Trade, Offshoring, and the Invisible Handshake

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Abstract. We study the effect of globalization on the volatility of wages and worker welfare in a model in which risk is allocated through long-run employment relationships (the ‘invisible handshake’). Globalization can take two forms: International integration of commodity markets (i.e., free trade) and international integration of factor markets (i.e., offshoring). In a two-country, two-good, two-factor model we show that free trade and offshoring have opposite effects on rich-country workers. Free trade hurts rich-country workers, while reducing the volatility of their wages; by contrast, offshoring benefits them, while raising the volatility of their wages. We thus formalize, but also sharply circumscribe, a common critique of globalization.

JEL Codes: F10, F16. Keywords: Offshoring, implicit contracts, invisible handshake.

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A key feature of globalization in recent years, paralleling increases in trade flows, has been the striking increase in international labor-market integration. This is manifested both in foreign direct investment, which allows a firm access to labor in several countries at once,\(^1\) and also in international offshoring\(^2\) of business services.\(^3\)

A parallel phenomenon has been the rise in income volatility for rich-country workers. This was documented in Gottschalk, Moffitt, Katz and Dickens (1994). Recent journalistic accounts of rising economic insecurity among US workers with evidence from individual case studies, survey data, and labor-market data are found in Gosselin (2004) and Hacker (2004). By some measures, volatility of individual earnings in the United States has doubled since the 1970’s. It should be underlined that this is not due merely to higher turnover or restructuring of the economy, as workers in one sector lose jobs and workers in other sectors gain them; Gottschalk et. al. (1994) show that a sharp rise in volatility can be observed even among workers who do not change jobs. A key theme in many accounts of current labor market trends is the claim that the nature of jobs has changed in such a way that jobs are less secure, and the loyalty felt by employers to workers is weaker, than in

\(^1\)In the US case, employment both by inward and outward multinational operations has increased markedly over the past generation. From 1977 to 2001, employment by majority-owned US affiliates of foreign companies grew by 4.7 million, and employment by foreign affiliates of US firms grew by 2.8 million (Mataloni (2004, pp. 53-54)). During this period employment by nonbank foreign affiliates grew from 1.7 percent to 5.6 percent of US employment (Zeile (2003, p.45)).

\(^2\)To clarify, in this paper ‘offshoring’ refers to the hiring of workers in one country by an employer in a different country, a practice sometimes called ‘outsourcing’ in the popular press.

\(^3\)Amiti and Wei (2006), for example, note that offshoring of business services by US manufacturers grew by 6.3 percent per annum in the 1990’s.
For example, sociologist Richard Sennett has described “the ‘casualization’ of the labor force” he has observed in interviews with workers over a thirty-year period (Sennett (2006, p. 48), accompanied by a decline in trust and loyalty in worker-employer relations (pp. 63-72). Journalistic accounts echoing these observations are common; see, for example, Uchitelle (2006, chapter 2) for a history of the rise and fall of long-run implicit contracts between employers and workers in the US labor market. Uchitelle argues that trade pressures of the 1970’s were a major influence in their demise. See also Meyerson (2006), Levine (2006), Uchitelle (2005), and Holstein (2005) for similar views.

We ask in this paper if it is possible that these phenomena may indeed be related, that is, if greater international integration may lead to greater volatility of wages by weakening employment relationships.

We explore this in the context of a simple model of risk-bearing in employment relationships in which complete contracts are unavailable for informational reasons. In this environment, the only way for an employer to share risk with a worker is to develop a long-run relationship in which the firm promises to smooth out (partially or completely) shocks to wages, and the worker in turn promises a long-run commitment to the firm. Such implicit contracts, often called the ‘invisible handshake,’ are enforceable only through the threat that if one reneges, he or she will lose the benefit of the trust on which the relationship was founded, and will need to suffer the whims of the market and search for a new worker (or employer, as the case may be). Integration of one’s country’s labor market with another can make it easier or harder to search for a worker, thus respectively reducing or increasing the potential for risk-sharing relationships, and thus increasing or reducing the volatility

\[ \text{(equation)} \]

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\footnote{These themes loom large in popular opinion, as well. For example, in a recent poll by Greenberg Quinlan Rosner Research (2004), 63% of respondents were ‘very concerned’ or ‘extremely concerned’ about ‘global economic competition and the outsourcing of American jobs’ (p.6). Further, 46% called it the most important or second most important issue concerning them (p.5). The respondents were only slightly more worried about the Iraq war.}
of wages as the case may be.

Here we will comment briefly on related work and then on the distinctive features of our approach.

**Related work.** The literature on international offshoring has followed several strands. Feenstra and Hanson (1996) analyze the allocation of tasks within a complex production process between a skill-abundant home country and a skill-poor foreign country. They show that movement of capital to the foreign country leads to a rise in the number of tasks allocated to foreign workers (‘offshoring’), while at the same time increasing the relative demand for skilled labor in both countries, a hypothesis borne out by the data. Later analyses of equilibrium international offshoring, such as Grossman and Helpman (2005), have been built on incomplete-contracting models; Spencer (2005) provides a survey. Grossman and Rossi-Hansberg (2006) show how offshoring tasks can confer a productivity benefit that can boost domestic wages, and Antràs, Garicano and Rossi-Hansberg (2006) employ a matching model with heterogeneous workers to examine the effects of offshoring on income distribution within both countries.

None of these approaches addresses issues of risk, and so none can shed light on the rising volatility of workers’ incomes – particularly within a job, as seen in the data. In contrast, we are able to do so by drawing on the economics of implicit contracts in the labor market. The importance of such contracts has been well documented empirically; see Beaudry and DiNardo (1991) and McDonald and Worswick (1999) for pioneering work, and Malcomson (1999, section 3) for a survey. Matusz (1985, 1986) explores implicit labor contracts in trade models, but with a different focus from this paper. His papers feature enforceable contracts and risk-averse workers with indivisible labor supply, resulting in rigid wages with positive rates of equilibrium unemployment.
This paper is related to an earlier one by McLaren and Newman (2004), which studied the effect of globalization on risk-sharing in an abstract economy with symmetric agents. Here, by contrast, the asymmetry between workers and employers is the focus, as well as the distribution of income between workers and employers. In addition, that paper, unlike the current paper, confined attention to stationary risk-sharing relationships, which are in general sub-optimal. Moreover, the two-good setup of the present paper allows us to analyze the effects of free trade, which was not possible with the earlier paper. See Kocherlakota (1996) for an extensive analysis of optimal history-dependant risk-sharing relationships. The argument is also related to the literature initiated by Ramey and Watson (2001), showing how improvements in search technology can have perverse effects on incentives.

This exercise is also close in spirit to Thomas and Worrall (1988). They analyze self-enforcing labor contracts between a risk-neutral employer and a risk-averse employee in the presence of an exogenous and randomly fluctuating labor spot market. The employer offers wage smoothing to the employee, implying wages above the spot wage in slumps; in return the worker accepts a wage below the spot market in booms. Both sides know that if either reneges on this agreement, both will be forced to use the spot market from then on. The presence of the spot market generally puts a binding constraint on the amount of insurance the employer can provide. By contrast, in this paper, there is no exogenous spot market, but rather a search pool which either employer or employee can enter at any time. The value of entering the search pool is endogenous, since it depends on how easy it is to find a match and also on how well cooperation works with the new partner once a match has

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6Our approach to finding the optimal contract with a risk-averse worker follows that paper. It should be pointed out that this project adds moral hazard, raising issues studied, for example, in MacLeod and Malcomson (1989).
been found. Thus, this is a general equilibrium exercise, while the Thomas and Worrall model is partial equilibrium in character. The aim is to ask how an increase in international openness would affect wage-smoothing within the firm.

This effect of globalization on wage volatility has been explored in various forms. For example, Rodrik (1997, chapter 2), in the course of a wide-ranging review of the risk effects of globalization, pointed out that globalization can change labor demand elasticities in such a way that the variance of spot wages is increased, an observation that has generated substantial subsequent work. This approach assumes away risk-sharing institutions implicitly. By contrast, we focus on how risk-sharing mechanisms, such as implicit labor contracts, which are endogenously imperfect, can themselves be affected by globalization.

An approach much more closely related to ours is Bertrand (2004). In her model, firms hit with stiff import competition have an increased risk of bankruptcy; a firm whose probability of bankruptcy is high has effectively a higher discount rate, and thus a diminished ability to promise wage-smoothing credibly. This leads to higher wage volatility within a given employment relationship. The effect is shown to have strong empirical support in US data.

**Our approach.** Our model is highly stylized, which is a necessity in order to treat optimal implicit contracts in an open-economy general equilibrium model. Once the logic of this stylized model is understood, it is not difficult to see what would happen in a more realistic and complicated extension. In our model, workers are risk-averse, while the employers are risk-neutral. There are two sectors, a ‘careers sector’ in which production is risky and requires unobservable effort by a

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7Traca (2005) offers an elegant general-equilibrium formalization along these lines, and Scheve and Slaughter (2004) summarize some of the empirical work that has followed.
worker and by an employer, and a ‘spot market sector’ with risk-free Ricardian technology. An employer in the ‘careers sector’ would like to commit credibly to a constant wage, in effect selling insurance at the same time as it purchases labor, but without enforceable contracts it can do so only by reputational means, and so is constrained by its incentive-compatibility constraints.

Workers seeking a career-sector job and employers seeking an employee search until they have a match. Because of the need to elicit effort, wage compensation in the ‘careers sector’ is ‘back-loaded’ in equilibrium; a worker puts in effort today in order to earn compensation that will be due to her tomorrow. This is the same principle analyzed by Lazear (1979) in his analysis of mandatory retirement: When non-contractible effort must be elicited from a worker, it is generally optimal to promise workers wages that increase over time, so that fear of losing high future wages deters shirking. This implies that senior workers receive quasi-rents from their employer. In Lazear’s analysis, this motivates the use of mandatory retirement. Shleifer and Summers (1988) show that the same principle can motivate hostile takeovers, if an acquiring firm is not bound by the commitments made by the incumbent employer to pay the promised high wages to the senior workers. In our paper, we show that this same principle can motivate increased variance of wages as a result of offshoring.

Because of this back-loading of wages, in the ‘careers sector,’ new workers are always cheaper than incumbent ones. This is the source of the firm’s problem: During adverse shocks, when the firm’s profitability is low, if it has promised to pay the same high wage as in good states, it will be tempted to renege, dumping the current worker and picking up a new, cheaper one instead. If it is easy to find a new worker quickly, workers will therefore know not to trust an employer’s promise of wage insurance, and, expecting a low wage in bad times, they will demand a high wage in good
Obviously, international offshoring can be modelled in many different ways. One approach is to assume a complex production process, requiring many tasks, some of which can be performed abroad, as in Feenstra and Hanson (1996) or Grossman and Rossi-Hansberg (2006). Changes in the environment can then raise the fraction of tasks done abroad. These can be thought of as models of partially-integrated labor markets; a rise in the degree of integration leads to more offshoring. In this paper, we simplify the treatment of offshoring by assuming that labor markets across borders are either completely segmented or completely integrated; offshoring is a move from the former to the latter. This simplification allows us to focus on the richness of the implicit contracts. It is clear that one could model offshoring with partially integrated labor markets so as to capture the same sort of effects, at the cost of greater complexity.

There is strong empirical support for effects of this sort. Amiti and Wei (2006) show that US manufacturing sectors that were in a better position during the 1990’s to take advantage of international service offshoring showed sharply better productivity gains than other sectors. This would, of course, be likely to lead to declines in the prices of output for those sectors as required by the story outlined here.

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country workers worse off; offshoring raises the volatility of rich-country wages, but makes rich-
country workers better off. We thus formalize, but also sharply circumscribe, a common critique of
globalization: Offshoring can indeed weaken the invisible handshake, raising the volatility of rich-
country wages, but general equilibrium effects raise expected wages by more than enough to
compensate.

The model can shed light on a number of empirical findings. First, as mentioned above,
Bertrand (2004) showed that US workers whose sector of employment saw a rise in import
penetration tended to see a rise in wage volatility compared to other sectors, which she interpreted
as a response to an increased probability of bankruptcy. In our paper, we show that exactly the same
effects can be obtained in a model without bankruptcy (and of course we analyze the effects of both
offshoring and trade in general equilibrium).10 Second, we can (subject to parameter values)
rationalize a rise in economy-wide average wage volatility as a result of globalization, as documented
(2004) have shown that British workers in sectors with more multinational activity are significantly
more prone to doubt their economic security than those in other sectors – even if the multinational
activity takes the form of inward foreign direct investment. Thus, it appears that labor market
integration raises workers’ perceptions of risk in a way that has nothing to do with the possibility of

10Bertrand (2004) shows a positive correlation between a sector’s import penetration and
wage volatility in that sector (measured as sensitivity of a worker’s wage to current labor-market
conditions). Strictly speaking, our model has only one sector with implicit contracts, the ‘careers
sector,’ and it is an export sector (hence has negative import penetration). However, it is easy to see
that if we had multiple careers sectors, with a role for the invisible handshake in each one, but the
US had a comparative advantage in some but not others, then clearly wage volatility in those sectors
with a comparative disadvantage would be increased by opening up trade and vice versa. This would
be exactly as in Bertrand’s empirical findings.
losing one’s job to a foreign worker. Scheve and Slaughter suggest that the presence of multinationals raises the elasticity of labor demand, thus raising the variance of wages for given variance of shocks; however, as noted above, this ‘elasticity’ interpretation assumes away the possibility of risk-sharing institutions between workers and employers, which themselves can be affected by globalization. In this paper, such institutions, namely the ‘invisible handshake,’ are the focus, and we show that the endogenous response of those institutions to globalization can themselves offer an explanation of the Scheve and Slaughter findings.

We present the formal model in the next section. In the following sections we characterize optimal wage contracts, derive the conditions under which those contracts will exhibit volatile wages, and study the comparative statics of wage volatility. Then, in the final section, we show how the general equilibrium is changed by free trade and offshoring.

1. The Model.

We analyze the questions at hand with a two-good, two-country, two-factor general equilibrium model. In this section, we will describe the key features of the closed-economy version in detail; we will treat the two-country version later.

(i) Production.

Consider first a closed-economy model with two types of agent, ‘workers,’ of which there are a measure $L$, and ‘employers,’ of which there are a measure $E$. There are two sectors. A risk-free
sector, Y, uses only workers, each of whom produces one unit of output per period employed in the sector. This is what was called the ‘spot-market sector’ in the introduction. A second sector, X, which will serve as a numeraire sector, employs both employers and workers. This is what was called the ‘careers sector’ in the introduction. In order for production to occur in this sector, one worker must team up with one employer. We will call a given such partnership a ‘firm.’ In each period, X production requires that a worker and employer must both put in one unit of non-contractible effort. Workers suffer a disutility from effort equal to \( k > 0 \), while employers suffer no such disutility. (Adding a disutility for employers would add to the notation without substantively changing results.) Within a given firm, denote the effort put in by agent \( i \) by \( e^i \in \{0, 1\} \), where \( i = W \) indicates the worker and \( i = E \) denotes the employer. The output generated in that period is then equal to \( R = x_e e^w e^E \), where \( e \) is an idiosyncratic iid random variable that takes the value \( e = G \) or \( B \) with respective probabilities \( \pi_e \), where \( \pi_G + \pi_B = 1 \) and \( x_G > x_B > 0 \). The variable \( e \) indicates whether the current period is one with a good state or a bad state for the firm’s profitability. Of course, since X is the numeraire, output and revenue are equal. The average revenue is denoted by \( \bar{x} = \pi_G x_G + \pi_B x_B \).

Production in the Y sector is straightforward. Each worker in that sector produces one unit of output per period, receiving an income of \( \omega^y \). Since this is a constant-returns-to-scale sector with only one factor, we must have \( \omega^y = p^y > 0 \), where \( p^y \) is the price of Y-sector output.

(ii) Search.

Workers seeking an X-sector employer and X-sector employers seeking a worker search until they have a match. Search follows a specification of a type used extensively by Pissarides (2000).
If a measure $n$ of workers and a measure $m$ of employers search in a given period, then $\Phi(n, m)$ matches occur, where $\Phi$ is a concave function increasing in all arguments and homogeneous of degree 1, with $\Phi(n, m) \leq \min(n, m)$ and $\Phi_{nm} = \Phi_{mn} > 0 \forall n, m$. It is convenient to denote by $Q^E$ the steady-state probability that a vacancy will be filled in any given period, or in other words, $Q^E = \Phi(n, m)/m$, where $m$ and $n$ are set at their steady-state values. Similarly, denote by $Q^W = \Phi(n, m)/n$ the steady-state probability that a searching worker will find an X-sector job in any given period. Search has no direct cost, but for those who are currently in X-sector firms it does have an opportunity cost: If an agent is searching for a new partner, then she is unable to put in effort for production with her existing partner. On the other hand, for workers in the Y sector, there is no opportunity cost to search.\(^{11}\)

Note that since a worker can produce Y without an employer, and can search simultaneously, any worker not currently in an X-sector firm produces Y.

There is also a possibility in each period that a worker and employer who have been together producing X output in the past will be exogenously separated from each other. This probability is given by a constant $(1 - \rho) \in (0, 1)$.

(iii) Preferences.

There is no storage, saving or borrowing, so an agent’s income in a given period is equal to that agent’s consumption in that period.

\(^{11}\)Thus, the X-sector jobs are more challenging jobs that require a worker’s full attention, while Y-sector jobs are more casual, and permit a worker to earn an income while searching for something else. Adding an opportunity cost to search in the Y sector would add complexity without adding anything of real importance.
Employers. All employers have the same linear homogeneous quasi-concave per-period utility function, \( U(c^X, c^Y) \), defined over consumption \( c^X \) and \( c^Y \) of goods X and Y, respectively. This yields indirect utility function \( v(I, p^X, p^Y) = I/\Gamma(p^X, p^Y) \), where \( I \) denotes income; \( p^X \) and \( p^Y \) denote the prices of the two goods respectively; and \( \Gamma \) is a linear homogenous function that generates the consumer price index derived from the utility function \( U \). (In other words, \( \Gamma(p^X, p^Y) \) is the minimum expenditure required to obtain unit utility with prices \( p^X \) and \( p^Y \).) Recalling that X is our numeraire sector, we have \( p^X = 1 \), and it is convenient to write the consumer price index as \( P(p^Y) = \Gamma(1, p^Y) \). Note that by Shephard’s Lemma, the elasticity of \( P(p^Y) \) with respect to \( p^Y \) is equal to good \( Y \)’s share in consumption.

Workers. All workers have the same per-period utility function \( \mu(U(c^X, c^Y)) \) over consumption of goods X and Y. The function \( \mu \) is a strictly increasing and strictly concave von-Neumann-Morgenstern utility function. Thus, using the notation developed just above, if in a given period a worker receives a wage \( \omega \) and faces a consumer price index of \( P = P(p^Y) \), then the worker’s utility for that period is given by \( \mu(\omega/P) \).

In other words, workers are risk-averse and employers are risk-neutral, but both will exhibit the same demand behavior for a given income.

(iv) Goods market clearing.

In each period, the total amount of each good produced must equal the amount consumed. Since given the relative price \( p^Y \) both workers and employers will consume X and Y in the same proportions, this amounts to the condition that \( p^Y = U_2(1, r)/U_1(1, r) \), where the subscripts denote
partial derivatives, and \( r \) denotes the ratio of \( Y \) production to \( X \) production. In other words, the relative price must be equal to the marginal rate of substitution between the two goods determined by the production ratio. We assume that \( U_2(1, r) \to -\infty \) as \( r \to 0 \), and \( U_1(1, r) \to -\infty \) as \( r \to \infty \), which (given that \( U \) is quasi-concave and hence the marginal rate of substitution is strictly decreasing in \( r \)) implies a unique, market-clearing value of \( p^r \in (0, \infty) \) for any \( r \in (0, \infty) \). Further, \( p^r \) is strictly decreasing in \( r \).

\( (v) \) Sequence of events.

The sequence of events within each period is as follows. (i) Any existing matched employer and worker in the \( X \) sector learn whether or not they will be exogenously separated this period. (ii) The profitability state \( \epsilon \) for each \( X \)-sector firm is realized. Within a given employment relationship, this is immediately common knowledge. The value of \( \epsilon \) is not available to any agent outside of the firm, however. (iii) The wage, if any, is paid (a claim on the firm’s output at the end of the period). (iv) The employer and worker simultaneously choose their effort levels \( e^i \). At the same time, the search mechanism operates. Within an \( X \)-sector firm, if \( e^i = 0 \), then agent \( i \) can participate in search. At the same time, all \( Y \)-sector workers search. (v) Each \( X \)-sector firm’s revenue, \( R \), is realized, and profits and consumption are realized.\(^{13} \) (vi) For those agents who have

\(^{12}\)Obviously, in the closed-economy version of the model \( r \) will refer to the ratio of domestic \( Y \) and \( X \) production, while in the open-economy version the world output ratio will be the relevant variable.

\(^{13}\)Strictly speaking, there is the possibility, off of the equilibrium path, that the firm’s output will be zero because one or the other party has shirked, raising the question of how the wage claim issued in sub-period (iii) can be redeemed. This issue could be eliminated by assuming that, rather than zero output, the employer is able to produce some positive output, say, \( x_{min} > 0 \), even without a worker. The wages can be paid out of that output at the end of the period. The interpretation of
found a new potential partner in this period’s search, new partnerships with a new self-enforcing agreement are formed. This is achieved by a take-it-or-leave-it offer made by the employer to the worker.

We will focus on steady-state equilibria. In such an equilibrium, the expected lifetime discounted profit of an employer with vacancy is denoted $V_{ES}$ and the expected lifetime discounted utility of a searching worker is denoted $V_{WS}$, where the ‘$S$’ indicates the state of searching. Similarly, we can denote by $V_{ER}$ and $V_{WR}$ the lifetime payoffs to employers and workers respectively evaluated at the beginning of a cooperative X-sector relationship. Naturally, we must have $V_{WR} \geq V_{WS}$ in equilibrium, or no worker will accept an X-sector job. The values $V_{ij}$ are endogenous, as they are affected by the endogenous probability of finding a match in any given period and by the endogenous value of entering a relationship once a match has been found. However, any employer will take them as given when designing the wage agreement. We can write:

\[
V_{WS} = \mu(w^y/P) + Q^w \rho \beta V_{WR} + Q^w(1 - \rho) \beta V_{WS} + (1 - Q^w) \beta V_{WS},
\]

\[
V_{ES} = Q^e \rho \beta V_{ER} + Q^e(1 - \rho) \beta V_{ES} + (1 - Q^e) \beta V_{ES}.
\]  

(1)

The Y-sector worker’s payoff from search is the current Y-sector wage plus the continuation values if the worker finds X-sector work and is not immediately separated, finds X-sector work and is immediately separated, or fails to find X-sector work. The payoff from search for an X-sector employer with vacancy is given by the continuation value if the employer finds a worker who is not

$x_G$ and $x_B$ is, then, the additional output that is produced in cooperation with a non-shirking worker. This would require carrying this additional piece of notation throughout the analysis, but would not change any of our qualitative results.
immediately separated, finds a worker who is immediately separated, or fails to find a worker. If an X-sector worker, or an X-sector employer who already has a worker, chooses to search, the payoff will be the same as in (1), except for a straightforward change in the first-period payoff.

Given those values, a self-enforcing agreement between a worker and an employer is simply a sub-game perfect equilibrium of the game that they play together. We assume that the employer has all of the bargaining power, so the agreement chosen is simply the one that gives the employer the highest expected discounted profit, subject to incentive constraints (we discuss briefly the consequences of relaxing this assumption in footnote 22, Section 5). Without loss of generality, we will assume that the ‘grim punishment’ is used, meaning here that if either agent defects from the agreement at any time, the relationship is severed and both agents must search for new partners. Thus, the payoff following a deviation would be $V^{ES}$ for an employer and $V^{WS}$ for a worker.

To sum up, risk-neutral employers search for risk-averse workers, and when they find each other, the employer offers the worker the profit-maximizing self-enforcing wage contract, which then remains in force until one party reneges or the two are exogenously separated. This pattern provides a steady flow of workers and employers into the search pool, where they receive endogenous payoffs $V^{WS}$ and $V^{ES}$. These values then act as parameters that constrain the optimal wage contract.

The analysis will proceed as follows. We will characterize optimal labor contracts in the X sector. It turns out that optimal contracts are very much affected by the values of $p^v$ and $Q^E$. We will show how they change as we vary $p^v$ and $Q^E$ exogenously, and then we will show how $p^v$ and $Q^E$ are determined endogenously, to complete the general equilibrium analysis. We then will examine how these two values change with international integration of: (i) goods markets, and then (ii) labor markets, to see how the behavior of wages is affected by globalization.
We first turn to the form of optimal contracts.

2. The form of optimal contracts in the X sector.

In general, optimal incentive-constrained agreements in problems of this sort can be quite complex because the specified actions depend on the whole history of shocks and not only the current one. (See Thomas and Worrall (1988) and Kocherlakota (1996).) In analyzing the equilibrium, it is useful to note that in our model the employment contracts offered by employers always take one of two very simple forms, which we will call ‘wage smoothing’ and ‘wage volatility.’ Derivation of this property is the purpose of this section.

The equilibrium can be characterized as the solution to a recursive optimization problem. Denote by \( \Omega (W) \) the highest possible expected present discounted profit the employer can receive in a subgame-perfect equilibrium, conditional on the worker receiving an expected present discounted payoff of at least \( W \). Arguments parallel to those in Thomas and Worrall (1988) can be used to show that \( \Omega \) is defined on an interval \([W_{\text{min}}, W_{\text{max}}]\) and is decreasing, strictly concave, and differentiable, where \( W_{\text{min}} \) and \( W_{\text{max}} \) are respectively the lowest and highest worker payoffs consistent with a subgame-perfect equilibrium of the game played by an employer-worker pair. This function must satisfy the following functional equation:
\[
\Omega(W) = \max_{\{\omega_{\epsilon}, \bar{W}_{\epsilon}\}} \sum_{\epsilon \in G,B} \pi_{\epsilon} \left( x_{\epsilon} - \omega_{\epsilon} + \rho \beta \Omega(\bar{W}_{\epsilon}) + (1 - \rho) \beta V^{ES} \right) \tag{2}
\]

subject to

\[
x_{\epsilon} - \omega_{\epsilon} + \rho \beta \Omega(\bar{W}_{\epsilon}) + (1 - \rho) \beta V^{ES} \geq V^{ES}, \tag{3}
\]

\[
\mu(\omega_{\epsilon}/P) - k + \rho \beta \bar{W}_{\epsilon} + (1 - \rho) \beta V^{WS} \geq V^{WS} - \mu(\omega_{\epsilon}/P) + \mu(\omega_{\epsilon}/P), \tag{4}
\]

\[
\sum_{\epsilon \in G,B} \pi_{\epsilon} \left[ \mu(\omega_{\epsilon}/P) - k + \rho \beta \bar{W}_{\epsilon} + (1 - \rho) \beta V^{WS} \right] \geq W, \tag{5}
\]

\[
W_{\text{min}} \leq \bar{W}_{\epsilon} \leq W_{\text{max}}, \text{ and} \tag{6}
\]

\[
\omega_{\epsilon} \geq 0. \tag{7}
\]

The right-hand side of (2) is the maximization problem solved by the employer. She must choose a current-period wage \(\omega_{\epsilon}\) for each state \(\epsilon\), and a continuation utility \(\bar{W}_{\epsilon}\) for the worker for subsequent periods following that state. Constraint (3) is the employer’s incentive compatibility constraint: If this is not satisfied in state \(\epsilon\), then the employer will in that state prefer to renege on the promised wage, understanding that this will cause the worker to lose faith in the relationship and sending both parties into the search pool. Constraint (4) is the worker’s incentive compatibility constraint. The left-hand side is the worker’s payoff from putting in effort in the current period, collecting the wage, and continuing the relationship.\(^{14}\) The right-hand side is the payoff from

\(^{14}\)Note that we are assuming that a worker cannot receive a Y-sector wage while searching if that worker is shirking on an X-sector job. This makes sense if, for example, effort is not observable and third-party verifiable but physical presence on the job site is, and a worker can search while physically at the X-sector job site but cannot produce Y-sector output while there. Thus, an
shirking and searching, in which case the worker’s payoff is the same as it would be if she were in
the Y sector except that in the current period her income is $\omega_e$ instead of $\omega^y$. If this constraint is not
satisfied, the worker will prefer to shirk by searching instead of working. Constraint (5) is the
target-utility constraint. In the first period of an employment relationship, the employer must
promise at least as much of a payoff to the worker as remaining in the search pool would provide.
Thus, in that case, denoting the target utility at the beginning of the relationship by $W_0$, we have
$W = W_0 = V^{WS}$ (and so $V^{ER} = \Omega (V^{WS})$). Thereafter, the employer will in general be bound by
promises of payoffs she had made to the worker in the past. Finally, (6) and (7) are natural bounds
on the choice variables.

Constraint (4) can be replaced by the more convenient form:

$$\tilde{W}^* \geq \tilde{W},$$

where

$$\tilde{W}^* = \left[ (1 - (1 - \rho)\beta \right] V^{WS} - \mu (\omega^y / P) + k] / \rho \beta.$$  \hspace{1cm} (4)'

The value $\tilde{W}^*$ is the minimum future utility stream that must be promised to the worker in order to
convince the worker to incur effort and forgo search. Given that $V^{WS} \leq V^{WR}$ in equilibrium, it is easy
to see from (1) that $\tilde{W}^* > V^{WS}$.

Let the Kuhn-Tucker multiplier for (3) be denoted by $\psi_e$, the multiplier for (4)' by $\nu_e$, and
the multiplier for (5) by $\lambda$. The first-order conditions with respect to $\omega_e$ and $\tilde{W}_e$ respectively are:

$X$-sector employer would be able to sue to recover the wage just paid if the worker was absent,
working another job, instead of on site at the location of the $X$ firm.

Throughout, we will assume that it is optimal to induce the worker to exert effort in each
state as long as the employment relationship continues. This is clearly the case in a substantial
portion of the parameter space, and so we are implicitly restricting attention to that portion. We will
comment in footnote 20, Section 5 on the parameter restrictions implicit in this assumption.
Formally, note that if the target-utility constraint (5) does not bind in the first period, so that 
the worker’s utility exceeds $V$, then $\psi = 0$, and (8) cannot bind with any positive wage. Therefore, 
the first-period wage will be equal to zero in both states. Then, by Lemma 1, if (5) does not bind, 
then the lower bound of (6) will not bind either, and so (9) will hold with equality; with $\psi = 0$, this 
implies $< 0$, which in turn implies that $W = W_{\text{max}}$ is never an optimal choice, and so we will ignore that case.)

The following lemma is proven in the appendix:

**Lemma 1.** $W_{\text{min}} = V_{\text{WS}}$.

In other words, it is feasible for the employer to push the worker’s payoff down to the opportunity payoff at the beginning of the employment relationship. Since it is in the interest of the employer to do so, Lemma 1 makes clear that workers joining X-sector employment receive the same payoff that they would receive in the Y sector, or in other words, $V_{WR} = V_{WS}$.\(^\dagger\) From (1), this immediately tells us:\(^\dagger\)

\[^\dagger\]Formally, note that if the target-utility constraint (5) does not bind in the first period, so that the worker’s utility exceeds $V_{WS}$, then $\lambda = 0$, and (8) cannot bind with any positive wage. Therefore, the first-period wage will be equal to zero in both states. Then, by Lemma 1, if (5) does not bind, then the lower bound of (6) will not bind either, and so (9) will hold with equality; with $\lambda = 0$, this implies $v > 0$, which in turn implies that $\bar{v} = \bar{v}$ in both states. Substituting this with the zero first-period wages into the left-hand side of (5) shows that the worker’s payoff will be below $V_{WS}$, a contradiction.

\[^\dagger\]Of course, this implies that, in equilibrium, Y-sector workers are indifferent between searching and not searching, so if a small search cost were imposed, there would be no search (this is a version of the Diamond search paradox). However, this feature would disappear if any avenue
and (4)' can be rewritten as:

\[ \bar{w}_c \geq \tilde{w}^*, \text{ where } \tilde{w}^* = \frac{\mu (\omega^y / P)}{1 - \beta} + \frac{k}{p\beta}. \]  

Further, since \( \tilde{w}^* > V^{WS} \), Lemma 1 tells us that (6) is redundant, so it will be ignored henceforth. As a result, (9) will always hold with equality.

To sum up, in each period the employer maximizes (2), subject to (3), (4)'', (5), and (7). In the first period of the relationship, the worker’s target utility \( W = W_0 \) is given by \( V^{WS} \), but in the second period it is determined by the values of \( \bar{w}_c \) chosen in the first period and by the first-period state, and similarly in later periods it is determined by choices made at earlier dates. We impose an assumption:

**Assumption 1.** In the first period of an employment relationship, the employer’s incentive-compatibility constraint (3) does not bind in either state.

We will discuss sufficient conditions for this later (in footnote 20 of Section 5). We can now
prove that under Assumption 1, the equilibrium always takes the same simple form: A one-period ‘apprenticeship’ in which the Y-sector wage $\omega^y$ is paid, followed by a time- and history-invariant but perhaps state-dependent wage. The key idea is that it is never optimal to promise more future utility than is required to satisfy the worker’s incentive constraint (4), so after the first period of the relationship, the worker’s target utility is always equal to $\tilde{w}^*$. This means that after the first period, the optimal wage settings by the firm are stationary. We can now establish a detailed proof through the following two propositions.

**Proposition 1.** Consider the first period of an employment relationship. If the employer’s incentive-compatibility constraint does not bind in either state, the first-period wage is set equal to $\omega^y$ in each state and the continuation payoff for the worker in each state is set equal to $W^#_\epsilon$.

**Proof.** Suppose, first, that the worker’s incentive compatibility constraint does not bind in state $\epsilon$ in the first period. Then $\psi_\epsilon = 0$, and since $\psi_\epsilon = 0$ because of Assumption 1, (9) becomes:

$$
\Omega'(\tilde{W}_\epsilon) + \lambda = 0.
$$

(Recall that as seen in the proof of Lemma 1, (9) holds with equality). Since by the envelope theorem, $\Omega'(W_0) = -\lambda$, this and the concavity of $\Omega$ imply that $\tilde{w}_\epsilon \leq W_0 = V^{ws}$. But since $V^{ws} < \tilde{w}^*$, this implies that the worker’s incentive compatibility constraint (4)” will be violated, a contradiction. Therefore, the worker’s incentive compatibility constraint must bind in each state, ensuring that
\( \bar{w}_e = \bar{w}^* \). Given that \( \bar{w}_e = \bar{w}^* \) and \( W_0 = V^{WS} \), the target utility constraint (5) is exactly satisfied by setting the wage in each state in the first period equal to \( \omega^\gamma \). Therefore, \( \omega^\gamma \) is the minimum first period wage required to make the worker willing to accept the job. The condition (8), with \( \psi_\epsilon = 0 \), then ensures that it is indeed optimal to pay the same wage in both states. \textbf{Q.E.D.}

Now we can use the fact that the worker’s target utility for the second period of the relationship (denoted as \( \bar{w} \) in (2)) is equal to \( \bar{w}^* \) to characterize the equilibrium from that point forward.

**Proposition 2.** Under the conditions stated for Proposition 1, there is a pair of values \( \omega^*_\epsilon \) for \( \epsilon = G, B \) such that in the second period and all subsequent periods of an X-sector employment relationship regardless of history (provided neither partner has shirked), the wage \( \omega^*_\epsilon \) is paid whenever the state is \( \epsilon \). In addition, the worker’s continuation payoff is always equal to \( \bar{w}^* \). Further, after the first period there are three possible cases:

(i) The employer’s incentive compatibility constraint (3) never binds, and \( \omega^*_G = \omega^*_B \).

(ii) The employer’s incentive compatibility constraint (3) binds in the bad states but not in the good states, and \( \omega^*_G > \omega^*_B \).

(iii) The employer’s incentive compatibility constraint (3) always binds, and \( x_G - \omega^*_G = x_B - \omega^*_B \).

**Proof:** See appendix.
As a result, we need concern ourselves with only two types of possible equilibrium wage contracts: The type that features $\omega^*_G = \omega^*_b$ after the first period, which we will call *wage-smoothing* agreements; and the type with $\omega^*_G > \omega^*_b$ after the first period, which we will call *fluctuating-wage* agreements.

To sum up, if the employer’s incentive constraint does not bind, the worker goes through an ‘apprenticeship period’ at the beginning of the relationship, followed by a constant wage. If the employer’s constraint ever binds, then it binds only (and always) in the bad state, resulting in a fluctuating-wage equilibrium. Otherwise, the wage is constant after the apprenticeship. Now, the natural question is under which conditions the employer’s bad-state incentive constraint will bind. We address this next.


In the case of a wage-smoothing agreement, the wage paid can be computed by substituting (4)" and (10) into (5) with equality (setting the target utility $W$ equal to $\bar{w}^*$). This determines the equilibrium wage as the unique solution to:

$$\mu(\omega^*/P) = \mu(\omega^y/P) + k/p\beta.$$  

(11)

We will henceforth call this the ‘efficiency wage,’ and denote it by $\omega^*$. This is clearly the lowest wage that, if credibly promised to be paid at all dates in the future conditional on no shirking, will
induce the worker to apply effort.

Here, we show that for given parameters if it is sufficiently difficult for an employer to find a new worker or if Y-sector output is sufficiently cheap, the equilibrium involves wage smoothing. Otherwise, it involves a fluctuating wage.

First, note that the wage-smoothing agreement is preferred by the employer whenever it is feasible (because with risk-averse workers, wage smoothing delivers the required incentives to workers with a lower expected wage). Therefore, if we assume a wage-smoothing equilibrium and then compute the values $V^{ES}$ and $\Omega (\bar{\nu}^*)$ that it implies, we can check if the bad-state employer’s incentive constraint (3) is satisfied. If it is, wage-smoothing will occur, and otherwise, it will not.

We can now find $V^{ES}$ as follows:

$$V^{ES} = Q^E \rho \beta (\Omega (\bar{\nu}^*) + \omega^* - \omega^y) + Q^E (1 - \rho)\beta V^{ES} + (1 - Q^E) \beta V^{ES}.$$  \hspace{1cm} (12)

Note in addition that:

$$\Omega (\bar{\nu}^*) = [\bar{x} - \omega^* + (1 - \rho)\beta V^{ES}] / (1 - \rho \beta).$$  \hspace{1cm} (13)

If we substitute (13) into (12) and rearrange, we get:

$$V^{ES} = \{Q^E \rho \beta / [(1 - \beta)[1 - \rho \beta (1 - Q^E)]]\}(\bar{x} - \rho \beta \omega^* - (1 - \rho \beta)\omega^y).$$ \hspace{1cm} (14)

It is easy to verify that this is increasing in $Q^E$ and decreasing in $\omega^y = p^y$. 

Now, the employer’s incentive constraint in the bad state is:

\[ x_b - \omega^* + \rho \beta \Omega(\tilde{w}^*) + (1 - \rho)\beta V^{ES} \geq V^{ES}. \]

Using (13), this becomes:

\[ x_b - \omega^* + \rho \beta (\bar{x} - x_b) \geq (1 - \beta) V^{ES}, \text{ or} \]
\[ x_b - \omega^* + \rho \beta \pi_g (x_g - x_b) \geq (1 - \beta) V^{ES}. \]  \hspace{1cm} (15)

This condition allows us to identify the conditions under which wage smoothing will occur:

**Proposition 3.** For given \( p^r \), there is a value \( Q^{E}(p^r) \) such that if \( Q^{E} < Q^{E}(p^r) \) a wage-smoothing equilibrium can be sustained, while if \( Q^{E} > Q^{E}(p^r) \) it cannot. Further, \( Q^{E}(p^r) \) is decreasing in \( p^r \).

**Proof.** The value \( Q^{E}(p^r) \) can be defined for any \( p^r \) as the solution for \( Q^{E} \) to

\[ x_b - \omega^* + \rho \beta \pi_g (x_g - x_b) = (1 - \beta) V^{ES}. \]

Taking total derivatives with respect to \( Q^{E} \) and \( p^r \), using (11) and \( \omega^r = p^r \) to obtain the derivative of \( \omega^r \) with respect to \( p^r \), gives the result. \textbf{Q.E.D.}

The function \( Q^{E}(p^r) \) is shown by the \( VV \) curve in Figure 1. Values of \( Q^{E} \) and \( p^r \) above and
to the right of this curve are points imply that equilibrium X-sector wages must be volatile.

At this point it may be useful to review how the pieces fit together. *Workers in the X sector are promised higher future wages in order to motivate current effort.* Thus, in a wage-smoothing equilibrium, the worker is paid the opportunity wage $\omega^r$ during the ‘apprenticeship’ of the first period, and then the higher efficiency wage $\omega^e$ thereafter. For this reason, an incumbent worker is always more expensive than a new one, although they have the same productivity. *Employers in the X sector thus are always to some degree tempted to shirk on their commitment to their incumbent workers and search instead for a new one; this temptation is strongest in bad states when the worker’s productivity is low.* If this temptation is strong enough, the wage-smoothing equilibrium is untenable, because workers will know that X-sector employers will not honor their promises. *This happens when it is easy to find a new worker, or when $Q^E$ is high.* That is why points to the right of the $VV$ curve imply equilibrium with wage volatility.

We turn to those fluctuating-wage equilibria next.

4. *Fluctuating-wage equilibria.*

In a fluctuating-wage equilibrium, the two state-dependant wages are determined by the worker’s binding incentive-compatibility constraint and the employer’s binding bad-state incentive constraint. This first of these conditions can be simplified by substituting (4)$''$ and (10) into (5) with equality (setting the target utility $W$ equal to $\bar{w}^*$) to obtain:
In other words, (16) states that the expected utility promised to an X-sector worker in any period after the first must be enough to compensate that worker next period, in expected value, for the current disutility of effort. Equation (16) is represented in Figure 2 by the downward-sloping curve WW. The figure measures the bad-state wage $\omega_b$ on the vertical axis and the good-state wage $\omega_g$ on the horizontal axis. This curve is strictly convex due to the worker’s risk aversion.

The second of these conditions can be derived from the employer’s binding bad-state incentive constraint:

$$x_b - \omega_b^* + \rho \beta \Omega (\bar{y}^*) + (1 - \rho) \beta V^{ES} = V^{ES}.$$ 

Developing expressions for $V^{ES}$ and $\Omega (\bar{y}^*)$ analogous to (14) and (13) and substituting them into (17) yields the equation:

$$\omega_b = [\rho \beta \pi_G \omega_g + Q^E \rho \beta \omega^y + x_b + \rho \beta (1 - Q^E) \pi_G (x_g - x_b)] / [1 - \rho \beta (\pi_G - Q^E)],$$

which is depicted in Figure 2 as the straight downward-sloping line EE.

The intersection of WW with the 45°-line is the efficiency wage, $\omega^*$, and any movement along the WW curve toward that point represents an increase in the employer’s profits, because it implies a lower expected wage. The downward-sloping line EE is the employer’s incentive-compatibility constraint in the bad state. Any equilibrium pair of wages must lie on or above WW and on or below
Of course, if the two curves do not intersect at all, no cooperation is possible. It will be shown that this occurs in a portion of the parameter space to the right of curve $BB$ in Figure 1. It is also possible that the two curves are tangent, which occurs only on the curve $BB$, and thus in a zero-measure portion of the parameter space. We focus our attention on the portions of the parameter space where $EE$ and $WW$ have a non-vanishing region of intersection, as shown.

We are focusing here on the fluctuating-wage case, so by assumption, the constant-wage outcome is not sustainable. Therefore, we know that the intersection of $EE$ with the 45°-line occurs below the intersection of $WW$ with the 45°-line. Further, since we have shown that in equilibrium the good-state wage is never below the bad-state wage, the two curves must intersect below the 45°-line. Given the concavity of $WW$ and the linearity of $EE$, there will clearly be two such intersections, but the one that will be chosen by the firm is the one closest to the 45°-line, as shown, because it will offer the lowest expected wage consistent with the constraints. This means that at the point of intersection that determines $\omega_g$ and $\omega_G$, $EE$ is flatter than $WW$. As a result, it is clear that anything that shifts the $EE$ line down without shifting $WW$ will raise $\omega_G$ and lower $\omega_g$. In addition, it is useful to note that, since the $WW$ curve is a worker indifference curve, holding $k$ constant, anything that shifts down the $WW$ line (whether or not it shifts the $EE$ line) lowers worker welfare.

It can easily be verified by differentiating (18) that a rise in $Q^E$ will shift the $EE$ down. Clearly, it has no effect on $WW$. Therefore, we have the following:

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$^{18}$Of course, if the two curves do not intersect at all, no cooperation is possible. It will be shown that this occurs in a portion of the parameter space to the right of curve $BB$ in Figure 1. It is also possible that the two curves are tangent, which occurs only on the curve $BB$, and thus in a zero-measure portion of the parameter space. We focus our attention on the portions of the parameter space where $EE$ and $WW$ have a non-vanishing region of intersection, as shown.
Proposition 4. If the equilibrium has fluctuating wages, an increase in $Q^E$ holding $\omega^y = p^y$ constant will raise $\omega_c$ and lower $\omega_b$, in the process raising average X-sector wages, but having no effect on worker welfare.

A rise in $Q^E$ increases the volatility of X-sector wages, by making it easier to find a replacement worker and thus sharpening the temptation to renege on promises to an incumbent worker in a bad profitability state. Thus, an improvement in the ease with which an employer can find a new worker has a negative indirect effect on profits in the form of higher expected wages, in addition to the positive direct effect.

At the same time, a rise in $p^y$ will shift both curves upward. The $WW$ curve shifts up because the worker’s opportunity cost has risen. The $EE$ curve shifts up because, for given $\omega_c$ and $\omega_b$, the rise in the workers’ opportunity cost lowers the degree to which new workers are cheaper than incumbents (recall that a new worker is paid her opportunity wage $\omega^y$ in the first period of employment). The net effect on wages can be signed as follows.

Proposition 5. If the equilibrium has fluctuating wages, an increase in $p^y$ will raise $\omega_c$ and lower $\omega_b$, in the process raising average X-sector wages and X-sector worker utility.

Proof: See appendix.

A rise in $p^y$ increases the volatility of X-sector wages, by increasing the opportunity cost of X-sector workers, which lowers the joint surplus available to a worker and employer in the X sector.
and also lowers the share of the surplus that can be captured by the employer. This sharpens the employer’s incentive-compatibility constraints. Note the striking force of the sharpened incentive constraint: Even though the worker’s opportunity wage increases, the wage paid by an X employer in the bad state falls. This is because the employer’s temptation to cheat is strongest in the bad state, and that temptation is increased by the rise in the worker’s opportunity cost.

These results can be summarized in Figure 1 by observing that any movement up and to the right from a point above the locus $VV$ must result in an increase in wage volatility. Further, any movement upward will raise the welfare of workers in both sectors, while any horizontal movement will leave worker welfare unchanged.

Note that if $Q^E$ and $p^y$ are close to the $VV$ curve in Figure 1, $\omega^*_G$ is close to $\omega^*_B$, so $x_G - \omega^*_G > x_B - \omega^*_B$. Further, from Proposition 4, as we increase $Q^E$ holding $p^y$ constant, $\omega^*_G$ rises and $\omega^*_B$ falls, so that either we reach the limit $Q^E = 1$ with the inequality $x_G - \omega^*_G > x_B - \omega^*_B$ still true, or there exists a value $Q^E_{BB}(p^y)$ such that $x_G - \omega^*_G = x_B - \omega^*_B$ at that value of $Q^E$ and $x_G - \omega^*_G < x_B - \omega^*_B$ for higher values. The function $Q^E_{BB}(p^y)$ is represented in Figure 1 by the curve $BB$. Clearly, the employer’s incentive-compatibility constraint will bind in both states if and only if the $Q^E$ and $p^y$ combination lies on the curve $BB$. Further, by Propositions 4 and 5, $BB$ must be downward-sloping.

We can now use the process of elimination to characterize equilibrium at each point in Figure 1. By Proposition 3, any point below $VV$ implies wage smoothing. Any point between $VV$ and $BB$ implies wage volatility, with the employer’s constraint binding in the bad state but not in the good state. Any point on $BB$ implies wage volatility with the employer’s constraint binding in both states. Any point to the right of $BB$ implies that equilibrium with X-sector production requires the
employer’s constraint to bind in the good state but not in the bad state, which by Proposition 2 is impossible. Therefore, under our assumptions it is not possible to have an equilibrium with $X$ production under all states for points to the right of $BB$.

Of course, in general equilibrium $Q^E$ and $p^y$ are both endogenous. We turn to this in the next section, which allows us to analyze the full equilibrium and how it changes with globalization.

5. General equilibrium, and the Effects of Globalization.

Suppose that we now have two countries. Call the first the ‘US’ and the second ‘India.’ The US has $E$ employers and $L$ workers, while India has $E^*$ employers and $L^*$ workers. Assume that

\[ E/L > E^*/L^*, \]

so that workers are relatively abundant in India.

There are three possible states to concern us: Autarky, in which there is no integration of goods or factor markets; free trade, in which goods markets but not factor markets are integrated; and full integration, in which both goods and factor markets are integrated. We will call the movement from the second to the third of these states ‘offshoring,’ since it simply means that now employers in one country are free to hire workers from another. Thus, globalization conceptually has two distinct components, and we will see that the effects of trade per se on wage volatility are very different from the effects of offshoring.
First, we will consider the steady state under autarky, which here means simply that American employers can match only with American workers; Indian employers can match only with Indian workers; and in each country, the quantities of each good produced must be equal to the quantities consumed.

We need to derive the equilibrium value of $Q^E$. Recall that the total number of employers searching for a worker in any one period is denoted $m$, the total number of workers searching for a new X-sector employer is denoted $n$, and in any period $\Phi(n, m)$ matches occur. Therefore, the fraction of searching employers who find workers is $Q^E = \Phi(n, m)/m = \Phi(n/m, 1)$, hence an increasing function of $n/m$. The steady-state level of searching employers therefore must satisfy the following equation:

$$m = m (1 - \Phi(n/m, 1)) + (1 - \rho) (E - m) + (1 - \rho) m \Phi(n/m, 1).$$

The first term on the right-hand side represents vacancies for which no worker was found; the second represents firms currently with workers who are exogenously separated from them; and the last term represents firms that find a worker to fill a vacancy but are immediately exogenously separated from them.

This can be simplified to:

$$m = E - (\rho / (1 - \rho)) m \Phi(n/m, 1).$$  \hspace{1cm} (19)

Similarly,
\[ n = L - (\rho / (1 - \rho ))m\Phi(n/m, 1). \] (20)

This can be used to show the following.

**Proposition 6.** For any value of \( E/L \), the steady-state value of \( n/m \) and hence \( Q^E \) is uniquely determined. We can thus write \( Q^E(E/L) \). Further, \( Q^E(E/L) \) is strictly decreasing.

**Proof:** See appendix.

Thus, holding other parameters constant, when workers are more scarce, it is more difficult for an employer to find one to match with.

Next, we need to determine \( p^\nu \). For this, given the identical and homothetic demands held by consumers in both countries, it will be sufficient to determine relative supplies of the two goods:

**Proposition 7.** Under autarky, the steady-state supply of \( X \) output is an increasing and linear homogeneous function of \( E \) and \( L \), while the steady-state supply of \( Y \) output is decreasing in \( E \), increasing in \( L \), and linear homogeneous in \( E \) and \( L \). Therefore, the relative supply of \( Y \)-sector output, \( r \), is a decreasing function of \( E/L \), and the relative price \( p^\nu \) of \( Y \)-sector output is an increasing function of \( E/L \).

**Proof:** See appendix.
Propositions 6 and 7 can be illustrated with the help of Figure 3, which is the same as Figure 1 except for the addition of the downward-sloping curve $MM$. This curve gives the combinations of $Q^E$ and $p^y$ obtained in an autarkic economy by varying $E/L$ over the positive real line. The $MM$ curve is, then, the locus of market-clearing values that complete the general equilibrium in the autarkic case. The fact that $Q^E$ is decreasing in $E/L$ while $p^y$ is increasing guarantees that $MM$ must indeed be downward-sloping. In other words, from the top left-hand of the $MM$ curve to the bottom right-hand end, we move from labor-scarce economies (with high $E/L$), where the labor-intensive good is expensive and it is difficult to find a worker, to labor-abundant ones.

Note that as goods $X$ and $Y$ become very close substitutes, $MM$ becomes arbitrarily flat, while as they approach the case of perfect complementarity it becomes arbitrarily steep. Therefore, the

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19More precisely, for a given value of $E/L$ for an autarkic economy, we can find the steady-state value of $Q^E$ (as in Proposition 6) and the steady-state value of the ratio of $Y$ to $X$ supplied, hence the equilibrium relative price $p^y$ (as in Proposition 7). Tracing out the $Q^E$ and $p^y$ values so generated produces the $MM$ curve as we vary $E/L$.

20We can now also clarify the conditions under which (i) it is optimal to elicit effort in all states, and (ii) Assumption 1 holds. (i) In the region of the parameter space where the firm’s incentive constraints do not bind, the optimal contracts we analyze are the same as would be chosen under full commitment. Therefore, for points on or to the left of the $VV$ curve, if it is optimal to elicit production under all states with full commitment, it is also optimal without commitment, as in our model. A sufficient condition for this is that $\rho \beta \omega^* < x_B$. (ii) It is easy to verify that the wage-smoothing condition (15) is a strictly stronger condition than Assumption 1, since the strictly higher worker target utility in the second and later periods of the relationship, compared with the first period, make it more likely that the employer’s incentive constraints will bind. Therefore, for the whole length of the $MM$ curve to the left of $VV$ and for at least a segment of positive length to the right of $VV$, Assumption 1 will be satisfied. If it is also true that at the intersection of $MM$ and $VV$, $\rho \beta \omega^* < x_B$ holds, then there is a segment of $MM$ including its intersection with $VV$ plus some distance on both sides in which Assumption 1 and the assumption that it is always optimal to elicit effort are both satisfied. We assume this condition, and focus our attention on that segment.

21If the elasticity of substitution implied by the utility function $U$ between $X$ and $Y$ is high, then a given rise in $E/L$ and consequent drop in $r$ will require only a small change in the relative price $p^y$ to restore market clearing. Conversely, a low elasticity of substitution will require a large
MM curve could be either flatter or steeper than the VV curve. It has been drawn flatter in this case for concreteness.

Now, we have all of the tools required to analyze the effects of globalization. First, we consider the effects of free trade, and then the effects of offshoring.

5.A. Free trade.

Free trade establishes a unified world market for goods X and Y, without allowing for movements of labor across borders. Given Proposition 7, if autarkic supplies of the two goods in the two countries are denoted $X^i$ and $Y^i$ respectively for country $i$, then the relative supply of Y will be equal to $r^{US} = Y^{US}/X^{US}$ for the US under autarky; $r^{IN} = Y^{IN}/X^{IN} > r^{US}$ for India under autarky; and $r^{FT} = (Y^{US} + Y^{IN})/(X^{US} + X^{IN}) > r^{US}$ under free trade (note that free trade does not change the quantities produced in either country). As a result, the free-trade value of $p^y$ will be lower than the autarkic US value. This will lower the real wage $\omega^y/P(p^y) = p^y/P(p^y)$ for US workers in the Y sector, and since US workers are indifferent between working in the two sectors, this also means that the steady-state welfare of US X-sector workers will fall. At the same time, by Proposition 5, we know that the variance of wages will fall. To sum up, we have the following:

**Proposition 8.** Free trade lowers the steady-state welfare of all US workers and raises the welfare of all workers in India. It also (weakly) lowers the variance of US wages and raises the variance of wages in India.

movement in relative price.
If the US economy is initially at point A in Figure 3, then from the point of view of US workers this change is represented by the move from point A to point B. (Note that the only reason for the qualifier ‘weakly’ in the proposition is the possibility that one or both countries may be in the wage smoothing regime both with and without trade.)

Note that this result is exactly in line with the empirical findings of Bertrand (2004). Those results are not about the effects of trade on *economy-wide average* wage volatility, but rather *inter-industry* comparisons of wage volatility across time. Bertrand finds a positive cross-industry correlation between increases in an industry’s import penetration ratio and weakening of its invisible handshake. In our model, the X industry’s import penetration *falls* with the opening of trade (it goes from zero to a negative value, since the X industry is a net exporter), and in that industry the invisible handshake is *strengthened*, in line with the Bertrand finding. Obviously, we could extend the model to have several career sectors, some of them import-competiting, and similar logic would continue to hold.

5.B. Offshoring.

Now, suppose that in addition to free trade we allow offshoring to occur. In that case we have arrived at full integration; the two economies will combine to form one large one with $E + E^*$ employers and $L + L^*$ workers.

Since full integration essentially creates an autarkic economy with $E + E^*$ employers and $L + L^*$ workers, comparing full integration with autarky is straightforward. The ratio $(E + E^*)/(L + L^*)$ necessarily falls between $E/L$ and $E^*/L^*$, so, again by Proposition 7, the free-trade
value of $p^y$ will be lower than the autarkic US value and above the autarkic Indian value. Thus, it is immediate that full integration has qualitatively the same effect on worker welfare in both countries, compared to autarky, as does free trade; US workers are worse-off under full integration than under autarky, while Indian workers are better-off. However, what is not straightforward is the marginal effect of offshoring on worker welfare, in other words, the difference in worker welfare between free trade and full integration. It can be shown that this effect is positive, for workers in both countries.

**Proposition 9.** The world relative supply of good Y, $r$, is lower under full integration than under free trade. Therefore, the relative price, $p^y$, is higher, and the welfare of workers in both countries is higher, under full integration than under free trade.

**Proof:** See appendix.

This change is represented by the move from point B to point C in Figure 3. The point is that offshoring allows for efficiencies in the matching of X-sector employers in the labor-scarce US market with workers in the worker-rich Indian market, thus allowing for the world X industry to increase its employment and output. More workers worldwide producing X also means fewer workers worldwide producing Y, so the world relative supply of Y falls, making Y relatively more expensive. This benefits workers producing Y, raising the opportunity cost of X-sector workers, and raising workers’ equilibrium utility in both countries.\(^{22}\)

\(^{22}\)To simplify the analysis, we have assigned all bargaining power to the employer. It is interesting to speculate how this result would change if the workers also had some bargaining power, which would add enormous complexity to the model. One point is clear, however: We have shown...
Further, from Proposition 6 it is clear that $Q^E$ rises in the US. From Propositions 4 and 5, the rise in $p^y$ and in $Q^E$ together imply an increase in the volatility of US X-sector wages. Thus, offshoring does indeed increase the variance of US workers’ earnings, even though we have just seen from the previous proposition that their welfare also rises. This implies that in response to offshoring *expected X-sector wages in the US go up by more than enough to compensate for the additional risk.*

Finally, a comment on the overall effects of globalization, the movement from point A to C in Figure 3. Note that the effects of free trade and offshoring on wage volatility run in opposite directions, and the net effect of globalization on wage volatility is therefore not obvious. That it is truly ambiguous can be seen from the figure. If the elasticity of substitution between X and Y consumption is very high, the MM curve will be flatter than the VV curve as shown, while if the elasticity is very low, it will be steeper. In the former case, it is possible that globalization takes the US from a point on MM in the wage-smoothing regime (in other words, to the left of VV), to a point on MM in the fluctuating-wage regime. In the latter case, the opposite is possible. More generally, the elasticity of substitution between X and Y in consumption will govern whether price effects or $Q^E$ effects will dominate. This provides our final result.

---

that the worker’s threat point is *improved* by allowing offshoring, because by raising the relative price of good Y, it raises the value of the worker’s outside option (producing Y). Thus, the worker’s bargaining position relative to the employer is strengthened by offshoring. This surprising result, running counter to the intuitive expectation of many observers, is a reminder of the importance of working through the general equilibrium ramifications of the change.

---

23The effect of offshoring on the volatility of wages in India is ambiguous, as $Q^E$ and $p^y$ move in opposite directions. However, it definitely raises Indian workers’ utility.
Proposition 10. If the elasticity of substitution between X and Y consumption is sufficiently small, globalization on balance lowers the volatility of US wages. If it is sufficiently large, it raises the volatility of US wages.

6. Conclusion.

In this paper, we have joined some insights from labor economics with tools from contract theory and international trade theory to examine the impact of globalization on implicit contracts. We have shown that globalization can indeed affect the nature of long-term employment relationships, and in particular the volatility of workers’ incomes within those relationships. By extension, these effects can influence the degree of income inequality among workers with identical characteristics, an important matter empirically that conventional trade models cannot address.

However, it is important to note that different types of globalization have quite different effects. We have focused on two kinds of globalization: Free trade (or integration of goods markets) and international offshoring (or integration of labor markets). In our model, we find that free trade strengthens implicit contracts for rich-country workers, lowering the volatility of workers’ incomes, but lowering their welfare due to relative-price effects. On the other hand, international offshoring weakens implicit contracts for rich-country workers, raising the volatility of workers’ incomes, but raising their welfare due to relative-price effects, as the improved productivity of the offshoring sector leads to a lower price for its output.

More generally, globalization affects implicit contracts through two channels. Relative-price
changes will tend to strengthen implicit contracts and lower wage volatility in sectors whose relative price rises, and have the opposite effect in sectors whose relative price falls. Thus, trade liberalization will tend to raise wage volatility in import-competing sectors and lower it in export sectors. At the same time, in any industry if it becomes easier for an employer to find a new worker (because the employer can now hire foreign workers as well as local ones), implicit contracts will be weakened, and if it becomes harder (because the employer must now compete with foreign employers), implicit contracts will be strengthened. Thus, holding relative prices fixed, offshoring will tend to raise wage volatility in the sectors that exhibit positive net offshoring and lower it in sectors that exhibit negative net offshoring (with foreign employers on balance hiring more domestic workers than vice versa, a case often called ‘insourcing’). Further, offshoring itself can give rise to relative-price effects due to improved productivity in the offshoring sector, and to the extent that this raises the relative price for labor-intensive industries, it can in general equilibrium provide an indirect welfare benefit to workers in all countries.

Thus, we simultaneously formalize and sharply limit one argument on the dangers of globalization.
Proof of Lemma 1.

First, observe that since a worker will never accept employment with payoff below $V^{WS}$, we must have $W_{min} \geq V^{WS}$. We will show that $W_{min} = V^{WS}$ by showing through contradiction that it is not possible to have $W_{min} > V^{WS}$. First, however, it will be useful to demonstrate that $W_{min} \leq \bar{w}^{*}$, which will allow us to ignore the constraint $W_{min} \leq \bar{w}^{*}_{e}$ and treat (9) as an equality.

Suppose, then, that $W_{min} > \bar{w}^{*}$. In this case, the worker’s incentive constraint (4)' can never bind, and so $v_{\epsilon} = 0$ for $\epsilon = G, B$. Consider the first-period decision. Then if the lower-bound constraint in (6) does not bind for state $\epsilon$, then the first-order condition (9) holds with equality, so from (9), we have $-\lambda = \Omega' (W_{0}) \leq \Omega' (\bar{w}_{e}) < 0$, so $W_{0} \geq \bar{w}_{e} > W_{min}$. But this would be a suboptimal choice by the employer, as the employer could choose a first-period wage and future worker payoffs to give the worker a current payoff $W_{0}$ equal to $W_{min}$; this would realize a higher profit, and would also satisfy the first-period target utility constraint (5) since $W_{min} \geq V^{WS}$. Therefore, we conclude that the lower-bound constraint in (6) must bind in the first-period decision for both states, and so $\bar{w}_{e} = W_{min}$ for $\epsilon = G, B$. Now, suppose that the target utility constraint (5) binds in the first period, recalling that the first-period target utility level $W$ is equal to $V^{WS}$. This immediately yields a contradiction, as it implies that a worker payoff of $V^{WS}$ can be realized in equilibrium, which contradicts the maintained assumption that $W_{min} > \bar{w}^{*}$ (since $\bar{w}^{*} > V^{WS}$). Therefore, the target utility constraint does not bind in the first period, and so $\lambda = 0$. But then the first-order condition (8) for the wage cannot be satisfied for any positive value of the wage, implying a wage of zero in the first period in both states. This
implies a first-period payoff for the worker equal to:

\[
\mu(0) - k + \rho \beta W_{\text{min}} + (1 - \rho) \beta V^{WS} < W_{\text{min}},
\]

since \(W_{\text{min}}\) cannot be less than \(\mu(0)/(1 - \beta)\) (which is the utility from permanent zero consumption; in no case could the worker receive lower utility than that).

But this is a contradiction, since by definition it is not possible to give a worker a payoff less than \(W_{\text{min}}\). Therefore, \(W_{\text{min}} \leq \tilde{\nu}^{*}\). As a result, the constraint \(W_{\text{min}} \leq \tilde{\nu}^{*}\) is redundant, and can be removed without changing the solution. Consequently, we can treat (9) as an equality.

Now, suppose that \(W_{\text{min}} > V^{WS}\). Then if the target utility constraint (5) binds in the first period, recalling that the first-period target utility level \(W\) is equal to \(V^{WS}\), then we have a contradiction as before, so suppose that the target utility constraint does not bind in the first period. Then \(\lambda = 0\), so the first-order condition (8) for the wage cannot be satisfied for any positive value of the wage, implying a wage of zero in the first period in both states. Note that with \(\lambda = 0\), the first-order condition (9) cannot be satisfied with equality unless \(\nu^{*} > 0\), so that the workers’ incentive constraint binds, and so \(\tilde{\nu}^{*} = \tilde{\nu}^{*}\). (We already know that the lower-bound constraint in (6) is redundant, because we have shown above that \(W_{\text{min}} \leq \tilde{\nu}^{*}\).) This implies a first-period payoff for the worker equal to:

\[
\mu(0) - k + \rho \beta \tilde{\nu}^{*} + (1 - \rho) \beta V^{WS}
\]

\[
= \mu(0) + V^{WS} - \mu(\omega^{*} / P)
\]

\[
< W_{\text{min}}.
\]
Of course, again, this is a contradiction, by the definition of $W_{min}$. We conclude that $W_{min} \leq V^{WS} < \tilde{w}^*$. Since we already know that $W_{min} \geq V^{WS}$, we conclude that $W_{min} = V^{WS}$. Q.E.D.

Proof of Proposition 2.

Consider the second-period problem. Under conditions of Proposition 1, we know that the target continuation payoff for the worker is $\tilde{w}^*$. We claim that the choice of next-period continuation payoff $\tilde{w}_e$ will be equal to $\tilde{w}^*$ for $e = G, B$. If $\nu_e > 0$, then complementary slackness implies that $\tilde{w}_e = \tilde{w}^*$. Therefore, suppose that $\nu_e = 0$. This implies that (9) becomes:

$$\Omega'(\tilde{w}_e) = (-\lambda) \left( \frac{\pi_e}{\pi_e + \psi_e} \right).$$

Since, by the envelope theorem, $-\lambda = \Omega'(W)$, and as we recall for the second-period problem the worker’s target utility $W = \tilde{w}^*$, this becomes:

$$\Omega'(\tilde{w}_e) = \Omega'(\tilde{w}_e^*) \left( \frac{\pi_e}{\pi_e + \psi_e} \right).$$

(21)

If $\psi_e = 0$, this implies through the strict concavity of $\Omega$ that $\tilde{w}_e = \tilde{w}^*$, and we are done. On the other hand, if $\psi_e > 0$, (21) then implies that $0 > \Omega'(\tilde{w}_e) > \Omega'(\tilde{w}^*)$, implying that $\tilde{w}_e < \tilde{w}^*$. However, this violates (4)*. Therefore, all possibilities either imply that $\tilde{w}_e = \tilde{w}^*$ or lead to a contradiction, and the
claim is proven.

Since \( \bar{w}_e = \bar{w}^* \), the optimization problem in the third period of the relationship is identical to that of the second period. By induction, the target utility for the worker in every period after the first, regardless of history, is equal to \( \bar{w}^* \), and so the wage chosen for each state in every period after the first, regardless of history, is the same.

Now, to establish the three possible outcomes, we consider each possible case in turn. Consider the optimization problem (2) at any date after the first period of relationship. First, suppose that the employer’s constraint does not bind in either state. In this case, \( \psi_e = 0 \) for \( e = G, B \). Condition (8) now becomes:

\[
-\pi_e + \lambda \pi_e \mu'(\omega_e / P) / P \leq 0.
\]

If this holds with strict inequality for some \( e \), then \( \omega_e = 0 \). This clearly cannot be true for both values of \( e \), because that would imply a permanent zero wage, and it would not be possible to satisfy (5). (To see this, formally, substitute \( W = \bar{w}^* \), the expression for \( V^{ws} \), and \( \omega_G = \omega_B = 0 \) into (4), and note that the constraint is violated.) Therefore, for at most one state, say \( e' \), can the inequality in (22) be strict. Denote by \( e'' \) the state with equality in (22). Then \( \mu'(0)/P < 1/\lambda = \mu'(\omega_{e''}/P)/P \). However, given that \( \omega_{e''} \) is non-negative and \( \mu \) is strictly concave, this is impossible. We conclude that (22) must hold with equality in both states, and therefore \( \omega_G = \omega_B \).

Next, suppose that we have \( \psi_G > 0 \) and \( \psi_B = 0 \), so that the employer’s constraint binds only in the good state. We will show that this leads to a contradiction. Recall from the previous proposition that \( \bar{w}_e = \bar{w}^* \) for both states, and note that, by assumption, (3) is satisfied by equality for
$e = G$. Since $x_B < x_G$, we now see that (3) must be violated for $e = B$ if $\omega_G \leq \omega_B$. Therefore, $\omega_G > \omega_B \geq 0$. This implies that (8) holds with equality in the good state. Applying (8), then, we have:

$$\frac{\mu' (\omega_G / P)}{P} = \frac{1}{\lambda} \left(1 + \frac{\psi_G}{\pi_G}\right) > \frac{1}{\lambda} \geq \frac{\mu' (\omega_B / P)}{P},$$

which contradicts the requirement that $\omega_G > \omega_B$. This shows that it is not possible for the employer’s constraint to bind in the good state.

Now suppose that we have $\psi_G = 0$ and $\psi_B > 0$, so that the employer’s constraint binds only in the bad state. We now wish to prove that in this case $\omega_G > \omega_B$. Suppose to the contrary that $\omega_G \leq \omega_B$. Since we already know that we cannot have a zero wage in both states, this implies that $\omega_B > 0$, so that (8) holds with equality in the bad state. Then, from (8):

$$\frac{\mu' (\omega_B / P)}{P} = \frac{1}{\lambda} \left(1 + \frac{\psi_B}{\pi_B}\right) > \frac{1}{\lambda} \geq \frac{\mu' (\omega_B / P)}{P},$$

which implies that $\omega_G > \omega_B$. Therefore, we have a contradiction, and we conclude that $\omega_G > \omega_B$.

Finally, suppose that the employer’s constraint binds in both states. Given that $\bar{\nu}_e = \bar{\nu}^*$ in both states, equality in both states for (3) requires that short-term profits $x_e - \omega_e^*$ are equal in the two states.

We have thus eliminated all possibilities aside from the two listed in the statement of the proposition. Q.E.D.
Proof of Proposition 5.

Recall that the $WW$ curve is given by:

$$
\pi_G \mu_G (\omega_G / P(p^y)) + \pi_B \mu_B (\omega_B / P(p^y)) = \mu(\omega^y / P(p^y)) + k/\rho \beta.
$$

If we take a total derivative of this condition, taking into account that $\omega^y = p^y$, we obtain:

$$
(\pi_G / P) \mu' (\omega_G / P) d\omega_G + (\pi_B / P) \mu' (\omega_B / P) d\omega_B - (\pi_G \omega_G P'/P^2) \mu' (\omega_G / P) dp^y
$$

$$
- (\pi_B \omega_B P'/P^2) \mu' (\omega_B / P) dp^y = [(P - p^y P')/P^2] \mu' (p^y / P) dp^y.
$$

This can be rearranged as:

$$
\pi_G \mu' (\omega_G / P) d\omega_G / dp^y + \pi_B \mu' (\omega_B / P) d\omega_B / dp^y
$$

$$
= (\pi_G \omega_G P'/P) \mu' (\omega_G / P) + (\pi_B \omega_B P'/P) \mu' (\omega_B / P) + [(P - p^y P')/P] \mu' (p^y / P). \quad (23)
$$

Recalling that $P(p^y)$ is the minimum expenditure required to obtain one unit of utility, given that the price of $Y$ is $p^y$, Shephard’s Lemma implies that $P' p^y / P = \alpha^y$, where $\alpha^y$ is the share of good $Y$ in consumer expenditure. This allows us to rewrite the total derivative as:

$$
\pi_G \mu' (\omega_G / P) d\omega_G / dp^y + \pi_B \mu' (\omega_B / P) d\omega_B / dp^y
$$

$$
= (\alpha^y / p^y) E \omega^y \mu' (\omega^y / P) + (1 - \alpha^y) \mu' (p^y / P).
$$
The $EE$ curve is given by:

$$\rho \beta \pi_G \omega_G + [1 - \rho \beta (\pi_G - Q^E)] \omega_B = Q^E \rho \beta \omega^y_B + x_B + \rho \beta (1 - Q^E) \pi_G (x_G - x_B).$$

If we take the total derivative of this condition, again taking into account that $\omega^y = p^y$, we obtain:

$$\rho \beta \pi_G \frac{d \omega_G}{dp^y} + [1 - \rho \beta (\pi_G - Q^E)] \frac{d \omega_B}{dp^y} = Q^E \rho \beta. \quad (24)$$

Equations (23) and (24) are then a system of two linear equations in two unknowns, $\frac{d \omega_G}{dp^y}$ and $\frac{d \omega_B}{dp^y}$. Solving, we obtain:

$$\frac{d \omega_B}{dp^y} = -\frac{\rho \beta \pi_G \left[ (\omega^y / p^y) \left[ \pi_G \omega_G (\omega_G / P) + \pi_B \omega_B (\omega_B / P) \right] + (1 - \alpha^y) \mu'(p^y / P) - Q^E \mu'(\omega_G / P) \right]}{D},$$

where

$$D \equiv \pi_G \left[ (1 - \rho \beta (\pi_G - Q^E)) \mu'(\omega_G / P) - \rho \beta \pi_B \mu'(\omega_B / P) \right] > 0.$$

is the determinant of the system, and is positive because at the equilibrium the $WW$ curve is steeper than the $EE$ curve. Note that
The first inequality holds because the condition defining the $WW$ curve implies that $\omega^y < \pi_G \omega_G + \pi_B \omega_B$, and the second holds because the middle expression is a weighted average of $\mu'(\omega_G/P)$ and $\mu'(\omega_B/P)$, of which the former is smaller. This implies that

$$\frac{\pi_G \omega_G \mu'(\omega_G / P) + \pi_B \omega_B \mu'(\omega_B / P)}{\pi_G \omega_G + \pi_B \omega_B} > \mu'(\omega_G / P).$$

Since $d\omega_B/d\omega^y < 0$, (23) requires that $d\omega_G/d\omega^y > 0$, and therefore $d(\omega_G - \omega_B)/d\omega^y > 0$.

Q.E.D.

Proof of Proposition 6.

The number of employers paired with a worker is equal to $E - m$, and the number of workers paired with an employer is equal to $L - n$. These must always be equal, so:

$$E - L = m - n.$$

Suppose that $E > L$. Dividing both sides by $L$ and using (20), we find:
\[ E / L = \{( m - n ) / [ ( \rho / (1 - \rho ))m\Phi(n/m, 1) + n] \} + 1, \] so:

\[ E / L = \{(1 - (n/m)) / [( \rho / (1 - \rho ))\Phi(n/m, 1) + (n/m)]\} + 1. \tag{25} \]

The right-hand side of (25) exceeds unity iff \( n/m < 1 \). Since we are assuming that \( E > L \), the right-hand side of (25) clearly needs to be greater than unity, so \( n/m \) must be less than unity. Therefore, at an equilibrium, the right-hand side of (25) is strictly decreasing in \( n/m \), so the equilibrium level of \( n/m \) is uniquely determined for a given value of \( E/L \) and \( \rho \). Furthermore, \( n/m \) is a locally decreasing function of \( E/L \) for given values of the other parameters.

Now, if \( E < L \), a parallel argument can be developed by dividing through by \( n \) instead of \( m \) and later by \( E \) instead of \( L \). \textbf{Q.E.D.}

\textit{Proof of Proposition 7.}

The number of employers producing output in period \( t \) is given by:

\[ E - m_t = \rho \left[ E - m_{t-1} + \Phi(n_{t-1}, m_{t-1}) \right]. \tag{26} \]

The number of employers producing output is equal to \( E - m_t = L - n_t \). Denote aggregate X-sector output in period \( t \) by \( x_t \). Since the average output of a functioning firm is equal to \( \bar{x} \), the number of employers producing output must also equal \( x_t / \bar{x} \). Therefore, we can rewrite (26) as follows:

\[ x_t / \bar{x} = \rho \left[ (x_{t-1} / \bar{x}) + \Phi(n_{t-1}, m_{t-1}) \right]. \tag{27} \]
In steady state, (27) becomes:

\[
x_{ss} / \bar{x} = \Phi(n, m) / (1 - \rho) = \Phi (L - (x_{ss} / \bar{x}), E - (x_{ss} / \bar{x})) / (1 - \rho).
\]

Then, we have:

\[
\Phi(L \bar{x} / x_{ss} - 1, E \bar{x} / x_{ss} - 1) = 1 - \rho. \tag{28}
\]

Thus, \(x_{ss}(E, L)\) is increasing in \(E\) and \(L\) and linear homogenous in \(E, L\). Now note that Y-sector output can be written as:

\[
y_t = L - (x_t / \bar{x}), \tag{29}
\]

where \(y_t\) is the output in the Y sector in period \(t\). In steady state, this can be rewritten as follows:

\[
y_{ss} = L - (x_{ss} / \bar{x}).
\]

Thus, from the properties just derived for \(x_{ss}\), we see that \(y_{ss}(E, L)\) is increasing in \(L\) and decreasing in \(E\) and linear homogenous in \(E, L\).

As a result, \(r \equiv (y_{ss} / x_{ss})\) is decreasing in \(E / L\). \textbf{Q.E.D.}
Proof of Proposition 9.

Recall from Proposition 7 that the steady-state values of X and Y output within one country can be written as functions $x_{ss}(E, L)$ and $y_{ss}(E, L)$ of $E$ and $L$. We can thus speak of the isoquants of these functions. For example, the slope of the $x_{ss}$ isoquant is given by $-(\partial x_{ss}/\partial E)/(\partial x_{ss}/\partial L)$. Taking derivatives of (28), we see that:

$$(\partial x_{ss}/\partial E)/(\partial x_{ss}/\partial L) = \Phi_E \left( L \tilde{x}/x_{ss} - 1, E \tilde{x}/x_{ss} - 1 \right) / \Phi_L \left( L \tilde{x}/x_{ss} - 1, E \tilde{x}/x_{ss} - 1 \right).$$

Notice that:

$$x_{ss}(E, L)/L = x_{ss}(E/L, 1).$$

Thus, $L \tilde{x}/x_{ss}$ is decreasing in $E/L$. Similarly,

$$x_{ss}(E, L)/E = x_{ss}(1, L/E),$$

so $E \tilde{x}/x_{ss}$ is increasing in $E/L$.

Therefore, the absolute value of the slope of the isoquant is smaller in a more labor-scarce economy. This is illustrated in Figure 4, which depicts a box whose height is the world supply of workers and whose length is the world supply of employers. In the figure, the US endowment of workers and employers is measured upward and rightward respectively from the lower left-hand origin, and India’s endowments are similarly measured down and to the left from the upper right-hand
origin. The allocation of the two factors between the two countries is given by the point $a$; the $x_{SS}$ isoquant for the US going through that point is marked $UU$; and the $x_{SS}$ isoquant for India going through that point is marked $II$. The finding that the absolute slope of the isoquant for a given country is decreasing in that country’s $E/L$ ratio implies that these isoquants are strictly convex, and in addition, at every possible allocation point below the main diagonal $OO$, $UU$ is flatter than the corresponding Indian isoquant at that point.

Now under free trade without integration of factor markets, consider the change in $Y$ output if we transfer workers from India to the US, at the same time reallocating employers from the US to India so that steady-state $X$ output in the US is unchanged. This can be represented as a movement left along $UU$ from point $a$. Suppose that we stop the process when the $E/L$ ratio in the two countries is the same (and therefore equal to the world $E/L$ ratio). In other words, we stop at point $b$. Since the US steady-state $X$ isoquant is flatter than the Indian one at every point along this process, the movement from $a$ to $b$ results in an increase in $X$ output in India, and therefore in the world. Given (29), this implies a reduction in worldwide $Y$ output, and hence a reduction in $r$. Finally note that, under free trade, a reallocation of workers and employers across countries that results in the same factor ratio in each country – as for example in point $b$ or any other point on the main diagonal – will replicate the outcome of integration of the labor markets. **Q.E.D.**
References.


