

Information Processing and Non-Bayesian Learning in Financial Markets

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Abstract

Ample empirical and experimental evidence documents that individuals especially learn out of self-experienced observations. I embed this availability bias in an overlapping generations equilibrium model. This model captures several empirical findings. First, it explains why returns on high trading volume dates tend to reverse. Second, it provides explanations for a high trading volume, a connection between trading volume and volatility, excess volatility, and overreaction and correction patterns. As empirically documented, young agents buy high and sell low, trade frequently, and obtain a lower return. For intraday trading it predicts a high trading volume around the opening hours especially for cross-listed stocks.

JEL Codes: G02, G12

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There is ample evidence in psychology and empirical finance that agents put a higher weight on information gained from own experience. First being mentioned by Kahneman, Slovic, and Tversky (1982) in the form of an availability bias, this finding has considerable support in the recent empirical finance literature. For example, Malmendier and Nagel (2011) show that the predictions of agents who lived throughout the Great Depression are more cautious. Graham and Harvey (2001) find that past market performance is positively correlated with the average CFO's one-year and ten-year perceived risk premium.

As agents differ with respect to their experience history they are not affected by the same experiences in the same way. Old investors have a longer experience history and are, therefore, biased by all their observations. In contrast younger agents have a shorter experienced data sample and are, as a result, more responsive to new observations. Empirically this is supported by Malmendier and Nagel (2013) and Vissing-Jorgensen (2003), who find that young investors are more sensitive to recent news. The resulting difference in the perception leads to disagreement among agents and can be considered as a source for heterogeneity.

In this paper, I introduce the idea of agents overvaluing self-experienced relative to not self-experienced information in an overlapping generations equilibrium model. In my model, the market entrance of individuals determines which past dividends were directly observed by the individuals and, therefore, also especially anchored in the individuals' mind. In my base-case model I distinguish between two groups of individuals. Young individuals have just entered the financial market. In contrast, the adult individuals have already been in the market in the prior period. In the case of availability bias this leads to a difference in the reaction to current and past news and, as a result, to heterogeneity across agents. The difference in opinion is observable for the agents through the deviation of the realized price from the perceived rational price. However, because the dividend stream is the only information accessible, and can be observed by everyone, the agents can reasonably believe in noise-trading. This leads to a willingness to trade.

My model explains several up to now explained and unexplained empirical findings. Moreover, it provides links between these findings by stating that one anomaly directly results out of another finding. In particular, my model provides an explanation for the overreaction and correction pattern: When the public signal strongly deviates from its long-term average, agents disagree with respect to the interpretation of this signal. As all agents in the market have experienced the last dividend the impact on prices is exaggerated, leading to overreaction. When agents leave the market after some time or make further experiences this impact fades, leading to reversal. In the context of my model, this overreaction and correction pattern is consistent with the finding of Greenwood and Shleifer (2013). The authors document that investors' expectations show a strong negative correlation with model-based expected returns. This can be explained by the fact that at times when the average investors' expectations are high, the prices are (too) high and, therefore, will mean-revert over time.

Extreme public signals lead to significant changes in the individuals' expectations and a higher disagreement. This results in a positive correlation between volatility and trading volume. Further, my model supports the empirical finding that returns on high-volume days tend to reverse as e.g. described in Gallant, Rossi, and Tauchen (1992) and Campbell, Grossman, and Wang (1993). The intuition behind this finding is that on high trading volume dates the price changes are more likely to be caused by an initial overreaction than by a reversal. As a consequence, in the time that follows, the overreaction fades out, which leads to reversed returns in the aftermath.

My model provides explanations for agent specific findings. Younger agents overreact most to new observed information because their observed data sample is very small. Therefore, the younger agents change their portfolio more drastically than older investors and tend to invest more in stock markets when recent signals have been positive. Because prices tend to mean-revert thereafter, the performance of younger agents on average is low. In summary, young agents tend to buy high and sell low, trade frequently, and thereby obtain a lower return. This is consistent with the equivalent

empirical findings of Barber and Odean (2001) and Vissing-Jorgensen (2003).

Depending on the interpretation of an agent's life cycle, my model not only provides insights for long term financial asset pricing phenomena but also provides predictions for intraday trading. Looking at cross-listed stocks, people enter the stock market at different points in time. Especially around the opening hours of a stock exchange, the model explains high trading volumes. In this context, my paper provides the testable hypothesis that the effect of a high trading volume around the opening hours is more pronounced for cross-listed stocks compared to single-listed stocks. Moreover, for cross-listed stocks, my model provides predictions regarding which agents sell and which agents buy in a transaction. On days with recent positive information, the agents that have already been in the market are more affected by this information than any agents just entering the market. Therefore, the entering investors generally sell their stock to the incumbents. On days with recent bad news the trading behavior is predicted to exhibit the opposite pattern.

The inclusion of availability bias or in other words learning out of experiences in the modeling of financial markets is very recent. There are two closely related papers, which after this work also followed the idea of incorporating this bias in an overlapping generations model. Ehling, Graniero, and Heyerdahl-Larsen (2014) adapt my model to the special case of log-utility and concentrate on specific predictions. Collin-Dufresne, Johannes, and Lochstoer (2013) describe an overlapping generations model in a production economy with Epstein Zin preferences. With a focus on pricing effects they find e.g. time-varying over- and undervaluation and a resulting under- and over-investment.

In a related paper Barberis, Greenwood, Jin, and Shleifer (2013) construct a model, in which investors form beliefs concerning future stock performance by extrapolating from past returns. In their model an infinitely lived investor overestimates the importance of recent news. In contrast, in my model agents react to new observations in different ways, depending on their history of past experiences. Therefore, apart from general predictions with respect to prices and volatilities, I also

obtain predictions regarding individual trading behavior.

Hirshleifer and Yu (2013), Alti and Tetlock (2013), Barberis and Shleifer (2003) and Fuster, Hebert, and Laibson (2011) also incorporate the extrapolation of past state variables (either returns or fundamentals) in their models . However, in none of these models the agents differ with respect to their past observations. Therefore, my overlapping generations model is able to explain agent specific trading behavior and makes predictions in environments in which the market entrance and exit of agents plays an important role.

My model relates to the literature that deals with differences in beliefs. Among others Bhamra and Uppal (2014), Schmedders and Kubler (2012), and Garleanu and Panageas (2012) investigate the effect of differences in beliefs on asset prices. Although e.g. Schmedders and Kubler (2012) and Garleanu and Panageas (2012) also use an overlapping generations model, they do not take learning out of experience into account.

In contrast to overconfidence and other branches of the behavioral finance literature (e.g. Dumas, Kurshev, and Uppal (2009), Daniel, Hirshleifer, and Subrahmanyam (1998), Daniel, Hirshleifer, and Subrahmanyam (2001), Gervais and Odean (2001), Odean (1998), and Scheinkman and Xiong (2003)) in my model, individuals are not required to believe in someone having the informational advantage for beating the market. They 'only' try to learn about the model's parameters, thereby committing the mistake of availability bias. Moreover, my model does not rely on private information, which is generally not observed by empiricists. Information only becomes heterogeneous through individuals' interpretations and their perceptions.

Finally, in contrast to several streams of the rational learning literature, in my model the effect of learning does not fade. In this way, learning still plays an important role, even after more than 70 years of market data have been observed.

1 Two-generation economy with availability bias

The following model introduces the different availability of information into an overlapping generations model. First, I provide the underlying intuition of the model in a two-generation setup. Then, I generalize my approach to the multi-generations model. This extension is a better representation of the real world and, therefore, also explains more anomalies than the two-generations model. However, the mechanisms in this model are better illustrated in the two-generations version.

1.1 Individuals and Assets

I consider a discrete time financial market with one risky and one riskless asset. The riskless asset pays the riskfree rate r_f in each period $t = 1, \dots, \infty$ and is in perfectly elastic supply. The risky asset is available in unit supply and trades in a frictionless capital market. It can be interpreted as the market portfolio. The dividends d_t paid by the risky asset in each period are normally distributed with mean δ and variance Σ

$$d_t \sim N(\delta, \Sigma). \tag{1}$$

The individuals populating this economy live for three periods. Each period a new generation of individuals enters the market and chooses its initial portfolio. The previous young generation becomes adult and changes the portfolio, taking prior experiences into account. Finally, the previous adult generation retires.

For simplicity I assume a constant population. There are equally many first period, second period and retired individuals in the market. Since the retired individuals do not buy assets for the next period any more, only the first two generations actively trade in the market. Thus, in the remainder of this paper I ignore the retired generation. I call the new generation that enters the market *Young* and the generation that has already been trading in the prior period *Adult*. Thus, the *Young* and the *Adult* agents represent half of the active population. The overlapping

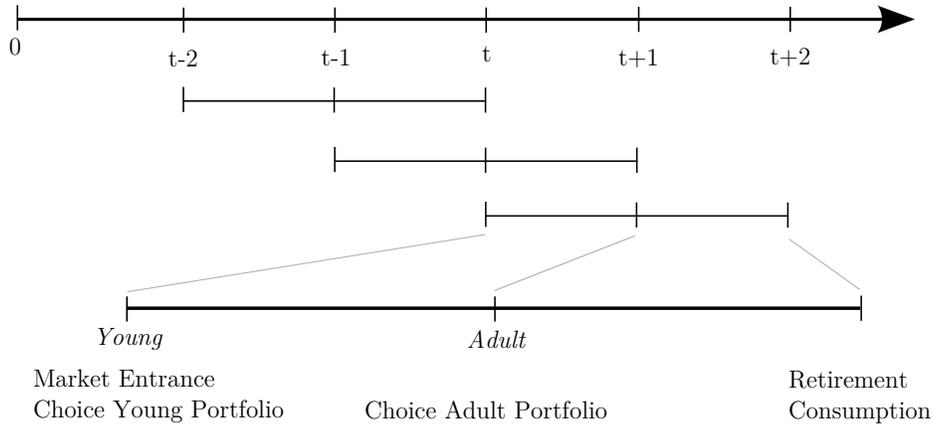


Figure 1: Overlapping Generations Model and an Individuals' Lifecycle. This figure shows the overlapping generations in my model. The young generation that enters in t is adult at time $t + 1$ and retires at time $t + 2$. Each point in time there is one young, one adult and one retired generation in the economy.

generations and an individual's life are illustrated in Figure 1.

1.2 Beliefs about Dividends

Individuals' expectations regarding dividends depend on both observed and unobserved dividends. But, and this is the key assumption, self-experienced dividends influence the perception of the dividend process more than the not self-experienced dividends. To define which past dividends have been observable when trade is taking place and prices are determined, I have to define whether period t dividends realize before or after price determination. I assume first dividends are paid and then the prices are formed. Thus, d_t is known when the prices are determined and date t dividends belong to the individuals who bought shares in period $t - 1$.

The *young generation* has just entered the financial market. Thus, the young agents have not observed the dividends themselves. Therefore, all past dividends are the same to the young generation. Weighting all past dividends equally, the young agents form rational expectations regarding the mean dividend. This assumption is equivalent to stating that when entering the market, agents have correct priors. However, this assumption is not crucial to the model because,

for more generations, the young agent's initial perception is not important to the result. As in the remainder of this paper not all expectations will be rational, I indicate subjective expectations using the superscript s followed by the first letter of the generation in which these expectations were formed

$$E_t^{s,y} [d_{t+1}] = \bar{d}_t. \quad (2)$$

The *adult generation* differs from the young generation in that the adult generation has already experienced the past period and the payment of the period t dividend. Therefore, the adult agents place special weight on this observation. To capture the availability bias, I assume that the adult generation's dividend expectations equals the young generation's dividend expectations, except that the adult generation over-weights the latest dividend observation by a constant factor m

$$E_t^{s,a} [d_{t+1}] = \frac{\bar{d}_t + md_t}{1 + m}. \quad (3)$$

For $m = 0$, the adult generation's expectations are rational. For $m > 0$ the only deviation from rationality is the over-weighting of the past dividends, which occurred after the individual's market entrance.

For simplicity I assume, as in Lewellen and Shanken (2002), that the variance Σ of the dividend process is common knowledge

$$Var_t^{s,y} (d_{t+1}) = Var_t^{s,a} (d_{t+1}) = \Sigma. \quad (4)$$

1.3 Optimal portfolio choices and prices

In the absence of intermediary consumption individuals maximize their utility out of terminal wealth, ω , by choosing the optimal portfolio x . The young and adult individuals differ in two

important aspects. First, the time horizon until retirement is shorter for the adult individuals (only one period left) than for the young individuals (two periods left). Second, adult individuals are assumed to put more weight on the most recent observation, especially more than on those observations prior to their entrance.

The *adult generation* is close to retirement and, therefore, has only an one period investment horizon. The adult agents maximize their next period utility out of wealth, given their current expectations (indicated by the operator $E_t^{s,a}$)

$$\max_{x_t^a} E_t^{s,a} [-\exp(-2\gamma w_{t+1}^r(x_t^a))]. \quad (5)$$

Terminal wealth equals

$$w_{t+1}^r = x_t^a (p_{t+1} + d_{t+1} - (1 + r_f) \cdot p_t) + (1 + r_f)w_t^a, \quad (6)$$

where w_t^a is the time t wealth of the adult individuals and w_{t+1}^r denotes the time $t + 1$ wealth of the retired generation. Moreover, x_t^a equals the number of shares bought and not the amount of money invested. This optimization problem results in the standard optimal portfolio share

$$x_t^a = \frac{1}{2\gamma \text{Var}_t^{s,a}(d_{t+1} + p_{t+1})} \cdot (E_t^{s,a}[p_{t+1} + d_{t+1}] - (1 + r_f)p_t). \quad (7)$$

This equation specifies the adult generation's investment given the generation's perception of the price and dividend process. The adult generation's perception of the dividend stream has already been specified in Equations (3) and (4). However, in contrast to the dividend process, the price process depends on the perception of both generations and, therefore, will be treated later.

For the *young generation* retirement is still two periods away. Thus, the young generation maximizes its wealth for two periods from now as a function of this and next periods portfolio

choice

$$\max_{x_t^y, x_{t+1}^a} E_t^{s,y} \left[-\exp \left(-2\gamma w_{t+2}^r (x_t^y, x_{t+1}^a) \right) \right]. \quad (8)$$

Terminal wealth is determined by

$$w_{t+2}^r = x_{t+1}^a ((p_{t+2} + d_{t+2}) - (1 + r_f)p_{t+1}) \quad (9)$$

$$+ (1 + r_f) ((1 + r_f)w_t^y + x_t^y ((p_{t+1} + d_{t+1}) - (1 + r_f)p_t)).$$

Now, I assume that the young generation does not take the influence of future prices on future portfolio choices into account. This assumption is equivalent to agents maximizing their next period discounted utility. This assumption is made for the comprehensiveness of the general idea of the model. However, as a result of this assumption, the qualitative results do not change. The computations, not assuming that the agents are myopic, can be found in Appendix B. Using this approach the optimal portfolio choice at time t is given by

$$x_t^y = \frac{E_t^{s,y} [p_{t+1} + d_{t+1}] - (1 + r_f)p_t}{2\gamma(1 + r_f) \cdot \text{Var}_t^{s,y} (p_{t+1} + d_{t+1})}. \quad (10)$$

As before, with the adult generation (compare Equation (7)), this equation specifies the young generation's investment given that generation's perception of the dividend and price process. Having derived general expressions for the demands of the young and adult generation for the risky asset, I now calculate prices by applying the market clearing condition

$$\frac{1}{2}x_t^a + \frac{1}{2}x_t^y = 1. \quad (11)$$

The resulting price process p_t has the form

$$p_t = \left(\frac{1 + r_f}{4\gamma \text{Var}_t^{s,a} (d_{t+1} + p_{t+1})} + \frac{1}{4\gamma \text{Var}_t^{s,y} (d_{t+1} + p_{t+1})} \right)^{-1} \quad (12)$$

$$\cdot \left(\frac{E_t^{s,a}[d_{t+1} + p_{t+1}]}{4\gamma \text{Var}_t^{s,a}(d_{t+1} + p_{t+1})} + \frac{E_t^{s,y}[p_{t+1} + d_{t+1}]}{4\gamma(1 + r_f)\text{Var}_t^{s,y}(d_{t+1} + p_{t+1})} - 1 \right).$$

This expression shows that the price depends on the young and adult generations' subjective perception of the dividend and price process. Because future prices are determined endogenously, agents' expectations about future prices do not only depend on their expectations about future dividends but also on the individuals' perception of the other agents' behavior.

1.4 Beliefs about prices and realized prices

Generally, it can be assumed that agents try to infer about the rational price by studying the underlying market mechanisms. Ultimately this results in a price process, which is structurally similar to that described in Equation (12). However, each generation does so without knowing the other generation's perceptions.

In my model, there is only one source of information, the dividend process, which everyone can observe. Thus, I assume that the agents believe that their conclusions based on their observations are correct and that, therefore, everyone else, who acts rationally, must have drawn the same consequences. In technical terms, this means, that individuals calculate the future price given by replacing the other agents' expectations by their own expectation.

However, because their dividend expectations differ, also the price expectations vary between the generations. Finally, this leads to a deviation of the realized prices from the subjectively expected prices. Because there is no private information in this market, the price is assumed to have no informative properties. Under the assumption, that agents trust their own expectations, the only sensible explanation for a deviation of the observed price from the subjective rational price is the existence of noise traders, whose perceived influence on price is denoted by ϵ .¹

¹Another potential assumption would be that agents are aware of the other individuals' perceptions but perceive them as wrong. This can be interpreted in the sense of Aumann (1976) that agents agree to disagree. This would lead to agents continuing to trade, although the noise trader variance would no longer occur.

To distinguish between the forces that actually drive the market and determine prices (meaning individual expectations and the market clearing mechanism) and the individual's perception of these forces, I now introduce a new sign B^y or B^a (inspired by the word belief, with the superscript of the respective generation), which refers to the individuals perception of the process.

Consequently, the idea described above, from the perspective of the young generation, can be written as

$$B_t^y(p_t) = \frac{1}{1+r_f} E_t^{s,y} [p_{t+1} + d_{t+1}] - \frac{4\gamma}{2+r_f} Var_t^{s,y} (p_{t+1} + d_{t+1}) + \epsilon_t^y. \quad (13)$$

It means that the young generation has replaced the expectations of the adult generation by its own expectations. The adult generation's subjective rational prices are determined accordingly

$$B_t^a(p_t) = \frac{1}{1+r_f} E_t^{s,a} [p_{t+1} + d_{t+1}] - \frac{4\gamma}{2+r_f} Var_t^{s,a} (p_{t+1} + d_{t+1}) + \epsilon_t^a. \quad (14)$$

The concrete expressions for the mean and variance expectations of the dividends have already been specified in the previous section.

As explained above, the additional ϵ -term in Equation (13) and Equation (14) describes the perceived noise in prices. For now I assume, and will later on show, that the perceived influence of noise traders' trading volume on price realization, meaning the deviation of the realized price from the subjectively rational price, is normally distributed with a mean of zero and variance Σ_N

$$\epsilon_t \sim N(0, \Sigma_N). \quad (15)$$

As with the dividend process, I assume that prior to their market entrance, agents are certain of the variance in prices, which can be attributed to noise trading.

All agents believe that they are rational. Therefore, the expected next-period price equals the current perceived *rational* price (meaning the price that is not distorted by perceived noise traders).

Applying this insight to the price process perceived by the young generation, as given by equation (13) results in

$$E_t^{s,y} [p_{t+1}] = \frac{1}{r_f} \bar{d}_t - \frac{4\gamma (1+r_f)}{r_f (2+r_f)} (\Sigma + \Sigma_N). \quad (16)$$

The noise trade perceived by the adult generation differs from that of the young generation. However, performing equivalent calculations as those for the young generation, I obtain the expected price of the adult generation as

$$E_t^{s,a} [p_{t+1}] = \frac{1}{r_f} \frac{(\bar{d} + md_t)}{1+m} - \frac{4\gamma (1+r_f)}{r_f (2+r_f)} (\Sigma + \Sigma_N). \quad (17)$$

Concerning the variance, there are two reasons for why the sum of the price and the dividends of the next periods can deviate from the perspective of the young generation.² The first reason is rational. It is a variation in the dividends, which is captured by Σ . The second reason can be found in the perceived noise traders' activity, which leads to a variation in price according to Σ_N . Thus, the total perceived variance in the future price plus dividend equals

$$Var_t^{s,y} (p_{t+1} + d_{t+1}) = \Sigma_N + \Sigma. \quad (18)$$

Now, I can insert these results into Equation (12) and receive the following prices

$$p_t = \frac{(1+r_f)}{r_f(2+r_f)} \left(\frac{\bar{d}_t}{1+r_f} + \frac{(\bar{d} + md_t)}{1+m} - 4\gamma (\Sigma + \Sigma_N) \right). \quad (19)$$

From this pricing function, some effects of the model become already obvious. For $m = 0$, the model describes the rational pricing mechanisms in mature markets. For $m > 0$, the model differs in one aspect: The period t dividend overly influences the price (through the adult generation), thereby creating excessive volatility.

²The adult generation can be treated in an analogous way.

Moreover, as already stated above, the price process described in Equation (19) varies over time with d_t . The first reason for this result is the variation of \bar{d}_t . This effect, though rational, decreases over time and, therefore, is negligible in mature markets. The second reason for the variation in prices is the result of availability bias. The term representing the availability bias equals md_t . Because all agents commit this behavioral bias after having entered the market, the effect of availability bias persists over time. Thus, even in mature markets, the importance of availability bias remains unchanged.

2 Multi-generation economy with availability bias

In the two generations model, the agents leave the market shortly after having learned about the dividends in the market. However, in real life, agents act on the market for longer periods. Thus, the agents have more time to observe data and revise their initial perceptions. Moreover, as agents live longer, the influence of past dividends also remains in the market for a longer period of time. Therefore, having illustrated the basic idea with the help of a two-generations model, I move forward to a multi generation model with more than two agents. This extended model is afterwards used to show the predictions made.

2.1 Beliefs about dividends

I consider a n generations model, in which individuals live for $n+1$ periods. As the nomenclature for the agents must now distinguish between more than two generations, I enumerate the generations, starting with the youngest agent (agent 1: a_1) and ending with the oldest agent (agent n : a_n). Moreover, now individuals have the opportunity to learn about the dividend process for more than one period. Therefore, I must specify the way in which individuals form their expectations in the subsequent periods.

As in the two-generations model, the youngest generation entering the market has rational expectations

$$E_t^{s,a_1} [d_{t+1}] = \bar{d}_t. \quad (20)$$

The expectations of the second youngest generation, which has been in the market for one period, correspond to those of the adult individuals in the two period model (compare Equation (3)). This is obviously the case, because both have the same experience background. For the older generations, which have been in the market for more than one period, I assume that these individuals place special weight on the average observed dividend. So taking these thoughts together we obtain the expectation of the older generation i , for $n \geq i \geq 2$ to be given by

$$E_t^{s,a_i} [d_{t+1}] = \frac{\bar{d}_t + m \cdot \bar{d}_{e,t}^{a_i}}{1 + m} \quad \text{for } n \geq i \geq 2. \quad (21)$$

In this notation, $\bar{d}_{e,t}$ is the mean observed dividend after the entrance of individuals at time t_e . Thus, for the second-youngest generation we obtain $\bar{d}_{e,t}^{a_2} = d_t$, for the third-youngest generation $\bar{d}_{e,t}^{a_3} = \frac{1}{2}(d_t + d_{t-1})$, for the fourth-youngest generation $\bar{d}_{e,t}^{a_4} = \frac{1}{3}(d_t + d_{t-1} + d_{t-2})$, and so forth. Thus, for the k th- youngest generation ($k \geq 2$), I obtain a mean observed dividend of

$$\bar{d}_{e,t}^{a_k} = \frac{1}{k-1} \sum_{i=2}^k d_{t-i+2}. \quad (22)$$

In this specification, a generation over-weights all past experienced dividends equally. However, this assumption is not of crucial importance and other weighting functions can be implemented easily. Moreover, as before in the two-generations model, I assume the volatility of the dividends and prices to be known

$$Var_t^{s,a_i} (p_{t+1} + d_{t+1}) = \Sigma_N + \Sigma \quad \text{for } 1 \leq i \leq n. \quad (23)$$

2.2 Portfolio choice and prices

The next step is to determine the optimal holdings in the risky asset with dependence on the individual's expectations. It is important to note that the youngest generation, which has the same expectations as the *young* generation in the two-generations model, now has an investment horizon of n periods. The second-youngest generation, which has the same expectations as the *adult* generation in the previous model, has a remaining investment horizon of $n - 1$ periods. As in the two period model, I assume that the agents do not take the interaction between future prices and their portfolio choice into account.³ I obtain the optimal amount of shares held by the generations as

$$x_t^{aj} = \frac{1}{2\gamma(1+r_f)^{(n-j)}\text{Var}_t^{aj}(p_{t+1} + d_{t+1})} \cdot (E_t^{s,aj}[p_{t+1} + d_{t+1}] - (1+r_f)p_t). \quad (24)$$

As in the two-generations model I calculate the prices by applying the market clearing condition. With n generations, it now equals

$$1 = \frac{1}{n} \sum_{j=1}^n x_t^{aj}. \quad (25)$$

I take the weighted average formulation, because it is generally insensitive to the number of generations in the economy. The resulting price process p_t has the form⁴

$$p_t = G \cdot \left(\left(\sum_{i=1}^n (H_i \cdot E^{s,ai}[p_{t+1} + d_{t+1}]) \right) - 1 \right). \quad (26)$$

³This assumption is made for the comprehensiveness of the general idea of the model. Through this assumption, the qualitative results do not change. The computations, not assuming the agents to be myopic, can be found in the Appendix. Appendix C treats the optimal portfolio choice in the case of N generations.

⁴The coefficient are given by $G = \left(\left(\sum_{i=1}^n \frac{1}{2n\gamma\text{Var}_t^{s,ai}(d_{t+1}+p_{t+1})(1+r_f)^{n-i}} \right) \cdot (1+r_f) \right)^{-1}$ and $H_i = \frac{1}{2n\gamma\text{Var}_t^{s,ai}(d_{t+1}+p_{t+1})(1+r_f)^{n-i}}$.

2.3 Beliefs about prices and realized prices

As before, individuals believe that they have drawn the correct conclusions from the observed dividends. Consequently, agents only take differences in the investment horizon into account, but not differences in opinion. When inferring about the price process, each agent assumes that all other agents share his/her opinion, apart from some (imaginary) noise traders, who distort prices. In my terminology, I can write this as

$$B^{a_j}(p_t) = \frac{E^{s,a_j}[p_{t+1} + d_{t+1}]}{1 + r_f} - \frac{2n\gamma}{r_f} \left(1 - \frac{1}{(1 + r_f)^n}\right) \text{Var}_t^{s,a_j}(d_{t+1} + p_{t+1}) + \epsilon_t^j. \quad (27)$$

For the multi-generation setup, I use the same solution approach as for the two-generations model. Again, every agent assumes to be rational. Therefore, each generation thinks that the expected price of the next periods equals the perceived *rational* price (meaning the price that is not distorted by the perceived noise traders)

$$E_t^{s,a_j}[p_{t+1}] = \frac{1}{r_f} E_t^{s,a_j}[d_{t+1}] - 2n\gamma \frac{(1 + r_f)^{(n-1)}}{(1 + r_f)^n - 1} (\Sigma + \Sigma_N). \quad (28)$$

The subjective expectations of future dividends are given by a weighted average of the mean dividend (from the beginning of the time series) and the mean observed dividend since market entrance. The final step in determining the equilibrium prices in this economy is inserting the expectations of individuals into the pricing Equation (26). In this way, I can specify the price at time t as a closed form expression of past dividends⁵

$$p_t = a \frac{m}{(1 + m)} \sum_{i=0}^{n-2} d_{t-i} b_i + c \cdot \frac{1}{(1 + m)} \bar{d}_t - f(\Sigma + \Sigma_N). \quad (29)$$

⁵The coefficients are given by $a = \frac{(1+r_f)^{n-1}}{(1+r_f)^n - 1}$, $b_i = \left(\sum_{j=(i+2)}^n \frac{1}{j-1} \frac{1}{(1+r_f)^{(n-j)}}\right)$, $c = \left(\frac{m}{(1+r_f)^{n-1}} + \frac{1}{r_f}\right)$ and $f = 2n\gamma \frac{(1+r_f)^{n-1}}{(1+r_f)^n - 1}$.

In this equation I already identify two effects that will become visible later in the simulation. First, the longer that individuals live, the longer that the extraordinary effect of a past dividend lasts. Second, as more generations become present, the initial overreaction decreases.

2.4 Trading volume and volatility

Because several anomalies of volatility and trading volume can be explained by my model, I now derive the corresponding analytic expressions.

Volatility

The variance is best calculated from Equation (29). I obtain⁶

$$Var(p_t) = \frac{t - n + 2}{t^2} \left(L_1 + \frac{1}{r_f(1+m)} \right)^2 \Sigma + \sum_{i=0}^{n-2} \left(L_1 + \frac{1}{r_f(1+m)} + L_2 M_i \right)^2 \frac{\Sigma}{t^2}. \quad (30)$$

This is generally larger than the price volatility in the case of full rationality. In the latter case ($m = 0$), the variance equals

$$Var^{m=0}(p_t) = \frac{1}{r_f^2 t} \Sigma. \quad (31)$$

Moreover, from Equation (30), I can see that the noise trader variance, which individuals infer prior to their market entrance, must be of order

$$\Sigma_N \approx \sum_{i=0}^{n-2} \left(\frac{(1+r_f)^{(n-1)}}{(1+r_f)^n - 1} \cdot \frac{m}{1+m} \cdot \left(\sum_{j=i+2}^n \frac{1}{j-1} \frac{1}{(1+r_f)^{(n-j)}} \right) \right)^2 \Sigma. \quad (32)$$

I ignore the randomness in \bar{d}_t because, in the strict sense, it is no *noise* and converges to zero for mature markets ($t \rightarrow \infty$).

Trading volume

⁶The coefficients are given by $L_1 = \frac{1}{(1+r_f)^{n-1}} \cdot \frac{m}{1+m}$, $L_2 = \frac{(1+r_f)^{(n-1)}}{(1+r_f)^{n-1}} \cdot \frac{m}{1+m}$ and $M_i = \sum_{j=i+2}^n \frac{1}{j-1} \frac{1}{(1+r_f)^{(n-j)}}$.

The trading activity of an individual can be defined as the change in his/her portfolio. Initially, individuals enter the market and choose their first portfolio, thereby maximizing their expected utility. After one period, the individual, now belonging to the second-youngest generation, has the opportunity to change his/her portfolio. The same happens after the second period with the individuals now belonging to the third youngest-generation. Generalizing this thought, I conclude that the trading volume of an individual after having been in the market for j periods (TV_j) can be expressed as

$$TV_t^j = x_{t+1}^{a_{j+1}} - x_t^{a_j}. \quad (33)$$

Deriving the trading volume as a function of dividends, I obtain

$$\begin{aligned} TV_t^j = & C \cdot \left(\frac{1}{1+m} F \left(\frac{1+r_f}{t+1} - \frac{1}{t} \right) \sum_{i=1}^t d_i + \frac{1}{1+m} F \frac{1+r_f}{t+1} d_{t+1} \right. \\ & + \frac{m}{1+m} F \left(\frac{1+r_f}{j} - \frac{1}{j-1} \right) \sum_{i=2}^j d_{t-i+2} + \frac{m}{1+m} F \frac{1+r_f}{j} d_{t+1} \\ & - E \cdot \left(\frac{1+r_f}{t+1} - \frac{1}{t} \right) \sum_{i=1}^t d_i + E \frac{1+r_f}{t+1} d_{t+1} \\ & \left. - B \cdot \sum_{i=0}^{n-2} A_i ((1+r_f)d_{t+1-i} - d_{t-i}) + H \right). \end{aligned} \quad (34)$$

The exact formulae for the terms A to H are derived in Appendix A. The first two lines of Equation (34) can be attributed to the change in the agent's expectation due to the observation of a new dividend. The new observed dividend affects the individuals' perception of the dividend process in two ways. The first is rationally founded, although decreasing over time (first line). The second can be attributed to availability bias. Nevertheless, it is not the absolute change in personal expectations that drives the trading volume but the change relative to the other agents. As with the individual expectation in the first two lines, again, two effects drive the average dividend expectation in the market. On the one hand, rational expectations changes due to the new observation (third line)

and on the other hand, the weighted irrational component in the market also differs (fourth line).

When going more into the details of this formula it becomes clear that there are mainly two factors that strongly influence the trading volume. The first factor is the deviation of the last observed dividend from the average. Also the deviation of prior dividends from the mean influences the trading volume, but the effect decreases for more distant observations. The second influencing factor on trading volume is the generation the agent is belonging to. Young agents trade more because they change their perception most relative to the average investor.

3 Predictions and effects of biased learning

The goal of this section is to show the predictions described in the introduction with the help of simulations. In this part I mainly focus on the long term asset pricing phenomena. These can be separated into pricing phenomena (e.g. overreaction and reversal), volatility patterns (excess volatility, volatility clustering, etc.) and connections between trading volume and volatility. Finally, I investigate agent specific predictions. I find that young investors trade more intensively than older investors and, as a result, obtain lower returns.

For the simulation I choose my parameters to best match empirical findings.⁷ However, in all cases, the results, especially the direction of the effects, are insensitive to the specific parameter values chosen. If not stated otherwise, I specify the dividend process by taking a mean dividend of 0.04 and a variance of $\Sigma = (0.01)^2$. Moreover, because my model does not consider inflation, I choose a real risk-free rate of $r_f = 0.01$. The absolute risk aversion parameter γ is calibrated such that I obtain a mean price of 1 for each number of generations n .⁸

⁷As reference papers I choose among others Campbell, Grossman, and Wang (1993), Shiller (1981), Bansal and Yaron (2004) and Goyal and Welch (2003).

⁸I choose γ in the basic calculations based on the number of generations between 1.8865 and 2.8770.

3.1 Prices overreact and revert

In this Section I provide an intuition for why the price process exhibits an overreaction and correction pattern. Moreover, the forces influencing the shape of the price process are identified.

When treating the two-generations model in Section 1 I observed that the price was perfectly correlated with the dividend stream. However, for more than two generations, this is no longer the case, because the effect of two (in the case of a three generations model) or more consequent dividend realizations overlay each other. To calculate the mean effect of one extreme dividend observation on the price process, I simulate $p = 1000$ price paths, which are independent from each other. Only at one point in time (after 100 periods) all paths are simulated to have one extreme dividend (two standard deviations below the mean). This separates the effect of one dividend realization on the price process. All other effects cancel out on average. I carry out this process for two-, three-, four-, five-, six- and eight-generations models. The simulated paths can be seen in Figure 2.⁹

When comparing the price path with availability bias and the price path without bias, I observe that with availability bias the price overreacts in the direction of the dividend deviation and bounces back in the subsequent periods. Moreover, the size of the overreaction and the length of the correction phase depends on the number of simulated generations.

While the overreaction in the period following the dividend payment directly stems from an overestimation of the dividend's relevance, the fading is caused by two different effects. First, in each period, one generation leaves, and one new, unbiased, generation enters the market. Thus, the effect of a period t dividend dies out as a result of old generations exiting the market. The second correcting effect can be attributed to the observation of new dividends in the subsequent periods. These new dividends reduce the effect of the time t dividend on the mean dividend while being

⁹Here, it should be noted that the dividends are multiplied by 25 to fit the values on the same scale.

in the market ($d_{e,t+i}$). The relative importance of these two effects depends upon the number of generations modeled or, more precisely on the length of an individual's trading life. For models with only a few generations, a relatively high percentage of the total population leaves the market in each period. So, in these models, the agents have already left the market before they have had the opportunity to learn something about the dividend stream based on their own observations. In contrast, for multiple-generations models, the dying out rate is significantly lower. As a result, most of the agents observe the following dividends and, thereby, correct their beliefs and bias. Taking both effects together it becomes clear, why the correction rate is higher in the periods directly following the overreaction, than in the periods that occur much thereafter. Nevertheless, although the effect diminishes a lot, the effect of the dividend observation only stops to overly influence the market, when the last agent, who observed the dividend, has retired.

3.2 Young agents trade most and obtain lower returns

In my model the youngest generation trades most (compare Table 1). In other words, the youngest generation disagrees most with the representative agent's interpretation of the new observation. This finding is consistent with the empirical results of Barber and Odean (2001), namely, that young investors trade more actively.

Having a closer look one observes that for the two-, three- and four-generation models, the trading volume decreases with age. In contrast, for the more than four generations models, the very old agents start trading again. They do not trade as much as the youngest investors, but still more than the median generation. This can be explained by the fact that older generations become increasingly insensitive to new dividend observations. As a result they also disagree with the representative investor, who places medium importance on the new observation. However, because in every period a new, rational generation enters the market, the representative agent is drawn back towards more rational attitudes. Consequently, the youngest generations trade most.

For the same reason a market with additional rational investors leads to an even higher trading volume of the young investors and decreases the trading volume of the older investors.

Young agents do not only trade more, but they also achieve lower returns. Because young agents' investment strategies are the most pro-cyclical, younger investors also suffer most from the mean reversion in prices. Therefore, the average return that a young agent obtains is low. Moreover, the average return increases with age, as illustrated in Table 2. The only exception is the very first generation, because the first generation is assumed to enter the market with rational expectations. Depending on this assumption, the performance of the first generation is not as low as that of the second generation. This initial assumption also influences the results of the two generations model. However, as I increase the number of generations the general pattern becomes more obvious: young agents tend to buy high and sell low, trade frequently, and thereby obtain lower returns. This is consistent with the empirical findings of Barber and Odean (2001) and Vissing-Jorgensen (2003).

3.3 Volatility and trading volume

Apart from the high trading volume (as shown in the previous part) also a high return volatility can be explained (compare Table 3). Because the most recent dividends are experienced by almost everyone in the market, these dividends overly influence current prices. This leads to an increase in price volatility and, consequently, more volatile returns. With my current model, an assumed dividend volatility of $(0.01)^2$, and availability bias parameters ranging between 0 and 0.3, I am able to obtain standard deviations of the returns of approximately 0.1 to 0.15. Assuming that in the real world, dividends are smoothed compared to earnings, I could obtain a high return volatility with even lower availability bias levels.¹⁰

The effect that returns vary more than they would under full rationality is shown in Figure 3. The variation in returns is much higher than the variation in dividends. Apart from a higher second

¹⁰For literature on dividend smoothing, see Lintner (1956).

central moment the distribution also has fatter tails with a kurtosis of greater than 3. However, the effect is not large in this basic model. Empirical evidence indicates that individuals memorize extreme events better than normal events. In this context large negative shocks are especially held in mind. Taking this into account, I obtain negatively skewed returns with a higher kurtosis (cf. Section 4).

Moreover, I obtain a significant autocorrelation in absolute returns for few-generations models (compare Table 4). This effect can be attributed to the mean reverting tendency in the price process. However, because the reaction to new observations decreases in the number of generations, the autocorrelation also decreases. To maintain the effect as clean as possible, in my model agents weight all of their observed dividends equally. However, it can be assumed that agents weight their latest observations more than their initial observations. In this way my model would still be able to maintain a high autocorrelation with a high number of generations.

The correlation between volatility (in my case measured by absolute return) and trading volume is positive (except for the two- and three-generations models). Thus, when returns are high, I can also expect a high trading volume. The correlation between return and trading volume has two influencing factors. First, when extreme dividends are realized, the agents disagree most regarding the valuation and influence of this observation. This leads to a high trading volume. Apart from this main effect there is another effect, which works in the opposite direction - especially for models with few generations. This can be attributed to agents entering with rational expectations and becoming more affected by observations. For example, in a two-generations model, when an extremely high dividend is followed by an extremely low dividend, the absolute return is very high, but the trading volume is low. This can be explained by the fact, that when the extremely high dividend has occurred in the previous period, a newly entering agent is less affected than the agents having experienced that dividend. Therefore, it does not buy a lot of shares. If the high dividend is then followed by a very low dividend, the agent now experiences it. As it is the only dividend observed

by the agent, he/she does not increase his/her number of shares. Therefore, the trading volume is low. However, in the real world, an agent is likely to observe more than two dividends in his/her lifetime. This favors the model with more generations in which the correlation is positive.

3.4 Returns on high trading volume dates tend to revert

Finally, I investigate the interaction between volatility and the direction of price movements. In this model I find that returns on high trading volume days tend to revert themselves. I define high trading volume dates as those, in which the trading volume lies in the upper decile. Table 6 shows the percentage of negatively correlated returns on subsequent days. I take that percentage on high trading volume days and compare it to the unconditional percentage of negatively correlated returns.

The unconditional percentage of negatively correlated returns decreases in the number of generations. This can be explained by the fact that for more generations the initial overreaction is lower and the fade out time lasts longer. As discussed above, in the fade out phase the returns are positively correlated. Therefore, the unconditional percentage of negatively correlated returns decreases.

The percentage of negatively correlated returns on high trading volume days is higher than the unconditional percentage of negatively correlated returns. The intuition behind this finding is that on high trading volume dates the price changes are more likely to be caused by an initial overreaction than by a reversal. However, with respect to the number of generations this effect is hump-shaped. The increasing part of the hump from two generations to four generations can be explained by the fact that the correction is (almost) as immediate as the overreaction. This causes a high trading volume in both parts. The decreasing part of the hump from four generations to eight generations, can be explained by a similar idea as for the unconditional probability. For really large deviations of the dividend from its mean, there is a high trading volume also in the early stages of

the reversion process. As this lasts longer, the number of positively correlated returns increases.

4 Extensions of the model and further explanations of financial anomalies

With the basic dividend model described in this paper, I wish to convey the effect of availability bias in the most general framework, namely, a dividend economy. Among others, I am able to explain several financial anomalies, such as excessive volatility, price overreaction and correction, and excessive trading volume. However, the explanatory potentials of this idea go far beyond those derived in this basic model.

Non-constant population (intraday trading)

Until now, I have mostly concentrated on the long-term effects of availability bias. However, I can also interpret the market entrance on an intraday basis. Then, we obtain the effect that, in particular, for multi-listed shares the “population” is not constant. Whenever a new market opens, a large number of new investors with different experience histories enters the market. My model predicts the trading volume and the volatility to be especially high throughout the opening hours of a stock exchange. This prediction holds for both the opening market and the markets that are already trading.

Availability through news

First, the basic model mainly addresses learning by experience. However, there are other factors that influence how available the market performance (in my model, in the form of dividends) of the respective dates is in the individuals’ mind. In the basic model, all past dividend experiences are equally weighted. However, the latest market performance should be more available than the market performance at the time of market entry (especially if the time of entry was several periods

beforehand).

Therefore, apart from the former factor $\bar{d}_{e,t}$, which is the mean dividend since market entrance, I introduce another factor $\bar{d}_{a,t}$, which is a weighted average of the latest 3 time periods. This can be interpreted as the availability created through news and current hot topics. Thus, the individual expectations for the youngest generation can be specified as

$$E_t^{s,a_1} = \frac{(1+m) \cdot \bar{d}_t + f \cdot \bar{d}_{a,t}}{1+f+m}. \quad (35)$$

The expectations of the older generations are consistently specified as

$$E_t^{s,a_j} = \frac{\bar{d}_t + m \cdot \bar{d}_{e,t} + f \cdot \bar{d}_{a,t}}{1+f+m}. \quad (36)$$

In this way, autocorrelation is relatively stable between models with different numbers of generations.

Differences in reactions to news

Empirical studies show that extreme information is more likely to be retained in individuals' mind than average events. Moreover, extremely negative shocks are memorized best (compare McGraw, Larsen, Kahneman, and Schkade (2010)). This can be incorporated into the basic model in such a way that extreme and, in particular, extremely negative events become specially weighted. In the context of my model, I could implement this insight by placing a greater weight on the observed dividends when they are extreme (with an even greater weight on extremely low dividends)

$$\bar{d}_{e,t}^{a_k} = \left(\sum_{i=2}^k w_{t-i+2} \right)^{-1} \left(\sum_{i=2}^k w_{t-i+2} \cdot d_{t-i+2} \right). \quad (37)$$

The weights are given by the following formula ($0 < a < b$)

$$w(d_t) = \begin{cases} (1+a) & d_t > \bar{d}_t + \sigma \\ 1 & \bar{d}_t - \sigma \leq d_t \leq \bar{d}_t + \sigma \\ (1+b) & d_t < \bar{d}_t - \sigma. \end{cases} \quad (38)$$

In this way, the return distribution becomes negatively skewed with even fatter tails than in the basic model.

Differences in the degree of availability bias

In markets, some investors are more familiar with background statistics than others and are better supported by financial software. In particular, private investors often must rely on their own intuition, which is very susceptible to biases, such as availability bias. This can be incorporated into the basic model by adding investors with different realizations of m to my economy. Thus, the market clearing condition (25) is modified to

$$1 = \frac{1}{n+r} \left(\sum_{j=1}^n x_t^{a_j} \right) + \frac{r}{n+r} x_t^{a_1}. \quad (39)$$

In this way, the phenomenon that many private investors start investing after some favorable news and returns can be explained.

5 Conclusion

This paper embeds the idea that agents mostly learn out of experienced data in an overlapping generations equilibrium model. The market entrance of individuals determines which past dividends were directly experienced by the individuals and, therefore, which dividends are especially anchored in the individuals' mind. Young generations have recently entered the market and thus have had

comparatively little time to learn and experience dividends in the market. Consequently, new dividend observations have a great impact on the young agents' perception of the dividend process. In contrast, older generations have had the opportunity to observe several dividends throughout their lives. Consequently, the older generation's perception of the dividend stream varies less when new information arises.

Because the dividend stream is the only information that is accessible and observable by everyone, every agent can assume that he/she has drawn the correct consequences out of his/her observations. Thus, the differences in reaction must be attributed to noise trading, resulting in a willingness to trade.

In this model setup a lot of long term asset pricing puzzles can be explained and new intraday predictions can be made. For examples it provides an intuition for why returns on high trading volume dates tend to reverse themselves. Moreover, it explains a connection between trading volume and volatility and other asset pricing patterns. Finally, it predicts a high trading volume around the opening hours of a stock exchange, which is especially pronounced for cross-listed stocks.

In contrast to other behavioral biases, such as overconfidence and self-attribution bias, the effects of heuristic learning and availability bias are rather unexplored in the financial research. However, as this paper demonstrates, incorporating this empirical evidence can explain many puzzles in financial markets. Therefore, further research in this area, and a deeper understanding of what information is most available to agents, can lead to better comprehension of financial markets.

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A Calculations for volatility and trading volume

The trading volume after period j can be described as

$$TV_t^j = x_{t+1}^{a_{j+1}} - x_t^{a_j}. \quad (40)$$

Now I insert the holdings in the risky asset, derived in Equation (24) into Equation (40) and get

$$TV_t^j = \frac{1}{2\gamma(1+r_f)^{(n-j-1)}Var_{t+1}^{a_{j+1}}(p_{t+2} + d_{t+2})} \cdot (E_{t+1}^{a_{j+1}}[p_{t+2} + d_{t+2}] - (1+r_f)p_{t+1}) \quad (41)$$

$$- \frac{1}{2\gamma(1+r_f)^{(n-j)}Var_t^{a_j}(p_{t+1} + d_{t+1})} \cdot (E_t^{a_j}[p_{t+1} + d_{t+1}] - (1+r_f)p_t).$$

First, I replace $E_t^{s,a_i}[p_{t+1}]$ by the Equation (28) and $Var_t^{s,a_i}(p_{t+1} + d_{t+1})$ by specification (23) to obtain

$$TV_t^j = \frac{1}{2\gamma(\Sigma + \Sigma_N)(1+r_f)^{n-j}} \cdot \left(-(1+r_f)^2 p_{t+1} + (1+r_f)p_t + (1+r_f)\left(1 + \frac{1}{r_f}\right)E_{t+1}^{a_{j+1}}[d_{t+2}] \right. \quad (42)$$

$$\left. - \left(1 + \frac{1}{r_f}\right)E_t^{a_j}[d_{t+1}] - 2n\gamma r_f \frac{(1+r_f)^{n-1}}{(1+r_f)^n - 1}(\Sigma + \Sigma_N) \right).$$

In other words, the trading volume depends on both, the changes in prices and the changes in expected dividends. For the prices I get the following expression by inserting Equation (29)

$$(1+r_f)p_t - (1+r_f)^2 p_{t+1} = \quad (43)$$

$$\frac{m(1+r_f)^n}{(1+m)((1+r_f)^n - 1)} \cdot \sum_{i=0}^{n-2} \left(\sum_{j=i+2}^n \frac{1}{j-1} \frac{1}{(1+r_f)^{n-j}} \right) (d_{t-i} - (1+r_f)d_{t+1-i})$$

$$+ \left(\frac{m}{(1+r_f)^n - 1} + \frac{1}{r_f} \right) \frac{1}{(1+m)} \left(\frac{1}{t} - (1+r_f)\frac{1}{t+1} \right) \sum_{i=1}^t d_i$$

$$- \left(\frac{m}{(1+r_f)^n - 1} + \frac{1}{r_f} \right) \frac{1}{(1+m)} (1+r_f) \frac{1}{t+1} d_{t+1} + 2n\gamma \frac{(1+r_f)^n}{(1+r_f)^n - 1} r_f (\Sigma + \Sigma_N).$$

The dividend expectations are specified in Equation (21). Consequently, I obtain

$$(1+r_f)E_{t+1}^{a_{j+1}}[d_{t+2}] - E_t^{a_j}[d_{t+1}] = \tag{44}$$

$$\frac{1}{(1+m)} \left(\frac{1+r_f}{t+1} - \frac{1}{t} \right) \sum_{i=1}^t d_i + \frac{(1+r_f)}{(1+m)(t+1)} d_{t+1} + \frac{m}{(1+m)} \left(\frac{1+r_f}{j} - \frac{1}{j-1} \right) \sum_{i=2}^k d_{t-i+2} \frac{m}{1+m} \frac{1+r_f}{j} d_{t+1}.$$

Inserting Equation (43) and (44) into Equation (42) I obtain

$$TV_t^j = C \cdot \left(B \cdot \sum_{i=0}^{n-2} A_i (d_{t-i} - (1+r_f)d_{t+1-i}) + E \cdot \left(\frac{1}{t} - \frac{1+r_f}{t+1} \right) \sum_{i=1}^t d_i + E \frac{1+r_f}{t+1} d_{t+1} \right) \tag{45}$$

$$+ \frac{1}{1+m} F \left(\frac{1+r_f}{t+1} - \frac{1}{t} \right) \sum_{i=1}^t d_i + \frac{1}{1+m} F \frac{1+r_f}{t+1} d_{t+1}$$

$$+ \frac{m}{1+m} F \left(\frac{1+r_f}{j} - \frac{1}{j-1} \right) \sum_{i=2}^j d_{t-i+2} + \frac{m}{1+m} F \frac{1+r_f}{j} d_{t+1} + H),$$

with

$$A_i = \sum_{j=i+2}^n \frac{1}{j-1} \frac{1}{(1+r_f)^{n-j}}, \quad B = \frac{m(1+r_f)^n}{(1+m)((1+r_f)^n - 1)}, \tag{46}$$

$$C = \frac{1}{2\gamma(\Sigma + \Sigma_N)(1+r_f)^{n-j}}, \quad E = \left(\frac{m}{(1+r_f)^n - 1} + \frac{1}{r_f} \right) \frac{1}{(1+m)}, \tag{47}$$

$$F = 1 + \frac{1}{r_f}, \quad H = 2n\gamma r_f^2 \frac{(1+r_f)^{n-1}}{(1+r_f)^n - 1} (\Sigma + \Sigma_N). \tag{48}$$

B Non-myopic agents - 2 generations

I solve this model in a rational expectations equilibrium with CARA utility functions and a Gaussian-Normal environment. The solution technique is similar to the one used in Albagli (2012).

I assume that

$$p_{t+1} = \bar{p} + \sigma_p \epsilon_{p1} \quad p_{t+2} = \bar{p} + \sigma_p \epsilon_{p2} \quad d_{t+1} = \bar{d} + \sigma_d \epsilon_{d1} \quad d_{t+2} = \bar{d} + \sigma_d \epsilon_{d2}$$

The resulting wealth at time $t = 2$ is given by:

$$\begin{aligned} w_{t+2} &= \frac{1}{\gamma(\sigma_p^2 + \sigma_d^2)} \cdot (\bar{p} + \bar{d} - (1+r) \cdot (\bar{p} + \sigma_p \epsilon_{p1})) \\ &\quad \cdot (\bar{p} + \sigma_p \epsilon_{p2} + \bar{d} + \sigma_d \epsilon_{d2} - (1+r_f)(\bar{p} + \sigma_p \epsilon_{p1})) \\ &\quad + (1+r_f)x_t^y((\bar{p} + \sigma_p \epsilon_{p1} + \bar{d} + \sigma_d \epsilon_{d1}) - (1+r_f)p_t) + (1+r_f)^2 w_t^y \end{aligned}$$

I obtain $w_{t+2} = c + b'\epsilon + \epsilon' A \epsilon$ where $\epsilon = \begin{pmatrix} \epsilon_{d1} & \epsilon_{d2} & \epsilon_{p1} & \epsilon_{p2} \end{pmatrix}^T$ with

$$\begin{aligned} c &= \frac{[\bar{d} - r_f \bar{p}]^2}{\gamma(\sigma_p^2 + \sigma_d^2)} + (1+r_f)^2 w_t^y + (1+r_f)(\bar{d} + \bar{p} - (1+r_f)p_t)x_t^y \\ b &= \begin{pmatrix} b_1^x x_t^y & b_2 & b_3^x x_t^y + b_3 & b_4 \end{pmatrix}^T \end{aligned}$$

with

$$b_1^x = (1+r_f)\sigma_d, b_2 = \frac{[\bar{d} - r_f \bar{p}]\sigma_d}{\gamma(\sigma_p^2 + \sigma_d^2)}, b_3^x = (1+r_f)\sigma_p, b_3 = -2\frac{(1+r_f)\sigma_p[\bar{d} - r_f \bar{p}]}{\gamma(\sigma_p^2 + \sigma_d^2)}, \text{ and } b_4 = \frac{[\bar{d} - r_f \bar{p}]\sigma_p}{\gamma(\sigma_p^2 + \sigma_d^2)}$$

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & a_1 & a_2 & 0 \\ 0 & 0 & a_3 & 0 \end{pmatrix}$$

with

$$a_1 = -\frac{(1+r_f)\sigma_p\sigma_d}{\gamma(\sigma_p^2 + \sigma_d^2)}, a_2 = \frac{(1+r_f)^2\sigma_p^2}{\gamma(\sigma_p^2 + \sigma_d^2)}, a_3 = -\frac{(1+r_f)\sigma_p^2}{\gamma(\sigma_p^2 + \sigma_d^2)}$$

From Vives (2010) I know

$$E[-\exp(-\gamma w)] = -(\det(\Sigma))^{-\frac{1}{2}}(\det(\Sigma^{-1} + 2\gamma A))^{-\frac{1}{2}} \cdot \exp(-\gamma(c - \gamma b'(\Sigma^{-1} + 2\gamma A)^{-1}b/2)) \quad (49)$$

In my case $\Sigma = I_4$ and I obtain

$$(I_4 + 2\gamma A)^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -\frac{2\gamma a_1}{1+2\gamma a_2} & \frac{1}{1+2\gamma a_2} & 0 \\ 0 & \frac{4\gamma^2 a_1 a_3}{1+2\gamma a_2} & -\frac{2\gamma a_3}{1+2\gamma a_2} & 1 \end{pmatrix}$$

When I want to derive the FOC for Equation (49) I must obtain that the derivative of $c - \frac{\gamma}{2}b'(\Sigma^{-1} + 2\gamma A)^{-1}b$ with respect to x_t^y should be 0. Therefore, I get

$$\frac{\partial c}{\partial x} - \frac{\gamma}{2}(2b_1^2 x_t^y - 2\gamma \frac{b_3^x a_1 b_2}{1+2\gamma a_2} + 2\frac{b_3 b_3^x}{1+2\gamma a_2} + 2\frac{(b_3^x)^2}{(1+2\gamma a_2)} x_t^y - 2\gamma \frac{a_3 b_4 b_3^x}{(1+2\gamma a_2)}) = 0$$

Solving for x_t^y I obtain $x_t^y = M[\bar{d} + \bar{p} - (1+r_f)p_t] + N$ where M and N are given by

$$M = \frac{(1+2\gamma a_2)}{\gamma(1+r_f)(\sigma_d^2(1+2\gamma a_2) + \sigma_p^2)}, \quad N = \frac{(1+r_f)}{\gamma(\sigma_d^2(1+2\gamma a_2) + \sigma_p^2)} \frac{\sigma_p^2}{\sigma_p^2 + \sigma_d^2} [\bar{d} - r_f \bar{p}]$$

C Non-myopic agents - N generations

I conjecture the value function

$$\begin{aligned} J(W_{t+1}, M_{t+1}, j+1, t+1) &= \exp(-\alpha_{j+1}W_{j+1} - \frac{1}{2}M_{t+1}V_{j+1}M_{t+1}) \\ &= \exp(-\alpha_{j+1}RW_j - \alpha_{j+1}Q_{t+1}X_{j,t} - \frac{1}{2}M_{t+1}V_{j+1}M_{t+1}) \end{aligned}$$

In this case M_t is the state vector $M_{t+1} = A_m M_t + B_m \epsilon_{t+1}$ and $M_t = \begin{pmatrix} 1 & \sigma_p \epsilon_t^p & \sigma_d \epsilon_t^d \end{pmatrix}$ with

$$A_m = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad B_m = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \sigma_p & 0 \\ 0 & 0 & \sigma_d \end{pmatrix}$$

Q_t is the excess return $Q_{t+1} = D_{t+1} + P_{t+1} - RP_t = A_Q M_t + B_Q \epsilon_{t+1}$ with

$$A_Q = \begin{pmatrix} \bar{d} - r_f \bar{p} & -(1+r_f) & 0 \end{pmatrix} \quad \text{and} \quad B_Q = \begin{pmatrix} 0 & \sigma_p & \sigma_d \end{pmatrix}.$$

I obtain

$$\begin{aligned} J(W_t, M_t, j, t) &= \exp(- \alpha_{j+1}RW_{j,t} - \alpha_{j+1}XA_Q M_t - \frac{1}{2}M_t^T V^{AA} M_t \\ &\quad + \frac{1}{2}(\alpha_{j+1}XB_Q + M^t V^{AB}) \Xi (V^{AB} M^t + B_Q X \alpha_{j+1}) \end{aligned}$$

Deriving it with respect to X I obtain

$$X = \frac{A_Q - B_Q \Xi V^{AB}}{\alpha_{j+1} \Gamma_{j+1}} M_t.$$

From the Value function I can infer the value of the variable V

$$\begin{aligned}
 V_t = & - \frac{1}{2\Gamma} V^{AB,T} \Xi^T B_Q^T B_Q \Xi V^{AB} - \frac{1}{2\Gamma} A_Q^T A_Q \\
 & + \frac{1}{\Gamma} A_Q^T B_Q \Xi V^{AB} - \frac{1}{2} V^{AA} + \frac{1}{2} V^{AB,T} \Xi V^{AB}
 \end{aligned}$$

where

$$V^{AB} = A_m^T V_{t+1} B_m, V^{AA} = A_m^T V_{t+1} A_m, \Gamma = B_Q^T \Xi B_Q, \text{ and } \Xi = (\Sigma + B_m V_{t+1} B_m)^{-1}.$$

Trading after Period	n-Generations Model					
	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 6$	$n = 8$
1	3.16	2.79	2.76	2.83	2.89	3.08
2	-	1.07	1.35	1.54	1.67	1.91
3	-	-	0.84	0.67	0.60	0.62
4	-	-	-	1.01	0.81	0.53
5	-	-	-	-	1.07	0.73
6	-	-	-	-	-	0.90
7	-	-	-	-	-	1.02

Table 1: Trading Volume. This table illustrates the average absolute trading volume after period j . The trading volume TV_j is defined as in Equation (33). For the simulation I choose the basic parametrization, i.e., a mean dividend of 0.04 and a standard deviation of 0.01. The availability bias parameter m equals 0.2. The risk aversion parameters are chosen such that the prices are normalized to 1.

Excess return when belonging to generation	n-Generations Model					
	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 6$	$n = 8$
1	0.292	0.235	0.184	0.149	0.118	0.092
2	-0.215	-0.106	- 0.069	-0.052	-0.052	-0.028
3	-	-0.018	- 0.005	-0.001	-0.010	0.001
4	-	-	0.031	0.026	0.014	0.018
5	-	-	-	0.046	0.030	0.030
6	-	-	-	-	0.042	0.038
7	-	-	-	-	-	0.045
8	-	-	-	-	-	0.051

Table 2: Individual Performance. This table illustrates the average individual excess return of an agent belonging to generation j . For the simulation, I select the basic parametrization, i.e., a mean dividend of 0.04 and a standard deviation of 0.01. The availability bias parameter m equals 0.2. The risk aversion parameters are chosen such that the prices are normalized to 1.

σ_{return} for overreaction parameter	n-Generations Model					
	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 6$	$n = 8$
$m = 0$	0.012	0.012	0.012	0.012	0.012	0.012
$m = 0.10$	0.077	0.069	0.062	0.055	0.052	0.043
$m = 0.20$	0.134	0.120	0.105	0.094	0.087	0.070
$m = 0.30$	0.187	0.165	0.144	0.127	0.118	0.094

Table 3: Return Standard Deviation. This table shows the standard deviation in returns for varying availability bias parameters. The dividend stream is simulated with a mean of 0.04 and a standard deviation of 0.01. The risk aversion parameters are chosen such that the prices are normalized to 1.

Autocorrelation in σ_{return}	n-Generations Model					
	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 6$	$n = 8$
lag = 1	0.214	0.078	0.058	0.049	0.041	0.036
lag = 2	0.000	0.036	0.014	0.012	0.010	0.010
lag = 3	0.000	0.000	0.010	0.007	0.006	0.005

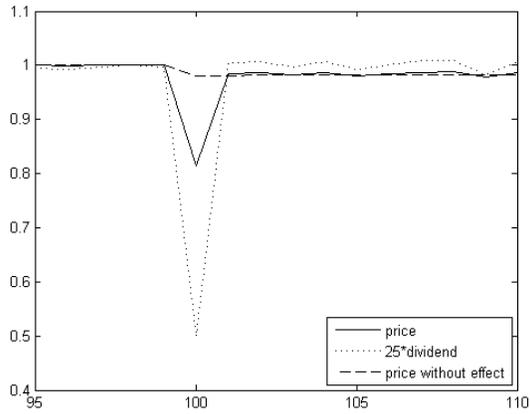
Table 4: Volatility Clustering. This table shows the autocorrelation in absolute returns for up to three lags. For the simulation I chose the basic parametrization, that is, a mean dividend of 0.04 and a standard deviation of 0.01. The availability bias parameter m equals 0.2. The risk aversion parameters are chosen such that the prices are normalized to 1.

σ_{return}	n-Generations Model					
	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 6$	$n = 8$
$\text{Cor}(TV, -r_a)$	0.01	0.09	0.40	0.62	0.73	0.84

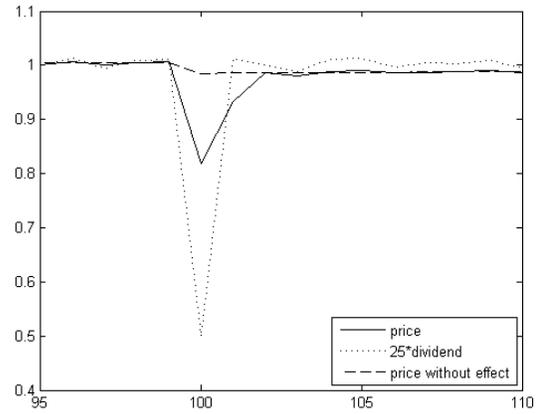
Table 5: Correlation between Trading Volume and Absolute Returns. This table shows the correlation between trading volume and absolute returns for a varying number of generations. The underlying dividend stream is simulated with a mean of 0.04 and a standard deviation of 0.01. The availability bias parameter is equal to $m = 0.2$. The risk aversion parameters are chosen such that the prices are normalized to 1.

Percentage of negatively correlated returns	n-Generations Model					
	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 6$	$n = 8$
High Trade	60.9%	68.4%	74.0%	73.7%	71.1%	66.4%
All	61.3%	54.2%	51.7%	49.9%	48.4%	45.0%

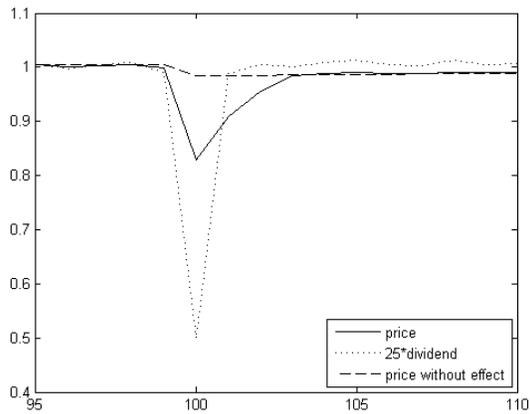
Table 6: High Trading Volume Returns Revert Themselves. This table shows the percentage of negatively correlated returns. I distinguish between high trading volume returns and all return. The dividend stream is simulated with a mean of 0.04 and a standard deviation of 0.01. The risk aversion parameters are chosen such that the prices normalize to 1.



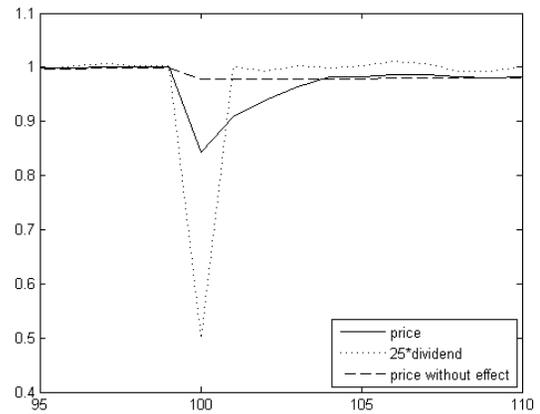
(a) Plot for Two Generations



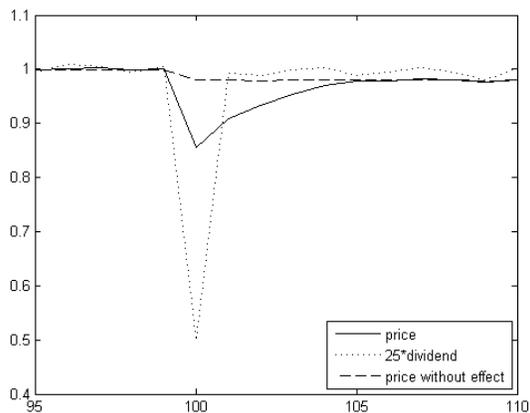
(b) Plot for Three Generations



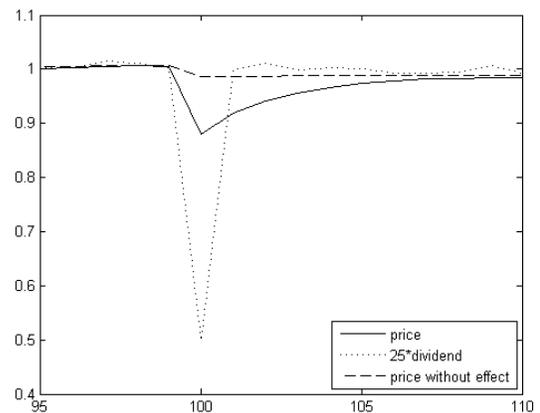
(c) Plot for Four Generations



(d) Plot for Five Generations

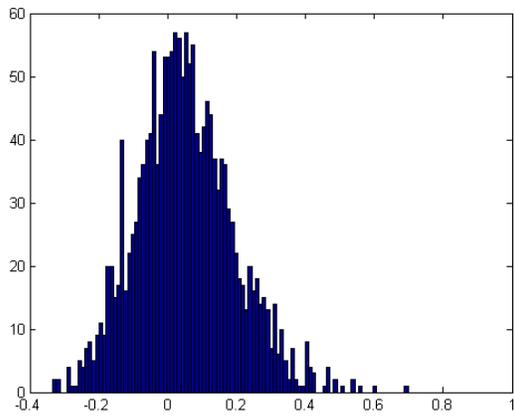


(e) Plot for Six Generations

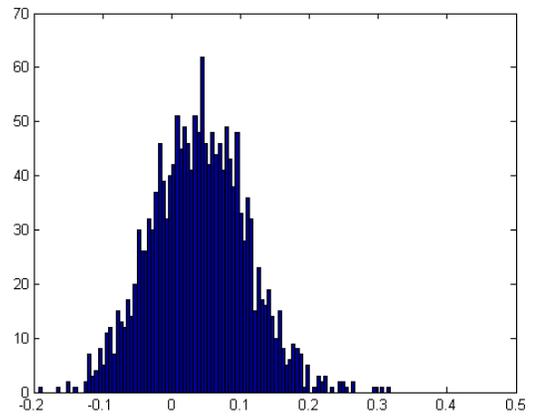


(f) Plot for Eight Generations

Figure 2: Simulation of Price Process. This figure illustrates the average price reaction to an extraordinary low dividend (2 standard deviations below the mean). The average is from $p = 1000$ simulations and after period 100. For the simulation I choose a mean dividend δ of 0.04 and a standard deviation of 0.01. The availability bias parameter m equals 0.2. The risk aversion parameters are chosen such that the prices are normalized to 1.



(a) Return Histogram for Two Generations



(b) Return Histogram for Eight Generations

Figure 3: Return Histogram. This figure shows a return histogram for the two- and the eight-generations models. The dividend stream is simulated with a mean of 0.04 and a standard deviation of 0.01. The risk aversion parameters are chosen such that the prices normalize to 1.