The Relative Income Hypothesis

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Abstract

Despite its theoretical dominance, the empirical case in favor of the permanent income hypothesis is at best a weak one. Contrary to one of its basic implications, a growing body of evidence suggests that rich households save a higher proportion of their permanent income than poor ones. Following Duesenberry (1949) we propose an overlapping generations economy where households care about relative consumption, the difference between their consumption and the consumption of their reference group. As a result, an individual’s consumption is driven by the comparison of his lifetime income and the lifetime income of his reference group, a permanent income version of the Duesenberry’s (1949) relative income hypothesis. Across households the saving rate increases with income while aggregate savings are independent of the income distribution. Positional concerns lead agents to consume and work above the welfare maximizing levels chosen by a planner. We propose a simple tax schedule that induces the competitive economy to achieve the efficient allocation.

JEL Classification: D62, E21, H21

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1 Introduction

James Duesenberry, in his seminal work *Income Saving and the Theory of Consumer Behavior* (1949), introduced the relative income hypothesis in an attempt to rationalize the well established differences between cross-sectional and time-series properties of consumption data. On the one hand, a wealth of studies based on the 1935-36 and 1941-42 budget surveys presented a saving rate that increased with income. On the other hand, the data on aggregate savings and income from 1869 to 1929 collected by Kuznets (1942) presented a trendless saving ratio. Duesenberry (1949) proposed an individual consumption function that depended on the current consumption of other people. As a result "for any given relative income distribution, the percentage of income saved by a family will tend to be a unique, invariant, and increasing function of its percentile position in the income distribution. The percentage saved will be independent of the absolute level of income. It follows that the aggregate saving ratio will be independent of the absolute level of income" (Duesenberry, 1949, pg. 3). Despite its empirical success, the relative income hypothesis was quickly replaced by the well-known permanent income hypothesis (Modigliani and Brumberg (1954), Friedman (1957)) as the economists’ workhorse to understand consumption behavior. According this view the cross-sectional correlation between saving and income is driven by transitory deviations from permanent income. While in the aggregate, most transitory components cancel out, leading to the close relation between consumption and income observed in time series data.

This paper presents a fully specified model of intertemporal choice that formalizes Duesenberry’s intuitions. We consider an overlapping generations economy where households differ in the initial bequest they inherit from their parents. Young households derive utility from leisure and the difference between their consumption and the consumption of others, i.e. relative consumption. In this context, the resulting consumption of a household is driven by the comparison of his lifetime income and the lifetime income of his reference group, a permanent income version of the relative income hypothesis. As in Duesenberry (1949), individual saving rates increase with relative income while aggregate savings are independent of the income distribution. Positional concerns lead agents to consume and work above the welfare maximizing levels chosen by a benevolent central planner. We propose a simple tax schedule that induces the competitive economy to achieve the efficient allocation. Along the lines anticipated by Frank (2007) it consists on a progressive tax on consumption.

Despite its overwhelming theoretical dominance, the empirical case in favor of the permanent income hypothesis is at best a weak one. Much of the early empirical work, Brady and
Friedman (1947) and Mayer (1966, 1972), presents strong evidence against the proportionality of savings rates. Recent empirical work casts further doubts. Browning and Lusardi (1996) conclude that the observed positive relationship between income and saving is difficult to rationalize in terms of consumption smoothing. Dynan, et al. (2004) use panel data to instrument permanent income by education, lagged and future earnings, and measures of consumption. Their careful analysis finds a strong positive relationship between saving rates and lifetime income. The literature on inter-generational saving, bequests and inter-vivos transfers, finds similar results. In recent work, Altonji and Villanueva (2007) estimate that, at the mean of permanent earnings, parents pass on about 2.5 cents of every extra dollar of lifetime resources to their children through bequests. Furthermore their estimate increases with income, hence wealthier households bequeth a larger proportion of their income that poor households do. If we are to believe this recent body of evidence we need to depart from the standard version of the permanent income hypothesis. Our model does so in an intuitive way, abandoning the independent preference assumption that underlies Friedman’s analysis. The resulting behavior, a mixture of permanent and relative income components, preserves the basic implications of the permanent income hypothesis while it is consistent with the empirical evidence we have just described.

The assumption that preferences are independent across households, although standard in the economic literature, is not particularly appealing. Indeed, social scientists have long stressed the relevance of status seeking as being an important characteristic of human behavior (see Cantril (1965), Schoeck (1966), Rawls (1971) and Argyle (1989)). In our discipline, the idea that the overall level of satisfaction derived from a given level of consumption depends, not only on the consumption level itself, but also on how it compares to the consumption of other members of society, is not new. Though origins of this proposition can be traced as far back as Smith (1759) and Veblen (1899), it was not until the work of Dueussenberry (1949) and Pollak (1976) that an effort was made to provide this idea with some

1Several authors have explored departures from the basic permanent income hypothesis to rationalize the cross-sectional variation in saving rates. Zeldes (1989) introduces liquidity constraints in an intertemporal optimization model. He finds that the inability to borrow against future labor income affects the consumption of a significant portion of the population. Ventura and Hugget (2000) analyze the impact on saving rates of the US social security system. Samwick (1998) considers a model where the subjective discount rate is correlated with income. Lawrance (1991) provides empirical evidence along these lines. Gentry and Hubbard (2000) assume that entrepreneurs enjoy better access to investment opportunities. As a result, if substitution effects dominate income effects, they will save more. Dynan, et al. (2002, 2004) explore the effects of the introduction of bequest motives and large medical expenses associated with health shocks. The introduction of these expenses implies that low-income households should save more than high-income households. Finally, Carroll (2000) considers the accumulation of wealth as an end in itself, the "capitalist spirit" model. He argues that the implications for saving of his model are virtually indistinguishable from those obtained in a model of interpersonal comparison.
micro-theoretic foundations. On the empirical side, Clark and Oswald (1996), using a sample of 5,000 British workers, find that workers’ reported satisfaction levels are inversely related to their comparison wage rates, supporting the hypothesis of positional externalities. Neumark and Postlewaite (1998) propose a model of relative income to rationalize the striking rise in the employment of married women in the U.S. during the past century. Using a sample of married sisters, they find that married women are 16 to 25 percent more likely to work outside the home if their sisters’ husbands earn more than their own husbands. Ravina (2007) estimates an Euler equation derived under interdependent preferences. Her results are consistent with preference specifications that place around one third of the weight on relative consumption. Finally, Frank (1985, 2000, 2007) provides a wealth of anecdotal evidence on the effects of positional externalities on individual behavior. On the theoretical side, there is a large literature that explores the effects of preference interdependence for asset pricing (Abel (1990), Gali (1994)), for short-run macroeconomic stabilization policy (Ljungqvist and Uhlig (2000)), for the interaction between saving and growth (Carroll, et al. (1997, 2000)), and for the process of capital accumulation (Fisher and Hof (2000), Alvarez-Cuadrado, et al. (2004), Liu and Turnovsky (2005)).

Our paper contributes to this literature exploring the interaction between consumption externalities and income inequality. In line with previous results, relative consumption concerns lead to an inefficiently low level of leisure, over-working. But in contrast with these results, in our specification consumption externalities are associated with an inefficiently low saving rate as opposed to the over-accumulation results obtained under under an infinitely lived representative agent (Fisher and Hof (2000), Liu and Turnovsky (2005)). Intuitively in a representative agent economy with an infinite planning horizon households want to keep up with the Joneses today and in every future period. Being forward-looking they anticipate that reducing current saving relative to their neighbors will lead to an undesirable low level of future consumption. This mechanism is behind the over-accumulation result reported by previous studies. Our framework, since it limits consumption externalities to young-age consumption, reverses their effect on the saving rate.

Finally, our work is closely related to the recent literature on self-reported well-being. Early work by Easterlin (1974, 1995) and Oswald (1997) found similar differences between the cross-section and time-series properties of happiness data than those reported on savings data more than fifty years before. Self-reported well-being data shows that within a country at a given point in time those with higher incomes are, on average, happier. However average happiness in developed countries has remained relatively constant over time despite sharp
increases in per capita GDP. Clark, et al. (2008) highlight the importance of interpersonal comparisons to account for the "Easterlin paradox". Recent work has tried to estimate the direct impact of interpersonal comparisons on self-reported well-being. Luttmer (2005) matches individual-level panel data on well-being from the U.S. National Survey of Families and Households to census data on local average earnings. After controlling for income and other own characteristics, he finds that local average earnings have a significantly negative effect on self-reported happiness. Ferrer-i-Carbonell (2005), using data from a large German panel, concludes that the income of the reference group is about as important as the own income for individual happiness. Dynan and Ravina (2007) find similar results for US households. Their estimates suggest that people’s happiness depends positively on how well they are doing relative to the average in their geographic area, even after controlling for the level of their own income. Finally, a growing body of experimental literature (Solnick and Hemenway (1998), Johansson-Stenman, et al. (2002), and Alpizar, et al. (2005)) highlights the importance of relative rather than absolute position for economic choices. Our work formalizes these insights and explores its implications for consumption, saving and leisure decisions.

The paper is organized as follows. Section 2 sets out the basic model. Section 3 compares the decentralized and centrally planned solutions under an homogeneous reference group. This section presents the basic implications of a life-cycle version of the relative income hypothesis. Section 4 extends the previous analysis allowing for heterogeneous reference groups. The conclusions are summarized in Section 5, while the Appendix provides some technical details.

2 The Model

Consider an overlapping-generations economy where a single composite good is produced. Time is discrete and infinite with $t = 0, 1, 2, \ldots \infty$.

Stevenson and Wolfers (2008) extend Easterlin’s (1974, 1995) country coverage to reassess his paradox. Their results suggest a positive link between GDP and average levels of subjective well-being across countries. These authors conclude that the role for relative income comparisons as drivers of happiness is minimal. We disagree with this interpretation of the evidence since cross-country comparisons of self-reported well-being are problematic. We believe that a definite rebuttal of Easterlin’s paradox requires a careful evaluation of time-series data from individual countries. In this respect, the evidence presented by this authors is mixed.
2.1 Production

Every period our economy produces a composite good that may be consumed or invested. Output, \( Y \), is produced combining physical capital, \( K \), labor, \( 1 - L \), and the available level of technology, \( A \), that for simplicity we assume to remain constant. The production function, \( F(K, L, A) \), is homogeneous of degree 1 in private inputs and satisfies the usual Inada conditions. Since markets are competitive factors are paid their marginal products and therefore,

\[
    w_t = f \left( \frac{K_t}{1 - L_t}, A \right) - \left( \frac{K_t}{1 - L_t} \right) f' \left( \frac{K_t}{1 - L_t}, A \right) \tag{1}
\]

\[
    r_t = f' \left( \frac{K_t}{1 - L_t}, A \right) - \delta \tag{2}
\]

where \( f \) denotes the production function in intensive form and capital is assumed to depreciate at the exponential rate \( \delta \).

Under the assumption that our economy is open and small the domestic capital-labor ratio is pinned down by (2) together with the constant world interest rate, \( r_t = r \). The degree of capital intensity, in turn, pins down the domestic wage rate at \( w \). Any changes in labor supply are accommodated by capital flows so that the domestic wage and the interest rate remain constant at \( w \) and \( r \) respectively. Following the convention we denote the gross return to capital by \( R = 1 + r \).

2.2 Households

Individuals live for two periods, "youth" and "old-age". At the end of their youth each individual gives birth to a single offspring and therefore at any point in time there are two generations alive. Each generation is composed by \( n \) individuals, indexed by \( i = 1, ..., n \). Our agents are altruistic toward their children, deriving a "warm-glow" from the bequests they leave to their descendents at the end of their lives (Adreoni (1989), Benabou (1996), Bertola, et al. (2006)). Within a generation, individuals differ only in their initial level of wealth captured by the bequest they inherit from their parent. The distribution of wealth in a particular generation \( t \) is given by a cumulative distribution function \( G_t(b) \) denoting the measure of individuals with wealth below \( b \). The initial distribution \( G_0(b) \) is given. Let’s focus on the \( i \)-th individual born in period \( t \). In the first period of his life he is endowed with one unit of time that he allocates between leisure, \( l^i_t \), and working. His labor income,
\( w(1 - l^i_t), \) together with the income inherited from his parent, \( b^i_t, \) is divided between current consumption, \( c^i_t, \) and saving, \( s^i_t. \) As a result his first period budget constraint is given by,

\[
w(1 - l^i_t) + b^i_t = c^i_t + s^i_t \tag{3}
\]

In the second period of his life our household is retired. His only source of income comes from the return on the savings he made when young, \( R_s^i. \) He allocates this income between old-age consumption, \( d^i_{t+1}, \) and bequests, \( b^i_{t+1}. \) We can write his old-age budget constraint as,

\[
R_s^i = d^i_{t+1} + b^i_{t+1} \tag{4}
\]

The preferences of an individual born in period \( t \) are given by the following life-cycle utility function,

\[
U_t\left(\hat{c}^i_t, l^i_t, d^i_{t+1}, b^i_{t+1}\right) = u\left(\hat{c}^i_t\right) + v\left(l^i_t\right) + \beta \left[u\left(d^i_{t+1}\right) + \phi\left(b^i_{t+1}\right)\right] \tag{5}
\]

where \( 0 < \beta < 1 \) is the psychological discount factor. We assume our three subutilities, \( u\left(\cdot\right), v\left(\cdot\right) \) and \( \phi\left(\cdot\right), \) are increasing, concave and satisfy the standard Inada conditions.

Our key behavioral assumption is that during youth, the satisfaction derived from a given level of consumption does not depend on the level of consumption itself but rather in how it compares to the consumption of some reference group. Following Ljungqvist and Uhlig (2000) we adopt an additive specification for relative consumption, \( \hat{c}^i_t = c^i_t - \gamma g^i_t, \) where \( g^i_t \) is the average consumption of the reference group of the \( i \)-th individual and \( 0 < \gamma < 1 \) is a measure of the relativity concerns\(^3\). This specification is consistent with the growing body of empirical evidence that highlights the role of interpersonal comparisons as a key determinant of behavior. Finally, the asymmetry we introduce in our modelling of the satisfaction derived from consumption along the life-cycle is justified along several grounds. First, the work of development psychologists and sociologists (Corsaro and Eder, 1990) suggests that interpersonal comparisons and peer effects are more relevant early in life. Second, we believe that the degree of social interactions is higher in the first period of our model. In this stage of life, people work, find partners, raise children, being exposed, and therefore influenced, by a wide variety of social networks. Our strategy takes the extreme position that old-age comparisons are irrelevant, although our results are still valid in an environment in which young-age comparisons are sufficiently stronger than old-age comparisons\(^4\).

\(^3\)We place restrictions on the initial endowments, \( G_0(b) \), so that every household consumes above his reference level.

\(^4\)Alternatively, we could assume that agents care about relative consumption in both periods of their
3 Homogeneous Reference Group

Following most of the literature on consumption externalities (Ljungqvist and Uhlig (2000), Liu and Turnovsky (2005)) we assume the reference group of any individual is composed by all the members of his own generation. Under this assumption all the young households share the same reference group and therefore the reference level of consumption is given by,

\[ \bar{g}_t^i = \bar{c}_t = \frac{1}{n} \sum_{j=1}^{n} c_j \]  

3.1 Competitive Solution

The \( i \)-th individual of the generation born in period \( t \) takes the initial wealth inherited from his parent, prices and the choices of the other members of his generation as given and chooses the amount he works, \((1 - l_t^i)\), his level of saving, \(s_t^i\) and the bequest he will transfer to his offspring \( b_{t+1}^i \) to maximize,

\[ u \left( w \left( 1 - l_t^i \right) + b_t^i - s_t^i - \gamma \bar{c}_t \right) + v \left( l_t^i \right) + \beta \left[ u \left( R s_t^i - b_{t+1}^i \right) + \phi \left( b_{t+1}^i \right) \right] \]

The solution to this problem is characterized by the following optimality conditions,

\[ u' \left( c_t^i - \gamma \bar{c}_t \right) = u' \left( \bar{c}_t \right) = \beta Ru' \left( d_{t+1}^i \right) \]  \( \text{(7)} \)

\[ u' \left( c_t^i - \gamma \bar{c}_t \right) w = u' \left( \bar{c}_t \right) w = v' \left( l_t^i \right) \]  \( \text{(8)} \)

\[ u' \left( d_{t+1}^i \right) = \phi' \left( b_{t+1}^i \right) \]  \( \text{(9)} \)

The interpretation of this conditions is standard. Nonetheless it is worth noticing the effects of interpersonal comparisons. An increase in the consumption of the reference group, \( \bar{c}_t \), increases the marginal utility of young-age consumption leading to a reduction in saving and leisure. As we will see, equations (7)-(9) together with the budget constraints, (3) and

\[ \gamma^y \text{ and } \gamma^o \text{ being the degree of relativity concerns while young and old respectively. The results presented in the following sections, obtained under logarithmic preferences and } \gamma^o = 0, \text{ are qualitatively equivalent to those obtained under the weaker restriction } \gamma^y > \gamma^o > 0. \]

\( ^5 \)See Abel (2005) for an overlapping generation model where the reference group is composed by a weighted average of young and old households. Our specification, in line with Frank’s (1985) arguments, limits interpersonal comparisons to agents belonging to the same generation.
(4), implicitly define the optimal choices of leisure, saving and bequests as functions of the relative income of the individual.

In order to keep the analysis tractable it is convenient to assume the following logarithmic specification for (5)

\[ U_t(c_t, l_t, d_{t+1}, b_{t+1}) = \ln(c_t - \gamma c_t) + \mu \ln(l_t) + \beta \left[ \alpha \ln(d_{t+1}) + (1 - \alpha) \ln(b_{t+1}) \right] \tag{10} \]

where \( \mu > 0 \) governs the importance of leisure when young and \( 0 < \alpha < 1 \) the relative importance of consumption versus bequests when old. The first order conditions under (10) become,

\[ \frac{d_{t+1}}{c_t - \gamma c_t} = \alpha \beta R \tag{11} \]

\[ \frac{l_t}{\mu (c_t - \gamma c_t)} = \frac{1}{w} \tag{12} \]

\[ \frac{d_{t+1}}{b_{t+1}} = \frac{\alpha}{1 - \alpha} \tag{13} \]

where the private marginal utility of consumption is given by,

\[ MUC^i = \frac{1}{c_t - \gamma c_t} \tag{14} \]

Let’s begin characterizing the optimal behavior of the average household, i.e. the household inheriting the average bequest, \( \bar{b}_t \). Combining (11), (13), (3) and (4) we reach the following expression that implicitly defines his optimal savings behavior,

\[ \frac{1}{(w + \bar{b}_t - \bar{s}_t)(1 - \gamma)} = \frac{\alpha \beta R}{d_{t+1}} = \frac{\beta}{\bar{s}_t} \tag{15} \]

using (11), (4) and (12) to solve for his optimal leisure choice we reach, \( \bar{l}_t = \frac{\mu}{w} \bar{s}_t \). Replacing this expression in (15) the optimal level of saving for the average individual born in period \( t \) is given by,

\[ \bar{s}_t = \frac{\beta (1 - \gamma)}{1 + (1 - \gamma)(\beta + \mu)} (w + \bar{b}_t) \equiv \beta (1 - \gamma) \psi \bar{y}_t \tag{16} \]

where \( \psi \equiv \frac{1}{1 + (1 - \gamma)(\beta + \mu)} \) and we can interpret \( \bar{y}_t \equiv (w + \bar{b}_t) \) as the potential life-time income of the average household of the generation born at \( t \), i.e. the lifetime income of the household that inherits the average bequest if he devotes all of his time endowment to work.
Savings is just a constant fraction, $0 < \beta (1 - \gamma) \psi < 1$, of this measure of lifetime income. Combining (16) with (3) and (4) we obtain,

$$c_t = \psi \bar{y}_t$$  \hspace{1cm} (17)

$$l_t = \frac{\mu (1 - \gamma)}{\psi} \bar{y}_t$$  \hspace{1cm} (18)

$$d_t = \alpha R \beta (1 - \gamma) \psi \bar{y}_t$$  \hspace{1cm} (19)

$$\bar{b}_{t+1} = (1 - \alpha) R \beta (1 - \gamma) \psi \bar{y}_t$$  \hspace{1cm} (20)

We can use the results for the average household to characterize the behavior of the $i$-th individual of the same generation. The counterpart of (15) for this individual is given by,

$$s_i = \frac{1}{w (1 - l_i) + b_i - s_i - \gamma c_i} = \frac{\beta}{s_i}$$  \hspace{1cm} (21)

and combining this expression with (11), (4), (12) and (17)

$$s_i = \frac{\beta}{1 + \mu + \beta} \left[ (w + b_i) - \frac{\gamma}{1 + (1 - \gamma)(\beta + \mu)} (w + b_i) \right] \equiv \frac{\beta}{1 + \mu + \beta} \left[ \bar{y}_i - \gamma \psi \bar{y}_i \right]$$  \hspace{1cm} (22)

where $\bar{y}_i \equiv (w + b_i)$ is the potential lifetime income, defined as before, of the $i$-th household of the generation born at $t$. Equation (22) shows that individual saving is a linear function of an individual and average income. Linearity ensures that the income distribution plays no role for aggregate saving and accumulation\(^6\). It is straight-forward to solve for the remaining optimal choices as functions of individual and average potential lifetime income,

$$c_i = \frac{1}{1 + \mu + \beta} \left[ \bar{y}_i + (\beta + \mu) \gamma \psi \bar{y}_i \right]$$  \hspace{1cm} (23)

$$l_i = \frac{\mu}{w (1 + \mu + \beta)} \left[ \bar{y}_i - \gamma \psi \bar{y}_i \right]$$  \hspace{1cm} (24)

$$d_{i+1} = \frac{\alpha R \beta}{1 + \mu + \beta} \left[ \bar{y}_i - \gamma \psi \bar{y}_i \right]$$  \hspace{1cm} (25)

\(^6\)Although, under endogenous leisure the wealth distribution affects labor supply choices which in turn will affect the market clearing wage. So in general aggregate dynamics are not independent of the income distribution.
Young-age consumption of the $i$-th household has a first component that increases in the household potential lifetime income and a second component that reflects the influence of interpersonal comparisons, increasing in the potential lifetime income of his reference group. As a result saving, labor supply and bequest choices depend on relative income rather than absolute income. When individual satisfaction depends on consumption comparisons across households, as a growing body of empirical evidence suggests, the relevant variable driving the saving decision is the comparison between individual $i$'s potential lifetime income and the potential lifetime income of his reference group. The agents populating our economy, are not only "disposed, as a rule and on the average, to be forward-looking animals" as those in Modigliani and Brumberg (1954, pg. 430) or Friedman (1957), but are also outward-looking animals with their choices being partially driven by the choices of other members of the community they live in. We can think of these results as a extension to an intertemporal framework of Duesenberry's (1949) relative income hypothesis. Finally our quasi-homothetic preferences under perfect capital markets imply that (22)-(26) are affine functions of the level of potential life-time income. This property of the model ensures that the distribution of wealth does not affect the aggregate evolution of the economy, provided that each household's income is above the reference consumption level, whereas the wealth distribution does change along the transitional path as in Chatterjee (1994), Caselli and Ventura (2000), and Alvarez-Pelaez and Diaz (2005).

### 3.2 The Life-cycle version of the Relative Income Hypothesis

Duesenberry (1949), building on work by Brady and Friedman (1947), proposed the relative income hypothesis to rationalize the well established differences between cross-sectional and time series properties of consumption. On the one hand, a wealth of budget studies presented a saving rate that increases with income. On the other hand, Kuznets’ (1942) time series data presented a trendless saving ratio. Duesenberry (1949) postulates an individual consumption function that depends on the current consumption of other people. As a result, the cross-sectional correlation between savings rate and income results from relative consumption concerns, the emulation effect, while the long run constancy of the saving rate arises when relativity concerns cancel out in the aggregation. The relative income hypothesis was quickly replaced by Friedman’s permanent income hypothesis (Modigliani and Brum-
berg (1954), Friedman (1957)) as the benchmark to understand consumption behavior. In this view, consumption is driven by an estimate of current and future income, permanent income, and as a result, saving is proportional to life-time resources. In the cross-section, the positive correlation between saving and income, is driven by transitory deviations from permanent income. In the aggregate, most transitory components cancel out, leading to the close relation between consumption and income observed in time series data. Despite the theoretical dominance of the permanent income hypothesis, recent empirical work finds important deviations from its basic predictions. Browning and Lusardi (1996) conclude that the observed positive relationship between income and saving is difficult to rationalize in terms of consumption smoothing. Dynan, et al. (2004) use panel data to proxy permanent income by education, lagged and future earnings, and measures of consumption and, contrary to the permanent income hypothesis, they find a strong positive relationship between saving rates and lifetime income. When we turn to measures of inter-generational saving, bequests and inter-vivos transfers, similar results arise. Recent work by Altonji and Villanueva (2007) reports that the propensity of these transfers increases with life-time income.

Our model, where agents not only care about permanent income, but also relative income provides a straightforward explanation for this evidence. In a representative agent framework Liu and Turnovsky (2005) and Alvarez-Cuadrado (2007) show that positional concerns lead households to choose levels of consumption and working hours above the welfare-maximizing levels. Our framework, in which consumption externalities interact with income inequality, allows for a more systematic exploration of the differential impact of relative consumption across the income distribution.

Since consumption (young and old) and bequests are normal goods their levels increase with wealth (income), although according to the permanent income hypothesis their rates should be a constant fraction of life-time resources independent of the position on the income distribution. In order to illustrate our permanent income version of the relative income hypothesis it is convenient to define (actual) lifetime income as

\[ y_t^i = w^i (1 - l_t^i) + b_t^i = \frac{1 + \beta}{1 + \mu + \beta} (w + b_t^i) + \frac{\mu \gamma \psi}{1 + \mu + \beta} (w + b_t^i) \tag{27} \]

and the saving and bequest rates out of (actual) lifetime income as the ratio of (22) and (26) to (27) respectively. Differentiating these ratios with respect to wealth, measured by

\[ y_t^i = w (1 - l_t^i) + b_t^i = \frac{1 + \beta}{1 + \mu + \beta} (w + b_t^i) + \frac{\mu \gamma \psi}{1 + \mu + \beta} (w + b_t^i) \]

It is worth to notice these authors’s ambivalent views on the issue; “... and finally, the evidence that we have cited seems to fit it (the Permanent Income Hypothesis) somewhat better ... however, this evidence is by no means sufficient to justify a firm rejection of the relative income hypothesis” (Friedman, 1957, pg. 169).
the initial bequest, we reach the following comparative static results,

\[ \frac{\partial s_i^j}{\partial y_i^j} = \frac{\gamma \psi (w + \beta)}{(1 - \gamma)(1 + \mu + \beta)(y_i^j)^2} > 0 \] (28)

\[ \frac{\partial b_i^{t+1}}{\partial b_i^t} = (1 - \alpha)R \frac{\partial s_i^j}{\partial y_i^j} > 0 \] (29)

Finally, differentiating (24) with respect to wealth we reach

\[ \frac{\partial l_i^j}{\partial y_i^j} = \frac{\mu}{w(1 + \mu + \beta)} > 0 \] (30)

So in the presence of consumption externalities the saving rate increases with life-time income as most empirical evidence suggests. The low intragenerational saving, from working life to retirement, is also coupled with a low intergenerational saving rate, with poor households transferring a lower fraction of their lifetime wealth in the form of bequests to their offsprings than richer households\(^8\). Poor household have lower intragenerational and intergenerational saving rates despite the fact that they work longer hours than their richer neighbors\(^9\). Finally, substantial anecdotal evidence coincides with the predictions of our model. For instance Newman and Chen (2007) portray the lifes of working poor families in America as holding multiple jobs per person while being unable to make ends meet.

Finally, Duesenberry’s (1949) dealt with one additional empirical regularity of consumption data; consumption is more stable than income over the business cycle. He introduced irreversibility of consumption choices, habit formation, to explain the short-run rigidity of consumption. In our intertemporal set up, this short time rigidity results naturally from consumption smoothing as in the permanent income hypothesis.

### 3.3 Efficient Solution

In a competitive equilibrium individual households ignore the effects that their consumption choices have on the utility of other members of their generation. As a consequence,

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\(^8\)Garcia-Penalosa and Turnovsky (2007) explore an economy populated by infinitely lived households in the presence of comparative consumption. As opposed to our framework, in the infinitely lived economy poor households save more than rich ones and the presence of consumption externalities reduces inequality in a growing economy. Their results seem at odds with the empirical evidence cited in this section.

\(^9\)In a version of our model without consumption externalities, poor households will also enjoy less leisure. The presence of consumption externalities only exacerbates this result. Bowles and Parker (2005) estimate that almost 60% of the difference in average working hours between Sweden and the US could be explained in terms of interpersonal comparisons and income inequality.
agents’ consumption, leisure and bequest may diverge from the socially optimal levels that would be chosen by a benevolent central planner. Let us consider a central planner that acknowledges that individual consumption choices create distortions through their effects on average consumption\(^{10}\). The planner chooses consumptions, labor efforts and bequests for each individual within a given generation to maximize the social welfare function,

\[
SW = \frac{1}{n} \sum_{i=1}^{n} \ln (c^i - \gamma \bar{c}) + \mu \ln (l^i) + \beta \left[ \alpha \ln (d^i) + (1 - \alpha) \ln (b^i) \right]
\]

subject to the individual’s budget constraints (3) and (4) and (6).

Since the planner realizes that each individual contributes to the externality by a fraction \(\gamma\) of his first-period consumption, the difference between the competitive and planned solution lies in the valuation of utility of consumption while young. The planner’s counterpart of (14), the social marginal utility consumption for the \(i\)-th household, is,

\[
MUC^{i,p} = \frac{1}{c^{i,p} - \gamma \bar{c}^p} - \left[ \frac{\gamma}{c^{1,p} - \gamma \bar{c}^p} + \ldots + \frac{\gamma}{c^{i,p} - \gamma \bar{c}^p} + \ldots + \frac{\gamma}{c^{n,p} - \gamma \bar{c}^p} \right]
\]

where the superscript \(p\) denotes the planner’s choices. Comparing (14) with (32) we see that the social marginal utility of first-period consumption is composed of two terms. The first term is just the private marginal utility of consumption. The second term, in square brackets, captures the negative impact that an additional unit of consumption of the \(i\)-th household has on his own welfare and on the welfare of other members of his generation through its impact on average consumption. Since this negative impact is independent of the level of consumption of the household, the second term, \(A \equiv \sum_{i=1}^{n} \frac{\gamma}{c^{i,p} - \gamma \bar{c}^p}\), is identical for all households of a given generation. As a result, the distortion introduced by relative consumption takes the form of an overvaluation, by a factor \(\frac{1 - \sum_{i=1}^{n} A c^i}{1 - A (c^i d - \gamma \bar{c}^i)}\) of the marginal utility of the young generation consumption. At this stage it is convenient to impose additional restrictions on the model to guarantee that the social marginal utility of consumption is always positive\(^ {11}\). In the analysis that follows we assume these restrictions are satisfied.

The overvaluation of young age consumption distorts the marginal rate of substitution between first-period consumption and leisure, the static distortion, and the marginal rate of

\(^{10}\)Given the focus of this paper, our planner abstracts from issues associated with intergenerational efficiency. As a result we drop the time subscripts.

\(^{11}\)This restriction plays a similar role than the one placed in representative agent versions of our model to guarantee that the marginal utility of consumption, even after taking into account external effects, is positive. See for instance, Liu and Turnovsky (2005) assumption 1 (i). In our circumstances this restriction implies that \(1 - A_t (c^i_t - \gamma \bar{c}_t) > 0\).
substitution between first-period consumption and saving. We refer to this second distortion as the *dynamic* distortion, since it affects the willingness to shift resources into the future, i.e. into old-age consumption and bequests. Combining (12) and (32) we obtain the following relation between the marginal rates of substitution between consumption and leisure of the decentralized and centrally planned economies for the $i$-th household,

$$MRS_{c,l}^d \equiv \frac{l^i}{\mu (c^i - \gamma \bar{c})} > \frac{l^i (1 - A (c^i - \gamma \bar{c}))}{\mu (c^i - \gamma \bar{c})} \equiv MRS_{c,l}^p \tag{33}$$

Similarly, combining (11), (13), (4) and (32) we reach the following relation between the marginal rates of substitution between consumption and saving of the two solutions,

$$MRS_{c,s}^d \equiv \frac{s^i}{(c^i - \gamma \bar{c})} > \frac{s^i (1 - A (c^i - \gamma \bar{c}))}{(c^i - \gamma \bar{c})} \equiv MRS_{c,s}^p \tag{34}$$

As a result of interpersonal comparisons, households overvalue young-age consumption, and therefore their willingness to substitute from leisure towards first-period consumption is too high and their willingness to postpone consumption is too low relative to the socially desirable levels. Both distortions lead to a competitive solution characterized by young age over-consumption, over-working and under-saving.

Furthermore, it is important to notice the differential impact of the distortion across the income distribution. Since the overvaluation factor, $\frac{1}{1 - A (c^{i,d} - \gamma \bar{c}^d)}$, increases exponentially with consumption (income), the relative size of the adjustment made by the planner on the private marginal utility of consumption is larger for high income households. This just reflects the fact that wealthy households, with their high levels of consumption, contribute in a disproportionate way to average consumption, inducing substantial welfare losses on themselves and their neighbors.

### 3.4 Optimal Tax Policy

A competitive economy, where young agents are concerned with relative consumption, is characterized by over-consumption, under-saving, and over-working. Under these circumstances the government can restore efficiency by means of distortionary taxation. Combining (3) and (4) we obtain the following life-time constraint for the $i$-th household,

$$w (1 - l^i) (1 - \tau^i_c) + b^i_t + T^i = c^i + s^i = (1 + \tau^i_c) c^i + (1 + \tau^i_s) \frac{d^i + b^i_{i+1}}{R} \tag{35}$$

where $\tau^i_c$, $\tau^i_c$ and $\tau^i_s$ are taxes (possibly transfers) on labor, first-period consumption, and saving respectively and $T^i$ is a lump sum tax used to balance the government budget.
Finding, under the proposed tax structure, the relevant marginal rates of substitution for the competitive solution and equating them to the efficient ones we reach,

\[
\frac{l^i (1 - \tau^i)}{\mu (c^i - \gamma \bar{c}) (1 + \tau^i)} = \frac{l^i (1 - A (c^i - \gamma \bar{c}))}{\mu (c^i - \gamma \bar{c})} \tag{36}
\]

\[
\frac{s^i (1 + \tau^i)}{(c^i - \gamma \bar{c}) (1 + \tau^i)} = \frac{s^i (1 - A (c^i - \gamma \bar{c}))}{(c^i - \gamma \bar{c})} \tag{37}
\]

that can be solved for, at least, two optimal tax packages: \(\tau^i_c = A \frac{1}{1 - A (c^i - \gamma \bar{c})}\) and \(\tau^i_l = \tau^i_s = 0\) or \(\tau^i_c = A (c^i - \gamma \bar{c})\) and \(\tau^i_s = -A (c^i - \gamma \bar{c})\), where choices are evaluated at the competitive solution. Since concerns for relative consumption lead to over-consumption, over-working and under-saving it is not surprising that the optimal fiscal policy penalizes the first two activities while subsidizing the last. The first package consists of a progressive tax on consumption\(^{12}\). Since high income households contribute to a disproportionate share of average consumption, their consumption is taxed at higher rates than the one of low income households. Frank (2007) proposes a similar tax structure and illustrates its practical implementation using only income and saving data.

The second package consists on a progressive tax on labor income combined with a regressive subsidy on saving. Wealthy households face higher labor income tax rates but their savings is also subsidized at a higher rate.

4 Heterogeneous Reference Groups

In the previous section we characterized the behavior of an heterogeneous agents economy under the limiting assumption that the reference group was common and equal to the average household in our economy. In this context it is natural to explore the implications of our model economy for the steady state distribution of wealth. Solving (20) with the initial condition, \(\bar{b}_0\), the time path of the average bequest is given by,

\[
\bar{b}_t = (\bar{b}_0 - \bar{b}_\infty) [z(1 - \gamma \psi)]^t + \bar{b}_\infty \tag{38}
\]

where provided, \(0 < z \equiv \frac{(1 - \alpha)R \beta}{1 + \mu + \beta} < 1\), the unique stable steady state is given by,

\(^{12}\)We define a progressive tax as one such that its effective rate increases with income. An alternative definition of a progressive tax is one which its effective rate increases as the tax base increases. It is worth noticing that we can use (17) and (23) to express the tax rates as functions of parameters and variables that are exogenous from the standpoint of the individual household.
\[ b_{\infty} = \frac{z(1 - \gamma \psi)}{1 - z(1 - \gamma \psi)} w \] 

(39)

Combining (26) and (38) we reach the following difference equation that governs the evolution of wealth (bequests) for the \( i \)-th dynasty,

\[ b_{i+1} = z [w + b_i - \gamma \psi (w + \bar{b}_t)] = z(1 - \gamma \psi) w + z b_i + z \gamma \psi \{ (\bar{b}_0 - \bar{b}_\infty) [z(1 - \gamma \psi)]^t + \bar{b}_\infty \} \quad (40) \]

that given the initial condition, \( b_0 \), has the following definite solution,

\[ b_t = (b_0 - \bar{b}_0) z^t - (\bar{b}_\infty - \bar{b}_0) [z(1 - \gamma \psi)]^t + \bar{b}_\infty \]

which implies that the steady state wealth distribution collapses to a single point, with every household in our economy inheriting the average bequest, \( \bar{b}_\infty \).

This outcome, although surprising at first sight, is just a restatement of the result presented by Stiglitz (1969). The intuition is best understood in the case of exogenous labor. In this case the stability condition on the evolution of bequests implies that savings out of labor income, which is equal for all the households, is larger than the reference level of consumption. Under these circumstances the rate of growth of bequests decreases on wealth and therefore the wealth distribution eventually collapses. See the Appendix for a formal proof. This controversial result, inequality disappears in steady state, is closely related to our simplifying assumption about the composition of the reference group.

Now we turn to explore the more general case where reference groups differ across households. As we will see the main results from the previous section carry through to this more realistic environment and the steady state wealth distribution does not degenerate. Solving the counterparts of (11), (12), (13), (3) and (4) for the \( i \)-th household born in period \( t \), where \( \bar{g}_t \) is the average consumption level of his reference group, we reach,

\[ s_t^i = \frac{\beta}{1 + \beta + \mu} [\bar{g}_t^i - \gamma \bar{g}_t] \quad (41) \]

\[ c_t^i = \frac{1}{1 + \beta + \mu} [\bar{g}_t^i + (\beta + \mu) \gamma \bar{g}_t] \quad (42) \]

\[ l_t^i = \frac{\mu}{w (1 + \beta + \mu)} [\bar{g}_t^i - \gamma \bar{g}_t] \quad (43) \]

\[ d_{t+1}^i = \frac{\alpha R \beta}{1 + \beta + \mu} [\bar{g}_t^i - \gamma \bar{g}_t] \quad (44) \]

\[ b_{t+1}^i = \frac{(1 - \alpha) R \beta}{1 + \beta + \mu} [\bar{g}_t^i - \gamma \bar{g}_t] \quad (45) \]
For the sake of illustration, suppose there are only two homogeneous income groups, say \( H \) (rich) and \( L \) (poor), and the population is evenly distributed between these two groups. Veblen (1899), Duesenberry (1949), and Frank (2007) eloquently argue that the behavior of successful individuals or groups set the standard for the rest of the community. Ferrer-i-Carbonell (2005) provides convincing microeconometric evidence on the importance of upward comparisons as a determinant of subjective well-being. In line with this evidence, we assume that the reference group of the rich households is made out only of rich households while the reference group of poor households is composed of a weighted average of poor and rich households, with \( \rho \) being the weight placed on poor households, so reference consumption for the two groups is given by,

\[
g^H_t = \bar{c}^H_t \quad (46)
\]

\[
g^L_t = \rho \bar{c}^L_t + (1 - \rho)\bar{c}^H_t \quad (47)
\]

Now we can proceed sequentially. First we solve (41)-(45) together with (46), noting that \( c^H_t = \bar{c}^H_t \), to reach the optimal choices of the rich households,

\[
c^H_t = \psi \tilde{y}^H_t \quad (48)
\]

\[
s^H_t = \beta (1 - \gamma) \psi \tilde{y}^H_t \quad (49)
\]

\[
l^H_t = \mu (1 - \gamma) \psi \tilde{y}^H_t \quad (50)
\]

\[
d^H_{t+1} = \alpha R \beta (1 - \gamma) \psi \tilde{y}^H_t \quad (51)
\]

\[
b^H_{t+1} = (1 - \alpha) R \beta (1 - \gamma) \psi \tilde{y}^H_t \quad (52)
\]

Once we have (48), we combine it with (47) and (41)-(45), noting that \( c^L_t = \bar{c}^L_t \), to reach the optimal choices of the poor households,

\[
c^L_t = \zeta \left[ \tilde{y}^L_t + (\beta + \mu) \gamma (1 - \rho) \psi \tilde{y}^H_t \right] \quad (53)
\]

\[
s^L_t = \beta \zeta \left[ (1 - \gamma \rho) \psi \tilde{y}^L_t - \gamma (1 - \rho) \psi \tilde{y}^H_t \right] \quad (54)
\]

\[
l^L_t = \frac{H \zeta}{w} \left[ (1 - \gamma \rho) \tilde{y}^L_t - \gamma (1 - \rho) \psi \tilde{y}^H_t \right] \quad (55)
\]

\[
d^L_{t+1} = \alpha R \beta \zeta \left[ (1 - \gamma \rho) \psi \tilde{y}^L_t - \gamma (1 - \rho) \psi \tilde{y}^H_t \right] \quad (56)
\]

\[
b^L_{t+1} = (1 - \alpha) R \beta \zeta \left[ (1 - \gamma \rho) \psi \tilde{y}^L_t - \gamma (1 - \rho) \psi \tilde{y}^H_t \right] \quad (57)
\]

where \( \zeta \equiv \frac{1}{1 + (1 - \gamma \rho) (\beta + \mu)} \). As before we calculate actual lifetime income as follows,
\begin{align}
y_t^H &= w(1 - L_t^H) + b_t^H = \frac{1 + (1 - \gamma) \beta}{1 + (1 - \gamma) (\beta + \mu)} \bar{y}_t^H = [1 + (1 - \gamma) \beta] \psi \tilde{y}_t^H \\
y_t^L &= w(1 - L_t^L) + b_t^L = \frac{1 + (1 - \rho \gamma) \beta}{1 + (1 - \rho \gamma) (\beta + \mu)} \bar{y}_t^L + \frac{\mu (1 - \rho) \gamma \psi}{1 + (1 - \rho \gamma) (\beta + \mu)} \bar{y}_t^H
\end{align}

and obtain the following relation between the saving rates, out of actual lifetime income, of poor and rich households,

\[\frac{s_t^H}{y_t^H} = \frac{\beta (1 - \gamma)}{1 + (1 - \gamma) \beta} > \frac{s_t^L}{y_t^L} = \frac{\beta (1 - \rho \gamma) \bar{y}_t^L - \beta \gamma (1 - \rho) \psi \bar{y}_t^H}{[1 + (1 - \rho \gamma) \beta] \bar{y}_t^L + \mu (1 - \rho) \gamma \psi \bar{y}_t^H}\]

Therefore, as in the presence of an homogeneous reference group, poor households work longer hours and save and bequeth smaller fractions of their lifetime income than rich households do. Finally it remains to be shown that under heterogeneous reference groups the steady state wealth (income) distribution does not degenerate. We can express (52) as a difference equation on \(b_t^H\),

\[b_{t+1}^H = z(1 - \gamma \psi) (w + b_t^H)\]

that implies that \(b_t^H\) converges monotonically to its steady state given by,

\[b_\infty^H = \frac{z(1 - \gamma \psi)}{1 - z(1 - \gamma \psi)} w\]

and starting from the initial condition, \(b_0^H\), its time path is given by,

\[b_t^H = (b_0^H - b_\infty^H) [z(1 - \gamma \psi)]^t + b_\infty^H\]

Combining this result with (57), we reach the following difference equation on \(b_t^L\) that characterizes the evolution of bequests for our poor households.

\[b_{t+1}^L = Q w + \lambda b_t^L - T [z(1 - \gamma \psi)]^t\]

where \(T \equiv (1 - \alpha) R \beta \zeta (1 - \rho) \psi (b_0^H - b_\infty^H)\), \(Q \equiv (1 - \alpha) R \beta \zeta \left[1 - \gamma \rho - \frac{\gamma (1 - \rho) \psi}{1 - z(1 - \gamma \psi)}\right]\), and \(\lambda \equiv \frac{(1 - \alpha) R \beta (1 - \gamma \rho)}{1 + (1 - \gamma \rho) (\beta + \mu)}\). Provided \(Q > 0\) and \(0 < \lambda < 1\) are satisfied, then \(b_t^L\) converges to a unique positive steady state given by,

\[b_\infty^L = \frac{Q}{1 - z(1 - \gamma \psi)} > 0\]
Proof. $Q > 0$ iff

$$\Phi(\rho) \equiv (1 - \gamma \rho) [1 - z(1 - \gamma \psi)] - \gamma (1 - \rho) \psi > 0$$

Since $\Phi(\rho)$ is monotonic, $\Phi(1) > 0$ and $\Phi(0) > 0$ then $\Phi(\rho) > 0$ for all $\rho \in [0, 1]$.

Starting from the initial condition, $b_0^L$, its time path is given by,

$$b_t^L = (b_0^L - b_\infty^L) \lambda^t + \frac{\zeta \gamma (1 - \rho) \psi}{\zeta (1 - \gamma \rho) - (1 - \gamma) \psi} (b_0^H - b_\infty^H) \left( [z(1 - \gamma \psi)]^t - \lambda^t \right) + b_\infty^H$$

with $b_\infty^H > b_\infty^L$. Notice that if $\rho = 1$, then $b_\infty^H = b_\infty^L$. It is easy to verify that if $(1 - \alpha) R \beta > \beta + \mu$ then $Q'(\rho) > 0$.

5 Conclusions

Despite the theoretical dominance of the permanent income hypothesis, there is a growing body of empirical evidence that finds important departures from its basic predictions. Specifically, recent work by Dynan, et al. (2004) and Altonji and Villanueva (2007) provides strong evidence of a saving rate that increases with permanent income, violating the proportionality hypothesis. Our approach departs from the standard version of the permanent income hypothesis in an intuitive way; in line with recent evidence on self-reported well-being, we abandon the independent preference assumption that underlies Friedman’s analysis. We consider an overlapping generations economy with heterogenous wealth levels. Young households derive utility from leisure and relative consumption. In this context, the resulting consumption of a household is driven by the comparison of his lifetime income and the lifetime income of his reference group, a permanent income version of the relative income hypothesis. As in Duesenberry (1949), individual saving rates increase with relative income while aggregate savings are independent of the income distribution. Positional concerns lead agents to consume and work above the welfare maximizing levels chosen by a benevolent central planner. We propose a simple tax schedule that induces the competitive economy to achieve the efficient allocation.

Finally, our specification replaces Keynes’ "fundamental psychological law" with the principle that men are disposed, as a rule and on average, to be not only "forward-looking" but also "outward-looking" animals.\(^\text{13}\)

\(^\text{13}\)This refers to Keynes’ (1936, pg. 96) well known observation about the "fundamental psychological law, upon which we are entitled to depend with great confidence both a priori from our knowledge of human nature and from the detailed facts of experience, is that men are disposed, as a rule and on the average, to increase their consumption as their income increases, but not by as much as the increase in their income"
6 Appendix

Under exogenous labor supply, $\mu = 0$, we construct a measure of the present value of the $i-th$ individual lifetime consumption as,

$$ C_i^t = c_i^t + \frac{d_{i+1}^t}{R} = \check{c}w + \check{c}b_i^t + \overline{c}_i $$  \hspace{1cm} (67) 

where $\check{c} \equiv \frac{(1 + \alpha \beta)(1 + \beta) - 2\alpha \beta \gamma}{1 + \beta(1 - \gamma)}$ is the average (and marginal) propensity to consume out of labor income, $\overline{c} \equiv \frac{1 + \alpha \beta}{1 + \beta}$ is the average propensity to consume out of inheritances and $\overline{c}_i \equiv \frac{\beta \gamma(1 - \alpha)}{(1 + \beta)(1 + \beta(1 - \gamma))} \overline{b}_t$ is a time-varying autonomous level of consumption. As in Stiglitz (1969) individual consumption is a linear function of individual income.

We can combine (3) and (4) to reach the lifetime budget constraint for the $i-th$ individual,

$$ \frac{b_{i+1}^t}{R} = w + b_i^t - C_i^t $$  \hspace{1cm} (68) 

Combining the previous two expressions we reach the following law of motion for bequests,

$$ \Delta b_{i+1}^t = R \left[ (1 - \check{c}) w - \overline{c}_i + \left(1 - \check{c} - \frac{1}{R}\right) b_i^t \right] $$  \hspace{1cm} (69) 

In order to explore the evolution of the distribution of wealth we can divide (69) by $b_i^t$ reaching,

$$ \frac{\Delta b_{i+1}^t}{b_i^t} = R \left[ \frac{(1 - \check{c}) w - \overline{c}_i}{b_i^t} + \left(1 - \check{c} - \frac{1}{R}\right) \right] $$  \hspace{1cm} (70) 

It becomes clear that the wealth distribution will eventually converge as long as $(1 - \check{c}) w - \overline{c}_i > 0$. Intuitively, since $(1 - \check{c} - \frac{1}{R})$ is proportional to bequests it has no effect on the evolution of the wealth distribution. On the other hand if saving out of labor income, which is the same for rich and poor households, is greater than the reference level of consumption, which is again the same accross households, then the bequest of a poor dynasty grows faster than the bequest of a rich dynasty, since $(1 - \check{c}) w - \overline{c}_i$ represents a higher fraction of the bequest of a poor household than of the bequest of a rich household.

Now we can find under what conditions does the "aggregate" bequest, $b_t$, reach a stable steady state, summing (69) accross households we reach,

$$ \Delta b_{t+1} = R \left[ (1 - \check{c}) w - \overline{c}_t + \left(1 - \check{c} - \frac{1}{R}\right) b_t \right] $$  \hspace{1cm} (71) 

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which implies that the average bequest eventually achieves a steady state iff \( 1 - \tilde{c} - \frac{1}{R} < 0 \). This stability condition together with the steady state condition, \( \Delta b_{t+1} = 0 \), implies that \((1 - \tilde{c}) w - \bar{c}_t > 0\), so that the wealth distribution eventually collapses to a single point.
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