

# Production-Based Term Structure of Equity Returns

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## Abstract

We study the link between timing of cash flows and expected returns in general equilibrium production economies. Standard neoclassical RBC models produce an upward-sloping term structure of equity returns. Our economy incorporates heterogeneous exposure to aggregate productivity shocks across capital vintages, yielding a downward-sloping term structure over a ten-year horizon, consistent with the empirical findings of Binsbergen et al. (2012a, b). This result is preserved after the introduction of an endogenous stock of growth options that enables us to reproduce the empirical negative relationship between cash-flow duration and expected returns in the cross section of book-to-market sorted stocks.

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# 1 Introduction

Working with synthetic aggregate cash-flow strips, Binsbergen et al. (2012a, b) and Boguth et al. (2012) provide novel empirical evidence suggesting that the expected return on aggregate dividends is decreasing in maturity over a horizon of up to seven years. The existence of a negative link between expected returns and timing of cash-flows has also been documented by earlier cross-sectional investigations. Among others, Dechow et al. (2004) and Da (2009) document that value stocks feature both higher expected returns and shorter cash-flow duration than growth stocks. Both of the above empirical findings are often interpreted as a challenge to the long-run risks model of Bansal and Yaron (2004), which predicts a monotonic upward-sloping term structure of equity returns.

In this article, we study general equilibrium models with long-run productivity shocks. Production economies generate dividend dynamics as an endogenous outcome and allow us to use empirical evidence on macroeconomic quantities to discipline the specification of the technology. Our analysis provides several new insights on the relationship between the timing of cash flows and expected returns.

First, we show that the long end of the term structure of equity obtained in production economies must agree with that obtained in endowment economies with the same consumption dynamics. The reason is that general equilibrium imposes an endogenous cointegration relationship along the balanced growth path between dividends and aggregate consumption. Therefore, in long-run risks models, the term structure of dividends and consumption must eventually converge over longer maturities and be upward sloping, as in Bansal and Yaron (2004).

This result is particularly important given the lack of direct and reliable data on dividend strips with medium-to-long maturity. In fact, the synthetic dividend strips of Binsbergen et al. (2012b) span a short maturity horizon of seven years, and the estimation of long-term cash-flow dynamics is known to be more susceptible to measurement error (Hansen et al. 2008). Given the tight restrictions imposed by production settings, our result cautions against generalizing the negative slope of the term structure of equity to long horizons.

Second, we observe that an upward-sloping short end of the term structure is a robust feature of standard real business cycle (RBC) models with or without adjustment costs and long-run risk. To

understand this result, note that in production economies the aggregate dividend payment equals total capital income less investment. Because investment responds positively and strongly to both long-run and short-run productivity shocks in all these settings, it is highly risky. As a result, short-term dividends provide a hedge against macroeconomic shocks and require a lower risk premium than long-term dividend strips.

Third, we turn our attention to a vintage capital model in which new investments are less exposed to productivity shocks than older capital vintages, consistent with the empirical evidence in Ai et al. (2012). This model resembles standard RBC models in terms of their success in generating co-movement among macroeconomic quantities over the business cycle, but it differs from them in that investment responds negatively to news about future productivity. Because investment drops upon the realization of positive news shocks, short-term dividends immediately rise. With Epstein and Zin (1989) preferences, the household requires a high risk compensation for this positive exposure to long-run shocks. As a result, short-term dividends are risky and our model generates a downward-sloping left tail of the term structure of equity returns.

The aforementioned negative response of investment to news shocks is key to our term structure analysis and can be explained as follows. Because young capital vintages are less exposed to aggregate productivity shocks than are older vintages of capital, good news about future aggregate productivity does not affect new investments. On one hand, this feature of the model implies that the substitution effect is small, and the agent has little incentive to increase investment upon news shocks. On the other hand, the income effect implies that it is optimal to immediately increase consumption, as the existing capital stock is expected to be more productive in the future. At the equilibrium, upon the realization of good news for the long run, our agent finds it optimal to delay investment. This feature of our model is consistent with recent empirical evidence in the macroeconomic news literature (Barsky and Sims 2011).

In the final step of our analysis, we introduce intangible capital and study the joint implications of our model for (i) the term structure of aggregate dividends, and (ii) the link between cash-flow duration and expected returns in the cross section. We choose the Ai et al. (2012) production economy as our main benchmark because of its ability to generate a negative relationship between cash-flow duration and expected returns in the cross section of stocks. In this setting, intangible capital is modeled as an

endogenous stock of growth options that can be produced and stored. At the equilibrium, low book-to-market ratio stocks (growth stocks) are intangible-capital intensive (Hansen et al. 2005 and Li 2009) and have both a lower long-run risk exposure and a longer cash-flow duration than high book-to-market ratio stocks (value stocks). These features of the model are consistent with empirical evidence in the long-run risks literature (Bansal et al. 2005, Bansal et al. 2007, Hansen et al. 2008), and the cash-flow duration literature (Dechow et al. 2004 and Da 2009).

We show that this model continues to produce a V-shaped term structure of equity returns, i.e., it is downward sloping for up to ten years and increases over longer maturity horizons. However, since at the equilibrium growth options are less risky than tangible assets, the introduction of intangible capital lowers significantly the entire term structure of aggregate equity returns. To improve the quantitative performance of the model, we consider two relevant modifications. First, we study a more general specification of the heterogeneous exposure of vintage capital and allow the risk exposure of new investments to increase gradually. Second, we account for dynamic financial leverage as in Belo et al. (2012). We show that both extensions reduce the quantitative gap between the model and the data.

Recently, Croce et al. (2007) have shown that the introduction of a learning friction in the Bansal and Yaron (2004) setting can produce a downward sloping term structure. Belo et al. (2012) obtain the same result by emphasizing the role of financial leverage as a device to reallocate exogenous cash-flow risk from the long end to the short end of the term structure of net equity payout. We differ from these papers in at least three respects. First, whereas previous studies focus on exogenously specified cash flows, we study dividends that are a direct outcome of investment and production decisions. This allows us to tie the asset pricing implications of our model to the dynamics of several macroeconomic quantities. More broadly, our production economy framework enables us to impose joint structural restrictions on cash flows and the pricing kernel using data on both macroeconomic aggregates and cross-sectional returns. Second, none of these studies attempts to jointly reconcile the evidence on the aggregate term structure of dividends with the properties of the returns in the cross section of book-to-market sorted stocks (value premium evidence). Third, our results suggest that the term structure of equity may be V-shaped, as opposed to being monotonic and decreasing over maturities. In the context of our model, long-term dividend strips are as risky as short-term strips.

Similarly to this paper, Berk et al. (1999), Gomes et al. (2003), and Carlson et al. (2004) study the cross section of returns by proposing equilibrium models with both tangible assets and growth options. However, we differ from this prior literature in at least two dimensions. First, in all these studies the term structure of equity either is not analyzed or is upward sloping. Also, in these models the relation between duration of cash-flows and excess returns is positive, in contrast to the duration evidence. More recently, Garleanu et al. (2012) presented a general equilibrium model consistent with the negative link between cash-flow duration and expected returns in the cross section; however, they did not study the term structure of equity.

Papanikolaou (2011), Kogan and Papanikolaou (2009, 2010, 2012), and Kogan et al. (2013) focus on investment-specific shocks to explain cross-sectional returns in production-based general equilibrium models. A broader review of the related literature is provided by Kogan and Papanikolaou (2011). Their models can produce a downward-sloping term structure, consistent with the Binsbergen et al. (2012a, b) findings. Our framework differs from the models in the abovementioned papers in several respects. First, our mechanism relies on a small but persistent component in productivity shocks (long-run risk), while their models typically feature sizeable and unpredictable shocks to the relative efficiency growth of the investment-goods versus consumption-goods sectors. Second, intangible capital in our framework is endogenously produced and storable, while the opposite is true in their framework. Third, our model, unlike theirs, suggests that the term structure of equity may change slope beyond the horizon for which data are available. In a recent paper, Favilukis and Lin (2013) study the asset pricing implications of nominal rigidities in the labor market. In their model, short-term dividends are riskier than long-term cash flows due to large countercyclical variations in labor share. Our model does not rely on nominal rigidities or labor share fluctuations.

The remainder of the paper is organized as follows. In the next section, we present our model. In section 3, we study the term structure of equity returns in a neoclassical RBC model with convex adjustment costs. We introduce capital vintages, intangible capital, and leverage in Sections 4, 5, and 6, respectively. Section 7 concludes.

## 2 General Formulation of Our Model

We consider three models: (i) an RBC model with homogeneous capital; (ii) a vintage capital model in which new investments are less exposed to aggregate productivity shocks than is capital of older vintages; and (iii) an intangible capital model with heterogenous vintages of physical capital. In this section, we describe a general framework that incorporates all three economies as special cases.

Across all the economies that we consider, equilibrium quantities in the decentralized economy coincide with those obtained under the central planner's problem. For this reason, in this section we describe only the key elements of the Pareto problem of our economies. Under the assumption that markets are complete, asset prices are recovered from the planner's shadow valuations.

**Preferences.** Time is discrete and infinite,  $t = 1, 2, 3, \dots$ . The representative agent has Kreps and Porteus (1978) preferences, as in Epstein and Zin (1989):

$$V_t = \left\{ (1 - \beta) u(C_t, N_t)^{1 - \frac{1}{\psi}} + \beta \left( E_t \left[ V_{t+1}^{1-\gamma} \right] \right)^{\frac{1-1/\psi}{1-\gamma}} \right\}^{\frac{1}{1-1/\psi}}, \quad (1)$$

where  $C_t$  and  $N_t$  denote, respectively, the total consumption and total hours worked at time  $t$ . For simplicity, we consider a Cobb-Douglas aggregator for consumption and leisure:

$$u(C_t, N_t) = C_t^o (1 - N_t)^{1-o}.$$

We normalize  $N_t = 1$  in the case of inelastic labor supply, i.e., when  $o = 1$ .

**Production technology.** Total output,  $Y_t$ , is produced according to a neoclassical Cobb-Douglas production technology:

$$Y_t = K_t^\alpha (A_t N_t)^{1-\alpha}, \quad (2)$$

where  $A_t$  is the labor-augmenting productivity shock, and  $K_t$  is the stock of physical capital. In economies with heterogenous capital vintages,  $A_t$  is interpreted as the productivity of the initial generation of capital vintage, and  $K_t$  is interpreted as the productivity-adjusted capital stock.

Total output comprises for consumption,  $C_t$ ; investment in physical capital,  $I_t$ ; and investment in intangible capital (if any),  $J_t$ :

$$C_t + I_t + J_t = Y_t. \quad (3)$$

The law of motion of the productivity process is specified as in Croce (2008) and captures both short-run and long-run productivity risks:

$$\begin{aligned} \log \frac{A_{t+1}}{A_t} \equiv \Delta a_{t+1} &= \mu + x_t + \sigma_a \varepsilon_{a,t+1}, \\ x_{t+1} &= \rho x_t + \sigma_x \varepsilon_{x,t+1}, \\ \begin{bmatrix} \varepsilon_{a,t+1} \\ \varepsilon_{x,t+1} \end{bmatrix} &\sim i.i.d.N \left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right), \quad t = 0, 1, 2, \dots \end{aligned} \quad (4)$$

According to the above specification, short-run productivity shocks,  $\varepsilon_{a,t+1}$ , affect contemporaneous output directly but have no effect on future productivity growth. Shocks to long-run productivity, represented by  $\varepsilon_{x,t+1}$ , carry news about future productivity growth rates but do not affect current output.

**Capital accumulation technologies.** We first specify the law of motion of intangible capital. Let  $S_t$  denote the total stock of intangibles available at time  $t$ . We follow Ai (2007) in modeling intangible capital as a stock of growth options:

$$S_{t+1} = [S_t - G(I_t, S_t)] \times (1 - \delta_S) + H(J_t, K_t), \quad (5)$$

where  $H(J_t, K_t)$  is a concave, constant return-to-scale adjustment cost function parameterized as follows:

$$H(J_t, K_t) = \left[ \frac{a_1}{1 - 1/\xi} \left( \frac{J_t}{K_t} \right)^{1-1/\xi} + a_2 \right] K_t,$$

and the parameters  $a_1$  and  $a_2$  are determined so that at the steady state  $\overline{H} = \overline{J}$  and  $\overline{\partial H / \partial J} = 1$ .

Each growth option can be used to build one unit of physical capital at a cost that depends on an *i.i.d.* shock distributed according to a density  $f$ . As shown in Ai (2007), under the optimal option-exercise rule, the total amount of physical capital created at time  $t$  depends only on  $I_t$  and  $S_t$  and can be represented

as a concave and constant return-to-scale aggregator,  $G(I_t, S_t)$ . Because each growth option can be used to construct exactly one unit of physical capital,  $G(I_t, S_t)$  is also the total amount of growth options exercised at time  $t$ .

Equation (5) can therefore be interpreted as follows. At time  $t$ , the agent has a mass  $S_t$  of available growth options. If options are exercised optimally and the total amount of investment goods used to exercise options is  $I_t$ , then  $[S_t - G(I_t, S_t)] \times (1 - \delta_S)$  is the total amount of growth options left at the end of the period after depreciation.  $H(J_t, K_t)$  is the amount of growth options newly produced in period  $t$ .

Following Ai et al. (2012), we focus on the class of cost distributions  $f$  that generate an aggregator  $G$  with constant elasticity of substitution between physical investment and intangible capital:

$$G(I, S) = \left( \nu I^{1-\frac{1}{\eta}} + (1-\nu) S^{1-\frac{1}{\eta}} \right)^{\frac{1}{1-1/\eta}}. \quad (6)$$

Ai (2007) shows that for each combination of  $\nu \in (0, 1)$  and  $\eta > 0$ , there exists a well-defined cost distribution  $f$ . Ai et al. (2012) show that their benchmark  $f$  distribution conforms well to the data on the cross section of book-to-market ratios.

We now turn our attention to tangible capital. We allow investments in different vintages of tangible capital to have heterogenous exposure to aggregate productivity shocks. The productivity processes are specified as follows. First, we assume that the log growth rate of the productivity process for the initial generation of production units,  $\Delta a_{t+1}$ , is given by equation (4).

Second, we impose the condition that the growth rate of the productivity of capital vintage of age  $j = 0, 1, \dots, t-1$  is given by

$$\frac{A_{t+1}^{t-j}}{A_t^{t-j}} = e^{\mu + \phi_j(\Delta a_{t+1} - \mu)}. \quad (7)$$

Under the above specification, production units of all generations have the same unconditional expected growth rate. We also set  $A_t^t = A_t$  to ensure that new production units are on average as productive as older ones. Heterogeneity is driven solely by differences in aggregate productivity risk exposure,  $\phi_j$ .

The empirical findings in Ai et al. (2012) suggest that  $\phi_j$  is increasing in  $j$ , that is, older production units are more exposed to aggregate productivity shocks than younger ones. To capture this empirical

fact, we adopt a parsimonious specification of the  $\phi_j$  function as follows:

$$\phi_j = \begin{cases} 0 & j = 0 \\ 1 & j = 1, 2, \dots \end{cases}$$

Under this specification, new production units are not exposed to aggregate productivity shocks in the initial period of their life, and their exposure to aggregate productivity shocks is identical to that of all other existing generations afterwards.

Let  $K_t$  denote the productivity-adjusted physical capital stock. As proven in Ai et al. (2012), aggregate production can be represented as a function of  $K_t$  and  $N_t$  as in equation (2), despite the heterogeneity in productivity. In addition, the law of motion of  $K_t$  can be written as

$$K_{t+1} = (1 - \delta_K) K_t + \varpi_{t+1} M_t, \quad t = 1, 2, \dots \quad (8)$$

$$\varpi_{t+1} = \left( \frac{A_{t+1}^t}{A_{t+1}} \right)^{\frac{1-\alpha}{\alpha}} = e^{-\frac{1-\alpha}{\alpha}(x_t + \sigma_a \varepsilon_{a,t+1})(1-\phi_0)} \quad \forall t,$$

where  $M_t$  is the total mass of new vintage capital produced at time  $t$ , and  $\varpi_{t+1}$  is an endogenous process that accounts for the productivity gap between the newest capital vintage and all older vintages. Note that when  $\phi_0 = 1$ , the new capital vintage has the same exposure to aggregate productivity shocks as older vintages. In this case,  $\varpi_{t+1} = 1$  for all  $t$ , and capital of all generations are identical.

The three models we consider in this study differ in terms of the dependence of  $M_t$  on investment  $I_t$  and in terms of our specifications of the  $\phi_j$  function. We detail these conditions below.

**Three economies.** First, we consider a classical RBC model with convex adjustment costs as in Jermann (1998) and homogeneous vintage capital ( $\phi_0 = 1$ ):

$$\begin{aligned} M_t &= \left[ \frac{\alpha_1}{1 - \frac{1}{\tau}} \left( \frac{I_t}{K_t} \right)^{1 - \frac{1}{\tau}} + \alpha_0 \right] K_t, \\ \varpi_{t+1} &= 1. \end{aligned} \quad (9)$$

The case of no adjustment costs corresponds to the parameter specification  $\tau = \infty$ . In this setting, the production of new investment goods does not require the exercise of growth options. At the equilibrium, there is no need to accumulate any intangible capital ( $S_t = J_t = 0$  for all  $t$ ), and hence physical capital is the only relevant stock.

The second model, the vintage capital model, abstracts away from both adjustment costs and intangible capital but allows for vintage capital. In this model, we continue to assume that physical capital accumulation does not require growth options and set  $\tau = \infty$  to remove adjustment costs. We also assume  $\phi_0 = 0$  and  $\phi_j = 1$  for all  $j \geq 1$ , so that new investments are less exposed to productivity shocks than capital of older vintages. The law of motion of productivity-adjusted tangible capital is written as

$$K_{t+1} = (1 - \delta_K) K_t + \varpi_{t+1} I_t. \quad (10)$$

Our final model, the intangible capital model, retains heterogenous risk exposure of physical capital vintages ( $\phi_0 = 0$ ) and incorporates intangible capital. Specifically, we assume that the accumulation of an extra unit of physical capital requires the exercise of a growth option, i.e., a unit of intangible capital. In this setting,  $M_t = G(I_t, S_t)$  and the accumulation of tangible capital is tightly related to the stock of growth options specified in equation (5):

$$K_{t+1} = (1 - \delta_K) K_t + \varpi_{t+1} G(I_t, S_t).$$

Because the accumulation of physical capital depends on the total amount of available growth options,  $S_t$ , the latter constitutes another state variable for the dynamic economy. In this case, the law of motion of intangible capital is given by equation (5).

**Term structures.** Given a sequence of cash flows,  $\{CF_t\}_{t=0}^{\infty}$ , the time  $t$  present value of the time  $t + n$  component of the cash-flow sequence is denoted by  $P_{t,t+n}$  and can be computed as follows:

$$P_{t,t+n} = E_t[\Lambda_{t,t+n} CF_{t+n}] \quad n = 1, 2, \dots,$$

where  $\Lambda_{t,t+n} = \Lambda_{t,t+1} \times \Lambda_{t+1,t+2} \times \dots \times \Lambda_{t+n-1,t+n}$  is the  $n$ -step-ahead discount factor that can be computed from the one-step-ahead stochastic discount factors:

$$\Lambda_{t,t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-1} \left( \frac{u_{t+1}}{u_t} \right)^{1-\frac{1}{\psi}} \left( \frac{V_{t+1}}{E_t \left[ V_{t+1}^{1-\gamma} \right]^{\frac{1}{1-\gamma}}} \right)^{\frac{1}{\psi}-\gamma}. \quad (11)$$

The one-period return of the claim to  $CF_{t+n}$  from period  $t$  to  $t+1$  is simply  $\frac{P_{t+1,t+n}}{P_{t,t+n}}$ . We are interested in studying the unconditional risk premium,  $RP(n)$ , on this return for different maturities  $n$ :

$$RP(n) = E \left[ \frac{P_{t+1,t+n}}{P_{t,t+n}} - r_t^f \right], \quad n = 1, 2, \dots,$$

where  $r_t^f = \frac{1}{E[\Lambda_{t,t+1}]}$  is the one-period risk-free interest rate. The term structure of a cash-flow sequence  $\{CF_t\}_{t=0}^{\infty}$  refers to the link between  $RP(n)$  and  $n$ .

We study the term structure of both the consumption claim and the dividend payout of the representative firm. In our setting, dividends equal capital income net of total investment costs, that is,  $\alpha Y_t - I_t - J_t$ . Given this consideration, to better understand the composition of dividend risk we also study the term structure of a claim to total investment,  $I_t + J_t$ . In addition, we report the implications for the term structure of real interest rates so that we can better disentangle dividend risk premia from bond risk premia over different horizons. In summary, in the rest of the paper we focus on the following cash-flow processes:

$$CF_t = \begin{cases} A_t & \text{Productivity} \\ Y_t & \text{Output} \\ C_t & \text{Consumption} \\ I_t + J_t & \text{Total Investment} \\ \alpha Y_t - I_t - J_t & \text{Dividends (Total Payout)} \\ 1 & \text{Real Bonds} \end{cases}.$$

In most of our analysis we abstract away from dynamic financial leverage. Therefore, we use the terms Dividends and Total Payout interchangeably. In section 6, we introduce dynamic financial leverage and distinguish between total payout and net equity payout (NEPO). Though dynamic leverage does not

alter our qualitative findings, it brings them quantitatively closer to the empirical findings of Binsbergen et al. (2012a, b).

### 3 Neoclassical Real Business Cycle Models

In this section, we focus on the neoclassical RBC model. Although the RBC setting features only one type of capital and hence makes no prediction about the link between cash-flow duration and the cross section of returns, it is a useful starting point from which to show that the term structure anomaly of Binsbergen et al. (2012a) applies to neoclassical production settings as well.

We document that the RBC model produces a counterfactual upward-sloping term structure of equity in three different capital-accumulation and risk-structure formulations. Specifically, in our RBC(1) model, productivity growth is assumed to be *i.i.d.*, and adjustment costs are null. We incorporate adjustment costs in model RBC(2) by setting  $\tau > 0$ . In model RBC(3), we additionally incorporate long-run productivity shocks. For simplicity, we assume a fixed labor supply in all three versions of the RBC model. We relax this assumption in the sections 4–6.

We report the calibrations adopted in our study in Table 1 and summarize the main statistics produced by the three formulations of the RBC model in Table 2. None of these models allows us to simultaneously match a low investment-output ratio and a low interest rate (see Tallarini 2000). This problem is resolved in our intangible capital model.

**Table 1: Calibrated Parameter Values**

		RBC(1)	RBC(2)	RBC(3)	Vintage Capital	Intangible Capital
<b>Preference parameters</b>						
Discount factor	$\beta$	0.974	0.974	0.974	0.9705	0.986
Effective risk aversion	$\gamma \cdot o$	10	10	10	10	10
Intertemporal elasticity of substitution	$\psi$	2.0	2.0	2.0	2.0	2.0
Leisure weight	$o$	1	1	1	0.345	0.345
<b>Technology parameters</b>						
Capital share	$\alpha$	0.3	0.3	0.3	0.35	0.3
Depreciation rate of physical capital	$\delta_K$	9.1%	9.1%	9.1%	8%	11%
Adjustment costs on tangibles	$\tau$	–	3.25	3.25	–	–
Depreciation rate of intangible capital	$\delta_S$	–	–	–	–	11%
Weight on physical investment	$\nu$	–	–	–	–	0.93
Elasticity of substitution	$\eta$	–	–	–	2.5	3.8
Adjustment costs on intangibles	$\xi$	–	–	–	–	5
<b>Total factor productivity parameters</b>						
Risk exposure of new investment	$\phi_0$	1	1	1	0	0
Average growth rate	$\mu$	2.0%	2.0%	2.0%	2.0%	2.0%
Volatility of short-run risk	$\sigma_a$	5.08%	5.08%	5.08%	5.08%	4.4%
Volatility of long-run risk	$\sigma_x$	–	–	0.86%	0.86%	0.75%
Autocorrelation of expected growth	$\rho$	–	–	0.925	0.925	0.925

This table reports the parameter values used for our annual calibrations. RBC(1) is a neoclassical real business cycle (RBC) model with short-run risk only. RBC(2) features adjustment costs. RBC(3) features both adjustment costs and long-run productivity shocks. In the vintage capital model, young capital vintages have lower exposure to productivity shocks,  $\phi_0$ . The intangible capital model is our benchmark and features both tangible and intangible capital, as well as heterogenous exposure to productivity shocks.

**Table 2: Neoclassical Real Business Cycle Models**

	Data	RBC(1)	RBC(2)	RBC(3)
Short-run risk		✓	✓	✓
Adjustment costs			✓	✓
Long-run risk				✓
$E[I/Y]$	00.15	00.27	00.26	00.29
$\sigma(\Delta c)$	02.51	01.83	02.76	03.22
$\sigma(\Delta i)$	16.40	10.05	05.58	06.15
$E[r^f]$	00.89	02.79	02.47	00.86
$E[r^{L,ex}]$	05.70	00.60	02.90	04.00

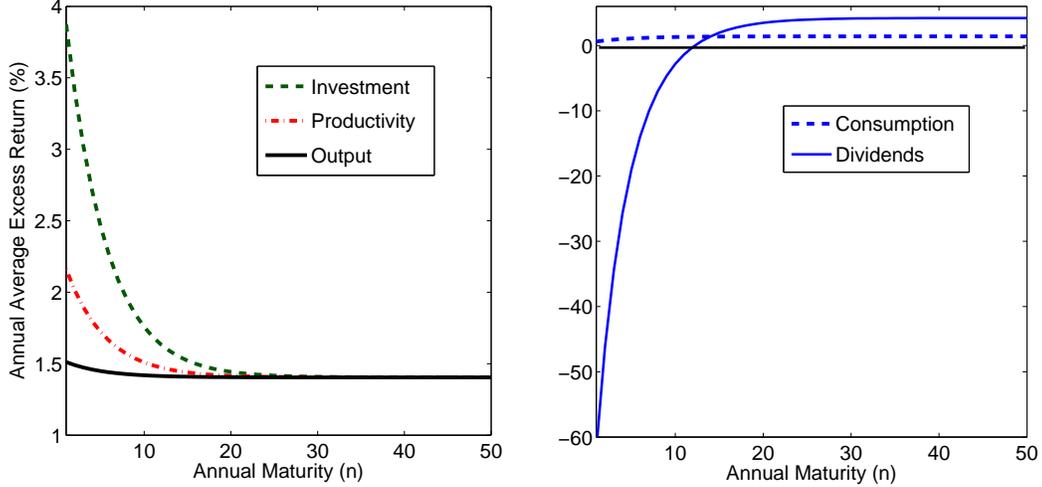
All entries for the models are obtained from repetitions of small samples. Data are from the U.S. and include pre-World War II observations (1930–2007). Our annual calibrations are reported in Table 1. Excess returns are levered by a factor of three, consistent with Garcia-Feijo and Jorgensen (2010).

**Short-run risk only.** Model RBC(1) refers to the neoclassical RBC model in its most essential form, i.e., without any investment frictions and subject only to short-run shocks. Even though this model produces counterfactual implications for asset prices, it is a useful starting point for understanding key features of the term structures in production economies.

We plot the term structure of the consumption, investment, dividends, output, and productivity of model RBC(1) in Figure 1. Several features of the term structures are salient. First, the long end of all term structures converges to the same level because of the cointegration relationship imposed by our general equilibrium model. Over long horizons, balanced growth implies that the dynamics of all quantities should be determined by productivity.<sup>1</sup> As a result, all risk premia must converge to the level of that of the exogenous productivity process. Under our calibration, the long end of the term structures is about 1.5% per year. This sizeable figure is due to the fact that our shocks are permanent in productivity levels.

Second, looking at the short end of the term structures, we see very different levels across macroeconomic quantities. This pattern is due to consumption smoothing. When a positive short-run shock materializes, output, consumption, and investment simultaneously increase. The response of consumption is moderate, whereas the investment response is much stronger, consistent with the volatility figures

<sup>1</sup>The long-term level of the term structure of dividends is higher than that of productivity just because excess returns are levered by a factor of three.



**Fig. 1: Term Structures in RBC(1)**

This figure shows average annual excess returns of zero-coupon equities for different maturities associated with different cash flows. Excess returns are multiplied by 100. Dividend returns are levered by a factor of three (Garcia-Feijo and Jorgensen 2010). Results are based on the calibration used for model RBC(1), as reported in Table 1.

reported in Table 2. As a result, investment cash flows are riskier in the short run than they are over long maturities and generate a downward-sloping term structure. The opposite is true for the consumption cash flow, as the claim to consumption is a position long in output and short in investment,  $C_t = Y_t - I_t$ , and hence investment works as a hedge. Since investment represents a small fraction of output, its hedging effect on consumption is limited, implying that short-term consumption carries a positive premium and the intercept of the consumption term structure is positive.

Similarly, the term structure of dividends has a positive slope, as the dividends claim is a position short on investment and long on capital income,  $D_t = \alpha Y_t - I_t$ . Because the share of capital income,  $\alpha$ , is close to the average investment-output ratio, the leverage effect from investment is much stronger than in the consumption case. As a result, the risk premium on short-term dividends is actually negative, since short-term dividends provide a strong hedge against short-run productivity shocks. The negative intercept of the dividends term structure and the low level of its long end reflect the well-known difficulty of generating a high equity premium in standard RBC models (Rouwenhorst 1995).

**Adjustment costs and long-run risk.** A common way to improve the asset pricing implications of a production economy is to introduce capital adjustment costs, as they allow for fluctuations in the marginal value of capital. We add adjustment costs in our model RBC(2).

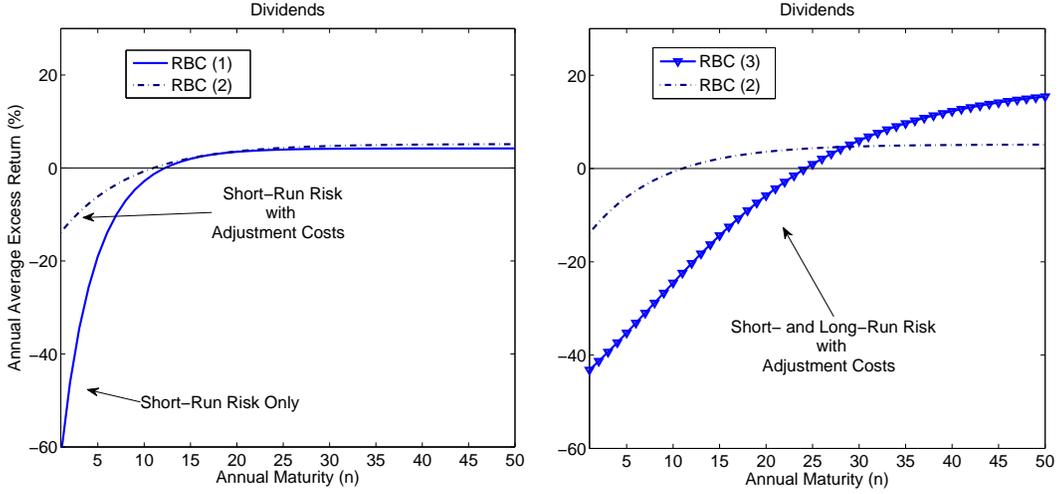
As documented by Croce (2008), this class of adjustment costs is unsatisfactory in a long-run risk framework because it creates a strong trade-off between the equity premium and investment volatility. Specifically, adding adjustment costs significantly reduces the response of investment to productivity shocks. In model RBC(2), for example, when we calibrate the adjustment costs to achieve a levered equity premium of about 3%, the volatility of investment growth is three times smaller than that in the data.

We depict the term structures generated with and without adjustment costs in the left panel of Figure 2. Compared to the model without adjustment cost, the term structure of investment in model RBC(2) is much flatter, precisely because investment is less responsive to productivity shocks and hence represents a weak hedge. Although less steep, the term structure of dividends remains counterfactually upward sloping.

In model RBC(3), we further incorporate long-run productivity shocks to increase the aggregate equity risk premium (Figure 2, right panel). This improvement on the aggregate excess return comes at the cost of making the term structure of equity returns more upward sloping, i.e., a greater departure from the empirical results of Binsbergen et al. (2012a, b). In the next section, we provide a resolution of the term structure anomaly by introducing vintage capital.

## 4 The Vintage Capital Model

In this section, we consider a vintage capital model incorporating the empirical fact that new investments are less exposed to aggregate productivity shocks than capital of older vintages, evidence for which is discussed in detail in Ai et al. (2012). Here, we remove adjustment costs ( $\tau = \infty$ ) and incorporate heterogeneous productivity of vintage capital by setting  $\phi_0 = 0$ , consistent with our equation (10). Our model with heterogeneous capital vintages generates a V-shaped term structure of equity returns. In particular, the dividend risk premium decreases with maturity up to ten years, consistent with the



**Fig. 2: The Role of Adjustment Costs and Long-Run Risk**

This figure shows annual average excess returns of zero-coupon equities for dividends with different maturities. Excess returns are in percentage units and are levered by a factor of three (Garcia-Feijo and Jorgensen 2010). Model RBC(1) is the neoclassical RBC model with short-run shocks only. Model RBC(2) features short-run risk and convex adjustment costs. Model RBC(3) features short- and long-run risk as well as adjustment costs. Calibrations are reported in Table 1.

empirical evidence documented in Binsbergen et al. (2012a). We first discuss the optimality conditions of the social planner's problem, and then examine the implications of our model.

**Optimality conditions.** Let  $q_{K,t}$  and  $p_{K,t}$  denote the ex- and cum-dividend prices, respectively, of one unit of productivity-adjusted aggregate physical capital. Given equilibrium quantities,  $q_{K,t}$  and  $p_{K,t}$  are jointly determined by the following recursions:

$$\begin{aligned}
 p_{K,t} &= \alpha \frac{Y_t}{K_t} + (1 - \delta)q_{K,t}, \\
 q_{K,t} &= E_t [\Lambda_{t+1} p_{K,t+1}].
 \end{aligned}
 \tag{12}$$

Optimality of investment implies

$$1 = E_t [\Lambda_{t+1} p_{K,t+1} \varpi_{t+1}].
 \tag{13}$$

The left-hand side of equation (13) measures the marginal cost of investment. In the absence of adjustment costs, the marginal cost is one. The right-hand side of this equation shows the marginal benefit of

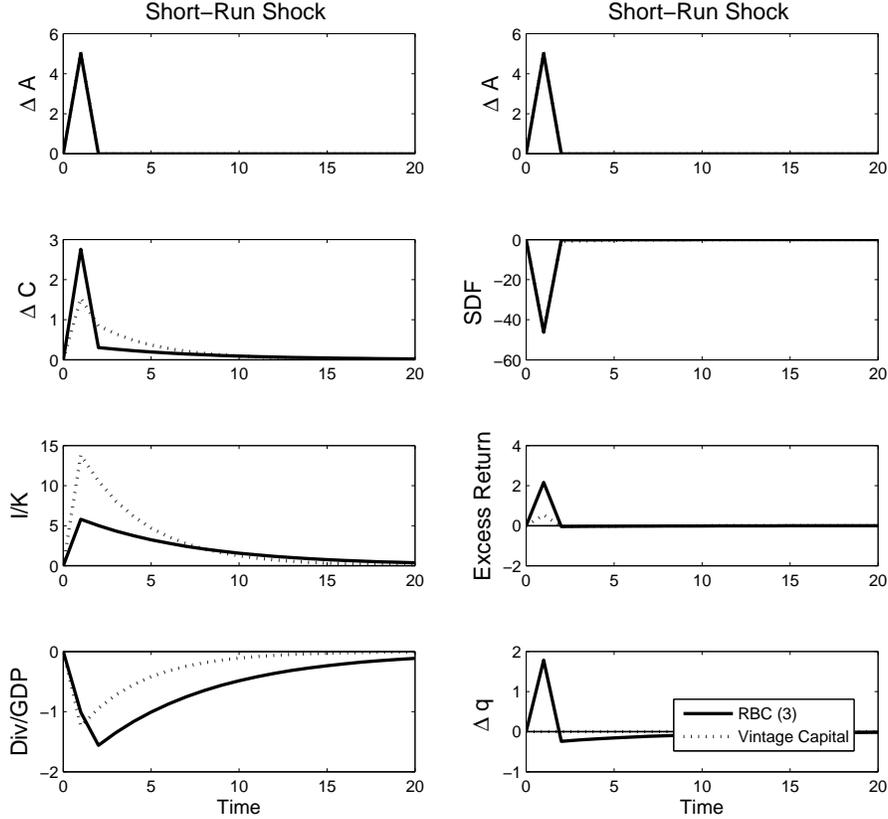
investment. In our vintage capital economy, the newly established capital vintage is less exposed to aggregate productivity shocks than older vintages. This heterogeneity creates a wedge between the productivity of new and older capital generations, measured by  $\varpi_{t+1}$ .

Specifically, after accounting for productivity differences, one unit of new vintage capital is equivalent to  $\varpi_{t+1}$  units of the existing capital stock, where the expression for  $\varpi_{t+1}$  is given by equation (8). Therefore, the marginal benefit of investment measured in terms of the time  $t$  consumption numeraire is  $E_t[\Lambda_{t+1}p_{K,t+1}\varpi_{t+1}]$ . This is the key feature that distinguishes our model from the standard RBC models discussed in the last section. Heterogenous exposure to risk across capital vintages is what improves our asset pricing implications.

**Asset price dynamics.** To understand the implications of vintage capital, we compare the impulse response functions of quantities and prices of both the vintage capital model and the RBC model with adjustment costs (RBC(3)). Figure 3 depicts the response of quantities (left panel) and asset prices (right panel) to short-run shocks. The responses to long-run productivity shocks are shown in Figure 4. The main asset pricing and macroeconomic moments of the model are summarized in Table 3.

Focusing on short-run productivity shocks, we note that the vintage capital model produces responses qualitatively similar to those in the RBC(3) model, but it features a considerably stronger movement in investment because of the absence of adjustment costs. As a result, our vintage capital model is as successful as the frictionless RBC(1) model in generating a high volatility of investment relative to consumption. As shown in Table 3, the volatility of investment growth is about six times higher than that of consumption growth, as in the data.

On the asset pricing side, short-run productivity shocks generate almost a null risk premium in the vintage capital model, as both the price of capital and the market return remain basically unaltered (right column of Figure 3, bottom two panels). Without adjustment costs, the price of new investment goods is always one, and the relative price of installed capital depends completely on the wedge between the productivity of old and new vintages of capital. Since short-run growth shocks are *i.i.d.*, they have no significant effect on the price of installed capital. In fact, in the absence of long-run shocks, the term  $\varpi_{t+1}$  does not depend on time  $t$  information and has a constant unconditional expectation of 1. Equations



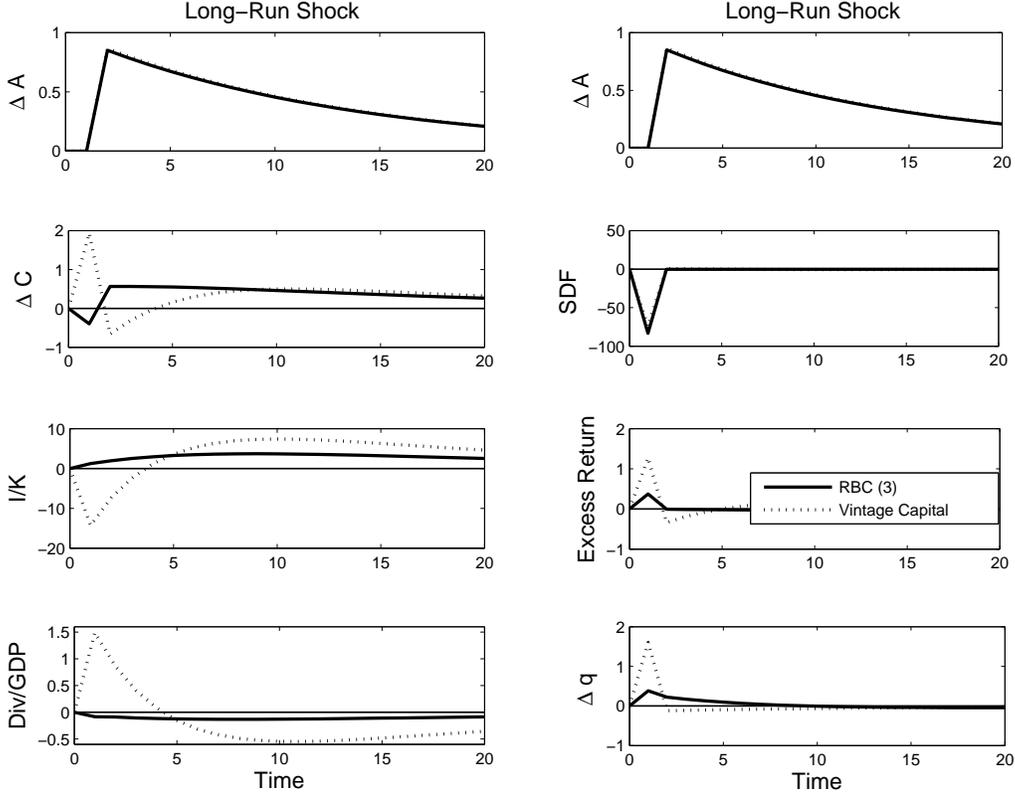
**Fig. 3: Impulse Response Functions for Short-Run Shocks**

This figure shows percentage annual log-deviations from the steady state upon the realization of a positive short-run shock. Tangible capital returns are not levered. Both the RBC(3) and vintage capital models feature short- and long-run productivity risk. RBC(3) has convex adjustment costs. The vintage capital model has heterogeneous capital vintages. Calibrations are reported in Table 1.

(12) and (13) together imply that  $q_{K,t}$  is approximately one, as in the model RBC(1) without adjustment cost.<sup>2</sup>

Long-run productivity shocks, on the other hand, produce a significant risk premium in our vintage capital model. As shown in the bottom-right panel of figure 4, the market value of existing capital,  $q_{K,t}$ , rises substantially and produces a spike in the capital excess return that is five times larger than that obtained under the RBC(3) model. Since long-run shocks carry a higher price of risk than short-run

<sup>2</sup>In the presence of short-run shocks, the condition  $q_{K,t} = 1$  does not hold precisely because  $\varpi_{t+1}$  is correlated with time  $t+1$  variables through general equilibrium channels. Quantitatively, this correlation is small. As a result, the response of  $q_K$  to short-run shocks is barely visible in Figure 3.



**Fig. 4: Impulse Response Functions for Long-Run Shocks**

This figure shows percentage annual log-deviations from the steady state upon the realization of a positive long-run shock. Tangible capital returns are not levered. Both the RBC(3) and vintage capital models feature short- and long-run productivity risk. RBC(3) has convex adjustment costs. The vintage capital model has heterogeneous capital vintages. Calibrations are reported in Table 1.

shocks, the equity premium under the vintage capital model is as large as that under model RBC(3).

In the vintage capital model, the price of capital goods,  $q_{K,t}$ , reacts strongly to long-run shocks because long-run shocks are persistent and significantly affect the expected productivity wedge between new investments and existing capital stock,  $E_t[\varpi_{t+1}] = \exp\{-\left(\frac{1}{\alpha}-1\right)x_t\}$ . Intuitively, a positive long-run shock increases the productivity of all existing generations of capital, but it affects a newly established vintage only with a delay. In relative terms, this means that old vintages become persistently more productive and hence more valuable than the new one. Because the price of new investment goods is constant, positive long-run shocks are associated with increases in the price of existing capital stock. In fact, ignoring the correlation terms due to short-run shocks, equations (12) and (13) imply  $q_{K,t} = \exp\left\{\left(\frac{1}{\alpha}-1\right)x_t\right\}$ .

**Table 3: Vintage and Intangible Capital Model**

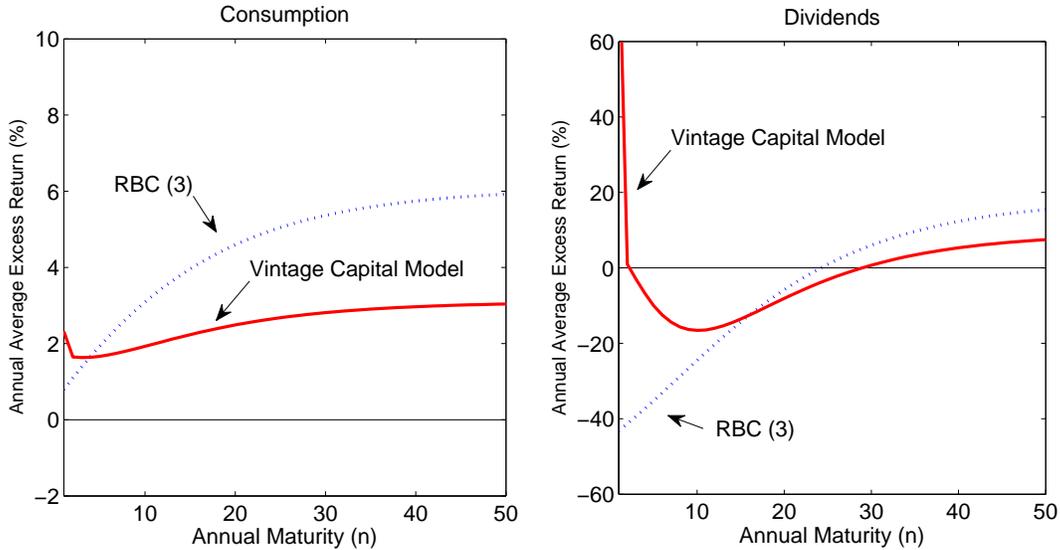
	Data	Model	
		Vintage Capital	Intangible Capital
$E[I/Y]$	00.15 (00.05)	00.31	00.18
$\sigma(\Delta c)$	02.53 (00.56)	03.05	02.58
$\sigma(\Delta i)/\sigma(\Delta c)$	05.29 (00.50)	06.91	05.84
$E[I/J]$	01.00 (00.10)	–	01.01
$\sigma(\Delta j)/\sigma(\Delta i)$	00.50 (00.07)	01.00	00.61
$\sigma(\Delta n)$	02.07 (00.21)	03.38	01.96
$\rho_{\Delta C, \Delta n}$	00.28 (00.07)	-00.31	00.05
$E[r^f]$	00.89 (00.97)	02.65	01.19
$E[r_K^{L,ex}]$	05.70 (02.25)	03.94	04.17
$E[r_K^L - r_S^L]$	04.32 (01.39)	–	04.76

All entries for the models are obtained from repetitions of small samples. Data refer to the U.S. and span the sample 1930–2007. Numbers in parentheses are GMM Newey-West adjusted standard errors.  $E[r_K^L - r_S^L]$  and  $E[r_K^{L,ex}]$  measure the levered spread between tangible and intangible capital returns, and the levered excess returns of tangible capital, respectively. Our leverage coefficient is three, consistent with Garcia-Feijo and Jorgensen (2010). Our annual calibrations are reported in Table 1.

**Investment dynamics and term structure.** In Figure 5, we depict the term structure of both dividends and consumption for the RBC model with adjustment cost and long-run productivity shocks [RBC(3)] and the vintage capital model. In the vintage capital model, the term structures of both consumption and dividends are V-shaped. This shape is especially pronounced for dividends.

To understand the negative slope of the short end of the term structure, recall that both consumption and dividends can be replicated by long positions in output and short positions in investment ( $C_t = Y_t - I_t$ , and  $D_t = \alpha Y_t - I_t$ ). In the vintage capital model, both short-term consumption and dividends are riskier than those in the RBC(3) model because investment responds negatively to long-run productivity shocks. As a result, the risk exposure of both short-run consumption and dividends to long-run shocks is amplified instead of being mitigated. This effect is stronger for dividends because investment fluctuations account for a large fraction of dividend dynamics.

The negative response of investment to long-run productivity shocks is completely driven by the heterogenous exposure to productivity shocks across capital vintages. In the RBC(3) model, all vintages have the same exposure to long-run productivity shocks. Hence upon the realization of a positive long-run shock, young vintages are expected to be more productive in the long-run as much as old vintages.



**Fig. 5: The Role of Vintage Capital**

This figure shows annual excess returns of zero-coupon equities for different maturities associated with different cash flows. Excess returns are multiplied by 100. Dividend returns are levered by a factor of three (Garcia-Feijo and Jorgensen 2010). All models feature both short- and long-run productivity risk. In model RBC(3) we assume the existence of convex adjustment costs. In the vintage capital model young capital vintages have low exposure to productivity shocks. Calibrations are reported in Table 1.

By substitution effect, it is optimal to reduce consumption and invest more to take advantage of higher future productivity. This explains the persistent increase of the investment-to-capital ratio, as shown in the third panel on the left in Figure 4.

In the vintage capital model, in contrast, new investments do not benefit from these shocks immediately and the substitution effect is initially small. Rather, all existing capital vintages are expected to be more productive, and the associated income effect encourages consumption as opposed to investment. On impact, aggregate consumption increases (second panel on the left in Figure 4), whereas investment drops and recovers only after a few periods, as the news shocks materialize as increases in output (third panel on the left in Figure 4).

The relevance of the negative response of investment with respect to news shocks is twofold. First, these investment dynamics are consistent with the recent empirical findings in Kurmann and Otrok (2010) and Barsky and Sims (2011).<sup>3</sup> Second, they provide a long-run risk-based general equilibrium mechanism

<sup>3</sup>Kurmann and Otrok (2010) and Barsky and Sims (2011) point out that labor also declines upon the realization of positive long-run news. Our vintage model is qualitatively consistent with this fact, but it overstates the quantitative magnitude of this negative co-movement. As a result, labor and output are slightly negatively correlated,

that produces a downward-sloping term structure of equity returns over a horizon of about ten years, as in Binsbergen et al. (2012b).

Over maturity horizons longer than ten years, the vintage capital model term structures become upward sloping like in model RBC(3). This result is consistent with the pattern observed in standard long-run risks models and is fully explained by the cointegration restrictions imposed by general equilibrium. In the long-run, in fact, the risk exposure of all cash flows converges to that of the productivity process. Because of long-run productivity risk, all term structures must have an upward-sloping right tail.

## 5 Intangible Capital

In this section, we incorporate intangible capital (Ai et al. 2012) into the vintage capital model to replicate the negative relationship between cash-flow duration and expected returns observed in the cross section (Dechow et al. 2004 and Da 2009). This is our benchmark model, as it provides a unified general equilibrium framework to study the relationship between expected returns and timing of the cash flows from both the time-series and the cross-sectional perspectives. We show that the benchmark model continues to produce a downward-sloping risk premium for dividend strips with maturity up to ten years. Below, we discuss the optimality conditions in the intangible capital model and then show our results.

**Optimality conditions.** We continue to denote the cum-dividend and ex-dividend prices of physical capital by  $p_K$  and  $q_K$ , respectively. We use  $p_S$  and  $q_S$  for the cum- and ex-dividend prices of intangible capital. By definition,  $q_K$  and  $q_S$  can be computed from  $p_K$  and  $p_S$  through the present value relation:

$$q_{K,t} = E_t[\Lambda_{t+1}p_{K,t+1}]$$

$$q_{S,t} = E_t[\Lambda_{t+1}p_{S,t+1}].$$

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as documented in Table 3. Our benchmark model with intangible capital discussed in the next section resolves this issue and produces a positive correlation of about 8%, close to that observed in U.S. data from 1930 to 2007.

Under our assumption of the functional form of the  $G(I, S)$  function (equation (6)), the envelope conditions on the social planner's problem can be used to relate cum-dividend prices to ex-dividend prices:

$$p_{K,t} = \alpha K_t^{\alpha-1} (A_t N_t)^{1-\alpha} + H_K(J_t, K_t) q_{S,t} + (1 - \delta_K) q_{K,t} \quad (14)$$

$$p_{S,t} = \frac{1 - \nu}{\nu} \left( \frac{I_t}{S_t} \right)^{\frac{1}{\eta}} + (1 - \delta_S) q_{S,t}, \quad (15)$$

where  $H_K(J, K)$  denotes the partial derivative of the adjustment cost function  $H(J, K)$  with respect to  $K$ . Both equations (14) and (15) have intuitive interpretations. The first two terms on the right-hand side of equation (14) are the marginal product of physical capital: one additional unit of physical capital increases total output by  $\alpha K_t^{\alpha-1} (A_t N_t)^{1-\alpha}$  and reduces the adjustment cost on intangible capital by  $H_K(J_t, K_t)$ . Similarly, the term  $\frac{1-\nu}{\nu} \left( \frac{I_t}{S_t} \right)^{\frac{1}{\eta}}$  is the marginal product of intangible capital.

Equations (14) and (15) are key to understanding the returns on physical and intangible capital. The payoff of physical capital is increasing in productivity,  $A_t$ , and therefore is directly exposed to aggregate productivity shocks. The marginal product of intangible capital, in contrast, does not depend directly on productivity, because intangible capital consists of growth options which produce no output unless they are exercised, that is, unless they are transformed into assets in place.

The key factor driving the marginal product of intangibles is the  $\frac{I_t}{S_t}$  ratio. On the one hand, the options payoff is increasing in  $I_t$ , because growth options and physical investment goods are complements: option exercise requires physical investment goods and hence states in which options are more likely to be exercised (“in-the-money”) are also states with high tangible investment. On the other hand, the options payoff decreases in  $S_t$ , i.e., it is high when the stock of growth options is scarce relative to available physical investment goods.

Optimality of investment in physical and intangible capital requires

$$E_t [\Lambda_{t,t+1} \varpi_{t+1} p_{K,t+1}] - \frac{1}{G_{I,t}} = (1 - \delta_S) E_t [\Lambda_{t,t+1} p_{S,t+1}], \quad (16)$$

$$q_{S,t} = 1/H_J(J_t, K_t). \quad (17)$$

The left-hand side of equation (16) measures the net marginal benefit of exercising an additional option:

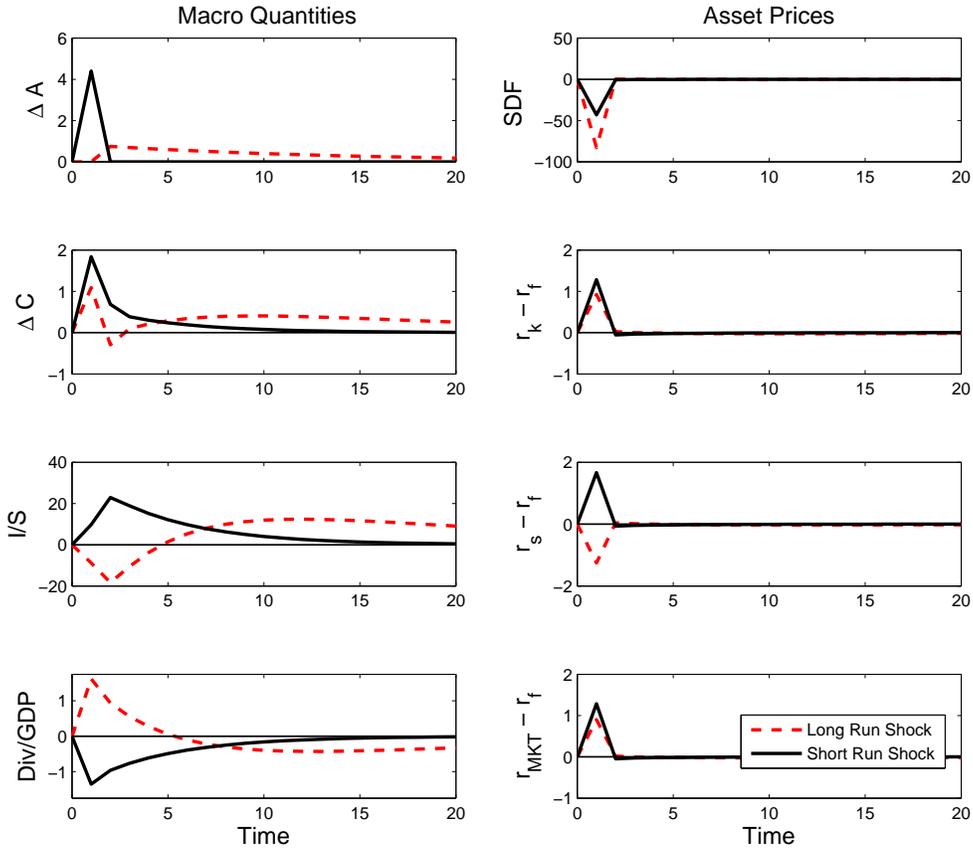
the representative investor obtains the present value of one additional unit of physical capital net of the  $\frac{1}{G_{I,t}}$  cost due to tangible investment. The right-hand side of equation (16) is, in contrast, the opportunity cost of exercising an additional option, i.e., the market value of the option adjusted for the death probability. Finally, equation (17) prescribes that intangible investment must be set so that the ex-dividend value of a marginal growth option equals its own marginal production cost.

**Macroeconomic quantity dynamics.** We calibrate the intangible capital model as in prior work (Ai et al. 2012) to match the first and second moments of macroeconomic quantities. We summarize key statistics on both quantities and asset prices in the last column of Table 3. Our benchmark model largely inherits the success of the vintage capital economy in terms of low volatility of consumption growth, high volatility of investment, high equity risk premium, and low and smooth risk-free interest rate.

In addition, the mean and volatility of intangible investments produced by our model roughly matches the empirical evidence provided in Corrado et al. (2006). In contrast to the vintage capital and the RBC models, our intangible capital model also produces a physical-investment-to-output ratio that matches closely the same moment in the data. In economies with tangible capital only, the physical-investment-to-output ratio tends to be excessively high when the calibration is designed to produce a low risk-free interest rate. This difficulty is resolved by the inclusion of intangible capital in our new model.

The intangible capital economy inherits the basic properties of the vintage capital model in terms of macroeconomic quantity dynamics. In particular, physical investment responds strongly to short-run productivity shocks and is very volatile, as in the data. Upon the realization of a positive long-run productivity shock, in contrast, investment drops and recovers slowly after several periods, exactly as in the vintage capital model. These features of the intangible capital model are evident in Figure 6, where we plot impulse response functions with respect to short-run productivity shocks and long-run productivity shocks. In what follows we explain in detail the implications of this investment behavior for the cross section and term structure of returns.

**Cash-flow duration and expected returns.** It is well documented that high book-to-market ratio stocks (value stocks) earn a higher average return than low book-to-market ratio stocks (growth stocks).



**Fig. 6: Impulse Response Functions for the Intangible Capital Model**

This figure shows percentage annual log-deviations from the steady state for the intangible capital model. We denote the returns on physical capital, intangible capital, and the aggregate equity market as  $r_K$ ,  $r_S$ , and  $r_{MKT}$ , respectively. Returns are not levered. Calibrations are reported in Table 1.

Value stocks are also known to have a shorter cash-flow duration. The negative relationship between cash-flow duration and expected returns in the cross section poses a challenge for general equilibrium asset pricing models. Leading asset pricing models, such as the habit model of Campbell and Cochrane (1999), the long-run risk model of Bansal and Yaron (2004), and the rare disaster model of Reitz (1988), Barro (2006), and Gabaix (2009), typically imply a positive relationship between duration of cash flow and expected return in time series. The difficulty of generating a negative relationship between cash-flow duration and expected return has been emphasized in Santos and Veronesi (2010) and Croce et al. (2007), among others.

The intangible capital model can be used to analyze the cross-section of returns because it features

two different types of capital stocks, intangible and physical capital. Intangible capital is a claim to growth options, whereas physical capital is an aggregate of installed assets in place. As documented by Ai et al. (2012), intangible capital has both longer cash-flow duration and lower expected returns than physical capital. Thus, this setting provides a rationalization for the negative relationship between cash-flow duration and expected returns in a general equilibrium production economy.

Growth options are long-duration assets because they produce cash flows only after option exercise, whereas assets in place have a shorter duration because they pay dividends immediately. To formalize this argument, we define the Macaulay duration,  $MD_t$ , of a stochastic cash-flow process,  $CF_t$ , as

$$MD_t = \frac{\sum_{s=1}^{\infty} s \cdot E_t [\Lambda_{t,t+s} CF_{t+s}]}{\sum_{s=1}^{\infty} E_t [\Lambda_{t,t+s} CF_{t+s}]} \quad (18)$$

Under our calibration, the Macaulay duration of physical capital is about 17 years. In contrast, the duration of intangible capital is more than two times longer, about 48 years. The magnitude of these numbers is economically plausible and comparable to the estimates of Dechow et al. (2004).

Standard real option models typically imply that growth options are riskier than assets in place. When these models are calibrated to match the value premium, they also imply a positive relationship between cash-flow duration and expected returns, in contrast to what we observe in the data. The intangible model solves this problem because growth options are less risky than assets in place. The spread between the levered return on physical and intangible capital is 4.5% per year, fairly close to its empirical counterpart (last column of Table 3).

The interaction between vintage capital and long-run shocks is the key to understanding the difference between the expected returns on growth options and assets in place. First, just as in the vintage capital model, assets in place are highly exposed to long-run productivity shocks and require a significant risk premium at the equilibrium. Upon the realization of a positive long-run shock, old assets in place become relatively more productive than young assets in place and as a result their market value,  $q_{K,t}$ , increases sharply and produces high market excess returns.

Also, growth options have negative exposure to long-run productivity shocks. This result is fully explained by the response of aggregate investment to long-run news. According to equation (15), the

payoff of growth options is an increasing function of the  $I_t/S_t$  ratio. In other words, options are more valuable when they are scarce relative to available physical investment goods. Because investment responds negatively to long-run productivity shocks, so does the excess return of growth options (Figure 6, right column, third panel).

In summary, both the negative response of investment with respect to news shocks documented in the macro-news literature and the higher exposure of value stocks to long-run shocks reported in the empirical long-run risk literature (Bansal et al. 2005, Bansal et al. 2007, and Hansen et al. 2008) are endogenous outcomes of the general equilibrium mechanism of our model.

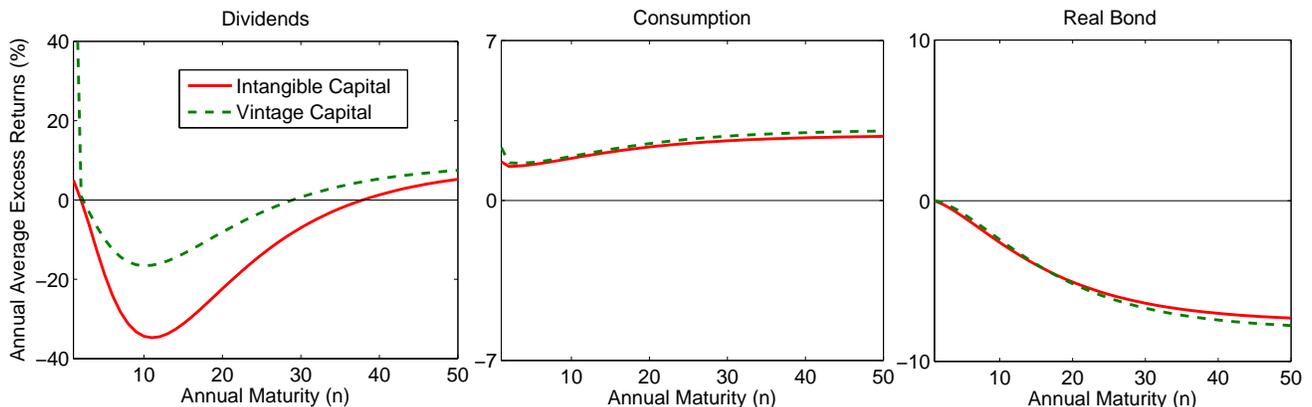
**Growth options and term structure.** In addition to explaining the negative link between duration and excess returns in the cross section, the investment dynamics in the intangible capital model also produce a downward-sloping term structure of equity returns over a ten-year maturity horizon.

In Figure 7, we plot the term structures of consumption, real bonds, and dividends in our intangible capital model. For comparison, we also depict the same term structures for the vintage capital economy discussed in the previous section.

We note that the main properties of macroeconomic dynamics in the intangible capital model are very similar to those in the vintage capital model (see Figure 6 and Table 3). This is especially true for consumption, and it implies that the dynamics of the stochastic discount factor are also similar. As a result, the term structures of consumption and real bonds in the vintage and intangible capital models are almost identical and are consistent with those obtained in the Bansal and Yaron (2004) endowment economy.

Second, the term structure of dividends in the intangible capital model features the same pattern obtained for dividends in the vintage capital model. This is because in these two economies total investment,  $I_t + J_t$ , and aggregate dividends,  $D_t = \alpha Y_t - I_t - J_t$ , follow qualitatively similar dynamics.

Third, from a quantitative point of view, we note that the term structure of equity returns in the model with intangible capital is lower than in the vintage capital model. Since our growth options are less risky than assets in place, the introduction of intangible capital lowers the aggregate equity risk premium and the term structure. More specifically, in our benchmark model intangible investment,  $J_t$ , responds



**Fig. 7: Term Structures in the Vintage and Intangible Capital Models**

This figure shows annual excess returns of zero-coupon equities for different maturities associated with different cash flows. Excess returns are multiplied by 100. Dividend returns are levered by a factor of three (Garcia-Feijo and Jorgensen 2010). The vintage capital model features heterogenous capital vintages. The intangible capital model features both tangible and intangible capital, as well as heterogenous capital vintages. Calibrations are reported in Table 1.

strongly and positively to short-run shocks. As a result, total investment,  $I_t + J_t$ , adjusts with a stronger intensity to short-run shocks. Since aggregate dividends are a short position in total investment, they become less exposed to short-run productivity risk, as can be seen by comparing Figures 3 and 6.

In summary, our long-run risk-based intangible capital model is consistent with (i) the negative relationship between cash-flow duration and expected returns in the cross section of returns, and (ii) a downward-sloping aggregate term structure of equity over a ten-year horizon. In contrast to endowment economy models, the risk exposure of the cash flows associated with tangible and intangible capital are endogenously determined by production and investment dynamics. Accounting for the heterogenous exposure of capital vintages in a general equilibrium model with production is the key to reconciling the long-run risk paradigm with the empirical evidence on value premium, cash-flow duration and the aggregate term structure of equity returns.

## 6 Smooth Transition and Financial Leverage

The main goal of our analysis is to show that the empirical evidence on the negative relationship between cash-flow duration and expected returns from both the time-series and the cross-sectional perspectives

can be jointly rationalized in a long-run risk-based model with production. As discussed in the previous section, our intangible capital model achieves this goal. From a quantitative point of view, however, our model is not fully satisfactory, as our term structure is lower than that observed in the data. In this section, we present two channels that enhance the quantitative performance of our approach.

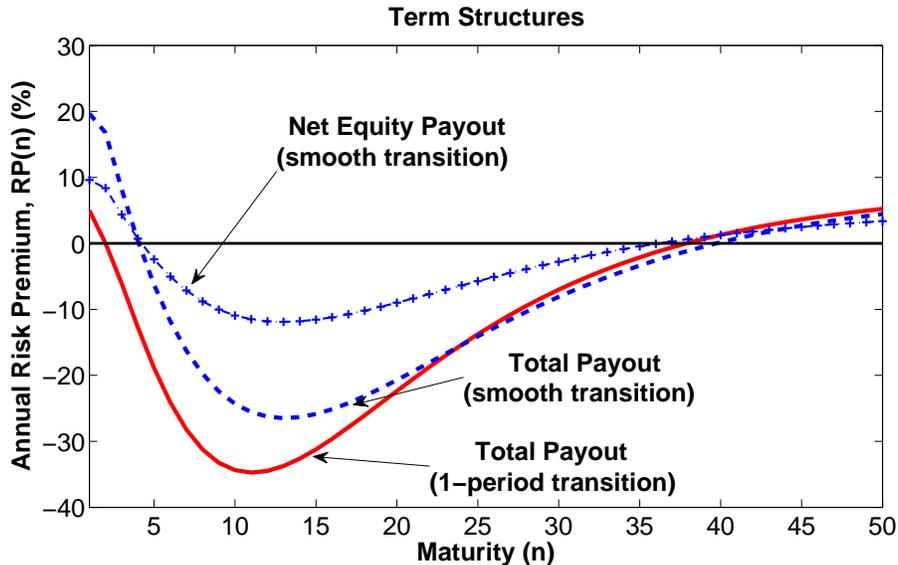
**Smooth time-varying exposure of capital vintages.** Our previous models assume that new investments have zero exposure to aggregate productivity shocks in their first period and are fully exposed afterwards. We have made this assumption to keep our model parsimonious, as more general specifications of the  $\phi_j$  function usually require keeping track of the age distribution of firms as a state variable. In the data, in contrast, this transition is smoother and takes several years (Ai et al. 2012).

In this subsection, we extend our benchmark model to allow for smoother transition in the  $\phi_j$  function. The extended model enhances the risk premium on tangible capital and mitigates the negative expected returns on dividend strips in the intangible capital model. Specifically, we allow for a two-period transition for the exposure of vintage capital to aggregate productivity shocks:

$$\phi_j = \begin{cases} 0 & j = 0 \\ 1/2 & j = 1 \\ 1 & j = 2, \dots \end{cases} .$$

As we show in the appendix, the recursive formulation of the social planner's problem requires an additional state variable,  $X_t$ , which can be interpreted as the time  $t$  productivity-adjusted measure of production units constructed at time  $t - 1$ . In this case, the equilibrium is represented by the same system of equations outlined in section 2, with equation (10) replaced by the following dynamics of the state variables:

$$\begin{aligned} K_{t+1} &= (1 - \delta_K)K_t + X_t [\pi_{t+1} - 1] + \varpi_{t+1}G(I_t, S_t), \\ X_{t+1} &= (1 - \delta_K)\varpi_{t+1}G(I_t, S_t), \\ \pi_{t+1} &= e^{-\frac{1-\alpha}{\alpha}(1-\phi_1)(x_t + \sigma_a \varepsilon_{a,t+1})} \quad \forall t, \end{aligned} \tag{19}$$



**Fig. 8: Two-Period Exposure Transition and Dynamic Financial Leverage**

This figure shows annual excess returns of zero-coupon equities for different maturities associated with different cash flows. Excess returns are multiplied by 100. Total payout returns are levered by a factor of three to account for both financial and operating leverage (Garcia-Feijo and Jorgensen 2010). Net equity payout (NEPO) returns are levered by a factor of 1.5 to account for operating leverage (Garcia-Feijo and Jorgensen 2010). The solid line refers to our baseline intangible capital model with a one-period exposure transition:  $\phi_0 = 0$ , and  $\phi_j = 1$  for  $j \geq 1$ . The dashed lines refer to a two-period exposure transition:  $\phi_0 = 0$ ,  $\phi_1 = 0.5$ , and  $\phi_j = 1$  for  $j \geq 2$ . The leverage parameters are  $\overline{lev} = 2/3$ ,  $\rho_l = .99$ ,  $\alpha_{lrr} = 1.6$ ,  $\alpha_{srr} = -0.21$ , and  $\alpha_0 = 1e^{-4}$ . All other parameters are calibrated to the values reported in Table 1.

In Figure 8, we compare the term structure of equity returns in the benchmark model to that in the extended model with the longer and smoother transition function,  $\phi_j$ . The latter is marked as “smooth transition.” The introduction of a smoother transition function shifts up the term structure for up to twenty years. In particular, the expected return on dividend strips remains positive for about 4.5 years.

**Dynamic financial leverage.** All the models discussed so far abstract away from financial leverage. Dividends in our models,  $\alpha Y_t - I_t - J_t$ , are best interpreted as the payout of the entire corporate sector. The empirical evidence presented by Binsbergen et al. (2012a, b), on the other hand, pertains to the expected return on dividend strips of equity claims, not including payout to corporate debt holders. We allow for dynamic financial leverage by specifying an exogenous dynamic process of capital structure as in Belo et al. (2012).

Specifically, we denote  $B_t$  as the total amount of risk-free debt of the representative firm at time  $t$ ,

and we define the leverage ratio,  $lev_t$ , as the ratio of debt to equity:

$$lev_t \equiv \frac{B_t}{q_{K,t}K_t + q_{S,t}S_t - B_{t-1}}.$$

We assume that the law of motion of the leverage process is given by

$$lev_t = (\overline{lev} + \alpha_0 x_t)(1 - \rho_l) + \rho_l lev_{t-1} - \alpha_{srr} \epsilon_{a,t} - \alpha_{lrr} \epsilon_{x,t}$$

and compute the net equity payout, NEPO, as follows:

$$NEPO_t = (\alpha Y_t - I_t - J_t) - [(1 + r_{t-1}^f)B_{t-1} - B_t],$$

where  $(\alpha Y_t - I_t - J_t)$  equals total payout and  $[(1 + r_{t-1}^f)B_{t-1} - B_t]$  equals the debt payout. We calibrate the leverage ratio parameters in equation (20) to have an average of  $2/3$  and an autocorrelation of  $.99$ , and we match a correlation between NEPO growth and output growth of  $10\%$ , consistent with the empirical evidence in Larraina and Yogo (2008). Our calibration, therefore, features a pro-cyclical net equity payout even though the correlation between total payout growth and output growth is about  $-30\%$ .

We present the term structure of net equity payout in our intangible capital model with two-period smooth transition function  $\phi_j$  in Figure 8. We note that the long end of the term structures converge to the same level due to cointegration, as in Belo et al. (2012). Also, the pattern of the term structure of equity returns in our benchmark model is preserved after the incorporation of financial leverage, while the presence of financial leverage makes the expected return on claims with short-to-medium maturity less negative than before.

## 7 Conclusion

Focusing on aggregate dividend strips with a maturity of up to ten years, Binsbergen et al. (2012a, b) and Boguth et al. (2012) document an inverse relation between zero-coupon equity expected returns and maturity. Dechow et al. (2004) and Da (2009) document a negative relationship between cash-flow duration and expected returns also in the cross section. These empirical facts represent challenges for

several leading endowment-based asset pricing models. This paper presents a long-run risk production-based general equilibrium model accounting for the negative empirical relationship between cash-flow timing and expected returns in both the time-series and the cross-sectional dimensions. Unlike endowment economy models, production economies make endogenous predictions on the cash-flow process of the corporate sector and provide a natural setup in which to examine the relationship between cash-flow duration and expected returns.

We show that the neoclassical real business cycle model produces a counterfactual upward-sloping term structure of equity returns. We resolve this issue by proposing a vintage capital model incorporating the empirical observation that new investments are less exposed to aggregate productivity shocks than is capital of older vintages. The vintage capital model creates endogenous co-movements between long-run productivity news and contemporaneous dividend payouts that make dividend claims with short maturity significantly riskier. In addition, we consider a production setting comprising both vintage capital and intangible capital. We model intangible capital as a stock of growth options and physical capital as a stock of vintages of assets in place. The intangible capital model reconciles the negative relationship between cash-flow duration and expected returns in the cross-section of book-to-market-sorted portfolios and produces a downward sloping term structure of dividends over a horizon of ten years.

Our analysis abstracts away from optimal choice of financial leverage. A fully specified general equilibrium model with endogenous capital structure choices is beyond the scope of this paper, but this represents an important topic for future research. Future studies should also analyze the term structure of equity in a multicountry version of our intangible capital model in order to shed light on the international co-movements documented in Binsbergen et al. (2012b).

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## Appendix

Ai et al. (2012) prove the following lemma, which is a useful starting point for our derivations:

**Lemma 1** *There are  $m$  types of production units. For  $i = 1, 2, \dots, m$ , the productivity of the type  $i$  production unit is denoted as  $A(i)$ , and the total measure of the type  $i$  production unit is denoted by  $K(i)$ . The production technology of the type  $i$  production unit is given by*

$$y(i) = [A(i)n(i)]^{1-\alpha},$$

where  $n(i)$  is the total amount of labor employed by production unit  $i$ . Suppose labor is perfectly mobile, and the total labor supply is  $N$ . Then total output is given by

$$Y = \left[ \sum_{i=1}^m K(i) \left( \frac{A(i)}{A(1)} \right)^{\frac{1-\alpha}{\alpha}} \right]^{\alpha} [A(1)N]^{1-\alpha}.$$

Denote the total measure of generation 0 production units as  $K_0$ , and denote the total measure of generation  $t$  production units as  $E_t$  (that is, the total measure of new production units built at time  $t$  is  $E_t$ ). Let  $A_t$  denote the productivity of generation 0 production units, and let  $A_t^i$  denote the generation  $i$ -specific productivity at time  $t$ . Using the above lemma, the total output of the economy at time  $t$  can be written as

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha},$$

where

$$K_t = (1 - \delta_K)^t K_0 + \sum_{j=0}^{t-1} (1 - \delta_K)^{t-j-1} \left( \frac{A_t^j}{A_t} \right)^{\frac{1-\alpha}{\alpha}} E_j \quad (\text{A1})$$

can be interpreted as the total amount of production units available at time  $t$  measured in generation 0 efficiency units. Using our assumption that  $A_t^t = A_t$ , we can rewrite equation (A1) using productivity growth rates as follows:

$$K_{t+1} = (1 - \delta_K)^{t+1} K_0 + \sum_{j=0}^t (1 - \delta_K)^{t-j} \prod_{i=j}^t \left( \frac{A_{i+1}^j / A_i^j}{A_{i+1} / A_i} \right)^{\frac{1-\alpha}{\alpha}} E_j.$$

In addition, under our specification of the two-period transition function,  $\frac{A_{i+1}^j/A_i^j}{A_{i+1}/A_i} = 1$  for  $i \geq j + 2$ , that is, generation  $j$  production units have the same exposure to aggregate productivity shocks as all previous generations after two periods. In this case, the above equation is simplified to

$$\begin{aligned}
K_{t+1} &= (1 - \delta_K)^{t+1} K_0 + \left( \frac{A_{t+1}^t/A_t^t}{A_{t+1}/A_t} \right)^{\frac{1-\alpha}{\alpha}} E_t \\
&\quad + \sum_{j=0}^{t-1} (1 - \delta_K)^{t-j} \left( \frac{A_{j+1}^j/A_j^j}{A_{j+1}/A_j} \right)^{\frac{1-\alpha}{\alpha}} \left( \frac{A_{j+2}^j/A_{j+1}^j}{A_{j+2}/A_{j+1}} \right)^{\frac{1-\alpha}{\alpha}} E_j \\
&= (1 - \delta_K)^{t+1} K_0 + \sum_{j=0}^{t-1} (1 - \delta_K)^{t-j} \omega^j (j+1) \omega^j (j+2) E_j + \omega^t (t+1) E_t,
\end{aligned}$$

where

$$\omega^j (j+1) = \left( \frac{A_{j+1}^j/A_j^j}{A_{j+1}/A_j} \right)^{\frac{1-\alpha}{\alpha}}$$

and

$$\omega^j (j+2) = \left( \frac{A_{j+2}^j/A_{j+1}^j}{A_{j+2}/A_{j+1}} \right)^{\frac{1-\alpha}{\alpha}}.$$

The process  $\omega^j (j+1)$  ( $\omega^j (j+2)$ ) represents the time  $j+1$  (time  $j+2$ ) productivity gap between generation 0 and generation  $j$ . Using the equation above, aggregate capital evolves as follows:

$$K_{t+1} - (1 - \delta_K) K_t = (1 - \delta_K) \omega^{t-1} (t) [\omega^{t-1} (t+1) - 1] E_{t-1} + \omega^t (t+1) E_t.$$

Define  $X_t = (1 - \delta_K) \omega^{t-1} (t) E_{t-1}$  to be the total measure of generation  $t - 1$  production units measured in generation 0 efficiency units. The dynamics of the state variables  $K_{t+1}$  and  $X_{t+1}$  can then be written recursively as follows:

$$\begin{aligned}
K_{t+1} &= (1 - \delta_K) K_t + [\omega^{t-1} (t+1) - 1] X_t + \omega^t (t+1) E_t \\
X_{t+1} &= (1 - \delta_K) \omega^t (t+1) E_t,
\end{aligned}$$

as stated in equation (19).