

Mortgage Risk and the Yield Curve*

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Abstract

We study the feedback from the risk of outstanding mortgage-backed securities (MBS) on the level and volatility of interest rates. We incorporate the supply shocks resulting from changes in MBS duration into a stylized equilibrium dynamic term structure model and derive two predictions that are strongly supported in the data: (i) MBS duration positively predicts excess bond returns, especially for longer maturities; (ii) MBS convexity increases interest rate volatility, and this effect has a hump-shaped term structure. Empirically, neither duration nor convexity are spanned by the first three yield factors.

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Mortgage-backed securities (MBS) and, more generally, mortgage loans expose investors to interest rate risk. Unlike with regular bonds, this exposure can change considerably with interest rate conditions.¹ This is the case because mortgages typically feature an embedded prepayment option that makes their convexity negative: Lower interest rates increase the probability that outstanding mortgages will be prepaid in the future and thereby considerably decrease their duration. This leaves financial institutions who invest in MBS short of duration exposure, until either interest rates revert back to higher levels, or the prepayment option becomes sufficiently in the money for a large number of households to refinance and take on new mortgages. Because households do not play an active role in bond markets and do not hedge their time-varying interest rate risk exposure, it is the position of financial institutions that determines the pricing of interest rate risk (see Gabaix, Krishnamurthy, and Vigneron (2007)). In other words, a fall in mortgage duration is equivalent to a negative transitory shock to the supply of long-term bonds and therefore can have an effect on their prices.² Moreover, mortgage investors who want to keep the duration of their portfolios constant after a drop in MBS duration (for hedging or portfolio rebalancing reasons) induce additional buying pressure on Treasuries and push interest rates down even further. Thus, negative convexity due to the prepayment option creates an amplification channel for interest rate shocks.

The negative convexity channel described above has attracted the attention of practitioners, policy makers, and empirical researchers alike.³ In this paper we build a parsimonious model that formalizes the intuition behind this channel and allows us to derive novel predictions about the effect of mortgage risk on the yield curve. The model implies a link between mortgage convexity and the volatility of interest rates with intermediate maturity, and between mortgage duration and expected returns on long-term bonds. We test these predictions and find strong support for them in the data.

¹In contrast with Treasuries, MBS duration can drop by more than 50%, e.g. from 5 to 2.5 years. See Figure 1. Taking into account the value of outstanding mortgage debt, we calculate that one monthly standard deviation shock to MBS duration is a dollar duration equivalent of a USD 368bn shock to the supply of 10-year Treasuries.

²See Lou, Yan, and Zhang (2013) on how financial intermediaries' supply shocks can affect Treasury bond prices.

³See Perli and Sack (2003), Chang, McManus, and Ramagopal (2005), Duarte (2008), Hanson (2013), and Li and Wei (2013) among others.

In our model the term structure of interest rates is determined by the interaction between (i) exogenous shocks to the short rate and (ii) changes in the net supply of long-term bonds that are endogenously driven by the interest rate risk exposure of mortgages. The term structure model we derive takes the form of a standard Vasicek (1977) short rate model augmented by an additional affine factor that captures the duration of outstanding MBS. This factor drives the market price of interest rate risk and affects the risk premia of long-term bonds but not the dynamics of the risk-free rate itself. Its contribution tends to zero if the risk-bearing capacity of financial institutions is high.

The model has two sets of testable predictions. First, the duration of outstanding MBS predicts bond excess returns. Moreover, this effect is stronger for longer maturity bonds. This happens because in the model the market price of interest rate risk is proportional to the quantity of duration risk that investors have to hold. Longer maturity bonds with a higher exposure to interest rate risk are more strongly affected through this channel.

Second, the average volatility of all yields is increasing in the convexity of outstanding MBS through the negative convexity amplification channel discussed above. This effect has a hump-shaped pattern across maturities with intermediate maturities being most strongly affected. In the model, supply shocks create transitory variations in the market price of interest rate risk. Short-maturity bonds are a close substitute to the short rate and are not very sensitive to variations in the market price of risk, while for long maturities, the market price of risk is expected to mean revert. As a result, the effect of negative convexity on yield volatility is strongest for intermediate maturities. The same intuition applies to swaption implied volatilities.

We test the aforementioned theoretical predictions in the data. We first regress bond excess returns onto measures of MBS duration and find a highly significant positive link between duration and excess returns. Moreover, the link becomes stronger as bond maturity increases. The relationship is also economically significant: A one standard deviation change in MBS duration implies a change of 273 basis points in the expected

one-year excess return on a 10-year bond.⁴ Both the statistical significance and the magnitude of the coefficients are robust to other documented predictors of bond risk premia.

For second moments of bond yields, we find that more negative MBS convexity significantly increases bond yield and swaption implied volatilities. As implied by the model, the effect is most pronounced for intermediate maturities between two and three years. For example, any one standard deviation change in MBS convexity increases bond yield volatility for these maturities by approximately 120 basis points. Moreover, this strong link remains if we add other determinants of interest rate volatility.

We also test how MBS convexity affects bond return volatility—both implied and realized—and find that the estimated slope coefficients are highly significant. To test how MBS convexity affects compensation for volatility risk in fixed income markets, we use measures of bond variance risk premia. In line with the intuition of the model we find that MBS convexity loads positively and highly significantly on these proxies of volatility risk.

Since both MBS duration and convexity depend on interest rate conditions, it is natural to ask whether duration and convexity contain any information above the one encoded in yields. We model MBS duration as a function of the refinancing incentive, i.e. the difference between the average coupon paid on outstanding mortgages and the current level of mortgage rates. As a result, duration depends not only on the current level of interest rates, but also on its past levels. While it is spanned by the cross-section of yields, it is not uniquely determined by the short rate factor that accounts for most of the variation in the shape of the yield curve in the model. Empirically, we find that neither duration nor convexity are spanned by the first three principal components of bond yields, which explain almost all variation in the cross section of yields. Running multivariate regressions from bond excess returns and bond volatility both on duration (or convexity) and the first three yield factors does not reduce the statistical and economic significance of the estimates.

⁴We find similar results when we use swap rather than Treasury data.

Our work is related to a series of empirical papers on the negative convexity channel. Li and Wei (2013) study a no arbitrage model of the term structure that includes an unspanned MBS supply factor. In our paper, the empirical analysis is guided by an equilibrium term structure model that, to the best of our knowledge, is the first to formalize the intuition behind the negative convexity channel. Moreover, our model has simultaneous implications for bond risk premia and interest rate volatility. Perli and Sack (2003), Chang, McManus, and Ramagopal (2005), and Duarte (2008) test the presence of a linkage between various proxies for MBS hedging activity and interest rate volatility, without considering the simultaneous effect that hedging activity can have on bond pricing.

In contemporaneous work, Hanson (2013) reports results similar to ours regarding the predictability of bond excess returns by MBS duration. The author's focus is mainly on documenting how MBS duration affects term premia and forward rates. Our paper is different along several dimensions. First, in contrast to his theoretical framework, ours allows us to derive formal predictions regarding the effects of mortgage risk on bond risk premia for different maturities. Second, our model also allows for an analysis of the negative convexity effect on interest rate and bond return volatilities across different maturities, and we provide novel predictions and empirical evidence in this regard.

Our work is also related to several strands in the asset pricing literature. We make use of the framework developed by Vayanos and Vila (2009). In their model, the term structure of interest rates is determined by the interaction of preferred habitat investors and risk-averse arbitrageurs, who demand higher risk premia as their exposure to long-term bonds increases. Thus, the net supply of bonds matters. Greenwood and Vayanos (2013) use this theoretical framework to study the implications of a change in the maturity structure of government debt supply, similar to the one undertaken in 2011 by the Federal Reserve during "Operation Twist". Our paper is different in at least three respects. First, in our model the variation in the net supply of bonds is driven endogenously by changing MBS duration and not exogenously by the government. Second, the supply factor in Greenwood and Vayanos (2013) explains low frequency variation in risk premia, because movements in maturity-weighted government debt to GDP occur at a

lower frequency than movements in the short rate. Our duration factor, on the other hand, explains variations in risk premia at a higher frequency than movements in the level of interest rates. Finally, Greenwood and Vayanos (2013) posit that the government adjusts the maturity structure of its debt in a way that stabilizes bond markets. For instance, when interest rates are high, the government will finance itself with shorter maturity debt and thereby reduce the quantity of interest rate risk held by agents. Our mechanism goes exactly in the opposite direction: Because of the negative convexity in MBS, the supply effect amplifies interest rates shocks.

Corporate debt constitutes another important class of fixed income instruments, which is shown to be negatively correlated with the supply of Government debt (see, e.g., Greenwood, Hanson, and Stein (2010) and Bansal, Coleman, and Lundblad (2011)). For example, Greenwood, Hanson, and Stein (2010) show that firms choose their debt maturity in a way that tends to offset the variations in the supply and maturity of Government debt. However, the authors find no relationship between corporate debt and MBS supply, which provides us with an additional motivation to focus on the latter.

This paper contributes to the literature on equilibrium term structure models, e.g., Le, Singleton, and Dai (2010), Xiong and Yan (2010), Ehling, Gallmeyer, Heyerdahl-Larsen, and Illeditsch (2013), Bansal and Shaliastovich (2013), Hong, Sraer, and Yu (2013), and Le and Singleton (2013), among others. Contrary to these papers, in which bond risk premia are determined by macroeconomic fundamentals or differences in beliefs about these fundamentals, we focus on the aggregate demand and supply of bonds as the main driver of risk premia. Le and Singleton (2013) also show that time-varying market prices of risk, in addition to time-varying quantities of risk, are required to explain bond risk premia. In our model, duration drives the variation in the market price of risk and hence, bond risk premia. Recent empirical work by Duffee (2011), Joslin, Pribsch, and Singleton (2012) and Le and Singleton (2013) documents the existence of unspanned factors that are unrelated to the cross-section of yields but have a significant impact on risk premia. In our model, duration is spanned by the cross section of yields, but not by the short rate factor that accounts for most of the variation in the shape of the yield curve. Empirically we find that MBS duration is not spanned by the first three

principal components that drive virtually all variation in the cross section of yields. Finally, Joslin, Le, and Singleton (2013) and Fenou and Fontaine (2013) develop term structure models that include additional lags in the dynamics of yield factors. Similarly, in our paper, MBS duration depends on both current and past yields.

There is an important literature that studies the optimal prepayment in the MBS market (see, e.g., Schwartz and Torous (1989), Stanton (1995), Stanton and Wallace (1998), Longstaff (2005) and, more recently, Agarwal, Driscoll, and Laibson (2013) for examples of prepayment models and optimal prepayment decisions). Furthermore, there is evidence that households' prepayment is too sluggish and that this non-optimal prepayment can be best explained using micro-level data (see, e.g., Campbell's 2006 AFA Presidential Address for an overview). In this paper we are interested in how the aggregate properties of prepayment affect the risk of mortgage-related portfolios of financial institutions. Closest to us are Gabaix, Krishnamurthy, and Vigneron (2007) who study the effect of limits to arbitrage in the MBS market. The authors show that, while mortgage prepayment risk resembles a wash on an aggregate level, it nevertheless carries a positive risk premium because it is the risk exposure of financial intermediaries that matters. Our paper is based on a similar premise. Different from these authors, however, we do not study prepayment risk, but changes in interest rate risk of MBS that are driven by the prepayment probability and their effect on the term structure of interest rates.

Finally, our paper is related to the literature that studies the effect of the recent Fed's purchase of long-term assets on interest rates (see, e.g., Gagnon, Raskin, Remache, and Sack (2010), Krishnamurthy and Vissing-Jorgensen (2011), D'Amico, English, Lopez-Salido, and Nelson (2012), D'Amico and King (2013)). While our model mainly focusses on the relationship between long-term yields and volatilities and MBS duration, other papers present evidence for alternative channels. For example, Krishnamurthy and Vissing-Jorgensen (2011) report that QE1, which involved major purchases of agency MBS, led not only to large reductions in mortgage rates, but also helped drive down Treasury yields and caused a drop in the default risk premium of corporate bonds. They interpret their findings as evidence for a long-term safety channel. D'Amico and King

(2013) emphasize a scarcity channel, i.e. a localized effect of supply shocks on yields of nearby maturities.

The remainder of the paper is organized as follows. Section 1 sets up a model of the term structure of interest rates and MBS duration. Section 2 describes the data used and Section 3 empirically tests our hypotheses. Section 4 concludes. Proofs are deferred to the Appendix. An Online Appendix contains additional institutional details on the MBS market.

1 Model

In this section we build a stylized equilibrium dynamic affine term structure model in which the supply of fixed income securities is driven by MBS duration. For instance, we interpret the shortening of MBS duration due to the increased probability of future refinancing as a negative shock to the supply of long-term bonds: before potential refinancing happens and investors have access to new mortgages, the interest rate risk profile of mortgage-related securities available to investors is that of relatively short maturity bonds.⁵

1.1 Bond market

Time is continuous and goes from zero to infinity. We denote the time- t price of a zero coupon bond paying one dollar at maturity $t + \tau$ by Λ_t^τ , and its yield by $y_t^\tau = -\frac{1}{\tau} \log \Lambda_t^\tau$. The short rate r_t is the limit of y_t^τ when $\tau \rightarrow 0$. We take r_t as exogenous and assume that its dynamics under the physical probability measure are given by

$$dr_t = \kappa(\theta - r_t) dt + \sigma dB_t, \tag{1}$$

where θ is the long run mean of r_t , κ is the speed of mean reversion, and σ is the volatility of the short rate.

⁵Market participants can invest in new mortgage loans by buying corresponding MBS. Up to 90 days before those MBS are issued, investors have access to them through the “to-be-announced” (TBA) market. We thank Douglas McManus for an insightful discussion; see also Vickery and Wright (2010).

At each date t , there exists a continuum of zero-coupon bonds with time to maturity $\tau \in (0, T]$ in total supply of s_t^τ . Bonds are held by financial institutions: they are competitive and have mean-variance preference over the instantaneous change in the value of their bond portfolio. If x_t^τ denotes their holdings in maturity- τ bonds at time t , the investors' budget constraint becomes

$$dW_t = \left(W_t - \int_0^T x_t^\tau \Lambda_t^\tau d\tau \right) r_t dt + \int_0^T x_t^\tau \Lambda_t^\tau \frac{d\Lambda_t^\tau}{\Lambda_t^\tau} d\tau, \quad (2)$$

and their optimization problem is given by

$$\max_{\{x_t^\tau\}_{\tau \in (0, T]}} \mathbb{E}_t [dW_t] - \frac{\alpha}{2} \text{Var}_t [dW_t], \quad (3)$$

where α is their absolute risk aversion. Since financial institutions have to take the other side of the trade in the bond market, the market clearing condition is given by

$$x_t^\tau = s_t^\tau, \quad \forall t \text{ and } \tau. \quad (4)$$

The supply of bonds is driven by households' mortgage liabilities. We assume a continuum of infinitely lived households who do not themselves invest in bonds but take fixed rate mortgage loans that are then sold on the market as MBS. Households refinance their mortgages when the incentive to do so is sufficiently high and individual refinancing thresholds can be heterogeneous across households. We limit ourselves to a stylized description of aggregate refinancing dynamics that captures the first order effects of prepayment on the supply of bonds.⁶ On aggregate, refinancing activity does not change the size of the mortgage pool: when a mortgage is prepaid, another mortgage is issued. However, the average coupon paid on outstanding mortgages, c_t , is affected by prepayment because the coupon of the newly issued mortgages is a function of the current level of mortgage rates. In the model, the latter is approximated by the long-

⁶The prepayment of mortgages has been the subject of much research in the past: Schwartz and Torous (1989), Hayre and Young (2004), Matthey and Wallace (2001), and LaCour-Little, Marschoun, and Maxam (2002) provide some econometric modeling, Stanton (1995), Stanton and Wallace (1998), Longstaff (2005) propose examples of optimization-based prepayment models.

term interest rate $y_t^{\bar{}}$.^{7,8} We assume that the evolution of the average coupon is described by:

$$dc_t = -\kappa_C(c_t - y_t^{\bar{}})dt, \quad (5)$$

where the parameter $\kappa_C > 0$ measures the speed of renewal of the mortgage pool as function of the refinancing incentive, i.e., the difference between the average coupon and current mortgage rate $c_t - y_t^{\bar{}}$: higher refinancing incentive leads to more prepayments, and new mortgages decrease the average coupon by more. Because our focus is the feedback between MBS markets and interest rates, we also assume that on aggregate there is no additional uncertainty about refinancing. The upper panel of Figure 1 provides empirical motivation for equation (5). It depicts the difference between long-term interest rates and the average MBS coupon, together with the subsequent change in the average coupon. The two series are closely aligned with the coupon reacting with a bit of delay to a change in the refinancing incentive.

The distinctive feature of mortgage-related securities is that their duration depends primarily on the likelihood that they will be refinanced in the future. We thus assume that the aggregate dollar duration of outstanding mortgages is a function of the refinancing incentive. The latter can be thought of as a measure of the average “moneyness” of mortgage prepayment options (see Richard and Roll (1989)). Omitting constant terms we have:

$$D_t = -\eta_y(c_t - y_t^{\bar{}}), \quad (6)$$

where $D_t \equiv -dMBS_t/dy_t^{\bar{}}$ is the observable sensitivity of the aggregate mortgage portfolio value to the changes in a reference long-maturity rate $y_t^{\bar{}}$ and $\eta_y > 0$. The middle panel of Figure 1 provides empirical motivation for equation (6). It depicts the difference between long-term interest rates and the average MBS coupon, together with aggregate

⁷Following Schwartz and Torous (1989) we approximate mortgage rates by long-term interest rates $y_t^{\bar{}}$. An argument against this choice is mentioned in Krishnamurthy (2010) who studies the spread between mortgage rates and interest rate swaps. He notes that especially during autumn 2008 there was a large disconnect between the two which can be attributed to a flight-to-liquidity episode from relatively illiquid mortgages to more liquid government bonds. Since these considerations are outside the scope of the model, we leave this to future research.

⁸We set the reference maturity $\bar{}$ to ten years. According to Hancock and Passmore (2010), it is common industry practice to use either the 5- or 10-year swap rate as a proxy for MBS duration.

MBS duration. The two series are very closely aligned. Overall, we note that our model captures well the key stylized properties of refinancing activity.

[Insert Figure 1 here.]

Combining equations (5) and (6) gives us the dynamics of D_t :

$$dD_t = -\kappa_D D_t dt + \eta_y dy_t^{\bar{r}}, \quad (7)$$

where $\kappa_D = \kappa_C$ and without loss of generality we work with de-meaned dollar duration series. From equation (7) dollar duration is driven both by changes in long-term interest rates and refinancing activity. In particular, there are two sources of mean reversion in aggregate duration: the mean-reversion in interest rates, $\eta_y E_t(dy_t^{\bar{r}})$, that affects the dollar duration of each individual mortgage, and the renewal in the aggregate pool of mortgages, $-\kappa_D D_t dt$. The parameter $\eta_y = dD_t/dy_t^{\bar{r}}$ is the negative of the dollar convexity: When $\eta_y > 0$, lower interest rates increase the probability of borrowers prepaying their mortgages in the future, leading to a lower duration. The lower panel of Figure 1 plots the MBS convexity series. Comparative statics with respect to η_y allow us to derive predictions regarding the effect of negative convexity on interest rate volatility for different maturities.⁹

Why do shocks to mortgage duration matter? First, note that while lower interest rates trigger a certain amount of refinancing, they also increase the probability of future prepayment and thus decrease the duration for all outstanding mortgages. There is ample empirical evidence that shows that households' refinancing is gradual and sluggish (see Campbell (2006)). The slowness in refinancing leaves financial institutions who invest in MBS on aggregate short of duration exposure.¹⁰ Second, there is no evidence suggesting that households are actively hedging the changes in the duration of their

⁹A model where η_y itself follows a stochastic process would not fall into one of the standard tractable classes of models. Appendix B presents a version of the model that accommodates time-varying convexity. While this model implies a quadratic instead of an affine term structure, it leads to identical qualitative predictions.

¹⁰Gabaix, Krishnamurthy, and Vigneron (2007) make a related point that from the perspective of financial intermediaries who are the marginal investors in MBS, this sluggishness creates an unhedgeable risk which is priced.

mortgage liabilities by trading bonds. Interest rate instruments are traded primarily by financial institutions and, moreover, trading volume in Treasuries and MBS is very high.¹¹ Consistent with this evidence, we assume that the supply of bonds is mainly held by financial institutions who actively rebalance their portfolios. The mortgage choice of households affects the supply of fixed income securities, s_t^τ , through the duration of mortgages, D_t , but in addition to this channel households are not present on either side of the market-clearing condition (4). As a result, changes in MBS duration matter for bond prices, and the MBS feedback channel is not a wash.

For tractability, and motivated by the size of MBS market relative to the Treasury market and the magnitude of the changes in MBS duration, we focus on modeling the MBS channel only.¹² The model allows us to make theoretical predictions regarding the effect of MBS markets on interest rate volatility and bond risk premia across different maturities of the term structure.

1.2 Equilibrium term structure

Before solving for equilibrium yields, we determine the market price of interest rate risk.

Lemma 1. *Given (1)-(4), the unique market price of interest rate risk is proportional to the dollar duration of the total supply of bonds:*

$$\lambda_t = \alpha\sigma \frac{d\left(\int_0^T s_t^\tau \Lambda_t^\tau d\tau\right)}{dr_t}. \quad (8)$$

¹¹Financial intermediaries and institutional investors hold approximately 25% of the total amount outstanding in Treasuries, and daily trading volume is almost 10% of the total amount outstanding. In addition, these financial intermediaries hold around 30% of the total amount outstanding in MBS, and daily trading is almost 25% of the total amount outstanding. Data are for the period 1997 to 2012; see Securities Industry and Financial Markets Association (2013) and Flow of Funds Tables of the Federal Reserve. The Online Appendix contains a summary of holdings and size of the MBS market.

¹²The three largest categories in the fixed income market in terms of total amount outstanding are MBS, Treasuries, and corporate debt with MBS constituting the largest fraction of around 25% (Treasuries and corporate bonds each hold around 20%). The total amount outstanding in MBS is a factor 1.35 larger than in Treasuries for the period 1997 to 2012; see SIFMA (2013). The relative size of the MBS to the Treasuries market is also economically relevant. As discussed above, taking into account the value of outstanding mortgage debt, we calculate that a one monthly standard deviation shock to MBS duration is a dollar duration equivalent of a USD 368bn shock to the supply of 10-year Treasuries.

Lemma 1 follows from the absence of arbitrage and implies that, regardless of the specific maturity composition of the supply of bonds, the market price of interest rate risk is determined by its total quantity. This means that in order to derive the equilibrium term structure, it is not necessary to explicitly model the full dynamics of the supply of bonds, but it is sufficient to capture its duration.

Because we are interested in the changes (rather than levels) of the market price of risk, and we assume only one source of variation in the duration of bond supply, namely the changes in MBS dollar duration, we can write¹³

$$\lambda_t = \alpha \sigma \frac{dMBS_t}{dr_t}. \quad (9)$$

Using a simple chain rule, $\frac{dMBS_t}{dr_t} = \frac{dMBS_t}{dy_t^\tau} \frac{dy_t^\tau}{dr_t}$, we rewrite (9) in terms of the sensitivity to the reference long-maturity rate y_t^τ :

$$\lambda_t = -\alpha \sigma_y^\tau D_t, \quad (10)$$

where $\sigma_y^\tau = \frac{dy_t^\tau}{dr_t} \sigma$, the volatility of y_t^τ , is a constant to be determined in equilibrium.

We now have all the ingredients to solve for the equilibrium term structure.

Theorem 1. *In the term structure model described by (1), (7) and (10), equilibrium yields are affine and given by*

$$y_t^\tau = \mathcal{A}(\tau) + \mathcal{B}(\tau) r_t + \mathcal{C}(\tau) D_t, \quad (11)$$

¹³Another justification for our exclusive focus on MBS duration is the lack of relation between MBS duration and other potential sources of variation in aggregate duration. We find little commonality between MBS duration and convexity on the one hand, and the Government debt supply factor proposed by Greenwood and Vayanos (2013) on the other. Greenwood, Hanson, and Stein (2010) argue that changes in the maturity structure of corporate debt, that could potentially constitute another important source of supply shocks, tend to offset the variations in the supply and maturity of Government debt but are not related to MBS.

where the functional forms of $\mathcal{A}(\tau)$, $\mathcal{B}(\tau)$, and $\mathcal{C}(\tau)$ are given by (A-6)-(A-8), the risk-neutral dynamics of r_t and D_t are given by

$$dr_t = (\kappa\theta - \kappa r_t + \alpha\sigma\sigma_y^{\bar{\tau}}D_t) dt + \sigma dB_t^{\mathbb{Q}}, \quad (12)$$

$$dD_t = (\delta_r\theta - \delta_r r_t - \delta_D D_t) dt + \eta_y\sigma_y^{\bar{\tau}}dB_t^{\mathbb{Q}}, \quad (13)$$

and parameters δ_r , δ_D , and $\sigma_y^{\bar{\tau}}$ solve

$$\delta_r = \frac{\kappa\eta_y\mathcal{B}(\bar{\tau})}{1 - \eta_y\mathcal{C}(\bar{\tau})}, \quad (14)$$

$$\delta_D = \frac{\kappa_D - \alpha\sigma\sigma_y^{\bar{\tau}}\eta_y\mathcal{B}(\bar{\tau})}{1 - \eta_y\mathcal{C}(\bar{\tau})}, \quad (15)$$

$$\sigma_y^{\bar{\tau}} = \frac{\eta_y\sigma\mathcal{B}(\bar{\tau})}{1 - \eta_y\mathcal{C}(\bar{\tau})}. \quad (16)$$

Equations (14)-(16) have a solution whenever α is below a threshold $\bar{\alpha} > 0$.

Theorem 1 motivates the inclusion of mortgage market factors in the term structure analysis. Equation (11) implies that long-term yields depend on two separate factors, the short rate and aggregate dollar duration of mortgages, even though the model has only one shock. This is because duration is a function of the average coupon paid on outstanding mortgages, and therefore depends on the past levels of interest rates, and not only on their current level. In other words, duration is driven both by changes in the level of interest rates and the renewal of the pool of mortgages, and both forces are important for the term structure.¹⁴

1.3 Model implications

Our model has a series of implications regarding the effect of MBS risk on bond risk premia, as well as bond price and yield volatilities. We summarize them in three propositions from which we derive our empirical hypotheses.

¹⁴Formally, when $\kappa_D \neq 0$, interest rates in our model are non-Markovian with respect to the short rate r_t alone. However, their history-dependence can be summarized by an additional Markovian factor, namely the duration D_t .

Proposition 1. *The dollar duration of MBS positively predicts excess bond returns for all maturities and the effect is stronger for longer maturities. Moreover, the predictive power of MBS duration remains the same when we control for the short rate.*

The market price of interest rate risk depends on the quantity of this risk that intermediaries hold to clear the net supply. In turn, bonds with higher exposure to interest rate risk are more affected. As a result, MBS duration should predict excess bond returns and the effect is stronger for longer maturity bonds.¹⁵

More formally, for the slope coefficient $\beta^{\tau,h}$ of the regression of excess returns on bonds with maturity τ over horizon h on the MBS duration factor D_t , we verify that $\lim_{\tau \rightarrow h} \beta^{\tau,h} = 0$ and $d\beta^{\tau,h}/d\tau > 0$ and hence, $\beta^{\tau,h}$ is positive and increasing across maturities.

Finally, the slope coefficient $\beta^{\tau,h}$ remains the same when we control for the level of interest rates by including the short rate or any one long-term yield in the predictive regression. This is the case because, while the level of interest rates and MBS duration are correlated, it is duration that drives the variation in bond risk premia. Moreover, because their unconditional correlation is imperfect, the two factors are not collinear.

Proposition 2. *The volatility of all yields is increasing in the negative dollar convexity of MBS and the effect is strongest for intermediary maturities, i.e. is hump-shaped.*

Higher MBS convexity implies that the quantity of duration risk and therefore the market price of risk is more sensitive to changes in interest rates. Moreover, because MBS convexity is negative, portfolio rebalancing by investors amplifies, rather than offsets, the effect that the initial shock to the short rate has on long-term interest rates. As a result, interest rate volatility is higher.

The link between convexity and volatility has a term structure dimension. Short-maturity yields are close to the short rate and therefore are not significantly affected by

¹⁵Note that the effect of MBS dollar duration on the level of yields is not necessarily monotonic in maturity. A yield depends on the average of risk premia over the life of the bond. Higher risk premia increase yields. However, because of mean reversion in interest rates and duration, we expect risk premia at longer horizons to be lower. We are not testing this implication empirically, because duration itself depends on yields, thus causing an endogeneity problem for identification.

the variations in the market price of risk. For long maturities, we expect the duration of MBS to revert to its long-term mean. At the limit, yields at the infinite horizon should not be affected by current changes in the short rate and MBS duration at all.¹⁶ As a result, the effect of MBS convexity on yield volatilities has a hump-shaped term structure.¹⁷

Finally, even for low levels of intermediary risk aversion α , negative convexity can cause potentially significant volatility in yields and risk premia. This is because in addition to the direct amplification effect, where the fluctuations in the market price of risk amplify the effect of short rate shocks on long rates, negative convexity also creates a feedback loop effect. Through the latter, movements in long rates caused by fluctuations in MBS duration themselves affect MBS duration.

More formally, the effect of MBS convexity on interest rate volatility can be understood in two steps. First, for the instantaneous volatility of the maturity- τ yield, σ_y^τ , and $\gamma \equiv -\frac{d^2 MBS_t}{dr_t^2}$, we verify that $d\sigma_y^\tau/d\gamma > 0$. Moreover, $\lim_{\tau \rightarrow 0} \sigma_y^\tau = \sigma$ and $\lim_{\tau \rightarrow \infty} \sigma_y^\tau = 0$ are both independent of γ . Hence, yield volatility is increasing in γ , and since it does not change for extreme values, the effect is hump-shaped across maturities.

Next, we relate the (negative of) the observed MBS convexity η_y to γ . Around $\alpha = 0$,

$$\gamma \approx \frac{h_0 \eta_y}{1 - \alpha h_1 \eta_y},$$

where both linearization coefficients are positive: $h_0, h_1 > 0$. Compared to the $\alpha = 0$ case where MBS duration has no effect on interest rates, MBS duration sensitivity to short-rate shocks increases by a factor $\frac{1}{1 - \alpha h_1 \eta_y} > 1$. In other words, because of the feedback between long rates and duration, a smaller shock to the short rate is required to produce an equilibrium change in duration of a given magnitude.

Proposition 3. *The effect of negative dollar convexity on the volatility of bond returns is positive and increasing in maturity.*

¹⁶This is just an application of a more general argument by Dybvig, Ingersoll, and Ross (1996) on why the yield volatility curve for long maturities should be downward sloping.

¹⁷When the negative convexity effect is strong enough, our model generates a hump-shaped term structure of unconditional yield volatilities.

Proposition 3 is a simple corollary to Propositions 1 and 2. According to the latter, the volatility of all interest rates is increasing in negative convexity. While this effect is weaker for longer maturities, long maturity bonds also have higher duration risk, and this second effect more than offsets the first. As a result, the volatility of longer-maturity bond returns is more affected by negative convexity.

Since negative convexity, η_y , is constant in the model we posit above, Propositions 2 and 3 are comparative static results. In Appendix B we show that our framework can be extended to a model with stochastic convexity. Moreover, in this less parsimonious version of the model we can derive time-series predictions that are equivalent to Propositions 2 and 3.

2 Data

We use data from several sources. From Bloomberg, we collect data on interest rate swaps. Furthermore, we use Treasury data from the Federal Reserve Board and agency bond index data from Datastream. Data are weekly and span the time period from January 1997 through December 2012 for a total of 835 observations.

2.1 Mortgage data

We use estimates of MBS duration, convexity, and average coupon from Barclays. The Barclays US MBS index covers mortgage-backed pass-through securities guaranteed by Ginnie Mae, Fannie Mae, and Freddie Mac. The index is comprised of pass-throughs backed by conventional fixed rate mortgages and is formed by grouping the universe of over one million agency MBS pools into generic pools based on agency, program (i.e., 30-year, 15-year, etc.), coupon (e.g., 6.0%, 6.5%, etc.), and vintage year (e.g., 2011, 2012, etc.). A generic pool is included in the index if it has a weighted-average contractual maturity greater than one-year and more than USD 250m outstanding.

The middle panel of Figure 1 depicts MBS duration. The average duration in our sample is around 4.5 years which is mainly due to the prepayment option in fixed rate

mortgages which reduces the duration of MBS. The lower panel plots MBS convexity which is negative throughout the whole sample and the average convexity is around -1.5 . We note that virtually all of the variation in dollar duration and dollar convexity of the index is driven by, respectively, duration and convexity themselves and not by the changes in the price. Therefore, for simplicity we use duration and convexity as our factors.

2.2 *Interest rates and excess returns*

We use the Gürkaynak, Sack, and Wright (2007, GSW henceforth), zero coupon yield data available from the Federal Reserve Board. Unlike the Fama and Bliss discount bond database from CRSP, the GSW data is available at the weekly frequency. We use the raw data to calculate annual Treasury bond excess returns for 2- to 10-year bonds. We also download interest rate swap data from Bloomberg from which we bootstrap a zero coupon yield curve.

We denote the return on a τ -year bond with price Λ_t^τ by $r_{t+1}^\tau = \log \Lambda_{t+1}^{\tau-1} - \log \Lambda_t^\tau$. The annual excess bond return is then defined as $rx_{t+1}^\tau \equiv r_{t+1}^\tau - y_t^1$, where y_t^1 is the one-year yield. As we have weekly data, the annual excess returns are overlapping by 51 weeks. From the same data, we also construct a tent-shaped factor from forward rates, the Cochrane and Piazzesi (2005) factor (CP factor, labeled cp_t).

We calculate the slope of the term structure as the difference between the 10-year and the one-year zero coupon yield (labeled $slope_t$).

2.3 *Bond volatility*

Using the GSW yields ranging from six months to ten years, we estimate a time-varying term structure of yield volatility. We sample the data at the weekly frequency and take log yield changes. We then construct rolling window measures of realized volatility which represent the conditional bond yield volatility. The resulting term structure of unconditional volatility exhibits a hump shape consistent with the stylized facts reported in Dai and Singleton (2010), with the volatility peak being at the two-year maturity.

Mueller, Vedolin, and Yen (2012) calculate measures of model-free implied and realized bond market volatilities for the one-month horizon using Treasury futures and options data from the Chicago Mercantile Exchange (CME). We use their data for the 30-year Treasury bond and henceforth label the realized volatility trv_t and the implied volatility tiv_t . Moreover, the difference between the expected variance under the risk-neutral (tiv^2) and physical probability measure (trv^2) is defined as the ex ante variance risk premium ($\text{vrp}_t \equiv \text{tiv}_t^2 - \text{trv}_t^2$). We also construct a measure of volatility of variance for both the realized and implied proxies. Rolling volatility is calculated using a 52 week window and we label the two time-series tivov_t and trvov_t , for implied and realized volatility of variance, respectively.

From Bloomberg, we also get implied volatility for at-the-money swaptions for different maturities ranging from one to ten years and we fix the tenor to ten years. We label these swaption implied volatilities by $\text{iv}^{\tau y 10y}$, where $\tau = 1, \dots, 10$.

Another way to capture bond price volatility risk is through returns on at-the-money straddles, which are portfolios mainly exposed to volatility risk (see Collin-Dufresne and Goldstein (2002)). We construct monthly straddle returns using at-the-money options written on the 30-year Treasury bond futures. We label this time-series straddle_t .

2.4 Other variables

Motivated by Hu, Pan, and Wang (2013) who report a link between swaption implied volatility and measures of liquidity in bond markets, we use their proxy of noise illiquidity, which measures an average yield pricing error from the Nelson, Siegel and Svensson model (see Svensson (2004)). To this end, we construct a daily measure of noise illiquidity from bond data available from Datastream. We also construct eight principal components from the cross-section of 132 macro factors (see Ludvigson and Ng (2009)).

3 Empirical analysis

In this section we test the predictions of the model in the data, and find that results are in line with the theory. To the best of our knowledge the literature has not proposed

an alternative explanation for the predictive power of MBS duration and convexity. Nevertheless, to address a potential omitted variable problem, we also control for other well-known determinants of bond risk premia and interest rate volatility but not explicitly accounted for in our model. We note that not only MBS duration and convexity remain statistically significant, but also the economic size of the coefficients stays stable across different specifications.

For simplicity, we use duration and convexity instead of dollar duration and dollar convexity in our benchmark results given that they drive virtually all the variation in the dollar time series. However, the results remain unchanged if we use dollar duration and dollar convexity. All regressions are standardized, meaning that we first de-mean and then divide each variable by its standard deviation to make slope coefficients comparable across different regressors. With each estimated coefficient, we report t-statistics adjusted for Newey and West (1987) standard errors. The lag length is determined using the Stock and Watson (2007) rule.

3.1 Bond risk premia

Hypothesis 1. *A regression of bond excess returns on the duration of MBS yields a positive slope coefficient for all maturities. Moreover the coefficients are increasing in bond maturity and remain significant when we control for the level of interest rates.*

This hypothesis is equivalent to Proposition 1. To test Hypothesis 1, we run linear regressions of annual excess returns on the duration factor. The regression is as follows:

$$rx_{t+1}^{\tau} = \beta_1^{\tau} \text{duration}_t + \beta_2^{\tau} \text{level}_t + \epsilon_{t+1}^{\tau},$$

where level_t is the first principal component of the yield curve at time t . The univariate results are depicted in the upper two panels of Figure 2, which plot the estimated slope coefficients of duration, i.e. $\hat{\beta}_1$ (upper left panel), and the associated adjusted R^2 (upper right panel). Both univariate and multivariate results are presented in Table 1.

[Insert Figure 2 and Table 1 here.]

The univariate regression results indicate that MBS duration is a significant predictor of bond excess returns at all maturities. The coefficient has the expected positive sign and is increasing with maturity. The estimated coefficients are also economically significant, especially for longer maturities. For example, for any one standard deviation increase in duration, there is a $0.381 \times 7.19\% = 273$ basis point increase in expected 10-year bond excess returns. Adjusted R^2 s range from 2% for the shortest maturity to 14% for the longest maturity.

Because duration is in part determined by the current level of interest rates, we include it as a control in our multivariate test. In our model duration and interest rate level are correlated but not collinear, and it is duration that is the relevant predictor of future excess returns. We find that the correlation between MBS duration and the first principal component is equal to 0.55. We note that the significance and the economic size of the coefficient for the duration factor in the multivariate regression remain similar to the univariate case.

3.2 Bond yield volatility

Proposition 2 states that bond yield volatility is increasing in the negative convexity of MBS. In addition, the effect should be strongest at intermediate maturities, meaning that we should observe a hump-shaped pattern. We therefore test the following hypothesis in the data.

Hypothesis 2. *A regression of conditional yield volatility on the negative convexity of MBS results in a positive slope coefficient for all maturities. Moreover, the coefficients are the largest for intermediate maturities.*

We run the following two univariate regressions from conditional bond yield volatility and swaption implied volatility onto convexity:

$$\text{vol}_t^\tau / \text{iv}_t^{\tau y^{10y}} = \beta_1^\tau \text{convexity}_t + \epsilon_t^\tau,$$

where vol_t^τ is the conditional bond yield volatility at time t of a bond with maturity $\tau = 1, \dots, 10$ years and $\text{iv}_t^{\tau y^{10y}}$ is the τ -year maturity implied volatility from swaptions

on the 10-year swap rate. The reason to include swaption implied volatility is that hedging activity could potentially affect both. For example, Wooldridge (2001) notes that non-government securities were routinely hedged in the Treasury market until the financial crisis of 1998 when investors started hedging their interest rate exposure in the swaptions market. This point is also made in Perli and Sack (2003), Duarte (2008), or Feldhütter and Lando (2008).

The univariate results are presented in the lower two panels of Figure 2 and Table 2. In line with our model predictions, we find a significant effect from convexity onto bond yield volatility and the effect is most pronounced for intermediate maturities. The estimated slope coefficients produce the hump shaped feature similar to the one observed in the unconditional averages of yield volatility. Adjusted R^2 s range from 6% for the shortest maturities, increase to 7% for the two and three year maturities and decrease again to 3% for longer maturities. Estimated coefficients are not only statistically but also economically significant: For the two-year maturity, any one standard deviation change in MBS convexity is associated with a 120 basis point increase in bond yield volatility.

[Insert Table 2 here.]

The same picture emerges for implied volatilities from swaptions: Higher convexity induces higher volatility on swaptions. Interestingly, we also find a hump-shaped pattern similar to the one observed in the bond yield volatility regressions.

An obvious concern with our regression results is that negative convexity could itself depend on volatility. Note, that it is a priori unclear in which direction volatility affects convexity as this depends on whether a particular mortgage is in-, out-, or at-the-money.¹⁸ For an at-the-money MBS, an increase in volatility will lead to an increase in negative convexity. Discount (i.e. low negative to positive convexity) and premium (negative convexity) mortgages will in general have a much lower sensitivity to changes in volatility, and the effect could go in the opposite direction.¹⁹

¹⁸This is analogous to the Zomma (sensitivity of an option's Gamma with respect to changes in the implied volatility) for equity options.

¹⁹We thank Bruce Phelps at Barclays Capital for insightful discussions on this.

We run the Granger causality tests between MBS convexity and volatility and present the results in Figure 3. In the left panel, we plot p-values from F-tests that assess the null hypothesis whether negative convexity does not Granger cause volatility. On the right panel, we plot the p-values of the reversed Granger regression, i.e. we test the null hypothesis whether volatility does not Granger cause negative convexity. We note that for standard confidence levels, we can reject the null of no Granger causality up to a maturity of seven years. On the other hand, volatility does not seem to Granger cause convexity, as we cannot reject the null hypothesis up to a maturity of five years.

[Insert Figure 3 here.]

As an additional robustness check we use the lagged values of convexity as instruments in an IV estimation. Running two-stage-least-square regressions, we find that the standard errors are in fact smaller if we lag the instrument further. In line with the empirical and theoretical findings in Stock, Wright, and Yogo (2002), we conclude that IV estimation performs worse than OLS and therefore report OLS results only (see also Krishnamurthy and Vissing-Jorgensen (2012)).

3.3 Other volatility regressions

As outlined in Proposition 3, our model implies that the volatility of bond returns is positively related to MBS convexity. We formulate a testable empirical prediction in the following hypothesis:

Hypothesis 3. *A regression of conditional volatility of bond returns, both implied and realized, on the negative convexity of MBS results in a positive slope coefficient for all maturities. Moreover, the associated volatility of volatility and variance risk premia are also increasing in MBS negative convexity.*

To test the above hypothesis, we run regressions of the following type:

$$\text{tiv}_t/\text{trv}_t/\text{vrp}_t/\text{straddle}_t/\text{tivov}_t/\text{trvov}_t = \beta_1 \text{convexity}_t + \epsilon_t,$$

where tiv_t (trv_t) is the implied (realized) volatility of the 30-year Treasury bond at time t , vrp_t is the associated variance risk premium defined as the difference between the implied and realized measure, $tivov_t$ ($trvov_t$) is the volatility of implied (realized) variance. The volatility of variance regressions are motivated by the results in Perli and Sack (2003) who find that mortgage hedging has not only a direct effect on the level of volatility but also on the changes in expected volatility, which is larger whenever hedging is intense. The results of these regressions are found in Table 2.

The data confirm our model predictions. The estimated coefficients are significant and also carry the expected sign: Higher MBS convexity implies a larger implied and realized volatility, a larger variance risk premium, and a larger volatility of variance. In economic terms, the regression results imply that for any one standard deviation increase in MBS convexity there is a 60 (40) basis point increase in implied (realized) volatility. MBS convexity also helps to explain returns of straddle strategies. The estimated coefficient for MBS convexity is significant, albeit only at the 10% confidence level.

3.4 *What do MBS duration and convexity capture beyond information in yields?*

Households refinance mortgages when interest rates drop and it is therefore tempting to assume that MBS duration is a mere reflection of information already contained in the yield curve. Our stylized model motivates us to take a closer look at this question. In the model, MBS duration drives bond risk premia and is spanned by the cross-section of yields. However, because duration depends on both current and past interest rates, it is not uniquely determined by the short rate factor that accounts for most of the variation in the shape of the yield curve.

Most models in fixed income decompose movements in yields into principal components and argue that the first three factors explain most of the variation in yields. In order to better understand the relationship between MBS duration and the shape of the yield curve we look at unconditional correlations between MBS duration and the first three principal components (PCs) given in Table 3 (Panel A). In addition to the correlation between MBS duration and the first PC equal to 0.55 discussed above, duration

is almost uncorrelated with the second and third PCs. To test more formally whether duration is spanned by these yield factors we run the following regression similar to Joslin, Pribsch, and Singleton (2012):

$$\text{duration}_t = \alpha + \beta_1 \text{level}_t + \beta_2 \text{slope}_t + \beta_3 \text{curvature}_t + \epsilon_t,$$

where level_t , slope_t and curvature_t are the first three PCs from the cross-section of yields. For our sample period, the projection of duration onto the first three PCs of yields gives us an adjusted R^2 of 25%. This means that 75% of the variation in MBS duration arises from risks which are distinct from these PCs. Moreover, a regression from the duration time series on bond yields should produce fitted residuals which are close to uncorrelated. In our regression, we find that the AR(1) coefficient of the residuals is 0.94 and the associated Durbin and Watson statistic is 0.1, which clearly rejects the null of zero autocorrelation.

This now begs the question of whether duration contains any information beyond these principal components to predict bond returns. We tackle this question by regressing bond excess returns on MBS duration and the first three PCs. Table 3 reports the results. The economic and statistical significance of the duration factor remains very close to the results reported in Table 1. We note that adding higher order PCs to the set of regressors (unreported results) starts decreasing the predictive power of MBS duration. This suggests that factors that do not account for a significant proportion of the variation in the shape of the yield curve could span the duration factor. We leave out the spanning of MBS duration by higher order yield PCs as a question for future research.

[Insert Table 3 here.]

It is also natural to investigate how MBS convexity is related to level, slope, and curvature of yields. Running a regression from MBS convexity onto the first three yield PCs, we find that the regressors explain around 40% of the variation, while the AR(1) coefficient of residuals is 0.85. We therefore conclude that—just as duration—convexity is not spanned by the first three yield factors. Since in our model, convexity

is mainly related to bond yield volatility, we repeat the same exercise using the PCs calculated from the cross-section of bond yield volatilities. Similar to the cross-section of yields, three factors are essentially enough to fully capture the dynamics of bond yield volatilities: The first three factors explain 91.8%, 6.5% and 1.15% of the overall variation, respectively. Running a regression from convexity onto the first three volatility PCs, we find that they explain little of the variation in convexity as the adjusted R^2 is only about 10%. Hence, we conclude that MBS convexity is also not spanned by yield volatilities.

3.5 MBS duration and other predictors of bond returns

The evidence on bond return predictability by MBS duration is consistent with our theoretical predictions. However, it could also be the case that duration simply captures predictability coming from other sources that are not explicitly accounted for in our model. To the best of our knowledge we are not aware of work that establishes, either theoretically or empirically, the correlation between MBS duration and some alternative predictor of bond returns. However, we control for this possibility by including the slope of the term structure and the CP factor, two well-known determinants of bond risk premia (see Cochrane and Piazzesi (2005)), in the predictability regression that now becomes:

$$rx_{t+1}^T = \beta_1^T \text{duration}_t + \beta_2^T \text{slope}_t + \beta_3^T \text{cp}_t + \epsilon_{t+1},$$

Results are reported in Table 4.

We find that including these additional regressors does not deteriorate the significance of duration, moreover, estimated slope coefficients on MBS duration remain remarkably stable. When we add the slope of the term structure and the CP factor to the regressions, the estimated coefficients on duration remain highly significant for maturities of five years and beyond. All three regressors combined explain between 21% and 46% of the time variation of annual bond excess returns.

[Insert Table 4 here.]

We conclude that there is a strong link between bond risk premia and MBS duration. The effect is more pronounced for longer maturity bonds and remains significant when we add other predictors to the regressions.

3.6 MBS duration and macro factors

Similarly, it could be the case that MBS duration depends on macroeconomic conditions and its predictive power is due to correlations with macro factors. Ludvigson and Ng (2009) exploit information in 132 different realized macroeconomic and financial series and explore the predictive content of these series for bond risk premia. The authors find that the main principal components extracted from this panel are statistically significant even in the presence of the CP factor and substantially improve predictability. To test whether our MBS duration factor is robust to the inclusion of these macroeconomic factors, we compute the eight static macroeconomic factors, F^j , $j = 1, \dots, 8$, for an updated data set through December 2012. In the following, we report regressions from bond risk premia onto duration and the macroeconomic factors at the monthly frequency. Table 4 presents the results.

While the effect of duration at the shortest maturities is negligible, at longer maturities, the impact remains almost unaltered compared to the results for weekly returns reported in Table 1. For longer maturities, macroeconomic factors become less significant, unlike duration. More importantly, the significance of the duration factor is virtually unchanged when moving from the univariate regression results to the regressions which include the macro factors. We summarize our findings as follows: MBS duration is a strong predictor of bond risk premia at longer horizons and its predictive power is not subsumed by macroeconomic factors.

3.7 MBS convexity and other determinants of yield and bond return volatility

In this section we control for additional determinants of yield and bond return volatility that have been documented in the literature. It is well-known that volatility tends to increase in periods of high illiquidity (see e.g., Hu, Pan, and Wang (2013)). In

our multivariate specification, we therefore add a proxy for illiquidity and a proxy of fixed-income implied volatility, similar to the VIX in equity markets.²⁰ We run the following regression from conditional bond yield volatility onto convexity and a set of other predictors:

$$\text{vol}_t^\tau = \beta_1^\tau \text{convexity}_t + \beta_2^\tau \text{illiq}_t + \beta_3^\tau \text{tiv}_t + \epsilon_t,$$

where vol_t^τ is the conditional bond yield volatility at time t of a bond with maturity $\tau = 1, \dots, 10$ years, illiq_t is the illiquidity factor at time t , and tiv_t is the Treasury-implied volatility at time t . Results are reported in Table 5. We find that when we add illiquidity into the regression, convexity still remains highly statistically significant. The estimated coefficients reveal that the effect is the largest for the intermediate maturities of two-three years as indicated by the size of the coefficient. Illiquidity has the expected positive sign as bond volatility tends to be high when markets are illiquid. However, the effect becomes insignificant as we add lagged values of yield volatility into the regression. All three factors together explain between 50% and 63% of the time variation in bond yield volatility across different maturities. The same picture emerges for implied volatilities from swaptions: Estimated slope coefficients on convexity are robust to the inclusion of other regressors.

[Insert Tables 5]

We repeat the exercise for the volatility of bond returns for which we run regressions of the following type:

$$\text{tiv}_t/\text{trv}_t/\text{vrp}_t/\text{straddle}_t/\text{tivov}_t/\text{trvov}_t/\text{iv}^{\tau y 10y} = \beta_1 \text{convexity}_t + \beta_2 \text{illiq}_t + \beta_3 \text{lagged}_{t-1} + \epsilon_t,$$

where tiv_t (trv_t) is the implied (realized) volatility of the 30-year Treasury bond at time t , vrp_t is the associated variance risk premium defined as the difference between the implied and realized measure, tivov_t (trvov_t) is the volatility of implied (realized)

²⁰Hu, Pan, and Wang (2013) document a strong link between their illiquidity proxy and a fixed-income implied volatility index, the Bank of America/Merrill Lynch MOVE index. Note that the MOVE is calculated from rather illiquid over-the-counter Treasury options while tiv_t is calculated using extremely liquid Treasury future options.

variance, $iv^{\tau y^{10y}}$ is the τ -year maturity implied volatility from swaptions on the 10-year swap rate, $illiq_t$ is the illiquidity measure and $lagged_{t-1}$ are the LHS variables lagged for one period that are included given the high persistence. The results of these regressions are found in Tables 5.

When we add additional regressors, $\hat{\beta}_1$ remains significant. Illiquidity is significant and carries the expected positive sign: Higher illiquidity implies higher volatility, a higher associated variance risk premium, and higher returns on a straddle strategy.

Overall, we conclude that both MBS duration and convexity are significantly linked to first and second moments of bond yields and bond returns. The empirical results are in line with the theoretical predictions and also hold when adding other control variables to the regressions.

3.8 *The impact of MBS duration and convexity over time*

Figure 4 plots the ratio of outstanding mortgages and GDP from 1990 to 2012 together with the amount outstanding in mortgages and Treasuries. As one can see, the importance of the mortgage market vis-à-vis both GDP and Treasuries has increased over the past 20 years, although both ratios peak in 2010 and since then have somewhat declined. If mortgage markets have become more important over time, one would expect the effect of MBS duration and convexity to increase over time as well. To control for the variation in mortgage volume, we run similar regressions as before but interact the duration and convexity measures with the mortgage to GDP ratio. The results are reported in Table 6.

[Insert Figure 4 and Table 6 here.]

The results indicate that MBS duration and convexity are still highly significant predictors of bond excess returns and bond yield volatility. For the bond yield volatility regressions, we find that the interaction term is able to explain 24% of the time variation in long term bond yield volatility. Overall, we conclude that MBS duration and convexity are robust predictors for levels and second moments of bond yields.

3.9 Robustness

Interest rate swaps: Interest rate risk is primarily hedged in either the Treasury or interest rate swap market and the main focus in the previous section has been on Treasury data. The reason for this is twofold. First, interest rate swap data contain a considerable credit risk component (see Feldhütter and Lando (2008)) which is outside the scope of our model to explain. Second, after the Lehman default in 2008, prices of interest rate swaps (especially at longer maturities) got possibly distorted due to a decline in arbitrage capital (see Krishnamurthy (2010)). In particular, our data sample also covers the time period where the swap spread, defined as the difference between the fixed rate on a fixed-for-floating 10-year swap and 10-year Treasury rate, turned negative. This is another feature in the data that goes beyond what the model is designed to capture. For robustness reasons, we also run bond risk premia regressions using swap rather than Treasury data and we report estimated coefficients in Table 7.²¹ We note that the size and significance of the estimated coefficients are almost identical to those reported for Treasuries. Adding explanatory factors such as the slope of the term structure or the CP factor does not deteriorate the significance of MBS duration.

[Insert Table 7 here.]

Bond portfolios: One issue with using annual bond excess returns is the short sample period. In our data sample of 16 years, we have a maximum of 16 independent observations. To address this issue, we use actual bond returns for different maturity bins available from CRSP. Data is monthly and represents an equally-weighted average of holding period returns for each bond in the portfolio. We calculate excess returns by subtracting the T-bill rate. Because of the large impact of monetary policy on T-bills in the past couple of years, we also use the one-month Eurodollar deposit rate as an alternative as suggested by Duffee (1996). Results are reported in Table 8.

[Insert Table 8 here.]

²¹We bootstrap a zero coupon curve from swap rates and calculate excess returns that are directly comparable to the Treasury excess returns we use in the benchmark results.

We note that while the adjusted R^2 s are almost halved compared to the previous results, the estimated coefficients for duration are still highly significant, even in the multivariate regressions. Moreover, these results hold whether we use the T-bill or the Eurodollar rate to construct the excess returns.

4 Conclusion

This paper studies the feedback from MBS risk on bond risk premia and interest rate volatility. We build an equilibrium model of bond supply shocks driven by changes in MBS duration and embed it into an otherwise standard one factor term structure model. Despite its simple structure, our model has interesting implications for first and second moments of interest rates: Our model is able to replicate the predictive power of MBS duration for bond excess returns and can accommodate a hump shaped term structure of bond yield and swaption implied volatilities.

We then empirically test our model predictions and find our hypotheses confirmed. For example, we find a strong positive link between MBS duration and bond excess returns. The relationship is not only statistically significant but also economically relevant. This relationship remains highly significant and stable when we add other standard regressors. We then proceed to study the relationship between MBS hedging and bond yield volatilities and bond variance risk premia. In line, with our theoretical predictions, we find that MBS convexity significantly affects bond yield volatilities and it produces the predicted hump shaped feature. A higher MBS convexity not only significantly increases yield volatility but also measures of bond return volatility, and bond variance risk premia.

While MBS duration and convexity are naturally related to information in the term structure of bond yields, we provide novel evidence that duration and convexity are not spanned by the usual bond yield factors (principal components), as well as theoretical motivation why this could be the case. An investigation of supply factors within a multi-factor reduced form term structure model and their relation to higher order yield factors is an exciting avenue which we leave to future research.

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Appendix A Proofs and derivations

Proof of Lemma 1. For notational simplicity let us suppose that bond prices are in the form

$$\frac{d\Lambda_t^\tau}{\Lambda_t^\tau} = \mu_t^\tau dt - \sigma_t^\tau dB_t. \quad (\text{A-1})$$

Substituting (A-1) into intermediaries' budget constraint, (2), we get

$$dW_t = \left[r_t W_t + \int_0^T x_t^\tau \Lambda_t^\tau (\mu_t^\tau - r_t) d\tau \right] dt - \left[\int_0^T x_t^\tau \Lambda_t^\tau \sigma_t^\tau d\tau \right] dB_t,$$

therefore (3) simplifies to

$$\max_{\{x_t^\tau\}_{\tau \in (0, T)}} \int_0^T x_t^\tau \Lambda_t^\tau (\mu_t^\tau - r_t) d\tau - \frac{\alpha}{2} \left[\int_0^T x_t^\tau \Lambda_t^\tau \sigma_t^\tau d\tau \right]^2. \quad (\text{A-2})$$

Because markets are complete, by no-arbitrage, there exists a unique market price of interest rate risk across all bonds that satisfies

$$\lambda_t = \frac{E_t \left(\frac{d\Lambda_t^\tau}{\Lambda_t^\tau} \right) / dt - r_t}{\frac{1}{\Lambda_t^\tau} \frac{d\Lambda_t^\tau}{dr_t} \sigma} = \frac{\mu_t^\tau - r_t}{-\sigma_t^\tau}, \quad (\text{A-3})$$

and introducing

$$x_t = \frac{d \left(\int_0^T x_t^\tau \Lambda_t^\tau d\tau \right)}{dr_t} = \int_0^T x_t^\tau \frac{d\Lambda_t^\tau}{dr_t} d\tau = -\frac{1}{\sigma} \int_0^T x_t^\tau \Lambda_t^\tau \sigma_t^\tau d\tau, \quad (\text{A-4})$$

for the total exposure of interest rate risk borne by intermediaries, their maximization problem (A-2) reduces to

$$\max_{x_t} \lambda_t x_t - \frac{\alpha \sigma}{2} x_t^2. \quad (\text{A-5})$$

The first order condition of (A-5) together with the market clearing condition (4) determine the equilibrium market price of risk and provides (8). \square

Proof of Theorem 1. First, note that (12) and (13) define a standard affine term structure model where bond yields are in the form (11), and coefficients $\mathcal{A}(\tau)$, $\mathcal{B}(\tau)$, and $\mathcal{C}(\tau)$ are characterized by the following system of ODEs:

$$\begin{aligned} 1 &= \tau \mathcal{B}'(\tau) + \mathcal{B}(\tau) + \kappa \tau \mathcal{B}(\tau) + \delta_r \tau \mathcal{C}(\tau), \\ 0 &= \tau \mathcal{C}'(\tau) + \mathcal{C}(\tau) + \delta_D \tau \mathcal{C}(\tau) - \alpha \sigma \sigma_y^\tau \tau \mathcal{B}(\tau), \\ 0 &= \tau \mathcal{A}'(\tau) + \mathcal{A}(\tau) - \kappa \theta \tau \mathcal{B}(\tau) + \delta_0 \tau \mathcal{C}(\tau) \\ &\quad + \frac{1}{2} \sigma^2 \tau^2 \mathcal{B}^2(\tau) + \frac{1}{2} (\sigma_y^\tau)^2 \tau^2 \mathcal{C}^2(\tau) + \sigma \sigma_y^\tau \tau^2 \mathcal{B}(\tau) \mathcal{C}(\tau). \end{aligned} \quad (\text{A-6})$$

with terminal conditions $\mathcal{A}(0) = \mathcal{B}(0) = \mathcal{C}(0) = 0$. In particular we have:

$$\tau \mathcal{B}(\tau) = \frac{1}{m_1} \left(\frac{-\delta_r w}{m_2 - m_1} (e^{-m_1 \tau} - e^{-m_2 \tau}) + \frac{\delta_D}{m_2} (1 - e^{-m_2 \tau}) \right), \quad (\text{A-7})$$

and

$$\tau\mathcal{C}(\tau) = w \frac{m_2 - \delta_D}{m_1} \left(\frac{1}{m_2 - m_1} (e^{-m_1\tau} - e^{-m_2\tau}) - \frac{1}{m_2} (1 - e^{-m_2\tau}) \right), \quad (\text{A-8})$$

where

$$m_1 = \delta_D - \delta_r w, \quad m_2 = \kappa + \delta_r w, \quad \text{and} \quad w = \frac{\delta_D - \kappa - \sqrt{(\delta_D - \kappa)^2 - 4\alpha\sigma\sigma_y^{\bar{\tau}}\delta_r}}{2\delta_r}.$$

Next, from (11) and (13) combined with (7) the parameters δ_r , δ_D , and $\sigma_y^{\bar{\tau}}$ solve (14)-(16).

The properties of the model can be shown parsimoniously for a special case when $E_t(dy_t^{\bar{\tau}}) \approx 0$, which can be seen as either the case where the effect of refinancing activity on duration dominates that of the mean reversion in interest rates, or simply as an approximation of the dynamics of D_t , where its drift depends only on duration itself. The proof in the general case follows the same steps but with two, rather than one state variables driving the mean-reversion of D_t . In the special case we have

$$\mathcal{B}(\tau) = \frac{1 - e^{-\kappa\tau}}{\kappa\tau}, \quad (\text{A-9})$$

$$\mathcal{C}(\tau) = -\frac{\alpha\sigma\sigma_y^{\bar{\tau}}}{\kappa - \kappa_D^{\mathbb{Q}}} \left(\frac{1 - e^{-\kappa\tau}}{\kappa\tau} - \frac{1 - e^{-\kappa_D^{\mathbb{Q}}\tau}}{\kappa_D^{\mathbb{Q}}\tau} \right), \quad (\text{A-10})$$

while $\delta_r = 0$, $\delta_D \equiv \kappa_D^{\mathbb{Q}} = \kappa_D - \alpha\eta_y (\sigma_y^{\bar{\tau}})^2$, and $\sigma_y^{\bar{\tau}}$ solves

$$\frac{\sigma_y^{\bar{\tau}}}{\sigma} = \frac{1 - e^{-\kappa\bar{\tau}}}{\kappa\bar{\tau}} - \frac{\kappa_D - \kappa_D^{\mathbb{Q}}}{\kappa - \kappa_D^{\mathbb{Q}}} \left(\frac{1 - e^{-\kappa\bar{\tau}}}{\kappa\bar{\tau}} - \frac{1 - e^{-\kappa_D^{\mathbb{Q}}\bar{\tau}}}{\kappa_D^{\mathbb{Q}}\bar{\tau}} \right). \quad (\text{A-11})$$

To complete the proof of the Theorem, we show that there exists $\bar{\alpha} > 0$ such that (A-11) has a meaningful solution for all $0 \leq \alpha < \bar{\alpha}$. First, we show that all $\sigma_y^{\bar{\tau}}$ solutions of (A-11) are non-negative. It is straightforward from (A-9) that $\mathcal{B}(\tau)$ is positive and decreasing. Also, as the function $x \mapsto \frac{1 - e^{-x}}{x}$ is decreasing, from (A-10) it must be that $\mathcal{C}(\tau)$ and $\sigma_y^{\bar{\tau}}$ have the same sign. However, as $\eta_y > 0$, the right-hand side of (A-11) is then positive, implying both $\sigma_y^{\bar{\tau}} \geq 0$ and $\mathcal{C}(\tau) \geq 0$.

Second, following Greenwood and Vayanos (2013), a sufficient condition for the existence of a solution to (A-11) is that the left-hand side is greater than the right-hand side when $\kappa_D^{\mathbb{Q}} = 0$ - this means that under the risk-neutral measure the model does not explode. After some algebra, this is equivalent to

$$\alpha < \frac{\kappa_D}{\eta_y \left(\frac{\kappa - \kappa_D}{\kappa} \frac{1 - e^{-\kappa\bar{\tau}}}{\kappa\bar{\tau}} + \frac{\kappa_D}{\kappa} \right)^2 \sigma^2}. \quad (\text{A-12})$$

Defining $\bar{\alpha}$ as the right-hand side of (A-12), which is certainly positive, (A-11) has at least one solution whenever $\alpha < \bar{\alpha}$. \square

Proof of Proposition 1. The theoretical slope coefficient of the regression of excess returns over horizon h on maturity- τ bonds on the duration factor D_t is equal to

$$\begin{aligned}\beta^{\tau,h} &= \tau\mathcal{C}(\tau) - h\mathcal{C}(h) - (\tau - h)\mathcal{C}(\tau - h)e^{-\kappa_D h}, \\ &= -\frac{\alpha\sigma\sigma_y^{\bar{\tau}}e^{-\kappa_D h}}{\kappa - \kappa_D^{\mathbb{Q}}} \left[\frac{1 - e^{-\kappa_D^{\mathbb{Q}}(\tau-h)}}{\kappa_D^{\mathbb{Q}}} \left(1 - e^{-(\kappa_D^{\mathbb{Q}} - \kappa_D)h}\right) - \frac{1 - e^{-\kappa(\tau-h)}}{\kappa} \left(1 - e^{-(\kappa - \kappa_D)h}\right) \right].\end{aligned}$$

Regarding the sign of $\beta^{\tau,h}$ and its behavior across maturities, first we have $\lim_{\tau \rightarrow h} \beta^{\tau,h} = 0$. Moreover,

$$\frac{d\beta^{\tau,h}}{d\tau} = -\frac{\alpha\sigma\sigma_y^{\bar{\tau}}e^{-\kappa_D h}}{\kappa - \kappa_D^{\mathbb{Q}}} \left[e^{-\kappa_D^{\mathbb{Q}}(\tau-h)} \left(1 - e^{-(\kappa_D^{\mathbb{Q}} - \kappa_D)h}\right) - e^{-\kappa(\tau-h)} \left(1 - e^{-(\kappa - \kappa_D)h}\right) \right].$$

We focus on the case where refinancing is the dominant force $\kappa_D > \kappa$. If also $\kappa_D^{\mathbb{Q}} > \kappa$, the term inside the bracket is positive, and $d\beta^{\tau,h}/d\tau > 0$. If, on the other hand, $\kappa_D^{\mathbb{Q}} < \kappa < \kappa_D$, the term in the bracket is negative and $d\beta^{\tau,h}/d\tau > 0$ again. Therefore, it is sufficient that $\kappa < \kappa_D$ to have $d\beta^{\tau,h}/d\tau > 0$, and hence $\beta^{\tau,h}$ being positive and increasing in maturities.

Note, that the unconditional covariance matrix of $[r_t \quad D_t]^\top$ is given by

$$\begin{pmatrix} \frac{\sigma^2}{2\kappa} & \frac{\sigma\sigma_D}{\kappa + \kappa_D} \\ \frac{\sigma\sigma_D}{\kappa + \kappa_D} & \frac{\sigma_D^2}{2\kappa_D} \end{pmatrix},$$

implying that in general the two factors are not collinear.

The effect of duration on yields, from (11), is given by $\mathcal{C}(\tau)$. From the Proof of Theorem 1, $\mathcal{C}(\tau) \geq 0$. Moreover, it is easy to show that

$$\lim_{\tau \rightarrow 0} \mathcal{C}(\tau) = \lim_{\tau \rightarrow \infty} \mathcal{C}(\tau) = 0,$$

which implies that the effect is either increasing across maturities if τ is small, or first increasing then decreasing if τ is sufficiently large. This completes the proof. \square

Proof of Proposition 2. From (11), bond yield volatility is given by

$$\sigma_y^\tau = \mathcal{B}(\tau)\sigma + \mathcal{C}(\tau)\eta_y\sigma_y^{\bar{\tau}}. \quad (\text{A-13})$$

We determine the effect of convexity $-\eta_y$ on yield volatilities in two steps.

First, we focus on $\frac{d\sigma_y^\tau}{d\gamma}$, where $\gamma \equiv -\frac{d^2 MBS_t}{dr_t^2}$. Straightforward from its definition, we have

$$\gamma\sigma^2 = \eta_y(\sigma_y^{\bar{\tau}})^2, \quad (\text{A-14})$$

and hence $\kappa_D^{\mathbb{Q}} = \kappa_D - \alpha\sigma^2\gamma$. Using (A-10) and (A-14) we rewrite $\mathcal{C}(\tau)\eta_y\sigma_y^{\bar{\tau}}$ as

$$\mathcal{C}(\tau)\eta_y\sigma_y^{\bar{\tau}} = -\frac{\alpha\sigma^3\gamma}{\kappa - \kappa_D^{\mathbb{Q}}} \left(\frac{1 - e^{-\kappa\tau}}{\kappa\tau} - \frac{1 - e^{-\kappa_D^{\mathbb{Q}}\tau}}{\kappa_D^{\mathbb{Q}}\tau} \right),$$

and since $\mathcal{B}(\tau)$ is only a function of κ , we obtain

$$\begin{aligned} \frac{d\sigma_y^\tau}{d\gamma} = & -\frac{\alpha\sigma^3}{\left(\kappa - \kappa_D^{\mathbb{Q}}\right)} \left(\frac{1 - e^{-\kappa\tau}}{\kappa\tau} - \frac{1 - e^{-\kappa_D^{\mathbb{Q}}\tau}}{\kappa_D^{\mathbb{Q}}\tau} \right) \\ & + \frac{\alpha\sigma^3 \left(\kappa_D - \kappa_D^{\mathbb{Q}}\right)}{\left(\kappa - \kappa_D^{\mathbb{Q}}\right)^2} \left(\frac{1 - e^{-\kappa\tau}}{\kappa\tau} - \frac{1 - e^{-\kappa_D^{\mathbb{Q}}\tau}}{\kappa_D^{\mathbb{Q}}\tau} + \left(\kappa - \kappa_D^{\mathbb{Q}}\right) \frac{1 - e^{-\kappa_D^{\mathbb{Q}}\tau} - \kappa_D^{\mathbb{Q}}\tau e^{-\kappa_D^{\mathbb{Q}}\tau}}{\left(\kappa_D^{\mathbb{Q}}\right)^2 \tau} \right). \end{aligned} \quad (\text{A-15})$$

As the function $f(x) = \frac{1 - e^{-x\tau}}{x\tau}$ is decreasing for any $\tau > 0$, the first term on the RHS is always positive. Moreover, it is also a convex function, hence

$$\frac{1 - e^{-\kappa\tau}}{\kappa\tau} - \frac{1 - e^{-\kappa_D^{\mathbb{Q}}\tau}}{\kappa_D^{\mathbb{Q}}\tau} + \left(\kappa - \kappa_D^{\mathbb{Q}}\right) \frac{1 - e^{-\kappa_D^{\mathbb{Q}}\tau} - \kappa_D^{\mathbb{Q}}\tau e^{-\kappa_D^{\mathbb{Q}}\tau}}{\left(\kappa_D^{\mathbb{Q}}\right)^2 \tau} \geq 0,$$

and the second term on the RHS of (A-15) is also positive. Therefore, $\frac{d\sigma_y^\tau}{d\gamma} > 0$.

Second, we relate γ to η_y by considering (A-14) around $\alpha = 0$. From (A-13) we have

$$\left(\frac{\sigma_y^\tau}{\sigma}\right)^2 = \left(\mathcal{B}(\bar{\tau}) + \mathcal{C}(\bar{\tau}) \frac{\sigma_D}{\sigma}\right)^2 \approx h_0 + \alpha h_1 \gamma, \quad (\text{A-16})$$

where

$$\begin{aligned} h_0 &\equiv \left(\mathcal{B}(\bar{\tau}) + \mathcal{C}(\bar{\tau}) \frac{\sigma_D}{\sigma}\right)^2 \Big|_{\alpha=0} = \mathcal{B}^2(\bar{\tau}), \text{ and} \\ h_1 &\equiv \frac{1}{\gamma} \frac{d\left(\mathcal{B}(\bar{\tau}) + \mathcal{C}(\bar{\tau}) \frac{\sigma_D}{\sigma}\right)^2}{d\alpha} \Big|_{\alpha=0} = -2\mathcal{B}(\bar{\tau}) \frac{\sigma^2}{\kappa - \kappa_D} \left(\frac{1 - e^{-\kappa\bar{\tau}}}{\kappa\bar{\tau}} - \frac{1 - e^{-\kappa_D\bar{\tau}}}{\kappa_D\bar{\tau}} \right). \end{aligned}$$

Combining (A-14) and (A-16), we obtain that around $\alpha = 0$,

$$\gamma = \frac{h_0 \eta_y}{1 - \alpha h_1 \eta_y}.$$

Since $h_0, h_1 > 0$, γ is increasing in η_y , and together with the first part of the proof we get

$$\frac{d\sigma_y^\tau}{d\eta_y} > 0. \quad (\text{A-17})$$

Third, we trivially verify that

$$\lim_{\tau \rightarrow 0} \sigma_y^\tau = \sigma \text{ and } \lim_{\tau \rightarrow \infty} \sigma_y^\tau = 0.$$

Therefore, the effect of negative convexity on yield volatilities tends to zero at very short and very long maturities, and hence it must be hump-shaped.

Finally, since bond return volatility satisfies $\sigma_t^\tau = \tau \sigma_y^\tau$, (A-17) also implies that bond return volatility increases when convexity becomes more negative. \square

Appendix B Time-varying convexity

Here we present a tractable way to relax the assumption of constant MBS convexity and capture the non-linearities inherent to the prepayment option. This version of the model allows for an additional degree of freedom and provides a better statistical description of MBS duration and convexity series. However, the qualitative implications of the model are identical to the ones outlined in Section 1. More precisely, we allow the sensitivity of outstanding MBS to the short rate to be quadratic:

$$\frac{dMBS_t}{dr_t} = z_t + \phi z_t^2, \quad \text{and} \quad (\text{A-18})$$

$$dz_t = -\kappa_z^{\mathbb{Q}} z_t dt + \sigma_z dB_t^{\mathbb{Q}}. \quad (\text{A-19})$$

In the data, when interest rates and MBS duration decrease, the negative convexity of MBS increases. In other words MBS duration has negative skewness. The skewness of the monthly series of MBS duration in our sample is equal to -1.32 compared to the 10 year yield which displays only a moderate skewness of -0.06 . The parameter ϕ can be calibrated to match this feature of the data. From an economic point of view negative skewness corresponds to the asymmetry in MBS duration response to changes in interest rates: it reacts more to falling than to rising interest rates.

In the model described by (1), (9) and (A-18)-(A-19) yields are given by

$$y_t^{\tau} = \mathcal{A}_z(\tau) + \mathcal{B}_z(\tau)r_t + \mathcal{C}_z(\tau)z_t + \mathcal{D}_z(\tau)z_t^2,$$

The key to quadratic closed form solution is that while quadratic terms appear under \mathbb{Q} in the dynamics of r_t , z_t is still affine under \mathbb{Q} (and therefore not affine under \mathbb{P}); see also Cheng and Scaillet (2007). Bond prices are given by

$$\Lambda_t^{\tau} = e^{-[A_z(\tau) + B_z(\tau)r_t + C_z(\tau)z_t + D_z(\tau)z_t^2]},$$

where $A_z(\tau) \equiv \mathcal{A}_z(\tau)\tau$, $B_z(\tau) \equiv \mathcal{B}_z(\tau)\tau$, $C_z(\tau) \equiv \mathcal{C}_z(\tau)\tau$, and $D_z(\tau) \equiv \mathcal{D}_z(\tau)\tau$. No-arbitrage pricing of bonds results in the following system of ODEs (where we remove the time-dependence and subscript to simplify the notation):

$$0 = A' - \kappa\theta B + \frac{1}{2}\sigma^2 B^2 + \frac{1}{2}\sigma_z^2 (C^2 - 2D) + \sigma\sigma_z BC,$$

and

$$1 = B' + \kappa B,$$

and

$$0 = C' + \left(\kappa_z^{\mathbb{Q}} + 2\sigma_z^2 D\right) C - \alpha\sigma^2 B + 2\sigma\sigma_z BD,$$

and

$$0 = D' + 2\kappa_z^{\mathbb{Q}} D + 2\sigma_z^2 D^2 - \alpha\sigma^2 \phi B,$$

together with the boundary conditions $A(0) = B(0) = C(0) = D(0) = 0$. The solution to the system above can be written in terms of J - and Y -type Bessel functions. To simplify, we can also solve for $A_z(\tau)$, $B_z(\tau)$, $C_z(\tau)$, and $D_z(\tau)$ recursively using a discrete time approximation of the dynamics of the state variables.

By Itô's lemma the second order dollar sensitivity of outstanding MBS to short rate shocks ($\frac{d^2 MBS_t}{dr_t^2} \equiv -\gamma$) is equal to:

$$\sigma_z + 2\phi\sigma_z z_t,$$

implying time-varying convexity. The instantaneous volatility of maturity- τ yield is given by

$$\mathcal{B}_z(\tau)\sigma + \mathcal{C}_z(\tau)\sigma_z + 2\mathcal{D}_z(\tau)\sigma z_t.$$

The code that calculates $A_z(\tau)$, $B_z(\tau)$, $C_z(\tau)$, and $D_z(\tau)$ and allows to verify Propositions 1, 2, and 3 in the context of stochastic convexity is available upon request.

Appendix C Tables

Table 1
Bond risk premia regressions Treasuries

This table reports estimated coefficients from regressing annual bond excess returns constructed from Treasuries, rx_{t+1}^T , onto a set of variables:

$$rx_{t+1}^T = \beta_1^T \text{duration}_t + \beta_2^T \text{level}_t + \epsilon_{t+1}^T,$$

where level_t is the first principal component from bond yields. t-Statistics presented in parentheses are calculated using Newey and West (1987). All variables are standardized to have mean zero and a standard deviation of one. Data is weekly and runs from 1997 through 2011.

	2y	3y	4y	5y	6y	7y	8y	9y	10y
duration	0.116 (1.63)	0.167 (2.28)	0.212 (2.88)	0.251 (3.38)	0.285 (3.80)	0.314 (4.16)	0.340 (4.46)	0.362 (4.72)	0.381 (4.96)
Adj. R^2	1.34%	2.78%	4.48%	6.29%	8.11%	9.87%	11.53%	13.07%	14.51%
duration	0.067 (0.83)	0.161 (1.94)	0.243 (2.91)	0.312 (3.78)	0.370 (4.54)	0.419 (5.18)	0.459 (5.72)	0.492 (6.16)	0.520 (6.54)
level	0.133 (1.48)	0.014 (0.16)	-0.083 (-0.96)	-0.164 (-1.89)	-0.230 (-2.63)	-0.282 (-3.20)	-0.322 (-3.60)	-0.353 (-3.89)	-0.375 (-4.07)
Adj. R^2	2.75%	2.67%	4.96%	8.50%	12.56%	16.63%	20.40%	23.71%	26.53%

Table 2
Bond volatility regressions

Panel A and B report estimated coefficients from regressing bond yield volatility, vol_t^τ , and τ -year maturity implied volatility of swaptions written on the 10-year swap rate, $\text{iv}_t^{\tau y 10y}$, onto convexity:

$$\text{vol}_t^\tau / \text{iv}_t^{\tau y 10y} = \beta_1^\tau \text{convexity}_t + \epsilon_t^\tau,$$

where $\tau = 1, \dots, 10$. Panel C reports estimated coefficients from regressing treasury implied volatility (tiv), realized volatility (trv), the bond variance risk premium (vrp), returns on a monthly straddle strategy (straddle), implied and realized measures of volatility of volatility (tivov/trvov) onto convexity. t-Statistics presented in parentheses are calculated using Newey and West (1987). All variables are standardized to have mean zero and a standard deviation of one. Data is weekly and runs from 1997 through 2011.

Panel A: Bond Yield Volatility

	1y	2y	3y	4y	5y	6y	7y	8y	9y	10y
convexity	0.248 (3.24)	0.264 (3.38)	0.258 (3.32)	0.247 (3.19)	0.231 (3.01)	0.213 (2.81)	0.195 (2.61)	0.180 (2.44)	0.166 (2.28)	0.154 (2.15)
Adj. R^2	6.17%	6.96%	6.66%	6.10%	5.34%	4.54%	3.82%	3.22%	2.75%	2.38%

Panel B: Swaption Implied Volatility

	1y	2y	3y	4y	5y	6y	7y	8y	9y	10y
convexity	0.283 (3.25)	0.289 (3.41)	0.308 (3.53)	0.314 (3.66)	0.313 (3.77)	0.240 (2.09)	0.323 (3.82)	0.251 (2.19)	0.256 (2.23)	0.319 (3.69)
Adj. R^2	8.02%	8.36%	9.51%	9.88%	9.77%	5.77%	10.45%	6.32%	6.55%	10.18%

Panel C: Bond Return Volatility

	iv	rv	vrp	straddle	tivov	trvov
convexity	0.186 (2.47)	0.146 (1.75)	0.187 (3.29)	0.090 (1.93)	0.152 (2.69)	0.159 (2.59)
Adj. R^2	3.45%	2.12%	3.48%	1.10%	2.32%	2.52%

Table 3
Duration and yield factors

Panel A reports the unconditional correlation between MBS duration and the first three principal components (PCs) of bond yields. Panel B reports estimated coefficients from regressing bond excess returns onto duration and the PCs.

$$rx_{t+1}^{\tau} = \beta_1^{\tau} \text{duration}_t + \beta_2^{\tau} \text{level}_t + \beta_3^{\tau} \text{slope}_t + \beta_4^{\tau} \text{curvature}_t + \epsilon_{t+1}^{\tau},$$

where level_t , slope_t and curvature_t represent the first three principal components of bond yields. t-Statistics presented in parentheses are calculated using Newey and West (1987). All variables are standardized to have mean zero and a standard deviation of one. Data is weekly and runs from 1997 through 2011.

Panel A: Unconditional Correlations

PC1	PC2	PC3
0.55	0.12	-0.14

Panel B: Bond Excess Return Regression

	2y	3y	4y	5y	6y	7y	8y	9y	10y
duration	0.064 (0.87)	0.144 (1.88)	0.216 (2.84)	0.278 (3.70)	0.331 (4.45)	0.374 (5.05)	0.409 (5.54)	0.438 (5.93)	0.462 (6.24)
level	0.420 (3.78)	0.318 (3.01)	0.233 (2.26)	0.164 (1.59)	0.108 (1.03)	0.063 (0.58)	0.028 (0.25)	0.001 (0.00)	-0.020 (-0.17)
slope	0.334 (2.68)	0.374 (2.94)	0.403 (3.20)	0.428 (3.50)	0.449 (3.81)	0.466 (4.13)	0.480 (4.43)	0.491 (4.70)	0.497 (4.93)
curvature	0.318 (2.75)	0.284 (2.46)	0.270 (2.37)	0.262 (2.38)	0.254 (2.42)	0.246 (2.47)	0.237 (2.51)	0.228 (2.52)	0.218 (2.51)
Adj. R^2	14.54%	14.36%	17.19%	21.49%	26.33%	31.07%	35.36%	39.04%	42.08%

Table 4
Bond risk premia regressions other

This table reports estimated coefficients from regressing bond excess returns, rx_{t+1}^τ , onto a set of variables:

$$rx_{t+1}^\tau = \beta_1^\tau \text{duration}_t + \beta_2^\tau \text{slope}_t + \beta_3^\tau \text{cp}_t + \sum_{i=1}^8 F_t^i + \epsilon_t,$$

where slope_t is the slope at time t , cp_t is the CP factor at time t , and F_t^i are the Ludvigson and Ng (2009) macro factors. t-Statistics presented in parentheses are calculated using Newey and West (1987). All variables are standardized to have mean zero and a standard deviation of one. Data is monthly and runs from 1997 through 2011.

	2y	3y	4y	5y	6y	7y	8y	9y	10y
duration	0.127 (0.96)	0.214 (1.60)	0.279 (2.12)	0.331 (2.60)	0.376 (3.03)	0.414 (3.43)	0.446 (3.80)	0.474 (4.12)	0.498 (4.41)
slope	-0.079 (-0.51)	0.042 (0.27)	0.130 (0.86)	0.198 (1.37)	0.253 (1.82)	0.298 (2.22)	0.333 (2.59)	0.361 (2.90)	0.381 (3.18)
cp factor	0.370 (2.58)	0.280 (1.99)	0.223 (1.63)	0.183 (1.40)	0.155 (1.23)	0.133 (1.10)	0.117 (0.99)	0.105 (0.91)	0.095 (0.84)
Adj. R^2	16.34%	15.59%	17.23%	19.87%	22.93%	26.07%	29.06%	31.75%	34.05%
duration	0.081 (0.63)	0.142 (1.04)	0.189 (1.38)	0.229 (1.67)	0.264 (1.96)	0.297 (2.23)	0.327 (2.51)	0.356 (2.78)	0.382 (3.04)
slope	-0.612 (-3.48)	-0.507 (-2.91)	-0.410 (-2.42)	-0.317 (-1.94)	-0.229 (-1.45)	-0.145 (-0.95)	-0.067 (-0.45)	0.004 (0.03)	0.066 (0.48)
cp factor	0.584 (5.86)	0.548 (5.88)	0.519 (5.73)	0.496 (5.56)	0.474 (5.37)	0.451 (5.15)	0.428 (4.90)	0.404 (4.61)	0.381 (4.28)
F^1	-0.588 (-5.21)	-0.535 (-4.88)	-0.476 (-4.52)	-0.418 (-4.15)	-0.361 (-3.76)	-0.307 (-3.34)	-0.257 (-2.91)	-0.211 (-2.45)	-0.168 (-1.98)
F^2	-0.111 (-2.75)	-0.111 (-2.60)	-0.110 (-2.57)	-0.108 (-2.57)	-0.105 (-2.56)	-0.101 (-2.52)	-0.096 (-2.44)	-0.089 (-2.31)	-0.082 (-2.13)
F^3	0.322 (3.58)	0.386 (4.13)	0.427 (4.56)	0.449 (4.80)	0.456 (4.88)	0.452 (4.82)	0.442 (4.66)	0.427 (4.42)	0.410 (4.14)
F^4	0.016 (0.17)	0.015 (0.15)	0.024 (0.23)	0.033 (0.32)	0.041 (0.40)	0.047 (0.47)	0.050 (0.53)	0.053 (0.57)	0.055 (0.62)
F^5	0.025 (0.49)	0.018 (0.37)	0.012 (0.25)	0.007 (0.16)	0.004 (0.09)	0.002 (0.05)	0.001 (0.01)	-0.001 (-0.01)	-0.002 (-0.04)
F^6	-0.067 (-0.83)	-0.091 (-1.12)	-0.095 (-1.20)	-0.090 (-1.16)	-0.080 (-1.07)	-0.068 (-0.93)	-0.056 (-0.78)	-0.045 (-0.63)	-0.034 (-0.49)
F^7	-0.116 (-1.44)	-0.121 (-1.48)	-0.123 (-1.52)	-0.125 (-1.56)	-0.125 (-1.58)	-0.124 (-1.58)	-0.121 (-1.56)	-0.117 (-1.53)	-0.111 (-1.48)
F^8	-0.014 (-0.22)	-0.017 (-0.25)	-0.020 (-0.28)	-0.021 (-0.29)	-0.020 (-0.29)	-0.018 (-0.26)	-0.015 (-0.22)	-0.011 (-0.17)	-0.007 (-0.11)
Adj. R^2	39.04%	37.12%	37.41%	38.76%	40.58%	42.52%	44.37%	46.01%	47.36%

Table 5
Bond volatility regressions other

Panel A and B report estimated coefficients from regressing bond yield volatility, vol_t^τ , and τ -year maturity implied volatility of swaptions written on the 10-year swap rate, $\text{iv}_t^{\tau y^{10y}}$, onto convexity and illiquidity:

$$\text{vol}_t^\tau / \text{iv}_t^{\tau y^{10y}} = \beta_1^\tau \text{convexity}_t + \beta_2^\tau \text{illiq}_t + \epsilon_t^\tau,$$

where $\tau = 1, \dots, 10$ and illiq_t is the illiquidity factor at time t . Panel C reports estimated coefficients from regressing treasury implied volatility (tiv), realized volatility (trv), the bond variance risk premium (vrp), returns on a monthly straddle strategy (straddle), implied and realized measures of volatility of volatility (tivov/trvov) onto convexity and illiquidity. t -Statistics presented in parentheses are calculated using Newey and West (1987). All variables are standardized to have mean zero and a standard deviation of one. Data is weekly and runs from 1997 through 2011.

Panel A: Bond Yield Volatility										
	1y	2y	3y	4y	5y	6y	7y	8y	9y	10y
convexity	0.301 (4.51)	0.312 (4.50)	0.307 (4.49)	0.299 (4.46)	0.286 (4.38)	0.271 (4.27)	0.257 (4.15)	0.243 (4.01)	0.231 (3.87)	0.220 (3.73)
illiq	0.478 (6.52)	0.438 (5.37)	0.450 (5.20)	0.474 (5.17)	0.505 (5.19)	0.535 (5.18)	0.560 (5.10)	0.579 (4.95)	0.592 (4.78)	0.601 (4.61)
Adj. R^2	28.64%	25.82%	26.54%	28.21%	30.40%	32.68%	34.69%	36.25%	37.34%	38.02%
Panel B: Swaption Implied Volatility										
	1y	2y	3y	4y	5y	6y	7y	8y	9y	10y
convexity	0.272 (2.81)	0.267 (2.61)	0.258 (2.63)	0.266 (2.94)	0.266 (3.23)	0.259 (3.47)	0.248 (3.63)	0.236 (3.70)	0.227 (3.71)	0.221 (3.71)
illiq	0.234 (4.14)	0.201 (3.12)	0.216 (3.34)	0.260 (4.07)	0.299 (4.26)	0.327 (3.73)	0.343 (3.19)	0.351 (2.85)	0.355 (2.65)	0.356 (2.54)
Adj. R^2	11.31%	9.87%	9.95%	12.15%	14.14%	15.38%	15.85%	15.91%	15.82%	15.69%
Panel C: Bond Return Volatility										
	iv	rv	vrp	straddle	tivov	trvov				
convexity	0.267 (3.65)	0.232 (2.87)	0.262 (4.41)	0.096 (1.95)	0.213 (4.11)	0.212 (3.86)				
illiq	0.684 (14.06)	0.700 (15.57)	0.628 (6.11)	0.106 (2.52)	0.545 (4.86)	0.473 (6.14)				
Adj. R^2	49.28%	50.18%	42.06%	1.30%	31.55%	24.55%				

Table 6
Regressions with interaction terms

This table reports estimated coefficients from regressing annual bond excess returns (Panel A) and bond yield volatility (Panel B) onto MBS duration and convexity interacted with the ratio between total mortgages outstanding and GDP. t-Statistics presented in parentheses are calculated using Newey and West (1987). All variables are standardized to have mean zero and a standard deviation of one. Data is quarterly and runs from 1997 through 2011.

Panel A: Bond Excess Return Regression									
	2y	3y	4y	5y	6y	7y	8y	9y	10y
duration \times ratio	0.131 (0.83)	0.216 (1.39)	0.270 (1.77)	0.305 (2.01)	0.326 (2.15)	0.339 (2.22)	0.346 (2.25)	0.350 (2.25)	0.351 (2.24)
Adj. R^2	1.70%	4.68%	7.31%	9.29%	10.65%	11.50%	11.99%	12.23%	12.31%
duration \times ratio	0.112 (1.60)	0.147 (2.07)	0.165 (2.27)	0.169 (2.30)	0.166 (2.23)	0.160 (2.11)	0.153 (1.99)	0.145 (1.87)	0.139 (1.76)
level	0.225 (3.18)	0.138 (2.00)	0.062 (0.90)	-0.006 (-0.08)	-0.066 (-0.92)	-0.115 (-1.59)	-0.156 (-2.11)	-0.187 (-2.50)	-0.211 (-2.78)
Adj. R^2	3.41%	1.62%	1.57%	2.38%	3.62%	5.02%	6.38%	7.56%	8.52%

Panel B: Bond Yield Volatility Regression										
	1y	2y	3y	4y	5y	6y	7y	8y	9y	10y
convexity \times ratio	0.404 (2.48)	0.422 (2.77)	0.439 (2.89)	0.453 (2.99)	0.461 (3.08)	0.467 (3.16)	0.472 (3.23)	0.477 (3.30)	0.484 (3.37)	0.492 (3.43)
Adj. R^2	16.35%	17.82%	19.30%	20.49%	21.23%	21.76%	22.25%	22.80%	23.44%	24.17%
convexity \times ratio	0.347 (3.25)	0.364 (3.83)	0.382 (4.10)	0.396 (4.30)	0.406 (4.47)	0.413 (4.62)	0.419 (4.76)	0.425 (4.87)	0.432 (4.97)	0.440 (5.06)
illiq	0.635 (10.02)	0.647 (10.44)	0.639 (10.08)	0.623 (8.91)	0.608 (7.82)	0.596 (7.14)	0.587 (6.74)	0.579 (6.52)	0.572 (6.40)	0.566 (6.36)
Adj. R^2	55.58%	58.52%	59.04%	58.25%	57.17%	56.25%	55.62%	55.26%	55.12%	55.14%

Table 7
Bond risk premia regressions swaps

This table reports estimated coefficients from regressing annual bond excess returns constructed from interest rate swaps, rx_{t+1}^τ , onto a set of variables:

$$rx_{t+1}^\tau = \beta_1^\tau \text{duration}_t + \beta_2^\tau \text{slope}_t + \beta_3^\tau \text{cp}_t + \epsilon_{t+1}^\tau,$$

where slope_t is the slope of the term structure at time t and cp_t is the CP factor at time t . t -Statistics presented in parentheses are calculated using Newey and West (1987). All variables are standardized to have mean zero and a standard deviation of one. Data is weekly and runs from 1997 through 2011.

	2y	3y	4y	5y	6y	7y	8y	9y	10y
duration	0.100 (1.17)	0.146 (1.76)	0.202 (2.52)	0.252 (3.26)	0.296 (3.95)	0.335 (4.57)	0.367 (5.09)	0.395 (5.56)	0.421 (5.99)
Adj. R^2	0.99%	2.13%	4.08%	6.33%	8.78%	11.23%	13.48%	15.60%	17.68%
duration	0.050 (0.54)	0.157 (1.73)	0.251 (2.92)	0.327 (4.04)	0.388 (5.02)	0.436 (5.84)	0.474 (6.50)	0.505 (7.07)	0.531 (7.53)
level	0.135 (1.37)	-0.031 (-0.33)	-0.133 (-1.48)	-0.204 (-2.33)	-0.248 (-2.88)	-0.273 (-3.20)	-0.288 (-3.39)	-0.297 (-3.50)	-0.299 (-3.53)
Adj. R^2	2.43%	2.08%	5.49%	9.82%	13.99%	17.57%	20.55%	23.10%	25.32%

Table 8
Bond portfolio regressions

This table reports estimated coefficients from regressing bond portfolio excess returns onto duration and level.

$$\text{rxpf}_{t+1}^r = \beta_1^r \text{duration}_t + \beta_2^r \text{level}_t + \epsilon_{t+j}^r,$$

where level_t is the first principal component from bond yields and rxpf_{t+1}^r are monthly excess returns on the CRSP bond portfolios with maturities between 5 and 10 years and larger than 10 years. Returns are in excess of either the 1-month T-bill or Eurodollar deposit rate. All variables are standardized to have mean zero and a standard deviation of one. Data is monthly and runs from 1990 through 2011.

	T-bill		ED rate	
	$\geq 5\text{y} < 10\text{y}$	$> 10\text{y}$	$\geq 5\text{y} < 10\text{y}$	$\geq 5\text{y} < 10\text{y}$
duration	0.159 (2.88)	0.163 (3.17)	0.156 (2.79)	0.161 (3.11)
Adj. R^2	2.53%	2.67%	2.42%	2.60%
duration	0.199 (2.43)	0.199 (2.72)	0.196 (2.42)	0.196 (2.70)
level	-0.054 (-0.61)	-0.038 (-0.48)	-0.056 (-0.64)	-0.039 (-0.49)
Adj. R^2	2.55%	2.77%	2.40%	2.68%

Appendix D Figures

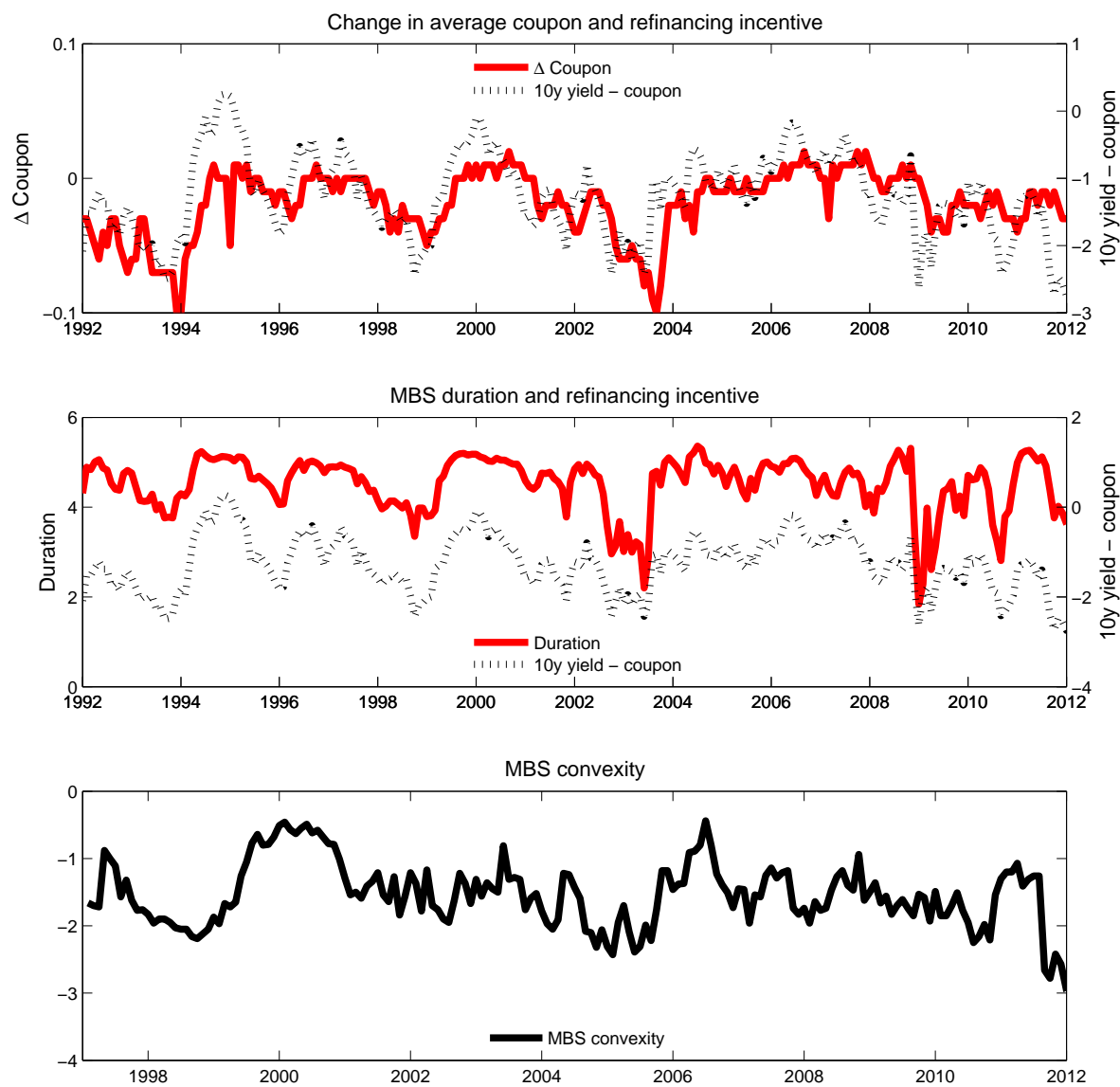


Figure 1. Average coupon, duration, and convexity

The upper panel plots the difference between 10-year yield and average MBS coupon together with the subsequent change in average MBS coupon. The middle panel plots the difference between 10-year yield and average MBS coupon together with MBS duration. The lower panel plots MBS convexity. Data is monthly and runs from January 1990 to December 2012 (upper and middle panels), resp. January 1997 to December 2012 (lower panel).

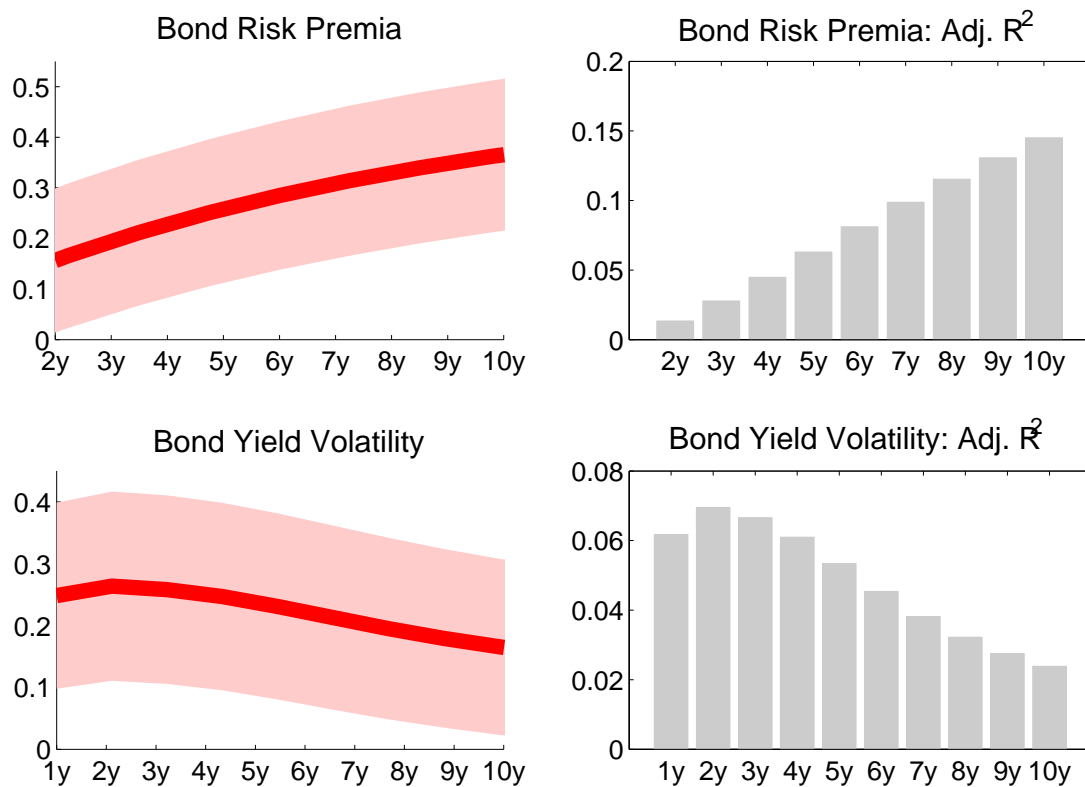


Figure 2. Univariate regression coefficients

This figure plots estimated coefficients and adjusted R^2 from univariate regressions of bond excess returns (upper panels) and bond yield volatilities (lower panels) onto MBS duration (bond excess returns) and MBS convexity (bond yield volatilities). All variables are standardized, i.e. have a mean of zero and a standard deviation of one. Data is weekly and runs from 1997 through 2011. Shaded areas represent confidence levels on the 95% level.

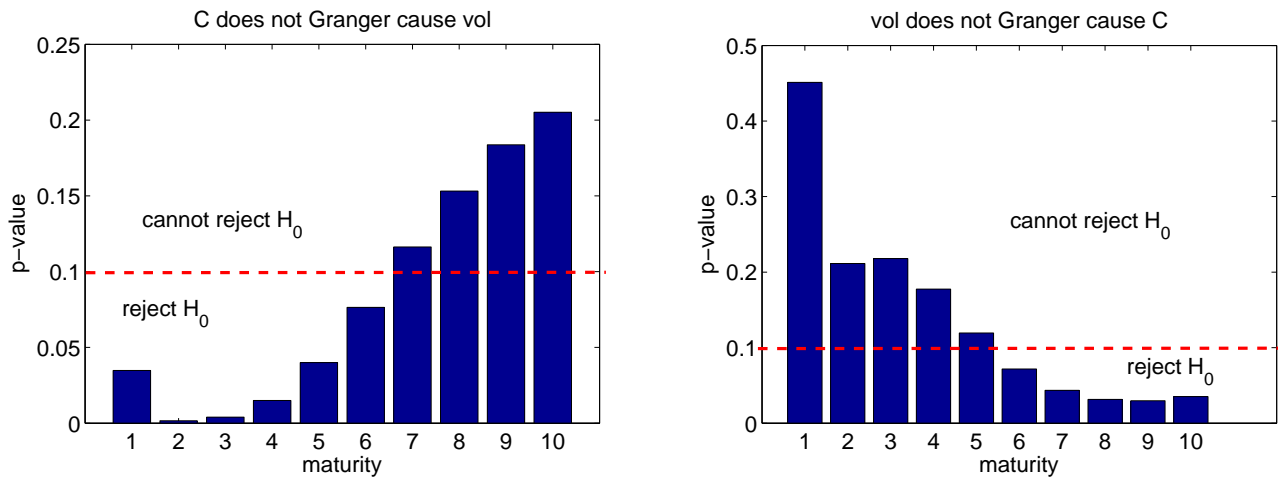


Figure 3. p-Values of Granger Causality test

These figures present results for Granger causality tests. The null hypothesis for the left (right) panel is that negative convexity (volatility) does not Granger cause bond yield volatility (convexity). The regressions are estimated on weekly data from 1997 to 2012.

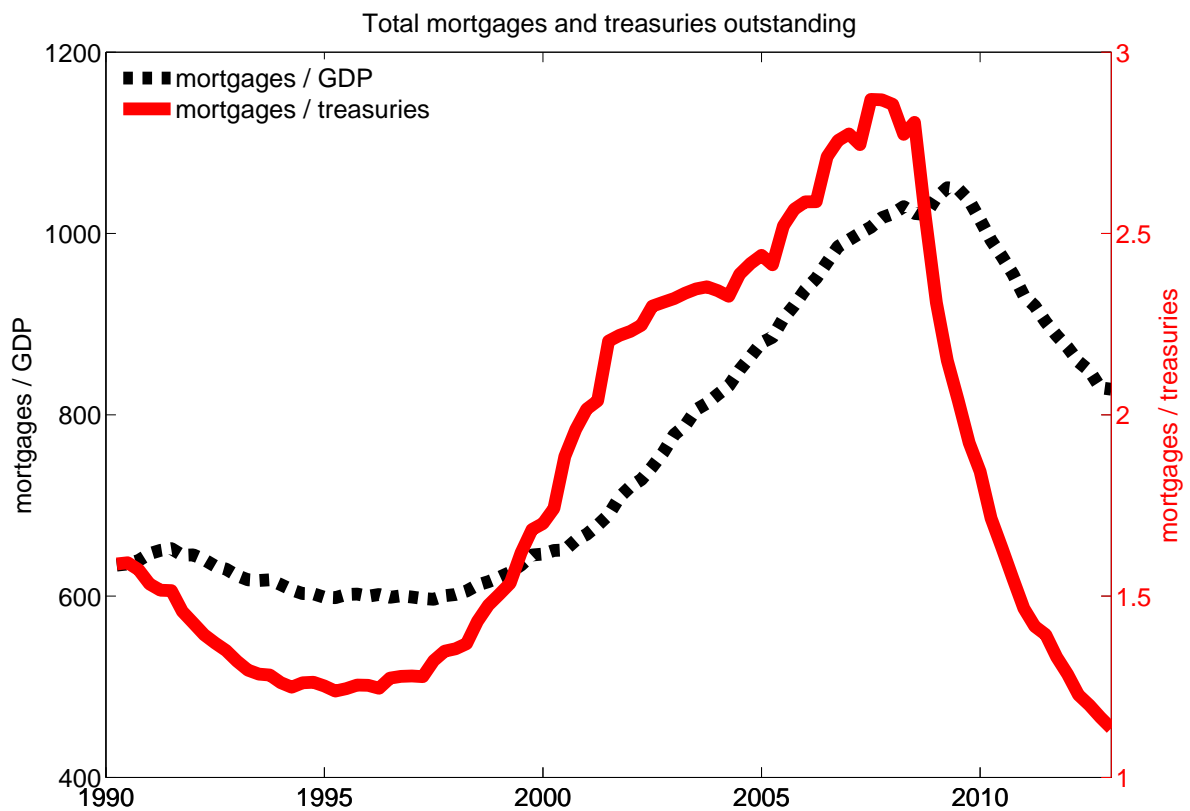


Figure 4. Total mortgages and treasuries outstanding

This figure plots total nominal value outstanding of all mortgages divided by GDP (left axis) and the total amount outstanding of mortgages divided by amount outstanding in Treasuries (right axis). Data is from the webpage of the Board of Governors of the Federal Reserve and its frequency is quarterly from 1990 to 2012.