Prospect Theory for the Stock Market: Empirical Evidence with Time-Series Data

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Abstract

Consumption based asset pricing models have been shown to have difficulties to match financial market characteristics such as the risk-free interest rate and equity premium. Prospect theory seems to be a new way to solve such puzzles. Based on the theoretical backgrounds of the loss aversion model of asset pricing, we show some empirical evidence of the prospect theory for the stock market with time-series data. We first undertake estimation with a Markov-switching coefficient and then consider a time-varying coefficient with different states. The estimation results show that prior gains and losses may have asymmetric effects on investment behavior because of the break-even effect, which has been ignored by the asset-pricing model with the prospect theory.

JEL: C60, C61, D90;
Keywords: Prospect theory, Loss aversion, Markov-switching, House-money effect, break-even effect.

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1 Introduction

Consumption based asset pricing models with time separable preferences, such as power utility or log utility have been shown to have serious difficulty to match financial market characteristics such as risk-free interest rate, the equity premium and the Sharpe ratio to time series data.

Therefore, recent development of asset pricing studies has turned to extensions of intertemporal models conjecturing that the difficulties to match real and financial time series characteristics may be related to the simple structure of the basic model. In order to match better asset price characteristics of the model to data, economic research has extended the baseline stochastic growth model to include different utility functions, such as non-separable preferences for example by habit formation. Moreover, adjustment costs of investment have also been built into the model. Yet, models with habit formation or adjustment costs have been shown to improve equity premium and Sharpe ratio weakly, see Grüne and Semmler (2004) and Lettau and Uhlig (2000, 2002) for example. They do not generate enough co-variance of consumption growth with asset returns so as to match the data. Therefore, research in new directions is in great demand, and the work by Barberis et al. (2001) seems to be a good progress in this field.

The work by Barberis et al. (2001) is based on the so-called “prospect theory” developed by Kahneman and Tversky (1979) to explore decision under risk. According to this theory, economic agents are concerned with changes in wealth rather than with its final state. Moreover, economic agents are more sensitive to losses than to gains. That is, an economic agent is more painful with a loss than he feels happy with a gain which is of the same size of the loss. It is further found that economic agents’ utility function is concave with respect to gains and convex with respect to losses, implying that economic agents are risk-averse with respect to gains and risk-seeking with respect to losses. This phenomenon has been called “loss aversion”.

Another important concept and theory employed by Barberis et al. (2001) is the “house-money” effect explored by Thaler and Johnson (1990) in detail. The house-money effect implies that economic agents’ decision under risk can be affected by their gains and losses in the prior period. In case they have obtained some gains in the prior period, they would be more ready to accept risk in the current period. That is, they become (more) risk-seeking tomorrow in case they obtain some gains today. The effect of losses in the prior period, however, can be more complicated than that of gains. Although experiments in Kahneman and Tversky (1979) show that people may become (more) risk-averse in the next period if they have some losses in the current period, this is not always true. That is, in some cases people
may also become (more) risk-seeking in the next period even if they have some losses in the current period. This is described as the “break-even” effect. That is, even if they have some losses in the prior period, people may be ready to accept further risks and accept gambles which offer them a chance to break even. Therefore, the effect of losses in the current period on the decision in the next period may not be so clear as that of gains. Barberis et al. (2001) do not consider the break-even effect and just assume that people become (more) risk-seeking after gains and (more) risk-averse in the aftermath of losses. The break-even effect would make their modelling much more complicated.

An important question concerned is how to measure losses and gains. Barberis et al. (2001) measure losses or gains by the difference of returns from risky and risk-free assets. Barberis et al. (2001) show that loss aversion alone can not justify the high equity premium, large stock volatility and weak correlation between asset returns and consumption, and that these puzzles can be better explained by way of the house-money effect. In the case of house-money effect the economic agent has a time-varying risk-aversion which influences their investment behavior on the stock market from period to period. Employing a special dynamic programming algorithm, Grüne and Semmler (2005) study the model of Barberis et al. (2001) numerically and confirm the findings of Barberis et al. (2001).

While Barberis et al. (2001) and Grüne and Semmler (2005) explore this topic theoretically, we would like to explore some empirical evidence concerning the prospect theory and asset pricing. Although there exists some empirical research on prospect theory in the literature, most of them has been undertaken with micro-panel data. Moreover, empirical evidence with macro time-series data may provide more appropriate advice to policy makers. As mentioned above, prior gains and losses may affect decision in the current period. This implies that persistence of investment behavior on the stock market may be affected by returns in the past. Based on some theoretical backgrounds, we will explore some empirical evidence of persistence of investment behavior on the stock market. In particular, we will explore whether there is any break-even effect in investment behavior on the stock market, which is not considered by Barberis et al. (2001).

The remainder of this paper is organized as follows. In the second section we undertake some replications of traditional studies. Section 3 sketches the asset pricing model of Barberis et al. (2001). Section 4 undertakes the estimation with the US time-series data. Section 5 concludes the paper.
2 Replication of traditional studies

Below we will present a traditional stochastic dynamic general equilibrium model (SDGE) and explore its implications for asset prices. The representative agent wants to maximize the following objective function:

\[
\max_{(C_t,K^*_t)_{t=0}^\infty} E \left[ \sum_{t=0}^\infty \rho^t C_t^{1-\delta} - 1 \right] \]

s.t.

\[
C_t + K^*_t = W_t N^*_t + R_t K^*_{t-1} \\
N^*_t = 1
\]

where \( C_t \) denotes real consumption, \( 0 < \rho < 1 \) the time discount factor, \( \delta > 0 \) the coefficient of relative risk aversion. \( N^*_t \) is labor supply and \( K^*_t \) is supply of capital. \( R_t \) is the gross return from capital which can be stochastic and \( W_t \) the real wage of labor.

The representative firm is assumed to maximize the following objective function

\[
\max_{(K^d_{t-1},N^d_t)} Z_t (K^d_{t-1})^\tau (N^d_t)^{1-\tau} + (1 - \kappa)K^d_{t-1} - W_t N^d_t - R_t K^d_{t-1}
\]

where

\[
\log Z_t = (1 - \omega)\log \bar{Z} + \omega \log Z_{t-1} + \epsilon_t, \quad \epsilon_t \sim iidN(0,\sigma^2),
\]

where \( Z_t \) denotes the technology shock, \( K^d_{t-1} \) the demand for capital, \( N^d_t \) the demand for labor and \( \kappa \) the depreciation rate of capital. \( 0 < \omega < 1 \). We do not consider leisure in the utility function and just assume \( N^*_t \) to be 1 for simplicity.

Assuming the markets to clear,\(^1\) we have the following first-order conditions of the representative firm

\[
W_t = (1 - \tau)Z_t K^d_{t-1} N^{-\tau} \\
R_t = \tau Z_t K^d_{t-1} N^{1-\tau} + (1 - \kappa).
\]

and the following first-order conditions of the representative household

\[
C_t + K_t - W_t - R_t K_{t-1} = 0 \\
C^{-\eta}_t - \lambda_t = 0 \\
-\lambda_t + \rho E_{t}[\lambda_{t+1}R_{t+1}] = 0
\]

\(^1\)Namely,

\[
N^d_t = N^*_t, \quad K^d_t = K^*_t = K_t, \quad \text{and} \quad C_t + K_t = Z_t K^d_{t-1} + (1 - \kappa)K_{t-1}.
\]
with $\lambda_t$ being the Lagrangian multiplier.

Combining these two sets of first-order conditions, one then obtains:

\[
C_t = Z_t K_{t-1}^\tau + (1 - \kappa) K_{t-1} - K_t
\]
\[R_t = \tau Z_t K_{t-1}^{\tau - 1} + (1 - \kappa)\]
\[1 = E_t \left[ \rho \left( \frac{C_{t+1}}{C_t} \right)^{-\delta} R_{t+1} \right] \]
\[
\log Z_t = (1 - \omega) \log \bar{Z} + \omega \log Z_{t-1} + \epsilon_t, \quad \epsilon_t \sim iidN(0, \sigma^2).
\]

One can compute the steady states of $C_t$, $R_t$ and $K_t$ with eq. (1) to (4) and then log-linearize these nonlinear equations around the steady states of $R_t$, $C_t$, $K_t$ and $Z_t$.

After log-linearizing these equations, one can then estimate the linear systems with the maximum likelihood method, generalized methods of moments, simulated methods of moments or indirect inference. One can also change the structural form of these log-linearized system into a reduced form with the methods proposed by Uhlig (1999) for example. This kind of analysis is out of the scope of this paper, and therefore, we leave it to the readers who are interested in this analysis. Below we will focus on eq. (3) which has some implications of asset prices.

Let $r_t$ denote $\log R_t - \log \bar{R}$ and $c_t$ denote $\log C_t - \log \bar{C}$, with $\bar{R}$ being the steady state of $R_t$ and $\bar{C}$ the steady state of $C_t$. We show the log-linearization of (3) in the appendix. The log-linearized equation reads

\[
r_{t+1} = \mu + \delta \Delta c_{t+1} + \eta_{t+1},
\]

with $\Delta c$ denotes growth rate of $C$, namely $\Delta c_{t+1} = \log C_{t+1} - \log C_t$. The error term is given by $\eta_{t+1} = r_{t+1} - E_t[r_{t+1}] - \delta(\Delta c_{t+1} - E_t[\Delta c_{t+1}])$.

Because the firm is assumed to be owned by the household, the return from capital can be considered as the return from equity and bonds issued by the firm. Therefore, one can take $R_t$ as the return from assets. The relation between asset returns and consumption is shown in eq. (5).\(^2\)

Assuming rational expectation, one knows that $\eta_{t+1}$ has zero mean. Because $\eta_{t+1}$ is correlated with $\Delta c_{t+1}$, the OLS is not an appropriate method to estimate the above equation. Therefore, Campbell, Lo and Mackinlay (1997, ch. 8) estimate this equation with two-stage least squares (TSLS). A problem with the instrument variables (IV) estimation here is that the instruments used in the estimation are only weakly correlated with the independent variables and the standard error of

\(^2\)The reader is also referred to Campbell, Lo and Mackinlay (1997, ch. 8) who take a consumption-based asset pricing model.
the coefficient tends to be too small for testing the overidentifying restrictions of the model. In order to see whether the orthogonality conditions hold, one can then reverse eq. (5) and obtains:

$$\Delta c_{t+1} = \nu + \varphi r_{t+1} + \xi_{t+1}. \quad (6)$$

In case that the orthogonality conditions hold, $\varphi$ will asymptotically be the reciprocal of $\delta$. Campbell, Lo and Mackinlay (1997, ch. 8) estimated the above two equations with the annual US data from 1889 to 1994. The instrument variables they used are lags of the real commercial paper rate, the real consumption growth rate and the log dividend-price ratio. Their estimation result is not convincing, since both $\delta$ and $\varphi$ have a negative sign, with significant or insignificant t-statistics, inconsistent with theoretical backgrounds.

Next, we would like to undertake some similar estimation for several countries. We will use both TSLS and the generalized methods of moments (GMM) to estimate the above two equations as a system. The instruments we use here include the two lags of stock return (log difference of share price index), real (aggregate private) consumption growth rate, short-term interest rate or treasury bill rate and a constant.

The estimation results are shown in Table 1 with t-statistics in parentheses. The determinant residual covariance in all cases are almost zero. It is clear that the estimation with TSLS is not essentially different from that with GMM. $\delta$ and $\varphi$ in some cases have negative signs and in other cases, they are positive but have relatively insignificant t-statistics. These results are relatively consistent with the findings of Campbell, Lo and Mackinlay (1997, ch. 8), that is, one can not find significant relationship between stock return and the growth rate of real consumption.

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3Because dividend data are unavailable, we just take the growth rate of share price approximately as stock return.

4In this model it is quite difficult to find appropriate instrument variables. As mentioned above, Campbell, Lo and Mackinlay (1997, ch. 8) use the lags of the real commercial paper rate, the real consumption growth rate and the log dividend-price ratio as instrument variables. The data of real commercial paper rate and dividend for the countries to be studied are unavailable, therefore, we just use a monetary policy instrument, the short-term interest rate (German call rate, Japanese call rate, Italian discount rate for example) or 3-month treasury bill rate (US, UK and France) as an instrument instead. Monetary policy may affect both share price and consumption. Data sources: OECD, International Statistical Yearbook and IMF.

5J-statistics are also presented to see the validity of the over-identifying restrictions, since the number of orthogonality conditions is larger than the number of the parameters to estimate. The J-statistic reported here is the minimized value of the objective function in the GMM estimation. Hansen (1982) claims that $N \cdot J \xrightarrow{D} \chi^2(m - q)$, with $N$ being the sample size, $m$ the number of moment conditions and $q$ the number of parameters to be estimated.
Table 1: Estimation of Eq. (5)-(6)

<table>
<thead>
<tr>
<th>Country</th>
<th>$\mu$</th>
<th>$\delta$</th>
<th>$\nu$</th>
<th>$\varphi$</th>
<th>Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>0.046</td>
<td>-0.941</td>
<td>0.032</td>
<td>-0.137</td>
<td>1962.4-2004.1</td>
</tr>
<tr>
<td></td>
<td>(2.887)</td>
<td>(1.825)</td>
<td>(22.281)</td>
<td>(2.291)</td>
<td></td>
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<tr>
<td>UK</td>
<td>0.021</td>
<td>0.059</td>
<td>0.040</td>
<td>0.037</td>
<td>1962.4-1999.1</td>
</tr>
<tr>
<td></td>
<td>(1.285)</td>
<td>(0.161)</td>
<td>(13.423)</td>
<td>(0.426)</td>
<td></td>
</tr>
<tr>
<td>Italy</td>
<td>0.018</td>
<td>0.098</td>
<td>0.052</td>
<td>0.028</td>
<td>1970.4-1998.4</td>
</tr>
<tr>
<td></td>
<td>(0.729)</td>
<td>(0.242)</td>
<td>(16.262)</td>
<td>(0.475)</td>
<td></td>
</tr>
<tr>
<td>Germany</td>
<td>0.059</td>
<td>-1.749</td>
<td>0.027</td>
<td>-0.206</td>
<td>1970.4-1998.4</td>
</tr>
<tr>
<td></td>
<td>(2.394)</td>
<td>(1.788)</td>
<td>(8.956)</td>
<td>(1.963)</td>
<td></td>
</tr>
<tr>
<td>France</td>
<td>0.045</td>
<td>-0.646</td>
<td>0.041</td>
<td>-0.249</td>
<td>1970.3-1998.4</td>
</tr>
<tr>
<td></td>
<td>(3.733)</td>
<td>(2.083)</td>
<td>(11.797)</td>
<td>(3.020)</td>
<td></td>
</tr>
<tr>
<td>Japan</td>
<td>0.011</td>
<td>0.119</td>
<td>0.027</td>
<td>0.062</td>
<td>1962.4-2004.1</td>
</tr>
<tr>
<td></td>
<td>(1.132)</td>
<td>(0.433)</td>
<td>(12.010)</td>
<td>(0.929)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Country</th>
<th>$\mu$</th>
<th>$\delta$</th>
<th>$\nu$</th>
<th>$\varphi$</th>
<th>Sample</th>
<th>J-St.</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>0.067</td>
<td>-1.537</td>
<td>0.031</td>
<td>-0.163</td>
<td>1962.4-2004.1</td>
<td>0.097</td>
</tr>
<tr>
<td></td>
<td>(5.532)</td>
<td>(4.090)</td>
<td>(19.500)</td>
<td>(5.844)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>UK</td>
<td>0.028</td>
<td>0.052</td>
<td>0.035</td>
<td>0.056</td>
<td>1962.4-1999.1</td>
<td>0.086</td>
</tr>
<tr>
<td></td>
<td>(1.859)</td>
<td>(0.144)</td>
<td>(8.163)</td>
<td>(0.654)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Italy</td>
<td>0.003</td>
<td>0.256</td>
<td>0.052</td>
<td>0.029</td>
<td>1970.4-1998.4</td>
<td>0.095</td>
</tr>
<tr>
<td></td>
<td>(0.106)</td>
<td>(0.470)</td>
<td>(3.091)</td>
<td>(0.496)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Germany</td>
<td>0.059</td>
<td>-3.307</td>
<td>0.028</td>
<td>-0.251</td>
<td>1970.4-1998.4</td>
<td>0.251</td>
</tr>
<tr>
<td></td>
<td>(6.944)</td>
<td>(7.313)</td>
<td>(14.266)</td>
<td>(6.823)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>France</td>
<td>0.094</td>
<td>-1.965</td>
<td>0.042</td>
<td>-0.334</td>
<td>1970.3-1998.4</td>
<td>0.515</td>
</tr>
<tr>
<td></td>
<td>(7.768)</td>
<td>(6.334)</td>
<td>(12.258)</td>
<td>(4.042)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Japan</td>
<td>0.009</td>
<td>0.240</td>
<td>0.028</td>
<td>0.090</td>
<td>1962.4-2004.1</td>
<td>0.088</td>
</tr>
<tr>
<td></td>
<td>(0.836)</td>
<td>(0.879)</td>
<td>(6.821)</td>
<td>(1.651)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note that in the production-based as well as the consumption-based asset pricing models, consumption refers to total (private) consumption. In empirical tests, however, economists usually employ the data of consumption of nondurables and services, as in Campbell et al. (1997, ch. 8). Here we employ the data of total private consumption for estimation. The time series of these two sets of consumption are usually highly correlated. The growth rates of total consumption and the consumption of nondurables and services in the US are shown in Figure 1, with the correlation coefficient as high as 0.860 (sample 1962.1-1998.4). The estimation result with the consumption of nondurables and services is found relatively similar to that with total consumption. Therefore, in the research below we keep using the data of total consumption since the data of nondurables and services consumption have smaller samples.

3 Asset pricing model with prospect theory

In this section we would like to make a brief sketch of the loss aversion model in Barberis et al. (2001). The representative agent would like to maximize the following
objective function:

\[ E \left[ \sum_{t=0}^{\infty} \left( \rho^t \frac{C_t^{1-\delta}}{1-\delta} + b_t \rho^{t+1} \nu(X_{t+1}, A_t, z_t) \right) \right], \quad (7) \]

where \( C_t \) is real consumption, \( \rho \) the discount factor and \( \delta \) the parameter of relative risk aversion. The second term in the parentheses stands for the effect of the change of wealth on the agent’s welfare. \( X_{t+1} \) is the change of wealth and \( A_t \) the value of the agent’s risky assets. \( z_t \) is a variable measuring the agent’s gains or losses prior to time \( t \) as a fraction of \( A_t \). Barberis et al. (2001) assume that \( X_{t+1} = A_t R_t + 1 - A_t R_{f,t} \), with \( R_t + 1 \) being the gross return of risky assets between time \( t \) and \( t+1 \), and \( R_{f,t} \) the gross return of risk-free assets between time \( t \) and \( t+1 \). The difference \( R_t + 1 - R_{f,t} \) can be positive, zero or negative and the variable \( z_t \) can be greater, equal or smaller than one. They further assume that in case \( z_t \leq 1 \)

\[ \nu(X_{t+1}, A_t, 1) = \begin{cases} A_t R_{t+1} - A_t R_{f,t} & \text{for } R_{t+1} \geq z_t R_{f,t} \\ A_t (z_t R_{f,t} - R_{f,t}) + \lambda A_t (R_{t+1} - z_t R_{f,t}) & \text{for } R_{t+1} < z_t R_{f,t} \end{cases}, \quad (8) \]

with \( \lambda > 1 \) and

\[ \nu(X_{t+1}, A_t, 1) = \begin{cases} X_{t+1} & \text{for } X_{t+1} \geq 0 \\ \lambda(z_t) X_{t+1} & \text{for } X_{t+1} < 0 \end{cases}. \quad (9) \]
in case $z_t > 1$ with $\lambda(z_t)$ defined by

$$
\lambda(z_t) = \lambda + k(z_t - 1),
$$

indicating the fact that a loss is more severe than a gain with $k > 0$, and

$$
z_{t+1} = \eta z_t \frac{\bar{R}}{R_{t+1}} + 1 - \eta
$$

with $\eta \in [0, 1]$ and $\bar{R}$ a fixed parameter which is chosen to be the long time average of the risk free interest rate. Moreover, it is presumed that

$$
b_t = b_0 \tilde{C}_t^{-\delta},
$$

with $b_0$, a scaling factor, and $\tilde{C}_t$ some aggregate consumption which will be specified below, so that the price-dividend ratio and the risky asset premium remain stationary. Hereby $b_0$ is an important parameter indicating the relevance that financial wealth has in utility gains or losses relative to consumption. In case $b_0 = 0$, we recover the consumption based asset pricing model with power utility.

Barberis et al. (2001) employ such a model of loss aversion and asset pricing to two stochastic variants of an endowment economy without production. In the first model variant there is only one stochastic pay-off for the asset holder, a stochastic dividend, whereby dividend pay-offs are always equal to consumption. In the other model variant dividends and consumption follow different stochastic processes. The Euler equation for the risky asset reads:

$$
1 = \rho E_t \left[ R_{t+1}(\tilde{C}_{t+1}/\tilde{C}_t)^{-\delta} \right] + b_0 \rho E_t [\tilde{\nu}(R_{t+1}, z_t)]
$$

with

$$
\tilde{\nu}(R_{t+1}, z_t) = \begin{cases} 
R_{t+1} - R_{f,t}, & R_{t+1} \geq z_t R_{f,t} \text{ and } z_t \leq 1 \\
(z_t - 1)R_{f,t} + \lambda(R_{t+1} - z_t R_{f,t}), & R_{t+1} < z_t R_{f,t} \text{ and } z_t \leq 1 \\
R_{t+1} - R_{f,t}, & R_{t+1} \geq R_{f,t} \text{ and } z_t > 1 \\
\lambda(z_t)(R_{t+1} - R_{f,t}), & R_{t+1} < R_{f,t} \text{ and } z_t > 1
\end{cases}
$$

We can now compare the two Euler equations (13) and (3). The first term of (13) is the same as (3). The difference is that (13) has an additional term $b_0 \rho E_t [\tilde{\nu}(R_{t+1}, z_t)]$ which expresses the effects of changes in the financial assets on the agent’s welfare. As can be seen from eq.(13), the Euler equation has now six variables, $R_{t+1}, C_{t+1}, C_t, R_{f,t}, \tilde{C}_t$ and $z_t$ which is a function of $R_{t-1}$ and $z_{t-1}$.
Considering the cases in eq. (14) separately, one sees that for each single case the right hand side of eq. (13) is affinely linear in $R_{t+1}$. More precisely, one can rewrite eq. (13) as

$$1 + \rho b_0 \alpha_2 R_{f,t} = E_t \left[ \left( \rho \left( \tilde{C}_{t+1}/\tilde{C}_t \right)^{-\gamma} + b_0 \alpha_1 \right) R_{t+1} \right]$$  \hspace{1cm} (15)$$

with $\alpha_1$ and $\alpha_2$ given by

$$\begin{align*}
\alpha_1 &= 1, \quad \alpha_2 = 1 \quad \text{for } R_{t+1} \geq z_t R_{f,t} \text{ and } z_t \leq 1 \\
\alpha_1 &= \lambda, \quad \alpha_2 = (\lambda - 1) z_t + 1 \quad \text{for } R_{t+1} < z_t R_{f,t} \text{ and } z_t \leq 1 \\
\alpha_1 &= 1, \quad \alpha_2 = 1 \quad \text{for } R_{t+1} \geq R_{f,t} \text{ and } z_t > 1 \\
\alpha_1 &= \lambda(z_t), \quad \alpha_2 = \lambda(z_t) \quad \text{for } R_{t+1} < R_{f,t} \text{ and } z_t > 1 
\end{align*}$$  \hspace{1cm} (16)$$

Using the equation

$$R_{t+1} = P_{t+1} + D_{t+1}/P_t$$

for the risky return with $P_t$ denoting the asset price and $D_t$ the dividend, which is chosen equal to $\tilde{C}_t$ in Grüne and Semmmler (2005) and plugging this equation into eq. (15) and using eq. (14) we obtain

$$P_t = E_t \left[ \frac{\rho \left( \tilde{C}_{t+1}/\tilde{C}_t \right)^{-\gamma} + b_0 \alpha_1}{1 + \rho b_0 \alpha_2 R_{f,t}} \left( \tilde{C}_{t+1} + P_{t+1} \right) \right]$$  \hspace{1cm} (17)$$

Taking $\tilde{C}_t$ as the aggregate consumption from the growth model of Brock and Mirman (1972), Grüne and Semmmler (2005) solve the above problem numerically with a special dynamic programming algorithm and find that the loss aversion model can explain the equity-premium puzzle better than models with habit formation and adjustment costs studied by Boldrin et al. (2001) and Jerman (1998), for example. To be precise, They obtain the $\tilde{C}_t$ from the following model

$$\begin{align*}
    \text{Max } \tilde{C}_t & \left( \sum_{t=0}^{\infty} \rho^t \frac{\tilde{C}_t^{1-\delta}}{1-\delta} \right) \\
\text{s.t.} & \\
    k_{t+1} &= y_t A k_t^\alpha - \tilde{C}_t \\
    \ln y_{t+1} &= \sigma \ln y_t + \epsilon_t
\end{align*}$$

10
with $\epsilon_t$ being an i.i.d. random variable. Taking $A = 5$, $\alpha = 0.34$, $\sigma = 0.9$, $\rho = 0.95$, $\delta = 1$, $\lambda = 10$, $\eta = 0.9$, $b_0 = 1$, $k = 3$, and assuming the SD of $\epsilon_t$ as 0.008, they have the following results (SR stands for the Sharpe-ratio):

Table 2: Numerical Result of Gr"une and Semmler (2005)

<table>
<thead>
<tr>
<th>$R_f$</th>
<th>$\text{Var}(R_f)$</th>
<th>$R_{t+1}$</th>
<th>$\text{Var}(R_{t+1})$</th>
<th>SR</th>
<th>$\text{Cov}(m_{f,t+1}, R_{t+1})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.05273</td>
<td>0.00001</td>
<td>1.05718</td>
<td>0.00941</td>
<td>0.47440</td>
<td>-0.00007</td>
</tr>
</tbody>
</table>

Gr"une and Semmler (2005) also undertake the numerical computation by varying the values of the parameters mentioned above and find some similar results. Taking $\rho = 0.98$ and leaving other parameters unchanged, for example, they find $R_f = 1.02453$ and $SR = 0.45096$.

While Gr"une and Semmler (2005) solve the loss aversion model numerically with a dynamic programming algorithm, below we would like to undertake some estimation of the model with actual data. To be precise, we would like to estimate the Euler equation (13). In principle, eq. (13) may also be log-linearized, as has been done for eq. (5). The problem is that eq. (13) has break points and, therefore, it is difficult to obtain a uniform expression of the log-linearized equation. However, eq. (13) at least tells us that $R_{t+1}$ is a function of $C_t$, $C_{t+1}$, $R_t$, $R_{f,t}$, $\tilde{C}_t$ and $z_t$. Therefore, one knows that

$$E_t F(R_{t+1}, C_t, C_{t+1}, R_t, R_{f,t}, \tilde{C}_t, z_t) = 0,$$

with $F(\bullet)$ being a measurable function. The most simplified assumption is, of course, that $r_{t+1}$ is a linear function of the variables in (18). In the research below, we just follow this simplification and assume that

$$r_t = \alpha + \beta \Delta c_t + \gamma r_{t-1} + \epsilon_t,$$

where $\epsilon_t$ is an i.i.d. noise with zero mean and constant variance. $r_t$ and $\Delta c_t$ have the same meanings as in eq. (5). Note that, $\epsilon_t$ may contain expectation errors which may be correlated with $\Delta c_t$, as discussed in section 2.

---

6Theoretically, the risk-free rate is also an independent variable in this equation. We tried the estimation with the GMM and found that the risk-free rate usually has a relatively insignificant t-statistic, although with a right sign. It is $-0.088$ with the t-st. being only 0.472 in the case of the US, for example. Therefore, in the estimation below we will not consider the effect of the risk free rate.
4 Empirical evidence of the prospect theory

Above we have made a brief sketch of the loss aversion model in Barberis et al. (2001) and explored the difference between the first-order conditions of the traditional model which considers only consumption in the utility function and the loss aversion model which considers also effects of wealth changes on welfare. A very important point of the loss aversion model with house-money effect is that the return from stock in the previous period may affect the agent’s risk aversion in the current period and, therefore, affects the investment behavior on the stock market, which in turn influences return from stock.

Barberis et al. (2001) assume that people become (more) risk-seeking in the aftermath of gains and (more) risk-averse with respect to prior losses. Applying this assumption to stock market, the following seems to hold: If the share price increases in the prior period, people may enlarge their investment in stocks and reinforce the booms and, as a result, stock prices may increase further. If the share price decreases, however, people become (more) risk-averse and will probably decrease their investment in stocks. This implies that the demand for stocks will decrease in the aftermath of losses, and as a result, stock prices may decrease further because of withdraw in stock investment. This kind of investment behavior will then enlarge volatility in asset prices and returns. Moreover, there should be a strong correlation between returns in the current period and in the next period. In the simplified equation (19) this may be expressed by the coefficient $\gamma$. To be precise, one would expect $\gamma$ to be relatively significant. Next, we will estimate $\gamma$ for several countries.

4.1 Some preliminary evidence

In this section we estimate eq. (19) with the GMM with the same instruments as for eq. (5)-(6): The estimation results are shown in Table 3 with t-statistics in parentheses. The covariance of $r_t$ and $\Delta c_t$ is also shown there.

It is clear that $\beta$ has a negative sign in most cases or a positive sign with insignificant t-statistics in others. The covariance of $r_t$ and $\Delta c_t$ is relatively low in all cases. On the contrary, $\gamma$ always has significant t-statistics with a positive sign. This supports the argument of Barberis et al. (2001) mentioned above. That is, investment in stocks may be greatly affected by stock returns in the past and as a result, returns from stocks may be greatly affected by their lags rather than by consumption.
Table 3: Estimation of Eq. (19)

<table>
<thead>
<tr>
<th>Country</th>
<th>$\beta$</th>
<th>$\gamma$</th>
<th>J-st.</th>
<th>$\text{Cov}(r_t, \Delta c_t)$</th>
<th>Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>-1.492</td>
<td>0.275</td>
<td>0.031</td>
<td>$3.88 \times 10^{-5}$</td>
<td>62.3-04.1</td>
</tr>
<tr>
<td></td>
<td>(2.422)</td>
<td>(3.439)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>UK</td>
<td>-0.438</td>
<td>0.258</td>
<td>0.033</td>
<td>$1.62 \times 10^{-4}$</td>
<td>62.3-99.1</td>
</tr>
<tr>
<td></td>
<td>(1.188)</td>
<td>(3.412)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>France</td>
<td>-0.472</td>
<td>0.169</td>
<td>0.063</td>
<td>-$1.06 \times 10^{-3}$</td>
<td>65.2-98.4</td>
</tr>
<tr>
<td></td>
<td>(1.279)</td>
<td>(3.886)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Germany</td>
<td>-1.988</td>
<td>0.105</td>
<td>0.027</td>
<td>-$1.57 \times 10^{-4}$</td>
<td>70.3-98.4</td>
</tr>
<tr>
<td></td>
<td>(2.156)</td>
<td>(1.816)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Italy</td>
<td>0.616</td>
<td>0.444</td>
<td>0.046</td>
<td>-$3.40 \times 10^{-5}$</td>
<td>70.2-98.4</td>
</tr>
<tr>
<td></td>
<td>(1.527)</td>
<td>(8.392)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Japan</td>
<td>0.034</td>
<td>0.343</td>
<td>0.040</td>
<td>$1.37 \times 10^{-4}$</td>
<td>62.3-04.1</td>
</tr>
<tr>
<td></td>
<td>(0.139)</td>
<td>(7.356)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4.2 Markov-regime changes in the coefficient: break-even effect?

As mentioned above, prospect theory argues that people’s investment behavior can be affected by their prior gains and losses. In case they have obtained some gains in the prior period, they would be more ready to accept risk in the current period. The effect of losses in the prior period, however, can be more complicated than that of gains. Kahneman and Tversky (1979) show that although in many cases people become (more) risk-averse in the next period if they have some losses in the current period, this is not always true. That is, they find that in some other cases people may become (more) risk-seeking in the next period even if they have some losses in the current period because of the break-even effect. That is, prior losses may induce risk-seeking if they fail to adapt to losses or to attain an expected gain. This is also clearly stated by Thaler and Johnson (1990, p.658) as follows:

Thus, while an initial loss may induce risk aversion for some gambles, other gambles, which offer the opportunity to break even, will be found acceptable.

Thaler and Johnson (1990, p.659) further mention the implication of the break-even effect in investment behavior as follows

Also, when options present the opportunity to “break even”, tendencies toward risk-seeking in the domain of losses might be enhanced. Thus, we would expect investments in failing enterprises to be particularly
prevailing when there is a hope, however dim, that one might eradicate existing losses. ...

Another potential domain for applying this research is in the study of investment behavior. The break-even effect suggests that individuals are averse to closing an account that shows a loss. This aversion can produce a reluctance to sell securities that have declined in value.

Therefore, the prior losses in stocks may have two kinds of effects (in opposite directions!) on the current investment behavior on the stock market. On the one hand, prior losses may induce risk aversion and make people decrease their demand for stocks and, as a result, strengthen the decreases in stock prices. On the other hand, in case there is a break-even effect, prior losses may induce risk seeking. In this case, people will not withdraw their investment in stocks and may even enlarge their demand for stocks. This break-even effect will obviously prevent the stock prices from falling further or slow down the decreases in stock prices. Therefore, how prior losses affect the investment behavior in stocks depends on which of the two above mentioned effects in opposite directions dominates. In case neither effect dominates, the effect of prior losses on the investment behavior may not be clear.

In short, the effects of prior gains on the investment behavior can be different from those of prior losses. While the former may induce a further increase in stocks, the latter may have some unclear effects.

Barberis et al. (2001), however, assume that people become (more) risk-seeking after gains and (more) risk-averse in the aftermath of losses, and do not consider the break-even effect. In their analysis, the effects of prior gains and losses on investment behavior seem to be symmetric. The break-even effect would, of course, make their modelling much more complicated. In the estimation above, we have followed the assumption of Barberis et al. (2001) and did not separate the effects of prior losses from those of prior gains, and found that $\gamma$ is relatively significant in the general case. Next, we will explore some empirical evidence of the break-even effect, to see whether the effects of prior gains are different from those of prior losses. To be precise, we will explore whether $\gamma$ in eq. (19) experiences regime changes in the cases of positive returns and negative returns of stocks.

Now, eq. (19) is modified as

$$r_t = \alpha + \beta \Delta c_t + \gamma S_t r_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma_{\varepsilon_t}^2),$$

(20)

with $S_t$ denoting the state of the stock markets. Here for simplicity we just assume two states, namely, $S_t = 0$ and $S_t = 1$. Moreover, we assume $S_t$ has a Markov-
switching process with the following transitional probabilities:

\[ Pr[S_t = 1|S_{t-1} = 1] = p \quad (21) \]
\[ Pr[S_t = 0|S_{t-1} = 0] = q. \quad (22) \]

One can employ the maximum likelihood estimation method to estimate the parameters. In the model above we assume that \( S_t \) is not observed. Therefore, we have to figure out the log likelihood function. Kim and Nelson (1999, ch. 4) shows that the log likelihood function can then be explored as follows. Let \( \psi_t \) denote the information up to \( t \) and \( f(.) \) the density function, the log likelihood function reads

\[
\ln L = \sum_{t=1}^{T} \ln \{ \sum_{s_t=0}^{1} f(r_t|S_t, \psi_t) Pr[S_t|\psi_t] \}
\]

with

\[
\sum_{s_t=0}^{1} f(r_t|S_t, \psi_t) Pr[S_t|\psi_{t-1}]
\]

\[
= \frac{1}{\sqrt{2\pi}\sigma_0^2} \exp \left( -\frac{(r_t - \alpha - \beta \Delta c_t - \gamma_0 r_{t-1})^2}{2\sigma_0^2} \right) Pr[S_t = 0|\psi_{t-1}]
\]

\[
+ \frac{1}{\sqrt{2\pi}\sigma_1^2} \exp \left( -\frac{(r_t - \alpha - \beta \Delta c_t - \gamma_1 r_{t-1})^2}{2\sigma_1^2} \right) Pr[S_t = 1|\psi_{t-1}] \quad (23)
\]

where

\[
Pr[S_t = j|\psi_{t-1}] = \sum_{i=0}^{1} Pr[S_t = j, S_{t-1} = i|\psi_{t-1}]
\]

\[
= \sum_{i=0}^{1} Pr[S_t = j|S_{t-1} = i] Pr[S_{t-1} = i|\psi_{t-1}], \quad i, j = 0, 1
\]

In order to obtain \( Pr[S_{t-1} = i|\psi_{t-1}] \), one should compute the following probability

\[
Pr[S_t = j|\psi_t] = Pr[S_t = j|\psi_{t-1}, r_t] = \frac{f(S_t = j, r_t|\psi_{t-1})}{f(r_t|\psi_{t-1})}
\]

\[
= \frac{f(r_t|S_t = j, \psi_{t-1}) Pr[S_t = j|\psi_{t-1}]}{\sum_{j=0}^{1} f(r_t|S_t = j, \psi_{t-1}) Pr[S_t = j|\psi_{t-1}]},
\]

where \( \psi_t = \{\psi_{t-1}, r_t\} \). Running the above iteration from period to period, one then obtains the time-varying probability of being in each state. In order to start
the iteration, one needs starting values for $Pr[S_t = j|\psi_t]$ (j=0,1). The steady-state probabilities of $S_t$ are taken as the starting values, namely, $Pr[S_0 = 0|\psi_0] = \frac{1-p}{2-p-q}$ and $Pr[S_0 = 1|\psi_0] = \frac{1-q}{2-p-q}$. The estimation with the quarterly US data from 1962.1 to 2004.1 is shown in Table 4 and Figure 2.\footnote{One may argue that the maximum-likelihood estimation method may not be the best way to estimate the equation above, since the noise $\varepsilon_t$ may be correlated with $\Delta c_t$, and the estimate of $\beta$ may not be consistent in this case. But as mentioned above, the most interesting parameter here is $\gamma$, not $\beta$, therefore, this would not make much trouble in interpreting the results.}

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\gamma_0$</th>
<th>$\gamma_1$</th>
<th>$\sigma_0$</th>
<th>$\sigma_1$</th>
<th>$p$</th>
<th>$q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate</td>
<td>0.025</td>
<td>-0.444</td>
<td>-0.061</td>
<td>0.375</td>
<td>0.096</td>
<td>0.043</td>
<td>0.750</td>
<td>0.0003</td>
</tr>
</tbody>
</table>


It is clear that $\beta$ has the wrong sign and an insignificant t-statistic, consistent with the estimate with the GMM. $\gamma$ is much lower in state 0 than in state 1 and, moreover, it has an insignificant t-statistic in state 0 (0.156). In state 1 it has a very significant t-statistic (5.612). What does state 0 or state 1 stand for? We may get some hints from Figure 2. Figure 2A is the return of share and Figure 2B is the probability of being in state 0, namely $Pr[S_t = 0|\psi_t]$. We find that if $r_t$ is positive (or in boom), $Pr[S_t = 0|\psi_t]$ is usually low, and if $r_t$ is negative (or in recession), $Pr[S_t = 0|\psi_t]$ is usually high. This indicates that boom is consistent with state 1 and recession is consistent with state 0. Therefore, $\gamma_1$ should be interpreted as the coefficient in a boom market and $\gamma_0$ the coefficient in a recession. This evidence supports the above statement that the effects of prior gains and losses may be asymmetric. Prior gains will enlarge investment in stocks, while the effects of prior losses may be unclear because of the break-even effect.

Barberis et al. (2001) measure losses or gains in wealth by the difference from stock return and bond return (or the equity premium). A higher equity premium in the prior period may induce people to enlarge their investment in stocks and the opposite holds with a lower one in the case of no break-even effect. In the above estimation we have just assumed the transition probabilities to be constant for simplicity, namely, $Pr[S_t = 1|S_{t-1} = 1] = p$ and $Pr[S_t = 0|S_{t-1} = 0] = q$. Next, we would like to explore the effects of the equity premium on the transition probability. Therefore, in the research below we assume that the transition probabilities are not constant but time-varying. To be precise, we assume that they are affected by
the gap between the stock return and bond’s return in the previous period, \( g_{t-1} \). Formally we have

\[
Pr[S_t = 1 | S_{t-1} = 1, g_{t-1}] = p(t) = \frac{\exp(\lambda_1 + \lambda_2 g_{t-1})}{1 + \exp(\lambda_1 + \lambda_2 g_{t-1})}
\]

\[
Pr[S_t = 0 | S_{t-1} = 0, g_{t-1}] = q(t) = \frac{\exp(\lambda_3 + \lambda_4 g_{t-1})}{1 + \exp(\lambda_3 + \lambda_4 g_{t-1})}.
\]

The effect of \( g_{t-1} \) on \( p(t) \) can be seen from follows

\[
\frac{dp(t)}{dg_{t-1}} = \frac{\lambda_2 \exp(\lambda_1 + \lambda_2 g_{t-1})}{[1 + \exp(\lambda_1 + \lambda_2 g_{t-1})]^2} \begin{cases} > 0, & \text{if } \lambda_2 > 0; \\ \leq 0, & \text{if } \lambda_2 \leq 0. \end{cases}
\]

If \( \lambda_2 \) is positive, a higher \( g_{t-1} \) may increase the probability of being in state 1 in the next period, given that the stock market is in state 1 in the current period, and vice versa. The effect of \( g_{t-1} \) on \( q(t) \) can be explored similarly. The estimation of eq. (20) with time-varying transition probabilities with the US quarterly data is shown in Figure 3 and Table 5. \( \bar{r} \) is the 3-month treasury bill rate.

---

8Below we prefer the gap between returns from stocks and bonds to the equity premium because we do not consider dividends from stocks when computing return from stocks.
Figure 3 looks quite similar to Figure 2. That is, we also find that $Pr[S_t = 0|\psi_t]$ is low when $r_t$ is positive (boom) and vice versa, implying that state 0 is recession and state 1 is boom. $\beta$ still has a wrong sign with the t-st. being 1.554. $\gamma_0$ is $-0.095$ with t-st. being as low as 0.229 and $\gamma_1$ is 0.362 with t-st. being as significant as 5.948. The estimation result is consistent with that of constant transition probabilities: the effects of prior gains and losses on the investment behavior are asymmetric, one may not ignore the break-even effect on the stock market.

From Table 5 we see that $\lambda_2$ is positive with t-st. being 2.234. This implies that $g_{t-1}$ has significant positive effects on $p(t)$. This indicates that the higher the return from shares relative to that from bonds, the higher the probability for the stock market to be in boom in the next period, given that it is also in boom in the current period. This also supports the idea that people may become (more) risk-seeking after gains. In contrast to the case of positive returns, $\lambda_4$ is negative with a relatively insignificant t-statistic, implying that $g_{t-1}$ might not have much effect on $q(t)$. This seems to support our earlier result that prior losses may not always induce risk-aversion because of the break-even effect.

We present the time-varying transition probabilities in Figure 4. Figure 4A is the gap between the return from shares and from bonds. Figure 4B is the path of
Table 5: Estimation of Eq. (20) with Time-Varying Transition Probabilities

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\gamma_0$</th>
<th>$\gamma_1$</th>
<th>$\sigma_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate</td>
<td>0.027</td>
<td>-0.524</td>
<td>-0.095</td>
<td>0.363</td>
<td>0.102</td>
</tr>
<tr>
<td></td>
<td>(2.595)</td>
<td>(1.554)</td>
<td>(0.229)</td>
<td>(5.948)</td>
<td>(6.038)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\sigma_1$</th>
<th>$\lambda_1$</th>
<th>$\lambda_2$</th>
<th>$\lambda_3$</th>
<th>$\lambda_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate</td>
<td>0.042</td>
<td>2.377</td>
<td>21.332</td>
<td>-38.028</td>
<td>-18.938</td>
</tr>
<tr>
<td></td>
<td>(12.376)</td>
<td>(2.742)</td>
<td>(2.234)</td>
<td>(0.038)</td>
<td>(0.018)</td>
</tr>
</tbody>
</table>

$p(t)$ ($[Pr[S_t = 1|S_{t-1} = 1, g_{t-1}]])$, and Figure 4C is the path of $q(t)$ ($[Pr[S_t = 0|S_{t-1} = 0, g_{t-1}]])$. It is clear that $g_{t-1}$ shows significant effects on $p(t)$, which increases as $g_{t-1}$ increases and decreases when $g_{t-1}$ decreases. $q(t)$ is almost constant and has not been affected significantly by $g_{t-1}$. This is consistent with the estimates of $\lambda_2$ and $\lambda_4$.

4.3 Time-varying coefficient in a state-space model with Markov-switching

In the previous section we have estimated the coefficient $\gamma$ with Markov-regime changes and found that the effects of prior gains and losses on the investment behavior are asymmetric. In this section we would like to explore this problem further and explore time-varying paths of $\gamma$. We will employ a state-space model with Markov-switching as follows:

$$r_t = \alpha + \beta \Delta c_t + \gamma S_t r_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma_{S_t}^2)$$

$$\gamma_{S_t} = \gamma_{S_{t-1}} + \phi \gamma_{S_{t-1}} + \eta_t, \quad \eta_t \sim N(0, \varrho^2)$$

with $E(\varepsilon_t \eta_t) = 0$. As in the previous section $\gamma$ may be different across states 0 and 1, and at the same time, $\gamma$ may also change over time and follows a path proposed by eq. (28). We also assume time-varying transition probabilities as shown in eq. (24) and (25). Next, we will estimate the time-varying $\gamma$ by way of the Kalman filter.\(^9\)

Let $\psi_{t-1}$ denote the vector of observations available as of time $t-1$. In the usual derivation of the Kalman filter in a State-Space model without Markov-Switching, the forecast of $\gamma_t$ based on $\psi_{t-1}$ can be denoted by $\gamma_{0|t-1}$. Similarly, the matrix

\(^9\)Theoretically, $\phi$ and $\varrho$ may also have Markov property. We will not consider this possibility because there are already 12 parameters to estimate and the efficiency of estimation may be reduced if more parameters are estimated.
Figure 4: Time-Varying Transition Probabilities

A: Gap Between Returns from Share and Returns from Bond

B: Pr(St=1|St-1=1; g(-1))

C: Pr(St=0|St-1=0; g(-1))

$\times 10^{-10}$
denoting the mean squared error of the forecast can be written as

\[ P_{t|t-1} = E[(\gamma_t - \gamma_{t|t-1})(\gamma_t - \gamma_{t|t-1})']|\psi_{t-1}], \]

where \( E \) is the expectation operator.

In the State-Space model with Markov-switching, the goal is to form a forecast of \( \gamma_t \) based not only on \( \psi_{t-1} \) but also conditional on the random variable \( S_t \) taking on the value \( j \) and on \( S_{t-1} \) taking on the value \( i \) (\( i \) and \( j \) equal 0 or 1):

\[ \gamma_{t|i,j} = E[\gamma_t|\psi_{t-1}, S_t = j, S_{t-1} = i], \]

and correspondingly the mean squared error of the forecast is

\[ P_{t|i,j} = E[(\gamma_t - \gamma_{t|t-1})(\gamma_t - \gamma_{t|t-1})']|\psi_{t-1}, S_t = j, S_{t-1} = i]. \]

Conditional on \( S_{t-1} = i \) and \( S_t = j \) (\( i, j = 0, 1 \)), the Kalman filter algorithm for our model is as follows:

\[ \gamma_{t|i,j} = \hat{\gamma}_j + \phi \gamma_{t-1|i,j}, \]  
\[ P_{t|i,j}^{(i,j)} = \phi^2 P_{t-1|i,j} + \vartheta^2, \]  
\[ \xi_{t|i,j}^{(i,j)} = r_t - \alpha - \beta \Delta c_t - r_{t-1}\gamma_{t|i,j}, \]  
\[ \nu_{t|i,j}^{(i,j)} = r_{t-1}^2 P_{t-1|i,j} + \sigma_j^2, \]  
\[ \gamma_t = \gamma_{t|i,j}^{(i,j)} + P_{t|i,j}^{(i,j)} [\nu_{t|i,j}^{(i,j)}]^{-1} \xi_{t|i,j}^{(i,j)}, \]  
\[ P_{t|i,j}^{(i,j)} = (1 - P_{t|i,j}^{(i,j)} r_{t-1}^2 [\nu_{t|i,j}^{(i,j)}]^{-1}) P_{t|i,j}^{(i,j)}, \]

where \( \xi_{t|i,j}^{(i,j)} \) is the conditional forecast error of \( r_t \) based on information up to time \( t-1 \) and \( \nu_{t|i,j}^{(i,j)} \) is the conditional variance of the forecast error \( \xi_{t|i,j}^{(i,j)} \). It is clear that \( \nu_{t|i,j}^{(i,j)} \) consists of two parts \( r_{t-1}^2 P_{t|i,j}^{(i,j)} \) and \( \sigma_j^2 \). When there is no Markov-Switching property in the shock variance, \( \sigma_j^2 \) is constant. In order to make the Kalman filter algorithm above operable, Kim and Nelson (1999) developed some approximations and managed to collapse \( \gamma_{t|i,j}^{(i,j)} \) and \( P_{t|i,j}^{(i,j)} \) into \( \gamma_t^{(i,j)} \) and \( P_{t|i,j}^{(i,j)} \) respectively.\(^{10}\) Zhang and Semmler (2005) employ a similar method to explore model and shock uncertainty with the US data. In the estimation below, however, we assume time-varying transition probabilities to explore whether the gap between the return from stock and

\(^{10}\) As for the details of the State-Space model with Markov-Switching, the reader is referred to Kim and Nelson (1999, ch. 5). The procedure applied below is modified from the Gauss procedures developed by Kim and Nelson (1999).
that from bonds has any effect on the investment behavior on the stock market.

The estimation results with the quarterly US data are shown in Table 6, with the likelihood function being 252.630.

Table 6: Estimation of Eq. (27)-(28)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\sigma_0$</th>
<th>$\sigma_1$</th>
<th>$\varphi$</th>
<th>$\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate</td>
<td>$(t-\text{st.})$</td>
<td>0.044</td>
<td>-0.964</td>
<td>0.099</td>
<td>0.041</td>
<td>0.00004</td>
</tr>
<tr>
<td>$\bar{\gamma}_0$</td>
<td>0.299</td>
<td>0.093</td>
<td>-0.016</td>
<td>0.429</td>
<td>2.449</td>
<td>26.500</td>
</tr>
<tr>
<td>$\bar{\gamma}_1$</td>
<td>(1.849)</td>
<td>(0.758)</td>
<td>(12.060)</td>
<td>(0.035)</td>
<td>(2.853)</td>
<td>(2.435)</td>
</tr>
</tbody>
</table>

The difference between $\bar{\gamma}_0$ and $\bar{\gamma}_1$ is obvious. It is about 0.3 with t-st. being 1.849 in state 0 and only 0.093 with t-st. being 0.758 in state 1. $\lambda_2$ has a relatively insignificant t-statistic, indicating that $g_{t-1}$ has little effect on the transition probability $p(t)$. $\lambda_4$, however, has a positive sign and a relatively significant t-statistic 2.435. This indicates that the gap between the stock return and bonds return affects the transition probability $q(t)$ positively. That is, in case the stock market is in state 0 in the current period, a higher $g_t$ indicates a higher probability for the stock market to be also in state 0 in the next period. This is consistent with the conclusion in the previous section in case that state 0 stands for boom and state 1 stands for recession in the stock market. We can explore briefly what state 1 and state 0 stand for from Figure 5.

Figure 5A is the time-varying $\gamma$ in states 0 and 1 over time. It is clear that $\gamma$ is relatively stationary and experiences relatively small changes in state 1. It experiences some significant changes in state 0, however. Figure 5B is the probability of being in state 0 given the information up to time t, namely, $Pr[S_t = 0|\psi_t]$. Figure 5C is the stock return. Comparing Figure 5B and Figure 5C, we find that most of the time $Pr[S_t = 0|\psi_t]$ is high when $r_t$ is positive and vice versa. This should indicate that state 0 stands for boom and state 1 for recession in the stock market. As a result, $\gamma_0$ is the reaction coefficient in boom and $\gamma_1$ in recession. As seen in Figure 5A, $\gamma_0$ is relatively higher than $\gamma_1$, implying that prior gains may have induced investment enlargement in stocks, while prior losses may not have induced significant withdraw of investment in stocks. This can also be seen from the mean of $\gamma_0$ and $\gamma_1$, with the former being 0.560 ($=0.299/(1-0.466)$) and the latter being only 0.174 ($=0.093/(1-0.466)$). This conclusion is consistent with that in the previous sections.
Figure 5: Time-Varying $\gamma$ across States
4.4 Empirical evidence of Japan

Next, we will show some empirical evidence with the data of Japan (1972.1-2004.2). The estimation of eq. (20) is shown in Table 7 and Figure 6, with the likelihood function being 158.464.

Table 7: Estimation of Eq. (20) for Japan

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\gamma_0$</th>
<th>$\gamma_1$</th>
<th>$\sigma_0$</th>
<th>$\sigma_1$</th>
<th>$p$</th>
<th>$q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate</td>
<td>0.015</td>
<td>-0.055</td>
<td>0.242</td>
<td>0.356</td>
<td>0.036</td>
<td>0.083</td>
<td>0.989</td>
<td>0.958</td>
</tr>
<tr>
<td>($t$-st.)</td>
<td>(1.747)</td>
<td>(0.228)</td>
<td>(1.356)</td>
<td>(3.738)</td>
<td>(5.835)</td>
<td>(12.436)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 6: Empirical Evidence of Japan

It is clear that $\beta$ has a wrong sign with an insignificant $t$-statistic. $\gamma_0$ is, however, not essentially different from $\gamma_1$. This is different from the case of the US where we find that $\gamma_0$ is $-0.061$ and $\gamma_1$ 0.375. From Figure 6 we find that the probability of
being in state 0 is very low when \( r_t \) is negative and relatively high when \( r_t \) is positive. This indicates that state 0 should be boom and state 1 recession. Moreover, there seems to be a clear regime change around 1990, from boom into recession. Note that \( \gamma_1 \) has a more significant t-statistic than \( \gamma_0 \). We have also tried the estimation with time-varying transition probabilities and found that \( \lambda_2 \) and \( \lambda_4 \) have relatively insignificant t-statistics.

In short, the empirical evidence in Japan seems somewhat different from that in the US. As shown in the previous section, the effects of prior gains and losses on the investment behavior in the current period seem relatively asymmetric because of the break-even effect. In the case of Japan, however, we find that the effects of prior gains and losses on the current decision are relatively symmetric, indicating a smaller break-even effect. This may have two implications: (1) Japanese are more risk-averse than Americans. They would be more reluctant to accept break-even opportunities in case they have experienced some losses in the past; (2) Japanese are not more risk-averse than Americans. The reason that they would not like to stay in the stock market after losses is that there has been little chance to break even after losses. The second possibility seems relatively consistent with the situation of the stock market in Japan. The return of stocks has been negative most of the time since the beginning of the 1990s. The stock market has been in such a long-time recession that people did not see good chance to break even, and as a result, would rather withdraw their investment from stocks in case they have had losses in the past. The situation in the US is, however, quite different. The stock return has been positive most of the time, especially after the 1990s. Therefore, there might have been a better chance to break even after losses. This would prevent people from withdrawing investment from stocks in a recession, and as a result, stop the stock prices from further decreasing in a recession.

The empirical evidence of Japan is more consistent with the assumption of Barberis et al. (2001) than the US. Barberis et al. (2001) just assume that prior gains induce risk-seeking and prior losses induce risk-aversion, and do not consider the break-even effect.

5 Conclusion

In this paper we have undertaken some estimation to explore empirical evidence of the prospect theory in asset pricing with time-series data. The asset pricing model with the prospect theory and house-money effect has been shown to be more successful in explaining some puzzles in financial markets such as equity premium.

Based on the theoretical backgrounds of the loss aversion model, we have first
undertaken estimation with a Markov-switching coefficient with constant as well as
time-varying transition probabilities and found that the effects of prior gains and
losses on investment in stocks may be asymmetric because of the break-even effect.
Moreover, the gap between returns from stocks and from bonds also seems to be
an important factor affecting persistence in investment behavior. We have then un-
dertaken estimation with a time-varying coefficient with two states. The estimation
result also favors the claim that persistence in investment behavior depends on the
state of the stock market (recessions or booms).

The main conclusion is that, although the asset-pricing model developed by Bar-
beris et al. (2001) with the prospect theory can explain the equity-premium puzzle
better than the habit-formation and adjustment-cost models, they have ignored the
break-even effect, which may have some obvious effect on investment behavior, es-
pecially in recessions.

**Appendix: Log-linearization of eq. (3)**

Define $x_t \equiv \log X_t - \log \bar{X}$, with $\bar{X}$ being the steady state of $X_t$. Then $X_t = \bar{X}e^{x_t}$.

In case $x_t$ is close to zero, one then usually employs the following log-linearization
rules (with $y_t$ having a similar definition of $x_t$), following Uhlig (1999):

- $x_t y_t \approx 0$
- $e^{x_t} \approx 1 + x_t$
- $X_t + aY_t \approx \bar{X}Y x_t + (\bar{X} + a)Y y_t + \text{constant}$.

We can then log-linearize eq. (3) as follows:

$$1 = E_t \left[ \rho \left( \frac{C_{t+1}}{C_t} \right)^{-\delta} R_{t+1} \right]$$

$$1 = \rho E_t \left[ \left( \frac{\bar{C} e^{c_{t+1}}}{C e^{c_t}} \right)^{-\delta} \bar{R} e^{r_{t+1}} \right]$$

$$1 = \rho \bar{R} E_t \left[ e^{r_{t+1}} - \delta (c_{t+1} - c_t) \right]$$

$$1 \approx \rho \bar{R} E_t [1 + r_{t+1} - \delta \Delta c_{t+1}]$$

where $\Delta c_{t+1} \equiv c_{t+1} - c_t = \log C_{t+1} - \log C_t$. Define $\mu \equiv \frac{1}{\rho \bar{R}} - 1$ and $\eta_{t+1} \equiv r_{t+1} - E_t [r_{t+1}] - \delta (\Delta c_{t+1} - E_t [\Delta c_{t+1}])$, we have

$$r_{t+1} = \mu + \delta \Delta c_{t+1} + \eta_{t+1}.$$
References


