# Investment and The Cross–Section of Equity Returns*


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This version: March 10, 2014
Preliminary and Incomplete
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**Abstract**

In absence of operating leverage, the neoclassical model of investment, equipped with a standard stochastic discount factor, produces (i) a counterfactual positive correlation between size and book-to-market and (ii) a counterfactual unconditional value discount. That is, so-called value firms (high book-to-market) earn on average a lower return than growth firms (low book-to-market). This happens because a greater fraction of the value of low book–to–market firms comes from growth options, which are riskier than assets in place. Adding operating leverage is necessary but not sufficient to overturn these results. With capital adjustment costs disciplined by the evidence on firms' investment rates and operating leverage set to match the unconditional average book–to–market, the model still fails to generate a value premium, both unconditionally and conditionally on size.



**Key words:** Asset Pricing, Value Premium, Operating Leverage.

**JEL Codes:** D24, D92, G12.



*We are grateful to Martin Boileau, Lars Lochstoer, Sydney Ludvigson, Stijn Van Nieuwerburgh, as well as seminar attendants at NHH Bergen, Boston University, Federal Reserve Bank of St. Louis, NYU, University of Southern California, Canadian Macro Study Group, EUI Macro Conference, NY/Philly Workshop on Quantitative Macroeconomics, Society for Economic Dynamics, European Finance Association, and Midwest Macro Meeting for their comments and insights. Marco Casiraghi provided excellent research assistance. All remaining errors are our own responsibility.



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# 1 Introduction

Beginning with Berk, Green, and Naik (1999), a rapidly amassing literature has tried to rationalize the cross–sectional variation in equity returns by characterizing the asset pricing implications of models of firm–level investment. Recent contributions to this body of work include Gomes, Kogan, and Zhang (2003), Carlson, Fisher, and Giammarino (2004), and Zhang (2005).

This paper contributes to the literature in two ways. First, we provide an exhaustive characterization of the implications for the cross–section of equity returns that derive from a rather canonical model of investment. The main insights are that counterfactually, the basic version of the model produces (1) a positive unconditional correlation between size and book-to-market and 2) a value discount – firms with low book–to–market earn higher expected returns than firms with high book–to–market; 3) operating leverage is a necessary although not sufficient condition for the model to generate a value premium; 4) when parameter values are chosen to match key cross-sectional moment of the distribution of investment rates and operating leverage is set to match the unconditional mean book-to-market value, the model still fails to generate a value premium; 5) if returns to scale are set counterfactually low, allowing for greater leverage, the model generates a small premium, due mostly to small-size firms in bad aggregate states.

The cross–sectional variation is driven by persistent shocks to idiosyncratic productivity. These shocks are orthogonal to innovations in the stochastic discount factor. Nonetheless, they are systematically associated with expected returns because they pin down the firm's relative exposure to current versus future cash–flow risk.

A firm with low current idiosyncratic productivity commands little capital and produces a small cash–flow, but has a high expected productivity growth. Its valuation derive for the most part from continuation cash–flows, which are riskier than current cash flows. In turn, this implies that on average low idiosyncratic productivity is associated with small size, low book–to–market, and high systematic risk.

Assuming that firms incur a constant and common operating cost changes the mapping between idiosyncratic productivity and book–to–market. Firms with low idiosyncratic productivity now have high book–to–market, especially conditional on the realization of an adverse aggregate state. This is necessary for the model to generate an unconditional value premium.

Whether operating leverage is large enough to generate a sizeable value premium, it is a quantitative question. Our calibration strategy follows the approach of macroeconomics studies with firm heterogeneity, i.e. we require that the model is consistent with key moments of the cross-sectional distribution of the investment rate and with average aggregate book–to–market. The result is that the model fails to generate non–negligible dispersion in expected returns.

When, say, returns to scale are set at a counterfactually low level, the calibration strategy allows operating leverage to grow to a level consistent with a value premium, although of a magnitude much smaller than in the data. In such scenario, the model also generates a value premium conditional on size. For given market capitalization, firms with higher book to market have greater capital and lower idiosyncratic productivity. Low productivity is associated with high risk, because it signals that (i) most value comes from growth options and (ii) operating leverage bites harder. However, these firms will be shedding capital with the objective of reaching a scale consistent with their productivity. The proceeds from the sale of capital will accrue to shareholders regardless of the aggregate state of nature, therefore lowering risk and expected return. A high enough operating leverage implies that the former of these channels prevails, inducing a conditional value premium.

Under these circumstances, the value premium originates mostly from small firms in bad aggregate states, consistent with the evidence.

The remainder of the paper is organized as follows. Section 2 presents a stylized model that helps shaping our intuition about the role played by operating leverage in generating the value premium. A fully–fledged model is introduced in Section 3. Its implications for the cross–section of equity returns are analyzed in Section 4. Section 5 is dedicated to comparative statics exercises. We identify parameter assumptions under which the model does generate a small value premium. Under those assumptions, Section 6 we describes the cyclical properties of the value premium. Section 7 concludes.

## 2 A Three–Period Model

In this section, we lay out a simple three–period model of investment and we explore analytically its implications for the cross–section of equity returns. The time periods are indexed by $t = 0, 1, 2$. For $t = 1, 2$, firms produce output by means of $y_t = e^{s_t+z_t}k_t^{\alpha}$, where $\alpha \in (0, 1)$ and $k_t$ denotes the capital stock installed at $t-1$. That is, we assume one–period time–to–build and full depreciation. Dividends equal cash flows minus investment.

The variables $s_t$ and $z_t$ denote the idiosyncratic and aggregate components of productivity, respectively. Both evolve according to first–order autoregressive processes and independent, normally distributed innovations. That is,

$$s_{t+1} = \rho_s s_t + \varepsilon_s, \quad \varepsilon_s \sim N(\mu_s, \sigma_s^2),$$
$$z_{t+1} = \rho_z z_t + \varepsilon_z, \quad \varepsilon_z \sim N(\mu_z, \sigma_z^2),$$

where $\rho_s, \rho_z \in (0, 1)$. Firms evaluate future cash flows according to the stochastic discount factor $M(z_t, z_{t+1})$, where

$$\log M(z_t, z_{t+1}) = \gamma(z_t - z_{t+1}), \quad \gamma > 0.$$

It follows that, conditional on capital $k_1$ and productivity levels $\{s_1, z_1\}$, the value of equity at $t = 1$ is

$$V_1(k_1, s_1, z_1) = \max_{k_2} \; e^{s_1+z_1}k_1^{\alpha} - k_2 + E_1[M(z_1, z_2)e^{s_2+z_2}k_2^{\alpha}],$$

where the linear operator $E_s$ denotes the expectation taken conditional on the information known at $t = s$. As of $t = 0$, equity is worth

$$V_0(k_0, s_0, z_0) = \max_{k_1} \; -k_1 + E_0[M(z_0, z_1)V_1(k_1, s_1, z_1)].$$

## 2.1 Characterization

Define the the expected equity return at time $t$ as the expected value of cash–flows to equity–holders divided by the ex–dividend market value. Then, we can write the expected return on equity at time $t = 0$ as

$$E_0[R_1] = \frac{E_0[V_1(k_1, s_1, z_1)]}{E_0[M(z_0, z_1)V_1(k_1, s_1, z_1)]}. \tag{1}$$

With some abuse of notation, rewrite (1) as

$$E_0[R_1] = \frac{E_0[y_1] + E_0[-k_2 + E_1[M_2 y_2]]}{E_0[M_1 y_1] + E_0[M_1[-k_2 + E_1[M_2 y_2]]]}.$$

The first term at the numerator is the expected value of $t = 1$ cash flow, while the second term is the expected value of $t = 2$ cash flows. The denominator is nothing but the market value of the two streams.

We can express equity as a portfolio of two risky assets which pay off at time $t = 1$ and $t = 2$, respectively. We will refer to them as current assets and continuation assets, respectively. The loading on current assets, which we denote as $x(s_0, z_0)$, is the fraction of equity value accounted for by current assets, or

$$x(s_0, z_0) = \frac{E_0[M_1 y_1]}{E_0[M_1 y_1] + E_0[M_1[-k_2 + E_1[M_2 y_2]]]}.$$

Since the expected return on current and continuation assets is independent of idiosyncratic productivity, the role of idiosyncratic productivity in shaping equity expected returns is given by the impact of $s_0$ on $x$. Idiosyncratic productivity being mean–reverting, its expected growth rate is decreasing in $s_0$. It follows that the loading $x$ is strictly increasing in $s_0$.

Whether expected equity returns increase or decrease with $s_0$ depends on the expected return commanded by current and continuation assets, respectively. It turns out that for most parameter values, continuation assets command a higher return. Therefore, the expected return on equity strictly declines with idiosyncratic productivity. These results are summarized in Lemma 1.[1]

**Lemma 1** *1. Equity is a portfolio of current and continuation assets, which pay off exclusively at $t = 1$ and $t = 2$, respectively;*

*2. The loading on current assets is an increasing function of idiosyncratic productivity;*

*3. The excess return of neither asset depends on idiosyncratic productivity;*

*4. Continuation assets are riskier than current assets if and only if $\gamma > 1 - \frac{\alpha}{1-\rho_z}$.*

Because of mean–reversion in idiosyncratic productivity, the book–to–market value is also increasing in the level of idiosyncratic productivity. See Lemma 2. It follows that, as stated formally in Proposition 1, this simple model is unable to reproduce the positive relation between equity returns and the book–to–market ratio – i.e. the value premium – observed in the data.

**Lemma 2** *The book–to–market ratio is is an increasing function of idiosyncratic productivity.*

**Proposition 1** *If $\gamma > 1 - \frac{\alpha}{1-\rho_z}$, the expected return on equity monotonically decreases with book-to-market.*

## 2.2 Operating leverage

We now amend our simple model by assuming that at $t = 1$ firms incur a fixed operating cost $c_f > 0$. We also assume that the $c_f$ is such that equity value is always non–negative. We now have

$$V_1(k_1, s_1, z_1) = \max_{k_2} \; e^{s_1+z_1}k_1^\alpha - c_f - k_2 + E_1[M(z_1, z_2)e^{s_2+z_2}k_2^\alpha].$$

[1]The proofs of all claims stated in this section can be found in Appendix A.

The expected return on equity at $t = 0$ is

$$E_0[R_1] = \frac{E_0[y_1 - c_f] + E_0[-k_2 + E_1[M_2 y_2]]}{E_0[M_1 y_1 - c_f] + E_0[M_1[-k_2 + E_1[M_2 y_2]]]},$$
$$= \frac{E_0[y_1] + E_0[-k_2 + E_1[M_2 y_2]] - c_f}{E_0[M_1 y_1] + E_0[M_1[-k_2 + E_1[M_2 y_2]]] - c_f/R^f}.$$

Equity is now a portfolio that includes the risk–free asset on top of current and continuation business assets. The loading on the risk–free asset is

$$-\frac{c_f/R^f}{E_0[M_1 y_1] + E_0[M_1[-k_2 + E_1[M_2 y_2]]] - c_f/R^f},$$

which is strictly decreasing in $c_f$. An increase in the fixed cost is equivalent to expanding the short position on the risk–free asset. Everything else equal, the expected return on equity is increasing in $c_f$.

Introducing operating leverage does not change the comparative statics of the expected return with respect to idiosyncratic productivity. A greater level of $s_0$ is accommodated by a decline in leverage, i.e. a smaller short position on the risk–free asset and a smaller long position on the portfolio of current and continuation assets. This property is stated formally in Lemma 3.

**Lemma 3** *With operating leverage, if $\gamma > 1 - \frac{\alpha}{1-\rho_z}$ the expected return on equity is still monotonically decreasing in the level of idiosyncratic productivity.*

Given Lemma 3, the shape of the mapping between book–to–market ratio and expected returns will depend on the relation between the former and idiosyncratic productivity. Lemma 4 shows that when one allows for operating leverage, in general this relation will be non–monotone. Furthermore, for $c_f$ high enough, there will be always an interval $s_0 \in (-\infty, \hat{s}_0]$, for some $\hat{s}_0$, such that the relation is decreasing.

**Lemma 4** *For every level of idiosyncratic productivity $\hat{s}_0$, there exists a value of $c_f$ such that the book–to–market ratio is decreasing in the current value of idiosyncratic productivity $s_0$ for all $s_0 \in (-\infty, \hat{s}_0]$.*

The right panel in Figure 1 illustrates a parametric example in which the relation is indeed non–monotone, conditional on low aggregate productivity. The solid lines depict expected returns and book–to–market for the model with operating leverage, in the case of low (blue) and high (red) aggregate productivity. The dashed lines, which are drawn for comparison purposes, depict the same magnitudes for the model without operating leverage.

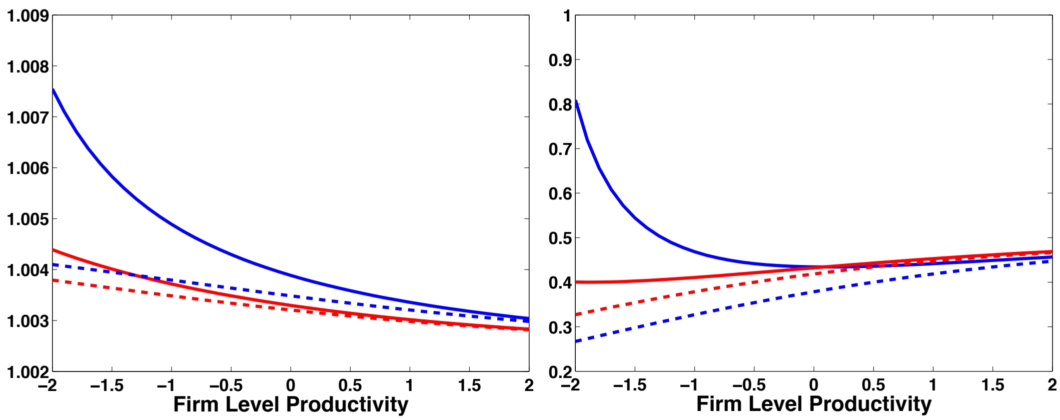


Figure 1: Expected return (left) and book-to-market over idiosyncratic productivity.

Notice that having book–to–market decreasing over $s_0$ is a necessary, although not sufficient condition for book–to–market and expected returns to be positively associated. In generalthe mapping between book–to–market and expected return, conditional on aggregate productivity, will be a correspondence. The range of book–to–market over which expected returns are increasing will depend on the second derivative of the mapping relating idiosyncratic productivity to expected returns.

Refer to the right panel of Figure 2, which illustrates the relation between book–to–market and expected returns conditional on low (blue) and high (red) aggregate productivity. Conditional on high aggregate productivity, expected returns are decreasing in idiosyncratic productivity, while book–to–market is decreasing. Therefore, the model generates a conditional value discount. Conditional on low aggregate productivity, however, a value premium may emerge.

Unfortunately, the three–period single–firm model cannot predict whether the premium will indeed emerge or not. This is the main reason why in Section 3, we will introduce a fully fledged equilibrium model which admits an ergodic distribution

Before turning to that framework, we wish to outline two important insights that emerge from the analysis of the three–period model. The first is that, should a value premium emerge, it will be generated mostly by small–cap firms in bad aggregate states of nature. The second is that the magnitude of the premium will be larger, the larger the second derivative of the mapping relating idiosyncratic productivity to expected returns (see the left panel in Figure 1).

Figure 1 reveals that expected equity returns are decreasing in the level of aggregate

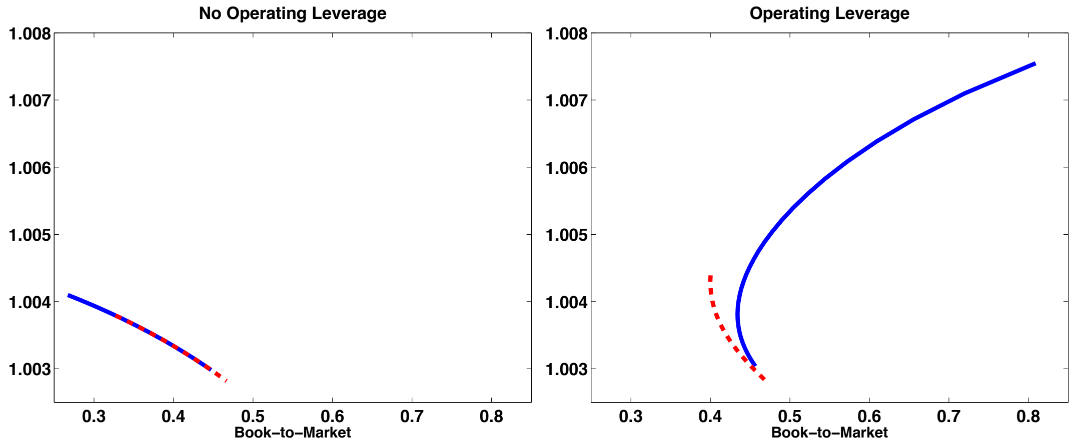


Figure 2: Expected return over book-to-market, with and without operating leverage.

productivity and that the expected return differential is greater for firms with low productivity. The reason is the same that explains the effect of adding financial leverage to a all–equity firm. When aggregate and idiosyncratic productivity are low, a constant coupon payment (fixed cost) lowers shareholders payoff more (in percentage terms) than in the case of high aggregate/idiosyncratic productivity.

A straightforward corollary is that the value premium should be generated mostly by small–cap firms, during recessions. In the remainder of the paper, we verify whether these predictions hold true in a fully fledged equilibrium model with heterogenous firms and capital accumulation.

## 3 A Fully Fledged Model

Time is discrete and is indexed by $t = 1, 2, ....$ The horizon is infinite. At every time $t$, a positive mass of price–taking firms produce an homogenous good by means of the production function $y_t = e^{z_t + s_t} k_t^{\alpha}$, with $\alpha \in (0, 1)$. Here $k_t$ denotes physical capital, while $z_t$ and $s_t$ are aggregate and idiosyncratic random disturbances, respectively.

The common component of productivity $z_t$ is driven by the stochastic process

$$z_{t+1} = \rho_z z_t + \sigma_z \varepsilon_{z,t+1},$$

where $\varepsilon_{z,t} \sim N(0, 1)$ for all $t \geq 0$. The dynamics of the idiosyncratic component $s_t$ is described by

$$s_{t+1} = \rho_s s_t + \sigma_s \varepsilon_{s,t+1},$$

with $\varepsilon_{s,t} \sim N(0, 1)$ for all $t \geq 0$. The conditional distribution will be denoted as $H(s_{t+1}|s_t)$.

Adjusting the capital stock by $x$ requires firms to incur a cost $g(x, k)$. Capital depreciates at the rate $\delta \in (0, 1)$. We also assume that each period firms need to pay a fixed operating cost $c_f > 0$. Think of that as overhead.

The supply of capital is infinitely elastic. We normalize its price at 1. The demand for the final good is given by the function $D(p) = p^{\eta}$, where $p$ denotes the price and $\eta < 0$.

Firms discount future cash flows by means of the discount factor $M_{t+1}$. Following Jones and Tuzel (2013), we assume that

$$\log M_{t+1} = \log \beta - \gamma_t \varepsilon_{z,t+1} - \frac{1}{2}\gamma_t^2 \sigma_z^2,$$

where $\log \gamma_t = \gamma_0 + \gamma_1 z_t$, $\beta > 0$, $\gamma_0 > 0$ and $\gamma_1 < 0$. This specification ensures that the price of risk is countercyclical, while the risk-free rate is constant.

At all $t \geq 0$, the distribution of firms over the two dimensions of heterogeneity is denoted by $\Gamma_t(k, s)$. Finally, let $\lambda_t \in \Lambda$ denote the vector of aggregate state variables and $J(\lambda_{t+1}|\lambda_t)$ its transition operator. Below we will show that $\lambda_t = \{\Gamma_t, z_t\}$.

Given the aggregate state $\lambda$, capital in place $k$, and idiosyncratic shock $s$, the incumbent's value function $V(\lambda, k, s)$ is the fixed point of the following functional equation:

$$V(\lambda, k, s) = \max_{k'} pe^{s+z}k^{\alpha} - x - g(x, k) - c_f + \int_{\Lambda}\int_{\Re} M(z, z')V(\lambda', k', s')dH(s'|s)dJ(\lambda'|\lambda),$$
$$\text{s.t. } k' = k(1 - \delta) + x.$$

### 3.1 Recursive Competitive Equilibrium

For given $\Gamma_0$, a recursive competitive equilibrium consists of (i) value functions $V(\lambda, k, s)$, (ii) policy functions $x(\lambda, k, s)$ and (iii) bounded sequences of product prices $\{p_t\}_{t=0}^{\infty}$ and firms' measures $\{\Gamma_t\}_{t=1}^{\infty}$ such that, for all $t \geq 0$,

1. $V(\lambda, k, s)$, and $x(\lambda, k, s)$ solve the firm's problem;
2. The product market clears: $\int e^{s+z}k^{\alpha}d\Gamma_t(k, s) = D(p_t)\ \forall\ t \geq 0$,
3. For all Borel sets $\mathcal{S} \times \mathcal{K} \in \Re \times \Re^+$ and $\forall\ t \geq 0$,

$$\Gamma_{t+1}(\mathcal{S} \times \mathcal{K}) = \int_{\mathcal{S}}\int_{\mathcal{B}(\mathcal{K},\lambda_t)} d\Gamma_t(k, s)dH(s'|s),$$

where $\mathcal{B}(\mathcal{K}, \lambda_t) = \{(k, s) \text{ s.t. } V(\lambda_t, k, s) > 0 \text{ and } k(1 - \delta) + x(\lambda_t, k, s) \in \mathcal{K}\}$.

### 3.2 Calibration

Investment adjustment costs are the sum of a fixed portion and of a convex portion:

$$g(x,k) = \chi(x)c_0 k + c_1\left(\frac{x}{k}\right)^2 k, \;\; c_0, c_1 \geq 0,$$

where $\chi(x) = 0$ for $x = 0$ and $\chi(x) = 1$ otherwise. Notice that the fixed portion is scaled by the level of capital in place and is paid if and only if gross investment is different from zero.

One period is assumed to be one quarter. Consistent with most macroeconomics studies, we set $\delta = 0.025$. Following Cooley and Prescott (1995), we let $\rho_z = 0.95$ and $\sigma_z = 0.007$. Finally, given that capital is the only factor of production, we set $\alpha = 0.6$.

The parameters of the process driving the idiosyncratic shock ($\rho_s$ and $\sigma_s$), along with those governing the adjustment costs ($c_0$ and $c_1$), were chosen to match the mean and standard deviation of the investment rate, the autocorrelation of investment, and the rate of inaction. The targets of our calibration are the moments computed in Appendix B by means of the perpetual inventory method and using all the firms in the sample.

Clementi and Palazzo (2014) show that a simpler version of the neoclassical investment model with lognormal disturbances – one without investment adjustment costs – has the interesting properties that (i) the mean investment rate is a simple non linear function of of the parameters $\rho_s$ and $\sigma_s$ and that (ii) the standard deviation of the investment rate is a simple non–linear function of the mean. It follows that in that framework, mean and standard deviation do not identify the pair $\{\rho_s, \sigma_s\}$. While these properties do not hold exact in our model, inspection reveals that a similar restriction between the two moments exists, leaving us with a degree of freedom.

We proceed to set $\rho_s = 0.9$, a value consistent with the estimates computed by Imrohoroglu and Tuzel (2014) and set the remaining three parameters to minimize a weighted average of the distances between the moments and their targets.

The parameters governing the stochastic discount are chosen to match the unconditional means of risk–free rate and Sharpe ratio. Finally, we set $c_f$ to produce a data–conforming average book–to–market ratio.

We list all parameters in Table 1 and the simulated moments in Table 2. In general, the model does a pretty good job in approximating the targets. The only exception is the first moment of the investment rate, which falls short of its target due to the restriction on mean and variance of the investment rate implied by the log-normal structure.

Table 1: Parameter Values

| Description | Symbol | Value |
|---|---|---|
| Capital share | $\alpha$ | 0.6000 |
| Depreciation rate | $\delta$ | 0.0250 |
| Persist. aggregate shock | $\rho_z$ | 0.9500 |
| Variance aggregate shock | $\sigma_z$ | 0.0070 |
| Persist. idiosync. shock | $\rho_s$ | 0.9000 |
| Variance idiosync. shock | $\sigma_s$ | 0.0090 |
| Fixed operating cost | $c_f$ | 0.00385 |
| Fixed cost of investment | $c_0$ | 0.00005 |
| Variable cost of investment | $c_1$ | 0.02000 |
| Parameter pricing kernel | $\beta$ | 0.9965 |
| Parameter pricing kernel | $\gamma_0$ | 3.1500 |
| Parameter pricing kernel | $\gamma_1$ | -15.7500 |

Table 2: Calibrated Targets

| **Investment Rate** | Data | Model |
|---|---|---|
| Mean | 0.044 | 0.029 |
| Standard Deviation | 0.089 | 0.087 |
| Autocorrelation | 0.308 | 0.315 |
| Inaction rate | 0.209 | 0.221 |
| Book–to–Market | 0.757 | 0.782 |
| **Risk-Free Rate and Sharpe Ratio** | Data | Model |
| Mean | 0.014 | 0.016 |
| Standard Deviation | 0.043 | 0.002 |
| Sharpe Ratio | 0.348 | 0.348 |

### 3.3 Computation

In order to formulate their choices, firms need to form expectations about the product price in the next period. The market clearing condition requires that at all $t$,

$$\log p_t = \frac{1}{\eta} z_t + \frac{1}{\eta} \Omega_t, \tag{2}$$

with $\Omega_t = \log\left[\int e^s k^\alpha d\Gamma_t(k,s)\right]$. The log–price is an affine function of the logarithm of aggregate productivity and of a moment of the firm distribution.

Since the dynamics of $\Omega_t$ depends on the evolution of the whole distribution, the vector of aggregate state variables $\lambda_t$ includes both $\Gamma_t$ and $z_t$. Faced with the formidable task of approximating an infinitely–dimensional object, we follow Krussel and Smith (1998)

Table 3: Baseline Scenario – Single–Sorted Portfolios

| Small Size | Medium | Large Size | L-S | Low BM | Medium | High BM | H-L |
|---|---|---|---|---|---|---|---|
| **Equity Returns** | | | | | | | |
| 0.050 | 0.040 | 0.033 | -0.017 | 0.043 | 0.040 | 0.039 | -0.003 |
| **Investment Rate** | | | | | | | |
| 0.006 | 0.028 | 0.049 | 0.043 | 0.156 | 0.028 | -0.089 | -0.245 |
| **Book-to-Market** | | | | | | | |
| 0.788 | 0.780 | 0.783 | -0.005 | 0.667 | 0.777 | 0.905 | 0.238 |
| **Size** | | | | | | | |
| 0.153 | 0.253 | 0.418 | 0.265 | 0.272 | 0.277 | 0.263 | -0.009 |
| **Capital** | | | | | | | |
| 0.120 | 0.198 | 0.328 | 0.207 | 0.181 | 0.216 | 0.238 | 0.057 |
| **Firm–Level Shock** | | | | | | | |
| -0.297 | -0.014 | 0.265 | 0.562 | 0.070 | 0.008 | -0.092 | -0.162 |

and conjecture that $\Omega_{t+1}$ is an affine function of $\Omega_t$ and $z_{t+1}$. Then, (2) implies that the equilibrium price follows the following law of motion:

$$\log p_{t+1} = \beta_0 + \beta_1 \log p_t + \beta_2 z_{t+1} + \beta_3 z_t + \varepsilon_{t+1}. \tag{3}$$

When computing the numerical approximation of the equilibrium allocation, we will impose that firms form expectations about the evolution of the price assuming that (3) holds true. This means that the aggregate state variables reduce to the pair $(p_t, z_t)$. The parameters $\{\beta_0, \beta_1, \beta_2, \beta_3\}$ will be set equal to the values that maximize the accuracy of the prediction rule. See Appendix D for a detailed description of the algorithm.

# 4 Results

For any state vector $\{\lambda, k, s\}$, the expected return on equity is given by

$$R_e(\lambda, k, s) = \frac{\int_\Lambda \int_\Re V(\lambda', k', s') dH(s'|s) dJ(\lambda'|\lambda)}{\int_\Lambda \int_\Re M(z, z') V(\lambda', k', s') dH(s'|s) dJ(\lambda'|\lambda)},$$

where $k'$ is the optimal choice of capital. The numerator is the expectation of the cum-dividend value at the next date, while the denominator is the current ex-dividend value.

Table 3 reports unconditional means for portfolios sorted on size and book-to-market, respectively. The upshot is that the model produces no appreciable unconditional cross–sectional variation in returns. In particular, there is no value premium.

## 4.1 Double Sorting

Table 4: Baseline Scenario – Double Sorted Portfolios

| | Low BM | Medium | High BM | H-L | Low BM | Medium | High BM | H-L |
|---|---|---|---|---|---|---|---|---|
| | **Size** | | | | **Book-to-Market** | | | |
| Small Size | 0.155 | 0.153 | 0.151 | -0.004 | 0.663 | 0.783 | 0.903 | 0.240 |
| Medium | 0.251 | 0.256 | 0.247 | -0.004 | 0.667 | 0.774 | 0.907 | 0.240 |
| Large Size | 0.410 | 0.423 | 0.412 | 0.002 | 0.670 | 0.782 | 0.904 | 0.234 |
| L-S | 0.255 | 0.271 | 0.261 | | 0.007 | -0.001 | 0.002 | |
| | **Equity Returns** | | | | **Investment Rate** | | | |
| Small Size | 0.052 | 0.050 | 0.049 | -0.004 | 0.136 | 0.004 | -0.098 | -0.235 |
| Medium | 0.042 | 0.040 | 0.038 | -0.004 | 0.153 | 0.028 | -0.093 | -0.246 |
| Large Size | 0.035 | 0.033 | 0.031 | -0.004 | 0.177 | 0.044 | -0.072 | -0.250 |
| L-S | -0.017 | -0.018 | -0.017 | | 0.041 | 0.041 | 0.026 | |
| | **Capital** | | | | **Firm–Level Shock** | | | |
| Small Size | 0.103 | 0.120 | 0.136 | 0.033 | -0.218 | -0.298 | -0.364 | -0.146 |
| Medium | 0.167 | 0.198 | 0.224 | 0.057 | 0.051 | -0.006 | -0.100 | -0.151 |
| Large Size | 0.275 | 0.331 | 0.373 | 0.098 | 0.329 | 0.271 | 0.183 | -0.147 |
| L-S | 0.172 | 0.211 | 0.237 | | 0.547 | 0.569 | 0.547 | |
| | **Mass of Firms** | | | | | | | |
| Small Size | 0.034 | 0.099 | 0.041 | 0.007 | | | | |
| Medium | 0.115 | 0.366 | 0.119 | 0.005 | | | | |
| Large Size | 0.046 | 0.136 | 0.044 | -0.002 | | | | |
| L-S | 0.012 | 0.037 | 0.003 | | | | | |

Table 4 reports the characteristics of portfolios double sorted on size and book–to–market. Here stocks are small–size if they belong to the bottom two deciles of the size distribution at portfolio formation. They are large–size when they belong to the top two deciles. All the other stocks are in the medium size category. Similarly, for book–to–market.

Conditional on size, portfolios with higher book–to–market consists of firms that, after an history of good idiosyncratic shocks which led them to accumulate a large capital stock, were recently hit by bad luck. Given persistence in the process driving idiosyncratic productivity, these are firms that are shedding capital with the objective of reaching a scale in line with their new, lower level of efficiency. The pace at which disinvestment takes place depends on the nature (fixed Vs. variable) and magnitude of adjustment costs.

Everything else equal, low idiosyncratic productivity means higher risk. The intuition is the same as in Section 2. When $s$ is small, most of the value comes from growth options, which are riskier than assets in place by assumption. However, the high level of capital calls for disinvestment. With reasonably calibrated adjustment costs, these firms will be disposing of capital and redistribute the proceeds to shareholders regardless of the aggregate state of nature. Given our assumptions on the stochastic discount factor, shareholders value these unconditional payments, depressing expected returns.

# 5 Comparative Statics

## 5.1 Returns to Scale

Given our calibration strategy, the degree of returns to scale turns to be an important parameter. Everything else equal, a lower level of $\alpha$ drives down the book-to-market ratio, allowing for a greater operating cost $c_f$. We performed an exercise in which $\alpha$ was set equal to 0.3, the value adopted by Zhang (2005). In one sub-scenario, we left $c_f$ at the baseline level and simply rearranged the adjustment cost in order to match our targets for the moments of the investment rate. This economy features an unrealistically low average book–to–market ratio. In another, we allowed $c_f$ to rise in order to hit the book–to–market target. The results for single sorting are reported in Table 5.

Simply changing the degree of returns to scale has no significant effect on the asset pricing cross–sectional properties of the model. However, when the operating cost rises to match the empirical average of book-to-market, the model produces a small, but non-negligible value premium. In table 6, we report the double sorting in the latter scenario. The model produces a tiny conditional value premium, mostly due to small-size firms. The greater operating leverage affects the map between idiosyncratic productivity and expected returns in a straightforward fashion. For given value of $s$ a higher fixed operating cost raises risk, much more so for small-size firms, and therefore returns.

## 5.2 Stochastic Discount Factor

In this section, we consider a different stochastic discount factor. Following Zhang (2005), we assume that

$$\log M_{t+1} = \log \beta + \left[\gamma_0 + \gamma_1 (z_t - \bar{z})\right] (z_t - z_{t+1}).$$

Table 5: Low $\alpha$ – Single–Sorted Portfolios

| | Small Size | Medium | Large Size | L-S | Low BM | Medium | High BM | H-L |
|---|---|---|---|---|---|---|---|---|
| | **Equity Returns** | | | | | | | |
| $c_f = 0.01008$ | 0.229 | 0.130 | 0.091 | -0.138 | 0.103 | 0.126 | 0.207 | 0.104 |
| $c_f = 0.00385$ | 0.094 | 0.090 | 0.086 | -0.008 | 0.093 | 0.090 | 0.086 | -0.007 |
| | **Investment Rate** | | | | | | | |
| $c_f = 0.01008$ | 0.015 | 0.028 | 0.040 | 0.025 | 0.133 | 0.021 | -0.048 | -0.181 |
| $c_f = 0.00385$ | 0.015 | 0.028 | 0.040 | 0.025 | 0.057 | 0.029 | 0.000 | -0.057 |
| | **Book-to-Market** | | | | | | | |
| $c_f = 0.01008$ | 1.082 | 0.788 | 0.697 | -0.385 | 0.629 | 0.780 | 1.102 | 0.473 |
| $c_f = 0.00385$ | 0.071 | 0.100 | 0.138 | 0.067 | 0.070 | 0.101 | 0.142 | 0.073 |
| | **Size** | | | | | | | |
| $c_f = 0.01008$ | 0.095 | 0.187 | 0.317 | 0.222 | 0.282 | 0.205 | 0.115 | -0.167 |
| $c_f = 0.00385$ | 1.357 | 1.449 | 1.579 | 0.221 | 1.369 | 1.455 | 1.572 | 0.202 |
| | **Capital** | | | | | | | |
| $c_f = 0.01008$ | 0.096 | 0.145 | 0.219 | 0.122 | 0.177 | 0.158 | 0.120 | -0.058 |
| $c_f = 0.00385$ | 0.096 | 0.145 | 0.219 | 0.122 | 0.096 | 0.147 | 0.225 | 0.129 |
| | **Firm–Level Shock** | | | | | | | |
| $c_f = 0.01008$ | -0.392 | -0.021 | 0.343 | 0.735 | 0.290 | 0.025 | -0.341 | -0.631 |
| $c_f = 0.00385$ | -0.390 | -0.020 | 0.343 | 0.734 | -0.323 | -0.006 | 0.304 | 0.627 |

The parameters $\beta$, $\gamma_0$, and $\gamma_1$ are set to match the unconditional mean and variance of the risk–free rate, as well as the unconditional mean Sharpe ratio. The result is a pricing kernel featuring countercyclical risk-free rate and price of risk. Table 7 shows that for $\alpha = 0.6$, the results do not differ in substantial ways from our baseline. In particular, the model still generates an unconditional value discount.

For $\alpha = 0.3$, the model generates a non-negligible value premium. The reason, once again, is that a lower $\alpha$ accommodates a larger value for the fixed cost $c_f$. We considered two sub-scenarios. One, labeled as I, shares the parameters $\sigma_s$ and $\rho_s$ with our baseline. In the other, labeled II, the autocorrelation coefficient $\rho_s$ is lowered to 0.8, and the conditional standard deviation $\sigma_s$ along with the parameters of the adjustment cost function are re-calibrated to approximate the cross-sectional moments of the investment rate. As a result, the unconditional standard deviation of idiosyncratic productivity is greater.

Table 8 reports the results of double-sorting in sub-scenario II. The conditional value

Table 6: Low $\alpha$ – Double–Sorted Portfolios

| | Low BM | Medium | High BM | H-L | Low BM | Medium | High BM | H-L |
|---|---|---|---|---|---|---|---|---|
| | | **Size** | | | | **Book-to-Market** | | |
| Small Size | 0.115 | 0.125 | 0.084 | -0.031 | 0.663 | 0.781 | 1.194 | 0.531 |
| Medium | 0.221 | 0.184 | 0.161 | -0.060 | 0.637 | 0.794 | 0.960 | 0.323 |
| Large Size | 0.358 | 0.292 | 0.295 | -0.064 | 0.619 | 0.742 | 0.923 | 0.303 |
| L-S | 0.243 | 0.167 | 0.211 | | -0.043 | -0.039 | -0.271 | |
| | | **Equity Returns** | | | | **Investment Rate** | | |
| Small Size | 0.190 | 0.174 | 0.250 | 0.060 | 0.270 | 0.115 | -0.023 | -0.293 |
| Medium | 0.117 | 0.131 | 0.140 | 0.023 | 0.148 | 0.019 | -0.085 | -0.233 |
| Large Size | 0.086 | 0.095 | 0.090 | 0.004 | 0.115 | -0.003 | -0.155 | -0.269 |
| L-S | -0.104 | -0.080 | -0.160 | | -0.156 | -0.118 | -0.132 | |
| | | **Capital** | | | | **Firm–Level Shock** | | |
| Small Size | 0.076 | 0.097 | 0.096 | 0.020 | -0.201 | -0.212 | -0.459 | -0.258 |
| Medium | 0.140 | 0.144 | 0.154 | 0.014 | 0.142 | -0.035 | -0.166 | -0.308 |
| Large Size | 0.222 | 0.216 | 0.272 | 0.050 | 0.473 | 0.266 | 0.201 | -0.272 |
| L-S | 0.146 | 0.119 | 0.176 | | 0.674 | 0.478 | 0.660 | |
| | | **Mass of Firms** | | | | | | |
| Small Size | 0.001 | 0.046 | 0.126 | 0.126 | | | | |
| Medium | 0.105 | 0.407 | 0.080 | -0.025 | | | | |
| Large Size | 0.087 | 0.146 | 0.002 | -0.086 | | | | |
| L-S | 0.087 | 0.099 | -0.125 | | | | | |

premium is now larger, but the mechanics appears to be the same as above.

# 6 Cyclical Properties of the Value Premium

One of the starkest implications of the three–period model discussed in Section 2 concerns the role played by the aggregate shock in shaping the unconditional mean value premium. Recall that in that simple setup, the value premium originates mostly from firms with low idiosyncratic productivity during periods of low aggregate productivity.

In this section we argue that, when the operating leverage is such that the model reproduces an unconditional value premium, these properties survive in the infinite horizon model with adjustment costs. Table 9 and Figure 3 document that this is indeed the case in the last sub-scenario considered in Section 5.

The value premium is highest during bad aggregate times and decreases monotonically

Table 7: Countercyclical risk-free rate – Single–Sorted Portfolios

| | Small Size | Medium | Large Size | L-S | Low BM | Medium | High BM | H-L |
|---|---|---|---|---|---|---|---|---|
| | **Equity Returns** | | | | | | | |
| $\alpha = 0.60$ | 1.332 | 0.960 | 0.709 | -0.623 | 1.029 | 0.951 | 0.878 | -0.152 |
| $\alpha = 0.30$ I | 4.949 | 1.344 | 0.995 | -3.953 | 1.136 | 1.287 | 1.787 | 0.650 |
| $\alpha = 0.30$ II | 1.647 | 1.216 | 0.963 | -0.684 | 1.114 | 1.200 | 1.345 | 0.231 |
| | **Investment Rate** | | | | | | | |
| $\alpha = 0.60$ | 0.027 | 0.049 | 0.070 | 0.043 | 0.182 | 0.046 | -0.062 | -0.245 |
| $\alpha = 0.30$ I | 0.029 | 0.043 | 0.054 | 0.025 | 0.167 | 0.037 | -0.053 | -0.220 |
| $\alpha = 0.30$ II | 0.016 | 0.042 | 0.065 | 0.049 | 0.165 | 0.038 | -0.057 | -0.222 |
| | **Book-to-Market** | | | | | | | |
| $\alpha = 0.60$ | 0.754 | 0.775 | 0.794 | 0.039 | 0.655 | 0.771 | 0.902 | 0.248 |
| $\alpha = 0.30$ I | 1.054 | 0.757 | 0.694 | -0.360 | 0.615 | 0.743 | 0.952 | 0.337 |
| $\alpha = 0.30$ II | 0.837 | 0.777 | 0.747 | -0.091 | 0.664 | 0.771 | 0.899 | 0.236 |
| | **Size** | | | | | | | |
| $\alpha = 0.60$ | 0.292 | 0.456 | 0.725 | 0.433 | 0.426 | 0.476 | 0.569 | 0.143 |
| $\alpha = 0.30$ I | 0.190 | 0.306 | 0.461 | 0.271 | 0.366 | 0.328 | 0.264 | -0.101 |
| $\alpha = 0.30$ II | 0.233 | 0.302 | 0.388 | 0.155 | 0.320 | 0.308 | 0.302 | -0.018 |
| | **Capital** | | | | | | | |
| $\alpha = 0.60$ | 0.222 | 0.361 | 0.590 | 0.368 | 0.290 | 0.376 | 0.507 | 0.217 |
| $\alpha = 0.30$ I | 0.149 | 0.222 | 0.322 | 0.173 | 0.228 | 0.238 | 0.223 | -0.005 |
| $\alpha = 0.30$ II | 0.179 | 0.228 | 0.289 | 0.109 | 0.213 | 0.232 | 0.253 | 0.041 |
| | **Firm–Level Shock** | | | | | | | |
| $\alpha = 0.60$ | -0.300 | -0.020 | 0.259 | 0.560 | 0.000 | -0.001 | 0.003 | 0.003 |
| $\alpha = 0.30$ I | -0.398 | -0.028 | 0.345 | 0.743 | 0.199 | 0.024 | -0.244 | -0.442 |
| $\alpha = 0.30$ II | -0.318 | -0.017 | 0.283 | 0.601 | 0.135 | 0.002 | -0.133 | -0.268 |

with output. When conditioning on size, the bulk of the value premium is generated by small firms. In particular, a sizable value premium emerges across small and medium–size firms in the bottom of the aggregate productivity distribution. Large firms always produce a value discount, no matter the aggregate conditions.

In the remainder of this section, we conduct a preliminary test of these predictions by means of the ten book–to–market and the twenty-five size and book–to–market portfolios available on Kenneth French's website.[2] Output is the gross real value added of

[2] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

Table 8: Countercyclical Risk–Free Rate – Double–Sorted Portfolios

| | Low BM | Medium | High BM | H-L | Low BM | Medium | High BM | H-L |
|---|---|---|---|---|---|---|---|---|
| | | **Size** | | | | **Book-to-Market** | | |
| Small Size | 0.204 | 0.196 | 0.168 | -0.036 | 0.637 | 0.754 | 0.995 | 0.358 |
| Medium | 0.327 | 0.291 | 0.278 | -0.048 | 0.609 | 0.742 | 0.893 | 0.283 |
| Large Size | 0.492 | 0.439 | 0.437 | -0.055 | 0.608 | 0.711 | 0.868 | 0.260 |
| L-S | 0.288 | 0.243 | 0.269 | | -0.030 | -0.043 | -0.127 | |
| | | **Equity Returns** | | | | **Investment Rate** | | |
| Small Size | 1.795 | 1.750 | 2.249 | 0.454 | 0.227 | 0.077 | -0.028 | -0.255 |
| Medium | 1.254 | 1.376 | 1.443 | 0.189 | 0.185 | 0.039 | -0.074 | -0.259 |
| Large Size | 0.969 | 1.051 | 1.008 | 0.039 | 0.163 | 0.035 | -0.104 | -0.267 |
| L-S | -0.826 | -0.699 | -1.240 | | -0.064 | -0.042 | -0.076 | |
| | | **Capital** | | | | **Firm–Level Shock** | | |
| Small Size | 0.129 | 0.143 | 0.147 | -0.036 | -0.239 | -0.302 | -0.486 | -0.247 |
| Medium | 0.201 | 0.210 | 0.235 | -0.048 | 0.125 | -0.038 | -0.143 | -0.268 |
| Large Size | 0.301 | 0.311 | 0.360 | -0.055 | 0.485 | 0.305 | 0.243 | -0.243 |
| L-S | 0.172 | 0.168 | 0.213 | | 0.724 | 0.607 | 0.729 | |
| | | **Mass of Firms** | | | | | | |
| Small Size | 0.018 | 0.066 | 0.082 | -0.036 | | | | |
| Medium | 0.120 | 0.370 | 0.109 | -0.048 | | | | |
| Large Size | 0.057 | 0.161 | 0.016 | -0.055 | | | | |
| L-S | 0.288 | 0.243 | 0.269 | | | | | |

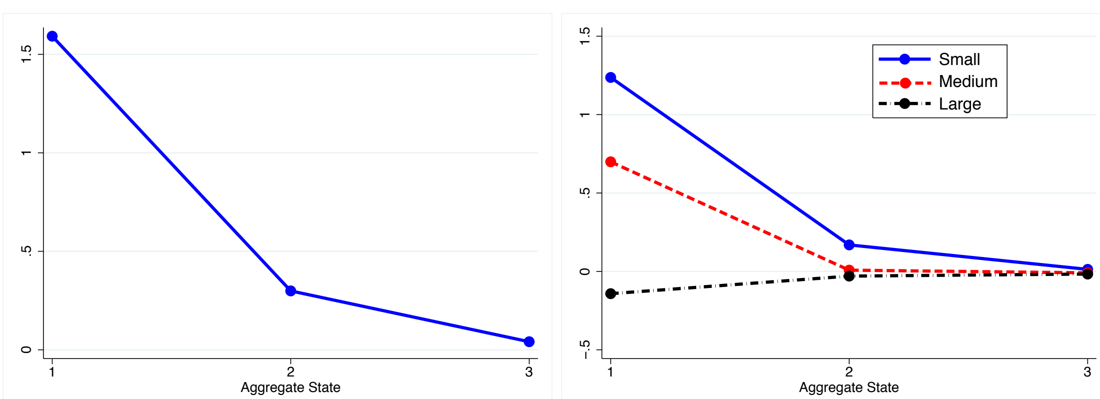


Figure 3: Cyclicality of the value premium: Model–generated data.

non–financial US corporate businesses. We consider quintiles of the distribution of log

Table 9: Cyclicality of the Value Premium: Model Generated Data

| Aggregate State | Uncond. | Small | Medium | Large |
|---|---|---|---|---|
| Low | 1.592 | 1.237 | 0.699 | -0.142 |
| Average | 0.299 | 0.169 | 0.008 | -0.030 |
| High | 0.041 | 0.013 | -0.009 | -0.017 |
| H–L | -1.551 | -1.225 | -0.708 | 0.124 |

deviations from its trend.

As a first pass, Figure 6 depicts the percent deviation of output from its H–P trend, together with the value premium over the period 1947q2–2012q2 – the one–quarter (Panel A) and one–year ahead value premium (Panel B). The grey bars denote NBER recessions. All variables are normalized to facilitate their comparison. Both series for the value premium are counteryclical. The correlation with output deviation is -0.14 (statistically significant at the 95% level) for the one–quarter ahead value, and -0.34 (significant at the 99% level) for the one-year ahead value.[3]

Table 10 reports average cumulative value premia earned by the different portfolios conditional on the state of the economy. In panel A, returns are computed for the quarter after portfolio formation. The data reported in panel B consists of average annualized return over the year following portfolio formation.

For portfolios formed in quarters whose output deviation falls in the bottom quintile, the average unconditional annualized value premium is 6.14%. That is 3.9 percentage points higher than for portfolios formed in quarters whose output deviation falls in the top quintile. Similarly, at the yearly frequency, the difference in recorded value premium between top and bottom quintile of log output deviation is $-3.75\%$. Figure 6 provides a graphical representation of the results.

The data supports the prediction that the value premium is highest when aggregate output is low. On the other hand, the data is less supportive of the model's implications when we condition on firm's size.

[3]When using the methodology proposed in Perron and Wada (2009) the correlations are -0.10 (not statistically different from zero) and -0.25 (statistically significant at the 99% level).

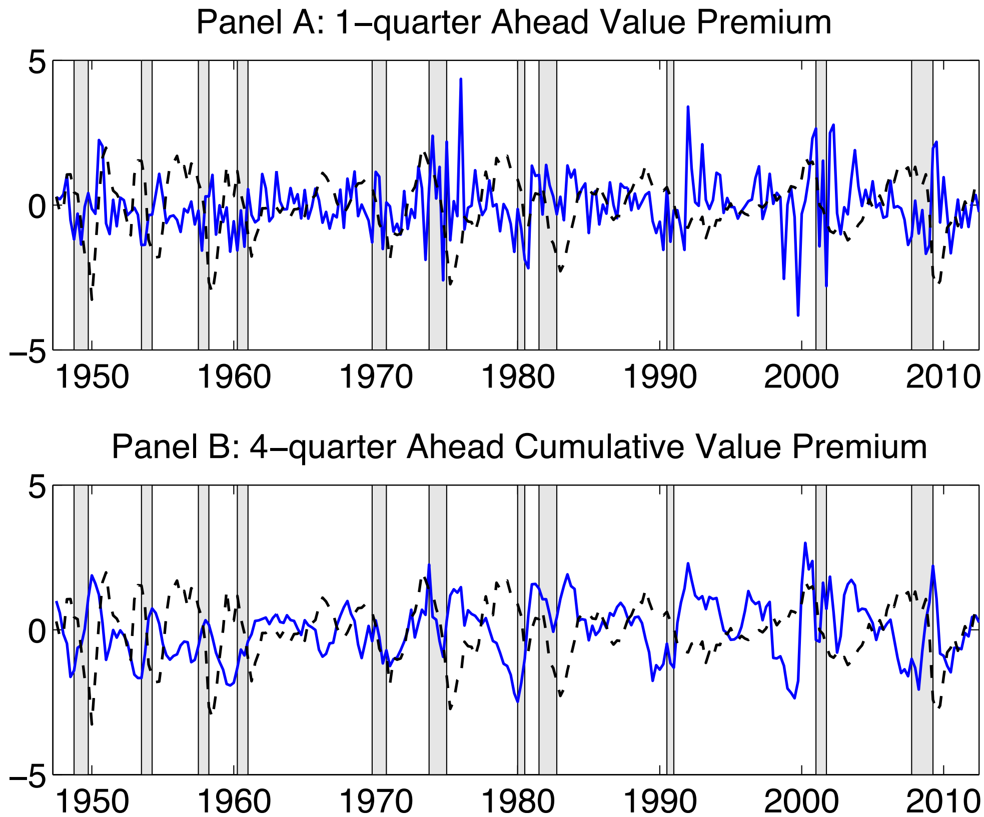


Figure 4: Cyclicality of the value premium: Time–series evidence

## 7 Conclusion

TBA.

Table 10: Cyclicality of the value premium: Data.

| | Panel A: $k = 1$ | | | | Panel B: $k = 4$ | | | |
|---|---|---|---|---|---|---|---|---|
| Quintile | Uncond. | Small | Medium | Large | Uncond. | Small | Medium | Large |
| 1 | 6.14 | 3.54 | 2.86 | 2.35 | 4.56 | 3.29 | 2.2 | 1.82 |
| 2 | 2.21 | 2.39 | 0.18 | 0.09 | 4.49 | 3.70 | 1.46 | 0.94 |
| 3 | 4.07 | 4.57 | 1.97 | 1.36 | 3.14 | 3.41 | 1.4 | 1.06 |
| 4 | 0.58 | 0.86 | 0.24 | -1.02 | 0.87 | 0.45 | 0.26 | -0.78 |
| 5 | 2.24 | 2.55 | 2.52 | 1.67 | 0.81 | 1.79 | 1.28 | 0.44 |
| 5-1 | -3.90 | -1.00 | -0.34 | -0.68 | -3.75 | -1.51 | -0.92 | -1.38 |

# A Proofs

**Proof of Lemma 1.**

The expected return on current assets is $\frac{E_0[y_1]}{E_0[M_1 y_1]}$, while $\frac{E_0[-k_2+E_1[M_2 y_2]]}{E_0[M_1[-k_2+E_1[M_2 y_2]]]}$ is the expected return from continuation. The linear equation

$$x\frac{E_0[y_1]}{E_0[M_1 y_1]} + (1-x)\frac{E_0[-k_2+E_1[M_2 y_2]]}{E_0[M_1[-k_2+E_1[M_2 y_2]]]} = \frac{E_0[y_1] + E_0[-k_2+E_1[M_2 y_2]]}{E_0[M_1 y_1] + E_0[M_1[-k_2+E_1[M_2 y_2]]]}$$

yields the solution

$$x = \frac{E_0[M_1 y_1]}{E_0[M_1 y_1] + E_0[M_1[-k_2+E_1[M_2 y_2]]]}.$$

Tedious algebra reveals that

$$\frac{E_0[M_1[-k_2+E_1[M_2 y_2]]]}{E_0[M_1 y_1]} = (1-\alpha)e^{-\frac{s_0\rho_s(1-\rho_s)+z_0(1-\rho_z)[\alpha\gamma+(1-\gamma)\rho_z]}{1-\alpha}} E_0\left[e^{\frac{\varepsilon_z[\gamma\alpha+(1-\gamma)\rho_z]+\varepsilon_s\rho_s}{1-\alpha}}\right].$$

Differentiating the latter expression with respect to $s_0$ delivers part 2) of the claim.

Now let the risk-free rate be $R^f_{t+1} = \frac{1}{E_t(M_{t+1})}$. Then, the excess returns of current and continuation assets are given by $e^{\gamma\sigma_z^2}$ and $e^{\gamma\sigma_z^2\frac{\gamma+(1-\gamma)\rho_z}{1-\alpha}}$, respectively. The latter is larger if and only if $\gamma > 1 - \frac{\alpha}{1-\rho_z}$. ■

**Proof of Lemma 2.**

Write the ratio of market to book value of capital as $\frac{E_0[M_1 y_1]+E_0[M_1[-k_2+E_1[M_2 y_2]]]}{k_1}$. Because of full depreciation, the optimal level of capital is $k_1 = \alpha E_0[M_1 y_1]$. It follows that the book-to-market ratio is simply $\alpha x$, where $x$ is the loading on current assets, and is therefore increasing in $s_0$. ■

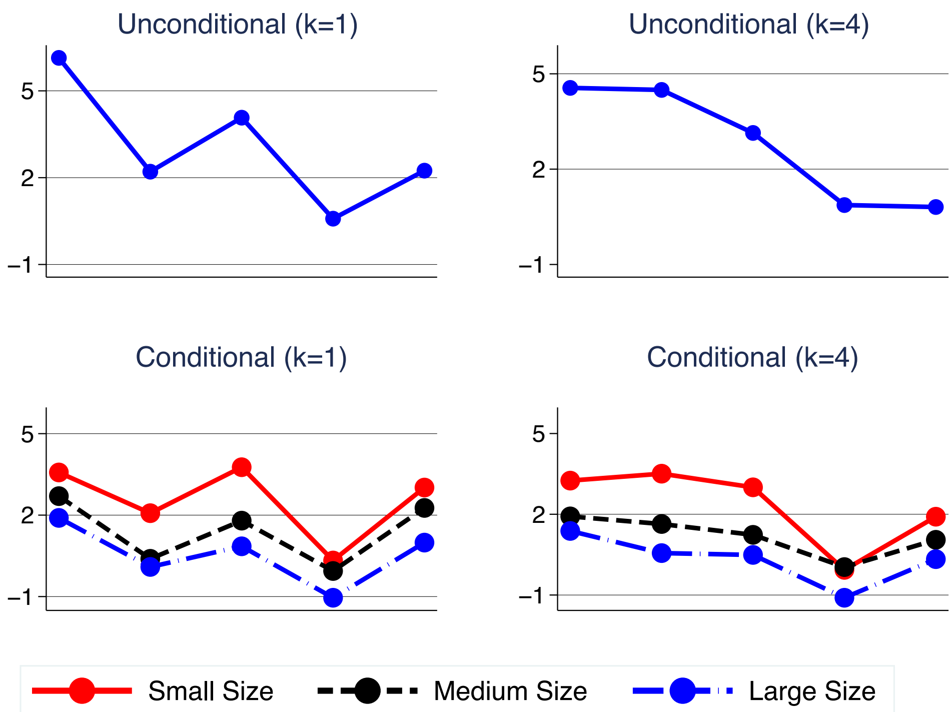


Figure 5: Cyclicality of the value premium: Data

**Proof of Lemma 3.**

Think of equity as being a portfolio consisting of the risk–free–asset as long as a composite of current and continuation business assets. The weight of the risk–free asset $-\frac{c_f/R^f}{E_0[M_1y_1]+E_0[M_1[-k_2+E_1[M_2y_2]]]-c_f/R^f}$ is negative and clearly increasing in $s_0$. That is, the short position on the risk–free asset declines with $s_0$. It follows that the long position on the composite of current and continuation assets also declines. Then the result follows from Lemma 1, which ensures that the return on the composite portfolio declines with $s_0$. ■

**Proof of Lemma 4.**

The ratio of market to book value is now $\frac{E_0(M_1y_1)+E_0[M_1[-k_2+E_1(M_2y_2)]]}{k_1}+\frac{-c_f/R_f}{k_1}$. Notice that the first addendum corresponds to market–to–book in the case with no leverage. It is decreasing in $s_0$ and independent from $c_f$. Differentiating the second addendum with respect to $s_0$ yields $\frac{c_f}{R_f}\frac{\rho_s(\alpha A_0)^{-\frac{1}{1-\alpha}}}{1-\alpha}>0$, where $A_0\equiv E_0[M_1e^{z_1+s_1}]$. Then the result follows from the observation that the derivative is increasing in $c_f$. ■

## B Evidence on Investment

In this appendix we characterize the investment process among public firms in the United States, along the lines of what accomplished by Cooper and Haltiwanger (2006) for the universe of manufacturing establishments. Our data is from the quarterly CRSP/Compustat Merged (CCM) dataset for the years 1985–2010.

We start in 1985 because it is the first year for which quarterly data on capital expenditures are available. We exclude financial firms (SIC codes 6000 to 6999), utilities (SIC codes 4900 to 4949), and firms with missing or non positive total assets value. To minimize the effect of acquisitions, we also eliminate firms that in a given quarter report a value of quarterly acquisitions to total assets ratio larger than 0.05 in absolute value (about 4.3% of the total observations).

We use the perpetual inventory method to convert the book value of the gross capital stock into its replacement value. In particular, we follow the steps described in Salinger and Summers (1983):

Step 1 - Estimate the average life of capital goods using the double declining balance method as

$$L_{i,t} = \frac{GK_{t-1} + I_t}{DEP_t},$$

where $GK_t$ is the book value of gross property, plant, and equipment in year $t$, $I_t$ is the reported spending on plant, property, and equipment in year $t$ (it does not include spending on acquisitions), and $DEP_t$ is the book depreciation in year $t$.

Step 2 - $L_{i,t}$ varies from firm to firm, and fluctuates from year to year. Therefore we take the cross-sectional average of $L_{i,t}$, that is $L_t$, and then we take the the average over time of $L_t$ to obtain a unique value $L^*$.

Step 3 - We use this average value, $L^*$, in the following formula to define the replacement value of the capital stock

$$K_t = \left( K_{t-1} \left( \frac{P_t}{P_{t-1}} \right) + I_t \right) \left( 1 - \frac{2}{L^*} \right)$$

where $P_t$ is the deflator for non-residential investment.

Step 4 - The investment rate is equal to

$$x_{i,t}^{PIM} = \frac{CAPX_{i,t} - SPPE_{i,t}}{K_{i,t-1}}.$$

For purpose of comparison, we compute two other measures of investment rate. Following Liu, Whited, and Zhang (2009), we compute the ratio of total quarterly capital expenditures (derived using year-to-date item CAPXY) net of sales of property, plant, and equipment (derived using year-to-date item SPPEY) and the previous period's total gross value of property, plant, and equipment:

$$x^G_{i,t} = \frac{CAPX_{i,t} - SPPE_{i,t}}{PPEG_{i,t-1}}.$$

Following Xing (2008), we also divide the net total quarterly capital expenditures by the previous period's total net value of property, plant, and equipment (item PPENTQ):

$$x^N_{i,t} = \frac{CAPX_{i,t} - SPPE_{i,t}}{PPEN_{i,t-1}}.$$

## B.1 Cross-sectional properties

Table 11 reports time–series averages of the cross–sectional moments that we use to calibrate our model: the average investment rate, the standard deviation of the investment rate, the fraction of firms with negative investment rate, the fraction of firms with investment rate in absolute value less than 0.01 (inaction rate), and the first order autocorrelation[4]. We truncate the investment rate observations at the top and bottom 0.1% using all the firm–quarter observations to limit the influence of outliers.

The average cross–sectional investment rate is in between 0.037 and 0.076, while the average cross–sectional standard deviation goes from 0.076 when we use $x^G$ to 0.150 when we use $x^N$. The implied coefficient of variation (not reported) is around 2.

The fraction of firms with negative investment is consistently between 4.0% and 5.0%, while the inaction rate when measured using $x^G$ and $x^{PIM}$ is almost twice as large as the value implied by $x^N$. The latter value is much smaller when we use $x^N$ because we net out the accumulated depreciation from the denominator of the investment rate. This also explains the large discrepancy in the mean and standard of the investment process between $x^G$ and $x^{PIM}$ on one hand and $x^N$ on the other. To conclude, we observe that the quarterly autocorrelation implied by the three different measures is between 0.24 and 0.32.

Table 11 shows that the universe of firms in our sample produces investment rate's moments very close to the ones generated using only manufacturing firms (SIC codes 2000 to 3999). In rest of the paper, we will limit our discussion to the cross–sectional properties of all the firms in the sample because this choice allows us to perform a cross–sectional equity returns analysis based on a larger number of observations.

[4] At each point $t$ in time, $t = 1, ..., T$, the cross-sectional autocorrelation at time $t$ is the coefficient on the lagged investment rate in the equation $x_{i,t} = \alpha_0 + \alpha_1 x_{i,t-1} + \varepsilon_{i,t}$, $i = 1, ..., N_t$, where $x_{i,t}$ is the investment rate of firm $i$ at time $t$ and $N_t$ is the total number of firms that have data for both $x_{i,t}$ and $x_{i,t-1}$ at time t. The value reported in Table 11 is the simple average of the T cross-sectional autocorrelations. Pooled ordinary least squares (OLS) deliver very similar values. We prefer the former measure because it allows one to study the investment rate autocorrelation's cyclical properties.

Table 11: Summary statistics (1985q1–2010q4)

| Variable | $x^G$ | $x^N$ | $x^{PIM}$ |
|---|---|---|---|
| **All Firms** | | | |
| Mean investment rate | 0.042 | 0.081 | 0.044 |
| Std. dev. investment rate | 0.090 | 0.144 | 0.089 |
| Inaction rate | 0.226 | 0.111 | 0.209 |
| Negative investment | 0.049 | 0.050 | 0.049 |
| Autocorrelation | 0.302 | 0.238 | 0.308 |
| Observations | 2579 | 4112 | 2460 |
| **Manufacturing Firms** | | | |
| Mean investment rate | 0.037 | 0.076 | 0.044 |
| Std. dev. investment rate | 0.076 | 0.15 | 0.081 |
| Inaction rate | 0.239 | 0.109 | 0.187 |
| Negative investment | 0.041 | 0.044 | 0.041 |
| Autocorrelation | 0.324 | 0.250 | 0.316 |
| Observations | 1347 | 2108 | 1291 |

## C Evidence on Equity Returns

This appendix describes the joint behavior of equity returns and investment rates in the cross–section. In particular, we perform univariate portfolio sorts along dimensions known to be related to equity returns like the market capitalization and the book–to–market. We also explore the joint behavior of equity returns and investment rates across portfolios double sorted along market capitalization and book-to-market.

To be consistent with the timing convention of the model, we rebalance portfolios at quarterly frequency. Following the portfolio formation strategy in Chen, Novy-Marx, and Zhang (2011) and Palazzo (2012), we use the quarterly accounting data in month $t$ in portfolios sorts starting at time $t+i+1$ if there has been an an earnings announcement (item RDQ) in month $t+i$, for $i = 1, 2, 3$.

We report values for equally and value weighted equity returns, market capitalization, and the book–to–market ratio.

Stock prices and quantities are from the Center for Research in Security Prices (CRSP), while accounting data is from CCM quarterly. We match the companies in CRSP with the companies in CCM that have the same security identifier PERMNO. We consider only ordinary common shares (share codes 10 and 11 in CRSP) and we exclude observations related to suspended, halted, or non–listed shares (exchange codes lower than 1 and higher than 3). We also require that a stock has reported returns for at least 24 months prior to portfolio formation. If a stock undergoes a performance delist after portfolio formation and the delisting return is missing, we follow Shumway (1997) and assign to the missing equity returns a value of -30%.

A firm's stock market value (SIZE) is defined as the value of its market capitalization at portfolio formation. The book–to–market ratio (BM) is the ratio of book value of equity to market value. Following Chen, Novy-Marx, and Zhang (2011), the book value is the book value of shareholders equity, plus balance sheet deferred taxes and investment tax credits (item TXDITCQ, if available), minus the book value of preferred stock. Shareholder equity stockholders' equity (item SEQQ). If the latter variable is not available, we use common equity (item CEQQ) plus the carrying value of preferred stock (item PSTKQ). If both shareholder equity and common equity are missing, we use total assets (item ATQ) minus total liabilities (item LTQ). The book value of preferred stock is its redemption value (item PSTKRQ) or, in alternative, its carrying value (item PSTKQ).

### C.1 Univariate portfolio sorts

In Table 12 we sort all the firms in the sample in quintiles according to their book-to-market value (Panel A) and according to their market capitalization at portfolio formation (Panel B). The reported equity returns are cumulative quarterly returns evaluated using the three monthly returns subsequent to portfolio formation.

The results show that high book-to-market firms (also known as value firms) earn a positive and significant excess return over low book-to-market firms (also known as growth firms) of 3.65% per quarter (value premium). The investment rate is decreasing across book–to–market categories and the quarterly difference between firms in the low book–to–market category and firms in the high book–to–market category is 0.036. Market capitalization is significantly decreasing across book–to–market categories.

Similarly to book-to-market sorted portfolios, small firms earn a positive and significant excess return over large firms. This difference (the size premium) equals 2.87% per quarter over the 1985q1–2010q4 period. The investment rate for large firms is significantly larger and the difference between large and small firms is, on average, three time smaller than the one documented across book–to–market sorted portfolios (0.013 vs. 0.036). Notice that the investment rate shows a hump shaped behavior across size sorted portfolios. Book–to–market is significantly decreasing across size categories, as expected.

Table 12: Size and Book-to-Market Sorted Portfolios

| Quintile | N | $R^{EW}$ | BM | Size | $x^{PIM}$ |
|---|---|---|---|---|---|
| | Panel A: portfolios sorted on book-to-market | | | | |
| 1 | 392 | 1.609 | 0.180 | 5.957 | 0.065 |
| 2 | 392 | 2.379 | 0.386 | 5.852 | 0.052 |
| 3 | 392 | 2.901 | 0.591 | 5.389 | 0.044 |
| 4 | 392 | 3.347 | 0.886 | 4.772 | 0.038 |
| 5 | 391 | 5.263 | 2.064 | 3.639 | 0.029 |
| 5-1 | | 3.654[b] | 1.883[a] | -2.318[a] | -0.036[a] |
| | Panel B: portfolios sorted on size | | | | |
| 1 | 392 | 5.101 | 1.422 | 2.278 | 0.033 |
| 2 | 392 | 3.011 | 0.917 | 3.860 | 0.045 |
| 3 | 392 | 2.739 | 0.718 | 5.002 | 0.051 |
| 4 | 392 | 2.408 | 0.579 | 6.197 | 0.053 |
| 5 | 391 | 2.233 | 0.467 | 8.279 | 0.046 |
| 5-1 | | -2.868[b] | -0.955[a] | 6.001[a] | 0.013[a] |

The 1%, 5%, and 10% significance levels are denoted with $a$, $b$, and $c$, respectively.

Table 13: Double Sorted Portfolios

| | Low BM | Medium | High BM | H-L | Low BM | Medium | High BM | H-L |
|---|---|---|---|---|---|---|---|---|
| | **Size** | | | | **Book-to-Market** | | | |
| Small Size | 2.561 | 2.383 | 2.092 | -0.469[a] | 0.174 | 0.693 | 2.463 | 2.289[a] |
| Medium | 5.139 | 5.106 | 4.527 | -0.612[a] | 0.176 | 0.631 | 1.764 | 1.587[a] |
| Large Size | 8.486 | 8.171 | 7.911 | -0.575[a] | 0.194 | 0.541 | 1.489 | 1.296[a] |
| L-S | 5.925[a] | 5.787[a] | 5.819[a] | | 0.020[a] | -0.152[a] | -0.974[a] | |
| | **Equity Returns** | | | | **Investment Rate** | | | |
| Small Size | 2.153 | 3.954 | 7.004 | 4.851[a] | 0.043 | 0.038 | 0.025 | -0.018[a] |
| Medium | 1.683 | 2.763 | 3.686 | 2.003[c] | 0.071 | 0.048 | 0.033 | -0.039[a] |
| Large Size | 2.015 | 2.351 | 2.966 | 0.951 | 0.060 | 0.040 | 0.035 | -0.025[a] |
| L-S | -0.138 | -1.603 | -4.038[a] | | 0.017[a] | 0.001 | 0.010[a] | |
| | **Total Assets** | | | | **Mass of Firms** | | | |
| Small Size | 2.011 | 2.664 | 3.465 | 1.455[a] | 0.021 | 0.091 | 0.088 | |
| Medium | 4.088 | 5.219 | 5.702 | 1.614[a] | 0.112 | 0.384 | 0.103 | |
| Large Size | 7.536 | 8.233 | 9.026 | 1.490[a] | 0.066 | 0.125 | 0.009 | |
| L-S | 5.525[a] | 5.569[a] | 5.561[a] | | | | | |

The 1%, 5%, and 10% significance levels are denoted with $a$, $b$, and $c$, respectively.

### C.2 Double–sorted portfolios

Table 13 documents the joint behavior of equity returns and investment rates across portfolios double sorted on size and book–to–market. Each quarter stocks are sorted independently in three size and book–to–market categories. Small size stocks belong to the bottom 20% of the size distribution at portfolio formation, while large size stocks belong to the top 20% of the size distribution at portfolio formation. All the other stocks are in the medium size category. Similarly, low book–to–market stocks belong to the bottom 20% of the book–to–market distribution, while high book–to–market stocks belong to the top 20% of of the book–to–market distribution. All the other stocks are in the medium book–to-market category.

The excess return of high book–to–market firms over low book–to–market firms is larger for small firms and it is decreasing in firm's size. In the large firms category, this excess returns is still positive but not significant. At the same time, the excess return of small size firms over large ones is increasing in book–to–market and it is significantly different from zero only for firms belonging to the high book–to–market category.

The investment rate is significantly decreasing in book–to–market across all the different size categories. Notice that the investment differential between high and low book–to–market firms shows a hump–shaped behavior: The difference is larger for firms in the medium size category. Large firms have on average a larger investment rate than small firms and this difference is significant only for firms in the low and high book–to–market categories.

We also report the behavior of the total book value of assets across the different size and book–to–market categories. Differently from the market capitalization, this quantity is significantly increasing in book–to–market across all size categories. To conclude, we report the distribution of firms over the nine portfolios. We notice that large firm tend to be low book–to–market firms and small firms tend to be large book–to–market firms. This is not surprising given the negative relation between this two variables documented in Table 12. Notice that small size–low book–to–market firms and large size–high book–to–market firms represent only 3% of all the observations.

### C.3 Summary of the empirical analysis

The empirical analysis suggests a number of stylized facts that should drive the construction of neoclassical investment models whose goal is to reproduce some basic cross–sectional features of the investment process and equity returns for U.S. public companies at a quarterly frequency.

When calibrated at a quarterly frequency using Compustat data over the period 1985q1 to 2010q4, a neoclassical investment model whose goal is to replicate the investment behavior of U.S. public firms should produce an average cross-sectional investment rate of 0.044, with a corresponding standard deviation of 0.089. In addition, the investment process should feature a quarterly inaction rate of 0.209, a negative investment of 0.049, and a first order autocorrelation of 0.309 (Table 11).

The model economy should also be able to replicate the negative and significant relation between the investment rate and book–to–market and the positive relation between the investment rate and market capitalization found in the data. The investment differential across book–to–market–sorted portfolios should also be much larger in absolute value (about three times larger) then the one across size–sorted portfolios.

Once calibrated to be consistent with the investment process, the model's pricing properties should be assessed using the empirical cross–section of equity returns as a benchmark. In particular, the model should be able to produce: (1) The positive relation between book–to–market and equity returns (value premium) and the negative relation between market value and equity returns (size premium) found in univariate portfolio sorts (Table 12); (2) A value premium which is decreasing in firm's size and a size premium which is increasing in book–to–market (Table 13).

# D Numerical Approximation

Our algorithm consists of the following steps.

1. Guess values for the parameters of the price forecasting rule $\hat{\beta} = \left\{\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3\right\}$;

2. Approximate the firm's value function;

3. Simulate the economy for $T$ periods, starting from an arbitrary initial condition $(z_0, \Gamma_0)$;

4. Obtain a new guess for $\hat{\beta}$ by running regression (3) over the time–series $\{p_t, z_t\}_{t=S+1}^{T}$, where $S$ is the number of observation to be scrapped because the dynamical system has not reached its ergodic set yet;

5. If the new guess for $\hat{\beta}$ is close to the previous one, stop. If not, go back to step 2.

## D.1 Approximation of the value function

The firm's value function is approximated by value function iteration.

1. Start by defining grids for the state variables $p, z, k, s$. Denote them as $\Psi_p$, $\Psi_z$, $\Psi_k$, and $\Psi_s$, respectively. The price grid is equally spaced and centered around the equilibrium price of the stationary economy. The capital grid is constructed following the method suggested by McGrattan (1996). The grids and transition matrices for the two shocks are constructed following Tauchen (1986). For all pairs $(s, s')$ such that $s, s' \in \Psi_s$, let $H(s'|s)$ denote the probability that next period's idiosyncratic shock equals $s'$, conditional on today's being $s$. For all $(z, z')$ such that $z, z' \in \Psi_z$, let also $G(z'|z)$ denote the probability that next period's aggregate shock equals $z'$ conditional on today's being $z$, and let $M(z, z')$ denote the associated discount factor.

2. For all triplets $(p, z, z')$ on the grid, the forecasting rule yields a price forecast for the next period (tildes denote elements not on the grid):

$$\log(\tilde{p}') = \hat{\beta}_0 + \hat{\beta}_1 \log(p) + \hat{\beta}_2 z' + \hat{\beta}_3 z.$$

   In general, $\tilde{p}'$ will not belong to the grid of prices. There will be contiguous grid points $(p_i, p_{i+1})$ such that $p_i \leq \tilde{p}' \leq p_{i+1}$. Now let $J(p_i|p, z, z') = 1 - \frac{\tilde{p}' - p_i}{p_{i+1} - p_i}$, $J(p_{i+1}|p, z, z') = \frac{\tilde{p}' - p_i}{p_{i+1} - p_i}$, and $J(p_j|p, z, z') = 0$ for all $j$ such that $j \neq i$ and $j \neq i+1$. This will allow us to evaluate the value function for values of the price which are off the grid, by linear interpolation;

3. For all grid elements $(p, z, k, s)$, guess values for the value function $V_0(p, z, k, s)$;

4. The revised guess of the value function, $V_1(p, z, k, s)$, is determined as follows:

$$\begin{aligned} V_1(p, z, k, s) = \max \Bigg[ 0, \max_{k' \in \Psi_k} \; & p e^{s+z} k^\alpha - x - c_0 k \chi - c_1 \left(\frac{x}{k}\right)^2 k - c_f + \\ & \sum_j \sum_i \sum_n M(z, z_j) V_0(p_i, z_j, k', s_n) H(s_n|s) J(p_i|p, z, z_j) G(z_j|z) \Bigg], \\ \text{s.t. } \; & x = k' - k(1-\delta), \\ & \chi = 1 \text{ if } k' \neq k \text{ and } \chi = 0 \text{ otherwise} . \end{aligned}$$

5. Keep on iterating until $\sup \left| \frac{V_{t+1}(p,z,k,s) - V_t(p,z,k,s)}{V_t(p,z,k,s)} \right| < 10.0^{-6}$. Denote the latest value function as $V_\infty(p, z, k, s)$.

### D.2 Simulation

1. Given the current firm distribution $\Gamma_t$ and aggregate shock $z_t$, compute the equilibrium price $\tilde{p}_t$ by equating product demand and product supply:

$$D(p) = e^{z_t} \sum_m \sum_n e^{s_n} k_m^{\alpha} \Gamma_t(s_n, k_m)$$

2. For all $z' \in \Psi_z$, compute the conditional price forecast $\tilde{p}_{t+1}(z')$ as follows:

$$\log[\tilde{p}_{t+1}(z')] = \hat{\beta}_0 + \hat{\beta}_1 \log(\tilde{p}_t) + \hat{\beta}_2 z' + \hat{\beta}_3 z_t.$$

   For every $z'$, there will be contiguous grid points $(p_i, p_{i+1})$ such that $p_i \leq \tilde{p}_{t+1}(z') \leq p_{i+1}$. Now let $J_{t+1}(p_i|z') = 1 - \frac{\tilde{p}_{t+1}(z') - p_i}{p_{i+1} - p_i}$, $J_{t+1}(p_i|z') = \frac{\tilde{p}_{t+1}(z') - p_i}{p_{i+1} - p_i}$, and $J_{t+1}(p_j|z') = 0$ for all $j$ such that $j \neq i$ and $j \neq i+1$;

3. Draw the aggregate productivity shock $z_{t+1}$;

4. Determine the distribution at time $t+1$. For all $(k, s)$,

$$\Gamma_{t+1}(k, s) = \sum_m \sum_n \Gamma_t(k_m, s_n) H(s|s_n) \Upsilon_{m,n}(p_t, z_t, k)$$

   where

$$\Upsilon_{m,n}(p_t, z_t, k) = \begin{cases} 1 & \text{if } k'(p_t, z_t, k_m, s_n) = k \\ 0 & \text{otherwise.} \end{cases}$$

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