Measuring Marginal q

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Abstract

Using asset prices I estimate the marginal value of capital under general assumptions about technology and preferences. The *state-space* measure of marginal q relies on the joint measurability of the value function, i.e. firm market value, and its underlying firm state variables. Unlike existing methodologies, the *state-space* measure of marginal q requires only very few general restrictions on the stochastic discount factor and the firm investment technology, and it uses only market values maximally correlated with the underlying state variables implied by the model. Consistently with a large class of neoclassical investment models, I construct the *state-space* marginal q using the firm capital stock and profitability shocks. I show how this new measure of real investment opportunities increases the correlation with investment and provides more plausible and robust estimates of capital adjustment costs.

Tobin's Q-theory of investment emphasizes a deep theoretical connection between financial markets and the real economy: a single (relative) market price, also known as Tobin's (average) Q, is a sufficient statistic to describe investment behavior (Hayashi, 1982). The use of the "observable" Q-ratio has since been very popular among researchers, despite a long-standing consensus that its restrictive underlying assumptions of perfect competition and homogeneity may be misspecified, particularly at the firm level. Under more general assumptions regarding the nature of technology and markets "unobservable" marginal q often becomes the relevant sufficient statistic for investment.[1] How do we measure marginal q in such more general economic environments?

I propose a general procedure to measure marginal q that is both theoretically justified, and useful, empirically. Under general regularity conditions for the existence and differentiability of the value function as well as for the measurement of the firm state variables, I show how the marginal value of capital or marginal q can be easily estimated as (market) price elasticity of capital using a two-stage procedure, which I refer to as *state-space* approach. First, I project the observable market values - i.e. value function - onto the measurable firm-level state-space, which includes also the firm capital stock, and then I differentiate the projected market values with respect to the firm capital stock to obtain the marginal value of capital. The key insight underlying the *state-space* measure of marginal q rests on the *joint measurability* of the value function - i.e. market values - and the underlying set of firm state variables. Unlike the demand side of the economy, where we get to observe *neither* the value function - i.e. indirect utility - *nor* its underlying state variables including above all household's wealth, in the production side of the economy we do get to observe, or at least easily measure in most cases, *both* the value function - i.e. market values - *and* its underlying state variables including above all the stock of capital. Although some of the firm's state variables are not directly available, they can be readily constructed from observables in most cases.

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[1] Possible departures from homogeneity include market power or decreasing returns to scale in production (Gomes, 2001; Cooper and Ejarque, 2003; Abel and Eberly, 2011), and inhomogeneous costs of investment (Abel and Eberly, 1994, 1997; Cooper and Haltiwanger, 2006).

Consistently with a large class of neoclassical investment models, I identify empirically the firm capital stock and profitability shock as the key state variables for market values and investment. I construct the state-space measure of marginal q, which can then be ultimately used to address several open questions in economics. In this paper I focus on the estimation of capital adjustment costs.

Abel and Blanchard (1986) and Gilchrist and Himmelberg (1995, 1998) offer an alternative methodology to measure marginal q. They propose using VAR-based forecasts of the future expected marginal profit of capital. However, their approach requires explicit functional forms for the marginal revenue product of capital, the marginal adjustment cost, and the stochastic discount factor. Furthermore, their approach also imposes important restrictions on the formation of expectations which are outside of the structural model. Differently, the *state-space* approach only requires standard regularity conditions for the existence and differentiability of the value function as well as for the measurement of the firm-level state variables. These conditions are generally satisfied in any well-behaved model of investment. In addition, the *state-space* approach imposes more discipline over the choice of firm-level state variables as implied by the structural model.

The Euler equation approach (e.g. Abel (1980), Shapiro (1986), and Whited (1992), among others) also provides an alternative methodology to estimate marginal q. Exploiting the first-order condition for investment, one can replace unobservable marginal q with a parameterized marginal investment cost in the Euler equation, and then estimate it using structural GMM. However, also this approach requires specific functional forms for the marginal profit of capital and the stochastic discount factor. In contrast, the *state-space* approach requires significantly fewer restrictions on the functional forms of technology and preferences, and can be implemented using fairly standard linear estimation methods rather than resorting to nonlinear GMM techniques.

More recently, Gala and Gomes (2013) propose a new methodology to estimate investment equation directly by approximating the optimal investment policy as function of a model's underlying state variables. The policy function approximation is particularly useful to describe firm investment behavior and quantify, through a statistical variance

decomposition, the importance of various state variables, and corresponding class of investment models. However, the policy function approximation, while requiring fewer structural restrictions than the *state-space* approach, does not provide direct identification of marginal q. The state-space approach, instead, provides direct identification and estimates of marginal q imposing only standard restrictions on the firm production technologies, and almost no restrictions on the functional forms of the stochastic discount factor and firm investment technologies.

Unlike these estimation methodologies, which do not rely on asset prices, the *state-space* measure of marginal q makes an efficient use of market values. In the construction of marginal q, I only use market values, which are maximally correlated with the underlying fundamentals or "instrumented" market values. As such, the *state-space* approach avoids measurement error problems induced, for instance, by potential stock market inefficiencies (Blanchard, Rhee and Summers (1993), Erickson and Whited (2000)). Therefore, the methodology I propose for testing investment equations is effectively a "differential IV estimation" procedure, whereby I test the standard first-order condition for investment using the first-order derivative of "instrumented" market values with respect to the capital stock.

While the focus in this paper is mainly on investment models only with frictions to capital adjustment, it is easy to modify the *state-space* approach to accommodate labor market frictions and financial market imperfections. Intuitively, any deviations from frictionless labor markets and/or from the Modigliani-Miller theorem imply that the value function and the optimal investment policy now depend on an augmented set of measurable state variables, which also include employment and/or net financial liabilities. Those additional state variables would indeed accommodate investment models with labor market frictions as in Merz and Yashiv (2007), and Bazdresch, Belo, and Lin (2013), and financial frictions as in Hennessy, Levy, and Whited (2004), Bustamante (2011), Bolton, Chen, and Wang (2012), and Bolton, Schaller, and Wang (2013).

This paper contributes to the literature in two significant ways. First, and foremost, it provides a new general methodology to estimate shadow prices in production economies.

Unlike existing methodologies, the state-space measure of marginal q makes an efficient use of market values, and requires only standard regularity conditions for the existence and differentiability of the value function. Imposing almost no restrictions on the functional forms of the stochastic discount factor and firm investment technologies makes the estimate of the shadow value of capital more robust to model misspecification. Second, I obtain estimates of capital adjustment costs, which are both more plausible and more robust to model misspecification than existing ones.

The rest of our paper is organized as follows. The next section describes the general model, the implied value and optimal investment functions, and how to estimate empirically the marginal value of capital as function of the key state variables. In Section 3, I describe the data and the empirical implementations including the main findings. I conclude with a discussion of the potential extensions and generalization of the approach in Section 4. The Appendices contain additional estimation details, and a detailed review of the alternative methodologies available to estimate marginal q.

I. A General Model of Investment

This section describes the general model of corporate investment. I use a generalized version of the model in Abel and Eberly (1994), which allows for a weakly concave production technology and asymmetric, non-convex and possibly discontinuous capital adjustment costs. This model is flexible enough to include the large majority of investment models in the literature as special cases.

A. Production and Investment Technologies

Consider a firm that uses capital and a vector of costlessly adjustable inputs, such as labor, to produce a nonstorable output. At each point of time, the firm chooses the amounts of costlessly adjustable inputs to maximize the value of its revenue minus expenditures on these inputs. Let $\Pi\left(K_{it}, A_{it}\right)$ denote the maximized value of this instantaneous operating

profit at time t, where K_{it} is firm i's capital stock at time t and A_{it} is a random variable representing uncertainty in technology, in the prices of costlessly adjustable inputs, and/or in the demand facing the firm.

Assumption 1. Profit. The function $\Pi : K \times A \to R$ satisfies: (i) $\Pi_K (K, A) > 0$ and $\Pi_A (K, A) > 0$; and (ii) $\Pi_{KK} (K, A) \leq 0$.

This formulation of the profit function allows the firm to be either a price-taker or a price-setter. The random variable A_{it} evolves according to a diffusion process:

$$dA_{it} = \mu_A (A_{it}, \Psi_t) \, dt + \sigma_A (A_{it}, \Psi_t) \, dW_{it}^A \tag{1}$$

where dW_{it}^A is standard Wiener process. The vector of aggregate random variables, Ψ_t, summarizes the state of the economy and evolves as

$$d\Psi_t = \mu_\Psi (\Psi_t) \, dt + \sigma_\Psi (\Psi_t) \, dW_t^\Psi \tag{2}$$

with dW_t^Ψ being a vector of standard Wiener processes independent of dW_{it}^A. The general formulation in (1) allows for common systematic variation in the shocks to technology, prices of costlessly adjustable inputs, and demand facing the firm.

Capital is acquired by undertaking gross investment at rate I, and the capital stock depreciates at a fixed proportional rate $\delta \geq 0$, so that the capital stock evolves according to

$$dK_{it} = (I_{it} - \delta K_{it}) \, dt. \tag{3}$$

When the firm undertakes gross investment, it incurs costs, which reduce operating profits. The adjustment costs are summarized by the function $\Phi (I, K)$.

Assumption 2. Adjustment Costs. The function $\Phi : I \times K \to R_+$ satisfies: (i) twice continuously differentiable for $I \neq I^* (K)$; (ii) $\Phi (I^* (K), K) = 0$; (iii) $\Phi_I (\cdot) \times (I - I^*(K)) \geq 0$; (iv) $\Phi_K (\cdot) \leq 0$; and (v) $\Phi_{II} (\cdot) \geq 0$.

Conditions (ii) and (iii) imply that capital adjustment costs are non negative and minimized at the natural rate of investment, $I^* (K)$. This is generally assumed to be either

0 or δK depending on whether adjustment costs apply to gross or net capital formation. Assumption 2 allows for the possibility of very general non-convex and discontinuous adjustment costs. A general function satisfying all the conditions in Assumptions 2 is:

$$\Phi\left(I,K\right) = \begin{cases} f^{+}K + p^{+}I + \frac{\gamma^{+}}{\varphi}\left(\frac{I-I^{*}(K)}{K}\right)^{\varphi}K & \text{if } I > I^{*}\left(K\right) \\ 0 & \text{if } I = I^{*}\left(K\right) \\ f^{-}K + p^{-}I + \frac{\gamma^{-}}{\varphi}\left(\frac{I-I^{*}(K)}{K}\right)^{\varphi}K & \text{if } I < I^{*}\left(K\right) \end{cases} \tag{4}$$

where the constants f^{+}, f^{-}, p^{+}, p^{-}, γ^{+}, γ^{-}, and φ are all non-negative. The constant $f^{+} \gtreqless f^{-} \geq 0$ denotes non-convex and discontinuous fixed cost of investment incurred whenever $I \neq I^{*}\left(K\right)$. The constant $p^{+} \geq p^{-} \geq 0$ denotes the purchase and sale price per unit of capital, and $\gamma^{-} \geq \gamma^{+} \geq 0$ allows for potentially asymmetric and convex ($\varphi \geq 2$) adjustment costs reflecting costly reversibility. The standard smooth quadratic adjustment costs are obtained as special case of (4) with $\varphi = 2$, $p^{+} = p^{-} > 0$, $\gamma^{-} = \gamma^{+} > 0$, and $f^{+} = f^{-} = 0$.

B. Optimal Investment Decisions

Each firm chooses optimal investment by maximizing the expected present value of operating profit, $\Pi\left(K,A\right)$ less total investment cost $\Phi\left(I,K\right)$. The value of the firm is thus

$$V\left(K_{it}, A_{it}, \Psi_{t}\right) = \max_{\{I_{t+s}\}} E_{t} \int_{0}^{\infty} \frac{\Lambda_{t+s}}{\Lambda_{t}}\left[\Pi\left(K_{it+s}, A_{it+s}\right) - \Phi\left(I_{it+s}, K_{it+s}\right)\right] ds \tag{5}$$

subject to the capital accumulation equation in (3), the firm shock process in (1), the dynamics for the vector of aggregate random variables in (2), and the pricing-kernel dynamics

$$\frac{d\Lambda_{t}}{\Lambda_{t}} = -r\left(\Psi_{t}\right) dt - \sigma_{\Lambda}\left(\Psi_{t}\right) dW_{t}^{\Psi} \tag{6}$$

where r_{t} denotes the instantaneous riskless rate, and $\sigma_{\Lambda}\left(\Psi_{t}\right)$ denotes the market prices of risks associated with the vector of aggregate systematic shocks, Ψ_{t}.[2]

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[2]The vector Ψ_{t} summarizes the aggregate state of the economy, which potentially includes aggregate shocks to productivity, wages, relative price of investment goods, and household preferences.

The firm value function $V(K, A, \Psi)$ satisfies the following Hamilton-Jacobi-Bellman (HJB):[3]

$$0 = \max_I \left\{ \Lambda \left[\Pi(K, A) - \Phi(I, K) \right] + \mathcal{D}[\Lambda V] \right\} \qquad (7)$$

with $\mathcal{D}[\cdot]$ denoting the infinitesimal generator of the Markov processes A and Ψ, and the process K

$$\mathcal{D}[M(\cdot)] = \mu_A(\cdot) M_A + \frac{\sigma_A^2(\cdot)}{2} M_{AA} + \mu_\Psi(\cdot) M_\Psi + \frac{\sigma_\Psi^2(\cdot)}{2} M_{\Psi\Psi} + (I - \delta K) M_K$$

applied to the discounted firm value ΛV, along with the transversality ("no bubble") condition:

$$\lim_{T \to \infty} \mathbf{E}_t \left[|\Lambda_{t+T} V_{it+T}| \right] = 0.$$

Substituting for $\mathcal{D}[\Lambda V]$ in (7), the optimal investment policy then satisfies

$$I^*(q, K) = \arg \max_I \left[qI - \Phi(I, K) \right] \qquad (8)$$

where the marginal value of capital $q \equiv V_K$, by the Fayman-Kac Theorem, is equal to

$$q(K_{it}, A_{it}, \Psi_t) = E_t \int_0^\infty e^{-\delta s} \frac{\Lambda_{t+s}}{\Lambda_t} \left[\Pi_K(K_{it+s}, A_{it+s}) - \Phi_K(I_{it+s}^*, K_{it+s}) \right] ds. \qquad (9)$$

As shown in Abel and Eberly (1994), the marginal q is the present value of the stream of expected marginal profit of capital which consists of two components: Π_K is the marginal operating profit accruing to capital, and $-\Phi_K$ is the reduction in the adjustment cost accruing to the marginal unit of capital.

C. Measuring Marginal q

The shadow value of capital or marginal q in (9) does not yield an explicit closed form solution under the general conditions under consideration. Hence, we cannot directly test the optimal investment policies in (8), unless we can measure the unobservable marginal q.

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[3]For simplicity of exposition, we have suppressed the firm and time subscripts i and t.

C.1 State-Space Approach

I propose a new methodology to measure marginal q that rests on the *joint measurability* of the firm value function, $V(\cdot)$, and its underlying state variables, $\Omega = \{K_{it}, A_{it}, \Psi_t\}$. Specifically, I can measure marginal q according to its definition as partial derivative of the *observable* value function - i.e. market value of the firm - with respect to its *observable* capital stock, $q \equiv V_K(\Omega)$.[4]

First, I approximate the (scaled) market value of firm i at time t, $Q_{it} \equiv V_{it}/K_{it}$, using a tensor product polynomial in the state variables as

$$v_{it} \equiv \log Q_{it} = \sum_{j_k=0}^{n_k} \sum_{j_a=0}^{n_a} \sum_{j_\Psi=0}^{n_\Psi} c_{j_k,j_a,j_\Psi} \times [k_{it}]^{j_k} \times [a_{it}]^{j_a} \times [\Psi_t]^{j_\Psi} + \epsilon_{it} \tag{10}$$

where $k_{it} \equiv \log K_{it}$, $a_{it} \equiv \log A_{it}$, and ϵ_{it} captures measurement error in market values.[5] Given the state-space variables k_{it}, a_{it} and Ψ_t, the coefficients c_{j_k,j_a,j_Ψ} are the subject of the estimation procedure. Then, I estimate the marginal q according to its definition of partial derivative of the value function as

$$\widehat{q}_{it} = \widehat{Q}_{it}\left(1 + \frac{\partial \log \widehat{Q}_{it}}{\partial \log K_{it}}\right) = \widehat{Q}_{it}\left(1 + \sum_{j_k=0}^{n_k} \sum_{j_a=0}^{n_a} \sum_{j_\Psi=0}^{n_\Psi} \widehat{c}_{j_k,j_a,j_\Psi} \times j_k \times [k_{it}]^{j_k-1} \times [a_{it}]^{j_a} \times [\Psi_t]^{j_\Psi}\right) \tag{11}$$

Rather than imposing additional restrictive conditions concerning the functional forms of the stochastic discount factor and adjustment cost functions, the *state-space* approach only requires general regularity conditions for the existence and differentiability of the value function as well as for the measurement of the *firm-level* state variables.

C.1.1 Measuring the State Variables

In order to estimate marginal q using the *state-space* approach, we need to measure the relevant state variables in Ω. First, we focus on

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[4] The identification of marginal q rests on the ability to identify the *exogenous* state variables, A and Ψ. Therefore, the selection of the relevant state variables for the representation of the value function should always include the *exogenous* state variables implied by the model (or any one-to-one transformation).

[5] Under the null of the model, the value function, V, depends only on the set of state variables Ω. Therefore, I estimate the value function under the standard assumption that firm intrinsic values are observed only with error by the econometrician.

the firm-level state variables K and A. The firm capital stock, K, is directly observable. Differently, the firm shocks, A, are not directly observable, but can be easily estimated using the theoretical restrictions imposed by the model. As standard in the literature, and in accordance with Assumption 1, the operating profit evaluated at the optimal choice of the costlessly adjustable inputs of production can be represented as:

$$\Pi(K, A) = AK^{\theta} \tag{12}$$

where θ denotes the share of capital in revenue. I estimate the firm shocks, A, following the procedure in Cooper and Haltiwanger (2006). See Appendix for more details.

Given a large panel of firms, one can also account for unobserved time-invariant heterogeneity across firms by allowing the constant term $c_{0,0,0}$ in (10) to be firm-specific.

The complete knowledge of the aggregate state variables in Ψ is not necessary for the purpose of estimating firm level marginal q. In fact, one can capture the impact of all unobserved aggregate state variables by allowing for time-specific polynomial coefficients in (10). Specifically, one can fit a separate cross-section of (scaled) firm market values for each year as

$$v_{it} \equiv \log Q_{it} = \sum_{j_k=0}^{n_k} \sum_{j_a=0}^{n_a} b_{j_k,j_a,t} \times [k_{it}]^{j_k} \times [a_{it}]^{j_a} + \epsilon_{it} \tag{13}$$

where I have suppressed the direct dependence on the aggregate state variables, Ψ, and I have allowed the polynomial coefficients $b_{j_k,j_a,t}$ to vary over time. For easy of exposition and comparison with the existing literature, I focus the empirical analysis on unobserved aggregate variation that affects only linearly the value function.[6]

D. Discussion

In general, we cannot directly observe shadow prices. Therefore, most of the literature follows Hayashi (1982) and assumes perfectly competitive firms with homogeneous profit

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[6]Gala (2012) allows aggregate state variables to enter non-additively the value function and investigates alternative state-space representations of asset prices.

9

function, $\Pi\left(\cdot\right)$, and investment cost function, $\Phi\left(\cdot\right)$, to derive expression for q in terms of observable variables. Specifically, Abel and Eberly (1994) prove that if $\Pi\left(\cdot\right)$ and $\Phi\left(\cdot\right)$ are homogeneous of degree ρ in both I and K, then marginal q is proportional to average or Tobin's Q: $q = \rho\frac{V}{K}$. Hayashi (1982) conditions of linear homogeneity follows as a special case with $\rho = 1$. The theoretical appeal of Q-theory lies on the fact that it is possible to summarize all relevant information about the state variables in a single (relative) market price. While this is often convenient and may well be a useful approximation in aggregate environments where homogeneity assumptions are more likely to hold, the identification and measurement of marginal q with (average) Tobin's Q under these restrictive homogeneity assumptions offer a fairly poor fit to the data at the firm level (e.g. Gala and Gomes (2013), among others).

I show instead how one can still use asset prices to estimate directly marginal q under general assumptions concerning technology and preferences. The *state-space* measure of marginal q rests on the *joint measurability* of the value function - i.e. market values - and the underlying set of state variables. As long as the observable value function depends on a set of measurable firm-level state variables, we can estimate marginal q by differentiating the projection of market values onto the firm level "state-space" with respect to the capital stock. The successful empirical identification of the marginal value of capital does not require the complete identification and measurement of *all* state variables affecting the value function, but only the identification of a subset affecting its partial derivative with respect to capital. As such, the *state-space* estimate of the marginal q is robust to the omission of any state variable that is independent of the firm capital stock. In addition, when using a large panel of firms, it is possible to allow for firm fixed effects to account for unobserved time-invariant firm heterogeneity. Similarly, one can allow for time-specific polynomial coefficients to fully account for the impact of any aggregate state variable.

Abel and Blanchard (1986) and Gilchrist and Himmelberg (1995) offer an alternative methodology to measure marginal q. They propose estimating marginal q using VAR-based forecasts of the future expected marginal profit of capital according to the

definition in (9) - see Appendix for more details. However, their approach requires explicit simplifying assumptions, suitable for linear VAR, concerning the functional forms for the marginal revenue product of capital, Π_K, the marginal adjustment cost, Φ_K, and the stochastic discount factor, Λ. Furthermore, their approach also imposes important restrictions on the formation of expectations which are outside of the structural model. In contrast, the *state-space* approach only requires standard regularity conditions for the existence and differentiability of the value function as well as for the identification and measurement of the *firm-level* state variables (indeed, it can accommodate a larger class of functional forms beyond those suitable for linear VARs). These conditions are generally satisfied in any well-behaved model of investment. In addition, the *state-space* approach imposes more discipline over the choice of firm-level state variables as implied by the structural model. Within a neoclassical model of investment, I focus on firm size and productivity shocks.

The Euler equation approach (e.g. Abel (1980), Shapiro (1986), and Whited (1992), among others) also provides an alternative methodology to measure marginal q. Exploiting the first-order condition for investment, which requires the marginal benefit of investment to equal its marginal cost, one can replace unobservable marginal q with a parameterized marginal investment cost in the Euler equation; and then use nonlinear GMM to estimate the underlying parameters of the model as well as marginal q (see Appendix for more details). However, also this approach requires specific functional forms for the marginal profit of capital and the stochastic discount factor. In addition, even though the estimation of the Euler equation allows to control for expectations without modelling them explicitly (under the assumption of rational expectations), in practice it still requires to make a (somewhat arbitrary) choice of variables among the set of valid instruments in the econometrician information set. In contrast, the *state-space* approach requires significantly fewer restrictions on the functional forms of technology and preferences, and can be implemented using fairly standard linear estimation methods rather than resorting to nonlinear GMM techniques. Furthermore, the *state-space* approach identifies explicitly the set of relevant variables in the econometrician information set

11

according to the structural model.

Unlike the existing estimation methodologies including the Euler equation approach and the VAR-based measure of marginal q, which do not rely on asset prices, the *state-space* measure of marginal q makes an efficient use of market values. In the construction of marginal q, I only use the variation in market values driven by *fundamental* state variables, that is I only use "instrumented" market values. As such, the *state-space* approach avoids measurement error problems induced, for instance, by potential stock market inefficiencies (Blanchard, Rhee and Summers (1993), Erickson and Whited (2000)).

II.   Empirical Implementation

I now describe the data used in the empirical analysis and additional issues concerning the state-space representation of marginal q. I then use the state-space measure of marginal q to estimate capital adjustment costs.

A.   Data

Our data comes from the combined annual research, full coverage, and industrial COMPUSTAT files. To facilitate comparison with much of the literature our sample is made of an unbalanced panel of firms for the years 1972 to 2010, that includes only manufacturing firms (SIC 2000-3999).

I keep only firm-years that have non-missing information required to construct the primary variables of interest, namely: investment, I, firm size, K, Tobin's Q, and sales revenues, Y. These variables are constructed as follows. Firm size, or the capital stock, is defined as gross property, plant and equipment. Investment is defined as capital expenditures in property, plant and equipment. Sales are measured by net sales revenues. These last two variables are scaled by the beginning-of-year capital stock. Finally, Tobin's Q is measured by the market value of capital (defined as market value of equity plus debt net

of current assets) scaled by gross property, plant and equipment.[7]

The sample is filtered to exclude observations where total capital, Tobin's Q and sales are either zero or negative. To ensure that the measure of investment captures the purchase of property, plant and equipment, I eliminate any firm-year observation in which a firm made an acquisition. Finally, all variables are trimmed at the 0.5th and 99.5th percentiles of their distributions to reduce the influence of any outliers, which are common in accounting ratios. This procedure yields a base sample of 29,564 firm-years observations. Table I reports summary statistics including mean, standard deviation and main percentiles for the variables of interest.

B. **Findings**

I now describe our main findings. I first examine the variation of market values and investment rates across portfolios sorted by firm size, K, and profitability shock, A. I then proceed to estimate marginal q and use it for the estimation of adjustment costs.

C. **Market Values and Investment by State-Variables Portfolios**

To gain some insights about the role of size and profitability shock in spanning the true underlying state-space for market values and investment rates, I sort all firms into 25 portfolios double-sorted on the empirical distribution of profitability shock conditional on firm size. Specifically, each firm is allocated annually first across five firm size quintiles, and then, within each size quintile, to five profitability shock quintiles. Table II reports the equally-weighted average market values and investment rates across the resulting 25 conditionally double-sorted portfolios.

Across all firm size quintiles the pattern in average market values and investment

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[7]Erickson and Whited (2006) show that using a perpetual inventory algorithm to estimate the replacement cost of capital and/or a recursive algorithm to estimate the market value of debt barely improves the measurement quality of the various proxies for Q.

rates shows a monotonic increasing relation with the productivity shock. This relation is statistically and economically significant. In the smallest firm size decile, the equal-weighted average market value increases from $0.02 billions for the lowest profitability shock quintile to $0.11 billions for the highest. Similarly, the average investment rates increases from 0.12 percent for the lowest profitability shock quintile to 0.32 percent for the highest. In the highest firm size quintile, average market values and investment rates increase from $1.83 billions and 0.10 percent for the lowest profitability shock quintile to $23.11 billions and 0.14 percent for the highest, respectively. These relations are statistically and economically significant across these portfolios.

Thus, the double-sort portfolio analysis confirms that there is substantial variation in market values and investment rates as functions of the underlying state-variables. I now turn to estimate the marginal q using the *state-space* approach.

D.   Value Function Approximation

I now turn to formally estimate firm market values (scaled by the capital stock) as function of the firm-level state-variables in (10). The goal is to find a parsimonious polynomial representation in terms of order of approximation that provides the best overall fit for market values empirically and can be used to evaluate marginal q.

Several practical questions arise when implementing these approximations in empirical work. The first issue is whether to use natural or orthogonal terms in the approximation. In this paper, I opt for natural polynomials as they make easier the interpretation of the estimated coefficients. The second issue concerns the order of the polynomial. The choice of the polynomial order can be made according to standard model selection techniques based on a measure of model fit such as Akaike information criterion (AIC). Using stepwise regression analysis, I find that a second order complete polynomial in k and a is often sufficient, and higher order terms are generally not necessary to improve the quality of the approximation.

Table III reports the empirical estimates for various specifications of the value function

14

polynomial regression in (10). As discussed above all estimates use time and firm fixed effects to account for potential aggregate shocks and unobserved firm heterogeneity.

I find that first and second order terms are all strongly statistically significant. The complete second order polynomial in k and a, which also includes the interaction term, explains up to 64% (including fixed effects) of the total variation in log (scaled) market values.[8] Based on the Akaike information criteria, I then choose the complete second order polynomial approximation in firm size and productivity shock (column 3) as the best parsimonious state-variable representation of market values empirically.[9]

D.1 Equivalence Between Marginal and Average Q

Given the estimates of the (scaled) value function approximation in (10), I can then compute the "Fitted" (average) Q and marginal q. The "Fitted" (average) Q is computed from the fitted values of the approximation in (10), and it provides a measure of Tobin's (average) Q which is maximally correlated with fundamentals (i.e. state variables). The firm marginal q is computed according to its definition as partial derivative of the value function with respect to the capital stock as in (11).

For each state variable representantion of the value function in terms of polynomials in k and a, I can test directly the equivalence between marginal q and average Q. Testing such an equivalence requires that $\partial \widehat{v}_{it}/\partial k_{it} = 0$, or equivalently that all coefficients corresponding to terms involving k are jointly equal to zero:

$$c_{j_k, j_a, j_\Psi} = 0 \text{ for } j_k = 1, ..., n_k; \text{ and } \forall j_a, j_\Psi. \tag{14}$$

The null hypothesis in (14) corresponds to a test of linear restrictions on the coefficient estimates. Such an hypothesis can be tested using a Wald statistic ("qQ-test"), which is distributed as χ_r^2 with degrees of freedom r equal to the number of restrictions.

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[8]I omit higher order terms because they are mostly insignificant and do not improve the overall quality of the approximation.

[9]Even if the quadratic and interaction terms do not increase substantially the overall fit of the value function, they are statistically significant and might be still important to explain variation in investment through marginal q.

The last row of Table III reports the p-values corresponding to the "qQ-test" for each polynomial specification. In all cases, I can *strongly* reject the null hypothesis that marginal q is equal to average Q.

E. Estimating Adjustment Costs

Under the optimal investment policy, the maximand in (8) - i.e. $qI - \Phi(I, K)$ - is nonnegative. As such, the estimate of marginal q provides an upper bound on the average capital adjustment costs, $\Phi(\cdot)/I$.

Table IV reports summary statistics for the distribution of estimated marginal q and therefore of the upper bound on average adjustment costs. I also report summary statistics for the observed Tobin's (average) Q and the "Fitted" (average) Q.

Marginal q is on average much lower and less volatile than both "Fitted" and Tobin's (average) Q. As such, the average adjustment costs (including the purchase price) do not exceed on average \$1.03 for each dollar of investment. In contrast, under the Hayashi (1982)'s assumptions of homogeneity and perfect competition, I would have estimated on average an upper bound on the average adjustment costs at about \$3.64 based on the implied equivalence between observed Tobin's (average) Q and marginal q. Therefore, these estimates of marginal q provide much tighter (and plausible) bounds on the average capital adjustment costs, regardless of the specific assumptions concerning the investment technology.

Figure 1 plots the empirical distributions of the logs of the ratio of Tobin's (average) Q to marginal q (left panel), and "Fitted" (average) Q to marginal q (right panel). Marginal q takes on values substantially lower and higher than observed Tobin's (average) Q. However, marginal q always exceeds the "Fitted" (average) Q. This is indeed consistent with the model-implied concavity of the firm value as a function of the firm capital stock.

E.1 Smooth Adjustment Costs

With smooth adjustment costs, the optimality condition for investment requires the marginal cost of investment to equal marginal q. As such, the distribution of marginal q corresponds to the distribution of marginal adjustment costs (under smoothness).

Under smooth adjustment costs, the average of the firm marginal adjustment cost of capital (including the purchase price) is only about $1.03 for each additional dollar of investment. This estimate is about 2.8 times smaller than the estimate under the "Fitted" (average) Q. Thus, if I were to use the "Fitted" (average) Q as a measurement error-free estimate of marginal q under the assumption of homogeneity, I would have substantially over-estimated the firm marginal adjustment costs. Even more so, if I were to use the observed Tobin's (average) Q. In such a case, I would have estimated the average marginal cost of investment at about $3.64, which is 3.6 times higher than under marginal q.

Therefore, while accounting for mismeasurement in marginal q (i.e. using "Fitted" (average) Q) is important, it is accounting for misspecification (i.e. using marginal q) that substantially improves the estimates of firm capital adjustment costs.

E.2 Quadratic Adjustment Costs

In this section I investigate further the empirical magnitudes of the investment-q elasticity using the new measure of marginal q. In order to facilitate the comparison with the estimates in the existing literature, I focus on smooth linear-quadratic adjustment costs. This is a special case of (4) with $\varphi = 2$, $p^+ = p^- = p > 0$, $\gamma^- = \gamma^+ = \gamma > 0$, and $f^+ = f^- = 0$. Under these assumptions, the optimal investment policy in (8) becomes linear in marginal q, and can be estimated as

$$q_{it} = \gamma \frac{I_{it}}{K_{it}} + \delta_i + \eta_t + \varepsilon_{it} \tag{15}$$

where I include firm (δ_i) and year (η_t) fixed effects to account for unobserved firm-specific and time-specific heterogeneity including differences in the depreciation rates

across firms and in the relative price of investment goods over time. The error term ε captures measurement/estimation error in the alternative measures of Q.

Table V reports the estimates of the adjustment cost parameter γ in (15) obtained with the widely used Tobin's Q (specifications (1)-(4)), "Fitted" (average) Q (specifications (2)-(5)), and the *state-space* measure of marginal q (specifications (3)-(6)).

I use the different measures of Q as dependent variables to mitigate concerns about attenuation bias due to measurement error in market values. Table V shows that the positive and significant coefficient on investment in specification (3) implies an adjustment cost parameter γ of only about 1.46. This is several order of magnitude smaller than the value implied by the use of Tobin's Q in specification (1), 12.18, and "Fitted" (average) Q, 6.65. Although many factors including fixed effects contribute to the overall adjustment costs implied by this linear-quadratic specification, one could also obtain a crude estimate of marginal adjustment costs by multiplying γ by 0.15, the sample mean for the investment rate. For $\gamma = 1.46$, the marginal cost of an additional dollar of investment is about \$0.21 dollars. For $\gamma = 6.65$, the implied marginal cost is \$1.00, and for $\gamma = 12.18$, the marginal cost is \$1.83. Assuming firm and time effects are roughly the same across these specifications, these numbers imply that estimated adjustment costs are nearly 4.8 times higher under the "Fitted" (average) Q specification, and 8.7 times higher under the observed Tobin's (average) Q. In addition, investment is substantially more correlated with marginal q than with the measures of Tobin's Q including the (measurement error-free) "Fitted" (average) Q.[10]

Finally, as reported in specifications (4)-(6) of Table V, I obtain similar results when I estimate the investment equation in (15) using first-differences rather than using the within-groups estimator.

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[10] Given the quadratic adjustment cost specification, I can also compare these estimates with previous studies. For instance, Gilchrist and Himmelberg (1995) find estimates of $\gamma = 20$ when using Tobin's (average) Q and $\gamma = 5.46$ when using the VAR-based measure of marginal q (i.e. Fundamental Q). These estimates, which correspond in dollar terms ($\simeq \gamma \times \overline{I/K} = \gamma \times 0.17$) to \$3.40 and \$0.93, respectively, are still an order of magnitude higher than the ones reported here.

While accounting for mismeasurement in marginal q (i.e. using "Fitted" (average) Q) helps to some extent, accounting for misspecification (i.e. using marginal q) substantially improves both the correlation with investment and the estimates of price-investment elasticities.

This evidence still rests on the strong (though common) assumption of a linear relationship between investment and marginal q. Given that the *state-space* estimation of marginal q is independent of the specific assumptions concerning adjustment costs, a full nonparametric investigation of nonlinearities and potential discontinuities in the relation between investment and marginal value of capital may further improve our understanding of the shape of the optimal investment policy function.

III.   Concluding Discussion and Extensions

A new and improved measure of firm marginal q - i.e. firm real investment opportunities - is surely a basis for a better understanding of corporate policies. Unlike much of the existing literature that focuses on the equivalence between marginal q and Tobin's Q by imposing strongly counterfactual assumptions about the nature of technology and markets, I show instead how to estimate directly marginal q using asset prices under general assumptions concerning technology and preferences. A general measure of marginal q can then ultimately be applied to address several open questions in economics. In this paper I focus only on the estimation of capital adjustment costs, and I show how an improved measure of marginal q can indeed provide more plausible estimates of capital adjustment costs.

For simplicity, this version of the paper only focuses on two firm-level state variables shared by a large class of neoclassical investment models: firm capital and profitability shock. While I show that these firm-level state variables capture already substantial variation in market values, the overall value function approximation can be further improved by augmenting the firm-level state space with additional measurable variables including employment and net financial liabilities. Those additional state variables would indeed

accommodate investment models with labor market frictions as in Merz and Yashiv (2007), and Bazdresch, Belo, and Lin (2013), and financial frictions as in Bond and Meghir (1994), Hennessy, Levy, and Whited (2004), Bustamante (2011), and Bolton, Chen, and Wang (2012). In such instances, the state-space methodology can be used not only to measure marginal q, but also to measure the marginal value of employment (as derivative of value function w.r.t. labor) and the marginal value of cash/debt (as derivative of value function w.r.t. cash/debt) so as to assess the overall contribution of technological versus financial market frictions on real investment. Furthermore, given any set of firm-level state variables, the overall value function approximation can be substantially improved by allowing for time-specific polynomial coefficients accounting for the full impact of any aggregate state variable.

Unlike the existing methodologies, the *state-space* measure of marginal q is independent of the assumptions concerning capital adjustment costs. As such, it provides an ideal measure to investigate nonparametrically the relation between investment and the marginal value of capital to identify potential nonlinearities and discontinuities. In addition, this independence can also be exploited to validate empirically alternative class of adjustment costs including those depending on the growth rate of investment as in Christiano, Eichenbaum, and Evans (2005) rather than on its level. Even more so, the *joint measurability* requirement in the state-space methodology also allows for a fully nonparametric kernel density estimation of marginal q, which seems a new direction of promising research.

IV.   Appendix A: Estimating Profitability Shocks

I follow Cooper and Haltiwanger (2006) to measure profitability shocks. I assume that each firm has a Cobb-Douglas revenue function $F(Z, K, N) = ZK^{\alpha_K} N^{\alpha_N}$, where Z denotes the productivity shock, K is physical capital, N is the variable factor(s), and W is the price of the variable factor(s). The equations that follow are based on one variable factor for expositional purposes but extend easily to multiple variable factors. Maximization of operating profit, $\Pi(Z, K, N) = F(Z, K, N) - WN$, over the flexible factor, N, leads to a reduced form profit function, $\Pi(K, A) = AK^{\theta}$, where $A = (1 - \alpha_N)[Z(\alpha_N/W)^{\alpha_N}]^{\frac{1}{(1 - \alpha_N)}}$ includes shocks to productivity as well as variations in factor prices and in demand. The exponent on capital θ is $\alpha_K/(1 - \alpha_N)$. Similarly, the revenue function evaluated at the optimal flexible factor takes the reduced form $F(K, A) = \frac{A}{(1 - \alpha_N)} K^{\theta}$.

The coefficient on K measuring the degree of returns-to-scale in capital (θ) in both the revenue and profit functions is the same. Moreover, the properties of the shocks to revenue and profits are the same up to a factor of proportionality. Thus, the estimation strategy is to estimate θ from either a quasi log-linear first-differenced profit or revenue regression on the capital stock. The latter seems preferred since there is potentially less measurement error involved.

Let $a_{it} = \ln(A_{it})$ have the following structure

$$a_{it} = \gamma_t + \epsilon_{it}$$

where γ_t is a common shock, and ϵ_{it} is a firm-specific shock, whose dynamics are given by

$$\epsilon_{it} = \eta_i + \rho_\epsilon \epsilon_{it-1} + \omega_{it}; \ \omega_{it} \sim MA(0)$$

where η_i is a firm-specific time-invariant effect capturing heterogeneity in the average firm profitability shocks. Taking logs and quasi-differencing yields

$$\pi_{it} = \rho_\epsilon \pi_{it-1} + \theta k_{it} - \rho_\epsilon \theta k_{it-1} + \gamma_t - \rho_\epsilon \gamma_{t-1} + \eta_i + \omega_{it}$$

or

$$\pi_{it} = \beta_1 \pi_{it-1} + \beta_2 k_{it} + \beta_3 k_{it-1} + \gamma_t^* + \eta_i + \omega_{it}$$

where $k_{it} = \ln(K_{it})$ and $\pi_{it} = \ln(\Pi_{it})$, $\beta_1 = \rho_\epsilon$, $\beta_2 = \theta$; $\beta_3 = -\rho_\epsilon \theta$, $\gamma_t^* = \gamma_t - \rho_\epsilon \gamma_{t-1}$. Whenever, the standard assumption on the initial conditions hold ($E[x_{i1}\omega_{it}] = 0$ for $t = 2, ..., T$), then by first differencing, we have

$$E[x_{it-s}\Delta\omega_{it}] = 0 \text{ where } x_{it} = (k_{it}, \pi_{it})$$

for $s \geq 2$ if $\omega_{it} \sim MA(0)$. This allows the use of suitably lagged levels of the variables as instruments, after the equation has been first-differenced to eliminate the firm-specific effects (cf. Arellano and Bond, 1991) as:

$$\Delta\pi_{it} = \beta_1 \Delta\pi_{it-1} + \beta_2 \Delta k_{it} + \beta_3 \Delta k_{it-1} + \Delta\gamma_t^* + \Delta\omega_{it}.$$

I estimate this equation via GMM using a complete set of time dummies to capture the aggregate shocks and using twice and thrice-lagged capital and twice and thrice-lagged revenue as instruments. Table A reports both the unconstrained and constrained GMM estimates. All coefficient estimates are statistically significant and the test of over-identifying restrictions (J-test) does not reject the model. In addition, the GMM distance test (D-test) also does not reject the nonlinear constraint on the coefficient estimates (i.e. $\beta_3 = -\beta_1\beta_2$). Having estimated a statistically significant $\widehat{\theta} = 0.51$ (0.13), I then recover a_{it} from $\Pi(A_{it}, K_{it}) = A_{it}K_{it}^\theta$.[11]

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[11] Although based on a different dataset - i.e. plant-level data - Cooper and Haltiwanger (2006) also obtain a similar estimate ($\theta = 0.59$).

V. Appendix B: Alternative Methodologies

I now review the alternative existing methodologies available to estimate marginal q: (i) VAR approach; and (ii) Euler equation approach.

A. VAR Approach

In their seminal work, Abel and Blanchard (1986) and Gilchrist and Himmelberg (1995) propose to estimate marginal q using VAR-based approximations of marginal q computed according to its definition as the expected present value of the future marginal profit of capital:

$$q_{it} = E\left[\int_0^\infty e^{-\delta s}\frac{\Lambda_{t+s}}{\Lambda_t}D_K\left(K_{it+s}, A_{it+s}\right)ds \mid \Theta_{it}\right] \tag{16}$$

where $D_K \equiv \Pi_K - \Phi_K$ denotes the marginal profit of capital, and Θ_{it} denotes the time t information set.

Estimation of marginal q then requires the specification of functional forms for the marginal profit of capital (i.e. Π and Φ), the stochastic discount factor, Λ, together with a method of evaluating the expected discounted stream of marginal profit of capital. To this end, one can assume that the marginal profit of capital and the stochastic discount factor are each generated as linear combinations of the elements of some observable vector which evolves according to a vector autoregressive process. The choice of some vector of fundamentals observable to the econometrician then identifies the relevant variables in the time t information set Θ_{it} for the computation of expectations.

In summary, the VAR-based approximation of marginal q requires: 1) (usually linear) approximations of marginal q (as function of Λ and D_K) to facilitate the computation of expectations of long products of stochastic variables; 2) specific functional forms for Π, Φ and Λ (usually, $\Pi_K = \Pi/K$; $\Phi_K = 0$; and $\Lambda \equiv$ weighted average of equity and debt discount factors); 3) the specification of the relevant observable variables (often not implied by the model) in the time t information set Θ_{it}; 4) assumptions on the dynamics of the relevant variables observable to the econometrician (usually linear VAR); 5) no

direct use of asset prices (unless potentially included in the time t information set Θ_{it}).

B. Euler Equation Approach

An alternative methodology available to estimate structural investment equations is based on the Euler equation (e.g. Abel (1980), Shapiro (1986), Whited (1992), and Bond and Meghir (1994)). We can rewrite the equation defining marginal q in (16) as

$$q_{it} = E\left[\int_0^\Delta e^{-\delta s}\frac{\Lambda_{t+s}}{\Lambda_t}D_K\left(K_{it+s}, A_{it+s}\right)ds + e^{-\delta\Delta}\frac{\Lambda_{t+\Delta}}{\Lambda_t}q_{it+\Delta}|\ \Theta_{it}\right] \qquad (17)$$

for any time interval $\Delta > 0$. This is the standard (continuos-time) Euler equation for investment: the shadow value of capital at time t equals its discounted expected value at time $t + \Delta$ plus any marginal profit flow generated over the time interval Δ.

Since marginal q is unobservable, one can exploit the optimality condition for investment in (8) - i.e. $q = \Phi_I$ at any times t and $t + \Delta$ when capital adjustment occurs - and substitute marginal q in the Euler equation (17) with the parameterized marginal cost of investment Φ_I:[12]

$$\Phi_I\left(I_{it}^*, K_{it}\right) = E\left[\int_0^\Delta e^{-\delta s}\frac{\Lambda_{t+s}}{\Lambda_t}D_K\left(K_{it+s}, A_{it+s}\right)ds + e^{-\delta\Delta}\frac{\Lambda_{t+\Delta}}{\Lambda_t}\Phi_I\left(I_{it+\Delta}^*, K_{it+\Delta}\right)|\ \Theta_{it}\right]. \qquad (18)$$

Equation (18) is now only a function of "observables" and its estimation then follows the procedure introduced by Hansen and Singleton (1982). The expectation operator in (18) is replaced with an expectational error uncorrelated with any information known at time t.[13] Ex-post errors based on specific functional forms for the marginal profit of capital (i.e. Π and Φ) and the stochastic discount factor, Λ, are calculated using

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[12]In models without non-convex adjustment costs, investment activity occurs each period. However, with non-convex adjustment costs, investment activity becomes infrequent, and the equivalence between marginal q and the marginal cost of investment holds only when investment activity takes place. Cooper, Haltiwanger, and Willis (2010) consider the Euler equation estimation in the presence of non-convex adjustment costs.

[13]If expectations are formed rationally, the difference between the realized and expected value should be orthogonal to information available at time t.

observations on capital and profit flows. Then structural parameters are estimated using Hansen's (1982) GMM based on appropriate orthogonality conditions. The choice of some vector of fundamentals observable to the econometrician then identifies the relevant variables in the time t information set Θ_{it} which can be used as instruments for the computation of orthogonality conditions.

In summary, the Euler equation approach requires: 1) specific functional forms for Π, Φ and Λ; 2) the specification of the relevant observable variables (often not implied by the model) in the time t information set Θ_{it}; 3) no direct use of asset prices (unless potentially included in the time t information set Θ_{it}); 4) (usually) nonlinear structural estimation methods.

Unlike the VAR-based approach, the Euler equation approach circumvents the direct estimation of marginal q by replacing it with the parameterized marginal cost of investment whenever investment activity takes place. One can then compute the implied marginal q from the optimality conditons for investment using the estimated structural parameters.

In models without non-convex adjustment costs, one can always compute the implied marginal q as investment activity takes place each period. However, in models with non-convex adjustment costs, where investment activity is infrequent, one can recover marginal q from the optimality conditons for investment only when investment activity takes place (Cooper, Haltiwanger, and Willis, 2010). As such the measurement of marginal q in the Euler equation approach is subject to the occurence of investment activity, unless one approximates the marginal value of capital in the remaining periods of inactivity by solving the dynamic programming problem under the estimated model parameters.

References

Abel, Andrew B., "Empirical Investment Equations: An Integrative Framework", *Carnegie-Rochester Conferences on Economic Policy*, vol 12, pp 29-91, 1980.

Abel, Andrew B. and Olivier Blanchard, "The Present Value of Profits and Cyclical Movements in Investments", *Econometrica*, vol 54, pp 249-273, 1986.

Abel, Andrew B. and Janice C. Eberly, "A Unified Model of Investment Under Uncertainty", *American Economic Review*, vol 84, pp 1369-1384, 1994.

Abel, Andrew B. and Eberly, Janice C., "An Exact Soultion for the Investment and Market Value of a Firm Facing Uncertainty, Adjustment Costs, and Irreversibility", *Journal of Economic Dynamics and Control* 21, 831-852, 1997.

Abel, Andrew B. and Janice C. Eberly, "Investment and q with Fixed Costs: An Empirical Investigation", unpublished manuscript, University of Pennsylvania, 2002.

Abel, Andrew B. and Janice C. Eberly, "How Q and Cash Flow Affect Investment without Frictions: An Analytic Explanation", *The Review of Economic Studies*, 78, 4, 1179-1200, 2011.

Blanchard, Olivier, Changyong Rhee, and Lawrence Summers, "The Stock Market, Profit and Investment", *Quarterly Journal of Economics* 108, 261-288, 1993.

Bazdresch, Santiago, Frederico Belo, and Xiaoji Lin, "Labor Hiring, Investment, and Stock Return Predictability in the Cross Section", *Journal of Political Economy*, forthcoming, 2013.

Bolton Patrick, Hui Chen, and Neng Wang, "A Unified Theory of Tobin's q, Corporate Investment, Financing, and Risk Management", *Journal of Finance*, forthcoming, 2012

Bolton Patrick, Huntley Schaller, and Neng Wang, "Financial Constraints, Corporate Savings, and the Value of Cash", Working Paper, 2013.

Bond, Stephen and Costas Meghir, "Dynamic Investment Models and the Firms' Financial Policy", *Review of Economic Studies* 61, 197-222, 1994.

Bustamante, Maria Cecilia, "How do Frictions affect Corporate Investment? A structural approach", Working Paper, 2011.

Caballero, Ricardo and Eduardo M.R.A. Engel, "Explaining Investment Dynamics in the U.S. Manufacturing: A Generalized (S,s) Approach", *Econometrica*, 67, 783-826, 1999.

Caballero, Ricardo J., Eduardo M.R.A. Engel, and John Haltiwanger, "Plant Level Adjustment and Aggregate Investment Dynamics", *Brookings Papers on Economic Activity*, vol 2, pp 1-39, 1995.

Caballero, Ricardo and John Leahy, "Fixed Costs and the Demise of Marginal q", NBER Working Paper #5508, 1996.

Chirinko, Robert S. and Huntley Schaller, "Why Does Liquidity Matter in Investment Equations", *Journal of Money, Credit and Banking,* vol 27, pp 527-548, 1995.

Christiano, Lawrence, Martin Eichenbaum, and Charles Evans, "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, 113: 1-45, 2005.

Cooper, Russell and Joao Ejarque, "Financial Frictions and Investment: A Requiem in Q", *Review of Economic Dynamics*, 6, 710-728, 2003.

Cooper, Russell W. and John C. Haltiwanger, "On the nature of capital adjustment costs", *Review of Economic Studies* 73, 611-633, 2006.

Cooper, Russell W., John C. Haltiwanger, and Jonathan L. Willis, "Euler-Equation Estimation for Discrete Choice Models: A Capital Accumulation Application", NBER Working Paper #15675, 2010.

Doms, Mark and Timothy Dunne, "Capital Adjustment Patterns in Manufacturing Plants", *Review of Economic Dynamics,* vol 1, pp 409-429 1998.

Eberly, Janice C., Sergio Rebelo, and Nicolas Vincent, "What explains the lagged investment effect?", *Journal of Monetary Economics*, 59(4), pp 370-380, 2012.

Erickson, Timothy and Toni Whited, "Measurement Error and the Relationship Between Investment and Q", *Journal of Political Economy*, 108, 1027-57, 2000.

Erickson, Timothy and Toni Whited, "On the Accuracy of Different Measures of Q", *Financial Management*, 35, 5-33, 2006.

Gala, Vito D., "A Fundamental Characterization of Asset Prices", Working Paper, London Business School, 2012.

Gala, Vito D. and João F. Gomes, "Beyond Q: Investment without Asset Prices", Working Paper, The Wharton School, 2013.

Gala, Vito D. and Brandon Julio, "Convergence in Corporate Investments", Working Paper, London Business School, 2011.

Gilchrist, Simon and Charles Himmelberg, "Evidence on the Role of Cash Flow in Investment", *Journal of Monetary Economics,* vol 36, pp 541-572, 1995.

Gilchrist, Simon and Charles Himmelberg, 1998, Investment: Fundamentals and Finance, in NBER Macroeconommics Annual, Ben Bernanke and Julio Rotemberg eds, MIT Press

Gomes, João F., "Financing Investment", *American Economic Review*, 90, 5, 1263-1285, 2001.

Hansen, Lars P., "Large Sample Properties of Generelized Method of Moments Estimators", *Econometrica*, 50, 1029–1054, 1982.

Hansen, Lars P. and Kenneth J. Singleton, "Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models", *Econometrica*, 50, 1269–1286, 1982.

Hayashi, Fumio, "Tobin's Marginal and Average q: A Neoclassical Interpretation", *Econometrica,* vol 50, pp 213-224, 1982.

Jorgenson, Dale W., "Capital Theory and Investment Behavior", *American Economic Review*, vol 33, 247-259, 1963.

Lucas, Robert E., "Econometric policy evaluation: a critique", *Carnegie-Rochester Conference Series*, 1, 19-46, 1976.

Lucas, Robert E., and Edward C. Prescott, "Investment Under Uncertainty", *Econometrica*, vol 39, pp 659-681, 1971.

Merz, Monika, and Eran Yashiv, "Labor and the Market Value of the Firm", *American Economic Review* 97, 1419–1431, 2007.

Shapiro, Matthew, "The dynamic demand for capital and labor", *Quarterly Journal of Economics* 101, 513-542, 1986.

Tobin, James, "A General Equilibrium Approach to Monetary Theory", *Journal of Money Credit and Banking,* vol 1, 15-29, 1969.

Whited, Toni, "Debt, liquidity constraints, and corporate investment: Evidence from panel data", *Journal of Finance* 47, 1425-1460, 1992.

Table I: Summary Statistics

This table reports summary statistics for the primary variables of interest from Compustat over the period 1972-2010. Investment rate, I/K, is defined as capital expenditures in property, plant and equipment scaled by the beginning-of-year capital stock. The capital stock, K, is defined as gross property, plant and equipment. Firm size, $\ln(K)$, is the natural logarithm of the beginning-of-year capital stock. The sales-to-capital ratio, $\ln(Y/K)$, is computed as the natural logarithm of end-of-year sales scaled by the beginning-of-year capital stock. Tobin's Q is defined as the market value of capital (market value of equity plus debt net of current assets) scaled by gross property, plant and equipment.

	Obs.	Mean	Std. Dev.	25th	50th	75th
I/K	29,564	0.15	0.16	0.06	0.10	0.18
$\ln(K)$	29,564	4.30	2.26	2.58	4.23	5.96
$\ln(Y/K)$	29,564	0.97	0.74	0.50	0.95	1.42
Q	29,564	3.64	7.09	0.55	1.39	3.54

Table II: Market Value and Investment by State-Variable Portfolios

This table reports equal-weighted averages of market values and investment rates for portfolios based on conditional sorts on firm size, K, and profitability shock, A. The sample period is 1972 to 2010.

Market Value, $V($\$$, bn)		Profitability Shock (A)				
		Q1	2	3	4	Q5
Firm Size (K)	Q1	0.02	0.02	0.03	0.04	0.11
	2	0.07	0.08	0.12	0.19	0.34
	3	0.18	0.27	0.34	0.44	0.90
	4	0.43	0.66	1.04	1.48	2.59
	Q5	1.83	3.35	5.54	11.20	23.11

Investment Rate, I/K		Profitability Shock (A)				
		Q1	2	3	4	Q5
Firm Size (K)	Q1	0.124	0.132	0.183	0.225	0.318
	2	0.111	0.138	0.148	0.205	0.264
	3	0.110	0.131	0.143	0.170	0.222
	4	0.106	0.119	0.128	0.136	0.169
	Q5	0.098	0.107	0.116	0.120	0.139

Table III: Value Function Approximation

This table reports estimates from the value function approximation:

$$v_{it} = \sum_{j_k=0}^{n_k} \sum_{j_a=0}^{n_a} c_{j_k,j_a} \times [k_{it}]^{j_k} \times [a_{it}]^{j_a} + \delta_i + \eta_t + \epsilon_{it} \tag{19}$$

where the left-hand-side is the log market value scaled by capital, $\ln V/K$, k is firm size, $\ln K$, a is profitability shock, $\ln A$, δ_i is a firm fixed effect, and η_t is a year fixed effect. Standard errors are clustered by firm and reported in parenthesis. R^2 denotes adjusted R-square and AIC is the Akaike Information Criterion. "qQ-test" is a Wald test of the equivalence between marginal q and average Q as described in (14). P-values are reported. The sample period is 1972 to 2010.

	(1)	(2)	(3)
$\ln K$	-0.60	-0.76	-0.67
	$(0.02)^{***}$	$(0.04)^{***}$	$(0.04)^{***}$
$\ln A$	1.01	0.50	0.42
	$(0.02)^{***}$	$(0.05)^{***}$	$(0.05)^{***}$
$\ln K^2$		0.02	0.03
		$(0.00)^{***}$	$(0.00)^{***}$
$\ln A^2$		0.09	0.15
		$(0.01)^{***}$	$(0.01)^{***}$
$\ln A \times \ln K$			-0.07
			$(0.01)^{***}$
$\overline{R}^2$	0.63	0.64	0.64
AIC	$77,507$	$76,919$	$76,861$
qQ-test	0.00	0.00	0.00

Table IV: Distribution of Marginal q

This table reports summary statistics for Tobin's (average) Q, and the estimated marginal q and fitted (average) Q. Fitted (average) Q is computed as the fitted value of the value function approximation in (19). Marginal q is computed according to its definition as first derivative of the value function approximation with respect to the capital stock. The sample period is 1972 to 2010.

	Obs.	Mean	Std. Dev.	25th	50th	75th
Tobin's Q	$29,564$	3.64	7.09	0.55	1.39	3.54
Fitted Q	$29,564$	2.85	5.14	0.62	1.28	2.93
Marginal q	$29,564$	1.03	1.64	0.26	0.53	1.13

Table V: Estimating Smooth Adjustment Costs

This table reports estimates from the following regression:

$$Y_{it} = \gamma \frac{I_{it}}{K_{it}} + \delta_i + \eta_t + \varepsilon_{it}$$

where the left-hand side variable Y_{it} is either marginal q, fitted (average) Q or Tobin's (average) Q, and the right hand side variables include the investment rate (I/K), δ_i is a firm fixed effect, η_t is a year fixed effect. Specification (1) and (4) report estimates using Tobin's (average) Q as dependent variable. Specification (2) and (5) report estimates using fitted (average) Q as dependent variable. Specification (3) and (6) report estimates using marginal q as dependent variable. In specifications (1)-(3) I remove firm fixed effects by de-meaning the actual variables, while in specifications (4)-(6) by first-differencing the actual variables. Standard errors are clustered by firm and reported in parenthesis. $\overline{R}^2$ denotes adjusted R-square. The sample period is 1972 to 2010.

	Within-Groups			First-Difference		
	(1)	(2)	(3)	(4)	(5)	(6)
Tobin's Q	12.18			5.56		
	$(0.73)^{***}$			$(0.46)^{***}$		
Fitted Q		6.65			4.17	
		$(0.54)^{***}$			$(0.39)^{***}$	
Marginal q			1.46			0.94
			$(0.13)^{***}$			$(0.10)^{***}$
$\overline{R}^2$	0.12	0.13	0.16	0.05	0.14	0.21

Table A: Estimating Productivity Shocks

This table reports GMM estimates of the following specification:

$$\Delta\pi_{it} = \beta_1\Delta\pi_{it-1} + \beta_2\Delta k_{it} + \beta_3\Delta k_{it-1} + \Delta\gamma_t + \Delta\omega_{it}$$

using a complete set of time dummies, twice and thrice-lagged capital and twice and thrice-lagged profits as instruments. Standard errors are reported in parenthesis. CRS-test is a test of constant returns to scale hypothesis $\beta_2 = 1$. J-test denotes the test of overidentifying restrictions, and D-test denotes the GMM distance test concerning the constraints on the coefficient estimates as described in Appendix. P-values are reported. The sample period is 1972 to 2010.

	Unconstrained GMM	Constrained GMM
β_1	0.54	0.63
	$(0.13)^{***}$	$(0.10)^{***}$
β_2	0.47	0.51
	$(0.12)^{***}$	$(0.13)^{***}$
β_3	-0.29	-0.32
	$(0.09)^{***}$	$(0.09)^{***}$
J-test	0.18	0.20
CRS-test	0.00	0.00
D-test		0.25

Figure 1: Empirical Distribution of Q Wedges

This figure plots the empirical distributions of the log ratio of Tobin's (average) Q to marginal q (left panel) and "Fitted" (average) Q to marginal q (right panel). The sample period is 1972 to 2010.