

# Disclosure, Learning, and Coordination

By H. HENRY CAO, YUAN MA, AND DONGYAN YE\*

Draft: June 10, 2013

*We analyze how public disclosure of informed investors' trades results in manipulation, which in turn affects coordination and competition in a duopolistic setting. We show that disclosure always increases market efficiency but its effect on informed investors' profit is ambiguous. When informed investors have very imprecise information, disclosure makes them coordinate their trades, so their expected profits are higher. Moreover, an informed investor with very imprecise information would prefer competition in the presence of disclosure as he learns more from the other informed investor than the market maker and make more profits than he would obtain in a monopolistic market.*

How would trade disclosure in financial markets affect informed investors' profits? Intuitively, one would expect that their profits would reduce. Indeed, many regulatory proposals and legislations have argued that disclosure would help level the playing field, reduce information asymmetry and benefit small investors. For example, corporate insiders are required to disclose their trades to the Securities and Exchange Commission (SEC). Section 16(a) of the SEC Act requires the insiders to report their trades to the Commission within ten days following the end of the month in which the trade occurs. Recently, SEC have made proposals to report high frequency trading on a timely basis. On April 14, 2013, SEC issued a

\* Cao: Cheung Kong Graduate School of Business (CKGSB), Oriental Plaza, Tower E3, 3F, One East Chang An Avenue, Beijing 100738; email: hncao@ckgsb.edu.cn. Ma: KCG group; email: mayuan@mayuan.com. Ye: CKGSB, Oriental Plaza, Tower E3, 3F, One East Chang An Avenue, Beijing 100738, China; email: dyeye@ckgsb.edu.cn. We thank seminar participants at Berkeley-Stanford joint seminar series, Ohio State University, University of Utah, and University of Maryland for helpful comments. Correspondence should be addressed to H. Henry Cao.

release proposing that certain large-volume, high frequency traders (classified as “large traders”) be required to self-identify to the SEC and that broker-dealers that effect transactions for “large trader” customers maintain and produce records of these customers’ trades to the SEC.

We study the effects of mandatory public disclosure of trading (after the fact) in a duopolistic setting. It is sensible that disclosure will reduce informed investors’ profits when they have the same piece of private information but it is less clear when they have different information. In the latter case, not only the market can learn from the public disclosure, informed investors can also learn more from the disclosure about each other’s signals. With diversely informed investors, trade disclosure can act as a communication device that affects learning among informed investors differently from that of the market maker. We examine the following issues: How would disclosure of informed investors’ trades affect market efficiency and market liquidity in a setting with differentially informed investors? Under what conditions, if any, would an informed investor prefer competition if they can learn more from each other through disclosure?

In particular, we consider a Kyle model of two informed investors each of whom is required to disclose his trade immediately after the trade is made. In discrete time, we derive a recursive formula for the equilibrium, which can be solved by numerical methods. In continuous time, we derive a closed-form formula for the equilibrium. To determine the impact of disclosure, we compare our closed-form equilibrium formula with that obtained in Back, Cao, and Willard (2000, BCW), whose model is the same except no disclosure is required there.

Disclosure of informed investors’ trades creates incentives for informed investors to manipulate in that they sometimes trade against their own valuation to mislead the market, so that the market maker cannot perfectly infer information from their trades. As a result, the informed investors randomize to manipulate the market maker’s belief until the last moment of trading. The mixed strategy allows informed investors to maintain an informational advantage over the mar-

ket for a longer period of time. We show that the combined random components in informed investors' trade equals in distribution to that of liquidity traders. This is intuitively appealing as informed investors and liquidity traders will each contribute to half of the trading volume. Too much randomization will cause informed investors to lose a lot from randomized trade and too little randomization will cause informed investors to lose their informational advantage too early. To camouflage themselves, informed investors randomize such that their combined trading volume equals that of liquidity traders'.

The effects of trade disclosure on market efficiency is unambiguous. Market is more efficient at all times after disclosure. As informed investors know more about each other's signal, their valuations converge more quickly and they trade more aggressively on their information, which in turn makes the market more efficient.

The effects on the expected profits of informed investors and market liquidity are more complicated. Public disclosure has three effects on informed investors' expected profits. The first is the randomization effect. As informed investors manipulate and add noise to their own trades, they lose money, which reduces their expected profits. The second is the coordination effect. With trade disclosure, informed investors learn twice more about each other than the market maker, which in turn increase their expected profits. The coordination effect is the most evident in the case when informed investors' signals are uncorrelated. In this case, the equilibrium is collusive, informed investors act in sync as a monopolist. The third is the market efficiency effect. Disclosure increases market efficiency, which reduces expected profits of informed investors.

When informed investors have very precise signals, they won't be able to learn from each other as much. In this case the coordination effect will be less important and disclosure decreases expected profits of informed investors. On the contrary, when investors have very noisy signals, they tend to wait until they know more from each other before they trade aggressively. Trade disclosure can reduce the

incentive to wait and make investors trade more aggressively. Informed investors learn more from disclosure than the market maker. The coordination effect could dominate other effects and result in higher expected profits of informed investors. Moreover, the coordination effect could be so strong such that an informed investor makes more profits in a duopolistic setting than what he would receive in a monopolistic setting. Therefore in the presence of disclosure, an informed investor could prefer to have competition. Indeed, an informed investor can even make more money in a duopolistic setting with disclosure than what he would expect in a monopolistic market without disclosure. Notice that this never happens when trading is not disclosed.

Similarly, the effects on market liquidity is also ambiguous. Randomization will reduce the informational content in the aggregate order flow and thus increase market liquidity. However, coordination among investors could reduce market liquidity. The reduction of asymmetric information would increase market liquidity. As a result, market liquidity can either increase or decrease depending on the parameters and the timing of the trades.

We extend the model to more than two informed investors. In this case, the gains from learning by informed investors over that of the market maker is reduced as now each informed investor only knows  $1/N$  of the randomized noise trades. As a result, disclosure always reduces informed investors' profits. Nevertheless, with disclosure, it is still possible for informed investor to make more profits in an oligopolistic setting than what he would receive in a monopolistic setting when the number of informed investors is strictly less than five and they have very imprecise information. Moreover, removing one informed investor from trading in the market can make the rest of informed investors worse off.

The most related paper is by Huddart, Hughes, and Levine (2001, HHL) who study disclosure effects in a discrete-time Kyle model with a monopolistic informed investor. They show that the informed investor uses a mixed strategy in which the informed investor attaches a random order flow, for hiding information,

to the information-based flow that is exactly the same as that in Kyle's model. In addition, mandatory disclosure unambiguously reduces informed investor's profits, increases market liquidity, and improves market efficiency. However, they do not analyze how disclosure affects informed investors' strategic trading behavior when there are more than one informed investor. Gong and Liu (2012) extend their results to multiple informed investors. In their model, informed investors to have homogeneous information and thus as trading frequency goes to infinity, information will be revealed in opening trades and the expected profits for insiders go to zero. Zhang (2004) show that when the informed investor is risk averse, trade disclosure can reduce market efficiency as the risk averse investor will be facing less price risk in the future when he unloads his positions and thus will not trade in a hurry.<sup>1</sup>

The rest of the paper is organized into sections as follows. The model is described in Section I. Section II discusses the condition for equilibrium with public disclosure in a discrete-time framework and offers a closed-form formula for the equilibrium in a continuous-time framework. Section III gives comparative statistics such as the effects of the number of informed investors and the correlation of their signals on the intensity of trading, the rate of information transmission, the depth of the market, and the expected profits of informed investors. Section IV extends the model from a duopolistic setting to a general multiple players setting. Section V concludes. All proofs are left to the appendices.

<sup>1</sup>The effect of disclosure rules on informed investors' trading has also been studied by a number of authors including Fishman and Hagerty (1995) and John and Narayanan (1997). Fishman and Hagerty (1995) study a two period model when an informed investor only possesses inside information with a certain probability. While an *informed* informed investor will never manipulate the market in their model, an *uninformed* informed investor can manipulate the market since the market may mistakenly believe that the *uninformed* informed investor is *informed*. In models with disclosure but with multiple trading periods, Chakraborty and Yilmaz (2004) show that when the market faces uncertainty about the existence of the insider in the market and when there is a large number of trading periods before all private information is revealed, long-lived informed investors will manipulate in every equilibrium. Brunnermeier (2005) shows how disclosure of intermediary public information can cause investors with short term noisy information to manipulate the market.

## I. The Model

We consider an economy with two informed investors who are required to disclose their trades based on the classic model of Kyle (1985). In our model, there are one risk-free asset and one risky asset. An announcement is made at time 1 that reveals the liquidation value of the asset. The risk-free rate is taken to be zero. There are 2 risk neutral informed investors and many liquidity traders who trade for liquidity reasons. Trading takes place over time interval  $[0, 1)$ . In the discrete-time version of the model, there are  $M$  periods over time  $[0, 1)$ , and the time between any two consecutive trading periods is  $\Delta t = 1/M$ .

Let  $v$  denote the liquidation value of the risky asset at time 1. Before any trading starts, each informed investor  $i$  ( $i = 1, 2$ ) receives a mean-zero signal  $s^i$  at time 0. We assume the signals and the liquidation value of the risky asset has a non-degenerate joint normal distribution that is symmetric in the signals.<sup>2</sup> More specifically, we have

$$(1) \quad s^1 = \frac{v + \epsilon}{2}; \quad s^2 = \frac{v - \epsilon}{2}; \quad v = \sum_{i=1}^2 s^i.$$

The variances of  $v, \epsilon$  are denoted  $\sigma_v^2$  and  $\sigma_\epsilon^2$  respectively.<sup>3</sup>

We use  $\rho$  to denote the correlation coefficient of  $s^1$  with  $s^2$ .<sup>4</sup>

$$\rho = \frac{\sigma_v^2 - \sigma_\epsilon^2}{\sigma_v^2 + \sigma_\epsilon^2}$$

In the special case of  $\sigma_\epsilon^2 = 0$ ,  $\rho = 1$ , each informed investor has perfect infor-

<sup>2</sup>Symmetry means that the joint distribution of the asset value and the signals  $s^1, s^2$  is invariant to a permutation of the indices.

<sup>3</sup>The assumption about the information structure is made without the loss of generality. Due to the assumption of normality, for any arbitrary symmetric signals, the sufficient statistic of  $s^1, s^2$  is  $s^1 + s^2$ . Risk neutrality indicates that all investors care is the conditional expectation of  $v$  given  $s^1, s^2$  which is a linear function of  $s^1 + s^2$ . Since the covariance of  $s^1 - s^2$  and  $s^1 + s^2$  is zero, we can always redefine  $v \equiv s^1 + s^2$  and rewrite the signals in the form of equation (1).

<sup>4</sup>We allow informed investors to have negative correlations. Negative correlation can happen, for example, when the firm release a public signal of  $y \equiv v - \eta_1 - \eta_2$ , with  $v, \eta_1, \eta_2$  jointly independent and normal. Informed investor  $i, i = 1, 2$ , privately observes the error in the public signal,  $\eta_i$ . Conditional on the release of the public signal, we have  $v = y + \eta_1 + \eta_2$  and  $\eta_1, \eta_2$  are conditionally negatively correlated.

mation about  $v$ . For convenience, we also introduce the following notation

$$(2) \quad \delta_0 \equiv \frac{\text{var}[v] - \text{var}[v|s^1]}{\text{var}[v]} = \frac{\text{var}^{-1}[v|s^1] - \text{var}^{-1}[v]}{\text{var}^{-1}[v|s^1]}.$$

This is a measure of the quality of private information of informed investor 1 and by the argument of symmetry, informed investor 2 as well. Specifically,  $\delta_0$  is the ‘‘R-squared’’ in the linear regression of  $v$  on  $s^i$  for an arbitrary  $i$ , i.e., it is the percent of the variance in  $v$  that is explained by a single informed investor’s information. Alternatively, it is also the percentage drop in precision of informed investors to that of the market maker. It is easy to check that  $\delta_0$  is related to  $\rho$  by the following equation

$$(3) \quad \delta_0 = \frac{1}{2} + \frac{1}{2}\rho = \frac{\sigma_v^2}{\sigma_v^2 + \sigma_\epsilon^2}.$$

Thus, when  $\sigma_v$  is larger than, equal to, or smaller than  $\sigma_\epsilon$ ,  $\delta_0$  is larger than, equal to, or smaller than  $1/2$ , and informed investors’ signals are positively correlated, uncorrelated or negatively correlated respectively. When  $\sigma_\epsilon$  is small (large), each informed investor has very precise (coarse) information of the liquidation value.

In each trading period  $m$ , a risk-neutral market maker receives the total order from all informed investors and liquidity traders. Based on such order flow information, the market maker adjusts the price  $P_{m-1}$  to a new price  $P_m$  at which he buys or sells the risky security to clear the market in period  $m$ . Since the market maker is assumed to be risk neutral, price  $P_m$  must be the conditional expectation given all public information. We use  $x_m^i$  to denote informed investor  $i$ ’s order, and use  $z_m^0$  to denote the total order by all liquidity traders. We assume that  $z_m^0$  are serially uncorrelated and normally distributed with mean zero and variance

$$E[z_m^0] = 0, \quad \text{and} \quad \text{var}[z_m^0] = \sigma_u^2 \Delta t, \quad \text{for all } m.$$

For simplicity, we assume  $\sigma_u = 1$ . In addition,  $z_m^0$  is independent of all other

random variables in the model. Moreover, we assume that informed investors are prevented from any market making activities, and hence when they submit their orders in period  $m$  they have no information about the  $m$ th-period order flow from any other party.

The only difference between a model with disclosure and a model without disclosure is whether or not each informed investor is required to disclose his  $m$ th period trade immediately after all trades are completed in period  $m$ . Technically, this implies the following difference in how each of the involved parties behaves in the model. Without disclosure, (1) the market maker sets his price  $P_m$  by observing the history of the aggregate order flow  $\{y_k : 1 \leq k \leq m\}$ , where

$$y_k \equiv z_k^0 + \sum_{1 \leq i \leq 2} x_k^i$$

and (2) each informed investor  $i$  decides his trade by observing his own past order flow  $\{x_k^i : 1 \leq k < m\}$ , his own signal  $s^i$ , and the past price history  $\{P_k : 1 \leq k < m\}$ . With disclosure, (1) the market maker sets his price by observing the breakdown of all traders' past order flow  $\{x_k : 1 \leq k < m\}$  and  $\{z_k^0 : 1 \leq k < m\}$  together with the current aggregate order flow  $\{y_m\}$ ; and (2) each informed investor  $i$  decides his trade by observing all traders' past order flow  $\{x_k : 1 \leq k < m\}$  and  $\{z_k^0 : 1 \leq k < m\}$ , in addition to his signal  $s^i$  and the past price history  $\{P_k : 1 \leq k < m\}$ . Note that in a model with disclosure, the breakdown of all the past order flow  $\{x_k : 1 \leq k < m\}$  and  $\{z_k^0 : 1 \leq k < m\}$  are made public through public disclosure and price history.

The description above has focused on the discrete-time version of the model. An intuitive way to think of the continuous-time model is simply to take the limit of the discrete-time model with  $M \rightarrow +\infty$ . More technical details will be given when it comes to the derivation of our results in continuous-time.



## II. The Solution

Under the disclosure requirement, informed investors announce their trades,  $\{x_m^i\}$ ,  $i = 1, 2$ , immediately after the trade is executed. The market maker then adjusts his belief of the asset value from  $P_m$  (the market price for the risky asset in period  $m$ ) to  $V_m$  which is defined to be the market maker's estimate of the fair value of the risky asset with all the information up to and including the disclosure made at the end of period  $m$ . We can think of  $V_m$  as the pseudo-price that market maker would have set for the  $m$ th period trading if he had observed informed investors' order before the execution of trades in the  $m$ th period. Although  $V_m$  is only a pseudo-price at which no trade ever takes place, it is important since it will be the starting point for the market maker to set  $P_{m+1}$  for the  $(m + 1)$ th period of trading. In particular, in a linear equilibrium model that we will focus on, it is  $P_{m+1} - V_m$  (as opposed to  $P_{m+1} - P_m$ ) that will be linear to the total order flow submitted in the  $(m + 1)$ th trading period.

Let  $\underline{x}_m^i$  denote the history of investor  $i$ 's trade in each past period before and including period  $m$  (i.e.,  $\{x_k^i : k = 1, \dots, m\}$ ), let  $\underline{y}_m$  denote the history of the net trade before and including period  $m$  (i.e.,  $\{z_k^0 + \sum_{1 \leq i \leq 2} x_k^i : k = 1, \dots, m\}$ ), and let  $\underline{P}_m$  denote the price history before and including period  $m$  (i.e.,  $\{P_k : k = 1, \dots, m\}$ ). With disclosure, informed investor  $i$ 's private information prior to trading in period  $m$  includes his own signal  $s^i$  and the history of all past trades and prices  $\underline{x}_{m-1}^1, \underline{x}_{m-1}^2, \underline{P}_{m-1}$ . Let

$$x_m^i = x_m^i(s^i, \underline{x}_{m-1}^1, \underline{x}_{m-1}^2, \underline{P}_{m-1})$$

represent the optimal strategy of informed investor  $i$ . Let

$$P_m = P_m(\underline{x}_{m-1}^1, \underline{x}_{m-1}^2, \underline{y}_m)$$

represent the optimal strategy of the market maker given the history of all orders

and the current aggregate order.

Let  $X^i$  and  $P$  denote the strategy functions for informed investor  $i$  and the market maker, respectively. Given the strategy functions for informed investors and the market maker, the profit of informed investor  $i$  from trading in period  $m$  and on can be written as:

$$\pi_m^i(X^1, X^2, P) = \sum_{k \geq m} (v - P_k) x_k^i.$$

An equilibrium of the trading game exists if there is an 3-dimension vector of strategies,  $(X^1, X^2, P)$  such that:

- 1) For any  $i = 1, 2$  and for all  $m = 1, \dots, M$ , if  $\hat{X}^i \neq X^i$ ,

$$E [\pi_m^i(X^i, X^j) | s^i, \underline{x}_{m-1}^1, \underline{x}_{m-1}^2, P_{m-1}] \geq E [\pi_m^i(\hat{X}^i, X^j) | s^i, \underline{x}_{m-1}^1, \underline{x}_{m-1}^2, P_{m-1}]$$

i.e., the optimal strategy is the best no matter which past strategies informed investor  $i$  may have played.

- 2) For all  $m = 1, \dots, M$ , we have

$$P_m = E [v | \underline{x}_{m-1}^1, \underline{x}_{m-1}^2, \underline{y}_m],$$

i.e., the market maker sets prices equal to the conditional expectation of the asset value given the order-flow history.

In this model, since investor  $i$ 's trade at period  $m$  will be disclosed afterwards, the pricing and trading strategies for the no-disclosure case cannot be an equilibrium in the new setting. To see this, suppose the informed investor follows a strategy of <sup>5</sup>

$$x_m^i = \beta_m \Delta t s^i + L_1(x_{m-1}^i) + L_2(x_{m-1}^1, x_{m-1}^2)$$

<sup>5</sup>We restrict our attention to symmetric linear equilibria.

where  $L_i$  is a linear function of all public information. Then the market maker would infer

$$v = \frac{\sum_{1 \leq i \leq 2} [x_m^i - L_1(x_{m-1}^i) - L_2(x_{m-1}^1, x_{m-1}^2)]}{\beta_m \Delta t}$$

and choose

$$P_{m+1} = \frac{\sum_{1 \leq i \leq 2} [x_m^i - L_1(x_{m-1}^i) - L_2(x_{m-1}^1, x_{m-1}^2)]}{\beta_m \Delta t}$$

in the next period. Hence, in the next period, the market depth would be infinity. Understanding this, informed investors would have incentive to choose  $\hat{x}_m^i \neq x_m^i$  which is inconsistent with the proposed equilibrium strategy.

We analyze a symmetric linear equilibrium. In particular, informed investor's trade can be written as

$$(\star) \quad x_m^i = \beta_m \Delta t s^i + L_1(x_{m-1}^i) + L_2(x_{m-1}^1, x_{m-1}^2) + z_m^i,$$

where (1)  $\beta_m \Delta t s^i$  represents a private-information based linear component, (2)  $L_1(x_{m-1}^i) + L_2(x_{m-1}^1, x_{m-1}^2)$  is a public-information based linear component, and (3)  $z_m^i$  is a noise component with  $z_m^i$  being normally distributed with mean 0 and variance  $\sigma_m^2 \Delta t$ . Since informed investors are prevented from market making activities, we further assume that  $z_m^i$  are independently distributed across agents. The market maker also uses linear rules for setting prices before disclosure and for updating his value estimate after disclosure. In particular,

$$P_m = V_{m-1} + \lambda_m \left( z_m^0 + \sum_{1 \leq i \leq 2} x_m^i \right), \quad \text{and}$$

$$V_m = V_{m-1} + \bar{\lambda}_m \left( \sum_{1 \leq i \leq 2} x_m^i \right).$$

The preceding equations imply that the random order from liquidity traders only has a temporary effect on price formation. In particular, liquidity traders' order in period  $m$  (i.e.,  $z_m^0$ ) only affects  $P_m$  but not  $P_k$  for any  $k \geq m + 1$ : Once the

$m$ th-period disclosure is made, the market maker immediately abandons  $z_m^0$  and adjusts his belief of asset value to  $V_m$ , which is not affected by  $z_m^0$  and will be the base for forming future prices  $P_k$  ( $k \geq m + 1$ ).

Before stating our result, we first introduce some notation. Let  $F_m$  and  $F_m^i$  denote the information set of the market maker and informed investor  $i$  respectively after disclosure has been made in period  $m$ . Define

$$\begin{aligned} V_m &\equiv E[v|F_m], & V_m^i &\equiv E[v|F_m^i], \\ \Sigma_m &\equiv \text{var}[v|F_m], & \Omega_m &\equiv \text{var}[v|F_m^i], & \text{and } \delta_m &\equiv \frac{\Sigma_m - \Omega_m}{\Sigma_m}. \end{aligned}$$

**THEOREM II.1:** *The necessary and sufficient conditions for a recursive linear symmetric equilibrium to exist are described below. For all  $m = 1, \dots, M - 1$  and for all informed investors  $i = 1, 2$ ,*

$$\begin{aligned} x_m^i &= \frac{\beta_m \Delta t}{2\delta_{m-1}} (V_{m-1}^i - V_{m-1}) + z_m^i \\ P_m &= V_{m-1} + \lambda_m \left( z_m^0 + \sum_{i=1}^2 x_m^i \right) \\ V_m &= V_{m-1} + \bar{\lambda}_m \sum_{i=1}^2 x_m^i \\ \bar{\lambda}_m &= \beta_m \Sigma_m / (2\sigma_m^2) \\ \lambda_m &= \beta_m \Sigma_{m-1} / (\beta_m^2 \Delta t \Sigma_{m-1} + 1 + 2\sigma_m^2) \\ V_m^i - V_{m-1}^i &= \frac{\Omega_{m-1} - \Omega_m}{\Omega_{m-1}} \left( v - V_{m-1}^i + \frac{z_m^i}{\beta_m \Delta t} \right) \\ V_m - V_{m-1} &= \frac{\Sigma_{m-1} - \Sigma_m}{\Sigma_{m-1}} \left( v - V_{m-1} + \sum_{1 \leq i \leq 2} \frac{z_m^i}{\beta_m \Delta t} \right) \\ \Omega_m^{-1} &= \Omega_{m-1}^{-1} + \beta_m^2 \Delta t / (\sigma_m^2) \\ \Sigma_m^{-1} &= \Sigma_{m-1}^{-1} + \beta_m^2 \Delta t / (2\sigma_m^2) \\ E[\pi_m^i | F_{m-1}^i] &= \alpha_{m-1} (V_{m-1}^i - V_{m-1})^2 + \zeta_{m-1} \end{aligned}$$

$$\begin{aligned}
\lambda_m &= \alpha_m \bar{\lambda}_m^2 \\
\lambda_m &= \frac{\bar{\lambda}_m}{2 - \bar{\lambda}_m \beta_m \Delta t (1 - 1/(2\delta_{m-1}))} \\
\alpha_{m-1} &= \alpha_m \left( 1 - \frac{\beta_m^2 \Delta t \Sigma_m}{2\sigma_m^2} \left( 1 - \frac{1}{2\delta_{m-1}} \right) \right)^2 \\
\zeta_{m-1} &= \zeta_m + \alpha_m \beta_m^2 \Delta t \left( \frac{\Omega_m}{\sigma_m^2} - \frac{\Sigma_m}{2\sigma_m^2} \right)^2 (\Omega_{m-1} \beta_m^2 \Delta t + \sigma_m^2)
\end{aligned}$$

subjecting to the boundary conditions

$$\begin{aligned}
\beta_M &= \sqrt{\frac{2\delta_{M-1}}{\Sigma_{M-1} \Delta t}}, \\
\lambda_M &= \frac{\sqrt{2\delta_{M-1} \Sigma_{M-1} / \Delta t}}{1 + 2\delta_{M-1}}, \\
\alpha_{M-1} &= \frac{1}{\lambda_M (1 + 2\delta_{M-1})^2}, \\
\zeta_{M-1} &= 0,
\end{aligned}$$

and the second order condition

$$\lambda_M > 0.$$

In the special case that  $\sigma_\epsilon = \sigma_v$ , the model can be solved in closed form:

$$\begin{aligned}
\lambda_m &= \sqrt{\Sigma_0}/2, & \beta_m &= 1/[2\lambda_m(M - m + 1)], \\
\bar{\lambda}_m &= 2\lambda_m, & \sigma_m^2 &= (M - m)/[2(M - m + 1)], \\
\alpha_m &= 1/(4\lambda_m), & \Omega_m &= (1 - m/M)\Omega_0, \\
\zeta_m &= 0, & \Sigma_m &= (1 - m/M)\Sigma_0.
\end{aligned}$$

The results in the special case that  $\sigma_\epsilon = \sigma_v$  are the same as the monopolistic

model derived by HHL (2001). This is in sharp contrast to results on imperfect competition of informed investors without disclosure. Foster and Viswanathan (1996), Cao (1995), and BCW (2000) have shown that competition causes the market to be very illiquid and inefficient near the end of trade when there is no disclosure. With disclosure, we find that informed investors act in the aggregate as a monopolist when their signals are uncorrelated. In the uncorrelated-signal-case, with disclosure, each informed investor knows his own random noise in the past. Consequently, each informed investors' conditional precision will remain to be twice of that of the market maker as they learn twice as faster. If informed investors' signals are uncorrelated to begin with, they remain uncorrelated due to public disclosure of trades after transaction is completed. Therefore, disclosure makes informed investors coordinate with each other to maximize their profits and they act like a monopolist in the aggregate. On the contrary, without disclosure, BCW (2000) show that the conditional correlation coefficient of informed investors' signals goes to  $-1$  even when the initial correlation coefficient is zero.

When the number of trading periods goes to infinity, the model approaches to the continuous-time model. Ignoring higher order terms of  $\Delta t$ , we have the following:

$$\begin{aligned}\bar{\lambda}(t) &= \beta(t)\Sigma(t), & \Delta\Omega(t)^{-1}/\Delta t &= 2\beta(t)^2, \\ \lambda(t) &= \beta(t)\Sigma(t)/2, & \Delta\Sigma(t)^{-1}/\Delta t &= \beta(t)^2, \\ \sigma(t)^2 &= 1/2, & \Delta\alpha(t)/\Delta t &= 2\alpha(t)\beta(t)^2\Sigma(t)[1 - 1/(2\delta(t))], \\ \bar{\lambda}(t) &= 1/(2\alpha(t)), & \Delta\zeta(t)/\Delta t &= -\alpha(t)\beta(t)^2[2\Omega(t) - \Sigma(t)]^2/2.\end{aligned}$$

In the limit, these difference equations converge to a set of differential equations which leads to closed form solutions described in Theorem II.2.

**THEOREM II.2:** *If  $\sigma_\epsilon > 0$ , i.e., informed investors' signals are not perfectly*

correlated, there is a unique symmetric linear equilibrium specified as follows

$$\begin{aligned}\beta(t) &= \frac{\sqrt{-\Sigma(t)'}}{\Sigma(t)} = \frac{1}{\sigma_\epsilon(1-t)}, \\ \lambda(t) &= \frac{\sqrt{-\Sigma(t)'}}{2} = \frac{\sigma_v^2 \sigma_\epsilon}{2[\sigma_v^2 t + \sigma_\epsilon^2(1-t)]}, \\ \bar{\lambda}(t) &= \sqrt{-\Sigma'(t)} = \frac{\sigma_v^2 \sigma_\epsilon}{\sigma_v^2 t + \sigma_\epsilon^2(1-t)},\end{aligned}$$

where  $\Sigma(t)$  is specified as

$$(4) \quad \Sigma(t) = \frac{\sigma_v^2 \sigma_\epsilon^2 (1-t)}{\sigma_v^2 t + \sigma_\epsilon^2 (1-t)}$$

In equilibrium, the expected profit of each informed investor  $\pi_D$  is

$$(5) \quad \pi_D = \frac{1}{2} \int_0^1 \lambda(t) dt = \frac{\sigma_v^2 \sigma_\epsilon [\log(\sigma_v) - \log(\sigma_\epsilon)]}{2(\sigma_v^2 - \sigma_\epsilon^2)}.$$

Investors' trading intensity  $\beta$  is proportional to  $1/\sigma_\epsilon$ . This is sensible as investors will trade more cautiously as they have noisier signals. Surprisingly,  $\lambda$  is finite through the trading period remains constant as long as  $\sigma_v = \sigma_\epsilon$ . This is in sharp contrast to the result in BCW (2000) who show that  $\lambda$  goes to infinity near the end of trading in the absence of disclosure.

### III. Comparative Dynamics

In this section, we use the closed-form solutions derived in the previous section to study the dynamics and comparative statics of trading, market efficiency and market liquidity.

While in most strategic trading models, the trading volume coming from informed investors is negligible compared to liquidity traders. However, when disclosure is required, informed investors' trades contains a component of positive quadratic variation that is comparable to that of liquidity traders:

COROLLARY III.1: *Informed investors contribute half of the trading volume in the market with disclosure.*

To mix with liquidity traders, the endogenous random trades of informed investors equal in distribution to that of liquidity traders.

We next examine the comparative statics of  $\Sigma(t)$ ,  $\beta(t)$ ,  $\lambda(t)$  with respect to time and the degree of noise in informed investors' signals, as measured by  $\sigma_\epsilon$ .

COROLLARY III.2: *The variables  $\Sigma(t)^{-1}$ ,  $\beta(t)$  both increase with  $t$  and decrease with  $\sigma_\epsilon$ . The variable  $\lambda(0) = \sigma_v^2/(2\sigma_\epsilon)$  decreases with  $\sigma_\epsilon$  and  $\lambda(1) = \sigma_\epsilon/2$  increases with  $\sigma_\epsilon$ . The price impact function  $\lambda(t)$  decreases (increases) over time when  $\sigma_\epsilon < \sigma_v$  ( $\sigma_\epsilon \geq \sigma_v$ ).*

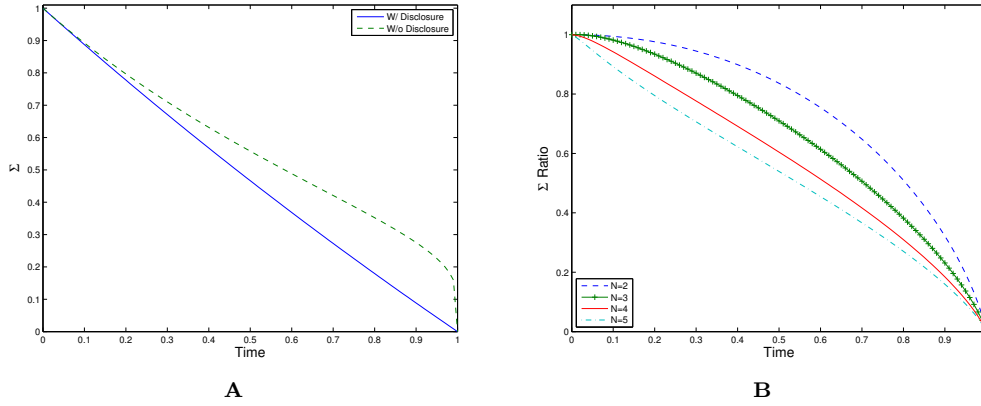


Figure 1. Figure 1A: Residual uncertainty  $\Sigma$  as a function of time for  $\sigma_\epsilon^2 = 0.875$  and  $N = 2$ . The solid line is for the case with disclosure and the dashed line is for the case without disclosure. Figure 1B: Ratio of  $\Sigma$  with and without disclosure as a function of  $t$  for  $\sigma_\epsilon^2 = 0.875$  and  $N = 2, 3, 4, 5$ .

In Figure 1A, the solid line plots the residual variance of the risky asset value for the market maker as a function of trading time  $t$ . As more information is revealed through trading and disclosure, clearly  $\Sigma(t)$  will decrease over time. In Figure 2A, the solid line, we plot the trading intensity  $\beta(t)$  with respect to



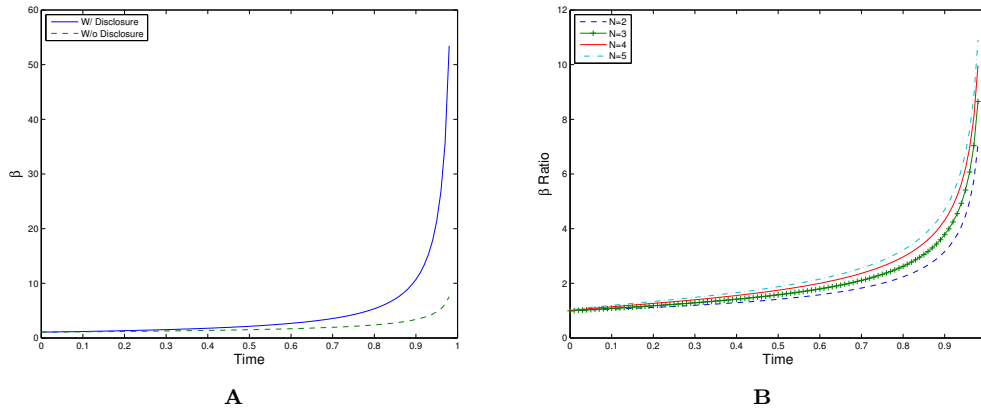


Figure 2. Figure 2A: Trading intensity  $\beta$  as a function of time for  $\sigma_\epsilon^2 = 0.875$  and  $N = 2$ . The solid line is for the case with disclosure and the dashed line is for the case without disclosure. Figure 2B: Ratio of  $\beta$  with and without disclosure as a function of  $t$  for  $\sigma_\epsilon^2 = 0.875$  and  $N = 2, 3, 4, 5$ .

$t$ . Similarly, as investors learn more from trading and disclosure and market becoming more efficient, the trading intensity increases over time as well. When  $\sigma_\epsilon$  is small, informed investors trade very aggressively with each other and thus  $\beta(t)$  is high and  $\Sigma(t)$  is low. Figure 4A and Figure 4B plot  $\Sigma(t)$  and  $\beta(t)$  as functions of  $t, \log(\sigma_\epsilon^2)$ . In Figure 4A, it is clear that  $\beta(t)$  decreases with  $\sigma_\epsilon$ . Coarser information makes investors compete with each other less intensively. As shown in Figure 4B, market also becomes less efficient as  $\sigma_\epsilon$  increases. Near the end of trading, conditional variance decreases almost as a straight line, like the monopolistic setting.

The comparative statics on  $\lambda(t)$  is more complicated. Following Kyle (1985), we use market depth,  $1/\lambda(t)$ , to measure market liquidity. When  $\sigma_\epsilon$  is small, each investor is very well informed and they trade very aggressively in the beginning. Thus  $\lambda(0)$  decreases with  $\sigma_\epsilon$ . Similarly, with very aggressive trading in the beginning, the market becomes more efficient later and thus  $\lambda(1)$  is low with small  $\sigma_\epsilon$ . Moreover, with small  $\sigma_\epsilon$ , higher market efficiency due to aggressive trading also means that market depth will increase over time. In Figure 3A, solid line,

we plot market depth over time when  $\sigma_\epsilon < \sigma_v$  and market depth increases over time. On the contrary when  $\sigma_\epsilon$  is high, investors will trade very cautiously initially and only increase their trades aggressively later on. This means that the market depth decreases over time. Figure 4C plots market depth as a function of  $t, \log(\sigma_\epsilon^2)$ . With low (high)  $\sigma_\epsilon$ , market depth increases (decreases) over time. When  $\sigma_\epsilon = \sigma_v$ ,  $\lambda(t)$  is a constant.

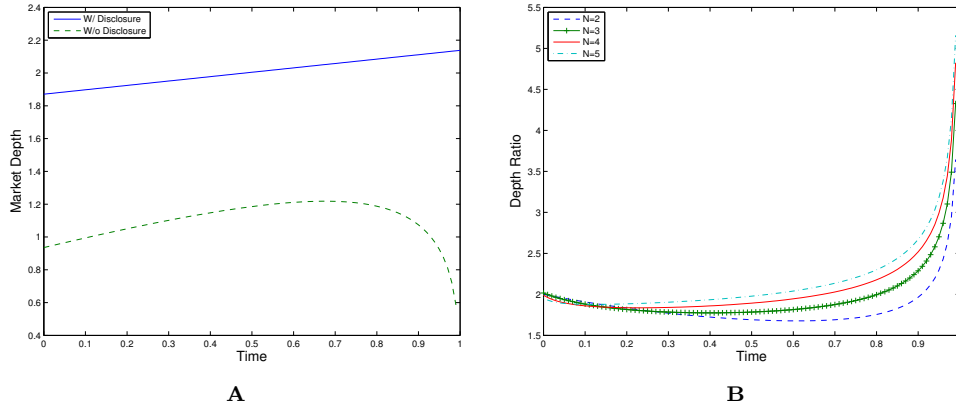


Figure 3. Figure 3A: Market depth  $1/\lambda(t)$  as a function of time for  $\sigma_\epsilon^2 = 0.875$  and  $N = 2$ . The solid line is for the case with disclosure and the dashed line is for the case without disclosure. Figure 3B: Market depth ratio with and without disclosure as a function of  $t$  for  $\sigma_\epsilon^2 = 0.875$  and  $N = 2, 3, 4, 5$ .

Disclosure not only increases market efficiency, it also affects how informed investors compete with each other. It is interesting to compare the trading strategy of informed investors in the aggregate to that of a monopolist. We have the following results.

**COROLLARY III.3:** *When  $\sigma_\epsilon^2 = \sigma_v^2$ , informed investors trade cooperatively like a monopolistic investor in the aggregate and their profits are maximized. Conditional correlation of investors' private valuation remains zero throughout the trading period.*

When informed investors' signals are uncorrelated initially, each informed in-

vestor's conditional precision is twice of that of the market maker. As trading goes on, since each informed investor knows his own random noise trades, the noise in the other informed investor's trades is also half of the variance of the noise in the market maker's observation. The conditional precision of each informed investor about the risky asset value remains twice of that of the market. As a result, informed investors' conditional correlation remains zero. Disclosure makes informed investors cooperate with each other. With uncorrelated signals, market efficiency and market liquidity are the same as if there exists a monopolistic informed investor with all the signals in the market. As shown in Figure 4E, informed investors' profits are maximized when  $\sigma_\epsilon = \sigma_v$ . Notice that in the setting without disclosure, informed investors' profits are maximized when  $\sigma_\epsilon$  is slightly larger than  $\sigma_v$ .

**COROLLARY III.4:** *When  $\sigma_\epsilon^2 \neq \sigma_v^2$ , as  $t \rightarrow 1$ ,  $2\delta(t) \rightarrow 1$  and informed investors' private valuations become uncorrelated and they all behave in the aggregate like a monopolistic informed investor with all the information in the economy. We have*

$$\lim_{t \rightarrow 1} \frac{\Sigma(t)}{\sigma_\epsilon^2(1-t)} = 1, \quad \frac{\beta(t)}{1/(\sigma_\epsilon(1-t))} = 1, \quad \lim_{t \rightarrow 1} \frac{\lambda(t)}{\sigma_\epsilon/2} = 1.$$

Even with correlated signals, informed investors learn to become cooperative. As discussed earlier, the increase in conditional precision of informed investors is twice of that of the market maker. As learning accumulates, the ratio of the conditional precision of informed investors and the market maker about the asset value converges to two. The conditional correlation among informed investors converges to zero. This is drastically different from the case without disclosure. In the BCW (2000) model without disclosure, near the end of trading, the ratio of the conditional precision of informed investors and that of the market maker about the asset value converges to 1 as the increase in conditional precision goes to infinity. This holds because the noise in the price comes from the liquidity traders

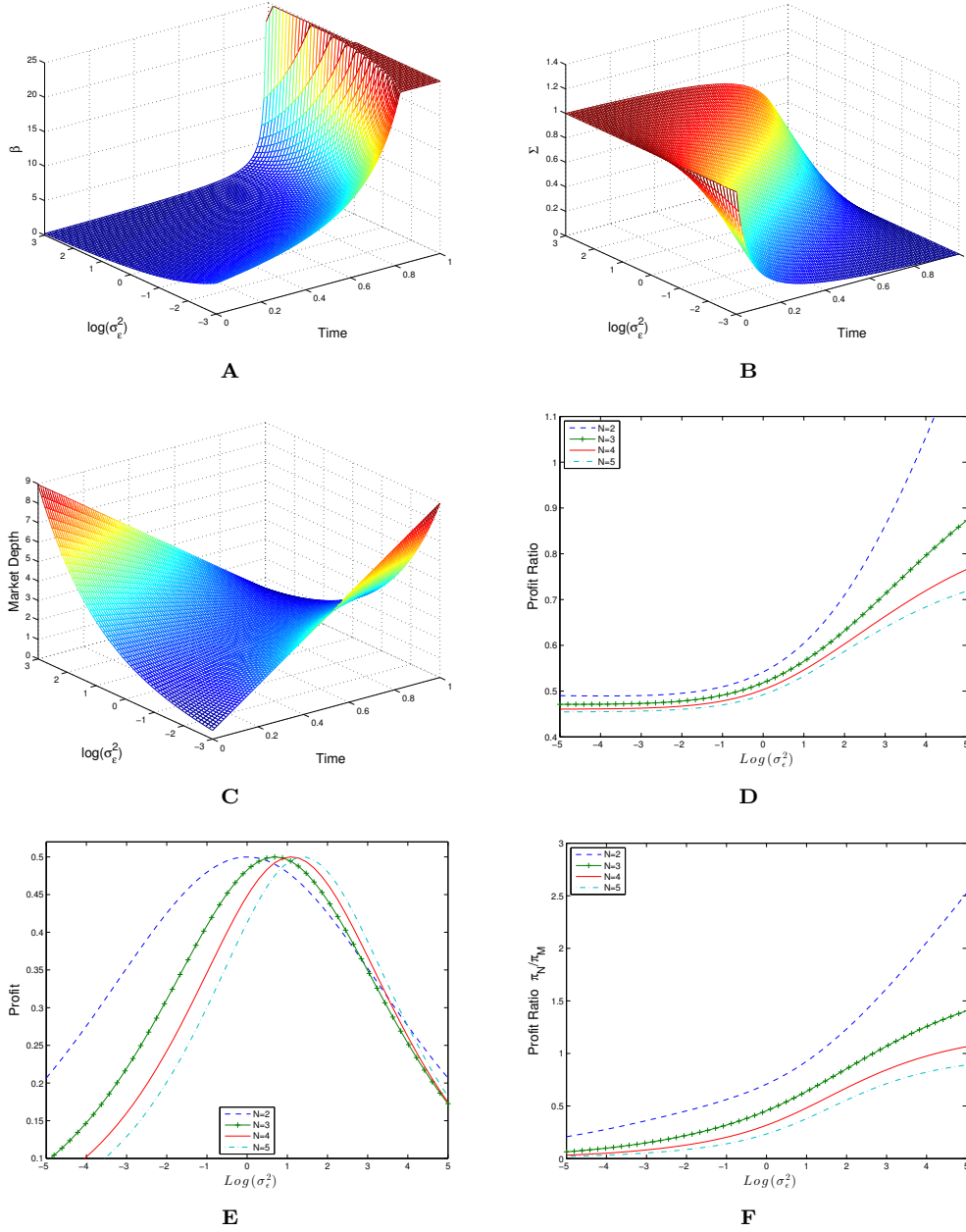


Figure 4. Figure 4A: Trading intensity  $\beta$  as a function of  $\log(\sigma_\epsilon^2)$ ,  $t$  for  $N = 2$  with disclosure. Figure 4B: Residual uncertainty  $\Sigma$  as a function of  $\log(\sigma_\epsilon^2)$ ,  $t$  for  $N = 2$  with disclosure. Figure 4C: Market depth  $1/\lambda$  as a function of  $\log(\sigma_\epsilon^2)$ ,  $t$  for  $N = 2$  with disclosure. Figure 4D: The ratio of informed investors' total profits  $\pi(0)$  with disclosure and without disclosure as a function of  $\log(\sigma_\epsilon^2)$  for  $N = 2, 3, 4, 5$ . Figure 4E: Informed Investors' expected profit  $\pi_N$  as a function of  $\log(\sigma_\epsilon^2)$  for  $N = 2, 3, 4, 5$  with disclosure. Figure 4F: The ratio of informed investors' total profits  $\pi_N$  with many competitive informed investors and  $\pi_M$  with a monopolistic investor as a function of  $\log(\sigma_\epsilon^2)$  for  $N = 2, 3, 4, 5$ .

and no one has any extra information about the noise trades. Therefore the increase in conditional precision is the same for the market maker and informed investors. As time goes to 1, the increase in conditional precision goes to infinity and the ratio of conditional precision between informed investors and market maker goes to 1. Informed investors has little informational advantage over the market maker, the conditional correlation of investors' private valuation goes to -1 and  $\lambda(t)$  goes to infinity. On the contrary, in continuous time trading with disclosure, investors learn to become cooperative. The conditional correlation of investors' private valuation goes to zero and  $\lambda(t)$  goes to a constant.

Next we compare the equilibrium with that obtained by BCW (2000) without disclosure. For comparison,  $\hat{\Sigma}(t), \hat{\beta}(t), \hat{\lambda}(t), \hat{\delta}(t)$  in the BCW (2000) economy without disclosure corresponds to the same parameters without the hat in the economy with disclosure.

**THEOREM III.1:** *In the continuous time trading model without public disclosure, there exists a unique symmetric linear equilibrium. In this equilibrium, informed investors submit a market order of*

$$dx^i(t) = \hat{\beta}(t) \left( s^i - \frac{V(t)}{2} \right) dt = \frac{\hat{\beta}(t)}{2\hat{\delta}(t)} (V^i(t) - V(t))dt,$$

and the market maker set the price according to

$$\begin{aligned} dP(t) &= \hat{\lambda}(t) \left( \sum_{i=1}^2 dx^i(t) + dz^0(t) \right), \\ \hat{\Sigma}(t) &= \frac{\sigma_v^2 \sigma_\epsilon^2}{\sigma_\epsilon^2 - \sigma_v^2 \log(1-t)}, \\ \hat{\beta}(t) &= \frac{1}{\sigma_\epsilon \sqrt{1-t}}, \\ \hat{\lambda}(t) &= \hat{\beta}(t) \hat{\Sigma}(t). \end{aligned}$$

In each period, the informativeness of informed investors' trade is measured by

$\beta(t)$  because the total information based trade in period  $t$  to  $t + dt$  is proportional to  $\beta(t)(v - V(t))dt$ . The variance of the aggregate randomization noise is  $dt$  and the increase in market maker's precision is  $\beta^2(t)dt$ . The derivative of market maker's conditional precision is  $(1/\Sigma(t))' = \beta^2(t)$ . The following describes how disclosure affects  $\beta(t)$ ,  $\Sigma(t)$ .

**COROLLARY III.5:** *Informed investors' information based trades are more aggressive and the market is more efficient, that is*

$$\frac{\beta(t)}{\hat{\beta}(t)} = \frac{1}{\sqrt{1-t}} > 1,$$

$$\frac{\Sigma(t)}{\hat{\Sigma}(t)} = \frac{\sigma_\epsilon^2 - \sigma_v^2 \log(1-t)}{\sigma_\epsilon^2 + \sigma_v^2 t/(1-t)} < 1,$$

Moreover as time approaches 1, we have,

$$\lim_{t \rightarrow 1} \frac{\beta(t)}{\hat{\beta}(t)} = \infty, \quad \lim_{t \rightarrow 1} \frac{\Sigma(t)}{\hat{\Sigma}(t)} = 0.$$

Disclosure makes the market more efficient. Since informed investors' information based trade is mixed with random noise trades, they trade more aggressively with respect to their signal. This effect is most profound near the end of trading as the ratio of  $\Sigma$  with and without disclosure goes to zero. Figure 2A shows the intensity of informed investors' trading in relation to that of informed trading without disclosure. The intensity is greater when disclosure is required. Figure 2B shows the ratio of trading intensity with and without disclosure. It is always larger than 1 and goes to infinity near the end of trade.

As a result of more aggressive trading by informed investors and the fact that the random order from all informed investors collectively equals, in distribution, that of liquidity traders, market becomes more efficient under the disclosure rule. This is clearly demonstrated in Figure 1.

Next we compare market depth,  $1/\lambda(t)$  and expected profits of informed in-

vestors in the two equilibria with and without disclosure. The expected profits  $\pi_D$  is related to market depth as described in Theorem II.2

$$\pi_D = \frac{1}{2} \int_0^1 \lambda(t) dt.$$

Notice that  $\lambda(t)$  represents the expected loss per unit of trade for liquidity traders arriving at time  $t$ . The previous relationship holds because the expected profits of informed investors equal to the expected loss of liquidity traders. The following describes the effects of disclosure on these variables.

**COROLLARY III.6:** *As time approaches 1, we have*

$$\lim_{t \rightarrow 1} \frac{1/\lambda(t)}{1/\hat{\lambda}(t)} = \infty,$$

*Moreover, when  $\sigma_\epsilon \leq \sigma_v$ , then  $1/\lambda(t) > 1/\hat{\lambda}(t)$ . In addition,  $\pi_D < \hat{\pi}_D$ .*

We can rewrite the ratio of market depth into the product of three components:

$$\frac{1/\lambda(t)}{1/\hat{\lambda}(t)} = \frac{2}{1} \times \frac{\hat{\beta}(t)}{\beta(t)} \times \frac{\hat{\Sigma}(t)}{\Sigma(t)},$$

Disclosure affect market liquidity in three ways. The first is the randomization effect which will increase market liquidity under disclosure. Other things being equal, this effect will double market liquidity. The second is the trading intensity effect due to private information which decrease market liquidity under disclosure. While both informed investor and the market maker learns from public disclosure. The noise in the publicly disclosed trades for the market maker is  $\sum_{i=1}^2 dW^i(t)/\sqrt{2}$  but the noise for each investor  $i$  is  $dW^j(t)/\sqrt{2}$ ,  $j \neq i$ . Therefore informed investor  $i$  learns more from the public disclosure than the market maker and this effect will decrease market liquidity. The third is the market efficiency effect which increase market liquidity under disclosure because of a lower residual uncertainty.

Figure 3 plots the market depth with positively correlated signals. As shown

in Figure 3A, when  $\sigma_v^2 \geq \sigma_\epsilon^2$  the last two effects roughly offset each other except near the beginning of trade. The first effect is dominant in early part of the trading period and market liquidity roughly doubles. In the latter part of the trading period, disclosure makes the market more efficient and the third effect is dominant which causes a higher market liquidity. Therefore, market is always more liquid with disclosure, which is shown clearly in Figure 3B.

In brief, when the noise in informed investors signals is small, informed investors do not learn from each other as much. As they trade more aggressively on their perceived differences from market expectation under disclosure, market depth is higher with disclosure due to randomization and higher market efficiency. It is interesting to observe that market depth changes over time in a pattern that is different from the no-disclosure case. Without disclosure, market depth first rises and then declines to 0 with positively correlated signals but market depth always rises with negatively correlated signals.

**COROLLARY III.7:** *For  $t > 3/4$ , there exists  $\sigma_\epsilon^* > \sigma_v$ , such that for  $\sigma_\epsilon > \sigma_\epsilon^*$ , that  $1/\lambda(t) < 1/\hat{\lambda}(t)$ . In addition, there exists  $\sigma_\epsilon^{**} > \sigma_v$ , such that for  $\sigma_\epsilon > \sigma_\epsilon^{**}$ ,  $\pi_D > \hat{\pi}_D$ .*

This is a rather surprising result. Intuitively, one would have expected that disclosure should always increase market liquidity. As discussed earlier, the effects of trade disclosure on market liquidity can be decomposed to three components: randomization effect, trade intensity effect and the market efficiency effect. When  $\sigma_\epsilon$  is very large, each informed investor on his own knows very little about the liquidation value of the risky asset. Therefore they learn a lot from the disclosure of informed investors' trades. Since the variance of noise in disclosed trades is  $2\sigma^2 dt$  for the market maker and  $\sigma^2 dt$  for each informed investor, informed investors learn faster from disclosed trades than the market maker. When  $\sigma_\epsilon$  is very large, the learning from public disclosure becomes very significant and this effect dominates the other two effects which causes the market liquidity to be higher for some  $t$ .



The reduction in market liquidity means that informed investors makes more profits in some trading periods with disclosure. A natural question is whether disclosure can increase expected profits of informed investors during the whole trading period, which we find possible when  $\sigma_\epsilon$  is large.

The effect of disclosure on informed investors' profits is ambiguous. Other things being equal, disclosure causes informed investors to lose half of their information based trading profits due to randomization. This results in a reduction of informed investors' profits when  $\sigma_\epsilon$  is small. With large  $\sigma_\epsilon$ , the results can be reversed. In the latter case, informed investors learn a lot from the disclosed trades about the asset value as they each have very imprecise signals in the beginning. In addition, informed investors learn more from the disclosed trades than the market maker. The increase of precision is twice that of the market maker. Consequently, the increase of learning by informed investors could more than offset the loss due to randomization and make them earn more profits than what they would receive in a setting without disclosure. Alternatively, we can view disclosure as an apparatus for coordination. Notice that informed investors' profits would be maximized if they could coordinate and trade at the same intensity as a monopolist with the same information. When each informed investor has very imprecise signals, they trade very cautiously, far from the level of a monopolist. Disclosure of trades releases information and make them trade more aggressively toward the level of a monopolist. The increase of trading intensity effectively coordinates their trading activity toward higher profits, and can offset the losses due to randomization when  $\sigma_\epsilon$  is small.

Figure 4D plots the ratio of informed investors' profits as a function of  $\log(\sigma_\epsilon^2)$ . Notice that informed investors' total expected profits could be larger under trade disclosure for large  $\sigma_\epsilon$ .

Disclosure makes informed investors learn to cooperate. Thus it is interesting to determine how disclosure affects an informed investor's profit with and without competition. Will an informed investor facing competition be better off? While

this can never happen in a setting without trade disclosure, it is possible with trade disclosure. Let  $\pi_M$  denote the expected profits of a single informed investor in a monopolistic setting and  $\hat{\pi}_M$  that of an informed investor in a monopolistic setting without disclosure. We have

**COROLLARY III.8:** *There exists  $\hat{\sigma}_\epsilon$  such that for  $\sigma_\epsilon > \hat{\sigma}_\epsilon$ ,  $\pi_D > \hat{\pi}_M > \pi_M$ . However, in the economy without disclosure, we always have  $\hat{\pi}_M > \hat{\pi}_D$ .*

With very large  $\sigma_\epsilon$ , investors have very noisy signals and are eager to learn from each other. Disclosure of trades lets investors to learn from each other about the market value at a speed (as measured by the increase in conditional precision) twice as fast as that of the market maker. Competition always reduces an informed investor's profit in the case without disclosure as informed investors learn at the same speed as the market maker. With very large  $\sigma_\epsilon$ , the benefit of learning can offset the loss due to competition and informed investors are better off with competition. Interestingly, learning from each other is so beneficial that an informed investor with disclosure and competition is better off than what he expects to receive with neither disclosure nor competition.

Our analysis indicates that learning can create synergies in the presence of disclosure. Suppose that each informed investor has to spend  $c$  to collect differential signals as described before and the act to collect information is observable by market participants, then we have the following herding result regarding information acquisition:

**PROPOSITION 1:** *When  $\sigma_\epsilon > \hat{\sigma}_\epsilon$  and  $\pi_D > c > \pi_M$ , there exists two information acquisition equilibria: (i) in the first equilibrium, no one would acquire any private signals; (ii) in the second equilibrium, both informed investors will acquire private signals.*

Herding in information acquisition happens because informed investors can learn more from each other than what the market can learn from informed investors.

#### IV. Extension

Our model can be extended to arbitrary number of informed investors with the following modification. Assuming that each informed investor  $i = 1, \dots, N$  receives a signal in the form of

$$s_i = \frac{v + \epsilon_i}{N}$$

in addition we have

$$\epsilon_i = \eta_i - \frac{\sum_{j=1}^N \eta_j}{N}$$

and that  $v, \{\eta_i, i = 1, \dots, N\}$  are multi-variate normally distributed and independent with mean zero. Moreover,  $\eta_i$  has variance  $\sigma_\eta^2$  for all  $i$ . Let  $\sigma_\epsilon^2$  denote the variance of  $\epsilon$ , it follows that

$$\sigma_\epsilon^2 = \frac{N-1}{N} \sigma_\eta^2.$$

Notice that since the  $\epsilon_i$ s sum up to zero, informed investors in aggregate know the liquidation value  $v$  of the risky asset. When  $N = 1$ , the informed investor knows  $v$  and our model reduces to a continuous time version of HHL (2001). For  $N > 1$ , let  $\rho$  denote the correlation of investor's private signals, it is easy to verify that

$$\rho = \frac{\sigma_v^2 - \sigma_\epsilon^2 / (N-1)}{\sigma_v^2 + \sigma_\epsilon^2}.$$

Given these notations, we present the discrete time model and continuous time model below:

**THEOREM IV.1:** *The necessary and sufficient conditions for a recursive linear symmetric equilibrium to exist are described below. For all  $m = 1, \dots, M-1$  and for all informed investors  $i = 1, \dots, N$ ,*

$$(6) \quad x_m^i = \frac{\beta_m \Delta t}{N \delta_{m-1}} (V_{m-1}^i - V_{m-1}) + z_m^i$$

$$(7) \quad P_m = V_{m-1} + \lambda_m \left( z_m^0 + \sum_{i=1}^N x_m^i \right)$$

$$(8) \quad V_m = V_{m-1} + \bar{\lambda}_m \sum_{i=1}^N x_m^i$$

$$(9) \quad \bar{\lambda}_m = \beta_m \Sigma_m / (N \sigma_m^2)$$

$$(10) \quad \lambda_m = \beta_m \Sigma_{m-1} / (\beta_m^2 \Delta t \Sigma_{m-1} + 1 + N \sigma_m^2)$$

$$(11) \quad V_m^i - V_{m-1}^i = \frac{\Omega_{m-1} - \Omega_m}{\Omega_{m-1}} \left( v - V_{m-1}^i + \sum_{j \neq i} \frac{z_m^j}{\beta_m \Delta t} \right)$$

$$(12) \quad V_m - V_{m-1} = \frac{\Sigma_{m-1} - \Sigma_m}{\Sigma_{m-1}} \left( v - V_{m-1} + \sum_{1 \leq j \leq N} \frac{z_m^j}{\beta_m \Delta t} \right)$$

$$(13) \quad \Omega_m^{-1} = \Omega_{m-1}^{-1} + \beta_m^2 \Delta t / ((N-1) \sigma_m^2)$$

$$(14) \quad \Sigma_m^{-1} = \Sigma_{m-1}^{-1} + \beta_m^2 \Delta t / (N \sigma_m^2)$$

$$(15) \quad E[\pi_m^i | F_{m-1}^i] = \alpha_{m-1} (V_{m-1}^i - V_{m-1})^2 + \zeta_{m-1}$$

$$(16) \quad \lambda_m = \alpha_m \bar{\lambda}_m^2$$

$$(17) \quad \lambda_m = \frac{\bar{\lambda}_m}{2 - \bar{\lambda}_m \beta_m \Delta t (1 - 1/(N \delta_{m-1}))}$$

$$(18) \quad \alpha_{m-1} = \alpha_m \left( 1 - \frac{\beta_m^2 \Delta t \Sigma_m}{N \sigma_m^2} \left( 1 - \frac{1}{N \delta_{m-1}} \right) \right)^2$$

$$(19) \quad \zeta_{m-1} = \zeta_m + \alpha_m \beta_m^2 \Delta t \left( \frac{\Omega_m}{(N-1) \sigma_m^2} - \frac{\Sigma_m}{N \sigma_m^2} \right)^2 (\Omega_{m-1} \beta_m^2 \Delta t + (N-1) \sigma_m^2)$$

subjecting to the boundary conditions

$$(20) \quad \beta_M = \sqrt{\frac{N \delta_{M-1}}{\Sigma_{M-1} \Delta t}},$$

$$(21) \quad \lambda_M = \frac{\sqrt{N \delta_{M-1} \Sigma_{M-1} / \Delta t}}{1 + N \delta_{M-1}},$$

$$(22) \quad \alpha_{M-1} = \frac{1}{\lambda_M (1 + N \delta_{M-1})^2},$$

$$(23) \quad \zeta_{M-1} = 0,$$

and the second order condition

$$(24) \quad \lambda_M > 0.$$

Similar to the case of two informed investors, the equilibrium can be solved recursively. When  $\Delta_t$  goes to zero, the system converges to a set of differential equations which leads to Theorem IV.2.

**THEOREM IV.2:** *In continuous time trading, there is a unique symmetric linear equilibrium specified as follows*

$$\beta(t) = \frac{\sqrt{-\Sigma'(t)}}{\Sigma(t)}, \quad \lambda(t) = \frac{\sqrt{-\Sigma'(t)}}{2}, \quad \bar{\lambda}(t) = \sqrt{-\Sigma'(t)},$$

where

$$\Sigma(t) = \begin{cases} \sigma_v^2(1-t) & \text{for } \sigma_\epsilon^2 = (N-1)\sigma_v^2 \text{ or } N=1, \\ \frac{\sigma_\epsilon^2 \sigma_v^2}{(N-1)\sigma_v^2 - \sigma_\epsilon^2} \left[ ((1-B)t + B)^{\frac{N}{4-3N}} - 1 \right] & \text{otherwise.} \end{cases}$$

with  $B = \left( \frac{\sigma_\epsilon^2}{(N-1)\sigma_v^2} \right)^{3-\frac{4}{N}}$ . In equilibrium, the expected profit of each informed investor is

$$(25) \quad \frac{1}{N} \int_0^1 \lambda(t) dt = \begin{cases} \frac{\sigma_v}{2N} & \text{for } \sigma_\epsilon^2 = (N-1)\sigma_v^2 \\ & \text{or } N=1, \\ \sqrt{\frac{(3N-4)\sigma_v^2 \sigma_\epsilon^2}{N(1-B)((N-1)\sigma_v^2 - \sigma_\epsilon^2)} \frac{|1-B|^{\frac{N-2}{3N-4}}}{2|N-2|}} & \text{otherwise.} \end{cases}$$

For the purpose of comparison, we restate the BCW (2000) result of continuous trading equilibrium without disclosure in the next theorem.

**THEOREM IV.3:** *If there is more than one informed investor ( $N > 1$ ) and their signals are perfectly correlated ( $\rho = 1$ ), then there is no symmetric linear equilibrium. Otherwise, there is a unique symmetric linear equilibrium. Consider*

the constant

$$(26) \quad k = \int_1^\infty x^{\frac{2(N-2)}{N}} e^{\frac{-2\sigma_\epsilon^2 x}{N\sigma_v^2}} dx.$$

For each  $t < 1$ , define  $\hat{\Sigma}(t)$  by

$$(27) \quad \int_1^{\hat{\Sigma}(0)/\hat{\Sigma}(t)} x^{\frac{2(N-2)}{N}} e^{\frac{-2\sigma_\epsilon^2 x}{N\sigma_v^2}} dx = kt.$$

The equilibrium is

$$(28) \quad \hat{\beta}(t) = \left(\frac{k}{\sigma_v^2}\right)^{1/2} \left(\frac{\hat{\Sigma}(t)}{\sigma_v^2}\right)^{(N-2)/N} \exp\left\{\frac{\sigma_\epsilon^2}{N\hat{\Sigma}(t)}\right\},$$

$$(29) \quad \hat{\lambda}(t) = \hat{\beta}(t)\hat{\Sigma}(t).$$

With respect to the comparative statics of the case with more informed investors, we have the following results:

**COROLLARY IV.1:** (i) *Informed investors contribute half of the trading volume in the market with disclosure; (ii) For  $N = 1$ , we have  $\beta(t) = \hat{\beta}(t)$ ,  $\Sigma(t) = \hat{\Sigma}(t)$ ,  $\lambda(t) = \hat{\lambda}(t)/2 = 1/(2\sigma_v)$ ; For  $N > 1$ , we have the following results:*

$$(iii) \lim_{t \rightarrow 1} \frac{\beta(t)}{\hat{\beta}(t)} = \infty, \quad \lim_{t \rightarrow 1} \frac{\Sigma(t)}{\hat{\Sigma}(t)} = 0, \quad (iv) \lim_{t \rightarrow 1} \frac{1/\lambda(t)}{1/\hat{\lambda}(t)} = \infty,$$

(v) *The conditional variance of the asset value  $\Sigma$  decreases with  $t$  and increases with  $\sigma_\epsilon$ . The initial market depth  $1/\lambda(0)$  increases with  $\sigma_\epsilon$  and the market depth in the end of trading,  $1/\lambda(1)$  decreases with  $\sigma_\epsilon$ . The variable  $\lambda(t)$  decreases over time when  $\sigma_\epsilon^2 < (N-1)\sigma_v^2$  while  $\lambda(t)$  increases over time when  $\sigma_\epsilon^2 \geq (N-1)\sigma_v^2$ ;*  
 (vi) *When  $\sigma_\epsilon^2 = (N-1)\sigma_v^2$ , informed investors trade in aggregate like a monopolistic investor and informed investors' profits are maximized. Therefore, market efficiency and market liquidity are the same as if there exists a monopolistic informed investor with all the signals in the market. Conditional correlation of investors' private valuation remains uncorrelated throughout the trading period;*

(vii) *When  $\sigma_\epsilon^2 \neq (N-1)\sigma_v^2$ , as  $t \rightarrow 1$ ,  $N\delta(t) \rightarrow 1$ , informed investors' private*

valuations become uncorrelated near the end of trading. They learn to cooperate and behave in aggregate like a monopolistic informed investor with all the information in the economy. We have

$$\lim_{t \rightarrow 1} \frac{\beta(t)}{1/(\sqrt{S_0}(1-t))} = 1, \quad \lim_{t \rightarrow 1} \frac{\Sigma(t)}{S_0(1-t)} = 1, \quad \lim_{t \rightarrow 1} \frac{\lambda(t)}{\sqrt{S_0}/2} = 1.$$

Here,  $S_0 = \frac{(1-\rho)(1-B)\sigma_v^2}{\rho(3N-4)}$ ,  $B$  is defined in Theorem IV.2.

Notice that our results on comparative statics obtained with two informed investors broadly hold for larger  $N$  with some notable exceptions. The conditional variance increases as investors receive noisier signals. Initial market depth is higher with noisier signals as investors trade cautiously initially. However, market depth in the end of trading will be lower with noisier signals as there will be more residual asymmetric information near the end. As a result, market depth will be decreasing with noisy signals and increasing with precise signals. Figure 4E plots informed investors' expected profits as a function of  $\log(\sigma_c^2)$ . Informed investors' profits will be maximized if they have uncorrelated signals in which case they coordinate and trade like a monopolist. Moreover, the conditional correlation goes to zero near the end of trading even when investors initially have correlated signals. Informed investors learn to be cooperative. Although we cannot prove that  $\Sigma(t) > \hat{\Sigma}(t)$  and  $\beta(t) > \hat{\beta}(t)$ , for all  $t$ , we prove it for  $t$  close to 1. Our numerical analysis shows that disclosure increases the intensity of informed trading and improves market efficiency in Figure 1B and Figure 2B. The increase in market efficiency due to disclosure also makes the market depth higher near the end of trading. Figure 3B shows that when investors' signals are positively correlated, disclosure always increases market liquidity.

Next we consider whether informed investors can be better off in the presence of more informed investors due to enhanced learning among informed investors. Let  $\pi_N$  denote what an informed investor would expected to receive in a setting with  $N$  informed investors. Let  $\pi_{N \rightarrow N-1}$  denote the profits each informed investor

would obtain if one informed investor leaves the market and the other  $N - 1$  informed investors will stay and trade in this market.

**COROLLARY IV.2:** *Then for any  $N > 1$ , there exists  $\bar{\sigma}_\epsilon$  such that  $\pi_N > \pi_{N \rightarrow N-1}$  for all  $\sigma_\epsilon > \bar{\sigma}_\epsilon$ . In addition, for  $1 < N < 5$ , there exists  $\hat{\sigma}_\epsilon$  such that for  $\sigma_\epsilon > \hat{\sigma}_\epsilon$ ,  $\pi_N > \pi_M$ . However, in the economy without disclosure, a monopolistic informed investor is always worse off in the presence of competition, i.e., we always have  $\hat{\pi}_M > \hat{\pi}_N$ , for all  $N > 1$ .*

Just like the case with two informed investors,  $N - 1$  informed investors can benefit from the participation of one more informed investor, if they collectively learn a lot from the new participant through trading. Indeed, learning can be so beneficial that a monopolist will be better off if  $N - 1$  informed investors all participate when  $N < 5$ . However, as  $N$  goes to infinity, each informed investor's profit goes to zero. In Figure 4**F**, we show numerically that for  $N = 5$ , a monopolist would prefer the other four informed investors not to participate in the market.<sup>6</sup>

With two informed investors, it is possible that disclosure increases the aggregate profits of informed investors. We show numerically in Figure 4**D** that this is impossible when  $N > 2$ . With larger  $N$ , each informed investor will learn at the speed  $N/(N - 1)$  times that of the market maker. However  $N/(N - 1)$  is decreasing in  $N$ , therefore, for larger  $N$  the benefit of learning from each other and coordinating with each other is not big enough to offset the loss due to randomization.

## V. Conclusion

How would disclosure of informed investors' trades affect market efficiency, market liquidity and expected profits of informed investors? In a setting with two informed investors, we show that informed investors will randomize their

<sup>6</sup>This holds also for  $N > 5$  numerically although we cannot provide an analytical proof for this result.



trades to hide their private information and to manipulate market maker's and others' beliefs. As a result, they sometimes trade against their own valuation. The instantaneous variance of informed investors' trade is the same as that of liquidity traders. Similar to the single informed investor model of HHL (2001), the market is more efficient with trade disclosure.

With more than one informed investor in the market, informed investors also learn from each other. Contrary to the model of BCW (2000) in which informed investors learn at the same speed (measured by the increase of conditional precision) as the market maker, in our model informed investors learn twice as fast as market maker as they know of the random component in their own trades. The ratio of conditional precision of informed investors to that of the market maker converges to two from above (below) when investors have positively (negatively) correlated signals. Learning makes informed investors cooperate and we show that when they have uncorrelated signals, they behave in aggregate like a monopolist. With very noisy signals, learning becomes so important that informed investors make more expected profits in the presence of disclosure. In addition, an informed investor could learn so much from disclosure that he makes more profits with competition than trading alone. Synergy in the gains from informed trading also implies that when there is cost in information collection, there could exist multiple information acquisition equilibria. In one equilibrium, no one would acquire information but in the other both investors would acquire information. Herding in information acquisition occurs because informed investors learn more from each other through disclosure than the market maker.

Disclosure also changes the inter-temporal patterns of the market liquidity. In BCW (2000), informed investors' conditional precision about asset value over that of the market maker converges to 1. Therefore conditional correlation goes to -1 and informed investors will eventually be on the other side of the market and market liquidity goes to zero as they cluster their trades near the end of trading. With precise signals, market liquidity will first increase and then decrease. With

noisy signals, market liquidity always decreases over time. On the contrary, in our model, market liquidity is always finite. When informed investors have very noisy signals they will trade more cautiously in the beginning. As time goes on, investors learn more and trade more aggressively, and market liquidity will decrease over time. With small noises in informed investors' private signals, informed investors will trade aggressively initially which results in a lower market liquidity that increases over time. Moreover, we show that initial market depth increases with  $\sigma_\epsilon$  while the end of period liquidity decreases with  $\sigma_\epsilon$ .

In the extension to three or more informed investors, each informed investor still learns more than the market maker. However the speed of learning measured by the derivative of conditional precision is  $N/(N - 1)$  of that of the market maker. Thus the relative advantage of learning through disclosure for informed investors over the market maker is decreasing with  $N$ . We show that for noisy signals, the first  $N - 1$  informed investors are better off if the  $N$ th informed investor is present in the market. The reduction in the relative speed of learning causes the gains informed investors receive from learning to be lower with higher  $N$ . Nevertheless, when  $N < 5$ , a monopolistic informed investor still prefers the presence of all remaining  $N - 1$  informed investors in the trading game when signals are very noisy, which will never happen in BCW (2000). However, for  $N > 2$ , disclosure always makes informed investors worse off. For larger  $N$ , potential gains through learning from each other is lower and is not enough to offset the losses due to random noise trades.

We considered only the case in which the signals have a symmetric structure. That is they all have the same correlation with each other and the same variance. In the future, it would be interesting to relax this restriction and it is possible that some informed investors benefit from disclosure while others would be worse off. Similarly, with asymmetric information structure, it is also possible that some informed investors may prefer more informed investors to learn from each other while others would be better off with less competition.

Our model provides the first example in which informed investors are better with more public information. It is worthwhile to examine if this also holds in cases of information disclosure of signals about asset value, which we leave that for future research.

## REFERENCES

- Attiyeh, G. M.** 1994. "A Theoretical Examination of Volatility Persistence in Strategic Settings." Unpublished.
- Back, K.** 1992. "Insider Trading in Continuous Time." *Review of Financial Studies*, 5(3): 387-409.
- Back, K., and H. Pedersen.** 1998. "Long-lived Information and Intraday Patterns." *Journal of Financial Markets*, 1(3): 385-402.
- Back, K., H. H. Cao, and G. A. Willard.** 2000. "Imperfect Competition among informed investors." *Journal of Finance*, 55(5): 2117-2155.
- Baruch, S.** 2002. "Insider Trading and Risk Aversion." *Journal of Financial Markets*, 5(4): 451 -464.
- Brunnermeier, M. K.** 2005. "Information leakage and market efficiency." *Review of Financial Studies*, 18(2): 417-457.
- Cao, H. H.** 1995. "Imperfect Competition in Securities Markets with Diversely informed investors." Unpublished.
- Chakraborty, A., and B. Yilmaz.** 2004. "Informed Manipulation." *Journal of Economic Theory*, 114(1): 132-152.
- Fishman, M. J., and K. M. Hagerty.** 1995. "The Mandatory Disclosure of Trades and Market Liquidity." *Review of Financial Studies*, 8(3): 637-676.
- Foster, F. D., and S. Viswanathan.** 1996. "Strategic Trading When Agents Forecast the Forecasts of Others." *Journal of Finance*, 51(4): 1437-1478.

- Foucault, Thierry, J. Hombert, and I. Rosu.** 2012. “News trading and speed.” SSRN working paper 2188822.
- Gong, F.Z., and H. Liu.** 2012. “Inside trading, public disclosure and imperfect competition.” *International Review of Economics & Finance*, 24: 200–223.
- John, K., and R. Narayanan.** 1997. “Market Manipulation and the Role of Insider Trading Regulations.” *Journal of Business*, 70(2): 217-247.
- Holden, C. W., and A. Subrahmanyam.** 1992. “Long-lived Private Information and Imperfect Competition.” *Journal of Finance*, 47(1): 247-270.
- Holden, C. W., and A. Subrahmanyam.** 1994. “Risk Aversion, Imperfect Competition and Long Lived Information.” *Economics Letters*, 44(1): 181-190.
- Huddart, S., J. Hughes, and C. Levine.** 2001. “Public Disclosure of and Dissimulation of Insider Trades.” *Econometrica*, 69(3): 665 -681.
- Kallianpur, G.** 1980. *Stochastic Filtering Theory*. New York: Springer-Verlag.
- Kyle, A. S.** 1985. “Continuous Auctions and Insider Trading.” *Econometrica*, 53(6): 1315-1335.
- Ostrovsky, Michael.** 2012. “Information aggregation in dynamic markets with strategic traders.” *Econometrica*, 80(6): 2595–2647.
- Protter, P.** 1990. *Stochastic Integration and Differential equations*. Berlin: Springer-Verlag.
- Revuz, D., and M. Yor.** 1991. *Continuous Martingales and Brownian Motion*. Berlin: Springer-Verlag.
- Zhang, W.D.** 2004. “Risk aversion, public disclosure, and long-lived information.” *Economics Letters*, 85(3): 327–334.

# Appendices

## PROOFS FOR SECTION II

### A1. Proof of Theorem II.1

We are here proving Theorem IV.1 the general case with  $N \geq 1$  and Theorem II.1 is included as a special case  $N = 2$ . We focus on proving the necessity of the claimed equations. The sufficiency of these equations can be established by reversing the necessity arguments (see the end of this proof for more details). So in the rest of this proof except in the last paragraph, we assume that a symmetric linear equilibrium exists, and we prove the claimed equations.

We first prove equations (11) to (14) simply by assuming that each informed investor follows Strategy  $\star$ . These equations will be used in the inductive proofs for other equations.

First, we can easily check the correctness of equations (11) and (12) by the fact that the expectation of a normal variable is precision-weighted average of all received signals. Moreover, the updating rule of normally distributed variables states that posterior precision equals prior precision plus the precision of the noise of the signals. Hence, we immediately establish the correctness of equations (13) and (14).

Before proving the rest of the desired equations, we first establish the following useful lemma.

LEMMA A.1: *Assume (1) each informed investor believes that all other informed investors follow Strategy  $\star$ , and (2) the market maker believes that all informed investors follow Strategy  $\star$ . Then,*

$$\sum_{1 \leq i \leq N} (V_m^i - V_m) = N\delta_m(v - V_m).$$

**Proof** First, it is easy to check the correctness of the following mathematical

identity by properties of normal variables

$$(A1) \quad \Omega_0 = \frac{N-1}{N}(1-\rho)\Sigma_0.$$

Using this relation and equations (13) and (14), we can easily check

$$(A2) \quad \frac{\Omega_m}{\Omega_0}(N-1)\rho + 1 = N\delta_m.$$

In what follows, define

$$U_m^i \equiv E[v - s^i | F_m^i]$$

where the expectation is computed after trade disclosures in period  $m$ . Equivalently, we could have defined  $U_m^i \equiv V_m^i - s^i$ .

Since the expected value of a normal variable is equal to the precision weighted average of all received signals, we have

$$(A3) \quad \begin{aligned} U_m^j &= \frac{\Omega_m}{\Omega_0} U_0^j + \Omega_m \sum_{1 \leq k \leq m} \left[ \left( \frac{1}{\Omega_k} - \frac{1}{\Omega_{k-1}} \right) \sum_{i \neq j} \left( s^i + \frac{z_k^i}{\beta_k \Delta t} \right) \right] \\ &= \frac{\Omega_m}{\Omega_0} U_0^j + \Omega_m \sum_{1 \leq k \leq m} \left[ \frac{\beta_k^2 \Delta t}{(N-1)\sigma_m^2} \sum_{i \neq j} \left( s^i + \frac{z_k^i}{\beta_k \Delta t} \right) \right], \end{aligned}$$

where the second equation follows from equation (13). (It is easy to verify that equation (13) holds when each informed investor merely *believes* all other informed investors follow Strategy  $\star$ .) Similarly,

$$(A4) \quad V_m = 0 + \Sigma_m \sum_{1 \leq k \leq m} \left[ \frac{\beta_k^2 \Delta t}{N\sigma_m^2} \sum_{1 \leq i \leq N} \left( s^i + \frac{z_k^i}{\beta_k \Delta t} \right) \right].$$

Summing up equation (A3) over  $j = 1, 2, \dots, N$ , we have

$$\sum_{1 \leq j \leq N} U_m^j = \frac{\Omega_m}{\Omega_0}(N-1)\rho \sum_{1 \leq j \leq N} s^j + \Omega_m \sum_{1 \leq k \leq m} \left[ \frac{\beta_k^2 \Delta t}{\sigma_m^2} \sum_{1 \leq i \leq N} \left( s^i + \frac{z_k^i}{\beta_k \Delta t} \right) \right]$$

$$\begin{aligned}
&= \frac{\Omega_m}{\Omega_0}(N-1)\rho v + N \frac{\Omega_m}{\Sigma_m} V_m \quad (\text{by equation (A4)}) \\
&= (N\delta_m - 1)v + N \frac{\Omega_m}{\Sigma_m} V_m \quad (\text{by equation (A2)}).
\end{aligned}$$

The last equation is only a slight variation of the equality claimed in the lemma.

□

We have thus completed the proof of Lemma A.1. Using the results established in proving the lemma, we next prove that Strategy 6 satisfies equation  $(\star)$ . In equation (6),  $x_m^i$  consists of a random component ( $z_m^i$ ), a component based on public information ( $\frac{\beta_m \Delta t}{N\delta_{m-1}} V_{m-1}$ ), and a private-information-related component ( $\frac{\beta_m \Delta t}{N\delta_{m-1}} V_{m-1}^i$ ). By equation (A3), the only private component in  $\frac{\beta_m \Delta t}{N\delta_{m-1}} V_{m-1}^i$  is equal to

$$\frac{\beta_m \Delta t}{N\delta_{m-1}} \left( s^i + \frac{\Omega_{m-1}}{\Omega_0} (N-1)\rho s^i \right) = \beta_m \Delta t s^i \quad (\text{by equation (A2)}).$$

This proves that Strategy 6 satisfies equation  $(\star)$ . Moreover, our arguments also imply that to support a symmetric linear equilibrium,  $x_m^i$  must have the following form:

$$(A5) \quad x_m^i - z_m^i = \frac{\beta_m \Delta t}{N\delta_{m-1}} V_{m-1}^i + \text{a public-information-based component.}$$

Using Lemma A.1 and equation (6), we have

$$(A6) \quad \sum_{1 \leq i \leq N} x_m^i = \beta_m \Delta t \left( v - V_{m-1} + \sum_{1 \leq i \leq N} \frac{z_m^i}{\beta_m \Delta t} \right).$$

Therefore, using equation (12) we immediately obtain equation (8) and equation (9) (the derivation of equation (9) also needs equation (14)). Note that in a symmetric linear equilibrium, the value updating rules must be of the form specified in equation (8). Our arguments in this paragraph together with equation (A5) also show that to support a symmetric linear equilibrium, equation (6)

must hold.

Using equation (A6) and the rules of conditional expectation of normally distributed variables, we immediately obtain equation (7) with

$$\begin{aligned}\lambda_m &= \frac{\text{cov}_{m-1} \left[ v, \sum_{1 \leq j \leq N} x_m^j + z_m^0 \right]}{\text{var}_{m-1} \left[ z_m^0 + \sum_{1 \leq j \leq N} x_m^j \right]} \\ &= \frac{\beta_m \Sigma_{m-1}}{\beta_m^2 \Delta t \Sigma_{m-1} + 1 + N \sigma_m^2}.\end{aligned}$$

The last equation is exactly equation (10).

We next proceed to prove equations (15) to (24) by backward induction on  $m$ , starting with the last period  $m = M$ . As there is no more trading opportunities after the last period, the maximization problem for each informed investor  $i$  is the same as the case without disclosure which has been derived in Foster and Viswanathan (1996) and Cao (1995). In particular, we know that the expected profit function of informed investor  $i$  has the form described in equation (15) with the boundary conditions specified in equations (20) to (24).

Thus, we have completed the base step. Next, we assume equations (15) to (19) are correct for period  $m + 1$  and prove them for period  $m$ . By the induction hypothesis, immediately after the  $m$ th period disclosure, the expected profits for future trades (i.e., trades from period  $m + 1$  onwards) can be written as,

$$E_m^i [\pi_{m+1}^i] \equiv E [\pi_{m+1}^i | F_m^i] = \alpha_m (V_m^i - V_m)^2 + \zeta_m.$$

Hence, the maximization problem of informed investor  $i$  immediately after the  $(m - 1)$ th period trade disclosure is:

$$(A7) \quad \max_{x_m^i} E_{m-1}^i \left[ x_m^i \left( v - V_{m-1} - \lambda_m \left( z_m^0 + \sum_{1 \leq j \leq N} x_m^j \right) \right) + \alpha_m (V_m^i - V_m)^2 \right] + \zeta_m$$

where the two terms inside the squared brackets represent the profit of the  $m$ th



trade and the total profit of all future trades.

For informed investor  $i$  to follow a random strategy, he must be indifferent between different values of  $x_m^i$ . Thus, the coefficients of  $(x_m^i)^2$  and  $x_m^i$  in Expression (A7) must be zero. These two restrictions respectively imply

$$(A8) \quad \lambda_m = \alpha_m \bar{\lambda}_m^2, \quad \text{and}$$

$$(A9) \quad E_{m-1}^i \left[ v - V_{m-1} - \lambda_m \sum_{j \neq i} x_m^j \right] = 2\alpha_m \bar{\lambda}_m E_{m-1}^i \left[ V_m^i - V_{m-1} - \bar{\lambda}_m \sum_{j \neq i} x_m^j \right].$$

Note that equation (A8) is the same as equation (16). In what follows, we show that equations (A8) and (A9) together imply equation (17). On the other hand, by Lemma A.1,

$$(A10) \quad \sum_{j \neq i} x_m^j = \beta_m \Delta t \left( v - V_{m-1} - \frac{1}{N\delta_{m-1}} (V_{m-1}^i - V_{m-1}) \right) + \sum_{j \neq i} z_m^j.$$

Hence,

$$E_{m-1}^i \left[ \sum_{j \neq i} x_m^j \right] = \beta_m \Delta t \left( 1 - \frac{1}{N\delta_{m-1}} \right) (V_{m-1}^i - V_{m-1})$$

Applying this relation to equation (A9), we obtain

$$\frac{1 - \lambda_m \beta_m \Delta t + \lambda_m \beta_m \Delta t / (N\delta_{m-1})}{1 - \bar{\lambda}_m \beta_m \Delta t + \bar{\lambda}_m \beta_m \Delta t / (N\delta_{m-1})} = 2\alpha_m \bar{\lambda}_m.$$

Now we multiply both sides of the preceding equation with the denominator of the left-hand side of the equation, and then we use equation (A8) to substitute all the  $\alpha_m \bar{\lambda}_m^2$  terms by  $\lambda_m$ . This leads to

$$2\alpha_m \bar{\lambda}_m = 1 + \lambda_m \left( \beta_m \Delta t - \frac{\beta_m \Delta t}{N\delta_{m-1}} \right).$$

Next, multiplying both sides of the above equation with  $\bar{\lambda}_m$  and using equation (A8) to substitute  $\alpha_m \bar{\lambda}_m^2$  by  $\lambda_m$ , we immediately obtain equation (17).

Since we have established that informed investor  $i$  is indifferent to  $x_m^i$ , Express-

sion (A7) can be simplified by setting  $x_m^i = 0$ . Thus,

$$\begin{aligned}
E_{m-1}^i[\pi_m^i] &= \alpha_m E_{m-1}^i \left[ (V_m^i - V_m)^2 \right] + \zeta_m \\
\text{(A11)} \quad &= \alpha_m \left( E_{m-1}^i [V_m^i - V_m] \right)^2 + \alpha_m \text{var}_{m-1}^i [V_m^i - V_m] + \zeta_m
\end{aligned}$$

On the other hand, since we have assumed  $x_m^i = 0$  in the profit calculation, using the updating rule for normal variables we have

$$\begin{aligned}
V_m^i &= \frac{\Omega_m}{\Omega_{m-1}} V_{m-1}^i + \frac{\Omega_{m-1} - \Omega_m}{\Omega_{m-1}} \left( v + \sum_{j \neq i} \frac{z_m^j}{\beta_m \Delta t} \right) \\
\text{(A12)} \quad &= \frac{\Omega_m}{\Omega_{m-1}} V_{m-1}^i + \frac{\Omega_m \beta_m^2 \Delta t}{(N-1)\sigma_m^2} \left( v + \sum_{j \neq i} \frac{z_m^j}{\beta_m \Delta t} \right),
\end{aligned}$$

where the second equation follows from equation (13). Moreover, using the pricing rules by market maker and applying equation (A10), we have

$$\begin{aligned}
V_m &= V_{m-1} + \bar{\lambda}_m \beta_m \Delta t \left( v - V_{m-1} - \frac{V_{m-1}^i - V_{m-1}}{N \delta_{m-1}} \right) + \bar{\lambda}_m \sum_{j \neq i} z_m^j \\
\text{(A13)} \quad &= V_{m-1} + \frac{\beta_m^2 \Sigma_m \Delta t}{N \sigma_m^2} \left( v - V_{m-1} - \frac{V_{m-1}^i - V_{m-1}}{N \delta_{m-1}} + \sum_{j \neq i} \frac{z_m^j}{\beta_m \Delta t} \right),
\end{aligned}$$

where the second equation follows from equation (9).

Now, using equations (A12) and (A13) and the fact that  $v$  is independent of  $\sum_{j \neq i} z_m^j$ , we have

$$\text{(A14)} \quad \text{var}_{m-1}^i [V_m^i - V_m] = \left( \frac{\Omega_m \beta_m^2 \Delta t}{(N-1)\sigma_m^2} - \frac{\beta_m^2 \Sigma_m \Delta t}{N \sigma_m^2} \right)^2 \left( \Omega_{m-1} + \frac{(N-1)\sigma_m^2}{\beta_m^2 \Delta t} \right)$$

Moreover,

$$E_{m-1}^i [V_m^i - V_m] = V_{m-1}^i - E_{m-1}^i [V_m]$$

$$(A15) \quad = \left( 1 - \frac{\beta_m^2 \Sigma_m \Delta t}{N \sigma_m^2} \left( 1 - \frac{1}{N \delta_{m-1}} \right) \right) (V_{m-1}^i - V_{m-1}),$$

here the last equation follows from equation (A13). Substituting equations (A14) and (A15) into equation (A11), we immediately see that equation (15) is correct for  $m$  with  $\alpha$  and  $\zeta$  satisfying equations (18) and (19). This completes our inductive step.

So far, we have proved all the desired equations as necessary conditions to support a symmetric linear equilibrium. In proving these equations, we have used (1) the rationality of the market maker's pricing rules and value updating rules, and (2) the optimality of all informed investors' trading strategies. Moreover, by reversing these arguments, we can easily check that when these equations indeed hold, (1) the pricing rules and value updating rules are indeed rational for the market maker, and (2) the trading strategies of all informed investors are indeed optimal. Therefore, all these equations collectively form a set of sufficient conditions to support a symmetric linear equilibrium.  $\square$

### Discussion on Solving the System of equations in Theorem IV.1

The whole recursive system of  $\alpha_m$ ,  $\beta_m$ ,  $\lambda_m$ ,  $\bar{\lambda}_m$ ,  $\Sigma_m$ ,  $\Omega_m$ , and  $\zeta_m$  can be numerically solved by first conjecturing a value of  $\Omega_{M-1}$  and then solving recursively for  $\Omega_{M-2}, \dots, \Omega_0$ . Given the conjectured  $\Omega_{M-1}$ , we can compute  $\delta_{M-1}$ , since the definition of  $\delta_M$  and equations (13) and (14) imply

$$N \delta_{M-1} = 1 + \frac{\Omega_{M-1}}{\Omega_0} (N \delta_0 - 1).$$

From  $\Omega_{M-1}$  and  $\delta_{M-1}$ , we can now derive  $\Sigma_{M-1}$ . From the boundary condition in equation (22), we can determine  $\alpha_{M-1}$ . Now again we conjecture a value for  $\Omega_{M-2}$ , which allows us to derive  $\delta_{M-2}$  and  $\Sigma_{M-2}$  as before. From equations (9) and (14),

$$\Sigma_{M-1}^{-1} = \Sigma_{M-2}^{-1} + \bar{\lambda}_{M-1} \beta_{M-1} \Delta t / \Sigma_{M-1}.$$

Consequently, we obtain  $\beta_{M-1} \bar{\lambda}_{M-1}$ . Comparing equation (10) and equation (16),

we arrive at

$$\beta_{M-1}\Sigma_{M-2}/(\beta_{M-1}^2\Delta t\Sigma_{M-2} + 1 + N\sigma_{M-1}^2) = \bar{\lambda}_{M-1}^2\alpha_{M-1}.$$

In the preceding equation, we can use the derived expression for  $\beta_{M-1}\bar{\lambda}_{M-1}$  to substitute  $\bar{\lambda}_{M-1}$  for  $\beta_{M-1}$ , and we can use equation (9) to substitute  $\bar{\lambda}_{M-1}$  for  $\sigma_{M-1}^2$ . Doing so results in an equation with  $\bar{\lambda}_{M-1}$  being the only unknown. Solving the resulting equation gives a formula for  $\bar{\lambda}_{M-1}$ . Next we can derive  $\beta_{M-1}$  from  $(\beta_{M-1}\bar{\lambda}_{M-1})/\bar{\lambda}_{M-1}$ ,  $\lambda_{M-1}$  from equation (16), and  $\sigma_{M-1}^2$  from equation (9). Given the expressions for  $\lambda_{M-1}$ ,  $\bar{\lambda}_{M-1}$ ,  $\beta_{M-1}$ , and  $\sigma_{M-1}^2$ , we can now check whether equation (17) holds or not. If it doesn't, we modify our initial value of  $\Omega_{M-2}$  until it holds. We repeat the procedure to derive  $\Omega_{M-3}, \dots, \Omega_0$ . If the derived  $\Omega_0$  is different from the initial given value, we adjust  $\Omega_{M-1}$  and repeat the whole procedure until the derived  $\Omega_0$  equals to the initial given value.

#### A2. Proofs for Theorem II.2

Here, we prove the general case Theorem IV.2 ( $N \geq 1$ ) and hence the proof of the special case Theorem II.2 ( $N = 2$ ) is covered.

#### MODEL SETUP

We use  $P(t)$  to denote the price set by the market maker for trading at time  $t$ , and we use  $V(t)$  to denote the market maker's adjusted belief of the risky asset value immediately after the disclosure of informed investors' trade at time  $t$ . Also, we use  $x^i(s^i, t)$  to denote the total order of informed investor  $i$  up to time  $t$ , and we use  $z^0(t)$  to denote the total order from all liquidity traders up to time  $t$ .<sup>7</sup>

For the price process, linearity means that there exist functions  $\lambda(t)$  and  $\bar{\lambda}(t)$  such that the market maker adjusts the risky asset's price and the post-disclosure value estimate by multiplying  $\lambda(t)$  and  $\bar{\lambda}(t)$  with the new orders from all traders

<sup>7</sup>In contrast, in the discrete-time model, we have used  $x_m^i$  to denote informed investor  $i$ 's instantaneous order at time  $m$ , rather than his cumulative order up to time  $m$ .

and those from all informed investors, respectively. More precisely, we have

$$(A16) \quad dV(t) = \bar{\lambda}(t) \sum_{1 \leq i \leq N} dx^i(t), \quad \text{and}$$

$$(A17) \quad P(t + dt) - V(t) = \lambda(t) \left( dz^0(t) + \sum_{1 \leq i \leq N} dx^i(t) \right).$$

It should be noted that although at any time  $t$ ,  $P(t)$  and  $V(t)$  only differ by an infinitesimal due to liquidity traders' trades,<sup>8</sup> this infinitesimal will be important in calculating the profit of an informed investor, as we will see in the proof of Lemma A.6.

We require that the trading strategy  $x^i(s^i, t)$  depends only on the trade history up to time  $t$  (e.g., it is independent of future value of  $x^j$  for any  $j = 1, \dots, N$ ). We also require that the trading strategies to be such that equation (A16) with boundary condition  $V(0) = 0$  has a unique solution  $V$ . Furthermore, we require the solution  $P$  to have a finite second moment and to have paths belonging to  $\mathcal{C}$ , where  $\mathcal{C}$  denotes the set of continuous functions  $f: [0, 1) \rightarrow \mathbb{R}$  such that  $\lim_{t \rightarrow 1} f(t)$  exists and is finite. This is a restriction on the strategy sets of the investors: given that agents  $i \neq j$  follow linear strategies to be described in equation (\*\*), we require agent  $j$  to follow a strategy such that equation (A16) has a solution with the desired properties.<sup>9</sup>

For the trading strategy, linearity means that the rate of purchase for informed

<sup>8</sup>It can be shown that  $V(t) - P(t) = \bar{\lambda}(t) \sum_{1 \leq i \leq N} dx^i(t) - \lambda(t) \left( dz^0(t) + \sum_{1 \leq i \leq N} dx^i(t) \right)$ , although we do not need this equation in deriving our equilibrium conditions.

<sup>9</sup>See, e.g., Protter (1990, §V.3) for conditions that guarantee the existence of unique solutions to stochastic differential equations. Our approach has the disadvantage of linking the feasible set for each investor to the strategies assumed to be chosen by the other investors and the market maker. In this respect, we are modeling a generalized game rather than a game. It would be better to define a feasible set for each investor and a set of  $\bar{\lambda}$  functions for the market maker such that, given any vector of choices from these sets, the stochastic differential equation defining the price has a unique solution with the desired limits existing. However, this approach would lead us into a thicket of technicalities that we prefer to avoid.

investor  $i$  can be specified as follows

$$(\star\star) \quad dx^i(s^i, t) = \beta(t)s^i dt + f(t)dt + dz^i(t) \quad \text{for } i = 1, \dots, N.$$

where  $f(t)$  is a certain function of all public information available up to time  $t$  and  $z^i(t)$  is a (non-standard) Brownian motion with instantaneous variance

$$(A18) \quad dz^i(t) = \sigma^i(t)dW^i(t) \quad \text{for } i = 1, \dots, N.$$

To be consistent with the discrete time model, we assume that  $dz^0(t)$  is a standard Brownian.

$$(A19) \quad \text{var}[dz^0(t)] = dt.$$

We restrict our attention to symmetric equilibria such that  $\sigma^i(t) = \sigma(t)$  for all  $i$  and in equilibrium, we show that  $\sigma(t) = 1/\sqrt{N}$ .

Since informed investors are assumed not to participate in market making activities, each  $dz^i$  is uncorrelated with both liquidity trader's trade  $dz^0$  and all other informed investors trade  $dz^j$  for all  $j \neq i$ .

While we have only included  $t$  in our notation  $f(t)$ , it should be emphasized that  $f(t)$  can be an arbitrarily complex function of all public information available before and including time  $t$ , such as the history of all the orders submitted by all informed investors and liquidity traders, which are revealed to the public through disclosures. We leave  $f(t)$  in this very general form for now and will make it more explicit later.

In most previous studies in the literature,  $f(t)$  is simply the asset price at time  $t$  multiplied by a certain function  $\alpha(t)$ , which solely depends on time  $t$  but no other information (see Kyle (1995), Back, Cao, and Willard (2000), and Huddart, Hughes, and Levine (2001)). In our current model, however,  $f(t)$  has to depend on more public information other than price. Indeed, it can be shown in the discrete-

time model that informed investors' order flows of the form  $\beta(t)s^i + \alpha(t)P(t)$  (where  $\alpha(t)$  is a function of time  $t$  only) does not constitute an equilibrium.

It may be natural to consider trading that is linear in an investor's updated estimate of the asset value rather than linear in an investor's initial signal. One difficulty with such an approach arises in calculating each informed investor's dynamic estimates of the asset value, because each investor's estimate would depend on other agents' trades, which depend on their estimates of the asset value, which depend on other agents' trades, etc. This is what is called the "forecasting the forecasts of others" problem (see Foster and Viswanathan (1996)). By specifying the trading strategy as a linear function in an investor's initial signal, we can avoid this problem. In the end, Strategy  $\star\star$  can be shown to be a linear functions of value estimates in equilibrium. In particular, although the trading strategy assumed in equation  $(\star\star)$  is quite different from that in Back, Cao, and Willard (2000) in that our strategy cannot even be expressed as a function of the *initial signals* and price, when expressed as a function of an investor's *estimate of the asset value* and the price, the deterministic component of our trading strategy follows the same formula as theirs (compare our equation (A33) and their equations (6) and (36)).

We define a symmetric linear equilibrium to be functions  $\beta(t)$  and  $\bar{\lambda}(t)$  such that (1) they are positive and continuous on  $[0, 1)$  and continuously differentiable on  $(0, 1)$ , (2)  $P(t)$  and  $V(t)$  calculated from equations (A16) and (A17) are both rational expectations of the asset value at all time  $t$ , and (3) the trading strategy in equation  $(\star\star)$  for each informed investor  $i$  is feasible and maximizes his expected profit over the set of feasible strategies.

#### VALUE ESTIMATES AND VARIANCES

In this part, we consider the filtering problems of the investors and market maker in detail. Throughout this section, we assume  $\beta(t)$  used in Strategy  $\star\star$  is a continuous and non-negative function.

Let  $\mathbf{F} \equiv \{\mathcal{F}(t) | 0 \leq t < 1\}$  denote the filtration generated by the aggregate informed investors' order process

$$\sum_{i=1}^N x^i(t).$$

We interpret  $\mathbf{F}$  as the market maker's information structure. Under the new notation,  $V(t) = E[v|\mathcal{F}(t)]$  where the conditional expectation is taken after the disclosure at time  $t$ . We set  $\Sigma(0) = \sigma_v^2$  and define  $\Sigma(t)$  as<sup>10</sup>

$$(A20) \quad \frac{1}{\Sigma(t)} \equiv \int_0^t \beta(u)^2 du + \frac{1}{\Sigma(0)}.$$

**LEMMA A.2:** *Assume each investor  $i$  follows a linear strategy as in equation (★★). Then  $\Sigma(t) = \text{var}[v|\mathcal{F}(t)]$ , where the variance is calculated after disclosure at time  $t$ . Define*

$$(A21) \quad W(t) \equiv \sum_{1 \leq i \leq N} z^i(t) + \int_0^t \beta(u) \{v - V(u)\} du.$$

*The process  $W$  is a Wiener process on the market maker's information structure  $\mathbf{F}$ . Furthermore,*

$$(A22) \quad V(t) = \int_0^t \beta(u) \Sigma(u) dW(u).$$

**Proof** Recall that each  $dz^i$  ( $1 \leq i \leq N$ ) is a non-standard Brownian motion and that  $dz^i$  and  $dz^j$  are independent for  $i \neq j$ . Hence, by equation (A18),  $\sum_{1 \leq i \leq N} dz^i$  is a standard Brownian motion with instantaneous variance  $dt$ . The correctness of the lemma then follows from the Kalman-Bucy filter (see, e.g.,

<sup>10</sup> In the discrete-time model, we define  $\Sigma(t)$  as the variance of the asset value conditional on the market maker's information. Here, we choose to define  $\Sigma(t)$  by a mathematical equation and then prove that it is equal to the same conditional variance under certain conditions (see Lemma A.2). Alternatively, we could define  $\Sigma(t)$  as the desired conditional variance and then prove Equality A20 in Lemma A.2. But such an alternative approach does not offer us a easy-to-use mathematical formula for  $\Sigma(t)$  when conditions in Lemma A.2 do not hold. Finally, we remark that it can be verified (see the proof of Theorem IV.2) that the function  $\Sigma(t)$  defined here is the same as that used in the statement of Theorem IV.2.



Kallianpur (1980)).  $\square$

The process  $W$  is called the “innovation” process for the market maker’s estimation problem. The differential

$$dW(t) = \sum_{1 \leq i \leq N} dz^i + \beta(t) \{v - V(t)\} dt$$

is the unpredictable part of the order flow from informed investors (recall that from the market maker’s viewpoint, the expected order from informed investors is 0). The lemma shows that the market’s estimate of  $v$  is revised according to  $dV = \beta \Sigma dW$ . Moreover, having the changes of both value estimates and prices proportional to orders as in equations (A16) and (A17) implies that these changes are unpredictable, as they must be when the market maker is risk neutral and competitive.

Consider an arbitrary informed investor  $j$  ( $j = 1, \dots, N$ ). Assume that each informed investor  $i$  ( $i \neq j$ ) follows a linear strategy as in equation ( $\star\star$ ), and assume that  $j$  follows an arbitrary strategy, which may or may not follow equation ( $\star\star$ ). Let  $\mathbf{F}^j \equiv \{\mathcal{F}^j(t) | 0 \leq t < 1\}$  denote the filtration generated by  $s^j$  and the order flow of all traders  $i$  ( $i \neq j$ ). This is informed investor  $j$ ’s information structure. We want to describe the conditional expectation and conditional variance of  $v$ , given his information. In particular, we define

$$U^j \equiv E[v - s^j | \mathcal{F}^j(t)], \quad V^j \equiv s^j + U^j$$

where the expectation is taken at time  $t$  after informed investors’ disclosure. We also define<sup>11</sup>

$$(A23) \quad \frac{1}{\Omega(t)} \equiv \frac{N}{N-1} \int_0^t \beta(u)^2 du + \frac{N}{(1-\rho)(N-1)\Sigma(0)}$$

<sup>11</sup>Here we choose to define  $\Omega(t)$  by a mathematical equation, rather than by defining it to be the conditional variance of the asset value as in the discrete-time model. The reason is the same as that given in Footnote 10 for  $\Sigma(t)$ .

LEMMA A.3: Consider an arbitrary informed investor  $j$  ( $j = 1, \dots, N$ ). Assume each investor  $i \neq j$  follows a linear strategy as in equation  $(\star\star)$ . Then,  $\Omega(t) = \text{var}[v|\mathcal{F}^j(t)]$ , where the variance is calculated after disclosure at time  $t$ . Define

$$(A24) \quad W^j(t) \equiv \sqrt{\frac{N}{N-1}} \left[ \sum_{i \neq j} z^i(t) + \int_0^t \beta(u) \{\tilde{v} - V^j(u)\} du \right].$$

The process  $W^j$  is a Wiener process on the information structure  $\mathbf{F}^j$ , and

$$(A25) \quad U^j(t) = (N-1)\rho s^j + \int_0^t \sqrt{\frac{N}{N-1}} \beta(u) \Omega(u) dW^j(u).$$

**Proof** Note that  $U^j(0) = (N-1)\rho s^j$  and that  $\sum_{i \neq j} z^i(t)$  is a Brownian motion with instantaneous variance equal to  $(N-1)dt/N$ . On the other hand, equation (A1) implies  $\Omega(0) = \text{var}[v|\mathcal{F}^j(0)]$  (since there is no trade at time 0, it makes no difference whether the variance is taken before or after disclosure at time 0). Now, the correctness of the lemma follows from the Kalman-Bucy filter (see, e.g., Kallianpur (1980)).  $\square$

The differential of the innovation process  $W^j$  is again the difference between the actual order and the expected order, but now we are computing the expected order using investor  $j$ 's information. The lemma shows that his estimate of the asset value  $v$  is revised as  $dV^j = \sqrt{N/(N-1)}\beta\Omega dW^j$ .

For ease of notation, we define

$$(A26) \quad \delta(t) \equiv \frac{\Sigma(t) - \Omega(t)}{\Sigma(t)} = \frac{\Omega^{-1}(t) - \Sigma^{-1}(t)}{\Omega^{-1}(t)}.$$

LEMMA A.4: Assume (1) each informed investor believes that all other informed investors follow Strategy  $\star\star$ , and (2) the market maker believes that all

informed investors follow Strategy  $\star\star$ . Then,

$$(A27) \quad \sum_{1 \leq i \leq N} (V^i(t) - V(t)) = N\delta(t)(\tilde{v} - V(t)).$$

**Proof** Consider an arbitrary informed investor  $j$  ( $1 \leq j \leq N$ ). If indeed all other investors follow Strategy  $\star\star$ , then from equations (A24) and (A25),

$$dU^j + \left( \frac{N}{N-1} \beta^2 \Omega \right) U^j dt = \frac{N}{N-1} \beta \Omega \left( \sum_{i \neq j} dz^i + \beta \sum_{i \neq j} s^i dt \right).$$

Together with equation (A23), this immediately implies

$$\frac{d}{dt} \left( \frac{1}{\Omega} U^j \right) = \frac{N}{N-1} \beta \left( \sum_{i \neq j} dz^i + \beta \sum_{i \neq j} s^i dt \right).$$

Since investor  $j$  believes all other investors follow Strategy  $\star\star$ , he expects  $dz^i + \beta s^i dt = dx^i - f(t)dt$ . Hence, he uses the following rule to update his  $U^j$ ,

$$\frac{d}{dt} \left( \frac{1}{\Omega} U^j \right) = \frac{N}{N-1} \beta \sum_{i \neq j} (dx^i - f(t)dt).$$

Therefore,

$$(A28) \quad U^j(t) = \frac{\Omega(t)}{\Omega(0)} U^j(0) + \Omega(t) \frac{N}{N-1} \int_0^t \beta(t) \sum_{i \neq j} (dx^i - f(t)dt)$$

$$(A29) \quad = \frac{\Omega(t)}{\Omega(0)} (N-1) \rho s^j + \Omega(t) \frac{N}{N-1} \int_0^t \beta(t) \sum_{i \neq j} (dx^i - f(t)dt).$$

Note that equation (A28) can also be directly derived by the fact that (under our normality assumption) the conditional expectation of the asset value is the precision-weighted average of all observable signals.

By exactly the same reasoning, the market maker, who believes that all in-

formed investors follow Strategy  $\star\star$ , has his estimate of asset value as

$$(A30) \quad V(t) = 0 + \Sigma(t) \int_0^t \beta(t) \sum_{1 \leq i \leq N} (dx^i - f(t)dt).$$

Summing up equation (A29) over all  $j$ , we obtain

$$(A31) \quad \begin{aligned} \sum_{1 \leq j \leq N} U^j(t) &= \frac{\Omega(t)}{\Omega(0)}(N-1)\rho v + \Omega(t)N \int_0^t \beta(t) \sum_{1 \leq i \leq N} (dx^i - f(t)dt) \\ &= (N-1) \frac{\Omega(t)}{\Omega(0)} \rho v + N \frac{\Omega(t)}{\Sigma(t)} V(t) \quad (\text{by equation (A30)}). \end{aligned}$$

Now the correctness of equation (A27) reduces to the following

$$(A32) \quad \frac{\Omega(t)}{\Omega(0)}(N-1)\rho + 1 = N \frac{\Sigma(t) - \Omega(t)}{\Sigma(t)}.$$

This equality can be directly verified by equations (A20) and (A23).  $\square$

The next lemma gives explicitly formula for each informed investor's trading strategy in equilibrium. It may be worth noting that, somewhat surprisingly, the deterministic part of an informed investor's order flow is identical to that in the no-disclosure case (see equations (1.6) and (3.11) in Back, Cao, and Willard (2000)).

**LEMMA A.5:** *Assume that each informed investor believes that all other informed investors follow Strategy  $\star\star$ . The following is the only trading strategy such that (1) it satisfies equation  $(\star\star)$  and (2) equation (A16) is a rational pricing rules for the market maker:*

$$(A33) \quad dx^i(t) = \frac{\beta(t)}{N\delta(t)} (V^i(t) - V(t)) dt + \sigma^i dW^i(t), \quad i = 1, \dots, N.$$

Moreover,

$$(A34) \quad \bar{\lambda}(t) = \beta(t)\Sigma(t),$$

and the trading strategy supports pricing rule given in equation (A17) with

$$(A35) \quad \lambda(t) = \beta(t)\Sigma(t) \frac{\sum_{1 \leq i \leq N} \text{var}[dz^i(t)]}{\text{var}[dz^0(t)] + \sum_{1 \leq i \leq N} \text{var}[dz^i(t)]}$$

**Proof** By equation (A29),  $V^j$  consists of two components.<sup>12</sup> The first component is based on private-information (i.e., it depends on  $s^j$ ) and is equal to

$$\left(1 + \frac{\Omega(t)}{\Omega(0)}(N-1)\rho\right) s^j.$$

The second component is purely based on public information. Hence, the private-information component in  $dx^j$  (i.e., the component dependent on  $s^j$ ) is equal to

$$\frac{\beta}{N\delta} \left(1 + \frac{\Omega(t)}{\Omega(0)}(N-1)\rho\right) s^j = \beta s^j,$$

where the equality follows from equation (A32) (which is purely a mathematical identity). This proves that Strategy A33 satisfies equation (\*\*). The above arguments also show that if a strategy satisfies equation (\*\*) and its deterministic part can be decomposed into a public-information component and another component involving  $V^j$ , then the latter component must be of the form specified in equation (A33).

On the other hand, if all informed investors indeed follow Strategy A33, then by Lemma A.2,

$$(A36) \quad dV(t) = \beta(t)\Sigma(t) \left[ \sum_{1 \leq i \leq N} dz^i + \beta(v - V(t))dt \right].$$

However, the market maker does not observe  $v$  directly. Hence, believing that all informed investors follow Strategy A33, he can use equation (A27) to substitute

<sup>12</sup>We can use this equation here since it is derived from merely assuming that each informed investor believes all other informed investors follow Strategy \*\*.

the  $v - V(t)$  term in the above equation and obtain

$$(A37) \quad dV(t) = \beta(t)\Sigma(t) \left[ \sum_{1 \leq i \leq N} dz^i + \frac{\beta(t)}{N\delta(t)} \sum_{1 \leq i \leq N} (V^i(t) - V(t)) dt \right].$$

Since we have already proved that each informed investor  $i$ 's information-based component has the form of  $\frac{\beta(t)}{N\delta(t)}V^i(t)$ , the above equation is consistent with equation (A16) if and only if the public-information component of each informed investor  $i$ 's deterministic trade is equal to  $\frac{\beta(t)}{N\delta(t)}V(t)$ . Hence, we conclude that Strategy A33 is the unique trading strategy with the claimed properties.

Now, comparing equations (A16) and (A37), we immediately obtain equation (A34). Finally, given that Strategy A33 supports the pricing rule in equation (A16) with  $\bar{\lambda}$  specified in equation (A34), a direct application of the Kalman-Bucy filter (see, e.g., Kallianpur (1980)) proves that Strategy A33 also supports the pricing rule in equation (A17) with  $\lambda$  specified in equation (A35).  $\square$

Given equation (A34), the entire equilibrium is determined by  $\bar{\lambda}(t)$ . To see this, note that

$$\bar{\lambda}(t)^2 = \beta(t)^2 \Sigma(t)^2 = -\Sigma'(t),$$

where the second equation follows from equation (A20). Therefore, the function  $\Sigma(t)$  is determined by  $\bar{\lambda}(t)$ . The condition  $\bar{\lambda}(t) = \beta(t)\Sigma(t)$  then determines  $\beta(t)$ .

To determine  $\bar{\lambda}(t)$  or, equivalently  $1/\bar{\lambda}(t)$ , which Kyle (1985) calls “the depth of the market,” we turn to the equilibrium condition that has not yet been exploited, namely, the requirement that each informed investor's trading strategy be optimal. In this subsection, we derive the optimality condition for an informed investor's trading rules. Such a condition turns out to be a restriction on market depth.

Throughout the subsection, we focus on an arbitrarily chosen investor, say investor  $j$ . Assume that each investor  $i \neq j$  follows Strategy A33. By Lemma A.5, investor  $j$ 's trading strategy can be written as  $x^j(s^j, t, P^{x^j})$ , where we use  $P^{x^j}$  to

emphasize that investor  $j$ 's strategy affects the price process. We define a trading strategy  $x^j$  to be feasible for investor  $j$  if there exists a unique solution  $P^{x^j}$  to equation (A16) (with boundary condition  $P^{x^j}(0) = 0$ ) for the given  $\bar{\lambda}$  and for the given  $\beta$  that characterizes the other investors' strategies and if

$$(A38) \quad \lim_{t \rightarrow 1} P(t) \text{ exists and is finite a.s.,}$$

$$(A39) \quad \int_0^1 dx^j(s^j, u, P^{x^j}) \text{ exists and is finite a.s., and}$$

$$(A40) \quad E \int_0^1 P^{x^j}(t)^2 dt < \infty.$$

Note that the integral in Expression A40 is the limit of the integral over  $[0, t]$  as  $t \rightarrow 1$ . The limits in Expressions A38 and A39 define, respectively, the price and number of shares held by investor  $j$  just before the announcement. Condition (A40) is the “no doubling strategies” condition introduced in Back (1992). Given the existence of the limits, the integral

$$(A41) \quad \int_0^1 (v - P^{x^j}(t + dt)) dx^j(s^j, t, P^{x^j}),$$

exists and equals to the profit of investor  $j$ . The formula is derived from the Merton-type wealth equation, and the existence of the integral can be verified by integrating by parts as in Back (1992).

LEMMA A.6: *Assume each investor  $i \neq j$  plays a linear strategy as in equation (A33). The conditions*

$$(A42) \quad \frac{d}{dt} \left( \frac{1}{\bar{\lambda}(t)} \right) = \frac{2\beta(t)}{N} \left( N - \frac{\Sigma(t)}{\Sigma(t) - \Omega(t)} \right),$$

$$(A43) \quad \lambda(t) = \frac{\bar{\lambda}(t)}{2}, \quad \text{and}$$

$$(A44) \quad \lim_{t \rightarrow 1} \Sigma(t) = 0 \quad \text{or} \quad \lim_{t \rightarrow 1} \bar{\lambda}(t) = +\infty$$

*are necessary and sufficient for Strategy A33 to create an optimal expected profit*

for investor  $j$ , which is equal to

$$(A45) \quad \frac{((1 + (N - 1)\rho)s^j)^2}{2\bar{\lambda}(0)} + \frac{N - 1}{2N} \int_0^1 \frac{1}{\bar{\lambda}(u)} \left( \bar{\lambda}(u) - \frac{N\beta(u)\Omega(u)}{N - 1} \right)^2 du.$$

**Proof** Since we will focus on an arbitrary informed investor  $j$  ( $1 \leq j \leq N$ ) throughout this proof, we use  $dx(t)$  as a shorthand for  $dx^j(s^j, V^j, V^{x^j})$ . Also, we rewrite  $V(t)$  and  $P(t)$  as  $V^x(t)$  and  $P^x(t)$ , respectively, to emphasize that trading strategy  $x$  affects the processes  $V$  and  $P$ . Using Expression A41 and the law of iterated expectations, we know that the objective of investor  $j$  is to maximize

$$E \int_0^1 (V^j(t) - P^x(t + dt)) dx(t)$$

under the dynamics of the state variables  $V^j$ ,  $V^x$ , and  $P^x$ .

From equation (A25),  $V^j$  follows the following dynamics

$$(A46) \quad dV^j(t) = \sqrt{\frac{N}{N - 1}} \beta(t) \Omega(t) dW^j(t).$$

On the other hand, the instantaneous order submitted by all traders  $i \neq j$  sum to

$$\begin{aligned} & \sum_{i \neq j} \frac{\beta}{N\delta} (V^i - V^x) dt + \sum_{i \neq j} dz^i \\ &= \left[ \beta(V - V^x) - \frac{\beta}{N\delta} (V^j - V^x) \right] dt + \sum_{i \neq j} dz^i \quad (\text{by equation (A27)}) \\ &= \sqrt{\frac{N - 1}{N}} dW^j + \beta \left( 1 - \frac{1}{N\delta} \right) (V^j - V^x) dt \quad (\text{by Lemma A.3}) \end{aligned}$$

Hence, by the pricing rule in equation (A16),

$$(A47) \quad \begin{aligned} dV^x(t) &= \bar{\lambda}(t) dx(t) + \bar{\lambda}(t) \sqrt{\frac{N - 1}{N}} dW^j(t) \\ &\quad + \bar{\lambda}(t) \beta(t) \left( 1 - \frac{1}{N\delta(t)} \right) (V^j(t) - V^x(t)) dt. \end{aligned}$$



The optimization problem is a Markovian stochastic control problem with state variables  $V^j(t)$ ,  $V^x(t)$ ,  $P^x(t)$ . Let  $J(t, s, V^j, V^x)$  denote a candidate for the following value function

$$\begin{aligned} & \sup_x E \int_t^1 (V^j(u) - P^x(u + du)) dx(u) \\ &= \sup_x E \int_t^1 (V^j(u) - V^x(u) - \lambda(u)dx(u)) dx(u), \end{aligned}$$

where the expectation is conditioned on  $F^j(t)$  and the equality follows from equation (A17). Note that we have dropped “ $-\lambda(dz^0(u) + \sum_{i \neq j} dx^i)$ ” in the parentheses on the right-hand side of the above equation. All of the dropped terms there are either a random variable uncorrelated with  $dx^i$  or a deterministic term of an order at least  $dt$ , and therefore they do not contribute to the expectation.

The Bellman equation for this control problem is

$$(A48) \quad 0 = \max_x E_t^j \left[ (V^j - V^x - \lambda dx) dx + J_t dt + J_{V^x} dV^x + J_{V^j} dV^j + \frac{1}{2} J_{V^x V^x} (dV^x)^2 + J_{V^x V^j} dV^x dV^j + \frac{1}{2} J_{V^j V^j} (dV^j)^2 \right].$$

Here,  $J_x, J_{xy}$  are the first- and second-order partial derivatives of  $J$  with respect to  $x$  and  $x, y$ . Intuitively, the Bellman equation states that the over  $x$  of the drift of  $J$  plus the instantaneous profit  $(V^j - P^x)x$  equals zero; i.e., the expected decline in future profit should be exactly offset by the realized current profit.

Note that the right-hand side of the above Bellman equation depends on  $x$  through a quadratic function of  $dx$ . In particular, since  $dx$  only appears in the dynamics of  $dV^x$  but not in the dynamics of  $dV^j$ , the only terms involving  $dx$  (except those of higher orders) on the right-hand side of the Bellman equation are:  $(-\lambda + \frac{1}{2} J_{V^x V^x} \bar{\lambda}^2)(dx)^2$  and  $(\bar{\lambda} J_{V^x} + V^j - V^x)dx$ . For investor  $j$  to follow a random strategy, he must be indifferent across the various possible orders induced by the random strategy; i.e., the coefficient of  $dx$  and  $(dx)^2$  must be zero. Reasoning

from the linear term, we have

$$(A49) \quad J_{V^x} = \frac{V^x - V^j}{\bar{\lambda}}.$$

Reasoning from the quadratic term, we have  $\frac{1}{2}J_{V^x V^x} \bar{\lambda}^2 = \lambda$ . Then, applying equation (A49), we obtain

$$(A50) \quad \lambda = \frac{\bar{\lambda}}{2}$$

which establishes equation (A43). Recall that our postulated equilibrium strategy in equation (A33) includes a stochastic term in investor  $j$ 's order flow. For investor  $j$  to follow such a random strategy, he must be indifferent across the various possible orders induced by the random strategy. The above two equations serves to ensure that investor  $j$  will be indeed indifferent.

From equations (A34), (A35) and (A50), we immediately know that  $\text{var}[dz^0(t)] + \sum_{1 \leq i \leq N} \text{var}[dz^i(t)] = 2 \sum_{1 \leq i \leq N} \text{var}[dz^i(t)]$ . This confirms our earlier claim that equation (A19) leads to

$$\sigma^i = \sigma = \frac{1}{\sqrt{N}}.$$

Using equations (A46), (A47), (A49) and (A50), we can simplify equation (A48) to

$$(A51) \quad 0 = J_t + J_{V^x} \bar{\lambda} \beta \left( 1 - \frac{\Sigma}{N(\Sigma - \Omega)} \right) (V^j - V^x) \\ + \frac{1}{2} J_{V^x V^x} \bar{\lambda}^2 \frac{N-1}{N} + J_{V^x V^j} \bar{\lambda} \beta \Omega + \frac{1}{2} J_{V^j V^j} \beta^2 \Omega^2 \frac{N}{N-1}.$$

By taking the derivatives of equation (A51) with respect to  $V^x$  and using equations (A49) and (A50) to simplify terms, we arrive at

$$0 = \frac{d}{dt} \left( \frac{V^x - V^j}{\bar{\lambda}} \right) + \frac{d}{dV^x} \left( \beta \left( 1 - \frac{\Sigma}{N(\Sigma - \Omega)} \right) (-1)(V^j - V^x)^2 \right).$$

It is straightforward to show that this is equivalent to equation (A42). Since Bell-

man equation is a necessary condition for the optimality of the trading strategy for investor  $i$ , the above arguments prove the necessity of equation (A42).

To prove the necessary and sufficient conditions for the optimality of the trading strategy, we can assume in the rest of the proof that equations (A42) and (A43) hold, and we only need to show that equation (A44) is necessary and sufficient for the optimality of investor  $j$ 's strategy.

First, straightforward calculations show that the following function  $J$  does satisfy the Bellman equation as specified in equations (A49) to (A51).

$$(A52) \quad J(t, s, V^j, V^x) = \frac{(V^x - V^j)^2}{2\bar{\lambda}(t)} + \int_t^1 \frac{N-1}{2N\bar{\lambda}(u)} \left( \bar{\lambda}(u) - \frac{N\beta(u)\Omega(u)}{N-1} \right)^2 du.$$

Reasoning with the above  $J$  as in Back (1992), we can show that an optimal strategy should not include discrete orders (this is due to the convexity of  $J$  as a function of  $V^j$  and  $V^x$ ). Given any trading strategy  $x$  with continuous orders, we can apply Ito's lemma to obtain

$$\begin{aligned} & J(1, s, V^j(1-), V^x(1-)) - J(0, s, V^j(0), V^x(0)) \\ &= \int_0^1 \left( J_t dt + J_{V^x} dV^x + J_{V^j} dV^j \right. \\ &\quad \left. + \frac{1}{2} J_{V^x V^x} (dV^x)^2 + J_{V^x V^j} dV^x dV^j + \frac{1}{2} J_{V^j V^j} (dV^j)^2 \right) \\ &= \int_0^1 g(t) dW^j(t) - \int_0^1 (V^j - V^x - \lambda dx(t)) dx(t) \end{aligned}$$

for some function  $g$  that depends on time  $t$  only and it's easy to verify

$$E \left[ \int_0^1 g(t)^2 dt \right] < \infty.$$

where the last equality holds since  $J$  satisfies equations (A49) to (A51). Thus,

$$E \left[ \int_0^1 (V^j - V^x - \lambda dx) dx \right] = J(0, s, V^j(0), V^x(0)) - E[J(1, s, V^j(1-), V^x(1-))].$$

By the definition of  $J$ ,  $-E[J(1, s, V^j(1-), V^x(1-))] \leq 0$ . Thus from the preceding equality, we see that the proposed trading strategy is optimal if and only if  $J(1, s, V^j(1-), V^x(1-)) = 0$ , *a.s.*, which is equivalent to

$$(A53) \quad \lim_{t \rightarrow 1} V^x(t) - V^j(t) = 0 \quad a.s. \quad \text{or} \quad \lim_{t \rightarrow 1} \bar{\lambda}(t) = +\infty.$$

To complete the correctness proof that equations (A42) and (A44) are indeed necessary and sufficient. We are left to prove that equations (A44) and (A53) are equivalent. First, if  $\lim_{t \rightarrow 1} \Sigma(t) = 0$ , then  $\lim_{t \rightarrow 1} V^x(t)$  is a precise estimate of  $v$  and so  $V^j(t)$  should also approach to  $v$ . On the other hand, by Lemma A.4, we know that  $\lim_{t \rightarrow 1} V^x(t) - V^j(t) = 0$  *a.s.* imply:

$$\begin{aligned} \lim_{t \rightarrow 1} \sum_j [V^x(t) - V^j(t)] &= \lim_{t \rightarrow 1} N\delta(t)(v - V^x) \\ &= 0 \end{aligned}$$

This must imply  $\lim_{t \rightarrow 1} \Sigma(t) = 0$ , otherwise we have  $\lim_{t \rightarrow 1} N\delta(t) \neq 0$  (by equation (A32)),  $\lim_{t \rightarrow 1} v - V^x \neq 0$ , a contradiction. Finally, to complete the proof, note that the above argument implies that the expected trading profit for Strategy A33

$$E[J(0, V^j(0), V^x(0))] = \frac{(V^j(0) - 0)^2}{2\bar{\lambda}(0)} + \int_0^1 \frac{N-1}{2N\bar{\lambda}(u)} \left( \bar{\lambda}(u) - \frac{N\beta(u)\Omega(u)}{N-1} \right)^2 du.$$

as claimed.  $\square$

If  $\sigma_\epsilon^2 = (N-1)\sigma_v^2$ , then the right-hand side of equation (A42) is zero. Therefore, market depth (which is  $1/\lambda = 2/\bar{\lambda}$ ) must be constant. If  $\sigma_\epsilon^2 > (N-1)\sigma_v^2$ , then the right-hand side of equation (A42) is negative. This implies that in such a case market depth  $1/\lambda$  must be declining over time. If  $\sigma_\epsilon^2 < (N-1)\sigma_v^2$ , then the right-hand side of equation (A42) is always positive. This implies that in such a case market depth  $1/\lambda$  must be rising over time, in contrast to the results in the

setting without disclosure obtained by Back, Cao, and Willard (2000), in which market depth first rises to its maximum and then fall to 0. The difference occurs because the conditional correlation in our model is positive when  $\sigma_\epsilon^2 < (N - 1)\sigma_v^2$  and never changes sign but in Back, Cao and Willard (2000), the conditional correlation will converges to -1 even when it was positive at time zero.

Condition (A42) is a local condition for optimality at each  $t < 1$ , which we will discuss below. Condition (A44) means there is no money “left on the table” an instant before the announcement. If the first condition of (A44) holds, then the market’s information about  $v$  is precise by the announcement date, and the asset will be correctly priced. If the second condition of (A44) holds, then the market is completely illiquid just before the announcement, so, even if the asset were mis-priced, there would be no profitable trades available. These conditions are not mutually exclusive. In fact, only the first condition holds in our case, which is contrasting with both conditions hold in Back, Cao, and Willard (2000).

**Proof of Theorem IV.2** This proof consists of two parts: (1) assuming  $\Sigma$  as given, we first prove the formulae for all other quantities; (2) then we prove that  $\Sigma$  exists if and only if  $\rho < 1$  or  $N = 1$  and that when  $\Sigma$  exists it is uniquely determined by the formula given in the theory.

First, we prove the formulae for all the other formulae assuming the correctness of the formula for  $\Sigma$ . The formula for  $\beta(t)$  as a function of  $\Sigma(t)$  follows directly from equation (A20). The formula for  $\bar{\lambda}(t)$  follows from the fact  $\bar{\lambda}(t) = \beta(t)\Sigma(t)$  (see Lemma A.5), and hence the formula for  $\lambda(t)$  follows from the fact that  $\lambda(t) = \bar{\lambda}(t)/2$ .

Since the market maker makes no profit, the expected profit of all informed investors is equal to the loss of liquidity traders, which is equal to

$$\int_0^1 \lambda(t) dz^0(t) dz^0(t) = \int_0^1 \lambda(t) dt.$$

By symmetry, each informed investor’s profit is  $1/N$  of the total expected profits

of all informed investors, and hence it is equal to  $\int_0^1 \lambda(t) dt/N$ , as claimed. To prove the correctness of Expression 25, note that

$$\begin{aligned}\lambda &= \frac{1}{2}\bar{\lambda} = \frac{1}{2}\beta\Sigma = \frac{1}{2}\sqrt{\left(\frac{1}{\Sigma}\right)'}\Sigma \\ &= \frac{1}{2}\sqrt{\Sigma(0)\left(\frac{1-\rho}{\rho}\right)\left(\frac{1-B}{3N-4}\right)((1-B)t+B)^{-\frac{2N-2}{3N-4}}},\end{aligned}$$

where the last equality follows from the formula for  $\Sigma$  in Theorem IV.2. Algebraic calculations then show that the integral of the last expression with respect to  $t$  from 0 to 1 is equal to Expression 25 times  $N$ , as desired.<sup>13</sup>

We have thus proved the correctness of all the formulae except the one for  $\Sigma(t)$ . Moreover, this means that the existence, non-existence, or uniqueness of the equilibrium is equivalent to the existence, non-existence, or uniqueness of  $\Sigma(t)$ , respectively. So in what follows, we only need to derive the formulae for  $\Sigma(t)$  or prove its non-existence. We will do so by solving the differential equation (A42) with boundary condition (A44).

From equation (A42), we have

$$\frac{d}{dt}\left(\frac{1}{\bar{\lambda}}\right) = \frac{d}{dt}\left(\frac{1}{\beta\Sigma}\right) = -\frac{\beta'}{\beta^2\Sigma} + \beta,$$

where we have used equation (A20) to derive the last equality. Thus, equation (A42) implies

$$(A54) \quad -\frac{\beta'}{\beta^3\Sigma} = 1 - \frac{2\Sigma}{N(\Sigma - \Omega)}.$$

On the other hand, the definitions of  $\Sigma(t)$  and  $\Omega(t)$  (equations (A20) and (A23))

<sup>13</sup>The absolute-value operator is needed for the case  $\rho < 0$ , which implies that the term inside the absolute value operator is negative. Also, we remark that the expected-profit formula can be alternatively derived by taking expectation (at time 0) of Expression A45.

implies

$$\frac{N-1}{N} \frac{1}{\Omega(t)} - \frac{1}{\Sigma(t)} = \frac{1}{(1-\rho)\Sigma(0)} - \frac{1}{\Sigma(0)} = \frac{A}{N},$$

where  $A = \frac{\rho N}{(1-\rho)\Sigma(0)}$  is a constant.

Substituting  $\Sigma(t)$  for  $\Omega(t)$  in equation (A54), we get

$$-\frac{\beta'}{\beta^3} = \left(1 - \frac{2}{N}\right) \Sigma + \frac{2(N-1)}{(-A-1/\Sigma)N}$$

In what follows, we let  $\Gamma = \frac{1}{\Sigma}$ . Using the fact  $\frac{d}{dt}\left(\frac{1}{\Sigma}\right) = \beta^2$ , we can rewrite the preceding differential equation as

$$(A55) \quad 0 = \frac{\Gamma''}{\Gamma'} + \left(2 - \frac{4}{N}\right) \frac{\Gamma'}{\Gamma} + \frac{4(N-1)\Gamma'}{(-A-\Gamma)N}$$

In the case  $N > 1$  and  $\rho = 1$ ,  $A = \infty$ , and hence the above equation implies<sup>14</sup>

$$0 = \frac{d}{dt} \left[ \log \left( \Gamma' \Gamma^{2-\frac{4}{N}} \right) \right].$$

Thus,

$$\Gamma' \Gamma^{2-\frac{4}{N}} = C_0 \text{ for some constant } C_0 > 0,$$

which in turns implies

$$(A56) \quad \Sigma(t) = \frac{1}{\Gamma(t)} = (C_1 t + C_2)^{\frac{-1}{3-\frac{4}{N}}} \text{ for some constants } C_1 \text{ and } C_2.$$

But when  $N > 1$ , there is no constants  $C_1 = C_0(3 - 4/N) > 0$  and  $C_2$  which can make the above  $\Sigma(t)$  satisfy either  $\Sigma(1) = 0$  or  $\lim_{t \rightarrow 1} \bar{\lambda}(t) = \sqrt{-\Sigma'(1)} = +\infty$ , as required by equation (A44). This completes the proof that a linear equilibrium does not exist for  $N > 1$  and  $\rho = 1$ .

In the rest of the proof, we assume either  $\rho \neq 1$  or  $N = 1$ . Under these assumptions, we prove that equation (A55) has a unique solution of  $\Sigma(t)$  as described in

<sup>14</sup>To be completely formal and to avoid dividing by 0, we should have directly derived the desired equation below. But this is a straightforward exercise by using the argument for obtaining equation (A55).

the theorem. Now, the only possible case with  $\rho = 1$  happens is when  $N = 1$ . But when  $N = 1$ , there is no competing informed investors, and  $\rho$  is irrelevant. Without loss of generality, we make the additional assumption  $\rho \neq 1$ . This will ensure a finite  $A$  in the rest of the proof.

By equation (A55),

$$0 = \frac{d}{dt} \left[ \log \left( \Gamma' \Gamma^{2-\frac{4}{N}} (\Gamma + A)^{-\frac{4(N-1)}{N}} \right) \right].$$

Hence,

$$\Gamma' \Gamma^{2-\frac{4}{N}} (\Gamma + A)^{-\frac{4(N-1)}{N}} = C_3 \text{ for some constant } C_3,$$

In the case of  $\rho = 0$ , we have  $A = 0$ . Hence, the above equation is equivalent to  $\Gamma^{-2}\Gamma' = C_3$ , which implies that  $\Sigma = 1/\Gamma$  is linear in  $t$ . Hence, the desired formula for  $\Sigma$  follows immediately from the boundary condition  $\Sigma(1) = 0$ .

For the case of  $\rho \neq 0$ , we can make a change of variable as  $\Gamma = A \frac{R}{1-R}$ , the above equation becomes

$$R^{2-\frac{4}{N}} R' = C_4.$$

From this and the boundary condition on  $R(0)$  and  $R(1)$ , we obtain

$$(A57) \quad \frac{1}{A\Sigma(t) + 1} = \left( \left[ \left( \frac{1}{A\Sigma(1) + 1} \right)^{3-\frac{4}{N}} - B \right] t + B \right)^{\frac{1}{3-\frac{4}{N}}}.$$

Taking derivatives with respect to  $t$  in the above equation, we know that  $\Sigma'(1)$  is bounded. Hence, from the proved formula  $\bar{\lambda}(t) = \sqrt{-\Sigma'(t)}$ , we know that  $\lim_{t \rightarrow 1} \bar{\lambda}$  is finite. Hence, from equation (A44), we must have  $\Sigma(1) = 0$ . Plugging  $\Sigma(1) = 0$  into equation (A57), we immediately arrive at the claimed formula for  $\Sigma(t)$ .  $\square$



## PROOFS FOR SECTION III

**Proof of Corollary III.1** This is a special case of part (i) in Corollary IV.1, the proof is omitted here.  $\square$

**Proof of Corollary III.2** It's straightforward to verify that  $\beta(t)$  is increasing in  $t$  and decreasing in  $\sigma_\epsilon^2$ , as  $\beta(t) = 1/[\sigma_\epsilon(1-t)]$ . And the proof for the other part is covered in the proof of (v) in Corollary IV.1.  $\square$

**Proof of Corollary III.3** This is a special case of part (vi) in Corollary IV.1, the proof is omitted here.  $\square$

**Proof of Corollary III.4** This is a special case of part (vii) in Corollary IV.1, the proof is omitted here.  $\square$

**Proof of Corollary III.5** This is a special case of part (iii) in Corollary IV.1, the proof is omitted here.  $\square$

**Proofs of Corollary III.6**

The ratio of market liquidity can be decomposed into three components:

$$\frac{1/\lambda(t)}{1/\hat{\lambda}(t)} = \frac{2}{1} \times \frac{\hat{\beta}(t)}{\beta(t)} \times \frac{\hat{\Sigma}(t)}{\Sigma(t)} = 2 \times \sqrt{1-t} \times \frac{\sigma_\epsilon^2 + \sigma_v^2 t/(1-t)}{\sigma_\epsilon^2 - \sigma_v^2 \log(1-t)}$$

As  $t$  approaches 1,  $\hat{\beta}(t)/\beta(t)$  goes to zero at the order of  $\sqrt{1-t}$  but  $\hat{\Sigma}(t)/\Sigma(t)$  goes to infinity at the order of  $1/[(1-t)\log(1-t)]$ . Thus, we must have

$$\lim_{t \rightarrow 1} \frac{1/\lambda(t)}{1/\hat{\lambda}(t)} = \infty.$$

When  $\sigma_\epsilon \leq \sigma_v$ , we have

$$\begin{aligned} \frac{\hat{\Sigma}(t)}{\Sigma(t)} &= \frac{\sigma_\epsilon^2 + \sigma_v^2 t/(1-t)}{\sigma_\epsilon^2 - \sigma_v^2 \log(1-t)} \\ &\geq \frac{1}{(1-t)(1 - \log(1-t))} \\ &\geq \frac{\sqrt{e}}{2\sqrt{1-t}}, \end{aligned}$$

where the last inequality holds because  $\sqrt{1-t}[1 - \log(1-t)]$  is maximized at  $t = 1 - 1/e$ . It follows that

$$\begin{aligned} \frac{1/\lambda}{1/\hat{\lambda}} &\geq 2 \times \sqrt{1-t} \times \frac{\sqrt{e}}{2\sqrt{1-t}} = \sqrt{e} > 1, \quad \text{and} \\ \pi_D &= \int_0^1 \lambda(t) dt < \int_0^1 \hat{\lambda}(t) dt = \hat{\pi}_D. \quad \square \end{aligned}$$

### Proofs of Corollary III.7

$$\frac{1/\lambda(t)}{1/\hat{\lambda}(t)} = \frac{2}{1} \times \frac{\hat{\beta}(t)}{\beta(t)} \times \frac{\hat{\Sigma}(t)}{\Sigma(t)} = 2 \times \sqrt{1-t} \times \frac{\sigma_\epsilon^2 + \sigma_v^2 t / (1-t)}{\sigma_\epsilon^2 - \sigma_v^2 \log(1-t)}$$

When  $t > 3/4$ ,  $2\sqrt{1-t} < 1$ . Moreover, as  $\sigma_\epsilon$  increases,  $\Sigma(t)/\hat{\Sigma}(t)$  goes to 1 since informed investors have very imprecise signals and thus are reluctant to trade, which causes very little information to be revealed to the market. As a result, market is less liquid in the presence of public disclosure for large  $\sigma_\epsilon$ , which means there exists a  $\sigma_\epsilon^* > \sigma_v$ , such that  $1/\lambda(t) < 1/\hat{\lambda}(t)$  for  $\sigma_\epsilon^* > \sigma_\epsilon$  and  $t > 3/4$ .

$$\begin{aligned} \frac{\hat{\pi}_D}{\pi_D} &= \int_0^1 \frac{4(\sigma_\epsilon^2 - 1)/\log(\sigma_\epsilon^2)}{[\sigma_\epsilon^2 - \log(1-t)]\sqrt{1-t}} dt \\ &\leq \int_0^1 \frac{4(\sigma_\epsilon^2 - 1)/\log(\sigma_\epsilon^2)}{\sigma_\epsilon^2 \sqrt{1-t}} dt \\ &= \frac{8(\sigma_\epsilon^2 - 1)}{\sigma_\epsilon^2 \log(\sigma_\epsilon^2)} \end{aligned}$$

So, we have  $\lim_{\sigma_\epsilon^2 \rightarrow \infty} \hat{\pi}_D/\pi_D = 0$ , which by the definition of limit means there exists a large enough  $\sigma_\epsilon^{**} > \sigma_v$  such that for  $\sigma_\epsilon > \sigma_\epsilon^{**}$ ,  $\pi_D/\hat{\pi}_D > 1$ .  $\square$

**Proof of Corollary III.8** Redefining  $\sigma_\epsilon^2$  as  $\sigma_\epsilon^2/\sigma_v^2$ , we have

$$\begin{aligned} \frac{\pi_D}{\pi_M} &= \frac{\sqrt{\sigma_\epsilon^2 \log(\sigma_\epsilon^2)}}{4(\sigma_\epsilon^2 - 1)} \bigg/ \frac{1}{2\sqrt{1 + \sigma_\epsilon^2}} \\ &= \frac{\sqrt{\sigma_\epsilon^2(1 + \sigma_\epsilon^2)} \log(\sigma_\epsilon^2)}{2(\sigma_\epsilon^2 - 1)} \end{aligned}$$

Taking derivative of  $\pi_D/\pi_M$  with respect to  $\sigma_\epsilon^2$  gives

$$\frac{\partial}{\partial \sigma_\epsilon^2} \frac{\pi_D}{\pi_M} = \frac{2(\sigma_\epsilon^4 - 1) - 3(\sigma_\epsilon^2 + 1) \log(\sigma_\epsilon^2)}{4(\sigma_\epsilon^2 - 1)^2 \sqrt{\sigma_\epsilon^2(1 + \sigma_\epsilon^2)}}$$

Considering the function  $2(\sigma_\epsilon^4 - 1) - 3(\sigma_\epsilon^2 + 1) \log(\sigma_\epsilon^2)$ , its first derivative with respect to  $\sigma_\epsilon^2$  is

$$\begin{aligned} & \frac{\partial}{\partial \sigma_\epsilon^2} \left[ 2(\sigma_\epsilon^4 - 1) - 3(\sigma_\epsilon^2 + 1) \log(\sigma_\epsilon^2) \right] \\ &= 4\sigma_\epsilon^2 - (3 + 1/\sigma_\epsilon^2) - 3 \log(\sigma_\epsilon^2) \\ &\geq 4\sigma_\epsilon^2 - (3 + 1/\sigma_\epsilon^2) - 3(\sigma_\epsilon^2 - 1) \\ &= \sigma_\epsilon^2 - 1/\sigma_\epsilon^2 \geq 0, \sigma_\epsilon^2 \geq 1. \end{aligned}$$

and its value is 0 at  $\sigma_\epsilon^2 = 1$ , which means  $\partial(\pi_D/\pi_M)/\partial \sigma_\epsilon^2 \geq 0$  for all  $\sigma_\epsilon^2 \geq 1$ . And also we have the ratio of  $\pi_D/\pi_M$  grows to  $\infty$  as  $\sigma_\epsilon^2$  goes to  $\infty$ ,

$$\lim_{\sigma_\epsilon^2 \rightarrow \infty} \frac{\pi_D}{\pi_M} = \lim_{\sigma_\epsilon^2 \rightarrow \infty} \frac{\log(\sigma_\epsilon^2)}{2} = \infty.$$

so there is a large enough  $\hat{\sigma}_\epsilon$  such that for  $\sigma_\epsilon > \hat{\sigma}_\epsilon$ , we have  $\pi_D > \hat{\pi}_M = 2\pi_M > \pi_M$ .

From equation (51) in BCW (2000) we have

$$\begin{aligned} \frac{\hat{\pi}_D}{\hat{\pi}_M} &= \int_0^1 \frac{\sqrt{\sigma_\epsilon^2(1 + \sigma_\epsilon^2)}/2}{[\sigma_\epsilon^2 - \log(1 - t)]\sqrt{1 - t}} dt \\ &\leq \int_0^1 \frac{\sqrt{\sigma_\epsilon^2(1 + \sigma_\epsilon^2)}/2}{[\sigma_\epsilon^2 + 1 - (1 - t)]\sqrt{1 - t}} dt \quad (\log(1 - t) \leq -t \text{ for } t \in [0, 1]) \\ &= \int_0^1 \frac{\sqrt{\sigma_\epsilon^2(1 + \sigma_\epsilon^2)}}{[\sigma_\epsilon^2 + 1 - s^2]} ds \quad (\text{Change of Variable: } s = \sqrt{1 - t}) \\ &= \sqrt{\sigma_\epsilon^2} \log \frac{1 + \sqrt{1 + \sigma_\epsilon^2}}{\sqrt{\sigma_\epsilon^2}} \\ &\leq 1. \quad \square \end{aligned}$$

## PROOFS FOR SECTION IV

**Proof of Theorem IV.1** This is covered in the proof of Theorem II.1.  $\square$

**Proof of Theorem IV.2** This is covered in the proof of Theorem II.2.  $\square$

**Proof of Theorem IV.3** The proof is in the Appendix of Back, Cao, and Willard (2000).  $\square$

**Proof of Corollary IV.1**

(i) From equations (A34), (A35) and (A50), we have  $\sum_{1 \leq i \leq N} \text{var}[dz^i(t)] = \text{var}[dz^0(t)]$ , which means informed investors contribute half of the total trading volume  $\sum_{1 \leq i \leq N} \text{var}[dz^i(t)] + \text{var}[dz^0(t)]$ .

(ii) From equation (A56), when  $N = 1$ , we have  $\Sigma(t) = C_1 t + C_2$  for some constants  $C_1, C_2$ . And only  $C_1 = -\Sigma_0, C_2 = \Sigma_0$  satisfies the initial condition and the condition  $\lim_{t \rightarrow 1} \Sigma(t) = 0$  or  $\lim_{t \rightarrow 1} \hat{\lambda}(t) = +\infty$ , required by equation (A44). Thus, we show

$$\Sigma(t) = \Sigma(0)(1 - t) = \hat{\Sigma}(t).$$

and it's trivial to show  $\beta(t) = \hat{\beta}(t), \lambda(t) = \hat{\lambda}(t)/2$ .

(iii) For ease of notation, we define

$$\begin{aligned} a_N &= 3 - 4/N, \\ b_{N\delta} &= \frac{2(1 - \delta(0))}{N\delta(0)}. \end{aligned}$$

$a_N \geq 1$  for  $N \geq 2$  and we can write  $\Sigma(t)$  as

$$\Sigma(t) = \frac{((1 - B)t + B)^{-1/a_N} - 1}{B^{-1/a_N} - 1} \Sigma(0).$$

and its derivative with respect to  $t$  as

$$(C1) \quad \frac{\partial \Sigma(t)}{\partial t} = - \frac{\Sigma(0)(1 - B)((1 - B)t + B)^{-1-1/a_N}}{a_N(B^{-1/a_N} - 1)}$$

Differentiating both sides of equation (27) gives

$$\frac{\partial \hat{\Sigma}(t)}{\partial t} = -\kappa \hat{\Sigma}(0)^{-a_N} \hat{\Sigma}(t)^{1+a_N} e^{b_{N\delta} \hat{\Sigma}(0)/\hat{\Sigma}(t)}$$

The information is gradually revealed as time flows and the public knows exactly the liquidation value  $v$  at the end of the trading period in both cases with disclosure and without disclosure. Both  $\Sigma(t)$  and  $\hat{\Sigma}(t)$  goes to 0 as  $t \rightarrow 1$ , so the L'Hospital's Rule is applied to calculate the following limit:

$$\begin{aligned} \lim_{t \rightarrow 1} \frac{\Sigma(t)}{\hat{\Sigma}(t)} &= \lim_{t \rightarrow 1} \frac{\partial \Sigma(t)/\partial t}{\partial \hat{\Sigma}(t)/\partial t} \\ &= \lim_{t \rightarrow 1} \frac{-\Sigma(0)(1-B)((1-B)t+B)^{-1-1/a_N} / [a_N(B^{-1/a_N} - 1)]}{-\kappa \hat{\Sigma}(0)^{-a_N} \hat{\Sigma}(t)^{1+a_N} e^{b_{N\delta} \hat{\Sigma}(0)/\hat{\Sigma}(t)}} \\ &= \frac{\Sigma(0) \hat{\Sigma}(0)^{a_N} (1-B)}{\kappa a_N (B^{-1/a_N} - 1)} \lim_{t \rightarrow 1} \frac{((1-B)t+B)^{-1-1/a_N} - 1}{\hat{\Sigma}(t)^{1+a_N} e^{b_{N\delta} \hat{\Sigma}(0)/\hat{\Sigma}(t)}} \\ \text{(C2)} \quad &= 0 \end{aligned}$$

The exponential function  $e^{b_{N\delta} \hat{\Sigma}(0)/\hat{\Sigma}(t)}$  grows much faster than the polynomial function  $\hat{\Sigma}(t)^{-1-a_N}$  as  $\hat{\Sigma}(t)$  goes to 0, so the denominator  $\hat{\Sigma}(t)^{1+a_N} e^{b_{N\delta} \hat{\Sigma}(0)/\hat{\Sigma}(t)}$  goes to  $\infty$  and the numerator  $((1-B)t+B)^{-1-1/a_N} - 1$  goes to 0 as time  $t \rightarrow 1$ , which proves the last equation.

Similarly, we have following result

$$\begin{aligned} \lim_{t \rightarrow 1} \frac{\beta(t)}{\hat{\beta}(t)} &= \lim_{t \rightarrow 1} \frac{\sqrt{-\Sigma'/\Sigma}}{\sqrt{-\hat{\Sigma}'/\hat{\Sigma}}} \\ &= \frac{\lim_{t \rightarrow 1} \sqrt{\Sigma'/\hat{\Sigma}'}}{\lim_{t \rightarrow 1} \Sigma'/\hat{\Sigma}'} \quad (\text{By L'Hospital's Rule}) \\ &= \lim_{t \rightarrow 1} (\Sigma'/\hat{\Sigma}')^{-\frac{1}{2}} \\ &= \infty \quad (\text{By equation (C2)}) \end{aligned}$$

(iv) Similarly, we have

$$\begin{aligned}
\lim_{t \rightarrow 1} \frac{1/\lambda}{1/\hat{\lambda}} &= \lim_{t \rightarrow 1} \frac{2/\sqrt{-\Sigma'}}{1/\sqrt{-\hat{\Sigma}'}} \\
&= \lim_{t \rightarrow 1} \frac{2}{\sqrt{\Sigma'/\hat{\Sigma}'}} \\
&= \infty \quad (\text{By equation (C2)})
\end{aligned}$$

(v) As time  $t \rightarrow 1$ , more and more information is revealed and the uncertainty about the liquidity value  $v$  decreases. This can be seen clearly from equation (C1). Given  $a_N \geq 1$ ,  $B-1$  and  $B^{-1/a_N}-1$  take opposite signs, so we have  $\partial\Sigma(t)/\partial t < 0$ .

For  $B$  is a increasing function of  $\sigma_\epsilon$ , we here prove the function's monotonicity with respect to  $B$  rather than  $\sigma_\epsilon$ . Taking derivative of  $\Sigma(t)$  with respect to  $B$ , we get

$$\frac{\partial\Sigma(t)}{\partial B} \propto (1 - B^{-1-1/a_N})(1-t) + B^{-1-1/a_N} - (t/B + 1 - t)^{1+1/a_N}$$

The derivative of  $(1 - B^{-1-1/a_N})(1-t) + B^{-1-1/a_N} - (t/B + 1 - t)^{1+1/a_N}$  with respect to  $B$  is  $(1/a_N + 1)B^{-2-1/a_N}t[(t + B(1-t))^{1/a_N} - 1]$ , which is larger than 0 if  $B \geq 1$  and smaller than 0 if  $B < 1$ . So  $(1 - B^{-1-1/a_N})(1-t) + B^{-1-1/a_N} - (t + B(1-t))^{1+1/a_N}$  reaches its minimum 0 at  $B = 1$  and we have  $\partial\Sigma(t)/\partial B \geq 0$  for all  $B > 0$ .

We write  $\lambda(t)$  as a function of  $B$ ,

$$\begin{aligned}
\lambda(t) &= \sqrt{\frac{\Sigma(0)(1-B)((1-B)t + B)^{-1-1/a_N}}{4a_N(B^{-1/a_N} - 1)}} \\
\lambda(0) &= \sqrt{\frac{\Sigma(0)(1-B)B^{-1-1/a_N}}{4a_N(B^{-1/a_N} - 1)}} \\
\lambda(1) &= \sqrt{\frac{\Sigma(0)(1-B)}{4a_N(B^{-1/a_N} - 1)}}
\end{aligned}$$

Taking derivative of  $\lambda(0)$  and  $\lambda(1)$  with respect to  $B$  gives:

$$\begin{aligned}\frac{\partial\lambda(0)}{\partial B} &= \frac{\Sigma(0) [(1-B)B^{1/a_N} + a_N(B^{1/a_N} - 1)]}{8\lambda(0)a_N^2B^2(1-B^{1/a_N})^2} \\ &\propto (1-B)B^{1/a_N} + a_N(B^{1/a_N} - 1) \\ \frac{\partial\lambda(1)}{\partial B} &= \frac{\Sigma(0)B^{1/a_N-1} [1 - (1+a_N)B + a_NB^{1+1/a_N}]}{8\lambda(1)a_N^2(1-B^{1/a_N})^2} \\ &\propto 1 - (1+a_N)B + a_NB^{1+1/a_N}\end{aligned}$$

The derivative of  $(1-B)B^{1/a_N} + a_N(B^{1/a_N} - 1)$  with respect to  $B$  is  $(1 + 1/a_N)(1-B)B^{1/a_N-1}$ , which is larger than 0 if  $B \leq 1$  and smaller than 0 if  $B > 1$ , so  $\partial\lambda(0)/\partial B$  reaches its maximum 0 at  $B = 1$ , i.e.,  $\partial\lambda(0)/\partial B \leq 0$ . The derivative of  $1 - (1+a_N)B + a_NB^{1+1/a_N}$  with respect to  $B$ ,  $(1+a_N)(B^{1/a_N} - 1)$  is larger than 0 if  $B \geq 1$  and smaller than 0 if  $B < 1$ , so  $\partial\lambda(1)/\partial B$  reaches its minimum 0 at  $B = 1$ , i.e.,  $\partial\lambda(1)/\partial B \geq 0$ .

From the definition of  $\lambda(t)$ , we have

$$\begin{aligned}\frac{\partial\lambda(t)}{\partial t} &= \frac{\partial(-\Sigma')/\partial t}{2\lambda(t)} \\ &= \frac{\Sigma(0)(1+1/a_N)(1-B)^2((1-B)t+B)^{-2-1/a_N}}{8\lambda(t)a_N(B^{-1/a_N}-1)} \\ &\propto 1/(1-B^{1/a_N})\end{aligned}$$

So,  $\lambda(t)$  is increasing in  $t$  when  $B < 1$  ( $\sigma_\epsilon^2 < (N-1)\sigma_v^2$ ) and decreasing in  $t$  when  $B \geq 1$  ( $\sigma_\epsilon^2 \geq (N-1)\sigma_v^2$ ).

(vi) When  $\sigma_\epsilon^2 = (N-1)\sigma_v^2$ , we have

$$\Sigma(t) = \Sigma(0)(1-t)$$

and hence

$$\beta = \frac{1}{\sqrt{\Sigma(0)}(1-t)}, \quad \lambda = \frac{1}{2}, \quad \bar{\lambda} = 1.$$

and the profits of informed investors are

$$\int_0^1 \lambda(t) dt = \int_0^1 \frac{1}{2} dt = \frac{1}{2}$$

Therefore, market efficiency, market liquidity, and profit are the same as if there exists a monopolistic informed investor with all the signals in the market. And from equation (A32) and equation (40) (i.e.,  $\delta(t) = (1 + (N-1)\rho(t))/N$ ) in Back, Cao and Willard (2000), we know  $\rho(t) = \rho\Omega(t)/\Omega(0)$  and hence the conditional correlation between private signals  $\rho(t)$  remains 0 throughout the trading period.

(vii) When  $\sigma_\varepsilon^2 \neq (N-1)\sigma_v^2$ , we first have

$$N\delta(t) = 1 + (N-1)\rho \frac{\Omega(t)}{\Omega(0)} \rightarrow 1$$

and hence  $\rho(t) \rightarrow 0$  (from equation (40) in Back, Cao and Willard (2000)) because  $\Omega(t) \leq \Sigma(t) \rightarrow 0$  as time  $t$  goes to 1. Further,

$$\begin{aligned} \lim_{t \rightarrow 1} \frac{\Sigma(t)}{1-t} &= \lim_{t \rightarrow 1} -\Sigma'(t) \quad (\text{By L'Hospital's Rule}) \\ &= \frac{(1-\rho)\Sigma(0)(1-B)}{\rho N a_N} \lim_{t \rightarrow 1} ((1-B)t + B)^{-1-1/a_N} \\ &= \frac{(1-\rho)\Sigma(0)(1-B)}{\rho N a_N} \\ &= S_0 \end{aligned}$$

from here we also have  $\lim_{t \rightarrow 1} \Sigma'(t) = -S_0$ .



It's straightforward to verify

$$\begin{aligned}
\lim_{t \rightarrow 1} \beta(t)(1-t) &= \lim_{t \rightarrow 1} \frac{\sqrt{-\Sigma'(t)}}{\Sigma(t)/(1-t)} \\
&= \frac{\lim_{t \rightarrow 1} \sqrt{-\Sigma'(t)}}{\lim_{t \rightarrow 1} \Sigma(t)/(1-t)} \\
&= \frac{\sqrt{S_0}}{S_0} \\
&= 1/\sqrt{S_0}, \quad \text{and} \\
\lim_{t \rightarrow 1} \lambda(t) &= \lim_{t \rightarrow 1} \frac{\sqrt{-\Sigma'(t)}}{2} \\
&= \frac{\lim_{t \rightarrow 1} \sqrt{-\Sigma'(t)}}{2} \\
&= \frac{\sqrt{S_0}}{2}
\end{aligned}$$

Writing the profit  $\pi(0)$  as a function of  $B$ , we have

$$\begin{aligned}
\pi(0) &= \sqrt{\frac{a_N \Sigma(0)}{4(N-2)^2} \frac{(1-B^{(1-1/a_N)/2})^2}{(1-B)(B^{-1/a_N}-1)}}, \quad \text{and} \\
\frac{\partial \pi(0)}{\partial B} &\propto \frac{\partial}{\partial B} \left[ \frac{(1-B^{(1-1/a_N)/2})^2}{(1-B)(B^{-1/a_N}-1)} \right] \\
&= -\frac{B^{(1/a_N-1)/2}(1-B^{(1-1/a_N)/2})(1-B^{(1+1/a_N)/2})}{a_N(1-B)^2(1-B^{1/a_N})^2} \\
&\quad \times \left[ a_N(1-B^{1/a_N}) - (1-B)B^{(1/a_N-1)/2} \right] \\
&\propto a_N(B^{1/a_N}-1) - (B-1)B^{(1/a_N-1)/2}
\end{aligned}$$

Again, taking derivative of  $a_N(B^{1/a_N}-1) - (B-1)B^{(1/a_N-1)/2}$  with respect to  $B$  gives  $B^{(1/a_N-3)/2} [B^{(1/a_N+1)/2} - (1/a_N+1)B/2 + (1/a_N-1)/2]$ . The derivative of the second term  $B^{(1/a_N+1)/2} - (1/a_N+1)B/2 + (1/a_N-1)/2$  is  $(1/a_N+1)(B^{(1/a_N-1)/2}-1)/2$ . Given  $a_N \geq 1$ ,  $B^{(1/a_N-1)/2}-1$  is negative if  $B > 1$  and positive if  $B \leq 1$ , so the derivative of  $a_N(B^{1/a_N}-1) - (B-1)B^{(1/a_N-1)/2}$  with

respect to  $B$  reaches its maximum 0 at  $B = 1$ , which means it decreases in  $B$  and equals to 0 at  $B = 1$ . So,  $\partial\pi(0)/\partial B$  is positive when  $B < 1$  ( $\sigma_\epsilon^2 < (N-1)\sigma_v^2$ ) and negative when  $B \geq 1$  ( $\sigma_\epsilon^2 \geq (N-1)\sigma_v^2$ ), i.e.,  $\pi(0)$  reaches its maximum at  $B = 1$  ( $\sigma_\epsilon^2 = (N-1)\sigma_v^2$ ).  $\square$

**Proofs of Corollary IV.2** When one of the  $N$  informed investors (without loss of generality, assume she is the  $N$ -th trader) leaves the market, the remaining  $N-1$  investors in aggregate don't know the true value of the asset  $v$ . Instead, the variable which the  $N-1$  informed investors and the market maker are interested in is informed investors' expectation of  $v$ :

$$v_{N \rightarrow N-1} = E[v | s_1, \dots, s_{N-1}] = \frac{N(N-1)}{(N-1)^2 + \sigma_\epsilon^2/\sigma_v^2} \sum_{i=1}^{N-1} s_i$$

Correspondingly, the expected profit each of the remaining  $N-1$  informed investors obtains

$$\pi_{N \rightarrow N-1} = \sqrt{\Sigma_{N \rightarrow N-1}(0) \frac{1-\rho}{\rho} \frac{3(N-1)-4}{1-B_{N \rightarrow N-1}}} \frac{\left| 1 - B_{N \rightarrow N-1}^{\frac{(N-1)-2}{3(N-1)-4}} \right|}{2(N-1)|(N-1)-2|}$$

here,

$$\begin{aligned} \Sigma_{N \rightarrow N-1}(0) &= \text{var}[v_{N \rightarrow N-1}] = \frac{(N-1)^2 \sigma_v^2}{(N-1)^2 + \sigma_\epsilon^2/\sigma_v^2} \\ &\sim O\left(\frac{(N-1)^2 \sigma_v^4}{\sigma_\epsilon^2}\right), \quad \sigma_\epsilon^2 \rightarrow \infty \\ B_{N \rightarrow N-1} &= \left(\frac{1-\rho}{1-\rho+(N-1)\rho}\right)^{3-4/(N-1)} = \left(\frac{N\sigma_\epsilon^2}{(N-1)^2 \sigma_v^2 + \sigma_\epsilon^2}\right)^{3-4/(N-1)} \\ &\sim O\left(N^{3-4/(N-1)}\right), \quad \sigma_\epsilon^2 \rightarrow \infty. \end{aligned}$$

Considering the limiting behaviour of the ratio of  $\pi_N$  and  $\pi_{N \rightarrow N-1}$  when  $\sigma_\epsilon^2 \rightarrow$

$\infty$ :

$$\begin{aligned}
& \lim_{\sigma_\epsilon^2 \rightarrow \infty} \frac{\pi_N^2}{\pi_{N \rightarrow N-1}^2} \\
&= \lim_{\sigma_\epsilon^2 \rightarrow \infty} \frac{\lim_{\sigma_\epsilon^2 \rightarrow \infty} \pi_N^2}{\lim_{\sigma_\epsilon^2 \rightarrow \infty} \pi_{N \rightarrow N-1}^2} \\
&= \lim_{\sigma_\epsilon^2 \rightarrow \infty} \frac{-\Sigma(0) \frac{1-\rho}{\rho} \frac{(3N-4)\sigma_v^2}{\sigma_\epsilon^2/(N-1)} / (4N^2(N-2)^2)}{\frac{(N-1)^2\sigma_v^4}{\sigma_\epsilon^2} \frac{1-\rho}{\rho} \frac{3(N-1)-4}{1-N^{3-4/(N-1)}} \frac{(1-N^{1-2/(N-1)})^2}{4(N-1)^2((N-1)-2)^2}} \\
&= \frac{(3N-4)(N-1)(N-3)^2(N^{3-4/(N-1)}-1)}{(3N-7)N^2(N-2)^2(1-N^{1-2/(N-1)})^2} \\
&> 1, \quad N \geq 4
\end{aligned}$$

In fact,  $\lim_{\sigma_\epsilon^2 \rightarrow \infty} \pi_N^2 / \pi_{N \rightarrow N-1}^2$  decreases to 1 as  $N$  goes to  $\infty$ .

For the case of  $N = 3$ , following similar steps, we get

$$\begin{aligned}
\lim_{\sigma_\epsilon^2 \rightarrow \infty} \frac{\pi_3^2}{\pi_{3 \rightarrow 2}^2} &= \lim_{\sigma_\epsilon^2 \rightarrow \infty} \frac{\Sigma(0) \frac{1-\rho}{\rho} \frac{5\sigma_v^2}{\sigma_\epsilon^2/2} / 36}{\frac{4\sigma_v^4}{64\sigma_\epsilon^2} \frac{1-\rho}{\rho} (\log(3))^2} \\
&= \frac{40}{9(\log(3))^2} \\
&= 3.68 > 1
\end{aligned}$$

So, we have  $\pi_N / \pi_{N \rightarrow N-1} > 1$  as  $\sigma_\epsilon^2$  grows to  $\infty$  for all  $N \geq 3$ , which means there exist a large enough  $\bar{\sigma}_\epsilon$  such that  $\pi_N > \pi_{N \rightarrow N-1}$  for all  $\sigma_\epsilon > \bar{\sigma}_\epsilon$ . The case of  $N = 2$  is covered in the proof of Corollary III.8.

Writing  $\pi_N$  and  $\pi_M$  in  $\sigma_\epsilon^2$  gives us:

$$\begin{aligned}
& \lim_{\sigma_\epsilon^2 \rightarrow \infty} \pi_N / \pi_M \\
&= \lim_{\sigma_\epsilon^2 \rightarrow \infty} \frac{\sqrt{3-4/N}}{2(N-2)} \sqrt{\frac{\Sigma(0)(1 - (\frac{\sigma_\epsilon^2/\sigma_v^2}{N-1})^{1-2/N})^2}{(1 - (\frac{\sigma_\epsilon^2/\sigma_v^2}{N-1})^{3-4/N})(\frac{N-1}{\sigma_\epsilon^2/\sigma_v^2} - 1)}} \bigg/ \frac{1}{2\sqrt{\sigma_v^2 + \sigma_\epsilon^2}} \\
&= \frac{\sqrt{3-4/N}}{2(N-2)} \lim_{\sigma_\epsilon^2 \rightarrow \infty} \frac{2\sqrt{N-1}(\frac{\sigma_\epsilon^2/\sigma_v^2}{N-1})^{1-2/N}(\frac{\sigma_\epsilon^2/\sigma_v^2}{N-1})^{1/2}}{(\frac{\sigma_\epsilon^2/\sigma_v^2}{N-1})^{3/2-2/N}}
\end{aligned}$$

$$= \frac{\sqrt{(3 - 4/N)(N - 1)}}{N - 2}$$

$\sqrt{(3 - 4/N)(N - 1)}/(N - 2)$  decrease in  $N$  and equals 1.8257 at  $N = 3$ , 1.2245 at  $N = 4$ , and 0.9888 at  $N = 5$ . So, for  $N = 3, 4$ , there exists  $\hat{\sigma}_\epsilon$  such that for  $\sigma_\epsilon > \hat{\sigma}_\epsilon$ ,  $\pi_N > \pi_M$ .

Denote by  $\hat{\pi}_N$  each of  $N$  informed trader's expected profit without disclosure. The case of  $N = 2$  is covered in the proof of Corollary III.8. From equations (41) and (57) in BCW (2000), we have the following result for  $N \geq 3$

$$\frac{\hat{\pi}_N}{\hat{\pi}_M} = \frac{\sqrt{1 + \sigma_\epsilon^2} \int_1^\infty x^{-2/N} e^{-x\sigma_\epsilon^2/N} dx}{N \left( \int_1^\infty x^{2(N-2)/N} e^{-2x\sigma_\epsilon^2/N} dx \right)^{\frac{1}{2}}}$$

Applying Cauchy-Schwartz Inequality, we get

$$\begin{aligned} \int_1^\infty x^{-2/N} e^{-x\sigma_\epsilon^2/N} dx &= \int_1^\infty x^{-1} x^{1-2/N} e^{-x\sigma_\epsilon^2/N} dx \\ &\leq \left( \int_1^\infty x^{-2} dx \right)^{\frac{1}{2}} \left( \int_1^\infty x^{2(1-2/N)} e^{-2x\sigma_\epsilon^2/N} dx \right)^{\frac{1}{2}} \\ &= \left( \int_1^\infty x^{2(1-2/N)} e^{-2x\sigma_\epsilon^2/N} dx \right)^{\frac{1}{2}} \end{aligned}$$

On the other hand, it is straightforward to verify

$$\begin{aligned} \int_1^\infty x^{-2/N} e^{-x\sigma_\epsilon^2/N} dx &\leq \int_1^\infty e^{-x\sigma_\epsilon^2/N} dx = \frac{N}{\sigma_\epsilon^2} e^{-\sigma_\epsilon^2/N}, \quad \text{and} \\ \int_1^\infty x^{2(N-2)/N} e^{-2x\sigma_\epsilon^2/N} dx &\geq \int_1^\infty e^{-2x\sigma_\epsilon^2/N} dx = \frac{N}{2\sigma_\epsilon^2} e^{-2\sigma_\epsilon^2/N} \end{aligned}$$

Combining the above inequalities gives:

$$\frac{\hat{\pi}_N}{\hat{\pi}_M} \leq \min \left\{ \frac{\sqrt{1 + \sigma_\epsilon^2}}{N}, \sqrt{\frac{2(1 + \sigma_\epsilon^2)}{N\sigma_\epsilon^2}} \right\} \leq \frac{\sqrt{1 + 2N}}{N} < 1. \quad \square$$