Stochastic Idiosyncratic Operating Risk and Real Options: Implications for Stock Returns

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Abstract

We combine real options and stochastic idiosyncratic operating risk in an equity valuation model of firms to capture the cross-sectional variation of stock returns associated with idiosyncratic volatility. An increase in idiosyncratic risk raises the firm value due to a greater option valuation and simultaneously decreases the sensitivity of firm value to systematic risk. The model explains two empirical anomalies: the positive contemporaneous relation between stock returns and changes in idiosyncratic return volatility, and the poor performance of stocks with high idiosyncratic volatility. The model further predicts that (i) returns correlate positively with idiosyncratic volatility during intervals between large changes in idiosyncratic volatility (the switch effect), (ii) and that the anomalies and the switch effect are stronger for firms with more real options and which undergo larger changes in idiosyncratic volatility. Empirical results support these predictions.

Keywords: Idiosyncratic return volatility, cross section of stock returns, asset pricing, real options, growth options, stochastic volatility, regime switching, mixed jump-diffusion processes.

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1 Introduction

Modern portfolio theory and the capital asset pricing model suggest that investors diversify idiosyncratic risks and only systematic risk is priced in equilibrium. The empirical evidence on idiosyncratic return volatility ($IVol$) and stock returns is not readily explained by this simple intuition. One strand of the literature (Duffee (1995); Grullon, Lyandres, and Zhdanov (2010)) establishes that changes in monthly realized $IVol$ is contemporaneously positively related with stock returns (positive $IVol$-return relation or anomaly hereafter), while a different strand (Ang, Hodrick, Xing, and Zhang (2006)) establishes that portfolios of high end-of-month $IVol$ stocks significantly under-perform their low $IVol$ counterparts (negative $IVol$-return relation or anomaly hereafter).[1] Yet, a third strand of the literature establishes that the negative $IVol$ anomaly is due to strong return reversals among a subset of small firms (Huang, Liu, Rhee, and Zhang (2009); Fu (2009)). Given the lack of consensus, it is not surprising that progress in delivering a unified explanation for these findings has been difficult.[2]

In this paper, we reconcile these seemingly disparate empirical regularities with an equity valuation model with growth options and stochastic idiosyncratic operating risk. The model formalizes the main ideas with analytical results highlighting the predictions which are then further explored via numerical simulations and verified empirically. We demonstrate that the presence of stochastic idiosyncratic risk imparts a close relationship between equity returns and $IVol$ and reconciles the two $IVol$ anomalies if firm valuations incorporate convexities in the firms' cashflows, a feature that we attribute to the firms' real options. The firm's currently producing assets – the assets-in-place – have linear valuations in cashflows, therefore they are invariant with respect to idiosyncratic operating risk. As a consequence, we argue that the relation between stock returns and $IVol$ is

[1] Other papers investigating the positive return-volatility relation are Spiegel and Wang (2006), Fu (2009) and Huang, Liu, Rhee, and Zhang (2009). The negative $IVol$ anomaly is also shown to exist in international stock markets by Ang, Hodrick, Xing, and Zhang (2009).

[2] Earlier empirical papers investigating idiosyncratic volatility and returns in the cross section are Lintner (1965), Tinic and West (1986) and Lehmann (1990).

entirely attributed to the firms' reliance on real options and their exposure to stochastic firm-specific operating risk.

The intuition follows from standard explanations in option pricing. Options are levered positions on the underlying asset, hence an increase in the volatility of the underlying asset increases the option value. If the source of risk is non-systematic, then the increase in value should be accompanied by a simultaneous decrease in the proportion of the option value that is exposed to systematic risk. In a similar vein, introducing a 2-regime Markov switching process to proxy for the idiosyncratic volatility of the firms' cashflows generates greater firm valuations and lower firm betas when idiosyncratic volatility is high.[3,4]

The regime dependency of equity value and the time-series pattern of the firm's operating risk help establish return dependency on $IVol$ that is empirically verifiable. The model suggests that the positive $IVol$-return relation is explained by the cross-sectional dispersion in returns driven by firms with growth opportunities that experience changes in idiosyncratic risk, while the negative $IVol$-return relation is explained by the dependency of the firm's beta on the volatility regime.[5] In portfolio-based tests, return realizations of $IVol$-sorted portfolios reflect differences in expected returns. Taken together, the model generates reversals in equity returns correlating positively with contemporaneous changes in $IVol$ and inversely with past realized $IVol$, reconciling the positive and the negative $IVol$ anomalies; anomalies previously not addressed jointly in a single framework.

The model also helps understand the findings that the negative $IVol$-return relation is largely explained by the return reversals of high $IVol$ stocks among a subset of small firms (Huang, Liu, Rhee, and Zhang (2009)); Fu (2009)). The model generates strong

[3]A 2-regime Markov switching process is assumed for tractability, but it is not with loss of generality. Qualitatively, our results should persist in a more general structure insofar as idiosyncratic risk exhibits mean reversion.

[4]Guo, Miao, and Morellec (2005) and Hackbarth, Miao, and Morellec (2006) also develop a 2-regime Markov switching process in state dynamics to investigate investment and capital structure decisions, respectively.

[5]The firm's equity beta, affected by both the value of assets-in-place and the growth option, is inversely related with the firm's idiosyncratic operating risk due to a low systematic component when the option value is high. This is in contrast with the embodied technology shocks modelled in Garleanu, Panageas, and Yu (2012), which impact the level of output flow.

return reversals through the risk dynamics embedded in the operations of the firms that possess growth opportunities. Therefore, we rely on a rational theory of firms that face uncertain operating environments which allows for observable firm-characteristics to explain dispersions in equity returns. In this sense, we depart from the explanations based on limits to arbitrage (Pontiff (2006)) or investors' cognitive biases and mispricings in financial markets (Daniel, Hirshleifer, and Subrahmanyam (1998)) for the negative $IVol$ anomaly.

We use the analytical results from the model in simulations to sharpen the intuition gained from the model. The simulations recreate the $IVol$ anomalies that are qualitatively similar to Duffee (1995) and Ang, Hodrick, Xing, and Zhang (2006), with more pronounced results when we specify larger spreads in volatility between regimes. When we specify a single regime – the standard specification in most real option models – we find that the model generates no statistical $IVol$-return relation, validating that our explanation is the driving mechanism behind the results.

The bulk of the empirical analysis is focused on verifying the predictions that the $IVol$-return relations rely on real options and idiosyncratic operating risks. For the positive $IVol$-return relation, we revisit Grullon, Lyandres, and Zhdanov (2010) by recreating many of their empirical proxies for growth options, and additionally, by creating some of our own proxies. We employ similar cross-sectional return regressions as well, and additional specifications in which we include the difference between the 70th and 30th percentile values of $IVol$ for each stock as a proxy for the spread in idiosyncratic volatility between regimes. For the negative $IVol$-return relation, we revisit Ang, Hodrick, Xing, and Zhang (2006) by creating $IVol$-sorted portfolios, and additionally, by sorting stocks based on the proxies for real option intensity and $IVol$ spread to compute portfolio returns. We find evidence for stronger positive and negative $IVol$-return relations for more real option intensive firms and which experience more extreme changes in $IVol$. These results lend strong support for our model.

The model offers additional testable predictions. Conditioned on a volatility regime, expected equity returns equate to the sum of a continuous drift term and a jump term that captures the expectation of a change in equity value in the event of a switch in volatility. Volatility regime that corresponds inversely with the jump term implies a positive correspondence with the continuous drift term. Therefore, to the extent that real options and stochastic idiosyncratic risk are incorporated into firm valuations, stock returns should correlate positively with $IVol$ in intervals between large changes in $IVol$. Using event studies methodology, we investigate the difference in 5-month average returns around the month in which stocks experience large changes in $IVol$. We find that the difference between post and pre-switch returns is positive for the up-switch sample, and negative for the down-switch sample, and that this 'switch effect' is stronger for more real option intensive firms and which experience more extreme changes in $IVol$. Here again the results are in strong agreement with the model.

Motivated by anomalies evidenced in the cross-section of stock returns, Berk, Green, and Naik (1999) were among the first to establish a linkage between corporate investments and expected equity returns.[6,7] Since then, the literature has been extended in many directions (Carlson, Fisher, and Giammarino (2004); Zhang (2005); Sagi and Seashole (2007); Cooper (2007)). A common theme in this literature focuses on the extent that growth options contribute to the beta of the firm relative to the firm's assets-in-place. We add to this literature by expanding the description of the firm's operating environment in an important way to reconcile the $IVol$ anomalies. In our model, idiosyncratic volatility serves as an additional state variable that affects the beta of the firm's real options, but not the beta of the firm's assets-in-place.

To the best of our knowledge few inroads have been made to link idiosyncratic risk to

[6]Fama and French (1992) provide evidence on the ability of size and book-to-market to explain returns. Fama and French (1996) provide a cross-sectional landscape view of how average returns vary across stocks. Anderson and Garcia-Feijoo (2006) offer empirical evidence on the relation between corporate investments and average returns.

[7]Firm-level investment in a real option context was first pioneered by MacDonald and Siegel (1985), MacDonald and Siegel (1986) and Brennan and Schwartz (1985), and later adopted and extended by many others. Dixit and Pindyck (1994) is a standard reference for a detailed analysis of the literature.

asset pricing. The exceptions are Babenko, Boguth, and Tserlukevich (2013) and Kogan and Papanikolaou (2013) who show that firm-specific shocks contain information about future priced risk. Babenko et al. (2013) view firms as portfolios of systematic and idiosyncratic divisions and rely on additive systematic and idiosyncratic cashflow shocks in the valuation of the firms to explain asset pricing anomalies. Kogan and Papanikolaou (2013), on the other hand, show that the investments of firms with high growth opportunities exhibit higher sensitivity to investment-specific-technology shocks earning a lower risk premia. While one can view these models as strongly complementary, our modeling approach explicitly considers idiosyncratic cashflow shocks with time-varying risk together with an optimal timing decision concerning growth option exercise. Hence, the underlying mechanism in our model is distinct from Babenko et al. (2013) and Kogan and Papanikolaou (2013), allowing us to propose a novel channel between the operating environment faced by the firms and equity returns. The distinct features of our model yield novel testable predictions on the correspondence between $IVol$ and stock returns such as the switch effect, which we test empirically in this paper.

The rest of the paper is organized as follows: Section 2 develops the model environment, Section 3 presents the analytical solutions and the implications of the model, Section 4 discusses the simulations and the results, Section 5 reports the empirical analysis, Section 6 concludes. The Appendix contains all the proofs and other technical details omitted in the main body of the paper.

2 Model

We construct a growth option model similar in spirit to the models in Garlappi and Yan (2008) and Carlson, Fisher, and Giammarino (2004).[8] This section describes the firms' economic environment.

[8]With no loss of generality, we rely specifically on growth options to incorporate convexity of firm valuations in the firms' output price. Other forms of real options that incorporate convexities would accommodate similar results.

2.1 The Environment

We consider two types of firms. Mature firms produce at full capacity. By contrast, young firms produce at a lower operating scale, but have the option to make an irreversible investment to increase production and become mature. Firms are all equity financed. Each firm produces a single commodity that can be sold in the product market at price P

$$P = XZ \tag{2.1}$$

where X and Z are respectively the idiosyncratic and the systematic components which have the following dynamics:

$$\begin{aligned} \frac{dX}{X} &= \sigma_{P,i} dB_1 \\ \frac{dZ}{Z} &= \mu dt + \sigma_A dB_2 \end{aligned} \tag{2.2}$$

μ denotes the growth rate, σ_A the market volatility, $\sigma_{P,i}$ the idiosyncratic volatility, and dB_1 and dB_2 are the increments of two independent Brownian motions. Time and firm subscripts throughout are omitted for convenience.

We allow firms to have random and time-varying potential to realize monopolistic rents by allowing idiosyncratic operating risk to be time varying.[9] $\sigma_{P,i}$ follows a 2-regime Markov switching process

[9]Dixit and Pindyck (1994) and Caballero and Pindyck (1996) show that idiosyncratic shocks translates to a firm's ability to retaining monopolistic rents – a firm that experiences a positive idiosyncratic technology shock experiences an advantage that cannot be stolen by its competitors, while a positive aggregate shock is shared with the firm's competitors. Some plausible micro-economic examples for a change in idiosyncratic operating risk are: shifts in consumer needs and wants, persistent changes in production technology, or changes in the general operating environment of the firm or the firm's industry, among others.

$$
\triangle\sigma_{P,i} = \begin{cases} \sigma_{P,H} - \sigma_{P,L} & , \text{ with prob. } \lambda_H dt, \quad \text{if } i = L \\ 0 & , \text{ with prob. } 1 - \lambda_H dt, \quad \text{if } i = L \\ \sigma_{P,L} - \sigma_{P,H} & , \text{ with prob. } \lambda_L dt, \quad \text{if } i = H \\ 0 & , \text{ with prob. } 1 - \lambda_L dt, \quad \text{if } i = H \end{cases} \tag{2.3}
$$

where $\sigma_{P,H} - \sigma_{P,L} > 0$, and λ_L and λ_H are known transition parameters between high and low volatility regimes H and L.[10] The switches between the two regimes $\triangle\sigma_{P,i}$ are independent Poisson processes and independent across firms. Both P and the volatility regime i are observable for any given firm.[11] We subscript quantities with $i \in \{H, L\}$ throughout to denote their dependence on the volatility regime.

Investors in the stock market can hedge market risk in the firms' operations by trading on two securities. Let M_t denote the price of the risk free asset with dynamics

$$
\frac{dM}{M} = rdt \tag{2.4}
$$

and let S be the price of a risky security with dynamics

$$
\frac{dS}{S} = \mu_S dt + \sigma_S dB_2 \tag{2.5}
$$

S has a beta equal to one and $\lambda = \frac{\mu_S - r}{\sigma_S}$ is the market price of risk. The proportion of S held in a replicating portfolio determines the beta of the portfolio. This greatly simplifies the valuation and the derivation of the beta of the firms.

[10] Assuming a 2-state Markov switching process is not without generality. A model with a more general volatility structure is possible, but at a cost of analytical tractability.

[11] Conditioned on being in the high volatility state, the probability that $\triangle\sigma_{P,i}$ will switch to the low volatility regime in the next short interval dt is $\lambda_L dt$. $\lambda_H dt$ is defined similarly. Based on standard properties of Poisson processes, the expected duration that the process dP will stay in the high volatility regime H and the low volatility regime L are λ_L^{-1} and λ_H^{-1}, respectively. The proportion of time spent in the high and low volatility regimes are $\frac{\lambda_H}{\lambda_H+\lambda_L}$ and $\left(1 - \frac{\lambda_H}{\lambda_H+\lambda_L}\right)$ respectively.

2.2 The Value of a Mature Firm

The value of a mature firm is composed of the profit stream from selling the output. The cost of producing a unit of output is c per unit of time. ξ_M denotes the scale of production, therefore the profit per unit of time is $\pi_M(P) = \xi_M(P - c)$. The equity value of a mature firm is as follows

$$V_M(P) = \xi_M \left(\frac{P}{r - \mu^*} - \frac{c}{r} \right) \tag{2.6}$$

where $\mu^* = \mu - \sigma_A \lambda < r$. The firm value is the present value of a growing risky perpetuity, less the present value of a riskless perpetuity.

2.3 The Value of a Young Firm

Young firms produce at a lower capacity than mature firms, i.e. $\xi_Y < \xi_M$, therefore the profit per unit of time is $\pi_Y(P) = \xi_Y(P - c)$, but possess a perpetual option to increase production scale by $\xi = \xi_M - \xi_Y$ upon making a one time irreversible investment of I. For simplicity, we assume that financing is done by equity. Young firms derive value from assets currently in production, or assets-in-place, and the growth option. The value of the assets-in-place have the same functional form as equation (2.6) with ξ_M replaced by ξ_Y. The total equity value of a young firm is as follows

$$V_{Y,i}(P) = \xi_Y \left(\frac{P}{r - \mu^*} - \frac{c}{r} \right) + GO_i(P) \tag{2.7}$$

where the value of the growth option $GO_i(P)$ obeys the following Bellman equation

$$GO_i(P) = e^{-rdt} E^{\mathbb{Q}} \left[GO_i(P + dP, \sigma_{P,i} + \triangle \sigma_{P,i}) \right] \tag{2.8}$$

with a value realization upon exercise net of cost of $\xi \left(\frac{P}{r - \mu^*} - \frac{c}{r} \right) - I$, and $E^{\mathbb{Q}}[.]$ denotes the expectation operator under the $\mathbb{Q}$ measure. The convexity of the option value with respect to P_t ensures that the firm value and the decision to expand are dependent on both P and the volatility regime i (Guo, Miao, and Morellec (2005)). Optimal exercise requires to choose when to invest, which occurs at time τ_i. Define P_i^* the price level at which a young firm exercises its growth option. The choice of P_i^* describes the strategy for a young firm, and the strategy chosen that satisfies the optimality conditions maximizes the value of the firm.

2.4 Equity betas and Expected Returns

Expected returns differ in the cross-section based on the firms' maturity, output price, and the idiosyncratic volatility regime in effect. For example, the expected return of a young firm can be expressed according to the CAPM as

$$E\left[\frac{\pi_Y(P)dt + dV_{Y,i}(P)}{V_{Y,i}(P)dt}\right] = r + \beta_{V_Y,i}(P)\sigma_S\lambda \tag{2.9}$$

where $\beta_{V_Y,i}(P)$ denotes the firm's CAPM beta conditional on i and P. Because the firm value and the decision to exercise the option are regime dependent, so is the equity beta of young firms.

3 Model Solution

In this section, we discuss the properties of the model solution and their empirical implications.

3.1 Valuation

The following proposition states the valuation and the exercise threshold of the investment option.

Proposition 1 *If the price process is given by (2.1) and (2.3), then the value of a growth option in the region of low values of P, $P \in (0, P_1)$, is*

$$F_H(P) = \frac{B_{L,1}P^{\beta_{2,1}}q_L(\beta_{2,1})}{\lambda_H} + \frac{B_{L,2}P^{\beta_{2,2}}q_L(\beta_{2,2})}{\lambda_H} \tag{3.1}$$

if P is in the high volatility regime, and

$$F_L(P) = B_{L,1}P^{\beta_{2,1}} + B_{L,2}P^{\beta_{2,2}} \tag{3.2}$$

if P is in the low volatility regime.

In the region of intermediate values of P, $P \in (P_1, P_2)$, the option value is

$$G_H(P) = \frac{\lambda_L}{\lambda_L + r}\left(\xi\left(\frac{P}{r-\mu^*} - \frac{c}{r}\right) - I\right) + C_{H,1}P^{\beta_{1,1}} + C_{H,2}P^{\beta_{1,2}} \tag{3.3}$$

if P is in the high volatility regime, and

$$G_L(P) = \xi\left(\frac{P}{r-\mu^*} - \frac{c}{r}\right) - I \tag{3.4}$$

if P is in the low volatility regime. Moreover, the optimal exercise boundaries P_1 and P_2 are the solution to the following system of equations

$$\begin{aligned} C_{H,1}P_1^{\beta_{1,1}} + C_{H,2}P_1^{\beta_{1,2}} - \frac{\lambda_L}{\lambda_L + r}\left(\xi\left(\frac{P_1}{r-\mu^*} - \frac{c}{r}\right) - I\right) \\ = \frac{B_{L,1}P_1^{\beta_{2,1}}q_L(\beta_{2,1})}{\lambda_H} + \frac{B_{L,2}P_1^{\beta_{2,2}}q_L(\beta_{2,2})}{\lambda_H} \end{aligned} \tag{3.5}$$

$$\begin{aligned} \beta_{1,1}C_{H,1}P_1^{\beta_{1,1}} + \beta_{1,2}C_{H,2}P_1^{\beta_{1,2}} + \frac{\xi P_1 \lambda_L}{(r-\mu^*)(\lambda_L + r)} \\ = \frac{\beta_{2,1}B_{L,1}P_1^{\beta_{2,1}}q_L(\beta_{2,1})}{\lambda_H} + \frac{\beta_{2,2}B_{L,2}P_1^{\beta_{2,2}}q_L(\beta_{2,2})}{\lambda_H} \end{aligned} \tag{3.6}$$

where the expressions for $B_{L,1}$,$B_{L,2}$,$C_{H,1}$, $C_{L,1}$, $q_L(\beta)$, $\beta_{1,1}$, $\beta_{1,2}$, $\beta_{2,1}$ and $\beta_{2,2}$ are given in the Appendix.

Proof: *See Appendix.* ∎

The proposition states that young firms have separate investment policies for each volatility regime. There are three distinct regions in the range of possible values of P to consider (Guo, Miao, and Morellec (2005)). In the region where $P \in (0, P_1)$, the investment value is below the investment cost in both volatility regimes, hence the option is kept alive and the option value is given by (3.1) and (3.2). In the region where $P \in (P_1, P_2)$, the investment value exceeds the cost in the low volatility regime only, while the option is kept alive in the high volatility regime.[12] The option value is given by (3.4) and (3.3) in this region. Lastly, in the region where $P > P_2$, the investment value exceeds the cost in both volatility regimes, therefore the value $\xi\left(\frac{P}{r-\mu^*} - \frac{c}{r}\right) - I$ reflects immediate investment and the value of the incremental increase in production scale.

More importantly, the proposition reveals that growth options have distinct valuations across volatility regimes, a feature that contrasts starkly from assets-in-place and mature firms (2.6). Therefore, the reliance of a firm's value on the volatility regime is attributed entirely to the growth option of the firm.

Insert Figure 1 here

Figure 1 provides a graphical illustration of Proposition 1 for different set of parameter values of $\sigma_{P,H}$ and $\sigma_{P,L}$. Comparing the graphs across panels reveals that the opportunity to expand has a larger valuation in regime $i = H$ than in regime $i = L$, and the difference is increasing in the spread between $\sigma_{P,H}$ and $\sigma_{P,L}$. The last panel reveals that the model results in a single valuation profile if $\sigma_{P,H} = \sigma_{P,L}$, which is the usual specification in standard growth option models.

[12] This property hinges on standard option pricing results that the value of an option is increasing in the volatility of the underlying asset.

3.2 Returns

Having derived the firms' valuation and their investment policies, this section explores the model's implications for equity returns.

Proposition 2 *If the price process is given by (2.1) and (2.3), and $F_i(P)$, $i \in \{H, L\}$, is given by (3.1) and (3.2), then $F_i(P)$ has the following dynamics*

$$\frac{dF_i(P)}{F_i(P)} = a_i(P)dt + b_i(P)dB_i + \nu_i(P)dz_i \tag{3.7}$$

where $dB_i = \frac{\sigma_{P,i}dB_1+\sigma_A dB_2}{\sigma_i}$, $\sigma_i = \sqrt{\sigma_{P,i}^2 + \sigma_A^2}$, dz_i is a Poisson process that is perfectly functionally dependent on $\Delta\sigma_{P,i}$, and the following relations hold between volatility regimes

$$a_H(P) > a_L(P) \tag{3.8}$$

$$b_H(P) > b_L(P) \tag{3.9}$$

$$\nu_H(P) < 0 < \nu_L(P) \tag{3.10}$$

The expected return of the growth option according to the CAPM is given by

$$E\left[\frac{dF_i(P)}{F_i(P)dt}\right] = a_i(P) + \lambda_{i'}\nu_i(P) \tag{3.11}$$

where $i' = L$ if $i = H$ and vice-versa, and the following holds for the option's CAPM beta

$$\beta_{F,H}(P) < \beta_{F,L}(P) \tag{3.12}$$

Expressions for $a_H(P)$, $b_H(P)$, $\nu_H(P)$, $\beta_{F,H}(P)$, $a_L(P)$, $b_L(P)$, $\nu_L(P)$ and $\beta_{F,L}(P)$ are given in the Appendix.

Proof: *See Appendix.* ■

Proposition 2 conveys the central idea of our paper. Growth options follow different dynamics between volatility regimes, a feature inherited from their values. Since firm valuations incorporate growth options, the value of a young firm obeys the laws of motion pertaining to the volatility regime in effect. In 'normal times' absent of a switch in volatility, the firm value dynamics is determined partly by the first two terms of (3.7), the drift and the diffusion terms $a_i(P)$ and $b_i(P)$ respectively. However, on average every $1/\lambda_{i'}$ units of time, a sudden and relatively large change in firm value contributed by $\nu_i(P)$ in (3.7) is activated by the arrival of a switch in regime, after which the dynamics obeys the law of motion pertaining to the new volatility regime i'. The dynamics stays the same until the next switch arrives. This feature is attributed to the firm's growth option and amplified by the spread between $\sigma_{P,H}$ and $\sigma_{P,L}$. By contrast, assets-in-place and mature firms are independent of the volatility regime and they have continuous returns that do not exhibit jumps. While the idiosyncratic return variance of a young firm is a weighted average of the idiosyncratic variance of assets-in-place and growth option, any correspondence between idiosyncratic volatility and expected equity returns is entirely attributed to the growth option.[13]

The model has important implications for the relation between idiosyncratic volatility and equity returns. The proposition reveals that $b_H(P) > b_L(P)$, establishing a positive correspondence between equity idiosyncratic return volatility and the idiosyncratic operating risk of the firm. Furthermore, the jump term $\nu_i(P)$ has the same sign as the switch in volatility, i.e., $\nu_H(P) < 0 < \nu_L(P)$. This suggests that, to the extent that real options and stochastic idiosyncratic risk are incorporated into firm valuations, returns should exhibit positive contemporaneous correlation with changes in idiosyncratic return volatility, contributing to the positive $IVol$-return empirical relation (Duffee (1995)). In our model, the positive $IVol$ anomaly is explained by the jumps in returns of growth option firms simultaneously experiencing a switch in idiosyncratic operating risk.

[13] Although the return dynamics for the assets-in-place and mature firms have a higher variance in the high volatility regime than in the low regime, the drift term of their value process is not regime dependent.

The proposition also reveals that $\beta_{F,H}(P) < \beta_{F,L}(P)$. The intuition follows from standard option pricing results. If the source of risk is non-systematic, then the greater option valuation when $i = H$ should be simultaneously accompanied by a lower proportion of the total value that is exposed to systematic risk.[14] In portfolio-based tests, sorting and grouping firms based on end-of-month realized $IVol$ is akin to grouping based on the firms' most recent idiosyncratic volatility regime. To the extent that real options and stochastic idiosyncratic risk are incorporated into firm valuations, return realizations of idiosyncratic volatility sorted portfolio reflect differences in expected returns, establishing a negative correspondence between future equity returns and current idiosyncratic risk (Ang, Hodrick, Xing, and Zhang (2006)). Taken together, the model generates reversals in equity returns correlating positively with contemporaneous changes in $IVol$ and inversely with past realized $IVol$, reconciling the positive and the negative $IVol$ anomalies. We further explore this feature of the model in the sequel with numerical simulations and empirical tests.

Lastly, the model offers the basis for a novel prediction on the relation between stock returns and idiosyncratic return volatility. The proposition shows that expected return equates to the sum of a continuous drift term $a_i(P)$ and a probability weighted jump term $\lambda_{i'}\nu_i(P)$. While expected return conditional on the volatility regime is based on the option's amount of systematic risk, it nonetheless includes the expectation of a change in the option value in the event of a switch in regime. Initial volatility regime i that corresponds inversely with the jump term $\nu_i(P)$ implies a positive correspondence with the continuous drift term $a_i(P)$, i.e., $a_H(P) > a_L(P)$. Therefore, to the extent that real options and stochastic idiosyncratic risk are incorporated into firm valuations, stock returns should correlate positively with $IVol$ during times between large changes in $IVol$. We also test this empirical prediction in the sequel.

[14]This result is consistent with Galai and Masulis (1976), who show that $\frac{\partial\beta}{\partial\sigma} < 0$ but does not consider stochastic volatility in the firm's assets, and with Johnson (2004), who shows that increasing uncertainty about the value of a firm's assets while holding the risk premium constant lowers the expected returns of levered firms.

Insert Figure 2 here

Figure 2 provides a graphical illustration of the ideas conveyed in Proposition 2 for different set of parameter values of $\sigma_{P,H}$ and $\sigma_{P,L}$. Panel (b) and (c) show that there is a positive difference in the diffusion terms $b_H(P) - b_L(P)$, and the continuous drift terms $a_H(P) - a_L(P)$ between regimes. Panel (d) of the figure shows that there is a negative difference in jump terms $\nu_H(P) - \nu_L(P)$. All the differences are increasing in the spread between $\sigma_{P,H}$ and $\sigma_{P,L}$, suggesting that the relation between returns and idiosyncratic volatility should be stronger the greater the variation in the firms' operating risk. Lastly, the difference in all quantities are identically zero if the volatility values are the same in both regimes, which is the usual specification in standard growth option models.

4 Simulations

In this section, we rely on numerical simulations in order to investigate if our model is able to simultaneously produce the positive and the negative $IVol$-return relations, reconciling the two $IVol$ anomalies is a single framework. Our goal is not to do a full calibration exercise with Simulated Methods of Moments. Instead, our goal is to create a laboratory in which to qualitatively analyze the effects of real options and stochastic idiosyncratic operating risk on the cross-sectional relation between stock returns and $IVol$.

Using the analytical solutions of the model, we simulate a large panel of daily firm values by first simulating a single path of S_t using (2.5). Then we simulate 2,500 separate paths of P_t and volatility values using processes (2.1) and (2.3).[15, 16] Each simulated path of P_t corresponds to the output price series for a single firm. The time horizon is 50 years with 20 trading days in each month, corresponding to a total of 12,000 daily observations

[15] After applying our filters, the data in our empirical study contains an average of 2,412 firms each month with non-missing sales growth observations.

[16] Hanson (2007) is a good reference for numerical simulations of diffusion and Poisson processes.

for each firm. Then, for each day and each firm, we compute firm values using equations (2.6), (2.7), (3.1), (3.2) and (3.3).

Initial maturities are drawn from a uniform distribution with equal probabilities of young and mature which are updated daily. To insure that mature firms do not dominate the sample overtime, mature firms exit the sample upon the arrival of an independent Poisson event with intensity $\lambda_{exit} = 0.01$ per unit of time or if the firm value reaches zero due to low realizations of P_t values. Exiting firms are replaced by new young entrants.

We compute daily abnormal returns relative to the $CAPM$ using the beta expressions for $F_i(P)$, $G_H(P)$, $V_M(P)$ and assets-in-place. Then for each firm and each month, we compute $IVol$ as the standard deviation of the abnormal returns. The beta of young firms is computed as a weighted average of the beta of the firms' assets-in-place and the beta of the firms' growth option where the weights are based on the proportion of firm value in the growth option.

We use the simulated returns to carry out the main analysis in Duffee (1995) and Ang et al. (2006) and store the results. Then, we repeat the entire process 99 more times in order to arrive at a set of 100 estimates allowing us to carry out t-tests in order to investigate the statistical significance of the results. To investigate the model's reliance on stochastic idiosyncratic risk, the simulation steps described thus far are repeated using three different sets of values for $\sigma_{P,H}$ and $\sigma_{P,L}$. Table 1 summarizes the set of parameters used to solve the model.

Insert Table 1 here

Using the baseline set of model parameters, Figure 3 shows the month-end values of a single simulated path of P, the corresponding $V(P)$ values, idiosyncratic volatility regimes, realized idiosyncratic return volatilities $IVol$ and realized returns. Panels (a) and (b) reveals that $V(P)$ follows a similar pattern as P, as expected. Panels (c) to (d) show that returns and $IVol$ appear to be regime dependent, consistent with Proposition 2.

Insert Figure 3 here

4.1 The Positive Return-Volatility Relation

Using the simulated data, we fit Fama and MacBeth (1973) monthly cross-sectional regressions of log return r_t on $\Delta IVol_t$ in order to investigate if the model can create the positive $IVol$-return relation. The cross sectional regression model for month t is

$$r_t = \gamma_{0,t}\iota + \gamma_{1,t}\Delta IVol_t + \eta_t \tag{4.1}$$

where ι is a vector of ones, r_t is a vector of $r_{j,t}$ and $\Delta IVol_t$ is a vector of $\Delta IVol_{j,t}$ of all the firms $j \in J$.

Insert Table 2 here

Table 2 reports the results. The table shows that if $\sigma_{P,H} > \sigma_{P,L}$ the model is able to produce the positive $IVol$-return relation. The table also shows that the positive $IVol$ anomaly is more pronounced for larger spreads between $\sigma_{P,H}$ and $\sigma_{P,L}$, but negligible and insignificant if $\sigma_{P,H} = \sigma_{P,L}$, confirming that the stochastic nature of idiosyncratic risk is crucial to generate the anomaly.

4.2 The Negative Return-Volatility Relation

Using the simulated data, we form portfolios based on $IVol$ in order to investigate if the model can create the negative $IVol$-return relation. At the end of each month, we sort firms based on $IVol$ into five equally sized groups. Then, we compute value-weighted one-month portfolio returns for each of the five groups. The portfolios are rebalanced at the end of each month.

Insert Table 3 here

Table 3 reports the results. $IVol$-sorted portfolios are reported across columns. The zero-cost (high minus low) $IVol$ portfolios are reported in the last column. Figure 4 provides a visual illustration of the average returns reported in the table. The zero-cost $IVol$ portfolio has a highly significant and negative average return if $\sigma_{P,H} > \sigma_{P,L}$, with more amplified results for larger spreads between $\sigma_{P,H}$ and $\sigma_{P,L}$. The model offers negligible and insignificant results if $\sigma_{P,H} = \sigma_{P,L}$, confirming that the stochastic nature of idiosyncratic risk is crucial to generate the anomaly.

Insert Figure 4 here

We conduct further analysis by fitting Fama and MacBeth (1973) monthly cross-sectional regressions of returns on lagged $IVol$. The regression model for month t is

$$r = \gamma_{0,t}\iota + \gamma_{1,t}IVol_{t-1} + \eta_t \tag{4.2}$$

Table 2 reports the results. There is a negative and highly statistically significant return-lag $IVol$ relation if $\sigma_{P,H} > \sigma_{P,L}$ with more amplified results for larger spreads between $\sigma_{P,H}$ and $\sigma_{P,L}$, but negligible and insignificant result if $\sigma_{P,H} = \sigma_{P,L}$. These results reaffirm the earlier portfolio results.

Taken together, the simulations confirm the prediction that real options and stochastic idiosyncratic operating risk play a significant role in reconciling the two $IVol$ anomalies.

5 Empirical Analysis

In this section, we test the empirical predictions of our model and show support in the data.

5.1 Data, Variable Descriptions and Summary Statistics

Daily and monthly stock returns are from CRSP. Daily and monthly factor returns and risk-free rates are from Ken French's website.[17] All accounting variables are from annual COMPUSTAT files. Our sample period is from January, 1971 to December, 2010 for all market-based variables.[18] We consider only ordinary shares traded on the NYSE, AMEX and Nasdaq with primary link to companies on COMPUSTAT with US data source. We eliminate utility (SIC codes between 4900 and 4999) and financial companies (SIC codes between 6000 and 6999), companies with less than one year of accounting data, stock price of zero and negative book equity values. In order to remove the effects of delisting, we eliminate return observations within one year of delisting if the delisting code has the first digit different from 1. The final sample size is over 1 million monthly observations with non-missing return and idiosyncratic return volatility values.

5.1.1 Idiosyncratic Volatility

Our empirical study requires a measure for the firms' idiosyncratic operating risk. Stock return volatility is commonly used as a proxy for the volatility of the firms' operations (Leahy and Whited (1996); Bulan (2005); Grullon, Lyandres, and Zhdanov (2010)). Following Ang, Hodrick, Xing, and Zhang (2006), for each firm j and month t, we estimate idiosyncratic return volatility $IVol$ as the standard deviation of the daily stock returns relative to the Fama and French 3 factor model:

$$r_{j,\tau} = \alpha_i + \beta_{j,MKT} MKT_\tau + \beta_{j,SMB} SMB_\tau + \beta_{j,HML} HML_\tau + \varepsilon_{j,\tau} \tag{5.1}$$

[17] http:// mba.tuck.dartmouth.edu/ pages/faculty/ken.french/data library.html

[18] The annual number of firms on COMPUSTAT with non-missing sales and net income observations is relatively low prior to the 70's after applying the reported filters.

where $IVol_{j,t} = \sqrt{var(log(1+\varepsilon_{j,\tau}))}$ and $\varepsilon_{j,\tau}$ for $\tau \in (t-1, t]$ are the residuals from fitting regression (5.1).[19] Furthermore, we define $\Delta IVol_{j,t}$ as the change in $IVol$ from previous month, i.e., $IVol_{j,t} - IVol_{j,t-1}$.

We also require an empirical proxy to capture the variability in the idiosyncratic risk of the firms. Towards this end, for each firm, we consider the stock's 70th and 30th percentile values of $IVol$ to be the thresholds that define the volatility regimes for the firm, and we denote the spread $\overline{\Delta IVol_j}$ to be to be the difference between the 70th and 30th percentile values.

5.1.2 Firm Characteristics

We require several variables shown in the literature to be determinants of stock returns as controls when conducting cross-sectional return regressions. They are: log market equity; log book-to-market; past stock returns; CAPM beta; and trading volume.[20]

[19]Following Grullon, Lyandres, and Zhdanov (2010), we use the logarithm of the residuals in order to mitigate the potential mechanical effects of return skewness on the relation between return and volatility (Duffee (1995); Chen, Hong, and Stein (2001); Kapadia (2007)).

[20]Following Fama and French (1993), market value of equity is defined as the share price at the end of June times the number of shares outstanding. Book equity is stockholders' equity minus preferred stock plus balance sheet deferred taxes and investment tax credit if available, minus post-retirement benefit asset if available. If missing, stockholders' equity is defined as common equity plus preferred stock par value. If these variables are missing, we use book assets less liabilities. Preferred stock, in order of availability, is preferred stock liquidating value, or preferred stock redemption value, or preferred stock par value. The denominator of the book-to-market ratio is the December closing stock price times the number of shares outstanding. We match returns from January to June of year t with COMPUSTAT-based variables of year $t-2$, while the returns from July until December are matched with COMPUSTAT variables of year $t-1$. This matching scheme is conservative and ensures that the accounting information-based observables are contained in the information set prior to the realization of the market-based variables. We employ the same matching scheme in all our matches involving accounting related variables and CRSP-based variables. We define past returns as the buy-and-hold gross compound returns minus 1 during the six-month period starting from month $t-7$ and ending in month $t-2$. Following Karpoff (1987), trading volume is trading volume normalized by the number of shares outstanding during month t. Lastly, stock CAPM beta is the estimated coefficient from rolling regressions of monthly stock excess returns on the market factor's excess returns. We use a 60-month window every month requiring at least 24 monthly return observations in a given window, and use the procedure suggested in Dimson (1979) with a lag of one month in order to remove biases from thin trading in the estimations.

5.1.3 Real Option Proxies

We also require empirical proxies for the extent that firms incorporate real options. We follow Grullon, Lyandres, and Zhdanov (2010) in the selection of our main growth option variables, and additionally, create some of our own.

The most common type of real options come in the form of future growth opportunities (Grullon, Lyandres, and Zhdanov (2010); Brennan and Schwartz (1985); MacDonald and Siegel (1986); Majd and Pindyck (1987); Pindyck (1988)). We consider firm size and firm age as inverse measures of growth opportunities because larger and older firms tend to be more mature and have larger proportions of their values from assets-in-place, while smaller and younger firms tend to derive value from future growth opportunities (Brown and Kapadia (2007); Carlson, Fisher, and Giammarino (2004); Lemmon and Zender (2010)). We define two measures of firm size: the book value of total assets and the market value of equity. Age is defined as the difference between the month of the return observation and the month in which the stock first appeared on CRSP.

Growth opportunities are revealed in growth capitalized in the future in the form of increased sales, profits or investments. Therefore, for our third set of growth variables, we define future sales growth as the sum of the sales growth rates starting 2 years and ending 5 years after the stock return observation. Future profit and future investment growth are defined similarly.[21]

We consider a novel proxy for real option intensity. The equity of a firm is akin to a call option on the firm's assets with the strike price amounting to the total value of the firm's debt (Merton (1974) and Merton (1992)). Since the vega of an option captures the option's sensitivity to the volatility of the underlying asset, the relation between $IVol$ and stock

[21] One caveat with these growth variables is the possibility of look-ahead bias. Following Grullon, Lyandres, and Zhdanov (2010), we are not concerned with potential issues related to look-ahead bias since the focus of our paper is on investigating the relation between return and volatility, and not on predicting future stock returns. Also, we alleviate concerns of spurious correlation between contemporaneous surprises in growth and monthly returns by merging month t returns with growth variables starting two years following the return observation.

returns should be stronger for firms with higher equity vegas. To test this hypothesis, for each firm j and year n, we utilize the firms' capital structure and the Black and Scholes' formula to define the firms' equity vega as follows:

$$vega_{j,n} = V_{j,n} N'(d_{j,n})\sqrt{5} \tag{5.2}$$

where $d_{j,n} = \frac{ln\left(\frac{V_{j,n}}{D_{j,n}}\right)+\left(r_{f,n}-\frac{\sigma_{j,n}^2}{2}\times 5\right)}{\sigma_{j,n}\sqrt{5}}$, $N'(x) = \frac{exp(-x^2/2)}{\sqrt{2\pi}}$, $r_{f,n}$ is the annualized risk free rate, $\sigma_{j,n}$ denotes firm j's annualized six-month rolling window idiosyncratic volatility based on the Fama French 3 factor model, $V_{j,n}$ denotes the sum of the firm's market equity value and book value of debt, and $D_{j,n}$ is the firm's book value of debt. For simplicity, we assume in (5.2) that firms have a debt maturity of 5 years. Option vegas are relatively invariant over most of the range of possible values for the underlying asset.[22] Therefore, we also classify firms based on equity vega values in relation to the other firms in the sample. To this end, we categorize high vega firms as firms with vegas in the top tercile based on breakpoint values found among NYSE firms in the sample.

We expand the set of proxies for option intensity described thus far by classifying firms as small, young, high sale growth, high investment growth and high profit growth if the corresponding option intensity proxies have values that fall in top or bottom tercile values based on breakpoint values found among NYSE firms in the sample.

Lastly, it is natural to think that firms in certain industries possess more growth options than others, and real option intensity may be captured by the firms' industry membership. Following Grullon, Lyandres, and Zhdanov (2010), we consider three main classifications of industries based on the 49 industries of Fama and French (1997). We define firms with membership in Fama and French (FF) industries 27 (precious metals), 28 (mining), and 30 (oil and natural gas) as natural resource firms. We classify firms in FF industries 22 (electrical equipment), 32 (telecommunications), 35 (computers), 36 (computer software),

[22]A call option's vega is greatest when the option is at the money, and relatively low and invariant over the remainder of possible prices for the underlying stock (see Hull (2011)).

37 (electronic equipment), and 38 (measuring and control equipment) as high-tech firms. Membership in FF industries 12 (medical equipment) and 13 (pharmaceutical products) are defined as biotechnology or pharmaceutical firms. Firms with membership in any one of these three industry classifications are defined as all-growth industry firms.

5.1.4 Summary Statitics

Table 4 reports summary statistics for the main variables in our study. Mean (median) excess return in our sample is 0.9976% (-0.41%) per month or about 11.9712% (-4.92%) per year. Mean (median) daily idiosyncratic stock return volatility $IVol$ is 2.9476% (2.2782%) or about 44.0171% (34.0208%) annually. Our $IVol$ estimates are similar to those reported in Ang, Hodrick, Xing, and Zhang (2006) and Grullon, Lyandres, and Zhdanov (2010). Mean (median) month-to-month change in $IVol$ is -.0023% (-0.011%). The standard deviation is 2.1096% and similar to the figure reported in Grullon, Lyandres, and Zhdanov (2010).

Insert Table 4 here

5.2 The Switch Effect

To the extent that real options and stochastic idiosyncratic risk are incorporated into firm values, the model predicts that stock returns should correlate positively with $IVol$ during intervals between large changes in $IVol$ (the switch effect hereafter) reflecting the dependence of the options' returns on the volatility regime. That is, post-switch returns should be greater than pre-switch returns for stocks that experience up switches in $IVol$, and lower for stocks that experience down switches in $IVol$.

We employ event studies methodology to verify this prediction. To this end, for each firm j and month t, we define an up switch in $IVol$ if $IVol_{j,t-1}$ was below the firm's 30th percentile value and if $IVol_{j,t}$ exceeds the firm's 70th percentile value, capturing the notion of an up switch in idiosyncratic volatility. A down switch event is defined similarly. Once all the up and down switch events are identified for each stock and each month in our sample, we compute the 5-month average return ending in the month prior to the month of the event, and the 5-month average returns beginning from the month after the event. Then we investigate how the difference in average returns around switch months relate to option intensity. More specifically, we risk-adjust monthly returns according to the Fama and French (1993) 3-factor model

$$r^*_{j,t} = r_{j,t} - r_{f,t} - \sum_{k=1}^{3} \widehat{\beta_{j,k}} F_{k,t} \tag{5.3}$$

where $r_{j,t}$ is the return on stock j in month t, $r_{f,t}$ is the risk-free rate, and $F_{k,t}$, $k \in [1, 3]$, denote the three Fama and French factors (market, size, and book-to-market factors).[23] Each month, we estimate the factor loadings $\widehat{\beta_{j,k}}$ for each stock using monthly rolling regressions with a 60-month window requiring at least 24 monthly return observations. The regressions use the Dimson approach with a lag of one month in order to remove biases from thin trading in the estimations (Dimson (1979)). Then, for each firm j and event month t, the difference in 5-month average returns is computed as follows:

$$r^{Diff}_{j,t} = \frac{1}{5} \sum_{\tau=t+1}^{t+6} r^*_{j,\tau} - \frac{1}{5} \sum_{\tau=t-6}^{t-1} r^*_{j,\tau} \tag{5.4}$$

We run separate Fama MacBeth cross-sectional return regressions for each real option proxy and for each of the up and the down switch samples. The regression model for month t is

$$r^{Diff}_{t} = \gamma_0 \iota + \gamma_1 RO_{t-1} + \eta_t \tag{5.5}$$

where r^{Diff}_t is a vector of differences in average returns around the switch month t, ι is a vector of ones, and RO_{t-1} is a vector of real option intensity values. Our model's predictions translate to tests that $\gamma_0 > 0$ and $\gamma_1 > 0$ (or $\gamma_1 < 0$ for inverse RO proxies) for the up

[23] The results using unadjusted returns are available from the authors upon request, but they are not materially different from the results using risk-adjusted returns.

switch sample, and $\gamma_0 < 0$ and $\gamma_1 < 0$ (or $\gamma_1 > 0$ for inverse RO proxies) for the down switch sample.

Insert Table 5 here

Table 5 reports the results. The estimates of $\gamma_0 > 0$ are positive for the up switch sample and negative for the down switch sample, and highly statistically significant in all specifications, offering evidence in agreement with the switch effect. The table also shows that the estimates of γ_1 on total asset size, market equity value and age are positive for the down switch sample, and negative for the up switch sample, highlighting a positive correspondence between the switch effect and real option intensity.

Using categorical proxies for real option intensity offers consistent results with greater significance for the up switch sample than for the down switch sample. The exception is when the high vega dummy is used as a proxy, whose coefficient estimate is positive and significant for the down switch sample. However, the estimate for the combined small and high vega dummy is significant and consistent with the model's predictions for both the up and down switch samples. The coefficient estimate for the combined young and high vega dummy is also in favor of the model predictions for the up switch sample, while it lacks statistical significance for the down switch sample. Based on these results, we argue that equity vega alone is not a strong measure for real options unless it is combined with other proxies such as size and age. [24]

Using industry dummies as proxies for option intensity offers consistent results as well. While natural resources, high tech or bio tech industries alone do not offer statistically significant estimates, the all-growth option industry dummy offers an estimate consistent with the switch effect for the up switch sample.

Next, we investigate how the switch effect relates to the variability in $IVol$ (spread) and the interaction between option intensity and spread. The regression model for month

[24] One way to view these results is that the levered equity of smaller and younger firms experience greater reactions to changes in operating risk than larger and more mature firms.

t is

$$r_t^{Diff} = \gamma_0\iota + \gamma_1\overline{\Delta IVol} + \gamma_2\overline{\Delta IVol} \times RO_{t-1} + \eta_t \tag{5.6}$$

where r_t^{Diff}, ι and RO_{t-1} are as defined previously, and $\overline{\Delta IVol}$ is a vector of $\overline{\Delta IVol_j}$. Our model's predictions translate to tests that $\gamma_1 > 0$ and $\gamma_2 > 0$ (or $\gamma_2 < 0$ for inverse RO proxies) for the up switch sample, and $\gamma_1 < 0$ and $\gamma_2 < 0$ (or $\gamma_2 > 0$ for inverse RO proxies) for the down switch sample.

Table 6 reports the results. The table shows that the coefficient estimates for $\overline{\Delta IVol}$ is positive for the up switch sample and negative for the down switch sample with statistically significant results in virtually all of the regression specifications. Hence, the switch effect is strongest among stocks that experience greater variability in $IVol$, consistent with our model's predictions.

The coefficient estimates for the interaction term between $\overline{\Delta IVol}$ and RO also support the model's predictions. The sign of the estimates are as predicted for age and size if measured as total assets in the up switch sample, while only size is significant in the down switch sample. The dummies for high future profit, sales and investment growth and their combinations with the small dummy all have positive estimates for the up switch sample with varying levels of significance. For the down switch sample, the estimates are not significant. As for the industry dummies, they are not statistically significant. A possible reason for this may be that industry classifications alone are weak proxies for real option intensity since firms within industries may vary widely in real option intensity. We conclude from these results that there is strong evidence for the switch effect which is more pronounced for more real option firms and which have more variable $IVol$, consistent with our model's predictions.

5.3 Positive $IVol$-Return Relation

To the extent that firm valuations reflect real options and are subject to changes in idiosyncratic volatility, our model predicts that the positive $IVol$ anomaly should be stronger for more option intensive firms and firms that experience larger changes in idiosyncratic volatility. In this section, we empirically test this prediction and provide supporting evidence.

We start by revisiting Grullon, Lyandres, and Zhdanov (2010) and estimating monthly return Fama and MacBeth (1973) regressions on changes in idiosyncratic volatility and growth option intensity. The regression model for month t is

$$r_t - r_{f,t} = \gamma_0 \iota + \gamma_1 \Delta IVol_t + \gamma_2 \Delta IVol_t \times RO_{t-1} + \gamma_3 X_{t-1} + \eta_t \tag{5.7}$$

where r_t, $r_{f,t}$, ι, $\Delta IVol_t$, RO_{t-1} are as defined before, and X_{t-1} is a matrix with columns of vectors of controls for firm size, book-to-market, past returns, trading volume and stock beta. Our model predictions translate to tests that $\gamma_1 > 0$ and $\gamma_2 > 0$ ($\gamma_2 < 0$ for inverse RO proxies)

Insert Table 7 here

Table 7 reports the results. Unsurprisingly, the coefficient estimates for stock beta and log book-to-market are both significantly positive, while the coefficient for log size are significantly negative in all specifications. The coefficient for trading volume is highly significant and positive, consistent with Karpoff (1987) and Grullon, Lyandres, and Zhdanov (2010). The coefficient for the past six month cumulative returns is insignificant and negative in all specifications, and consistent with some specifications reported in Cooper, Huseyin, and Schill (2008) and Grullon, Lyandres, and Zhdanov (2010).[25]

The table also reports a highly significant and positive $IVol$-return relation ($\gamma_0 > 0$) for all specifications. As for the relation with respect to option proxies, firm size (both equity

[25]Grullon, Lyandres, and Zhdanov show that the coefficient on past returns is sensitive to the set of other independent factors included in Fama Macbeth regressions.

market value and total asset value) offers highly significant and negative estimates of γ_2. While γ_2 has the predicted negative sign for age, it is not statistically significant.

Focusing on the categorical variable for real option intensity, the γ_2 estimates for the high equity vega dummy is positive and highly statistically significant. This result is interesting because equity vega is the only proxy for real option intensity that is not necessarily related to growth. The high investment and high sales growth dummies also offer similar results. While the high profit growth dummy estimate is not significant, the combined high profit growth and small dummy has a highly significant and positive γ_2 estimate. Similar results apply to the combined dummies for high investment growth, high sales growth, and high equity vega dummies when combined with the small size dummy, implying that combining option proxies may capture real option intensity better.

Focusing on the industry dummies, while the γ_2 estimates are positive for natural resources, high technology and bio technology firms, only the natural resources industry dummy offers statistically significant results. However, the all-growth industry dummy has a highly significant and positive estimate in line with the predictions. In sum, the results thus far support a positive $IVol$-return relation that is stronger for more option intensive firms. These results are consistent with the findings in Grullon, Lyandres, and Zhdanov (2010) as well as our model.

Next, our model also predicts that the positive $IVol$-return relation should be stronger for more real option intensive firms and firms with larger spreads in idiosyncratic volatility. We test this hypothesis with the following regression:

$$r_t - r_{f,t} = \gamma_0 \iota + \gamma_1 \overline{\Delta IVol} + \gamma_2 \Delta IVol_t + \gamma_3 \overline{\Delta IVol} \times \Delta IVol \times RO + \gamma_3 X_{t-1} + \eta_t \tag{5.8}$$

where r_t, $r_{f,t}$, ι, $\Delta IVol_t$, RO_{t-1}, X_{t-1} and $\overline{\Delta IVol}$ are as defined previously. Our model prediction translates to tests that $\gamma_3 > 0$ (or $\gamma_3 < 0$ for inverse RO proxies).

Insert Table 8 here

The results are reported in Table 8. The table reports estimates of γ_3 that are highly significant and positive (negative for inverse real option proxies) for virtually all the regression. The only exceptions apply when age and the dummies for young, small and young, and young and high vega are used where the results are statistically insignificant. The remainder of the table reports the results for the industry dummies. While γ_3 estimates are positive for natural resources, high technology and bio technology firms, only natural resources offers significant results. However, the all-growth industry dummy offers highly significant results in line with the predictions.

Collectively, the results are in strong agreement with our model. The positive $IVol$-return relation is more pronounced for more real option firms and firms that experience larger variability in idiosyncratic volatility.

5.4 The Negative $IVol$-Return Relation

Ang, Hodrick, Xing, and Zhang (2006) report that portfolios of high $IVol$ stocks significantly under-perform their low $IVol$ counterparts. Our model predicts that this negative $IVol$-return relation should be more pronounced for more real option intensive firms and firms with larger $IVol$ spreads. We test this prediction and provide empirical support in this section.

At the end of each June, sort and rank firms into three equally-sized groups based on each one of our real option proxies. For categorical variables, firms are separated into two groups. We merge the rankings with monthly $IVol$ and stock returns, and for each month, we sort and rank stocks into three equally-sized groups based on $IVol$. Then we compute value-weighted portfolio returns for each of the two-way classifications of $IVol$ and option intensity and assess their performance over the following month. All portfolios are rebalanced monthly. This approach corresponds to the 1/0/1 (formation period/waiting period/holding period) strategy of Ang, Hodrick, Xing, and Zhang (2006) which most of their analysis is concentrated on.

The performance of the portfolios are assessed on a risk-adjusted basis relative to the Fama and French 3 factor model:

$$r_t - r_{f,t} = \gamma_0 + \gamma_1 MKTRF_t + \gamma_2 SMB_t + \gamma_3 HML_t + \epsilon_t \tag{5.9}$$

where r_t is the portfolio return, $r_{f,t}$ is the riskless rate, $MKTRF$, SMB, and HML are the Fama and French (1993) three factors that proxy for the market risk premium, size and book-to-market factors respectively. In order to investigate the extent to which real option intensity contributes to the negative $IVol$ anomaly, we also estimate the regression for the zero-cost $IVol$ portfolios for each rank of option intensity.[26] A larger intercept γ_0 estimate translates to a greater average risk-adjusted return.

Insert Table 9 here

Tables 9 to 10 report results. Each panel of the tables corresponds to a different real option proxy, with $IVol$ ranks reported across columns. The last column of each panel reports the estimates for the zero-cost $IVol$ portfolios. The real option ranks are listed down the rows. The reported estimates are annualized to facilitate the interpretation of the economic significance. All other reported figures are unadjusted.

Table 9 reveals that the negative $IVol$ anomaly is more pronounced and statistically more significant for the two lowest firm size groups according to total asset value. The negative $IVol$ anomaly for the largest group is not significant. Size according to market equity value and age offer similar patterns, lending strong support for our model predictions. The anomaly is also stronger for high equity vega firms than for low vega firms. This finding is enlightening because equity vega is the only option proxy that is not necessarily related to future growth opportunities.

[26] For the zero-cost $IVol$ portfolios, we use portfolio returns instead of portfolio excess returns on the left hand side of regression (5.9).

Insert Table 10 here

The evidence for the negative $IVol$ anomaly is even stronger among small and high equity vega firms than for high equity vega firms alone. Hence, evidence for the negative $IVol$ anomaly is even stronger when proxies for real option intensity are combined, lending credence to our option based-explanation for the anomaly. The other panels point to that conclusion as well. While the negative $IVol$ anomaly is not conclusively stronger for high profit, high sale or high investment growth firms, it is stronger for these firms if they are also small in size, and similarly for younger firms and firms that are younger and have high equity vega.

As for the negative $IVol$ anomaly in relation to the firms' industries, the negative $IVol$ anomaly is more pronounced for natural resources and high technology stocks, while bio tech and all growth-industries offer inconclusive evidence. As mentioned earlier, industry membership alone may be a weak proxy for real option intensity because firms within industries can vary widely in their real option intensity. In sum, we find that there is considerable evidence that the $IVol$-return relation relates to real option intensity.

Next we investigate how the negative $IVol$-return relation relates to the spread in idiosyncratic volatility. In addition to the two-way independent sorts based on $IVol$ and each of the real option proxies, we independently sort stocks into three equally-sized groups based on $\overline{\Delta IVol}$. Then, for each of the two-way rank classifications of real option intensity and $\overline{\Delta IVol}$, we assess the value-weighted returns of the zero-cost $IVol$ portfolios relative to the Fama and French 3 factor model.

Insert Table 11 here

Tables 11 and 12 report the results. The negative $IVol$ anomaly is monotonically stronger and more significant for the top $\overline{\Delta IVol}$ group for the size and age proxies. The table also shows that the negative $IVol$ anomaly is stronger among the youngest firms and firms that have the largest $\overline{\Delta IVol}$. These results support our predictions that the negative $IVol$ anomaly should be more pronounced for growth firms that experience more extreme changes in $IVol$. The table also reveals that the negative $IVol$ anomaly seems to be more pronounced for larger firms among the top $\overline{\Delta IVol}$ stocks. While these results are not in direct support of our model, the negative $IVol$-return remains both statistically and economically significant for small firms.

The main conclusions are similar for high profit, high sale and high investment growth firms. While there is a stronger negative $IVol$-return relation for the high $\overline{\Delta IVol}$ stocks independently of the real option characteristics, the anomaly seems to be weaker for high future growth firms. One reason for these findings may be that the negative $IVol$-return relation could be confounded by the positive returns of high future growth stocks. This is likely to be the case if information on high future growth is reflected in stock returns during the portfolio evaluation period.

Insert Table 12 here

Now focusing on the combined real option proxies, Table 12 shows that the negative $IVol$-return relation is stronger for small and high growth, small and young, and small and high equity vega firms. Hence, the negative $IVol$ anomaly is more evidently related with real options and $\overline{\Delta IVol}$ for the combined real option proxies. In relation to industry membership, the table shows that the negative $IVol$ anomaly is monotonically stronger and statistically more significant for larger $\overline{\Delta IVol}$ independently of industry membership. Natural resources, bio tech and all-growth industry firms within the high $\overline{\Delta IVol}$ have stronger negative $IVol$-return relation, lending support for our model predictions. While high tech stocks exhibit a weaker $IVol$-return relation than low tech stocks within the high $\overline{IVol}$ group, the anomaly still remains significant for the high tech stocks.

Overall, these results demonstrate that the stocks of firms that experience more extreme changes in $IVol$ and incorporate more real options exhibit stronger $IVol$-return relations, lending support for our model.

6 Conclusions

Recent empirical evidences on the correspondence between stock returns and idiosyncratic return volatility at the firm-level have been mixed at best. In this paper, we propose a new economic explanation for the conflicting findings in a simple equity valuation model of firms involving real options and stochastic operating risk. More generally, we motivate why idiosyncratic risks may appear to be priced in the cross-section of stock returns.

In this paper, we introduce a 2-regime Markov switching process in order to incorporate uncertainty in idiosyncratic operating risk. The option value is convex in the output price and it does not distinguish between systematic and idiosyncratic risks, a feature that contrasts starkly from the firm's assets-in-place. This gives rise to regime dependency of the firm's equity return and beta. The time-series dynamics of the volatility structure results in an interplay between returns and idiosyncratic return volatility consistent with what has been observed empirically in the cross-section of stocks. We verify our intuition with numerical simulations, followed by supportive empirical evidences.

To maintain tractability, our model is devoid of a more general structure for the firms' idiosyncratic operating risk. In the 2-regime structure, the operating volatility of a firm leaps between two values to generate the results aligned with the empirical evidences on stock returns and idiosyncratic volatility. While qualitatively our results should persist in a more general structure insofar as idiosyncratic operating risk exhibits mean-reversion, work establishing this conjecture seems to be an interesting extension of our paper.

Previous literature has relied on iliquidity and other market microstructure related explanations for the distributional properties of stock returns related to heteroscedasticity, discontinuities or jumps, and heavy tails. Our model has the capability to parsimoniously generate these features in return distributions from the operating environment that firms face, providing fertile grounds for additional research.[27] Further research in this direction is highly merited.

Lastly, our model suggests that jumps in stock returns should coincide with large changes in idiosyncratic return volatility in predictable ways, potentially shedding new insights on the three-way relation between stock returns, idiosyncratic return volatility and expected return skewness (Boyer, Mitton, and Vorkink (2010)). Furthermore, the features of our model may help establish predictability akin to return continuation amenable with the findings of Jegadeesh and Titman (1993), and reversals reported in Jegadeesh (1990). We leave these other interesting extensions for future research.

[27]The literature has recognized that asset returns must exhibit both stochastic volatility and discontinuous jumps to fit their empirical distributions (Das and Sundaram (1999))

References

Anderson, C. and L. Garcia-Feijoo. 2006. Empirical Evidence on Capital Investment, Growth Options, and Security Returns. *Journal of Finance* 171–194.

Ang, Andrew, Robert J. Hodrick, Yuhang Xing, and Xiaoyan Zhang. 2006. The Cross-Section of Volatility and Expected Returns. *Journal of Finance* .

———. 2009. High Idiosyncratic Volatility and Low Returns: International and Further U.S. Evidence. *Journal of Financial Economics* 91:1–23.

Babenko, Ilona, Oliver Boguth, and Yuri Tserlukevich. 2013. Idiosyncratic Cash Flows and Systematic Risk. *Working Paper, Arizona State University* .

Berk, Jonathan, Richard Green, and Vasant Naik. 1999. Optimal Investment, Growth Options, and Security Returns. *Journal of Finance.* 54:1553–1607.

Bloom, Nicholas. 2009. The Impact of Uncertainty Shocks. *Econometrica* 77 (3):623–685.

Boyer, Brian, Todd Mitton, and Keith Vorkink. 2010. Expeced Idiosyncratic Skewness. *Review of Financial Studies* 23 (1):169–202.

Brennan, Michael and Eduardo Schwartz. 1985. Evaluating Nature Resource Investments. *Journal of Business* 58:135–157.

Brown, Gregory and Nishad Kapadia. 2007. Firm-Specific Risk and Equity Market Development. *Journal of Financial Economics* 84:358–388.

Bulan, Laarni. 2005. Real Options, Irreversible Investment and Firm uncertainty: New Evidence from U.S. Firms. *Review of Financial Economics* 14:255–279.

Caballero, Ricardo J. and Robert S. Pindyck. 1996. Uncertainty, Investment, and Industry Evolution. *International Economic Review* 37.

Carlson, Murray, Adlai Fisher, and Ron Giammarino. 2004. Corporate Investment and Asset Price Dynamics: Implications for the Cross-Section of Returns. *Journal of Finance.* 59:2577–2603.

Chen, Joseph, Harrison Hong, and Jeremy C. Stein. 2001. Forecasting Crashes: Trading Volume, Past Returns, and Conditional Skewness in Stock Prices. *Journal of Financial Economics* 61:345–381.

Constantinides, George M. 1978. Market Risk Adjustment in Project Valuation. *Journal of Finance* 33:603–616.

Cooper, Ilan. 2007. Asset Pricing Implications of NonConvex Adjustment Costs and Irreversibility of Investment. *Journal of Finance* 61:139–170.

Cooper, Michael, Gulen Huseyin, and Michael Schill. 2008. Asset Growth and the Cross-Section of Stock Returns. *Journal of Finance* 1609–1652.

Daniel, Kent, David Hirshleifer, and Avanidhar Subrahmanyam. 1998. Investor Psychology and Security Market Under- Over-Reactions. *Journal of Finance* 1839–1886.

Das, Sanjiv Ranjan and Rangarajan K. Sundaram. 1999. Of Smiles and Smirks: A Term Structure Perspective. *Journal of Financial and Quantitative Analysis* 34 (2):211–239.

Dimson, Elroy. 1979. Risk Measurement When Shares are Subject to Infrequent Trading. *Journal of Financial Economics* 7:197–226.

Dixit, Avinash K. and Robert S. Pindyck. 1994. *Investment under Uncertainty*. Princeton University Press.

Duffee, Gregory R. 1995. Stock Returns and Volatility. A Firm-Level Analysis. *Journal of Financial Economics* 37:399–420.

Fama, Eugene and Kenneth R. French. 1992. The Cross-Section of Expected Stock Returns. *Journal of Finance* 47:427–465.

———. 1993. Common Risk Factors in the Returns on Common Stocks and Bonds. *Journal of Financial Economics* 25:2349.

———. 1996. Multifactor Explanations of Asset-Pricing Anomalies. *Journal of Finance* 51:55–84.

———. 1997. Industry Costs of Equity. *Journal of Financial Economics* 49:153–193.

Fama, Eugene and James D. MacBeth. 1973. Risk Return, and Equilibrium: Empirical Tests. *Journal of Political Economy* 71:607–636.

Fu, F. 2009. Idiosyncratic Risk and the Cross-Section of Expected Stock Returns. *Journal of Financial Economics* 91:2437.

Galai, Dan and Ron Masulis. 1976. The Option Pricing Model and the Risk Factor of Stock. *Journal of Financial Economics* 3:53–81.

Garlappi, Lorenzo and Hong Yan. 2008. Financial Distress and The Cross-Section of Equity Returns. *Working Paper* .

Garleanu, Nicolae, Stavros Panageas, and Jianfeng Yu. 2012. Technological Growth and Asset Pricing. *Journal of Finance* 61.

Grullon, Gustavo, Evgeny Lyandres, and Alexei Zhdanov. 2010. Real Options, Volatility, and Stock Returns. *Journal of Finance Forthcoming* .

Guo, X. 2001. An Explicit Solution to an Optimal Stopping Problem with Regime Switching. *Journal of Applied Probability* 38:464–481.

Guo, X. and Q. Zhang. 2004. Closed-Form Solutions for Perpetual American Put Options with Regime Switching. *SIAM Journal of Applied Mathematics* 64.

Guo, Xin, Jianjun Miao, and Erwan Morellec. 2005. Irreversible Investment with Regime Shifts. *Journal of Economic Theory* 122:37–59.

Hackbarth, Dirk, Jianjun Miao, and Erwan Morellec. 2006. Capital Structure, Credit Risk, and Macroeconomic Conditions. *Journal of Financial Economics* 82:519–550.

Hanson, Floyd B. 2007. *Applied Stochastic Processes and Control for Jump-Diffusions: Modelling, Analysis and Computation*. Society for Industrial and Applied Mathematics.

Huang, Wei, Qiangqiu Liu, S. Ghon Rhee, and Liang Zhang. 2009. Return Reversals, Idiosyncratic Risk, and Expected Returns. *Review of Financial Studies* .

Hull, John. 2011. *Options, Futures, and Other Derivatives*. Prentice Hall.

Jegadeesh, N. 1990. Evidence of Predictable Behavior of Security Returns. *Journal of Finance* 45:881–898.

Jegadeesh, N. and Sheridan Titman. 1993. Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency. *Journal of Finance* 48:65–91.

Johnson, Timothy C. 2004. Forecast Dispersion and the Cross Section of Expected Returns. *Journal of Finance* 79:1957–1978.

Kapadia, Nishad. 2007. The Next Microsoft? Skewness, Idiosyncratic Volatility, and Expected Returns. *Working Paper* .

Karpoff, Jonathan. 1987. The Relation Between Price Changes and Trading Volume: A Survey. *Journal of Financial and Quantitative Analysis* 45:1161–1187.

Kogan, Leonid and Dimitris Papanikolaou. 2013. Firm Characteristics and Stock Returns: The Role of Investment-Specific Shocks. *Review of Financial Studies* 26:2718–2759.

Leahy, John and Toni Whited. 1996. The Effect of Uncertainty on Investment: Some Stylized Facts. *Journal of Money, Credit, and Banking* 28:64–83.

Lehmann, Bruce N. 1990. Residual Risk Revisited. *Journal of Econometrics* 45:71–97.

Lemmon, Michael and Jaime Zender. 2010. Debt Capacity and Tests of Capital Structure Theories. *Journal of Financial and Quantitative Analysis* 45:1161–1187.

Lintner, John. 1965. Security Prices, Risk, and Maximal Gains from Diversification. *Journal of Finance* 20:587–616.

MacDonald, Robert and Daniel Siegel. 1985. Investment and Valuation of Firms When There is an Option to Shut Down. *International Economic Review* 26:331–349.

———. 1986. The Value of Waiting to Invest. *Quarterly Journal of Economics* 101:707–727.

Majd, Saman and Robert S. Pindyck. 1987. Time to Build Option Value, and Investment Decisions. *Journal of Financial Economics* 18:7–27.

Malliaris, A. G. 1988. *Stochastic Methods in Economics and Finance.* Elsevier Science.

Merton, Robert C. 1974. On the Pricing of Corporate Debt: The Risk Structure of Interest Rates. *Journal of Finance* 29:449–470.

———. 1976. Option Pricing When Underlying Stock Returns are Discontinuous. *Journal of Financial Economics* 3:125–144.

———. 1992. *Continuous-Time Finance.* Basil Blackwell.

Newey, Whitney K. and Kenneth D. West. 1987. A Simple, Positive Semidefinite, Heteroskedastcity and Autocorrelation Consistent Covariance-Matrix. *Econometrica* 55:703–708.

Pindyck, Robert S. 1988. Irrevesible Investment, Capacity Choice, and the Value of the Firm. *American Economic Review* 78:969–985.

Pontiff. 2006. Costly Arbitrage and the Myth of Idiosyncratic Risk. *Journal of Accounting and Economics* 42:35–52.

Sagi, Jacob S. and Mark S. Seashole. 2007. Firm-Specific Attributes and the Cross-Section of Momentum. *Journal of Financial Economics* 84:389–434.

Spiegel, Matthew and Xiaotong Wang. 2006. Cross-sectional Variation in Stock Returns: Liquidity and Idiosyncratic Risk. *Working Paper, Yale School of Management* .

Tinic, Seha M. and Richard R. West. 1986. Risk, Return, and Equilibrium - A Revisit. *Journal of Political Economy* 94:126–147.

Zhang, Lu. 2005. The Value Premium. *Journal of Finance.* 60:67–103.

7 Appendix

This section describes the valuation approach followed by the proofs of the propositions stated in Section 3 of the paper.

7.1 Valuation Approach

We derive the fundament differential equation that asset values must solve. It simplifies valuation if we reexpress the dynamics of the price (2.1) more concisely by letting

$$\frac{dP}{P} = \mu dt + \sigma_i dB_i \tag{7.1}$$

where $dB_i = \frac{\sigma_{P,i} dB_1 + \sigma_A dB_2}{\sigma_i}$ and $\sigma_i = \sqrt{\sigma_{P,i}^2 + \sigma_A^2}$. Then it can be shown that $\mathrm{Cov}\left(dB_i, dB_2\right) = \frac{\sigma_A}{\sigma_i} dt$, $\mathrm{Cov}\left(\frac{dP}{P}, \frac{dS}{S}\right) = \sigma_S \sigma_A dt$, and $\rho_i = \frac{\sigma_A}{\sigma_i}$.

Denote $Y(P, \sigma_{P,i}) = Y_i(P)$ the value function of an asset that is twice-differentiable in P where P follows the process in equation (7.1) and $\sigma_{P,i}$ follows the process given by equation (2.3). At this stage, $Y_i(P)$ can be the value of a growth option. The generalized Ito's Lemma (Malliaris (1988)) implies that $Y_i(P)$ has the following process:

$$\frac{dY_i(P)}{Y_i(P)} = [\mu_{Y_i}(P) - \lambda_i \gamma_i(P)]dt + \sigma_{Y_i}(P) dB_i + \gamma_i(P) dz_i \tag{7.2}$$

where

$$\mu_{Y_i}(P) = \left[\frac{\mu P Y_i'(P) + \frac{1}{2} P^2 \sigma_i^2 Y_i''(P)}{Y_i(P)}\right] + \lambda_i \gamma_i(P) \tag{7.3}$$

$$\sigma_{Y_i}(P) = \frac{\sigma_i P Y_i'(P)}{Y_i(P)} \tag{7.4}$$

$$\gamma_i(P) = \frac{Y_{i'}(P) - Y_i(P)}{Y_i(P)} \tag{7.5}$$

The first two terms on the right hand side of the equation are the standard form for Ito's Lemma. The third term is the jump of the value of $Y_i(P)$ when $\sigma_{P,i}$ switches from regime

i to regime i'.

Following Merton (1976), denote w_1, w_2 and $w_3 = 1 - w_1 - w_2$ the proportions of portfolio value invested in the market asset S, asset $Y_i(P)$ and the riskless asset M respectively. It is not possible to make this portfolio riskless.[28] Instead, we choose the portfolio weights w_1^* and w_2^* to eliminate market risk only, which translates to the following conditions:

$$w_1^*(\mu_S - r) + w_2^*(\mu_{Y_i}(P) - r) + r = r \tag{7.6}$$

$$w_1^*\sigma_S + w_2^*\sigma_{Y_i}(P)\frac{\sigma_A}{\sigma_i} = 0 \tag{7.7}$$

where we have used the knowledge that $dB_i = \frac{\sigma_{P,i}dB_1 + \sigma_A dB_2}{\sigma_i}$. After solving for w_1^* and w_2^*, equation (7.6) together with equation (7.7) implies that

$$\sigma_S \mu_{Y_i}(P) = -\frac{r\sigma_A\sigma_{Y_i}(P)}{\sigma_i} + \frac{\sigma_A\mu_S\sigma_{Y_i}(P)}{\sigma_i} + r\sigma_S \tag{7.8}$$

Substituting equations (7.3) and (7.4) into (7.8), and simplifying gives the fundamental valuation equation

$$\frac{1}{2}P^2\sigma_i^2 Y_i''(P) + (\mu - \sigma_A\lambda)\, PY_i'(P) + \lambda_{i'}\left(Y_{i'}(P) - Y_i(P)\right) = rY_i(P) \tag{7.9}$$

where we have also substituted in the market Sharpe ratio $\lambda = \frac{\mu_S - r}{\sigma_S}$. This differential equation serves as the backbone for the derivation of all valuations in Section 3 of the paper.[29]

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[28] As in Merton (1976), the jump risk in the hedge portfolio is unhedgeable.

[29] An alternative and more direct approach to deriving the valuation equation (7.9) for any asset $Y_i(P)$ is based on Constantinides (1978). The first step in the approach calls for the replacement of the drift of $\frac{dS}{S}$, μ, by $\mu^* = \mu - \lambda \mathrm{Corr}\left(\frac{dP}{P}, \frac{dS}{S}\right)\sigma_i = \mu - \lambda\rho_i\sigma_i = \mu - \lambda\sigma_A$. The second step evaluates the stream of cash flows of $Y_i(P)$ as if the market price of risk were zero, i.e., discount expected cash flows at the riskfree rate. To this end, the Bellman equation for asset Y_i is given by

$$Y_i(P) = \frac{1}{1 + r\triangle t}E\left[Y(P + \triangle P, \sigma_i + \triangle\sigma_i)\right] \tag{7.10}$$

7.2 Value of Assets-in-Place and Mature Firms

To value a mature firm, it requires that we evaluate the present value of the cashflows

$$V_M(P) = \mathbb{E}^{\mathbb{Q}}\left[\int_0^\infty e^{-rs}\xi_M(P_{t+s}-c)ds\right] \tag{7.12}$$

where $\mathbb{E}^{\mathbb{Q}}$ is the expectation under the $\mathbb{Q}$ measure. Evaluating the integral results in (2.6). The value of the assets-in-place and incremental increase in the value from exercising the option have the same functions with ξ_M replaced by ξ_Y and ξ respectively.

7.3 Value of a Growth Firm

Let $G_i(P)$ denote the value of the growth option in the region where $P \in (P_1, P_2)$ and when the volatility regime i is in effect. In this region of P values, the growth option is in-the-money only if the low volatility regime is in effect, in which case is exercised immediately and it's payoff is

$$G_L(P) = \xi\left(\frac{P}{r-\mu^*} - \frac{c}{r}\right) - I \tag{7.13}$$

Using valuation equation (7.9), the value of the growth option in the high volatility regime obeys the following differential equation

$$\frac{1}{2}P^2\sigma_i^2 G_H''(P) + (\mu - \sigma_A\lambda)\,PG_H'(P) + \lambda_L\left[\xi\left(\frac{P}{r-\mu^*} - \frac{c}{r}\right) - I - G_H(P)\right] = rG_H(P) \tag{7.14}$$

The expectation on the right hand side evaluates to

$$E\left[Y_i(P+\triangle P)\right] = \{\lambda_{i'}\triangle t E\left[Y_{i'}(P+\triangle P)\right] + (1-\lambda_{i'}\triangle t)E\left[Y_i(P+\triangle P)\right]\} \tag{7.11}$$

The first term is the asset's probability weighted expected value if there is a switch in volatility regime and the second term corresponds to the asset's probability weighted expected value under the current volatility regime. One can arrive at equation (7.9) after substituting equation (7.11) into (7.10), multiplying both sides by $1 + r\triangle t$, letting $\triangle t$ go to zero, applying Ito's Lemma, and substituting μ by μ^*. Looked another way, the traded assets M and S allow us to define a new measure under which the process $dB_i^* = \rho_i\lambda dt + dB_i$ is a brownian motion under the $\mathbb{Q}$ measure. Under this risk neutral measure, the price dynamics follows $\frac{dP}{P} = \mu^* dt + \sigma_i dB_i^*$, where $\mu^* = \mu - \sigma_i\rho_i\lambda = \mu - \sigma_A\lambda$.

Equation (7.14) is the standard valuation equation commonly seen in the growth option literature with the exception of the last term. The last term corresponds to the probability weighed change in the value of the option due to a change in volatility from the high regime to the low regime and immediate activation. The payoff from activation, net of investment cost I and opportunity cost $G_H(P)$, is $\left[\xi\left(\frac{P}{r-\mu^*}-\frac{c}{r}\right)-I-G_H(P)\right]$.

Now we address the region of low values of P where $P \in (0, P_1)$. In this region the option is out-of-the money in both volatility regimes and kept alive. Let $F_i(P)$ denote the value of the growth option in this region when i is the regime in effect. Following the same steps as above that led to (7.14), we arrive at the following pair of simultaneous differential equations:

$$\frac{1}{2}P^2\sigma_H^2 F_H''(P) + (\mu - \sigma_A\lambda)\, PF_H'(P) + \lambda_L\,(F_L(P) - F_H(P)) = rF_H(P) \tag{7.15}$$

$$\frac{1}{2}P^2\sigma_L^2 F_L''(P) + (\mu - \sigma_A\lambda)\, PF_L'(P) + \lambda_H\,(F_H(P) - F_L(P)) = rF_L(P) \tag{7.16}$$

As before, the differential equations are similar to those in standard diffusion models with the exception that they include an additional component that captures the possibility of a change in volatility regime. This term equals $\lambda_L\,(F_L(P) - F_H(P))$ if the high volatility state is in effect, and $\lambda_H\,(F_H(P) - F_L(P))$ otherwise. With this last pair of valuation equations, we have all the tools required for all the valuations in the paper.

7.4 Proof of Proposition 1

The solution method follows Guo (2001) and Guo and Zhang (2004). Consider first the pair of differential equations (7.15) and (7.16). It is easy to show that the system has the following characteristic function

$$q_H(\beta) \times q_L(\beta) - \lambda_L\lambda_H = 0 \tag{7.17}$$

where $q_H(\beta)$ and $q_L(\beta)$ are given by the following quadratic equations

$$q_H(\beta) = -\beta\mu^* - \frac{1}{2}(\beta - 1)\beta\sigma_H^2 + \lambda_L + r \tag{7.18}$$

$$q_L(\beta) = -\beta\mu^* - \frac{1}{2}(\beta - 1)\beta\sigma_L^2 + \lambda_H + r \tag{7.19}$$

The characteristic function has four distinct roots (Guo (2001)) such that the general form of the solutions to (7.15) and (7.16) are given by[30] $\beta_{2,1} > \beta_{2,2} > 0 > \beta_{2,3} > \beta_{2,4}$

$$F_H(P) = \sum_{i=1}^{4} B_{H,i}(P)P^{\beta_{2,i}} \quad \text{and} \quad F_L(P) = \sum_{i=1}^{4} B_{L,i}(P)P^{\beta_{2,i}}$$

The valuation problem is greatly simplified if we reduce the number of terms. Given the signs of $\beta_{2,1}$, $\beta_{2,2}$, $\beta_{2,3}$, and $\beta_{2,4}$, and the property that the option value must approach zero if P approaches zero, the solutions take the simpler form given by

$$F_H(P) = B_{H,1}P^{\beta_{2,1}} + B_{H,2}P^{\beta_{2,2}} \tag{7.20}$$

$$F_L(P) = B_{L,1}P^{\beta_{2,1}} + B_{L,1}P^{\beta_{2,2}} \tag{7.21}$$

Substituting equations (7.20) and (7.21) into the differential equations (7.15) and (7.16) results in the following equations

$$0 = P^{\beta_{2,1}}\left(\lambda_L B_{L,1} - q_H(\beta_{2,1})B_{H,1}\right) + P^{\beta_{2,2}}\left(\lambda_L B_{L,2} - q_H(\beta_{2,2})B_{H,2}\right)$$

$$0 = P^{\beta_{2,1}}\left(\lambda_H B_{H,1} - q_L(\beta_{2,1})B_{L,1}\right) + P^{\beta_{2,2}}\left(\lambda_H B_{H,2} - q_L(\beta_{2,2})B_{L,2}\right)$$

The next step involves solving for the unknown constants. Equation (3.1) in the proposition is found by solving for $B_{H,1}$ and $B_{H,2}$ and substituting into $F_H(P)$.

[30]The roots of quartic equations can be found in standard math textbooks.

Now turning out attention to equation (7.14), the solution has the following form

$$G_H(P) = C_{H,1}P^{\beta_{1,1}} + C_{H,2}P^{\beta_{1,2}} + \phi(P) \tag{7.22}$$

where $C_{H,1}$ and $C_{H,2}$ are constants of integration, $\phi(P)$ is a particular solution and $\beta_{1,1}$ and $\beta_{1,2}$ are the two real roots of the following quadratic equation

$$q_H(\beta) = -\beta\mu^* - \frac{1}{2}(\beta-1)\beta\sigma_H^2 + \lambda_L + r$$

In particular, if $q_H(0) = r + \lambda_L \neq 0$ one can choose

$$\phi(P) = \frac{\lambda_L}{\lambda_L + r}\left(\xi\left(\frac{P}{r-\mu^*} - \frac{c}{r}\right) - I\right) \tag{7.23}$$

and the complete solution is given by (3.3) in the proposition.

It remains to determine the constants of integration $B_{L,1}$, $B_{L,2}$, $C_{H,1}$, $C_{H,2}$, and the exercise policies P_1 and P_2. To complete the solution, we make use of the following boundary conditions

$$H(P_1) - I = F_L(P_1) \tag{7.24}$$

$$H'(P)|_{P=P_1} = F_L'(P)|_{P=P_1} \tag{7.25}$$

$$H(P_2) - I = G_H(P_2) \tag{7.26}$$

$$H'(P)|_{P=P_2} = G_H'(P)|_{P=P_2} \tag{7.27}$$

$$G_H(P_1) = F_H(P_1) \tag{7.28}$$

$$G_H'(P)|_{P=P_1} = F_H'(P)|_{P=P_1} \tag{7.29}$$

where $H(P) = \xi\left(\frac{P}{r-\mu^*} - \frac{c}{r}\right)$. The value matching conditions (7.24) and (7.26) impose an equality between the option's intrinsic value and the option's value at the optimal exercise values of P in the two volatility regimes. These conditions merely mean that upon activation the owner foregoes the value of the option in exchange for the net benefits of exercising the option, $H(P) - I$. The smooth pasting conditions (7.25) and (7.27) ensure the optimality of the exercise policies P_1 and P_2 (Dixit and Pindyck (1994)). Lastly, the conditions (7.28) and (7.29) ensure that the value of the option is continuous and smooth around P_1.

We turn to each of the conditions (7.24)–(7.29) above. At $P = P_1$, the two conditions (7.24) and (7.25) can be written as

$$\xi\left(\frac{P_1}{r-\mu^*} - \frac{c}{r}\right) - I = B_{L,1}P_1^{\beta_{2,1}} + B_{L,2}P_1^{\beta_{2,2}} \tag{7.30}$$

$$\frac{\xi P_1}{r-\mu^*} = \beta_{2,1}B_{L,1}P_1^{\beta_{2,1}} + \beta_{2,2}B_{L,2}P_1^{\beta_{2,2}} \tag{7.31}$$

At $P = P_2$, the two conditions (7.26) and (7.27) can be written as

$$\xi\left(\frac{P_2}{r-\mu^*} - \frac{c}{r}\right) - I = C_{H,1}P_2^{\beta_{1,1}} + C_{H,2}P_2^{\beta_{1,2}} + \frac{\lambda_L}{\lambda_L + r}\left(\xi\left(\frac{P_2}{r-\mu^*} - \frac{c}{r}\right) - I\right) \tag{7.32}$$

$$\frac{\xi P_2}{r-\mu^*} = \beta_{1,1}C_{H,1}P_2^{\beta_{1,1}} + \beta_{1,2}C_{H,2}P_2^{\beta_{1,2}} + \frac{\lambda_L}{\lambda_L + r}\left(\frac{\xi P_2}{r-\mu^*}\right) \tag{7.33}$$

We can use the first four conditions (7.30)–(7.33) to solve for $B_{L,1}$, $B_{L,2}$, $C_{H,1}$ and $C_{H,2}$. The expressions in closed form are

$$B_{L,1} = -P_1^{-\beta_{2,1}}\left(\frac{\beta_{2,2}}{\beta_{2,1} - \beta_{2,2}}\right)\left[\frac{\xi P_1\left(1 - \frac{1}{\beta_{2,2}}\right)}{(r-\mu^*)} - \frac{\xi c}{r} - I\right] \tag{7.34}$$

$$B_{L,2} = P_1^{-\beta_{2,2}}\left(\frac{\beta_{2,1}}{\beta_{2,1} - \beta_{2,2}}\right)\left[\frac{\xi P_1\left(1 - \frac{1}{\beta_{2,1}}\right)}{(r-\mu^*)} - \frac{\xi c}{r} - I\right] \tag{7.35}$$

$$C_{H,1} = -P_2^{-\beta_{1,1}}\left(\frac{\beta_{1,2}}{\beta_{1,1} - \beta_{1,2}}\right)\left[\frac{\xi P_2\left(1 - \frac{1}{\beta_{1,2}}\right)}{(r-\mu^*)} - \frac{\xi c}{r} - I\right] \tag{7.36}$$

$$C_{H,2} = P_2^{-\beta_{1,2}}\left(\frac{\beta_{1,1}}{\beta_{1,1} - \beta_{1,2}}\right)\left[\frac{\xi P_2\left(1 - \frac{1}{\beta_{1,1}}\right)}{(r-\mu^*)} - \frac{\xi c}{r} - I\right] \tag{7.37}$$

The continuity and the smoothness conditions of the value functions at $P = P_1$ are the equations (3.5) and (3.6) in the proposition.

Conditions (3.5) and (3.6) and the constants of integration $B_{L,1}$, $B_{L,2}$, $C_{H,1}$ and $C_{H,2}$ are expressed in terms of the exercise boundaries P_1 and P_2. Therefore, the equations compose a system of two equations and two unknowns variables, P_1 and P_2, which are solved numerically for each set of parameter values of the model. This completes the proof. ∎

7.5 Proof of Proposition 2

Direct application of Ito's Lemma on $F_H(P)$ and $F_L(P)$ results in equation (3.7) where

$$a_H(P) = \left(\frac{1}{2}\sigma_H^2 P^2 F_H''(P) + \mu P F_H'(P)\right) \Big/ F_H(P) \tag{7.38}$$

$$b_H(P) = \sigma_H P F_H'(P)/F_H(P) \tag{7.39}$$

$$a_L(P) = \left(\frac{1}{2}\sigma_L^2 P^2 F_L''(P) + \alpha P F_L'(P)\right) \Big/ F_L(P) \tag{7.40}$$

$$b_L(P) = \sigma_L P F_L(P)/F_L(P) \tag{7.41}$$

$$\nu_H(P) = (F_L(P) - F_H(P))/F_H(P) \tag{7.42}$$

$$\nu_L(P) = (F_H(P) - F_L(P))/F_L(P) \tag{7.43}$$

Substituting in the value functions (3.1) and (3.2) and their derivatives into expressions (7.38) to (7.43) results in

$a_H(P) =$

$$\frac{\beta_{2,1} B_{L,1} P^{\beta_{2,1}} q_L(\beta_{2,1}) \left(\frac{1}{2}\left(\beta_{2,1}-1\right)\sigma_H^2 + \mu\right) + \beta_{2,2} B_{L,2} P^{\beta_{2,2}} q_L(\beta_{2,2}) \left(\frac{1}{2}\left(\beta_{2,2}-1\right)\sigma_H^2 + \mu\right)}{B_{L,1} P^{\beta_{2,1}} q_L(\beta_{2,1}) + B_{L,2} P^{\beta_{2,2}} q_L(\beta_{2,2})} \tag{7.44}$$

$$b_H(P) = \frac{\sigma_H \left(\beta_{2,1} B_{L,1} P^{\beta_{2,1}} q_L(\beta_{2,1}) + \beta_{2,2} B_{L,2} P^{\beta_{2,2}} q_L(\beta_{2,2})\right)}{B_{L,1} P^{\beta_{2,1}} q_L(\beta_{2,1}) + B_{L,2} P^{\beta_{2,2}} q_L(\beta_{2,2})} \tag{7.45}$$

$$\nu_H(P) = \frac{B_{L,1}P^{\beta_{2,1}}\left(\lambda_H - q_L(\beta_{2,1})\right) + B_{L,2}P^{\beta_{2,2}}\left(\lambda_H - q_L(\beta_{2,2})\right)}{B_{L,1}P^{\beta_{2,1}}q_L(\beta_{2,1}) + B_{L,2}P^{\beta_{2,2}}q_L(\beta_{2,2})} \tag{7.46}$$

$$a_L(P) = \frac{\beta_{2,1}B_{L,1}P^{\beta_{2,1}}\left(\frac{1}{2}\left(\beta_{2,1}-1\right)\sigma_L^2 + \mu\right) + \beta_{2,2}B_{L,2}P^{\beta_{2,2}}\left(\frac{1}{2}\left(\beta_{2,2}-1\right)\sigma_L^2 + \mu\right)}{B_{L,1}P^{\beta_{2,1}} + B_{L,2}P^{\beta_{2,2}}} \tag{7.47}$$

$$b_L(P) = \frac{\sigma_L\left(\beta_{2,1}B_{L,1}P^{\beta_{2,1}} + \beta_{2,2}B_{L,2}P^{\beta_{2,2}}\right)}{B_{L,1}P^{\beta_{2,1}} + B_{L,2}P^{\beta_{2,2}}} \tag{7.48}$$

$$\nu_L(P) = \frac{B_{L,1}P^{\beta_{2,1}}\left(q_L(\beta_{2,1}) - \lambda_H\right) + B_{L,2}P^{\beta_{2,2}}\left(q_L(\beta_{2,2}) - \lambda_H\right)}{\lambda_H\left(B_{L,1}P^{\beta_{2,1}} + B_{L,2}P^{\beta_{2,2}}\right)} \tag{7.49}$$

The dynamics for $G_H(P)$, $V_M(P)$ and the assets-in-place of young firms are omitted but they can be derived in the same way.

The conditional CAPM beta can be found by forming a replicating portfolio with state dependent and time varying weights in the traded assets S and M that exactly reproduces the systematic risk of the option.[31] The proportion of portfolio value held in S determines the beta of the option. To this end, take equation (7.2) and substitute in $dB_i = \frac{\sigma_{P,i}dB_1 + \sigma_A dB_2}{\sigma_i}$ and $Y_i(P) = F_i(P)$. By inspection we can see that the diffusion term of the common risk factor can be eliminated by holding $\frac{F_i'(P)\sigma_A P}{\sigma_S S}$ units of the stock in the hedge portfolio. Multiplying the number of stocks by S and dividing by the portfolio value $F_i(P)$, we get the weight of the hedge portfolio invested in the tradeable asset. Since the tradeable asset has a beta of one, the beta of the growth option is given by $\beta_{F,i}(P) = \frac{F_i(P)'\sigma_A P}{\sigma_S S}\frac{S}{F_i(P)} = \frac{F_i'(P)\sigma_A P}{\sigma_S F_i(P)}$. Substituting in $F_i(P)$ from equations (3.1) and (3.2) and their derivative gives

$$\beta_{F,H}(P) = \frac{\sigma_A}{\sigma_S}\left[\frac{\beta_{2,1}B_{L,1}P^{\beta_{2,1}}q_L(\beta_{2,1}) + \beta_{2,2}B_{L,2}P^{\beta_{2,2}}q_L(\beta_{2,2})}{B_{L,1}P^{\beta_{2,1}}q_L(\beta_{2,1}) + B_{L,2}P^{\beta_{2,2}}q_L(\beta_{2,2})}\right] \tag{7.50}$$

$$\beta_{F,L}(P) = \frac{\sigma_A}{\sigma_S}\left[\frac{\beta_{2,1}B_{L,1}P^{\beta_{2,1}} + \beta_{2,2}B_{L,2}P^{\beta_{2,2}})}{B_{L,1}P^{\beta_{2,1}} + B_{L,2}P^{\beta_{2,2}})}\right] \tag{7.51}$$

[31] Alternatively, one can find the conditional CAPM beta by computing the option's return elasticity with respect to the returns of the tradeable asset. The elasticity is $\frac{\mathrm{Cov}[dF_i(P)/F_i(P), dS/S]}{\mathrm{Var}[dS/S]} = \frac{\sigma_{F,i}\sigma_A}{\sigma_i\sigma_S}$. Substituting in (7.4), $F_i(P)$ from equations (3.1) and (3.2), and their derivative gives (7.50) and (7.51). There is yet a third approach as shown in Sagi and Seashole (2007) to compute the CAPM beta. Sagi and Seashole show that the expected excess return is given by $(\mu - \mu^*)\frac{\partial \log F_i(P)}{\partial \log P} = (\mu - \mu^*)\frac{F_i'(P)}{F_i(P)}$ where $(\mu - \mu^*)$ is the difference between the unadjusted and risk-adjusted mean returns of P. In our set up, $(\mu - \mu^*) = \rho_i\sigma_i\lambda = \sigma_A\lambda$. Substituting in $F_i(P)$ from equations (3.1) and (3.2), and dividing by $\sigma_S\lambda$ results in (7.50) and (7.51).

The beta for $G_H(P)$, $V_M(P)$ and the assets-in-place $AP(P)$ of young firms can be derived in the same way. The beta for $G_H(P)$ is

$$\beta_{G,H}(P) = \frac{\sigma_A\left(\beta_{1,1}C_{H,1}P^{\beta_{1,1}} + \beta_{1,2}C_{H,2}P^{\beta_{1,2}} + \frac{\xi P\lambda_L}{(r-\mu^*)(\lambda_L+r)}\right)}{\sigma_S\left(C_{H,1}P^{\beta_{1,1}} + C_{H,2}P^{\beta_{1,2}} + \frac{\lambda_L\left(\frac{\xi P}{r-\mu^*}-\frac{c\xi}{r}-I\right)}{\lambda_L+r}\right)} \tag{7.52}$$

and the beta for $V_M(P)$ and $AP(P)$ is

$$\beta_M(P) = \beta_{AP}(P) = \frac{\sigma_A Pr}{\sigma_S\left(Pr + c(\mu^* - r)\right)} \tag{7.53}$$

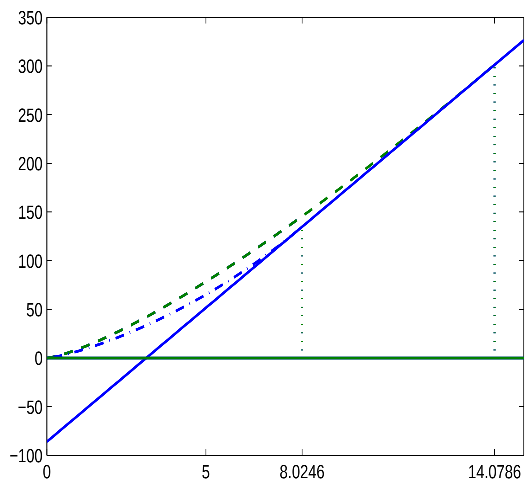
Expression (3.11) is consistent with the expected return given by the CAPM. To show this, substitute in (7.44), (7.47), (7.46) and (7.49) into the right hand side of (3.11) for $i = H$ and $i = L$. The resulting expressions equate to the expressions from substituting (7.50) and (7.51) into $r + \beta_{F,i}(P)\sigma_S\lambda$ and simplifying.

The proofs for (3.8), (3.9), (3.10) and (3.12) are trivial and merely rely on the knowledge that $\beta_{2,1} > \beta_{2,2}$ and $P < P_1$ if the investment option has not been extinguished. This completes the proof. $\blacksquare$

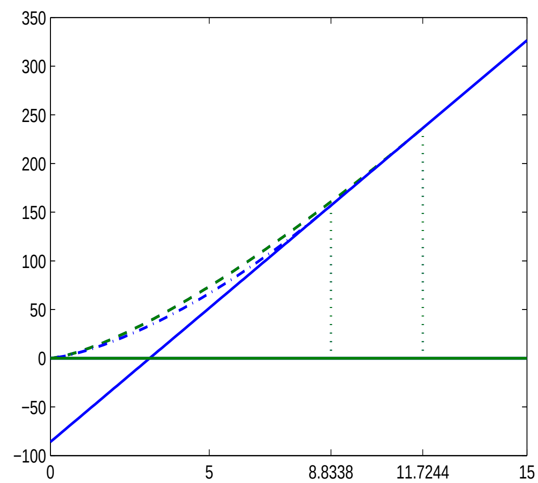
Figure 1: Model's Properties and Solution: Dependence of Growth Option Value on the Idiosyncratic Volatility Regime

The figure shows growth option values for various values of P across idiosyncratic volatility regimes and the exercise thresholds P_1 and P_2. The solid 45 degree line corresponds to the intrinsic value of the growth option. Option values in the high and low volatility states are depicted by dashed and dashed dotted curves, respectively. The exercise thresholds are depicted by the vertical dotted lines where the lower threshold corresponds to the exercise threshold P_1, and the higher threshold corresponds to the exercise threshold P_2. Panel (a) corresponds the model solution with parameters $\sigma_{P,H} = 0.5, \sigma_{P,L} = 0.1$, panel (b) corresponds to the model solution with parameters $\sigma_{P,H} = 0.4, \sigma_{P,L} = 0.2$, and panel (c) corresponds to the model solution with parameters $\sigma_{P,H} = 0.3, \sigma_{P,L} = 0.3$.

(a) $\sigma_H = 0.5$, $\sigma_L = 0.1$



(b) $\sigma_H = 0.4$, $\sigma_L = 0.2$



(c) $\sigma_H = 0.3$, $\sigma_L = 0.3$

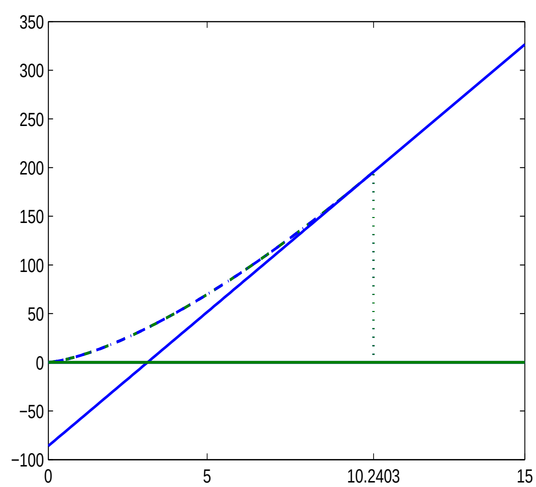
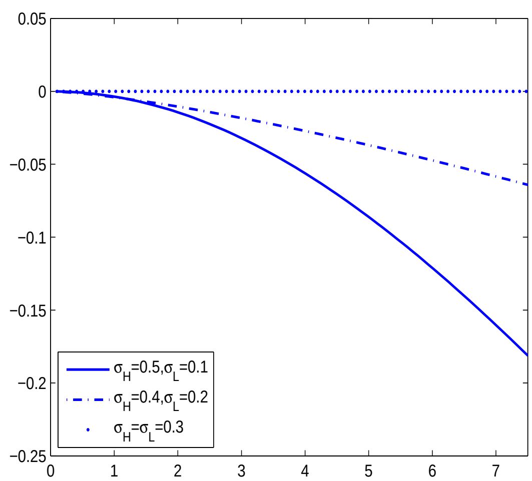


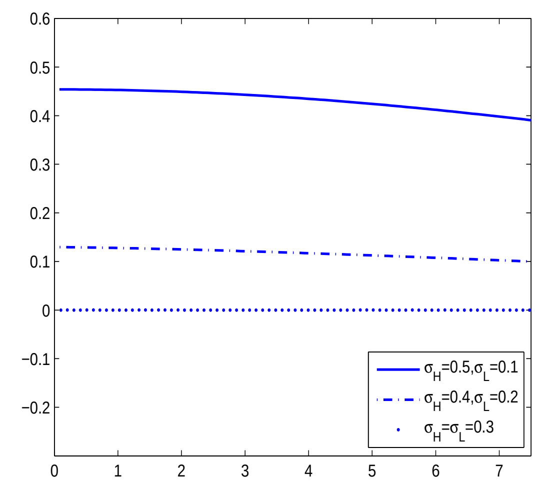
Figure 2: Model's Properties and Solution: Dependence of Return on Idiosyncratic Volatility Regime

The figure shows differences in the growth option's sensitivity to the systematic shock variable, the drift, the jump, and the diffusion terms of the option's value process (3.7) between the high and low volatility regimes for various values of P based on the model developed in Section 3 of the paper. Panel (a) shows differences in the sensitivity to the systematic shock variable, panel (b) shows differences in the diffusion term, panel (c) shows differences in the drift term, and panel (d) shows differences in the jump term. The figure shows separate results for each set of model parameter values ($\sigma_{P,H} = 0.5, \sigma_{P,L} = 0.1$; $\sigma_{P,H} = 0.4, \sigma_{P,L} = 0.2$; $\sigma_{P,H} = \sigma_{P,L} = 0.3$).

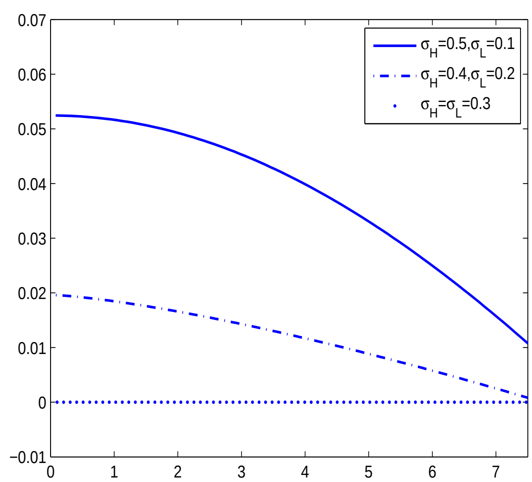
(a) $\beta_H(P) - \beta_L(P)$



(b) $b_H(P) - b_L(P)$



(c) $a_H(P) - a_L(P)$



(d) $\nu_H(P) - \nu_L(P)$

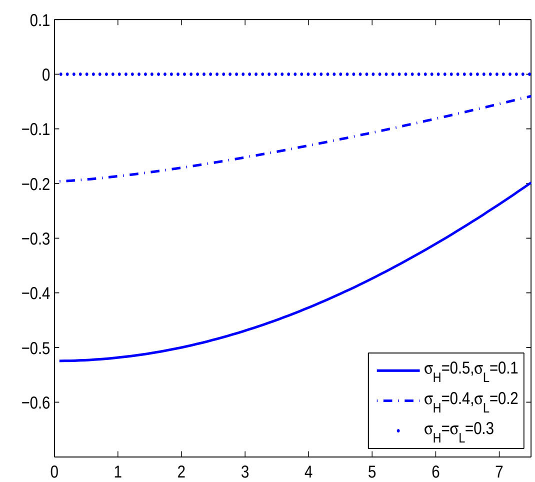
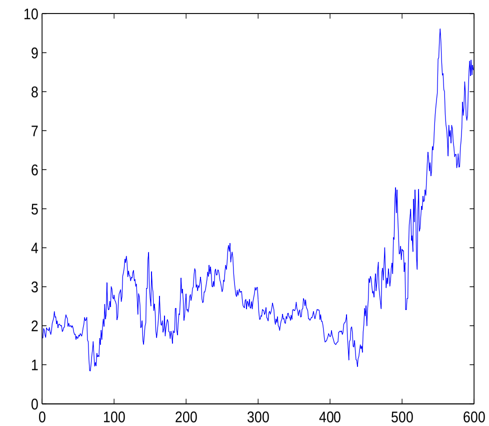


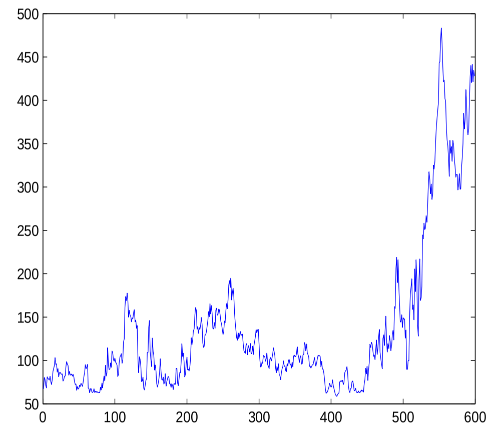
Figure 3: Model Properties and Solution: Time-Series Plots

Simulation Results. The figure shows a sample path of simulated variables based on the model developed in Section 3 of the paper for $\sigma_{P,H} = 0.5, \sigma_{P,L} = 0.1$. Panel (a) shows end-of-month values for the output price P, panel (b) shows the corresponding firm values $V(P)$, panel (c) shows the corresponding end-of-month idiosyncratic volatility regimes where $i = 1$ denotes the high volatility regime and $i = 0$ denotes the low volatility regime, panel (d) shows the end-of-month realized return volatility, and panel (e) shows the end-of-month realized returns.

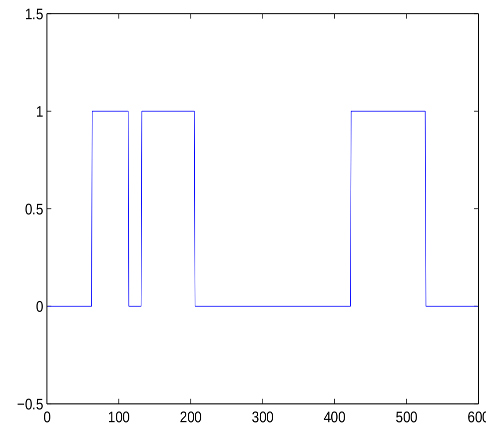
(a) P



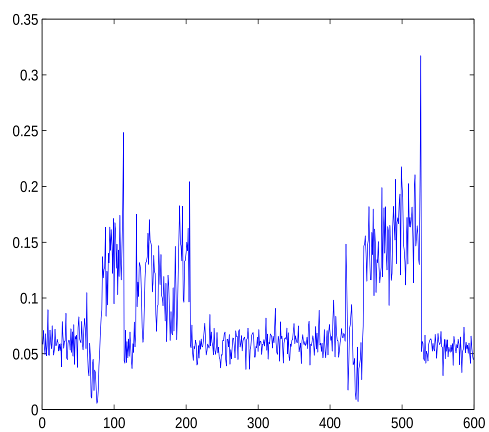
(b) $V(P)$



(c) State i: $1 = H, 0 = L$



(d) return volatility



(e) return

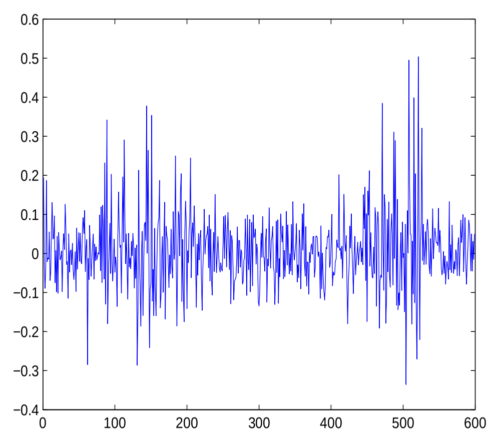


Figure 4: Simulation Results: Idiosyncratic Return Volatility Portfolio Returns

The figure shows the mean value-weighted returns of the portfolios formed after sorting stocks based on the past month return volatility $IVol$ using the simulated data based on the analytical solutions of the model developed in Section 3 of the paper. At the end of each month, stocks are sorted into five equally sized groups based on the past month $IVol$, then value-weighted one-month holding period portfolio returns are computed. The portfolios are rebalanced at the end of each month. The figure shows separate results for each set of model parameter values ($\sigma_{P,H} = 0.5, \sigma_{P,L} = 0.1$; $\sigma_{P,H} = 0.4, \sigma_{P,L} = 0.2$; $\sigma_{P,H} = \sigma_{P,L} = 0.3$).

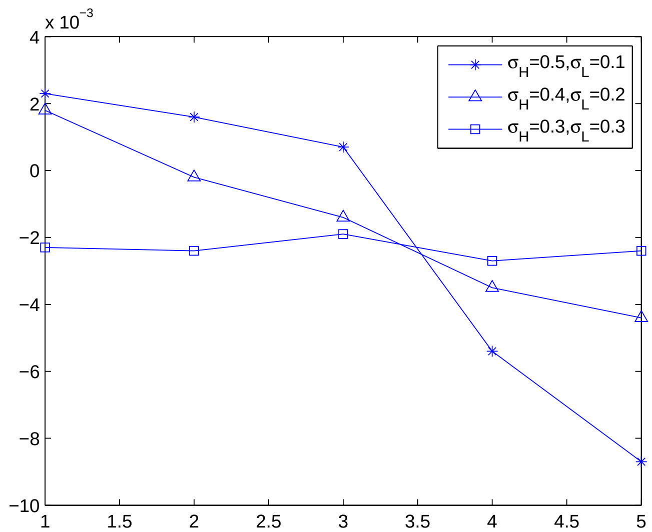


Table 1: Simulation Model Parameters

The table reports the parameter values used to solve and simulate the model developed in Section 3 of the paper. Base case parameter values are distinguished with an asterisk * if more than one value is reported for a variable.

Model Parameters		
Price Dynamics	Variable Description	Values
$\sigma_{P,H}$	Output price idiosyncratic volatility in the high regime	0.3,0.4,0.5*
$\sigma_{P,L}$	Output price idiosyncratic volatility in the low regime	0.1*,.02,0.3
λ_H	Transition parameter from low to high volatility regime	0.1
λ_L	Transition parameter from high to low volatility regime	0.1
μ	Drift term of the output price process	0.04
σ_A	Systematic volatility of the output price process	0.15
Market	Variable Description	Values
r	Riskless rate	0.05
μ_S	Drift term of tradeable asset (Market)	0.1
σ_S	Diffusion term of tradeable asset (Market)	0.25
Firm's Profit Function	Variable Description	Values
c	Variable cost per unit of output	0.5
ξ_Y	Production scale for young Firms	1
ξ	Difference in production scales between mature and young firms	1.1
I	Investment cost	$\frac{1.5\times(\xi-1)\times 20}{r-\mu^*}$
Simulations	Variable Description	Values
N	Number of samples	100
n	Number of firms in each sample	2500
T	Number of years	50
nt	Number of trading days in each month	20
λ_{exit}	Exit parameter for mature firms	0.01

Table 2: Simulation Results: Cross-Sectional Regressions

The table reports coefficient estimates for the regression model $r_t = \gamma_{0,t}\iota + \gamma_{1,t}\Delta IVol_t + \eta_t$ in the first column of each panel, and estimates for the regression model $r_t = \gamma_{0,t}\iota + \gamma_{1,t}IVol_{t-1} + \eta_t$ in the second column of the panels using the simulated from the analytical solutions from the model developed in Section 3 of the paper. Panels (a), (b) and (c) report separate model estimates corresponding to the simulated samples where $\sigma_{P,H} = 0.5$,$\sigma_{P,L} = 0.1$, $\sigma_{P,H} = 0.4$,$\sigma_{P,L} = 0.2$ and $\sigma_{P,H} = \sigma_{P,L} = 0.3$ respectively. T-statistics are reported in square brackets.

	(a) $\sigma_{P,H} = 0.5$,$\sigma_{P,L} = 0.1$		(b) $\sigma_{P,H} = 0.4$,$\sigma_{P,L} = 0.2$		(c) $\sigma_{P,H} = 0.3$,$\sigma_{P,L} = 0.3$	
Intercept	0.0056***	0.0083***	0.0056***	0.0077***	0.0050***	−0.0030***
	[40.1311]	[39.7255]	[44.5101]	[32.1002]	[27.9953]	[-11.7416]
$\Delta\sigma_t$	0.1178***		0.0884***		0.0006	
	[16.1998]		[19.1783]		[0.7927]	
σ_{t-1}		−0.1007***		−0.0880***		-0.0001
		[-39.2915]		[-31.0568]		[-0.5524]

Table 3: Simulation Results: $IVol$ Portfolio Returns

The table reports the mean $IVol$ portfolio returns using simulated data based on the analytical solutions of the model developed in Section 3 of the paper. Stocks are sorted into five equally sized groups based on past month $IVol$, then value-weighted one-month holding period portfolio returns are computed. The portfolios are rebalanced at the end of each month. $IVol$ portfolios are reported across columns, and the last column reports the mean return of the zero-cost $IVol$ portfolio. The table reports separate results for each set of model parameter values ($\sigma_{P,H} = 0.5, \sigma_{P,L} = 0.1$; $\sigma_{P,H} = 0.4, \sigma_{P,L} = 0.2$; $\sigma_{P,H} = \sigma_{P,L} = 0.3$). T-statistics are reported in square brackets.

	σ_P Portfolios					
	1	2	3	4	5	5-1
$\sigma_{P,H} = 0.5, \sigma_{P,L} = 0.1$	0.0023^{***}	0.0016^{***}	0.0007^{***}	-0.0054^{***}	-0.0087^{***}	-0.011^{***}
	[14.1267]	[8.7286]	[3.0931]	[-15.493]	[-25.1895]	[-33.7894]
$\sigma_{P,H} = 0.4, \sigma_{P,L} = 0.2$	0.0018^{***}	-0.0002	-0.0014^{***}	-0.0035^{***}	-0.0044^{***}	-0.0062^{***}
	[11.1684]	[-0.9962]	[-5.7645]	[-11.3485]	[-14.3503]	[-21.0366]
$\sigma_{P,H} = 0.3, \sigma_{P,L} = 0.3$	-0.0023^{***}	-0.0024^{***}	-0.0019^{***}	-0.0027^{***}	-0.0024	0
	[-7.2206]	[-8.0168]	[-7.8058]	[-9.5375]	[-9.0741]	[-0.0484]

Table 4: Smaple Summary Statistics

This table reports sample summary statistics for excess stock returns, idiosyncratic return volatilities $IVol$, month-to-month $IVol$ changes $\Delta IVol$, and the real option intensity proxies. The sample period is from January, 1971 to December, 2010 for all the market-based variables. Excess return is the difference between end-of-month stock return and the risk-free rate. Stock return volatility $IVol$ refers to the end-of-month volatility of the log daily returns risk-adjusted based on the Fama and French 3-factor model. Market equity and total assets are in millions of dollars. Firm age is expressed in months since the firms' first appearance on CRSP. Investment, profit and sale growths are expressed as the sum of the $t + 2$ to $t + 5$ growth rates where t is the fiscal year of the return observation. $vega$ is computed for each firm according to equation (5.2).

market variables	Mean	StdDev	P5	Median	P95	N
excess return	0.009976	0.180828	-0.22309	-0.0041	0.272627	1041266
$IVol$	0.029476	0.024979	0.0079	0.022782	0.072884	1038601
$\Delta IVol$	-2.3E-05	0.021096	-0.02552	-0.00011	0.026111	1035935

Real Option variables	Mean	StdDev	P5	Median	P95	N
log(market equity)	4.694734	2.106019	1.5389081	4.521163	8.389149	1040478
log(total assets)	4.804593	2.009753	1.789757	4.62188	8.352702	1041266
log(age)	3.953142	1.540425	0	4.290459	5.746203	1041266
investment growth	0.996235	18.22423	-0.64226	0.225036	2.237907	871778
profit growth	-0.55037	80.99137	-6.71653	0.353252	4.689659	871779
sales growth	1.579677	79.57993	-0.46927	0.29381	1.83045	868519
$vega$	2.84E-69	1.49E-67	9.63E-110	9.89E-81	1.88E-70	1041104

Table 5: Fama MacBeth Regressions: Return Difference Between Post and Pre-Switch Months Following Large Switches in Idiosyncratic Volatility and Its Dependence on Real Options.

This table reports regression results of differences in 5-month average returns between post and pre-switch months on real option proxies for stocks that experience switches in idiosyncratic volatility. The regression equation is $r_t^{Diff} = \gamma_0 \iota + \gamma_1 RO_{t-1} + \eta_t$. The construction of the real option proxies is described in the paper. Each column corresponds to results for a separate proxy for real options. Separate regression results are reported for the up and the down switch samples. The reported estimates are time-series averages of the monthly coefficient estimates. Newey and West (1987) robust t-statistics are reported in square brackets. RSQR refers to the average of monthly R-squareds.

switch	Coeff.	size (total asset)	size (mkt equity)	age	high vega	high profit	high sale	young	high inv	small high vega	small growth
down	Intercept	-0.0201***	-0.0175***	-0.0111**	-0.0062***	-0.0068***	-0.0079***	-0.0079***	-0.0080***	-0.0076***	-0.0074***
		[-6.3508]	[-5.4812]	[-2.0936]	[-6.8905]	[-6.6695]	[-7.8954]	[-7.7187]	[-8.7388]	[-8.6914]	[-8.1784]
	RO	0.0024***	0.0018***	0.0005	0.0063***	-0.0016	0.0021	-0.0007	0.0025	-0.0116**	-0.0071**
		[4.4752]	[3.1156]	[0.3552]	[-3.1565]	[-1.0178]	[1.2240]	[-0.3579]	[1.5724]	[-2.5072]	[-2.1897]
	RSQR	0.0326***	0.0329***	0.0265***	0.0268***	0.0266***	0.0252***	0.0288***	0.0244***	0.0482***	0.0376***
up	intercept	0.0158***	0.0182***	0.0150***	0.0052***	0.0040***	0.0036***	0.0044***	0.0030**	0.0048***	0.0047***
		[5.3102]	[6.9155]	[3.2173]	[5.2992]	[3.0968]	[3.0411]	[3.9123]	[2.4548]	[4.9601]	[4.7785]
	RO	-0.0022***	-0.0027***	-0.0024**	0.0023	0.0039***	0.0059***	0.0053***	0.0076***	0.0082**	0.0095***
		[-4.4951]	[-6.2128]	[-2.0830]	[1.2730]	[2.7997]	[3.8766]	[3.1845]	[5.2559]	[2.2245]	[3.4609]
	RSQR	0.0283***	0.0279***	0.0285***	0.0252***	0.0251***	0.0269***	0.0255***	0.0261***	0.0408***	0.0349***

switch	Coeff.	small high profit	small high sale	small young	small high inv	young high vega	Natural Resources	High Tech	Bio Tech	All Growth Industry
down	Intercept	-0.0062***	-0.0068***	-0.0069***	-0.0069***	-0.0077***	-0.0083***	-0.0085***	-0.0079***	-0.0082***
		[-7.1963]	[-7.6766]	[-7.6615]	[-8.1033]	[-8.3749]	[-9.2471]	[-8.5055]	[-8.8771]	[-8.5319]
	RO	-0.0088***	-0.0045	-0.0108***	-0.0036	0.001	-0.0015	0.0013	-0.001	0.0002
		[-3.1267]	[-1.4208]	[-3.5391]	[-0.8044]	[0.2391]	[-0.3588]	[0.5947]	[-0.3122]	[0.1036]
	RSQR	0.0394***	0.0388***	0.0384***	0.0439***	0.0357***	0.0333***	0.0273***	0.0288***	0.0296***
up	Intercept	0.0045***	0.0043***	0.0051***	0.0036***	0.0054***	0.0056***	0.0057***	0.0056***	0.0041***
		[3.9771]	[4.0485]	[5.1847]	[3.2879]	[5.2951]	[5.1394]	[5.5452]	[5.5774]	[3.8737]
	RO	0.0088***	0.0107***	0.0077***	0.0164***	0.0079***	0.002	0.0025	0.0028	0.0043**
		[3.0261]	[3.4846]	[2.6551]	[4.7291]	[2.7409]	[0.5846]	[1.3052]	[0.8773]	[2.1847]
	RSQR	0.0365***	0.0368***	0.0324***	0.0397***	0.0325***	0.0299***	0.0237***	0.0265***	0.0290***

Table 6: Fama MacBeth Regressions: Return Difference Between Post and Pre-Switch Months Following Large Switches in Idiosyncratic Volatility and Its Dependence on Real Options and Size of the Switch.

This table reports regression results of differences in 5-month average returns between post and pre-switch months on real option proxies and the 70th and 30th percentile spread in $IVol$ for stocks that experience switches in idiosyncratic volatility. The regression equation is $r_t^{Diff} = \gamma_0 \iota + \gamma_1 \overline{\Delta IVol} + \gamma_2 \overline{\Delta IVol} \times RO_{t-1} + \eta_t$.. The construction of the real option proxies is described in the paper. Each column corresponds to results for a separate proxy for real options. Separate regression results are reported for the up and the down switch samples. The reported estimates are time-series averages of the monthly coefficient estimates. Newey and West (1987) robust t-statistics are reported in square brackets. RSQR refers to the average of monthly R-squareds.

switch	Coeff.	size (total asset)	size (mkt equity)	age	high vega	high profit	high sale	young	high inv	small high vega	small growth
down	Intercept	-0.0018	0.0011	0.0015	0.0002	-0.0003	0	0.0023	0.0004	0.0014	0.0011
		[-0.6873]	[0.4085]	[0.7341]	[0.1085]	[-0.1137]	[0.0180]	[1.0268]	[0.1644]	[0.6783]	[0.5057]
	$\overline{\Delta IVol}$	-1.0054***	-0.8526***	-0.2454	-0.6433***	-0.5962**	-0.7080***	-1.0003***	-0.7249***	-0.8262***	-0.8361***
		[-3.8117]	[-3.4519]	[-0.4000]	[-2.8142]	[-2.5609]	[-3.1400]	[-4.6543]	[-3.2998]	[-3.9831]	[-3.8818]
	$\overline{\Delta IVol} \times RO$	0.1072*	-0.0046	-0.1626	-0.1921	0.0088	0.1432	0.2439	0.1334	-0.2349	0.0228
		[1.7705]	[-0.0742]	[-1.0051]	[-1.2213]	[0.0611]	[0.9226]	[1.4789]	[0.8459]	[-1.0029]	[0.1089]
	RSQR	0.0882***	0.0912***	0.0877***	0.0871***	0.0967***	0.0964***	0.0898***	0.0958***	0.0959***	0.0962***
		[19.6250]	[16.1655]	[17.3481]	[17.1345]	[18.9544]	[19.4351]	[17.9502]	[17.5652]	[17.1672]	[19.9459]
up	Intercept	-0.0003	0.0037	-0.0035	-0.0043*	-0.0066***	-0.0056**	-0.003	-0.0059**	-0.0038*	-0.0024
		[-0.1080]	[1.4158]	[-1.6219]	[-1.8809]	[-2.8511]	[-2.4964]	[-1.2945]	[-2.4858]	[-1.7371]	[-1.1340]
	$\overline{\Delta IVol}$	0.9565***	0.9889***	1.3251**	0.8610***	0.9418***	0.8203***	0.6773***	0.7773***	0.7808***	0.6350***
		[3.9990]	[4.4507]	[2.1528]	[4.0885]	[4.4276]	[4.2999]	[3.1748]	[3.9858]	[3.9437]	[3.2559]
	$\overline{\Delta IVol} \times RO$	-0.1229**	-0.2472***	-0.2528	-0.0527	0.2297*	0.4044***	0.1471	0.6317***	0.0379	0.2846
		[-2.2675]	[-3.9057]	[-1.0236]	[-0.3575]	[1.7174]	[2.7888]	[1.2512]	[4.7608]	[0.1882]	[1.5988]
	RSQR	0.0721***	0.0775***	0.0740***	0.0736***	0.0821***	0.0810***	0.0689***	0.0810***	0.0809***	0.0784***

switch	Coeff.	small high profit	small high sale	small young	small high inv	young high vega	Natural Resources	High Tech	Bio Tech	All Growth Industries
down	Intercept	-0.0007	0.0001	0	-0.0002	0.0015	0.0009	0.0003	0.0006	0.0005
		[-0.2803]	[0.0229]	[0.0216]	[-0.0799]	[0.6820]	[0.4145]	[0.1555]	[0.2746]	[0.2131]
	$\overline{\Delta IVol}$	-0.5243**	-0.6459***	-0.7449***	-0.6130***	-0.8933***	-0.7969***	-0.7817***	-0.7619***	0.8022***
		[-2.2498]	[-2.8497]	[-3.5372]	[-2.8459]	[-4.1053]	[-3.6453]	[-3.6182]	[-3.5135]	[-3.6652]
	$\overline{\Delta IVol} \times RO$	-0.209	-0.0493	-0.1334	-0.0914	0.3063	-0.0335	0.0778	0.0886	0.1131
		[-1.0722]	[-0.2414]	[-0.7354]	[-0.3688]	[1.3479]	[-0.0943]	[0.4351]	[0.3732]	[0.7114]
	RSQR	0.1005***	0.1007***	0.0930***	0.1045***	0.0941***	0.0937***	0.0913***	0.0888***	0.0934***
		[20.2486]	[20.3574]	[18.3941]	[18.4894]	[16.2996]	[17.5190]	[19.7699]	[19.0352]	[17.9798]
up	Intercept	-0.0059**	-0.0054**	-0.0032	-0.0048**	-0.0031	-0.0037*	-0.0042*	-0.0037	-0.0032
		[-2.3840]	[-2.1412]	[-1.4104]	[-2.0238]	[-1.4008]	[-1.6704]	[-1.6530]	[-1.4657]	[-1.4908]
	$\overline{\Delta IVol}$	0.9227***	0.8880***	0.7471***	0.7969***	0.7031***	0.7761***	0.8545***	0.7441***	0.6572***
		[4.2713]	[4.2814]	[3.7131]	[4.1374]	[3.4753]	[4.0162]	[3.7309]	[3.6893]	[3.5573]
	$\overline{\Delta IVol} \times RO$	0.1712	0.3161*	0.0841	0.6159***	0.0601	0.339	0.0486	0.0432	0.1858
		[0.9805]	[1.7296]	[0.5751]	[3.2721]	[0.3100]	[1.0305]	[0.3155]	[0.1791]	[1.1833]
	RSQR	0.0847***	0.0841***	0.0701***	0.0842***	0.0777***	0.0753***	0.0723***	0.0711***	0.0773***

Table 7: Fama MacBeth Regressions: Dependence of the Return-$\Delta IVol$ relation on Growth Options.

This table reports the coefficient estimates of Fama and MacBeth (1973) cross-sectional regressions of firm level excess returns on the loading on the market factor (β^{CAPM}), log book-to-market ($Log(BM)$), log market equity ($Log(ME)$), six-month lagged return ($Lag(r)$), monthly trading volume normalized by the number of shares outstanding ($trade$), month-to-month change in firm level idiosyncratic volatility ($\Delta IVol$), and the interaction between real option proxy and $\Delta IVol$. The construction of the real option proxies are described in the paper. The regression model is $r_t - r_{f,t} = \gamma_0 \iota + \gamma_1 \Delta IVol_t + \gamma_2 \Delta IVol_t \times RO_{t-1} + \gamma_3 X_{t-1} + \eta_t$. The real option proxies are reported across columns. The reported estimates are the time series average of the monthly coefficient estimates. Newey-West robust t-statistics are reported in square brackets. RSQR refers to the average of monthly R squared.

Coeff.	size (mkt equity)	size (total assets)	age	high vega	high profit	high sale	young	high inv	small high vega	small growth
Intercept	0.0402***	0.0404***	0.0405***	0.0404***	0.0417***	0.0414***	0.0404***	0.0417***	0.0403***	0.0403***
	[6.7509]	[6.8148]	[6.8501]	[6.8507]	[7.0746]	[7.0291]	[6.8406]	[7.0813]	[6.8330]	[6.8257]
$Log(BM)$	0.0057***	0.0059***	0.0057***	0.0057***	0.0052***	0.0052***	0.0057***	0.0052***	0.0057***	0.0058***
	[7.7458]	[8.0586]	[7.7807]	[7.6285]	[6.8186]	[6.8100]	[7.7920]	[6.7996]	[7.7387]	[7.7815]
$Log(ME)$	-0.0034***	-0.0035***	-0.0035***	-0.0035***	-0.0037***	-0.0037***	-0.0035***	-0.0037***	- -0.0035***	-0.0035***
	[-5.5311]	[-5.6697]	[-5.7620]	[-5.7513]	[-6.1040]	[-6.0733]	[-5.7558]	[-6.1205]	[-5.7169]	[-5.7682]
β^{CAPM}	0.0015**	0.0015**	0.0015**	0.0016**	0.0014*	0.0015*	0.0015**	0.0014*	0.0016**	0.0015**
	[2.2340]	[2.1974]	[2.0700]	[2.2294]	[1.8310]	[1.9049]	[2.1691]	[1.8073]	[2.2730]	[2.1722]
$trade$	0.0157***	0.0156***	0.0155***	0.0155***	0.0154***	0.0153***	0.0155***	0.0154***	0.0155***	0.0155***
	[14.4667]	[14.3834]	[14.2095]	[14.3831]	[13.4373]	[13.4022]	[14.2209]	[13.4620]	[14.3394]	[14.2345]
$Lag(r)$	-0.0031	-0.0031	-0.0031	-0.0031	-0.0031	-0.003	-0.0031	-0.0032	-0.0032	-0.003
	[-1.1424]	[-1.1612]	[-1.1747]	[-1.1768]	[-1.0903]	[-1.0371]	[-1.1534]	[-1.1049]	[-1.1820]	[-1.1219]
$\Delta IVol$	2.3623***	2.2474***	1.1341***	1.0903***	1.2728***	1.1797***	1.1865***	1.1680***	1.1066***	1.2181***
	[17.5356]	[15.8777]	[6.9660]	[9.8669]	[12.1527]	[11.3973]	[10.1845]	[10.9610]	[10.5451]	[12.3035]
$RO \times \Delta IVol$	-0.3125***	-0.2647***	-0.0094	0.3748***	0.0217	0.2497***	0.0321	0.3273***	0.5297***	0.3139***
	[-10.790]	[-9.8134]	[-0.1847]	[5.0322]	[0.3632]	[3.7902]	[0.3178]	[5.0967]	[5.2866]	[3.4861]
RSQR	0.1150***	0.1128***	0.1099***	0.1107***	0.1104***	0.1111***	0.1099***	0.1110***	0.1127***	0.1107***

Coeff.	small high profit	small highs sale	small young	small high inv	young high vega	Natural Resources	High Tech	Bio Tech	All Growth Option Industries
Intercept	0.0416***	0.0413***	0.0405***	0.0416***	0.0403***	0.0403***	0.0403***	0.0403***	0.0404***
	[7.0506]	[7.0154]	[6.8674]	[7.0715]	[6.8187]	[6.8169]	[6.8440]	[6.8730]	[6.8530]
$Log(BM)$	0.0053***	0.0052***	0.0058***	0.0052***	0.0058***	0.0057***	0.0057***	0.0058***	0.0057***
	[6.8579]	[6.8384]	[7.8430]	[6.7970]	[7.7727]	[7.7483]	[7.7597]	[7.8780]	[7.7787]
$Log(ME)$	-0.0037***	-0.0036***	-0.0035***	-0.0037***	-0.0035***	-0.0035***	-0.0035***	-0.0035***	-0.0035***
	[-6.0680]	[-6.0359]	[-5.7614]	[-6.0951]	[-5.7286]	[-5.7625]	[-5.7303]	[-5.7558]	[-5.7524]
β^{CAPM}	0.0015*	0.0015**	0.0016**	0.0014*	0.0015**	0.0016**	0.0015**	0.0015**	0.0015**
	[1.8461]	[2.0167]	[2.2027]	[1.8043]	[2.1244]	[2.2400]	[2.1177]	[2.0997]	[2.1287]
$trade$	0.0154***	0.0153***	0.0155***	0.0154***	0.0155***	0.0154***	0.0155***	0.0155***	0.0154***
	[13.4108]	[13.4492]	[14.2725]	[13.4390]	[14.1888]	[14.1937]	[14.2384]	[14.3645]	[14.2776]
$Lag(r)$	-0.003	-0.0029	-0.003	-0.003	-0.003	-0.0032	-0.0032	-0.0031	-0.0031
	[-1.0544]	[-1.0095]	[-1.1569]	[-1.0560]	[-1.1339]	[-1.1729]	[-1.1851]	[-1.1741]	[-1.1711]
$\Delta IVol$	1.1829***	1.1282***	1.1573***	1.1504***	1.2322***	1.2437***	1.2796***	1.2852***	1.1634***
	[11.4545]	[10.8841]	[11.6522]	[10.8892]	[12.0615]	[12.0660]	[12.8921]	[12.8305]	[11.8727]
$RO \times \Delta IVol$	0.4097***	0.6017***	0.3011***	0.5906***	0.1640**	0.3629***	0.0536	0.0483	0.2804***
	[4.6175]	[7.4742]	[3.5690]	[7.7597]	[2.0284]	[3.5133]	[0.6618]	[0.4384]	[4.2373]
RSQR	0.1116***	0.1122***	0.1111***	0.1122***	0.1108***	0.1106***	0.1102***	0.1101***	0.1102***

Table 8: Fama MacBeth Regressions: Dependence of the + $IVol$-Return relation on Real Options and Switch Size.

This table reports the coefficient estimates of Fama and MacBeth (1973) cross-sectional regressions of firm level excess returns on the loading on the market factor (β^{CAPM}), log book-to-market ($Log(BM)$), log market equity ($Log(ME)$), six-month lagged return ($Lag(r)$), monthly trading volume normalized by the number of shares outstanding ($trade$), month-to-month change in firm level idiosyncratic volatility ($\Delta IVol$), the 70th and 30th percentile spread in $IVol$ ($\overline{\Delta IVol}$), real option proxy, and the interaction between real option proxy and $\overline{\Delta IVol}$. The construction of the real option proxies are described in the paper. The regression model is $r_t - r_{f,t} = \gamma_0 \iota + \gamma_1 \overline{\Delta IVol} + \gamma_2 \Delta IVol_t + \gamma_3 \overline{\Delta IVol} \times \Delta IVol \times RO + \gamma_3 X_{t-1} + \eta_t$. The real option proxies are reported across columns. The reported estimates are the time series average of the monthly coefficient estimates. Newey-West robust t-statistics are reported in square brackets. RSQR refers to the average of monthly R squared.

Coeff.	size (mkt equity)	size (total assets)	age	high vega	high profit	high sale	young	high inv	small high vega	small growth
Intercept	0.0620***	0.0627***	0.0635***	0.0636***	0.0628***	0.0622***	0.0633***	0.0626***	0.0635***	0.0633***
	[12.6667]	[12.9225]	[13.0910]	[13.1015]	[13.0393]	[12.8871]	[12.9775]	[12.9719]	[13.0683]	[12.9677]
$Log(BM)$	0.0043***	0.0044***	0.0044***	0.0043***	0.0039***	0.0040***	0.0043***	0.0039***	0.0043***	0.0044***
	[5.9892]	[6.0253]	[6.0236]	[5.8810]	[5.2759]	[5.3703]	[6.0379]	[5.3164]	[5.9482]	[6.1117]
$Log(ME)$	-0.0058***	-0.0059***	-0.0059***	-0.0059***	-0.0059***	-0.0058***	-0.0059***	-0.0059***	-0.0059***	-0.0059***
	[-11.668]	[-11.886]	[-11.888]	[-11.951]	[-12.321]	[-12.203]	[-11.845]	[-12.268]	[-11.906]	[-11.880]
β^{CAPM}	0.0016**	0.0017**	0.0017**	0.0017***	0.0016**	0.0017**	0.0017**	0.0017**	0.0017***	0.0017***
	[2.3965]	[2.4694]	[2.5253]	[2.5956]	[2.1575]	[2.2686]	[2.4378]	[2.1917]	[2.5962]	[2.5885]
$Lag(r)$	-0.0042	-0.0041	-0.0041	-0.0041	-0.0039	-0.0038	-0.0041	-0.004	-0.0041	-0.0041
	[-1.6123]	[-1.5606]	[-1.5916]	[-1.5743]	[-1.4138]	[-1.3623]	[-1.6018]	[-1.4368]	[-1.5860]	[-1.5573]
$trade$	0.0158***	0.0158***	0.0159***	0.0159***	0.0157***	0.0157***	0.0158***	0.0157***	0.0159***	0.0158***
	[14.8382]	[14.7683]	[14.8815]	[14.9759]	[14.1370]	[14.0990]	[14.8001]	[14.1068]	[14.9459]	[14.8546]
$\overline{\Delta IVol}$	-0.9410***	-0.9714***	-1.0111***	-1.0136***	-0.9507***	-0.9369***	-0.9915***	-0.9412***	-1.0108***	-1.0003***
	[-9.4354]	[-9.6958]	[-10.129]	[-10.100]	[-8.9019]	[-8.7162]	[-9.9101]	[-8.8688]	[-10.092]	[-9.9407]
$\Delta IVol$	1.8416***	1.6986***	1.0350***	1.1713***	1.2416***	1.1677***	1.2704***	1.1767***	1.1665***	1.2368***
	[16.2397]	[12.7028]	[8.1893]	[10.8873]	[12.1765]	[11.3403]	[12.5016]	[11.2056]	[11.3075]	[12.3132]
$\overline{\Delta IVol} \times \Delta IVol \times RO$	-10.149***	-7.0635***	-2.5813	10.4454***	5.7354*	15.9548***	-1.0437	18.5515***	13.4981***	11.4536***
	[-6.4869]	[-5.3356]	[-0.5099]	[2.6592]	[1.6515]	[4.5518]	[-0.2828]	[5.1675]	[3.1356]	[2.6497]
RSQR	0.1176***	0.1164***	0.1166***	0.1180***	0.1170***	0.1175***	0.1169***	0.1173***	0.1191***	0.1173***

Coeff.	small high profit	small highs sale	small young	small high inv	young high vega	Natural Resources	High Tech	Bio Tech	All Growth Option Industries
Intercept	0.0629***	0.0623***	0.0636***	0.0628***	0.0634***	0.0627***	0.0627***	0.0627***	0.0631***
	[13.0392]	[12.9103]	[13.0859]	[12.9952]	[13.0304]	[12.9122]	[12.8832]	[13.0103]	[13.0997]
$Log(BM)$	0.0040***	0.0040***	0.0044***	0.0039***	0.0044***	0.0044***	0.0044***	0.0044***	0.0044***
	[5.2952]	[5.3880]	[6.0411]	[5.3075]	[5.9657]	[6.0308]	[6.0725]	[6.1201]	[6.0726]
$Log(ME)$	-0.0059***	-0.0058***	-0.0059***	-0.0059***	-0.0059***	-0.0059***	-0.0059***	-0.0058***	-0.0059***
	[-12.312]	[-12.215]	[-11.882]	[-12.270]	[-11.959]	[-11.903]	[-11.811]	[-11.871]	[-11.952]
β^{CAPM}	0.0017**	0.0017**	0.0017**	0.0017**	0.0018**	0.0018***	0.0018**	0.0016**	0.0017**
	[2.1808]	[2.3133]	[2.4464]	[2.1928]	[2.5693]	[2.6381]	[2.5412]	[2.3988]	[2.4645]
$Lag(r)$	-0.0038	-0.0037	-0.0041	-0.0039	-0.0041	-0.0042	-0.0042	-0.0041	-0.0041
	[-1.3929]	[-1.3437]	[-1.5979]	[-1.4052]	[-1.5618]	[-1.6174]	[-1.5995]	[-1.6019]	[-1.5898]
$trade$	0.0157***	0.0157***	0.0158***	0.0157***	0.0158***	0.0158***	0.0157***	0.0158***	0.0158***
	[14.1190]	[14.1104]	[14.8476]	[14.1089]	[14.8011]	[14.8076]	[14.8051]	[14.9660]	[14.8848]
$\overline{\Delta IVol}$	-0.9580***	-0.9432***	-1.0027***	-0.9531***	-1.0019***	-0.9704***	-0.9810***	-0.9709***	-0.9873***
	[-8.9675]	[-8.7742]	[-9.9681]	[-8.9661]	[-9.9511]	[-9.6355]	[-9.7776]	[-9.5971]	[-9.6676]
$\Delta IVol$	1.2115***	1.1590***	1.2214***	1.1849***	1.2578***	1.2651***	1.2737***	1.2776***	1.1733***
	[11.9304]	[11.2169]	[12.0872]	[11.3227]	[12.4545]	[12.3600]	[12.7952]	[12.8883]	[11.9514]
$\overline{\Delta IVol} IVol \times RO$	14.6836***	24.4176***	5.9097	22.5754***	3.2036	19.3544***	5.77	8.6091	13.0368***
	[3.3967]	[6.0754]	[1.3540]	[5.6070]	[0.7889]	[3.3040]	[1.4192]	[1.2756]	[4.2008]
RSQR	0.1178***	0.1180***	0.1176***	0.1179***	0.1173***	0.1158***	0.1168***	0.1160***	0.1167***

Table 9: – $IVol$-Return Relation and Real Options

The table reports Fama and French (1993) portfolio alphas along with robust Newey and West (1987) t-statistics in square brackets for the portfolios of stocks sorted by idiosyncratic return volatility $IVol$ and the proxies for real option intensity. $IVol$ portfolios are reported across columns. Real option portfolios are reported down the rows. The columns labeled '3-1' correspond to the zero-cost $IVol$ portfolios within each classification of real option intensity. Idiosyncratic return volatility is computed relative to FF-3. All portfolios are value-weighted and rebalanced monthly. Portfolio alphas are annualized.

Real Option	$IVol$ Portfolios											
Portfolios	1	2	3	3-1	1	2	3	3-1	1	2	3	3-1
	age				size (market equity)				size (total asset)			
1	0.7141	1.4974	-5.4860**	-6.2001**	3.4891**	7.1679***	-1.4118	-4.9009**	4.0975**	4.0300**	-6.5365***	-10.6340***
	[0.5620]	[0.8703]	[-2.0730]	[-2.2095]	[2.1164]	[4.2677]	[-0.6068]	[-2.0704]	[2.4911]	[1.9908]	[-2.8647]	[-4.3472]
2	0.0977	2.2573	-6.8932***	-7.0886***	2.0518**	2.4464***	-5.7961***	-7.8479***	4.1199***	2.5739*	-5.4489***	-9.5688***
	[0.0690]	[1.1694]	[-2.9674]	[-3.0333]	[2.2206]	[2.6878]	[-3.0515]	[-3.3100]	[4.0848]	[1.8638]	[-2.7191]	[-4.1640]
3	0.2338	1.0879	-1.3004	-0.999	1.1703**	1.1891	-4.3537	-5.5241*	1.1176**	1.0809	-3.3977	-4.5153
	[0.1729]	[0.7286]	[-0.4450]	[-0.3091]	[2.0816]	[0.8592]	[-1.5122]	[-1.8321]	[1.9880]	[0.7594]	[-1.0931]	[-1.4095]
	high vega				high profit				high sale			
0	1.4045**	2.0348*	-2.361	-3.7655*	0.9203	-0.3679	-3.9303	-4.8505*	0.5376	-2.7208**	-8.0188***	-8.5564***
	[2.2822]	[1.7913]	[-1.1747]	[-1.6816]	[1.3045]	[-0.3151]	[-1.6076]	[-1.8132]	[0.7579]	[-2.2648]	[-3.3133]	[-3.3929]
1	-1.1113	-2.3174	-10.4032***	-9.2920***	2.1091*	2.2445	-2.0649	-4.1739	4.8059***	6.1847***	2.9237	-1.8821
	[-0.7644]	[-1.2174]	[-3.6044]	[-2.9722]	[1.8580]	[1.1791]	[-0.7984]	[-1.5930]	[3.9921]	[3.2661]	[1.0957]	[-0.6379]
	high investment				young				small high vega			
0	-1.6825**	-5.2831***	-11.3864***	-9.7039***	1.3132	1.1363	-3.6190*	-4.9322**	1.3118**	1.443	-4.1226**	-5.4345**
	[-2.3719]	[-3.9863]	[-4.4539]	[-3.5168]	[1.2116]	[0.6678]	[-1.7697]	[-2.1477]	[2.2125]	[1.2015]	[-2.0369]	[-2.4447]
1	8.6144***	10.8817***	8.8514***	0.237	0.4351	-0.1468	-7.3830***	-7.8181***	7.2256***	-1.3516	-9.4742***	-16.6999***
	[9.5657]	[6.3204]	[3.5488]	[0.0895]	[0.3662]	[-0.0634]	[-2.7331]	[-2.9091]	[3.1348]	[-0.5017]	[-2.7450]	[-5.1945]

Table 10: $-$ $IVol$-Return Relation and Real Options

The table reports Fama and French (1993) portfolio alphas along with robust Newey and West (1987) t-statistics in square brackets for the portfolios of stocks sorted by idiosyncratic return volatility $IVol$ and the proxies for real option intensity. $IVol$ portfolios are reported across columns. Real option portfolios are reported down the rows. The columns labeled '3-1' correspond to the zero-cost $IVol$ portfolios within each classification of real option intensity. Idiosyncratic return volatility is computed relative to FF-3. All portfolios are value-weighted and rebalanced monthly. Portfolio alphas are annualized.

Real Option	$IVol$ Portfolios											
Portfolios	1	2	3	3-1	1	2	3	3-1	1	2	3	3-1
	small and high profit				small and high sale				small and young			
0	1.1321*	0.9205	-4.8299**	-5.9620***	1.1484*	0.8917	-4.9013**	-6.0497**	1.3810**	1.2084	-3.6206*	-5.0016**
	[1.9005]	[0.7740]	[-2.2867]	[-2.6013]	[1.8996]	[0.7600]	[-2.2452]	[-2.5556]	[2.3321]	[1.0050]	[-1.7557]	[-2.2157]
1	6.6400***	5.1844	-5.9994	-12.6394***	6.9615***	7.2097***	-4.2678	-11.2293***	3.2628	-0.4925	-12.5558***	-15.8186***
	[3.2080]	[1.5296]	[-1.4277]	[-3.1488]	[3.5611]	[2.6765]	[-1.1800]	[-3.1720]	[1.4685]	[-0.2145]	[-3.7289]	[-5.5878]
	small and high investment				young and high vega				natural resources			
0	1.0777*	0.8581	-5.3113**	-6.3890***	1.2383**	1.8331	-3.2394	-4.4778*	0.7477	0.4518	-3.7675*	-4.5152*
	[1.7810]	[0.7173]	[-2.4564]	[-2.7210]	[2.1660]	[1.4618]	[-1.5166]	[-1.9169]	[0.9722]	[0.3995]	[-1.7082]	[-1.8697]
1	13.4407***	15.6392***	-0.1405	-13.5812***	-2.2457	-1.7817	-11.4735***	-9.2278***	3.2293	2.3808	-4.5156	-7.7450**
	[6.5670]	[4.2039]	[-0.0450]	[-4.3761]	[-1.0604]	[-0.6608]	[-3.4854]	[-2.6007]	[1.5656]	[0.6126]	[-0.9362]	[-2.0684]
	high tech				bio tech				all growth			
0	1.4748*	0.7158	-2.7805	-4.2553*	0.6634	1.2607	-5.4760**	-6.1394***	0.0523	-0.6041	-5.3537***	-5.4061***
	[1.9199]	[0.5762]	[-1.3525]	[-1.8781]	[1.0983]	[0.8937]	[-2.4337]	[-2.6350]	[0.0518]	[-0.5033]	[-3.2074]	[-2.9169]
1	1.4316	1.2535	-6.0880*	-7.5196**	5.4876***	5.0094	0.8018	-4.6858	2.8762***	3.0485	-2.9505	-5.8266*
	[0.8085]	[0.4088]	[-1.7998]	[-2.4055]	[3.1616]	[1.5461]	[0.2237]	[-1.3247]	[2.9638]	[1.2202]	[-0.9131]	[-1.8956]

Table 11: Dependence of the $-IVol$-Return Relation on Switch Size and Real Options

The table reports Fama and French (1993) portfolio alphas, along with robust Newey and West (1987) t-statistics in square brackets, for the zero-cost $IVol$ portfolios for each of the two-way classifications of real option proxies and $\overline{\Delta IVol}$. The columns labeled 'mean' correspond to the equally-weighted portfolio on the zero-cost $IVol$ portfolios for each real option intensity group within each classification of $\overline{\Delta IVol}$. Idiosyncratic volatility is computed relative to FF-3. All portfolios are value-weighted and rebalanced monthly. Portfolio alphas are annualized.

$\overline{\Delta IVol}$	Real Option Portfolios											
Portfolios	1	2	3	mean	1	2	3	mean	1	2	3	mean
	age				size (market equity)				size (total asset)			
1	1.1583	-2.4428	-3.479	-0.2096	-0.1546	0.8774	-0.7373	-0.0048	1.2953	1.9408	-1.7919	0.4814
	[0.4656]	[-0.8672]	[-1.6132]	[-0.1219]	[-0.1097]	[0.6937]	[-0.3732]	[-0.0040]	[0.7783]	[0.9964]	[-0.8756]	[0.3398]
2	0.5376	-9.0395***	-3.8205	-3.8516*	0.6286	0.2541	-6.2849**	-1.8008	-1.1459	-5.6068**	-5.9637**	-4.2388**
	[0.1828]	[-3.0131]	[-1.1623]	[-1.9194]	[0.2874]	[0.1028]	[-2.2975]	[-1.0100]	[-0.3936]	[-2.1743]	[-2.0455]	[-2.0982]
3	-13.2842***	-9.7927**	-10.4336**	-11.6222***	-6.3840**	-12.7608***	-19.9126***	-13.0192***	-9.1933**	-16.1224***	-14.8518***	-13.3892***
	[-3.2330]	[-2.5055]	[-2.3812]	[-3.8341]	[-2.0049]	[-4.3610]	[-5.4313]	[-5.0374]	[-2.5293]	[-5.0924]	[-3.6702]	[-4.9292]

Table 12: Dependence of the $-IVol$-Return Relation on Switch Size and Real Options

The table reports Fama and French (1993) portfolio alphas, along with robust Newey and West (1987) t-statistics in square brackets, for the zero-cost $IVol$ portfolios for each of the two-way classifications of real option proxies and $\overline{\Delta IVol}$. The columns labeled 'mean' correspond to the equally-weighted portfolio on the zero-cost $IVol$ portfolios for each real option intensity group within each classification of $\overline{\Delta IVol}$. Idiosyncratic volatility is computed relative to FF-3. All portfolios are value-weighted and rebalanced monthly. Portfolio alphas are annualized.

$\overline{\Delta IVol}$	Real Option Portfolios									
Portfolios	0	1	0	1	0	1	0	1	0	1
	high vega		high profit		high sale		high investment		young	
1	0.147	-8.1421	-0.943	-7.1614	-5.8249**	3.3435	-6.8576**	2.6811	-2.6092	1.1289
	[0.0618]	[-1.3157]	[-0.3316]	[-1.5875]	[-1.9994]	[0.6184]	[-2.0994]	[0.6673]	[-0.9174]	[0.2094]
2	-3.505	-5.6724	-1.9006	1.2129	-1.9143	-1.607	-4.5404	0.7809	-5.2328**	-1.3772
	[-1.4618]	[-1.4065]	[-0.6468]	[0.3879]	[-0.6081]	[-0.5251]	[-1.5994]	[0.2560]	[-1.9887]	[-0.4292]
3	-6.8298*	-5.0545	-12.9659***	1.8703	-12.3146***	-2.904	-9.8621**	-4.5532	-8.4579**	-8.0120**
	[-1.9367]	[-1.2537]	[-3.1592]	[0.4470]	[-2.7325]	[-0.7771]	[-2.3589]	[-1.1472]	[-2.3154]	[-2.2161]
	small high vega		small growth		small high sale		small and young		small high investment	
1	-2.4541	-27.9701*	-1.2082	4.5421	-4.1531	-5.0725	-1.8499	-0.1049	-3.6428	-0.432
	[-1.0159]	[-1.8613]	[-0.4787]	[0.5815]	[-1.5812]	[-0.6850]	[-0.7736]	[-0.0137]	[-1.3841]	[-0.0552]
2	-2.187	-25.0583***	-3.1672	-11.2681***	-2.8776	-5.5105	-2.7867	-8.3822*	-2.9884	-8.5901*
	[-0.9645]	[-3.7224]	[-1.2881]	[-2.6043]	[-1.1796]	[-1.0362]	[-1.1933]	[-1.7600]	[-1.2031]	[-1.7891]
3	-9.4226***	-12.0750***	-7.5165**	-16.2439***	-7.0473*	-7.3758*	-8.4557**	-10.3204***	-7.0524*	-9.6920**
	[-2.7477]	[-3.3645]	[-1.9739]	[-5.2359]	[-1.8367]	[-1.8315]	[-2.2478]	[-2.8822]	[-1.8104]	[-2.5248]
	young and high vega		natural resources		high tech		bio tech		all growth	
1	-0.8212	-12.0280*	-1.6231	6.8209	0.3569	0.7912	-0.8241	7.5278	-0.8772	6.3261
	[-0.3315]	[-1.7938]	[-0.5745]	[1.3028]	[0.1429]	[0.1257]	[-0.2945]	[1.2412]	[-0.4017]	[1.0666]
2	-3.0975	6.1775	-1.7178	-2.7294	-0.8473	-8.2843**	-3.0634	-4.2983	-0.7051	-6.5709**
	[-1.2775]	[1.1195]	[-0.6962]	[-0.6421]	[-0.3146]	[-2.2199]	[-1.2751]	[-1.0584]	[-0.3291]	[-2.1681]
3	-8.8177***	0.4351	-7.0048**	-11.2290**	-10.9527***	-8.6281*	-7.7035**	-12.9701**	-12.3964***	-13.3725***
	[-2.5898]	[0.0944]	[-2.1141]	[-2.5374]	[-3.1950]	[-1.6827]	[-2.1873]	[-2.2989]	[-4.1275]	[-3.3579]