup> The result does not depend on the initial asset holdings  $n_t$  and therefore applies to all investing entrepreneurs.<sup>13</sup>

□

The intuition for the sufficient part is as follows. If  $q_t > 1$  and the financial constraint does not bind, then the solution to the problem does not exist because there will be arbitrage opportunities for investing entrepreneurs. At any allocation an investing entrepreneur will find it profitable to increase  $i_t$  by  $\Delta$  and consumption by  $(q_t - 1)\Delta$  as discussed above.

We now proceed to solve the entrepreneurs' problem and derive the aggregate entrepreneurs' allocations. Our model is one with heterogeneous entrepreneurs. Entrepreneurs differ in their wealth depending on their individual sequences of the idiosyncratic investment opportunity shocks. However, we can solve for aggregate dynamics without having to keep track of the whole wealth distribution. The reason is that the log utility and linearity of the right-hand side of the budget constraint imply linear decision rules. We omit the proof of this well-known result summarized by the following lemma.<sup>14</sup>

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<sup>12</sup>If  $\theta_t = 1$ , then  $q_t = 1$  as discussed above. In that case, the financial constraint cannot bind, since it would have to bind for all investing and noninvesting entrepreneurs, which is inconsistent with market clearing.

<sup>13</sup>If an entrepreneur does not have an investment opportunity at time  $t$ , he must set  $i_t = 0$ . His FC takes the form  $n_{t+1} \geq 0$  and it never binds.

<sup>14</sup>See Samuelson (1969).

**Lemma 4.2.** *The policy functions describing an individual entrepreneur's optimal decisions are linear and can be written as follows:*

$$(4.4) \quad c_t^i = (1 - \beta)n_t[r_t + (1 - \delta)q_t]$$

$$(4.5) \quad q_t^R n_{t+1}^i = \beta n_t[r_t + (1 - \delta)q_t]$$

$$(4.6) \quad c_t^s = (1 - \beta)n_t[r_t + (1 - \delta)q_t]$$

$$(4.7) \quad q_t n_{t+1}^s = \beta n_t[r_t + (1 - \delta)q_t],$$

where  $n_t$  denotes the initial asset holdings of an entrepreneur. Superscript  $i$  denotes the state in which this entrepreneur has an investment opportunity in period  $t$ , and superscript  $s$  denotes the state in which he does not have an investment opportunity in period  $t$ . Equations (4.6)-(4.7) summarize the behavior of the noninvesting entrepreneurs as implied by log utility. Equations (4.4)-(4.5) summarize the behavior of the investing entrepreneurs. They follow from the fact that we can express their budget constraint as follows (plugging in for  $i_t$  from the FC assuming the FC binds):

$$c_t^i + q_t^R n_{t+1}^i \leq n_t[r_t + (1 - \delta)q_t],$$

where  $q_t^R$  is the replacement cost of capital defined as

$$q_t^R := \frac{1 - \theta_t q_t}{1 - \theta_t}.$$

If the FC does not bind, then  $q_t^R = q_t = 1$ , individual investment is indeterminate, and equations (4.4)-(4.5) apply as well.

With linear policy rules, prices are functions of aggregate quantities only, which we can easily solve for. We denote the aggregate quantities with capital letters and use the fact that the arrival of the investment opportunity is i.i.d. This implies that entrepreneurs with an

investment opportunity hold a fraction  $\pi$  of the total asset holdings in the economy at the beginning of period  $t$ , and investors without an investment opportunity hold a fraction  $1 - \pi$  of all assets at the beginning of period  $t$ . Integrating over individual policies thus yields the following for the aggregate entrepreneurs' allocations:

$$(4.8) \quad C_t^i = (1 - \beta)\pi N_t[r_t + (1 - \delta)q_t]$$

$$(4.9) \quad q_t^R N_{t+1}^i = \beta\pi N_t[r_t + (1 - \delta)q_t]$$

$$(4.10) \quad C_t^s = (1 - \beta)(1 - \pi)N_t[r_t + (1 - \delta)q_t]$$

$$(4.11) \quad q_t N_{t+1}^s = \beta(1 - \pi)N_t[r_t + (1 - \delta)q_t].$$

### 4.3. Solving for Equilibrium.

Workers' allocations are determined by the productivity shock,  $A_t$ , and the aggregate capital,  $K_t$ , as discussed above. Aggregate capital  $K_t$  is determined by the entrepreneurs' problem. By definition, aggregate capital  $K_t$  is equal to the aggregate amount of equity  $N_t$ . Therefore, the dynamics of aggregate capital is determined by the aggregate equity holdings of investing and noninvesting entrepreneurs:  $N_{t+1} = N_{t+1}^i + N_{t+1}^s$ . The equilibrium is unique, but the price of equity can be  $q_t = 1$  or  $q_t > 1$  depending on the state.<sup>15</sup>

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<sup>15</sup>When we solve for the equilibrium, we need to make sure that we are in the correct regime. We start by assuming that  $q_t = 1$ , compute the implied  $I_t$  along with  $N_{t+1}^i$ , and verify whether the FC is satisfied. If not, we recompute the allocations assuming  $q_t > 1$ . If capital can be converted back to consumption  $q_t < 1$  will not be an equilibrium outcome. With capital irreversibility, i.e., an  $I_t \geq 0$  constraint,  $q_t$  would be smaller than one if and only if the constraint binds. This could happen only if capital levels were very high, which does not happen in our quantitative analysis. We therefore ignore the possibility that  $q_t < 1$  and do not take a stand on capital reversibility.

If  $q_t = 1$ , the equilibrium aggregate quantities are determined by the aggregate policy function for capital (one can get the equation below by adding equations (4.9) and (4.11)):

$$\begin{aligned} N_{t+1} &= \beta N_t [r_t + (1 - \delta)] \\ r_t &= A_t^{\frac{1+\eta}{\gamma+\eta}} \gamma \left[ \frac{1-\gamma}{\omega} \right]^{\frac{1-\gamma}{\gamma+\eta}} N_t^{\frac{\eta(\gamma-1)}{\gamma+\eta}}. \end{aligned}$$

These two equations fully describe the aggregate behavior of the model. The second equation determines  $r_t$  through the workers' problem. The rest of the variables are determined using the derived policy functions.

If  $q_t > 1$  the equilibrium is determined by the aggregate policies for  $N_{t+1}^i, N_{t+1}^s$ , market clearing conditions, and the financial constraint aggregated over investing entrepreneurs. It is useful to think about the equilibrium in terms of the demand for equity by the noninvesting entrepreneurs and the supply of equity by investing entrepreneurs. The net demand for equity by noninvesting entrepreneurs can be written as

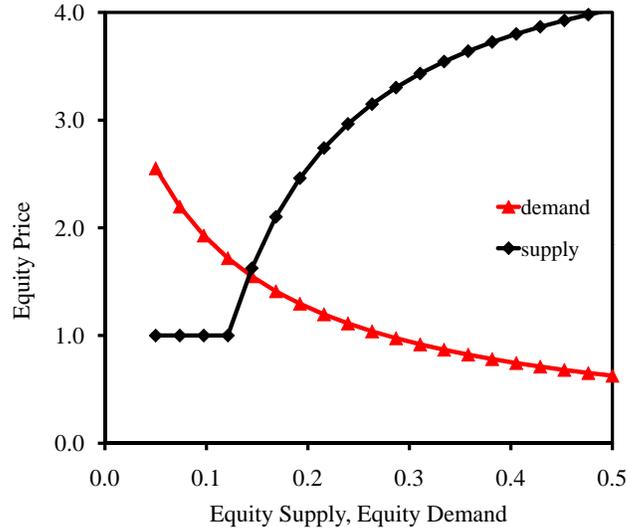
$$D_t^e : = N_{t+1}^s - (1 - \delta)(1 - \pi)N_t = \beta(1 - \pi)N_t \left[ \frac{r_t}{q_t} + 1 - \delta \right] - (1 - \delta)(1 - \pi)N_t$$

$D_t^e$  is a downward-sloping demand function since  $\frac{\partial D_t^e}{\partial q_t} < 0$ . The net supply of equity by the investing entrepreneurs is given by

$$\begin{aligned} S_t^e : &= \pi(1 - \delta)N_t + I_t - N_{t+1}^i = \pi(1 - \delta)N_t + \frac{\theta_t}{(1 - \theta_t)} N_{t+1}^i \\ &= \pi(1 - \delta)N_t + \frac{\theta_t}{(1 - \theta_t q_t)} \beta \pi N_t [r_t + (1 - \delta)q_t]. \end{aligned}$$

$S_t^e$  is an upward-sloping supply function since  $\frac{\partial S_t^e}{\partial q_t} > 0$ . Market clearing implies that in equilibrium  $S_t^e = D_t^e$ . Since  $r_t$  is a function of states  $N_t, A_t, \theta_t$  only, we can solve for  $q_t$  as a function of these states and then use (4.9) and (4.11) to compute  $N_{t+1}(N_t, A_t, \theta_t)$ , which is

FIGURE 1. Model Demand and Supply of Equity as a Function of the Price of Equity



This figure shows the demand and supply of equity as a function of the price of equity in our model. The model parameters are  $\beta = 0.99$ ,  $\eta = 1$ ,  $\delta = 0.0226$ ,  $A = 1$ ,  $\gamma = 0.36$ ,  $\pi = 0.01$ , and  $\theta = 0.2$ . These numbers are similar to those that we use in the quantitative analysis. We set  $\omega = 7.5257$ , which guarantees that workers spend 30% of their time working in steady state, and we set  $N = 9.9522$ , which is the steady state value.

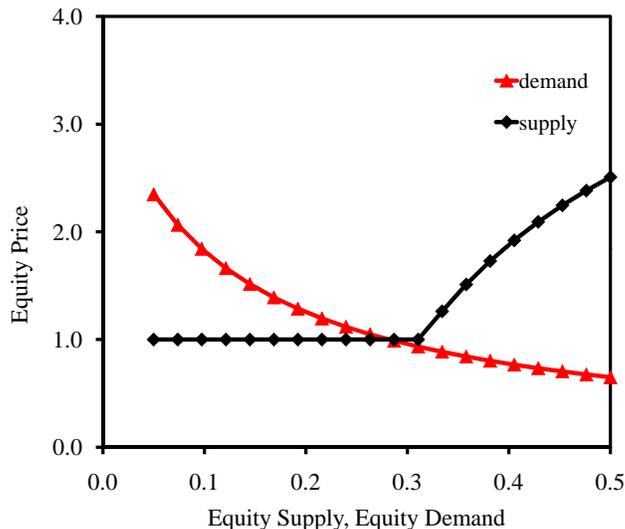
the only endogenous state variable we need to keep track of.<sup>16</sup>

#### 4.4. Steady State Equilibria.

A steady state equilibrium in our model is defined as an equilibrium in which the values of the exogenous shocks  $A_t$  and  $\theta_t$  as well as aggregate endogenous variables, in particular the aggregate capital and equity holdings  $N_t$ , are constant. For a given set of parameters the (unique) steady state equilibrium is either an equilibrium in which the financial constraint

<sup>16</sup>The market clearing condition has exactly one solution for  $q_t$  in the interval  $(0, \frac{1}{\theta_t})$ . This is the unique equilibrium unless it is smaller than 1. Then  $q_t = 1$  is the unique equilibrium. Note that the unique equilibrium does not depend on the expectations of  $\theta_t$  and  $A_t$ . Neither does the equilibrium  $q_t$  depend on the expectations of future  $q$ . This is a feature of log utility.

FIGURE 2. Model Demand and Supply of Equity as a Function of the Price of Equity



This figure shows the demand and supply of equity as a function of the price of equity in our model. The model parameters are  $\beta = 0.99$ ,  $\eta = 1$ ,  $\delta = 0.0226$ ,  $A = 1$ ,  $\gamma = 0.36$ ,  $\pi = 0.02$ , and  $\theta = 0.2$ . We set  $\omega = 8.2232$ , which guarantees that workers spend 30% of their time working in steady state, and we set  $N = 12.7304$ , which is the steady state value.

binds and the price of equity is greater than one, or an equilibrium in which the financial constraint does not bind and the price of equity is equal to one. Theorem 4.3 summarizes the conditions on model parameters under which each of these equilibria exist.

**Theorem 4.3.** *In steady state the financial constraint binds and the price of equity  $q_t$  is greater than one if and only if  $\theta < \frac{\delta - \pi}{\delta}$ .*

Proof: See Appendix A.

An illustrative example in which  $q > 1$  is shown in Figure 1. In this example, both  $\theta$  and  $\pi$  are relatively small, the inequality in Theorem 4.3 is satisfied, and the financial constraint binds. The intuition for the fact that  $q > 1$  is as follows. If  $q$  was equal to 1, investing entrepreneurs would not be willing to produce enough new capital without violating the

financial constraint to cover the demand for equity by the noninvesting entrepreneurs. Since at price one demand exceeds supply, the price of equity must increase. Therefore,  $q > 1$  and the financial constraint binds.

An illustrative example in which  $q = 1$  in a steady state is shown in Figure 2. The parameter  $\pi$  is now twice the size of the previous example. The figure shows that at any price  $q > 1$ , the supply of equity exceeds the demand. At price  $q = 1$ , investing entrepreneurs are willing to supply any amount of equity that will not violate their financial constraint (any amount on the flat part of the supply curve). Individual supply is indeterminate, and asset trades are determined by demand.

#### 4.5. Comparative Statics in $A_t$ and $\theta_t$ .

We now analyze what happens when  $A_t$  or  $\theta_t$  changes. This is a comparative statics exercise in the following sense. We fix the states  $N_t, A_t, \theta_t$  and derive the asset supply and demand and the equilibrium price  $q_t$ . Then we redo the exercise for a different value of  $\theta_t$  or  $A_t$ .

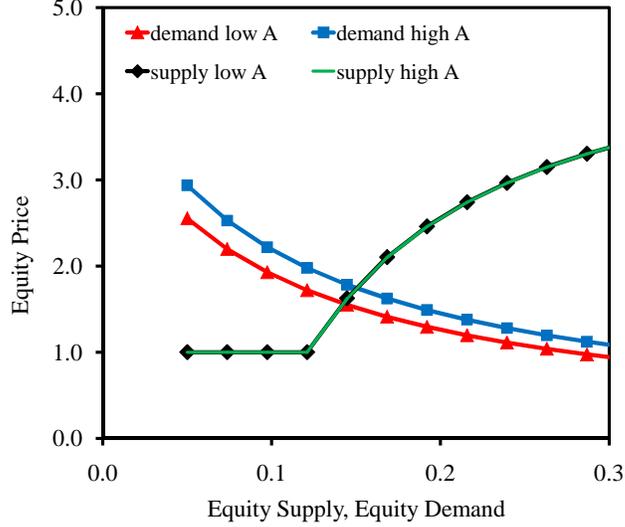
Consider a productivity shock, i.e.,  $\Delta A_t$ . The equity demand curve and the equity supply curve move up if  $\Delta A_t > 0$  because

$$(4.12) \quad \frac{\partial D_t^e}{\partial A_t} = \frac{\beta(1-\pi)N_t}{q_t} \frac{\partial r_t}{\partial A_t} > 0$$

$$(4.13) \quad \frac{\partial S_t^e}{\partial A_t} = \frac{\beta\pi N_t \theta_t}{(1-\theta_t q_t)} \frac{\partial r_t}{\partial A_t} > 0.$$

These claims are true since  $\frac{\partial r_t}{\partial A_t} > 0$ . Thus, the volume of equity traded increases unambiguously with  $A_t$ . As for the price of equity, equations (4.12) and (4.13) imply that as long as  $1 - \pi - \theta_t q_t > 0$ , the demand curve moves more than the supply curve, implying an increase in the price of equity.<sup>17</sup> An illustrative example that shows the effects of a change in  $A_t$  is presented in Figure 3. The shift of the supply curve is very small, and the two supply curves are not distinguishable.

<sup>17</sup>Numerically, we find this to be the case around the equilibrium for small values of  $\pi$ .

FIGURE 3. Model Demand and Supply of Equity for Various Levels of  $A$ 

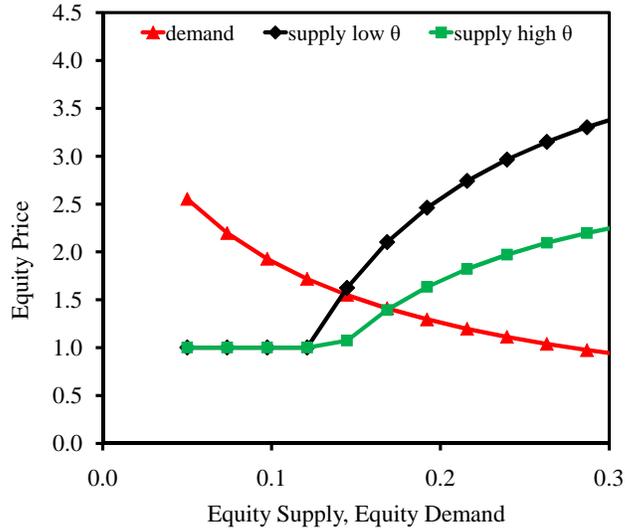
This figure shows the demand and supply of equity as a function of the price of equity in our model. The model parameters are  $\beta = 0.99, \eta = 1, \delta = 0.0226, \gamma = 0.36, \pi = 0.01, \theta = 0.2, \omega = 7.5257$ , and  $A_{low} = 1, A_{high} = 1.1$ . We set  $N = 9.9522$ , which is the steady state value for  $A_{low}$ .

Next consider a financial shock, i.e.,  $\Delta\theta_t$ . The equity demand curve  $D^e$  does not move. The shift in the equity supply curve  $S_t^e$  is determined by

$$\frac{\partial S_t^e}{\partial \theta_t} = \frac{1}{(1 - \theta_t q_t)^2} \beta \pi N_t [r_t + (1 - \delta)q_t] > 0.$$

Therefore, if  $\Delta\theta_t > 0$ , the equity supply curve moves up. As a result, the price of equity decreases and the quantity of equity traded increases. An illustrative example that shows the effects of a change in  $\theta_t$  is presented in Figure 4.

This analysis shows that productivity shocks as well as financial shocks cause changes in the price of equity  $q_t$ . We conclude that these shocks are candidate sources of asset price volatility. In the next section, we assess their quantitative importance.

FIGURE 4. Model Demand and Supply of Equity for Various Levels of  $\theta$ 

This figure shows the demand and supply of equity as a function of the price of equity in our model. The model parameters are  $\beta = 0.99$ ,  $\eta = 1$ ,  $\delta = 0.0226$ ,  $A = 1$ ,  $\gamma = 0.36$ ,  $\pi = 0.01$ ,  $\omega = 7.5257$ , and  $\theta_{low} = .2$ ,  $\theta_{high} = .3$ . We set  $N = 9.9522$ , which is the steady state value for  $\theta_{low}$ .

## 5. QUANTITATIVE ANALYSIS

In this section we first present the calibration of our model. We then study the implications of the model for business cycles and asset prices. Our main results are as follows. Our benchmark model generates high asset price volatility, namely, about 80% relative to the aggregate U.S. stock market. The equity premium implied by the model is more than 70% relative to the data. At the same time, the model generates the investment volatility observed in the data, unlike models with adjustment costs. Finally, a version of the model with productivity shocks and no financial shocks ( $\theta$  constant) generates only modest asset price volatility as in other models of this kind, such as Gomes, Yaron, and Zhang (2003).

### 5.1. Model Calibration and Solution Method.

We divide variables in the model into two groups. The first group consists of utility and technology parameters. The second group consists of: (1) the parameter  $\pi$  representing the fraction of firms with investment opportunities, (2) the stochastic process for the financial shock,  $\theta_t$ , and (3) the stochastic process for the productivity shock,  $A_t$ .

5.1.1. *Utility and Technology Parameters.* We divide the utility and technology parameters into two groups: (1) parameters that we take from the literature: share of capital in output production  $\gamma = 0.36$ , subjective quarterly discount factor  $\beta = 0.99$ , the workers' labor supply elasticity parameter  $\eta = 1$ ,<sup>18</sup> (2) parameters that we choose so that our model in steady state matches chosen moments in the data. We set the quarterly depreciation rate  $\delta = 2.26\%$  so that the average annual investment to capital ratio in the steady state of our model is 9.35% as in the data for the period from 1964 to 2008. We set the scaling parameter of the workers' utility function  $\omega = 8.15$  so that the labor supply in steady state is  $l_s = 0.3$ .

5.1.2. *Investment Opportunity Parameter.* Parameter  $\pi$  represents the fraction of firms with an investment opportunity. Using data from the Longitudinal Research Database (LRD), covering 12,000 U.S. manufacturing plants over the period 1972-1989, Doms and Dunne (1998) document several aspects of "infrequent" and "large" capital adjustment.<sup>19</sup> Although this type of capital adjustment has typically been taken as evidence of the existence of fixed costs of investment, it can also be thought of as evidence of an infrequent arrival of

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<sup>18</sup>We perform a sensitivity analysis on  $\beta$  and  $\eta$ . We report these results in Appendix C.

<sup>19</sup>For example, more than half of the establishments exhibit capital growth close to 50% in a single year, and more than 25% of an average plant's gross investment over the 17-year period is concentrated in a single year/project. One can expect that aggregation over plants to the company level smooths some of the discreteness observed for individual plants. However, using the COMPUSTAT database for the period 1974-1993, Abel and Eberly (2002) find that the distribution of investment rates is positively skewed in a large sample of publicly traded U.S. companies. In other words, investment is also lumpy at the firm level. See also Becker and Haltiwanger (2006), Caballero, Engel, Haltiwanger, Woodford, and Hall (1995), Cooper, Haltiwanger, and Power (1999), Cooper and Haltiwanger (2006), Khan and Thomas (2003), Khan and Thomas (2008), and Thomas (2002), among others.

TABLE 1. Average Fraction of Firms with an Investment Spike in 1965-2008

Investment spike threshold $\widehat{\frac{investment}{ppent}}$	Average fraction of firms with $\frac{investment_t}{ppent_{t-1}} > \widehat{\frac{investment}{ppent}}$
20%	10.6%
25%	7.2%
30%	5.2%
35%	4.3%

investment opportunities. Our estimation of parameter  $\pi$  is based on this interpretation of the observed lumpy investment.

In our model firms with an investment opportunity generally invest a lot relative to their size. Therefore, we calibrate  $\pi$  by matching it to the fraction of firms with an investment spike in the data. However, the definition of an investment spike in the literature is not unique. Gourio and Kashyap (2007) use two definitions: investment exceeding 20% and investment exceeding 35% of the beginning of the period capital. We follow their definitions and construct our own firm-level measures of investment spikes.

In particular, we construct the time series for investment as an increase in “Net property, plant and equipment,” i.e., variable  $ppent$  in the COMPUSTAT database,  $investment_t = ppent_t - ppent_{t-1}$ . We then determine the fraction of firms whose investment at time  $t$  exceeds a given fraction of  $ppent_{t-1}$ . As in Gourio and Kashyap (2007), we weigh firms by beginning of the period capital  $ppent_{t-1}$ . Table 1 reports the average fraction of firms with an investment spike over the period 1965-2008 for several alternative definitions of the investment spike. We set the annual  $\pi = 0.06$  as a benchmark. However, we do think that this parameter is hard to pin down. Therefore we perform a detailed sensitivity analysis in Appendix C.

5.1.3. *Productivity and Financial Shock Processes.* We construct the time series for productivity shocks,  $A_t$ , using the time series of output, capital, and labor with the assumption of a Cobb-Douglas production technology with the capital output share of  $\gamma = 0.36$ . We define

$\hat{z}_t := \log(A_t)$ , and use  $z_t$ , the linearly detrended version of  $\hat{z}_t$ , as a realization of the shock process for the consumption good production technology.

We recover the time series for  $\theta_t$  directly from the data in a similar procedure as for the productivity shock, which makes the model  $\theta_t$  consistent with the data. There is one caveat. In our model, the variable  $\theta_t$  represents the fraction of investment in period  $t$  that is financed externally. In the model, equity financing is the only external financing option that firms have, but in reality firms use both equity and debt to raise capital. Therefore, we bring our model to the data based on the total amount of outside financing. Specifically, we construct the time series of  $\theta_t$  for the nonfinancial corporate sector using Flow of Funds data (for precise definitions of these variables see Appendix B). We define

$$\theta_t = \frac{(\text{Funds Raised in Markets})_t}{(\text{Fixed Investment})_t}.$$

Having constructed the quarterly time series for  $z_t$  and  $\theta_t$  for the time period 1964-2008, we estimate the stochastic processes for  $z_t$  and  $\theta_t$  as

$$\begin{aligned} z_{t+1} &= \rho_z z_t + \varepsilon_{z,t} \\ \theta_{t+1} &= \mu_\theta + \rho_\theta(\theta_t - \mu_\theta) + \varepsilon_{\theta,t} \\ E \begin{pmatrix} \varepsilon_{z,t} \\ \varepsilon_{\theta,t} \end{pmatrix}^2 &= \begin{bmatrix} \sigma_{\varepsilon_z}^2 & \text{corr}(\varepsilon_z, \varepsilon_\theta) \\ \text{corr}(\varepsilon_z, \varepsilon_\theta) & \sigma_{\varepsilon_\theta}^2 \end{bmatrix}, \end{aligned}$$

where  $\varepsilon_{z,t}$  and  $\varepsilon_{\theta,t}$  are i.i.d. with standard deviations  $\sigma_{\varepsilon_z}$  and  $\sigma_{\varepsilon_\theta}$ , respectively. We omit the cross terms in the bivariate AR(1) process estimation because when included they are insignificant. Table 2 summarizes the parameters in our model.

TABLE 2. Parameters in the Model

Parameter	Symbol	Value
Capital share in output	$\gamma$	0.36
Capital depreciation rate	$\delta$	0.0226
Quarterly discount factor	$\beta$	0.99
Workers risk-aversion parameter	$\eta$	1
Scaling factor in workers' utility	$\omega$	8.15
Fraction of firms with investment opportunity	$\pi$	0.015
Average of financial shock	$\mu_\theta$	0.284
Persistence of financial shock	$\rho_\theta$	0.651
Standard deviation of financial shock	$\sigma_{\varepsilon_\theta}$	0.167
Persistence of productivity shock	$\rho_z$	0.95
Standard deviation of productivity shock	$\sigma_{\varepsilon_z}$	0.006
Correlation between innovations of $\theta$ and $z$	$corr(\varepsilon_z, \varepsilon_\theta)$	-0.073

5.1.4. *Solution Method.* We are able to solve our model exactly, and therefore do not rely on log-linear approximations.<sup>20</sup> Having solved the model, we simulate it by generating random series of the primitive shocks  $A_t$  and  $\theta_t$  using the estimated parameters for these processes.<sup>21</sup> Having simulated the model, we compute a set of statistics and compare them with the data. We should emphasize that we do not target any of the statistics reported in Tables 3 and 4, an approach taken, for example, by Boldrin, Christiano, and Fisher (2001), who pick the discount factor so as to match the mean observed risk-free rate. We now proceed to the detailed discussion of our results.

## 5.2. Standard Business Cycle Statistics.

Table 3 reports the standard business cycle statistics in the model. The data column reports the U.S. statistics for the period 1964-2008.<sup>22</sup> Column (1) reports the statistics for a version

<sup>20</sup>Other quantitative papers that use variants of the Kiyotaki and Moore (2011) model such as Del Negro, Eggertsson, Ferrero, and Kiyotaki (2011) and Ajello (2012) do rely on log-linear approximations around steady state. Shi (2011) analyzes a simplified version of the model without relying on log-linear approximations. He provides a detailed analysis of impulse responses, i.e. equilibrium responses to shocks, in several versions of the model. In contrast, we are primarily interested in business cycle properties of our model.

<sup>21</sup>We approximate the processes on a 25-point grid in the  $z \times \theta$  space using the Tauchen approximation method (see, Tauchen (1986)), and then use  $A_t = \exp(z_t)$ .

<sup>22</sup>We obtain quarterly data from the Current Employment Statistics provided by the Bureau of Labor Statistics, National Income and Product Accounts and Fixed Asset Tables provided by the Bureau of Economic

of the model without the financial constraint. This model is closely related to the standard stochastic one sector growth model. Column (2) reports the statistics for a version of the model with the financial constraint and productivity shocks, i.e. stochastic  $A_t$ . In this version of the model, there are no financial shocks, i.e.  $\theta_t$  is constant at its mean level. Column (3) reports the statistics for the model with the financial constraint and both productivity shocks and financial shocks, i.e both  $A_t$  and  $\theta_t$  are stochastic.

This table shows that financial shocks do not affect output volatility and persistence.<sup>23</sup> This indicates that the process for output in our model is determined by the process for the productivity shock (assumed to be the same in all three versions of the model). As discussed in Section 4, labor and output are perfectly correlated, and their relative volatility is determined by the parameter  $\eta$ . Therefore, the properties of labor are not affected by financial shocks either.

Table 3 also shows that financial shocks significantly affect investment. In the version of the model with no financial constraint, shown in column (1), investment is significantly less volatile than in the data. This result is not at odds with the results of the standard one-sector growth model, in which productivity shocks generate the investment volatility observed in the data. Our model in column (1) is similar to the standard one sector growth model with one exception. In our model, investment is determined by entrepreneurs only. Log utility implies that they save a fixed fraction of their income  $r_t K_t + (1 - \delta)K_t$ , which is significantly less volatile than workers' income. This results in low investment volatility in this version of our model. In the version of the model with the financial constraint but no financial shocks, shown in column (2), investment volatility is further decreased by the endogenous changes in the price of equity. When productivity increases, the asset demand increases, as discussed

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Analysis, COMPUSTAT, Flow of Funds, CRSP, and Global Financial Data. Details of the construction of the time series can be found in Appendix B.

<sup>23</sup>Jermann and Quadrini (2012) build a model in which financial shocks affect output significantly. However, their model does not generate much asset price volatility.

TABLE 3. Standard Business Cycle Statistics in the Model

Statistic	Data	(1)	(2)	(3)
		Without FC	With FC	With FC
		$A$ stochastic	$\theta$ constant $A$ stochastic	$\theta$ stochastic $A$ stochastic
$\sigma_Y$	1.52	1.18	1.19	1.18
$\sigma_I$	5.00	1.70	1.32	5.12
$\sigma_C$	0.85	1.01	1.15	1.93
$\sigma_L$	1.73	0.59	0.60	0.59
$\rho_Y$	0.87	0.68	0.67	0.67
$\rho_I$	0.85	0.68	0.67	0.40
$\rho_C$	0.90	0.68	0.67	0.47
$\rho_L$	0.92	0.68	0.67	0.67
$\rho(Y, I)$	0.90	1.00	1.00	0.23
$\rho(Y, C)$	0.85	1.00	1.00	0.61
$\rho(Y, L)$	0.87	1.00	1.00	1.00

This table reports the business cycle statistics for three versions of our model. Statistics are computed based on 100 replications of size 180. The symbol  $\sigma_x$  represents the standard deviation of variable  $x$ ,  $\rho_x$  represents the autocorrelation of  $x$ , and  $\rho(x, y)$  represents the correlation between  $x$  and  $y$ . All variables are logged and HP filtered before statistics are computed. Standard deviations are measured in percentage terms. The data column reports statistics for quarterly U.S. data for the period 1964:1-2008:4. Column (1) reports the statistics for a version of the model with no financial constraint. Column (2) reports the statistics for a version of the model in which the financial constraint is constant at its mean of 0.28 and is binding. Column (3) reports statistics for a version of the model in which the financial constraint is binding and its tightness is stochastic.

in Section 4. However, the increase in the equilibrium quantity demanded will be smaller than if supply was infinitely elastic (column (1)). Therefore, relatively less new capital will be produced and investment will be less volatile.

In the version of the model with financial shocks, shown in column (3), investment is significantly more volatile than in the other versions of the model. In fact, investment volatility is very close to the observed volatility in the data, even though it was not targeted. This is an important improvement relative to models with habit persistence, such as Guvenen (2009), and models with factor immobility, such as Boldrin, Christiano, and Fisher (2001). These models do not generate the investment volatility observed in the data.

These results indicate that in our model, financial shocks play a more important role in investment fluctuations than productivity shocks. This finding is further supported by the relatively low persistence of investment coming from the lower persistence of  $\theta_t$  relative to that of  $A_t$ . In contrast, we have argued above that output dynamics is driven by productivity shocks only. The low correlation between  $\theta_t$  and  $A_t$  that we estimated from the data therefore translates into a relatively low correlation between investment and output.

### 5.3. Asset Price Statistics.

In our model the return on equity is determined by

$$r^e = \frac{r_t + (1 - \delta)q_t}{q_{t-1}} - 1.$$

The corresponding counterpart in the data is the real value weighted stock return.

We denote the total market value in the model as  $val_t = q_t N_t$ . The corresponding counterpart in the data is the series *totval* from the CRSP database.

We construct the model shadow risk-free rate as follows. The shadow price of a risk-free asset, which is not directly traded in our model, is given by

$$p_t(s^t) = \beta E_t \left[ \frac{u'(c_{t+1})}{u'(c_t)} \right].$$

Noninvesting entrepreneurs are not constrained in their asset holdings, whereas the investing entrepreneurs would like to sell more assets, but they cannot because of the financial constraint. Therefore, we construct the risk-free rate as the shadow risk free rate of the unconstrained noninvesting entrepreneurs.<sup>24</sup>

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<sup>24</sup>This approach is similar to the one taken in Gomes, Yaron, and Zhang (2003). Alternatively, we could rationalize our choice by thinking about borrowing-constrained entrepreneurs. The risk-free rate would then be determined by the shadow risk free rate of the noninvesting entrepreneurs, because investing entrepreneurs find investing and selling equity more profitable than buying the risk-free asset.

We get the following equation:

$$r_t^f = \frac{1}{p_t(s^t)} = \frac{1}{q_t E_t \left[ \frac{1}{r_{t+1} + (1-\delta)q_{t+1}} \right]}$$

5.3.1. *Equity Premium and Asset Price Volatility.* Table 4 reports our results for quarterly asset prices and returns. The definition of each column is the same as in Table 3.

Table 4 shows that the version of the model with no financial constraint, column (1), generates very little volatility in equity returns and a very small equity premium. Recall that in this version of the model, the price of equity is  $q_t = 1$  at all times. Therefore, there are no capital gains and the return volatility comes solely from the modest volatility in  $r_t$ .

This table also shows that the version of the model with a financial constraint, but no financial shocks, column (2), generates a small volatility in equity returns. The standard deviation of the equity return is 0.77%, which is similar to Gomes, Yaron, and Zhang (2003). They get an equity return volatility of about 1% in a model with endogenous borrowing constraints and productivity shocks as the only source of uncertainty. This result highlights that in our model (and in theirs as well), the financial constraint is not a strong propagator of productivity shocks.<sup>25</sup>

Our most important result shown in Table 4 is that financial shocks generate a high asset price volatility and asset return volatility, which is comparable to the data. The benchmark model of column (3) generates over 80% of the observed volatility in asset prices and total market value, namely 9.69% in the model relative to 11.85% in the data in quarterly frequency for the asset price  $q_t$ . In addition, the model generates a quarterly equity premium of 0.85%, i.e. over 70% of what we observe in the data, which is particularly interesting considering

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<sup>25</sup>We find that our model is able to generate high asset price volatility even with a constant  $\theta$ . However, we need to significantly increase the volatility of productivity shocks, which would generate a counterfactually high volatility in investment and output. In “disaster” papers, such as Gourio (2012), this does not happen because the driving force is not shocks to the levels of productivity but rather to the beliefs about a large future productivity and capital level drop.

TABLE 4. Asset Price Statistics in the Model

Statistic	Data	(1)	(2)	(3)
		Without FC <i>A</i> stochastic	With FC $\theta$ constant <i>A</i> stochastic	With FC $\theta$ stochastic <i>A</i> stochastic
$\sigma_q$	11.85	0	0.93	9.69
$\sigma_{r^e}$	8.63	0.07	0.77	10.8
$\sigma_{val}$	10.72	0.13	0.92	9.74
$\rho_q$	0.74	1	0.67	0.38
$E(r^e)$	1.49	1.01	0.89	1.36
$E(r^f)$	0.30	1.01	0.88	0.51
$\sigma_r^f$	0.68	0.06	0.16	5.45
$E(r^e) - E(r^f)$	1.19	0.00	0.008	0.85

This table reports the asset price statistics for three versions of our model. Statistics are computed based on 100 replications of size 180. The symbol  $\sigma_x$  represents the standard deviation of variable  $x$ ,  $\rho_x$  represents the autocorrelation of  $x$ , and  $\rho(x, y)$  represents the correlation between  $x$  and  $y$ . All variables with the exception of the returns are logged and HP filtered before statistics are computed. Standard deviations and returns are measured in percentage terms. The data column reports statistics for quarterly U.S. data in the period 1964:1 - 2008:4. Column (1) reports the statistics for a version of the model with no financial constraint. Column (2) reports the statistics for a version of the model with the financial constraint and shocks to productivity only. The financial shock  $\theta$  is constant at its mean of 0.28. Column (3) reports statistics for a version of the model with the financial constraint and both productivity and  $\theta$  stochastic.

that entrepreneurs in our economy have logarithmic utility.<sup>26</sup> These results are driven by the fact that in our model entrepreneurs determine the asset prices. Our model generates high asset price volatility and a large equity premium, because the volatility of entrepreneurs' consumption is large relative to the volatility of workers' consumption.<sup>27</sup> The important difference between entrepreneurs and workers in our model is that entrepreneurs participate in asset markets whereas workers do not.<sup>28</sup> In the data the consumption of stock markets

<sup>26</sup>In their classic paper, Mehra and Prescott (1985) show that in a representative agent model one can generate only a very small equity premium with a risk aversion parameter smaller than 10.

<sup>27</sup>The standard deviation of entrepreneurs' consumption is 9.37% while the standard deviation of workers' consumption is 1.18% in the benchmark model with both productivity and financial shocks.

<sup>28</sup>In this respect our model is similar to Guvenen (2009)'s model with limited stock market participation.

participants is very volatile as documented by e.g. Mankiw and Zeldes (1991), Vissing-Jorgensen (2002), Attanasio, Banks, and Tanner (2002) and Malloy, Moskowitz, and Vissing-Jorgensen (2009). Guvenen (2009), provides a short survey documenting that consumption of stock market participants is much more volatile than that of non-participants. Our model is thus in line with this empirical evidence.

One might think that the high asset price volatility generated by our benchmark model of column (3) follows simply from the high volatility of the financial shocks. We should emphasize that this is not the case. In our model asset price volatility follows from the interaction of financial shocks and the random arrival of investment opportunities. If all entrepreneurs had investment opportunities in each time period, the financial constraint would not bind, the price of equity would be one, and the volatility of financial shocks would be irrelevant.

*5.3.2. Risk Free Rate Volatility.* Our benchmark model with financial shocks generates a volatility in the (shadow) risk free rate which is higher than the one observed in the data. This shortcoming is not specific to our model. It is also the case for most models with capital supply frictions (adjustment costs, factor immobility) that generate high equity premia. However, our model does improve relative to that literature. For example, Boldrin, Christiano, and Fisher (2001) build a model with habit persistence and factor immobility. Their model generates an annual risk free rate volatility of 24.6%. Our model on the other hand generates an annual risk free rate volatility of 15.3%. We conclude that our model is able to generate high volatility in asset returns with a much smaller volatility in the risk free rate than Boldrin, Christiano, and Fisher (2001).<sup>29</sup>

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<sup>29</sup>We conjecture that one could further improve on this dimension by assuming that entrepreneurs have a recursive Epstein-Zin utility function. This is suggested by the results obtained in Guvenen (2009) who uses the Epstein-Zin utility function and generates a less volatile risk free rate. This extension of the model would most likely not change the main conclusion of our paper, which is that time varying financial shocks play a significant role in generating asset price volatility.

5.3.3. *Asset Return Predictability.* We further investigate whether logged (cumulative) asset returns are predictable. Frequently, researchers ask whether returns are predictable by the logged price-to-dividend ratio. However, we do not explicitly model dividends in our model. Therefore, we use the marginal product of capital, i.e. the return on capital  $r_t$ , instead of dividends.<sup>30</sup> We investigate whether the price-to-capital return ratio predicts the (cumulative) return 1 quarter and 5 quarters ahead. We report our results and compare them to Guvenen (2009)'s in Table 5. We find that in our benchmark model with both productivity and financial shocks the price-to-capital return ratio is negatively correlated with the return. In addition, the predictive power is increasing with the time horizon. These properties are consistent with what has been found in the data for the price-to-dividend ratio.

In the last two columns of Table 5 we report the results for a version of our model with a financial constraint, but with shocks to productivity only. We find that the variability in the price-to-capital return ratio is limited and hence the predictive power is small. In addition, the regression coefficients are positive, which is counterfactual. Therefore, adding financial shocks to the model significantly improves its performance along the predictability dimension.

5.3.4. *Asset Price Cyclicalities.* The correlation between aggregate asset prices and output in the data is 0.39. In our benchmark model with both productivity and financial shocks the correlation is 0.11. This is an improvement relative to the version of the model with no financial constraint, in which asset prices are constant and hence uncorrelated with output. It is also an improvement relative to the version of the model with a financial constraint, but without financial shocks, in which asset prices and output are perfectly correlated. Therefore,

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<sup>30</sup>One can also think of price-to-return on capital ratio as a measure of the value-to-earnings ratio, which has also been used in predictability regressions. In our model the firm value would be  $q_t k_t$  and earnings would be  $r_t k_t$ . The value-to-earnings ratio is therefore equal to the price-to-return on capital ratio:  $\frac{q_t k_t}{r_t k_t} = \frac{q_t}{r_t}$ .

TABLE 5. Asset Return Predictability

Horizon	Data		Guvenen (2009)		Benchmark		$\theta$ constant	
	Coefficient	$R^2$	Coefficient	$R^2$	Coefficient	$R^2$	Coefficient	$R^2$
1	-0.21	0.07	-0.19	0.09	-0.49	0.26	0.33	0.05
5	-0.70	0.23	-0.77	0.28	-0.99	0.51	1.37	0.19

This table reports the asset return predictability for the data, our benchmark model and Guvenen (2009)'s model. The data column is taken from Guvenen (2009) and contains predictability of logged cumulative stock returns by the logged price-to-dividend ratio. Column 'Guvenen (2009)' reports the same statistics for his model. Column 'Benchmark' reports the predictability of logged cumulative stock return by the logged price-to-capital return ratio for a version of our model with the financial constraint and both productivity shocks and financial stochastic. Column ' $\theta$  constant' reports the same statistics for a version of the model with the financial constraint and shocks to productivity only. The statistics are computed as means of 100 replications of the quarterly model of size 180. In order to run these regressions, we do not HP filter the model generated data so as to be able to take logs.

adding financial shocks helps explain the relatively low correlation between asset prices and output that we observe in the data.

Our model generates a negative correlation between asset prices and investment. This feature is not specific to our model. In this class of models, changes in the tightness of the financial constraint,  $\theta_t$  in our model, directly affect the amount of investment but do not affect the productivity of the existing capital. A tighter constraint implies less investment and less new capital, which makes old capital (and new capital as well) more valuable to agents in the economy. Therefore, a tighter constraint implies higher asset prices.<sup>31</sup>

In the data the correlation between asset prices and investment is positive over the whole sample period. Investigating the subperiods, we find that the correlation is small and positive in the subsample ending in 1982, negative in the subsample of 1983 - 1997 and large and positive thereafter (the pattern is in fact similar for the correlation between asset prices and

<sup>31</sup>We have found this to be true in the original Kiyotaki and Moore (2011) model in which the friction takes the form of limited resaleability. In this model an entrepreneur can sell only a fraction of his assets at a point in time to finance new investments. Tightening this constraint implies a decrease in investment and an increase in the asset price by the same logic. Shi (2011) discusses this result in detail in several versions of the Kiyotaki and Moore (2011) model. A similar logic would apply to shocks to monitoring costs in models with costly monitoring or to shocks in the value of default in limited commitment models.

output). Therefore, our model accounts better for the eighties and nineties than for the most recent period. To account for the changes in the relationship between asset prices and investment we would, most likely, need a richer model with several types of shocks affecting the economy in different ways.<sup>32</sup>

#### 5.4. Sensitivity Analysis and Extensions.

We next provide a brief summary of the results of our sensitivity analysis. A detailed discussion along with all tables can be found in Appendix C. In terms of our main result regarding high asset price volatility, we find the following. (1) We find that the model generates substantial asset price volatility for a wide range of processes for the financial shock. However, we also find that asset price volatility is quite sensitive to the parameters of the financial shock process  $\theta_t$ . (2) In a similar vein we find that asset price volatility is substantial for a set of different values for the probability of having an investment opportunity  $\pi$ . In general, we find that asset price volatility is quite sensitive to the choice of  $\pi$ . (3) Asset price volatility is unaffected by changes in the labor supply elasticity parameter  $\eta$ . (4) Asset price volatility does not change significantly if one uses standard balanced growth path preferences for the workers. (5) Asset price volatility increases with the discount factor  $\beta$ , but is high even for smaller  $\beta$ 's such as  $\beta = .98$ .

To assess whether our results are robust to adding more assets to the model, we solve a version of the model with fiat money as in Kiyotaki and Moore (2011). In this version of the model entrepreneurs are allowed to hold any non-negative amount of fiat money, which is available in fixed positive net supply.<sup>33</sup> We find that a monetary equilibrium does not exist

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<sup>32</sup>In a recent paper Gertler and Kiyotaki (2010) propose a ‘capital destruction shock’ to explain the recent recession. This type of shock leads to a drop in output and the total value of capital in our environment as well. However, it is not clear how to recover the time series for this shock from the data in a way consistent with our estimation of the productivity and financial shock processes.

<sup>33</sup>When we numerically solve the stochastic versions of the model with fiat model, we find that the non-negativity constraint on money holdings binds for the investing entrepreneurs in each state. This has been shown by Kiyotaki and Moore to be true in steady state.

for our benchmark parameters, i.e. in equilibrium money has no value. The reason is that the financial constraint is not tight enough. Therefore, the dynamics of the model is exactly the same as in our model without money. In this sense, our benchmark results are robust to adding money to the model. To compare our model to the model with money in case when the monetary equilibrium exists we tighten the financial constraint by increasing the depreciation rate  $\delta$  to 4%. In this case fiat money is valued in equilibrium. We keep the rest of the parameters the same. We find that the dynamics of the monetary model is still very similar to our benchmark model.<sup>34</sup> Most importantly, the asset price volatility is almost the same as in our benchmark model without money.

Adding bonds to the model is much more involved. If we allow firms to issue bonds, we need to think about the limitations in bond issuance relative to equity issuance that firms face. With no limits on the issuance of bonds entrepreneurs will try to avoid the constraint on equity issuance by issuing more bonds. Assuming that bonds are available in positive net supply (e.g. issued by the government) and a non-negativity constraint on bond holdings solves this issue. However, aggregation becomes impossible, because the non-negativity constraint on bond holdings might bind depending on the individual and aggregate states. While this is an interesting extension of our benchmark model, we leave analyzing more complex models with multiple assets as a task for future research.

## 6. CONCLUSION

In this paper we quantify the role of financial shocks in business cycle fluctuations and asset price volatility. We build a production economy model with heterogeneous entrepreneurs that distinguishes itself from the standard neoclassical growth model along two dimensions.

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<sup>34</sup>These results are available upon request.

First, in each period only a fraction of entrepreneurs can start new investment projects. Second, a new investment project cannot be fully financed externally due to a financial constraint. We introduce financial shocks that affect the tightness of the financial constraint over time.

We first show that both financial shocks and productivity shocks result in fluctuations in the price of equity. We then calibrate the model to the U.S. data and focus on a comparison of two models: one with productivity shocks but without financial shocks and one with both shocks. We find that the model without financial shocks generates only low asset price volatility, similar to a related model in Gomes, Yaron, and Zhang (2003). We find that our model, which includes financial shocks, generates about 80% of the asset price volatility relative to the aggregate stock market. The quarterly equity premium in the model is 0.85%, i.e. over 70% of what we observe in the data. At the same time, our model fits key aspects of the behavior of aggregate quantities. In particular, our model matches the volatility of aggregate investment, unlike the existing models with convex adjustment costs, such as Guvenen (2009).

In contrast to other models, high asset price volatility in our model originates from financial frictions and financial shocks. Therefore, the equilibrium in our model is not first best and there is a role for a Pareto-improving government interventions. More specifically, the government might want to participate in financial markets and relax the financial constraints of the investing entrepreneurs. A formal consideration of the role of government in models like ours remains a topic for future research.

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## APPENDIX A. PROOF OF THEOREM 4.3

Theorem 4.3: In steady state the financial constraint binds and the price of equity,  $q_t$ , is greater than one if and only if  $\theta < \frac{\delta - \pi}{\delta}$ .

Proof: (i)  $\Leftarrow$  We prove this part by contradiction. Assume that  $q_t = 1$ . Denote by  $J_t$  the set of indexes identifying entrepreneurs with an investment opportunity at time  $t$  and use capital letters for aggregate variables. Since the arrival of the investment opportunity is i.i.d., investing entrepreneurs will hold a fraction  $\pi$  of the total equity holdings at the beginning of period  $t$ :  $\int_{j \in J_t} n_t^j dj = \pi N_t$ . Since they solve the same problem as noninvesting entrepreneurs, investing entrepreneurs will hold a fraction  $\pi$  of total equity holdings in period  $t+1$  as well:  $\int_{j \in J_t} n_{t+1}^j = \pi N_{t+1}$ . In any equilibrium, the financial constraint must be satisfied for all investing entrepreneurs. Integrating over the set  $J_t$ , we get

$$(A.1) \quad \begin{aligned} \int_{j \in J_t} n_{t+1}^j dj &\geq (1 - \theta) \int_{j \in J_t} i_t^j dj \\ \pi N_{t+1} &\geq (1 - \theta) I_t \end{aligned}$$

In steady state  $N_t = N_{t+1} = N$  and  $I_t = \delta N$ . Thus, we can rewrite equation (A.1) as

$$\pi - (1 - \theta)\delta \geq 0.$$

This contradicts our assumption, and thus it must be that  $q_t > 1$ . By Lemma 4.1, this implies that the financial constraint binds. The proof also makes it clear that  $\theta \geq \frac{\delta - \pi}{\delta} \iff \exists$  a steady state equilibrium with  $q = 1$  and the FC slack.

(ii)  $\implies$  Suppose in steady state the financial constraint binds and  $q > 1$ . Using the steady state conditions  $N_{t+1} = N_t$  and  $I_t = \delta N_t$  and dropping the time indexes, we can rewrite equations (4.9) and (4.11) from Section 4 along with the binding financial constraint

as

$$\begin{aligned}
 q^R N^i &= \beta \pi N [r + (1 - \delta)q] \\
 q N^s &= \beta (1 - \pi) N [r + (1 - \delta)q] \\
 N^i &= (1 - \theta) \delta N \\
 N &= N^s + N^i.
 \end{aligned}$$

This can be simplified to

$$(A.2) \quad q^R (1 - \theta) \delta N = \beta \pi N [r + (1 - \delta)q]$$

$$(A.3) \quad q [1 - \delta(1 - \theta)] N = \beta (1 - \pi) N [r + (1 - \delta)q].$$

Plugging in for  $\beta N [r + q(1 - \delta)]$  from equation (A.2) into equation (A.3) and simplifying further, we get

$$\begin{aligned}
 \pi q [1 - \delta(1 - \theta)] &= (1 - \pi)(1 - \theta q) \delta \\
 \pi q [1 - \delta(1 - \theta)] &= (1 - \pi) \delta - \theta \delta (1 - \pi) q \\
 q [\pi - \pi \delta + \pi \delta \theta + \theta \delta - \pi \theta \delta] &= (1 - \pi) \\
 q &= \frac{(1 - \pi) \delta}{\pi - \pi \delta + \theta \delta} \\
 q &= \frac{(1 - \pi) \delta}{(1 - \pi) \delta + \pi - \delta + \theta \delta}
 \end{aligned}$$

Both the numerator and the denominator are positive, and therefore  $q > 1$  implies that  $\pi - \delta + \theta \delta < 0$ , which is equivalent to  $\theta < \frac{\delta - \pi}{\delta}$ .  $\square$

**Theorem A.1.** *Suppose workers' discount factor is  $\beta$ . Then they do not save in a steady state in which the FC binds and  $q > 1$ .*

Proof: To prove this claim, we show that the return on equity in steady state is lower than the time preference rate, i.e.

$$\frac{r + (1 - \delta)q}{q} < \frac{1}{\beta}$$

Simplifying equation A.3 above and using the fact that  $\theta < \frac{\delta - \pi}{\delta}$  in a steady state in which  $q > 1$  as shown in theorem 4.3 we can write

$$\frac{r + (1 - \delta)q}{q} = \frac{1 - \delta(1 - \theta)}{\beta(1 - \pi)} = \frac{1 - \delta + \delta\theta}{\beta(1 - \pi)} < \frac{1 - \delta + \delta\frac{\delta - \pi}{\delta}}{\beta(1 - \pi)} = \frac{1}{\beta}$$

□

## APPENDIX B. CONSTRUCTION OF THE TIME SERIES

### B.1. Macroeconomic variables.

Databases used for 1964q1-2008q4:

- (1) CES-BLS: Current Employment Statistics survey published by the Bureau of Labor Statistics.
- (2) FAT-BEA: Fixed Asset Tables published by the Bureau of Economic Analysis.
- (3) NIPA-BEA: National Income and Product Accounts published by the Bureau of Economic Analysis.
- (4) Flow of Funds.

Series generated:

- (1) Hours  $L$ : from CES-BLS:
  - Hours = average weekly hours · average number of workers.

- Average weekly hours: in private sector, series CES0500000036.
  - Average number of workers: average number of workers in private sector over a quarter computed using monthly data in series CES0500000001.
- (2) Real capital  $K$  : we generate quarterly data by interpolating the yearly “Fixed assets and consumer durable goods,” line 2 in Table 1.2 in FAT-BEA.
- (3) Output  $Y$ : real GDP, line 1 in table 1.1.6 in NIPA-BEA.
- (4) Productivity series  $A_t$ : generated from the capital and hours series as

$$A_t = \frac{Y_t}{L_t^{.64} \cdot K_t^{.36}}$$

- (5) Nominal capital  $NK$  : we generate quarterly data by interpolating the yearly “Fixed assets and consumer durable goods,” line 2 in Table 1.1 in FAT-BEA.
- (6) Nominal Investment  $NI$  = nominal private fixed investment + nominal durable consumption good expenditure + nominal government gross investment.
- nominal private fixed investment: line 7 in Table 1.1.5 in NIPA-BEA.
  - nominal durable consumption good expenditure: line 4 in Table 1.1.5 in NIPA-BEA.
  - nominal government gross investment: line 3 in Table 3.9.5 in NIPA-BEA, does not include investment in inventories.
- (7) Real investment  $I$ : Nonfarm nonfinancial corporate businesses fixed investment, line 12 in the Flow of Funds Table F.102 deflated using the deflator for gross private domestic investment constructed using line 7 in NIPA-BEA 1.1.5 and 1.1.6. We choose this series because we use it to estimate  $\theta$ . The time-series properties of various real investment measures are very similar. Excluding government, inventories, and durable consumption makes the series slightly more volatile (standard deviation of 5.00% versus 4.41%).

(8) Real consumption  $C = \text{Nondurable goods} + \text{Services}$ .

- Nondurable goods: line 4 in Table 1.1.5 in NIPA-BEA.
- Services: line 6 in Table 1.1.5 in NIPA-BEA.

The real counterparts of these nominal series are only reported starting in 1995. To generate the real series, we deflated these nominal series by a personal consumption expenditure deflator constructed from line 2 in Tables 1.1.5 and 1.1.6. The correlation between the deflator for personal consumption expenditure and nondurable goods and services from 1995 onwards is .991 and .997, respectively.

## B.2. Financial Variables.

- (1) Asset price  $q$  was constructed from the broad Wilshire 5000 Total Market Index for the period 1974-2008. We have constructed the same series for the S&P 500 Composite Price Index for the 1964-2008 period. The time-series properties of HP-filtered logged versions of these indexes are very similar. The Wilshire 5000 is slightly more volatile. Both raw series were recovered from the Global Financial Data database and computed as averages over the given quarter.
- (2) Asset return  $r^e$ : series *vwretd* from the CRSP database (Center for Research in Security Prices), value-weighted returns including distributions from NYSE, AMEX, and NASDAQ. We constructed quarterly data from monthly observations.
- (3) Total market value *val*: series *totval* from the CRSP database. We constructed quarterly data as averages over monthly observations.
- (4) Real risk-free rate  $r^f$  is the three-month T-bill as priced on the secondary market recovered from the Global Financial Data database deflated by CPI.
- (5) CPI: nominal returns are deflated using the CPI series from the BLS database, series ID: CPI-U, BLS CUUR0000SA0.

### B.3. Construction of $\theta_t$ .

We construct the time series for  $\theta_t$  from the data as follows. The variable  $\theta_t$  in the model represents the fraction of investment in period  $t$  that is financed externally. We construct the time series of  $\theta_t$  for the nonfinancial corporate sector using Flow of Funds data by

$$\theta_t = \frac{(\text{Funds Raised in Markets})_t}{(\text{Fixed Investment})_t}.$$

The variables are:

- Net funds raised in markets: line 38 in Table F.102, equals: net new equity issuance (line 39) plus credit market instruments (line 40) for nonfarm nonfinancial corporate businesses.
- Fixed investment: line 12 in Table F.102, for nonfarm nonfinancial corporate businesses.

## APPENDIX C. SENSITIVITY ANALYSIS

This section discusses the sensitivity of our results to the choice of various parameters.

### C.1. Sensitivity analysis to the parameters of the financial shock process.

The stochastic process underlying the financial shock process is

$$\theta_{t+1} = \mu_\theta + \rho_\theta(\theta_t - \mu_\theta) + \varepsilon_{\theta,t}.$$

We report the results of our sensitivity analysis exercise to the mean and variance of this process in Table 6.<sup>35</sup> The asset price dynamics is driven by fluctuations in  $\theta$ . Therefore, one

<sup>35</sup>We find that changes in  $\text{corr}(\varepsilon_z, \varepsilon_\theta)$  do not play any role for asset price volatility. Increasing  $\rho_\theta$  slightly increases the asset price volatility. We do not report these results here, but they are available upon request from the authors.

would expect the asset price volatility to be increasing in the variance of  $\theta$ . This is what we see in column (2) of Table 6. We decrease the variance of innovations of the  $\theta$  process to 50% of the benchmark value. Asset price volatility decreases, but the model still generates over 60% of the asset price volatility observed in the data. Volatility of investment decreases as well, but note that volatility and persistence of output remain unaffected.

Next we analyze to what extent our results depend on the chosen mean level of  $\theta$ . We increase the mean  $\theta$  by 50% and keep all other parameters unchanged. The financial constraint binds if the realization of  $\theta$  is relatively small, and it does not bind if it is relatively large. In the latter case, no entrepreneurs are constrained.<sup>36</sup> Consequently, we find that the asset price volatility decreases to about 50% relative to the data. These results are reported in column (3) of Table 6.

Finally, we change the mean of  $\theta$  and the volatility of innovations of the  $\theta$  process at the same time. We decrease  $\sigma_{\varepsilon_\theta}^2$  and increase  $E[\theta]$  by 50%. These results are reported in column (4) of Table 6. The model generates about a quarter of the observed volatility in asset prices and over 40% of the observed volatility in asset returns. This is much more than, for example, that found in Gomes, Yaron, and Zhang (2003)'s paper.

We conclude that in our model fluctuations in financial frictions explain a large fraction of the observed asset price volatility for a wide range of parameters of the  $\theta$  process.

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<sup>36</sup>Because of the linearity of entrepreneurs' decision rules, our model can never have the financial constraint binding for some investing entrepreneurs and not binding for others.

### C.2. Sensitivity analysis to the fraction of firms with investment opportunities.

In Theorem 4.3 we showed that in steady state  $\theta < \frac{\delta - \pi}{\delta} \iff q > 1$ . Note that if  $\delta < \pi$ , then  $q = 1$  in steady state independently of  $\theta$ . Recall that investing entrepreneurs are able to sell all their current equity holdings at any point in time. If  $\delta < \pi$ , investing entrepreneurs hold a relatively large share of aggregate wealth and sales of current equity are enough to cover the demand for equity by noninvesting entrepreneurs. Therefore,  $q = 1$  and the financial constraint does not bind. Although we cannot directly map a steady state statement into our dynamic equilibrium analysis, it is clear that if  $\pi > \delta$  we should not expect the financial constraint to bind. If  $\pi$  is smaller than but close to  $\delta$ ,  $\theta$  would have to be very small in order for the financial constraint to bind and  $q > 1$ . In a dynamic equilibrium as  $\pi \rightarrow \delta$  the financial constraint will bind ( $q_t > 1$ ) or not bind ( $q_t = 1$ ) depending on the realization of  $\theta_t$ . Therefore, we should expect the volatility of asset prices and investment to decrease as  $\pi$  increases.

This is exactly what we see in Table 7. This table reports our sensitivity analysis to the parameter  $\pi$  representing the fraction of firms with investment opportunities. The table shows that the model still generates about 25% of the observed asset price volatility for  $\pi = 0.02$  (corresponding to an annual  $\pi \sim 8\%$ ). For  $\pi = 0.025$  the financial constraint does not bind and  $q_t = 1$  in every period.

This result should not be viewed as a shortcoming specific to our model. Other models with borrowing constraints have a similar feature (see, for instance, Gomes, Yaron, and Zhang (2003)). To avoid a first best solution and generate interesting dynamics, they need to make sure that borrowing-constrained entrepreneurs do not accumulate enough assets to escape the borrowing constraint. Therefore, these models assume that entrepreneurs discount the future more heavily than households, which guarantees that entrepreneurs will remain borrowing constrained.

In our model we argued that it is reasonable to assume that entrepreneurs cannot start new projects instantly, but we have abstracted from other factors that would make the financial constraint tighter. For instance, allowing workers to hold equity would decrease the fraction of wealth held by entrepreneurs, and a larger fraction of them would need to have an investment opportunity in order to provide enough new capital for workers and noninvesting entrepreneurs. Therefore, we could increase  $\pi$  and still have the financial constraint binding. Decreasing the discount factor  $\beta$  would enhance this effect by making the share of wealth held by entrepreneurs even smaller.

### C.3. Sensitivity analysis to the labor supply elasticity.

Our choice of the benchmark  $\eta = 1$  implies a Frisch labor supply elasticity<sup>37</sup> of 1. Although this value is in the range that has been considered in the literature, it is by no means a consensus in the profession. Table 8 reports our sensitivity analysis to the choice of  $\eta$ .

Section 4 shows that labor and output are related as follows:

$$\begin{aligned} \text{corr}(\log L_t, \log Y_t) &= 1 \\ (1 + \eta)^2 \text{var}(\log L_t) &= \text{var}(\log Y_t). \end{aligned}$$

Therefore, changing  $\eta$  changes the relative volatility of labor and output. Table 8 shows that increasing (decreasing)  $\eta$  decreases (increases) both the volatility of output and labor. The same is true for consumption as workers' consumption accounts for 64% of aggregate output and even more of aggregate consumption. Table 8 also shows that the asset prices are not affected by the labor supply elasticity.

### C.4. Sensitivity analysis to the functional form of workers' utility.

In our model workers consume all of their income period by period and adjust their labor

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<sup>37</sup>With these preferences  $\eta = \frac{1}{\text{Frisch elasticity}}$ .

supply depending on the current wage only. With balanced growth preferences, workers' labor supply will be constant because for these preferences, the income and substitution effects associated with a change in the wage rate cancel out. Therefore, this version of the model is equivalent to one in which workers' labor supply is fixed. We report our results for this version of the model in Table 8. This table shows that although the volatility of output and consumption decreases, other statistics are unaffected. In particular, the asset price behavior is almost identical to the benchmark version of our model with elastic labor supply.

#### C.5. Sensitivity analysis to the discount factor.

Other studies have used very high discount factors in order to account for both asset prices and macroeconomic quantities.<sup>38</sup> We report the results of our sensitivity analysis exercise to the choice of  $\beta$  in Table 9. This table shows that our model generates a high asset price volatility for a range of  $\beta$ 's. We consider  $\beta \in \{0.98, 0.99, 0.999, 0.99999\}$ . As  $\beta$  increases, entrepreneurs become more patient, postponing consumption into the future. Since their share in output is constant at 0.36, investment becomes more correlated with output. As a consequence, investment becomes less volatile. On the other hand, the share of workers' consumption in aggregate consumption increases, which increases the correlation between aggregate consumption and output (recall that the correlation between workers' consumption and output equals one). This is the logic behind the dynamics shown in the table. Importantly, asset price volatility is not affected very much. It increases with  $\beta$ , but even for  $\beta = 0.98$  it is still about two-thirds of the level that we observe in the data.

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<sup>38</sup>See, for example, Jermann (1998) or Boldrin, Christiano, and Fisher (2001).

TABLE 6. Sensitivity Analysis to the Parameters of the Financial Shock Process

Statistic	Data	(1)	(2)	(3)	(4)
		$\sigma_{\varepsilon_\theta} = 0.17$ $E(\theta) = .28$	$\sigma_{\varepsilon_\theta} = 0.12$ $E(\theta) = .28$	$\sigma_{\varepsilon_\theta} = 0.17$ $E(\theta) = .43$	$\sigma_{\varepsilon_\theta} = 0.12$ $E(\theta) = .43$
$\sigma_Y$	1.52	1.18	1.17	1.17	1.18
$\sigma_I$	5.00	5.12	3.77	2.93	2.07
$\sigma_C$	0.85	1.93	1.63	1.36	1.16
$\sigma_L$	1.73	0.59	0.58	0.59	0.59
$\rho_Y$	0.87	0.67	0.68	0.68	0.67
$\rho_I$	0.85	0.40	0.41	0.38	0.48
$\rho_C$	0.90	0.47	0.52	0.53	0.61
$\rho_L$	0.92	0.67	0.68	0.68	0.67
$\rho(Y, I)$	0.90	0.23	0.30	0.48	0.73
$\rho(Y, C)$	0.85	0.61	0.72	0.80	0.92
$\rho(Y, L)$	0.87	1.00	1.00	1.00	1.00
$\sigma_q$	11.85	9.69	7.39	5.36	3.08
$\sigma_{r^e}$	8.63	10.77	8.15	6.37	3.71
$\sigma_{val}$	10.72	9.74	7.42	5.38	3.10
$\rho_q$	0.74	0.38	0.39	0.28	0.27
$E(r^e)$	1.49	1.36	1.18	1.13	1.04
$E(r^f)$	0.30	0.51	0.63	0.83	0.92
$\sigma_r^f$	0.68	5.45	4.08	3.54	2.19
$E(r^e) - E(r^f)$	1.19	0.85	0.55	0.30	0.11

This table reports the business cycle statistics and asset prices statistics in the model. Statistics are computed based on 100 replications of size 180. The symbol  $\sigma_x$  represents the standard deviation of variable  $x$ ,  $\rho_x$  represents the autocorrelation of  $x$  and  $\rho(x, y)$  represents the correlation between  $x$ , and  $y$ . All variables are logged and HP filtered before statistics are computed. Standard deviations are measured in percentage terms. The data column reports statistics for quarterly U.S. data for the period 1964:1-2008:4. Columns (1), (2), (3), and (4) report results for various choices of parameters for the mean and the standard deviation of the financial shock.

TABLE 7. Sensitivity Analysis to  $\pi$ 

Statistic	Data	(1)	(2)	(3)	(4)
		$\pi = .01$	$\pi = .015$	$\pi = .02$	$\pi = .025$
$\sigma_Y$	1.52	1.21	1.18	1.19	1.19
$\sigma_I$	5.00	9.99	5.12	2.10	1.71
$\sigma_C$	0.85	3.12	1.93	1.16	1.02
$\sigma_L$	1.73	0.60	0.59	0.59	0.60
$\rho_Y$	0.87	0.68	0.67	0.68	0.67
$\rho_I$	0.85	0.42	0.40	0.48	0.67
$\rho_C$	0.90	0.47	0.47	0.59	0.67
$\rho_L$	0.92	0.68	0.67	0.68	0.67
$\rho(Y, I)$	0.90	0.03	0.23	0.74	1.00
$\rho(Y, C)$	0.85	0.47	0.61	0.92	1.00
$\rho(Y, L)$	0.87	1.00	1.00	1.00	1.00
$\sigma_q$	11.85	16.02	9.69	3.07	0
$\sigma_{r^e}$	8.63	17.31	10.77	3.84	0.07
$\sigma_{val}$	10.72	16.12	9.74	3.09	0.13
$\rho_q$	0.74	0.43	0.38	0.21	1
$E(r^e)$	1.49	1.88	1.36	1.05	1.01
$E(r^f)$	0.30	-0.26	0.51	0.93	1.01
$\sigma_r^f$	0.68	8.16	5.45	2.32	0.06
$E(r^e) - E(r^f)$	1.19	2.14	0.85	0.12	0.00

This table reports the business cycle statistics and asset prices statistics in the model. Statistics are computed based on 100 replications of size 180. The symbol  $\sigma_x$  represents the standard deviation of variable  $x$ ,  $\rho_x$  represents the autocorrelation of  $x$  and  $\rho(x, y)$  represents the correlation between  $x$ , and  $y$ . All variables are logged and HP filtered before statistics are computed. Standard deviations are measured in percentage terms. The data column reports statistics for quarterly U.S. data for the period 1964:1-2008:4. Columns (1), (2), (3), and (4) report results for various values of  $\pi$ .

TABLE 8. Sensitivity Analysis to the Labor Supply Elasticity in the Model

Statistic	Data	(1) $\eta = .5$	(2) $\eta = 1$	(3) $\eta = 2$	(4) constant labor
$\sigma_Y$	1.52	1.41	1.18	1.03	0.79
$\sigma_I$	5.00	5.02	5.12	5.05	5.19
$\sigma_C$	0.85	2.75	1.93	1.55	1.79
$\sigma_L$	1.73	0.94	0.59	0.35	0
$\rho_Y$	0.87	0.68	0.67	0.67	0.67
$\rho_I$	0.85	0.41	0.40	0.40	0.39
$\rho_C$	0.90	0.46	0.47	0.51	0.43
$\rho_L$	0.92	0.68	0.67	0.68	1
$\rho(Y, I)$	0.90	0.26	0.23	0.16	0.13
$\rho(Y, C)$	0.85	0.52	0.61	0.69	0.46
$\rho(Y, L)$	0.87	1.00	1.00	1.00	0
$\sigma_q$	11.85	9.49	9.69	9.71	10.07
$\sigma_{r^e}$	8.63	10.45	10.77	10.71	11.10
$\sigma_{val}$	10.72	9.54	9.74	9.76	10.12
$\rho_q$	0.74	0.40	0.38	0.39	0.39
$E(r^e)$	1.49	1.35	1.36	1.36	1.42
$E(r^f)$	0.30	0.66	0.51	0.48	0.62
$\sigma_r^f$	0.68	5.36	5.45	5.45	5.61
$E(r^e) - E(r^f)$	1.19	0.69	0.85	0.88	0.80

This table reports the business cycle statistics and asset prices statistics in the model. Statistics are computed based on 100 replications of size 180. The symbol  $\sigma_x$  represents the standard deviation of variable  $x$ ,  $\rho_x$  represents the autocorrelation of  $x$  and  $\rho(x, y)$  represents the correlation between  $x$ , and  $y$ . All variables are logged and HP filtered before statistics are computed. Standard deviations are measured in percentage terms. The data column reports statistics for quarterly U.S. data for the period 1964:1-2008:4. Columns (1), (2), and (3) report results for the model with various values of  $\eta$ . Column (4) reports results for a version of the model with constant labor supply.

TABLE 9. Sensitivity Analysis to the Discount Factor in the Model

Statistic	Data	(1)	(2)	(3)	(4)
		$\beta = .98$	$\beta = .99$	$\beta = .999$	$\beta = .9999$
$\sigma_Y$	1.52	1.21	1.18	1.18	1.20
$\sigma_I$	5.00	7.85	5.12	1.36	1.20
$\sigma_C$	0.85	2.07	1.93	1.23	1.20
$\sigma_L$	1.73	0.60	0.59	0.59	0.60
$\rho_Y$	0.87	0.68	0.67	0.68	0.68
$\rho_I$	0.85	0.40	0.40	0.61	0.68
$\rho_C$	0.90	0.48	0.47	0.64	0.68
$\rho_L$	0.92	0.68	0.67	0.68	0.68
$\rho(Y, I)$	0.90	0.13	0.23	0.87	1.00
$\rho(Y, C)$	0.85	0.60	0.61	0.96	1.00
$\rho(Y, L)$	0.87	1.00	1.00	1.00	1.00
$\sigma_q$	11.85	7.78	9.69	12.67	13.00
$\sigma_{r^e}$	8.63	8.54	10.77	14.27	14.68
$\sigma_{val}$	10.72	7.85	9.74	12.68	13.00
$\rho_q$	0.74	0.40	0.38	0.37	0.37
$E(r^e)$	1.49	2.21	1.36	0.85	0.80
$E(r^f)$	0.30	1.63	0.51	-0.51	-0.63
$\sigma_r^f$	0.68	4.42	5.45	7.20	7.44
$E(r^e) - E(r^f)$	1.19	0.58	0.85	1.36	1.44

This table reports the business cycle statistics and asset prices statistics in the model. Statistics are computed based on 100 replications of size 180. The symbol  $\sigma_x$  represents the standard deviation of variable  $x$ ,  $\rho_x$  represents the autocorrelation of  $x$  and  $\rho(x, y)$  represents the correlation between  $x$ , and  $y$ . All variables are logged and HP filtered before statistics are computed. Standard deviations are measured in percentage terms. The data column reports statistics for quarterly U.S. data for the period 1964:1-2008:4. Columns (1), (2), (3), and (4) report the statistics for the model with various values of  $\beta$ .