Abstract

The purpose of this work is to formalize what are the conditions under which a marginally progressive income tax emerges in the political competition between two parties when labor is elastically supplied and candidates are uncertain on voters’ choice decisions. If we assume that the elasticity of labor is a decreasing function of marginal wage, surprisingly if we model political competition as in Coughlin and Nitzan (1981) we show that only marginal progressive taxes are played by both candidates in equilibrium. If we adopt the model of Lindbeck and Weibull (1989), the equilibrium tax schedule will be progressive if the political power of the rich voter is sufficiently small. The degree of progressivity is decreasing with population polarization.

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1 Introduction

The question, "Why do progressive taxes emerge in industrialized countries?", dates from Mirrlees seminal paper (1971). He showed that marginal progressive tax schedules, as we have in industrialized societies, were hardly optimal, unless we had a small elasticity of labor supply. Why such tax schedules are chosen in industrialized democracies? A growing literature on political economy of taxation inspired by Roberts (1977), Romer (1975) and Meltzer and Richard (1981), questioned whether high marginal taxes could be part of a political equilibrium. Very few of them considered disincentive effects from taxation to study tax progressivity (De Donder and Hindriks, 2003) the main literature built under the exogenous income hypothesis, see for example Marhuenda and Ortuño-Ortín (1995), Roemer (1999) and Carbonell and Klor (2003) and Carbonell and Ok (2007).

The starting literature on the political economy of taxation assumed a proportional income tax. Under some conditions on preferences such as single-crossing a Condorcet winner (CW) exists and both Downsian candidates play the CW tax rate in equilibrium. in the absence of labor disincentives (inelastic labor supply) if the median income is below the mean, then, the equilibrium marginal tax rate equals 100%. Even if endogenous labor supply is assumed taxes are still strictly positive and increasing with inequality, defined as the ratio of average income to median income.

Moreover, assuming a linear income tax schedule could not help us to understand the fact that in most industrial economies the tax rate is increasing with income. The aim of this paper is to understand why there is a democratic demand for income tax progressivity. In order to have a tax schedule that allows for (marginal) progressivity we need at least three parameters to vote for.\footnote{Note that if we restrict the policy space to tax functions ordered by Lorenz dominance, that is the case is after-tax income can be represented as } \[ x_i = (y_i)^{1-\tau} (\bar{y})^\tau \], where \(\bar{y}\) is common to all agents (it is determined so that average post-tax income equals per-capita income), \(y_i\) is pre-tax income and \(\tau\) is
transfer (or level of public good), a second one which is the linear tax rate parameter and the last capturing the concavity (regressivity) or convexity (progressivity) of the tax schedule. Thus, we are facing a multidimensional voting model. Conditions to have a CW in models with a multidimensional policy space are known to be very restrictive. For the quadratic tax function De Donder and Hindriks (2003) and Hindriks (2001) show that it is hard to avoid voting cycles. Other approaches than the direct democracy approach should be considered.

Carbonell and Klor (2003) developed a citizen candidate model and found that under some conditions only marginal increasing tax rates are implemented in equilibrium. Roemer (1999) developed the Party Unanimity Nash Equilibrium (PUNE) political equilibrium concept. The platform chosen by the party is the outcome of intraparty negotiation among party members. In equilibrium both parties announce marginal progressive taxes. In this paper we adopt the probabilistic voting model introduced by Coughlin and Nitzan (1981) and Coughlin (1992) and microfunded afterwards by Lindbeck and Weibull (1987). When voters cast their ballots in favor of one or another candidate they consider issues other than the economic issue, for instance ideology. The higher the difference in economic utility, the higher the probability that a given voter will favor the candidate that brings him the highest (economic) utility. Conditions for existence of an equilibrium are less restrictive in the Coughlin model. A CW needs not to exist and, when it does exist, the equilibrium tax schedule does not need to coincide with the CW tax schedule.

The probabilistic model can be understood as the outcome of a political process where voters choose probabilistically between candidates, with the probability to vote for one candidate increasing in the utility difference. The outcome of such a political process, as stressed by Coughlin and Nitzan (1981) in their Theorem 1, involves the tax parameter. A single parameter is enough to describe whether the tax schedule is marginal-rate progressive \(0 \leq \tau \leq 1\), or regressive \((\tau < 0)\). See for instance Benabou (2000).
maximization of a Nash welfare function.

When labor disincentives are taken into account we introduce another reason for which a given vote is easier to buy. If the elasticity of labor is decreasing in marginal wage then, we can tax heavily the rich voter relative to the poor since the former decreases less his labor supply in response of an increase in the tax. In this sense there is more scope for tax progressivity than in the fixed (exogenous) income model. It is worth noting that there are little estimates on the elasticity of labor supply by income groups. Saez (2004) finds that upper middle income families and individuals do not appear to be sensitive to taxation. Significant elasticities are found at the very top of the income distribution. Nevertheless, the increase of the shares of total wages accruing to top wage income earners, can not be explained solely by the evolution of marginal tax rates given the heterogeneity in size of responses overtime. Moreover we find that the progressivity degree is increasing in the political power of the middle class, the class that favors the most tax progressivity.

Note that we adopt the Coughlin and the Lindbeck and Weibull probabilistic model. In both models the equilibrium income tax is the income tax schedule that maximizes some welfare function. In this sense the equilibrium income tax is efficient. We show which conditions on the welfare function and the labor supply need to be satisfied for a marginal progressive tax to emerge in equilibrium.

The probabilistic voting model would bring credible predictions in either of these three scenarios: Voters vote probabilistically, candidates are uncertain about the voters choice or we have interest groups representing voters that compete for influence.

The rest of the paper goes as follows: Section 2 presents the model. Section 3 describes the labor supply decision of voters given the implemented tax schedule. In section 4 we describe the preferences of the different income groups over tax schedules. In section 5 we describe the political competition stage and the main results of the
paper. Section 6 concludes.

2 The Model

We develop a static model of political competition between two Downsian parties, $A$ and $B$. Candidates or parties are uncertain about how the economic preferences of voters translate into party preferences. Parties announce simultaneously a policy platform $t^C$, $C = A, B$, a vector of marginal income tax rates, that maximizes their probability of winning. They commit to the platform announced. The party holding the majority of votes wins the election. Once the equilibrium platform is implemented voters make labor decisions. We solve the model backwards.

2.1 The probabilistic voting model

The probabilistic voting model developed by Coughlin relaxes one of the assumptions of the traditional Downsian model: Candidates are certain about what voters choices will be in response of their announced platforms.

In Coughlin and Nitzan (1981) and Coughlin (1992) even after voters have learned the decisions of both of the candidates in the race, candidates are uncertain about what these choices will be. This would also be the case if the choices of voters were stochastic in nature.

Two possible interpretations of the Coughlin model are that voters do not vote deterministically but they are still rational. They vote with higher probability for the candidate whose policy platform brings them the highest utility.

This raises the question on why voters do not vote according to their economic preferences. This leads to the second interpretation of the Coughlin model where voters indeed vote deterministically but there are other issues in addition to the economic
issue that makes them to vote not necessarily for the party that promises them the best economic platform. Here voters are *ideological*.

Candidates use a logit model to infer voters’ selection probabilities. In an economy with $J$ voters the probability that a voter of group $j$ votes for candidate $A$ equals the relative utility $j$ derives from $A$ platform with respect to the utility he derives from the $B$ platform,

$$
\pi^A_j(t^A, t^B) = \frac{U_j(t^A)}{U_j(t^A) + U_j(t^B)}
$$

Note that voters do not abstain, they either vote for $A$ or for $B$. Indeed $\pi^A_j(t^A, t^B) + \pi^B_j(t^A, t^B) = 1$. The higher the economic utility from platform $A$, the higher will be the probability that group $j$ (or a representative voter in group $j$) will vote for $A$. Parties are office-motivated. They choose simultaneously the policy platform that maximizes $C(t^A, t^B) = \sum_{j=1}^{J} \pi^C_j(t^A, t^B)$ with $C = A, B$. Among the main findings of the probabilistic voting model we cite the following:

1.- Equilibrium existence and uniqueness. There exist an equilibrium (a saddle point to $\pi^A$) as long as $U_j(t^C)$ is quasiconcave on $t^C$. Note this is a multidimensional problem and a median voter need not to exist.

2.- Policy convergence. Both candidates face a similar problem $\pi^A(t^A, \bar{t}^B)$ for $A$ and $\pi^B(\bar{t}^A, t^B)$ for $B$. This implies that they both choose the same policy platform and the probability of winning equals $\frac{1}{2}$.

3.- The outcome of the political competition game is the social alternative that maximizes a Nash social welfare function (Theorem 1 of Coughlin and Nitzan, 1981).

Lindbeck and Weibull give a microfundation (generalizes) to the Coughlin model. They introduced ideology. A voter may cast their ballot in favor of a candidate that gives him lower economic utility if the utility from the non-economic issue overweights the economic loss. Although voters may indeed vote deterministically the Lindbeck
and Weibull model is called a probabilistic voting model because of the close link it has with the Couglin model. We take the Lindbeck et al. approach here (section 6 explains the model in more detail) and discuss how our predictions would change if the political competition would be modelled à la Coughlin.

2.2 Preferences

Voters are divided in 3 groups: poor (P), middle class (M), rich (R). The size of the population is normalized to one. We assume the proportion of voters in group P equals the proportion of voters in group R which is \( \alpha \), proportion of M voters is then, \( 1 - 2\alpha \). Groups are differentiated by their marginal wage (ability) parameter \( w_j \), with \( j = P, M, R \), such that \( 0 < w_P < w_M < w_R \).

Total income of individuals in each group \( j \) is \( y_j = w_j l_j \), where \( l_j \) is labor effort chosen by voter \( j \). Consumption equals after-tax income, \( c_j = y_j - T(y_j) \), with \( T(y_j) \) being total tax payment by the \( j \)-voter.

We denote by \( U_j(c_j, l_j) \) the utility of a member of group \( j \) with consumption \( c_j \), and labor supply \( l_j \in [0, L] \), and assume that \( U_j \) is increasing in consumption \( (c_j) \) and decreasing in labor \( (l_j) \) which can be seen as labor effort or hours worked per week, the last being an imperfect measure of labor effort. For simplicity we assume the utility function is quasilinear in consumption, \( U_j = u(c_j + v(L - l_j)) \), \( U_j \) is well behaved: \( u' > 0 \), \( u' < 0 \), \( v' > 0 \) and \( v' < 0 \). This utility specification will allow us to make straightforward comparisons between the outcomes of the two probabilistic voting models. We assume that the elasticity of labor is decreasing in marginal wage, \( \frac{\partial c_j}{\partial w_j} \leq 0 \), with \( \varepsilon_l = \frac{\partial l_j}{\partial w_j} \frac{w_j}{l_j} \).
2.3 The Tax Schedule

Each group $j$ pays a marginal tax rate of $t_j$ on income. The tax schedule is purely redistributive, and all the income tax collected finances a lump-sum transfer, $G$. The government budget condition is:

$$G(t) = \sum_{j=P,R} \alpha_j t_j y_j + (1 - 2\alpha) t_M y_M$$  \hspace{1cm} (1)

For simplicity we assume that $t_P = 0$, and $0 \leq t_M, t_R \leq 1$. This reduces the dimensionality of the economic platform to $t = (t_M, t_R)$, define $t \in T : [0, 1] \times [0, 1]$ as the set of possible income tax rates. The tax schedule is marginal progressive whenever marginal income tax rates are increasing in income. This means, for our particular example, that $t_R - t_M > 0$. Further conditions should be given to guarantee that indeed $y_R \geq y_M \geq y_P$. Given the disincentive effects from taxation we do not expect $t_M$ or $t_R$ to be larger than the income tax rates that maximizes $G$.

Given our tax schedule, from the government budget constraint balance condition, the political struggle takes place between two tax parameters: $(t_M, t_R)$, and the lump-sum transfer is:

$$G(t) = (1 - 2\alpha) t_M y_M + \alpha t_R y_R$$  \hspace{1cm} (2)

Since labor is endogenously supplied, the tax schedule should satisfy two additional conditions:

$$y_R(t) \geq y_M(t)$$  \hspace{1cm} (3)

$$y_M(t) \geq y_P(t)$$  \hspace{1cm} (4)

Once the winning platform is in place, voters choose their before tax income given the parameters of the tax function $(t_M, t_R, G)$, this is equivalent to choose their labor
3 Optimal labor supply

Once the winning platform is implemented voters make labor decisions. For the particular income tax schedule specified above the after-tax income is $w_pl_p + G$, $(1 - t_M) w_M l_M + G$ and $(1 - t_R) w_R l_R + G$ for the poor, middle and rich voter, respectively. Given the tax parameters, $(t_M, t_R, G)$ voter-$j$ decides over consumption and labor supply:

$$\max_{c_j, l_j} u (c_j + v(L - l_j))$$

s.t.

$$c_j \leq (1 - t_j)w_j l_j + G$$

(5)

The optimal labor supply:

$$((1 - t_j)w_j - v' (L - l_j)) u'(x_j) = 0$$

$$l_j = L - h ((1 - t_j) w_j)$$

(6)

(7)

Where $x_j = c_j + v(L - l_j)$ is consumption plus leisure, $v'^{-1} = h$ with $h(.)$ decreasing in its argument and $u'(x_j) > 0$. For the quasilinear specification of $x_j$ there are no income effects, which implies that $\frac{\partial l_j}{\partial c_j} < 0$ ($\frac{\partial l_j}{\partial w_j} > 0$) and $\frac{\partial l_j}{\partial G} = 0$. All voters supply strictly positive units of labor or for all $j$, $(1 - t_j)w_j - v' (L) > 0$. That will be the case for any $w_j$ if $v' (L) = 0$.

Given this optimal labor supply the tax schedule feasibility constraints (3) and (4)
can be rewritten as,

\[
L (w_R - w_M) \geq h ((1 - t_R) w_R) w_R - h ((1 - t_M) w_M) w_M \tag{8}
\]

\[
L (w_M - w_P) \geq h ((1 - t_M) w_M) w_M - h (w_P) w_P \tag{9}
\]

The above feasibility conditions give an upper bound to \( t_R \) and \( t_M \).

The indirect utility of a voter in group \( P, M \) or \( R \) given the tax schedule \((t_M, t_R, G)\):

\[
V_P (t_M, t_R) = u (w_P l_P + G (t_M, t_R) + v (L - l_P))
\]

\[
V_M (t_M, t_R) = u ((1 - t_M) w_M l_M + G (t_M, t_R) + v (L - l_M))
\]

\[
V_R (t_M, t_R) = u ((1 - t_R) w_R l_R + G (t_M, t_R) + v (L - l_R))
\]

4 Preferences over tax schedules

Given \( G (t_M, t_R) \) specified in (2) with \( y_M = (L - h ((1 - t_M) w_M)) w_M \) and \( y_R = (L - h ((1 - t_R) w_R)) w_R \). Next we derive the group specific preferences over \((t_M, t_R)\).

The preferred tax schedule of group \( P \)

Group \( P \) does not pay income taxes by assumption. Thus, group \( P \) objective is to maximize the lump-sum transfer \( G \).

\[
G (t_M, t_R) = (1 - 2\alpha) t_M y_M + \alpha t_R y_R
\]

The f.o.c. for a maximum,

\[
\hat{t}_M : (1 - 2\alpha) w_M \left( l_M + t_M \frac{\partial l_M}{\partial t_M} \right) = 0 \tag{10}
\]

\[
\hat{t}_R : \alpha w_R \left( l_R + t_R \frac{\partial l_R}{\partial t_R} \right) = 0 \tag{11}
\]
The $P-$voter maximizes utility under the tax schedule $(\hat{t}_M, \hat{t}_R)$ which is the peak of the Laffer curve. It is easy to show that $G(t_M, t_R)$ is concave in $(t_M, t_R)$ as long as $\frac{\partial^2 \varepsilon_j}{\partial t^2_j} \leq 0$, $j = M, R$, since $\frac{\partial^2 G}{\partial t^2_M} = (1 - 2\alpha) w_M \left(2 \frac{\partial l_M}{\partial t_M} + t_M \frac{\partial^2 l_M}{\partial t^2_M}\right) < 0$, $\frac{\partial^2 G}{\partial t^2_R} = \alpha w_R \left(2 \frac{\partial l_R}{\partial t_R} + t_R \frac{\partial^2 l_R}{\partial t^2_R}\right) < 0$ and $\frac{\partial^2 G}{\partial t_M \partial t_R} = 0.$

Rearranging terms in (10) and (11), the tax schedule that maximizes redistribution satisfies,

$$|\varepsilon_M| = |\varepsilon_R| = 1$$  \hspace{1cm} (12)

Where $|\varepsilon_j| = \left|\frac{\partial l_j}{\partial t_j}\right|$ is the elasticity of labor supply to changes in the tax rate $t_j$, with $j = M, R$. Whether the $P-$voter preferred tax schedule would be proportional or marginal rate progressive (regressive) depends upon the specific utility function. We assume that $\frac{\partial^2 l_j}{\partial t^2_j} \leq 0$, then, $\varepsilon_j$ will be decreasing in $t_j$.\footnote{Given that there are only substitution effects from taxation $\left(\frac{\partial l_j}{\partial t_j} < 0\right)$ and by assumption $\frac{\partial^2 l_j}{\partial t^2_j} < 0$, the partial derivative of $\varepsilon_j$ with respect to $t_j$ is negative: $\frac{\partial \varepsilon_j}{\partial t_j} = \frac{\partial^2 l_j}{\partial t^2_j} t_j + \frac{\partial l_j}{\partial t_j} \left(t_j - \frac{\partial l_j}{\partial t_j} t_j\right) < 0.$}

After some computations $\varepsilon_j = -\frac{t_j}{1-t_j} \varepsilon_l$. By assumption $\varepsilon_l$ is decreasing (specifically non increasing) in the marginal wage which implies that $\varepsilon_j$ is increasing in the marginal wage $w_j$.

These properties altogether imply that condition (12) can not be satisfied for a regressive tax schedule. Since $\varepsilon_j$ is increasing in $w_j$ at the proportional tax $t_M = t_R = t$ we have that $\varepsilon_M(t) \leq \varepsilon_R(t)$. At any regressive tax schedule with $t_M > t_R = t$ since $\varepsilon_j$ is decreasing in $t_j$ we have that $\varepsilon_M(t_M) < \varepsilon_M(t) \leq \varepsilon_R(t)$. This proves that the preferred tax schedule of group $P$ is either proportional ($t_M = t_R$) or progressive ($t_M < t_R$).

The preferred tax schedule of group $M$

Group $M$ pays income taxes at rate $t_M$ and receives the lump-sum transfer $G$. Thus group $M$ preferred income tax minimizes his tax burden.

$$V_M (t_M, t_R) = (1 - t_M) w_M l_M + G(t_M, t_R)$$
The indirect utility $V_M(t_M, t_R)$ reaches a maximum at $(t_M, t_R) = (0, \hat{t}_R)$. For any value $\alpha$, $(t_M, t_R) = (0, \hat{t}_R)$ is a global maximum.

$$V_M(0, \hat{t}_R) - V_M(t_M, t_R) = w_M(l_M(0) + (1 - 2t_M(1 - \alpha))l_M(t_M)) + \alpha (\hat{t}_Rl_R(\hat{t}_R) - t_Rl_R(t_R)) \geq 0$$

for all $t_M, t_R$ belonging to $t = [0, 1] \times [0, 1]$. Remember that $\hat{t}_R$ maximizes $G(t_M, t_R)$ for a given $t_M$.

Note that such tax schedule is marginal rate progressive since $t_R - t_M = \hat{t}_R > 0$.

The preferred tax schedule of group $R$

Group $R$ pays income taxes at rate $t_R$ and receives the lump-sum transfer $G$. Thus group $R$ preferred income tax minimizes his tax burden.

$$V_R(t_M, t_R) = (1 - t_R) w_Rl_R + G(t_M, t_R)$$

The $R$–voter maximize utility under the regressive tax schedule, i.e. when $t_M = \hat{t}_M$ and $t_R = 0$.

$$V_R(\hat{t}_M, 0) - V_R(t_M, t_R) = (l_R(0) - (1 - t_R(1 - \alpha))l_R(t_R)) w_R + (1 - 2\alpha)(\hat{t}_Ml_M(\hat{t}_M) - t_Ml_M(t_M)) w_M \geq 0$$

for all $t_M, t_R$ belonging to $t = [0, 1] \times [0, 1]$. Remember that $\hat{t}_M$ maximizes $G(t_M, t_R)$ for a given $t_R$.

The following picture shows the bliss-points of the three different groups of voters.\footnote{The plot was made for the particular utility function $U_j = c_j - \frac{1}{2}l_j^2$. For this utility function $\hat{t}_M = \hat{t}_R = \frac{1}{2}$.}
Figure 1: $P, M$ and $R$ bliss points.

It should be noted that no CW winner exists in our voting game, as we can see in figure one. Any point in rectangle 0RPM can be defeated by a coalition of two groups. The shaded areas in Figure 1 represent the alternatives that can defeat alternative "o", which can be defeated by other alternatives generating a cycle (the voting paradox).

In the next section we give conditions under which in the probabilistic voting only progressive taxes emerge in equilibrium.

5 Political Competition

We consider electoral competition between two office-motivated parties, $A$ and $B$. They differ in their fixed ideology position and may differ in the income tax schedule they announce. Parties commit themselves to the platform announced.

We have a continuum of voters in each group differing in their ideological position, measured as their relative preference from one party over the other. In order to combine the economic and ideological side of voters’ utility we assume that a voter $i$ in group $j$ will vote for party $A$ if the extra "economic" utility he gets from the $A$’s platform
exceeds his ideological preference for \( B \) relative to \( A \). We may capture this preferences over parties in a parameter \( \sigma_{ij} \), which is the location of the individual \( i \) in group \( j \) along the real line. If \( \sigma_{ij} \) is positive (negative) the \( i \) individual in group \( j \) prefers \( B \) to \( A \) (\( A \) to \( B \)) for the same platform announced. Voters with \( \sigma_{ij} \) around zero are ideological neutral, they only care about the economic benefits the different tax schedule policies give to them.

The utility of a \( ij \)-voter is simply \( V_j(t^A) \) if party \( A \) wins and it is \( V_j(t^B) + \sigma_{ij} \) if party \( B \) wins.

Individual \( i \) in group \( j \) will vote for \( A \) if:

\[
V_j(t^A) > V_j(t^B) + \sigma_{ij}
\]  

We assume that \( \sigma_{ij} \) has a group-specific cumulative distribution function \( F_j \) with density \( f_j \) with support on the real line \([-\infty, \infty]\). The density function around zero summarizes how much ideologically neutral voters we have in each group. We next introduce conditions that guarantee a unique pure strategy Nash equilibrium of the electoral game. Those conditions from Lindbeck and Weibull (1987), Enelow and Hinich (1989) and Couglhin (1992) were unified by Banks and Duggan (2004). To those, we need to add an additional condition since in our model \( F_j \) is not independent of \( j \).

**Condition 1** On \( F_j \) and aggregate \( V_j \):

1. \( C1 \) \( F_j \) is continuous and strictly increasing.

2. \( C2 \). Aggregate concavity holds, for any \( t^{-C} \), \( \pi^C(t^A, t^B) \) is strictly concave on \( t^C \),
\[ C = A, B : \]

\[
\pi^A (t^A, t^B) = \sum_{j=P,M,R} n_j F_j (V_j (t^A) - V_j (t^B))
\]

\[
\pi^B (t^A, t^B) = 1 - \sum_{j=P,M,R} n_j F_j (V_j (t^A) - V_j (t^B))
\]

where \( n_j \) is the proportion of voters in group \( j \).

3. \( \text{Laussel and Le Breton (2002). This guarantees that there is no profitable deviation at the political equilibrium of this game.} \)

\[ \forall j \text{ } f_j \text{ is symmetric around zero and } f_j(0) > 0. \]

We define the swing voter in group \( j \) as the voter that is indifferent between party \( A \) and party \( B \) given the policies announced. Let’s call it \( \sigma_j \), we have \( \sigma_j = V_j (t^A) - V_j (t^B) \). Voters in group \( j \) with an ideological parameter smaller (higher) than \( \sigma_j \) will vote for party \( A \) (respectively \( B \)). We assume there is no abstention.

The total share of votes of party \( A \) in group \( j \), \( \pi^A_j \) is:

\[
\pi^A_j (t^A, t^B) = n_j \Pr(\sigma_{ij} < \sigma_j) = n_j F_j (\sigma_j).
\] (14)

Total voting share of party \( A \) is:

\[
\pi^A (t^A, t^B) = \alpha F_P (\sigma_P) + (1 - 2\alpha) F_M (\sigma_M) + \alpha F_R (\sigma_R) \] (15)

The probability of winning is an increasing function of the voting share. We assume for simplicity that it is a 1 to 1 relation then the probability of winning equals aggregate voting share. The main reason why we make such assumption is that it allows us to compare the outcome of the Coughlin game (where parties maximize the probability of
winning) with the outcome of the Lindbeck and Weibull game (were parties maximizes their expected voting share).

The outcome of the game presented above will be called Lindbeck and Weibull game (LW). We assume for such a game that $u(x_j)$ is logarithmic with $x_j = c_j + v(L - l_j)$.

**Lemma 1** Assume conditions $C_1$, $C_2$ and $C_3$ are satisfied. We prove that:

1. The bliss points of group $P$, $M$ and $R$ are never part of a political equilibrium.

**Proof.** 1.a) The bliss point of $P$ is not an equilibrium. If it were an equilibrium then for a given $t^B$, $\frac{\partial \pi(t^A, t^B)}{\partial t_M} dt_M + \frac{\partial \pi(t^A, t^B)}{\partial t_R} dt_R |_{t_M = \hat{t}_M, t_R = \hat{t}_R} = 0$ for $dt_M, dt_R < 0$. We have that,

$$\frac{\partial \pi(t^A, t^B)}{\partial t_M} |_{t_M = \hat{t}_M, t_R = \hat{t}_R} = (1 - 2\alpha) f_M(\sigma_M) \left( \frac{1}{x_M \frac{\partial t_M}{\partial t_M}} \right) |_{t_M = \hat{t}_M} < 0$$

Which proves that the preferred income tax of the poor group is not an equilibrium.

1.b) The bliss point of $M$ is not an equilibrium.

$$\frac{\partial \pi(t^A, t^B)}{\partial t_R} |_{t_M = 0, t_R = \hat{t}_R} = \alpha f_R(\sigma_R) \left( \frac{1}{x_R \frac{\partial t_R}{\partial t_R}} \right) |_{t_M = 0, t_R = \hat{t}_R} < 0$$

Which proves that the preferred income tax of the middle class group is not an equilibrium.

1.c) The bliss-point of $R$ is not an equilibrium.

$$\frac{\partial \pi(t^A, t^B)}{\partial t_M} |_{t_M = \hat{t}_M, t_R = 0} = (1 - 2\alpha) f_M(\sigma_M) \left( \frac{1}{x_M \frac{\partial t_M}{\partial t_M}} \right) < 0$$

Which proves that the preferred income tax of the rich group is not an equilibrium.

The above proposition proves that under this political process the outcome will
never correspond with the ideal income tax schedule of some group. This is because the probabilistic model implies some compromise between the voters involved, this explains the fact that none of the ideal income tax schedules constitutes an equilibrium. The fact that the probabilistic model in a multidimensional space picks a policy that is different from the ideal of some voter was already stressed in Casamatta, Cremer and Pestieau (2006). Indeed the equilibrium outcome in our setting is the outcome of a Nash bargaining between the three groups of voters if the utility was linear, \( u(x) = x \), with the political (bargaining) power of group \( P, M \) and \( R \) given by \( f_P(0), (1 - 2\alpha) f_M(0) \) and \( \alpha f_R(0) \) respectively.

**Proposition 1** Assume C1, C2 and C3 are satisfied. Assume that for all \( t_R \leq \hat{t}_R \) and \( t_M \leq \hat{t}_M \) the feasibility constraints (8) and (9) are satisfied. This will be true for sufficiently high wage differential between groups \( R, M \) and \( M, P \). There exist a unique interior equilibrium. By symmetry of the game at this equilibrium we have policy coincidence \( t^A = t^B = t \) with \( t_M < \hat{t}_M \) and \( t_R < \hat{t}_R \).

**Proof.** Uniqueness comes from the fact that we maximize a strictly concave function (C2) under a convex set \( T \). By symmetry of the model if \((t^A, t^B)\) is an equilibrium so it is \((t^B, t^A)\), from uniqueness necessarily \( t^A = t^B = t \).

At the equilibrium we maximize a weighted sum of voters utilities (Lindbeck and Weibull, 1987). Then, any tax \( t_j > \hat{t}_j \) with \( j = M, R \), will be played in equilibrium since it is Pareto dominated. From Lemma 1 we know that the equilibrium tax schedule is different from \((\hat{t}_M, \hat{t}_R)\), the bliss-point of group \( P \).

The feasibility constraints (8) and (9) guarantees that at equilibrium \( y_R(t_R) > y(t_M) \) and \( y_M(t_M) > y_P \). Since \( y_j \) is decreasing in \( t_j \) we have that \( t_R : y_R(t_R) - y(t_M) = 0 \) is increasing in \( t_M \). The lowest possible \( t_R \) is \( \tilde{t}_R : y_R(\tilde{t}_R) - y_M(0) = 0 \). Similarly for \( t_M \), we have \( \tilde{t}_M : y_M(\tilde{t}_M) - y_P = 0 \). If \( \tilde{t}_R \geq \hat{t}_R \) and \( \tilde{t}_M \geq \hat{t}_M \) both feasibility constraints (8) and (9) can be omitted (they wont be binding in equilibrium), moreover \( \tilde{t}_R, \tilde{t}_M \) are
higher the higher is $w_R$ and the lower is $w_P$. ■

Clearly the outcome of the Lindbeck and Weibull game maximizes the following social welfare

\[ S^{LW}(t) = \alpha f_P V_P(t) + (1 - 2\alpha) f_M V_M(t) + \alpha f_R V_R(t) \]

where $f_j = f_j(0), j = P, M, R$.

Since we are interested in tax progressivity next we develop conditions under which a progressive income tax emerges as the outcome of the Lindbeck and Weibull game. Note that our assumption on the elasticity of labor supply (we assume that $\varepsilon_l$ is decreasing in $w$) facilitates the implementation of a progressive income tax since the labor response of the $R$-group to changes on the marginal tax rate they pay is lower than that of the $M$ group. Moreover the decreasing marginal utility of consumption (net of labor disincentives) facilitates also the emergence of a progressive income tax schedule since it increases the political power of the $P$ and $M$ groups compared to the $R$ group. The marginal utility loss from an increase in the tax rate $t_R$ is lower for group $R$ than it would be a proportional increase in $t_M$ for the utility of $M$, which makes group $M$ more reactive to changes on $t_M$. Despite all this a proportional or even marginal regressive income tax may arise in equilibrium if the proportion of ideologically neutral voters $f_R$ is sufficiently large.

The equilibrium income tax satisfies the following first order constraints,

\[ \Phi(t_M, t_R)(1 - |\varepsilon_M(t_M)|) x_M(t_M, t_R) - f_M = 0 \]  \hspace{1cm} (16)

\[ \Phi(t_M, t_R)(1 - |\varepsilon_R(t_R)|) x_R(t_M, t_R) - f_R = 0 \]  \hspace{1cm} (17)

where $\Phi(t_M, t_R) = \frac{\alpha f_P}{x_P(t_M, t_R)} + \frac{(1 - 2\alpha) f_M}{x_M(t_M, t_R)} + \frac{\alpha f_R}{x_R(t_M, t_R)}$.  

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The tax schedule will be marginal rate progressive if at the proportional tax schedule there is a profitable deviation to a more progressive schedule (higher \( t_R \)). From uniqueness of the equilibrium this would imply that the proportional tax is not an equilibrium and that there is no profitable deviation from moving toward a regressive tax (lower \( t_R \)). The condition is,

\[
\frac{(1 - |\varepsilon_M(t)|) x_M(t, t)}{(1 - |\varepsilon_R(t)|) x_R(t, t)} < \frac{f_M}{f_R}
\]

(18)

Since \( \varepsilon_j \) is increasing in \( w_j \) the expression \( \left( \frac{1 + \varepsilon_M(t)}{1 + \varepsilon_R(t)} \right) \) < 1. We have that \( x_j \) is increasing in \( w_j \left( \frac{\partial x_j}{\partial w_j} = (1 - t_j) l_j > 0 \right) \), which implies that the term \( \frac{x_M(t, t)}{x_R(t, t)} < 1 \). If \( f_M \geq f_R \), only marginal progressive taxes emerge in equilibrium.

**Proposition 2** The equilibrium income tax is marginal progressive as long as the inequality below holds,

\[
\frac{(1 - |\varepsilon_M(t)|) x_M(t, t)}{(1 - |\varepsilon_R(t)|) x_R(t, t)} < \frac{f_M}{f_R}
\]

Proof. If from the proportional tax a progressive tax is a profitable deviation for Party A, then \( [(\Phi(t, t)(1 - |\varepsilon_M(t)|) x_M(t, t) - f_M) dt_M + (\Phi(t, t)(1 - |\varepsilon_R(t)|) x_R(t, t) - f_R) dt_R] > 0 \) for \( dt_M < 0 \) and \( dt_R > 0 \). Dividing the both sides by the RHS of the expression in brackets and rearranging terms we find condition for progressivity in the Lindbeck and Weibull model. ■

We now take the approach of Coughlin (1992) from Coughlin and Nitzan (1981) we know that the outcome of the electoral competition game is the social alternative that maximizes a Nash social welfare function. For simplicity we assume that the \( V_j(t_M, t_R) = x_j(t_M, t_R) \).

Following Coughlin and Nitzan (1981), the party’s objective function is then,

\[
S^{CN}(t) = \alpha \ln x_P(t_M, t_R) + (1 - 2\alpha) \ln x_M(t_M, t_R) + \alpha \ln x_R(t_M, t_R)
\]

(19)
Note that the equilibrium outcome of this game is the result of a Nash bargaining between the three groups of voters with the political (bargaining) power of group $P$, $M$ and $R$ given by $\alpha$, $(1 - 2\alpha)$ and $\alpha$ respectively. Since in this game the political power of the poor and the rich is the same the first prefers a progressive (or proportional) income tax and the $M$– group unambiguously prefer a progressive income tax only marginally progressive taxes emerge in equilibrium.

**Proposition 3** Only marginal rate progressive taxes emerge in equilibrium.

**Proof.** Note that the CN game is equivalent to the LW game for $F_j$ independent of $j$ (this was previously stressed by Banks et al., 2003). In such case $f_j = f$, i.e. $f_M = f_R$. As proved in Proposition 2 this is a sufficient condition for marginal progressive taxes.

When voters choose a candidate (party) probabilistically the best response for both candidates is to announce a marginal rate progressive tax schedule. This is because when competing for elections they try to attract the *swing* voters, the ones that by increasing their consumption level a little, parties increase a lot the probability to vote for the party. Such swing voters are the lower income voters provided that the probability to vote for a given party, say left, is concave in their consumption level since $V_j$ is linear in $c_j$, being $\ln V_j$ a proxy for the probability of voting for the party. The preferred tax schedule of those low income is either proportional (in some cases for group $G$) or progressive under the assumption of decreasing elasticity of labor on wages. This guarantees that a move from (proportionality) regressivity toward progressivity is profitable. It captures more votes from swing voters, since the marginal gain in increasing consumption for group $M$ is higher than the one of group $R$. 

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5.1 Comparative Statics

We wonder at this point how the degree of progressivity changes as a result of a change in the parameters of the model. Assume (8) and (9) are satisfied at the solution of $S^{CN} : (t_M^*, t_R^*)$.

**Proposition 4**

1. In the CN game an increase in population polarization, measured by $\alpha$ decreases the progressivity degree.
2. An increase in $f_M$ ($f_R$) decreases the equilibrium tax rate $t_M$ ($t_R$). Redistribution is increasing in $f_P$.

**Proof.** At the equilibrium tax schedule $(t_M^*, t_R^*)$ we have that,

$$\frac{\partial \pi}{\partial t_M} = \alpha \left( \frac{1}{x_P} \frac{\partial x_P}{\partial t_M} + \frac{1}{x_R} \frac{\partial x_R}{\partial t_M} \right) + (1 - 2\alpha) \left( \frac{1}{x_M} \frac{\partial x_M}{\partial t_M} \right) = 0 \quad (20)$$

$$\frac{\partial \pi}{\partial t_R} = \alpha \left( \frac{1}{x_P} \frac{\partial x_P}{\partial t_R} + \frac{1}{x_R} \frac{\partial x_R}{\partial t_R} \right) + (1 - 2\alpha) \left( \frac{1}{x_M} \frac{\partial x_M}{\partial t_R} \right) = 0 \quad (21)$$

We know that for $t_M^* > 0$, $\frac{\partial x_M}{\partial t_M} < 0$. Then $\left( \frac{1}{x_P} \frac{\partial x_P}{\partial t_M} + \frac{1}{x_R} \frac{\partial x_R}{\partial t_M} \right) > 0$ to satisfy equation (20). While for $t_R^* < \hat{t}_R$, $\frac{\partial x_M}{\partial t_M} > 0$. Then, necessarily $\left( \frac{1}{x_P} \frac{\partial x_P}{\partial t_R} + \frac{1}{x_R} \frac{\partial x_R}{\partial t_R} \right) < 0$ to satisfy equation (21).

Consider now a different economy with $\alpha' > \alpha$.

At $(t_M^*, t_R^*)$ it can be easily showed that a higher $\alpha$ gives a higher weight to the negative part of the f.o.c. condition (21) and a lower weight to the positive part $\frac{\partial x_M}{\partial t_R}$. Then, we have,

$$\frac{\partial \pi}{\partial t_R} = \alpha' \left( \frac{1}{x_P} \frac{\partial x_P}{\partial t_R} + \frac{1}{x_R} \frac{\partial x_R}{\partial t_R} \right) + (1 - 2\alpha) \left( \frac{1}{x_M} \frac{\partial x_M}{\partial t_R} \right) < 0$$

From concavity of $\pi(.)$, this implies that the equilibrium tax rate $t_R$ at the economy $\alpha'$ is lower than $t_R^*(\alpha)$, i.e. $t_R^*(\alpha') < t_R^*(\alpha)$. 

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Applying an analogous reasoning we have that at \((t_M^*, t_R^*)\) \(\frac{\partial \pi}{\partial t_M} > 0\) for a higher \(\alpha\),

\[
\frac{\partial \pi}{\partial t_M} = \alpha' \left( \frac{1}{x_P \partial t_M} + \frac{1}{x_R \partial t_M} \right) + (1 - 2\alpha) \left( \frac{1}{x_M \partial t_M} \right) > 0
\]

From concavity of \(\pi (.)\), this implies that the equilibrium tax rate \(t_M\) at the economy \(\alpha'\) is higher than \(t_M^* (\alpha)\), i.e. \(t_M^* (\alpha') > t_M^* (\alpha)\).

Finally,

\[
t_R^*(\alpha') < t_R^*(\alpha) \quad \text{and} \quad t_M^* (\alpha') \geq t_M^* (\alpha)
\]

\[
\implies t_R^*(\alpha') - t_M^* (\alpha') < t_R^*(\alpha) - t_M^* (\alpha)
\]

The progressivity degree is lower the higher is the population polarization.

2. In the LW model applying the implicit function theorem to foc conditions (16) and (17), we have:

\[
\frac{\partial t_M}{\partial f_M} = - \left( \frac{\partial \Phi}{\partial f_M} (1 - |\varepsilon_M (t_M)|) x_M (t_M, t_R) - 1 \right) / D
\]

where \(D = \left( \frac{\partial \Phi}{\partial t_M} (1 - |\varepsilon_M (t_M)|) x_M (t_M, t_R) + \Phi \frac{\partial x_M}{\partial t_M} x_M (t_M, t_R) + \Phi (1 - |\varepsilon_M (t_M)|) \frac{\partial x_M}{\partial t_M} \right) < 0\) by concavity of \(\pi (t)\) on \(T\). Substituting \(\frac{\partial \Phi}{\partial f_M} = \frac{(1 - 2\alpha)}{x_M (t_M, t_R)}\) in the above equation:

\[
\text{sgn} \left( \frac{\partial t_M}{\partial f_M} \right) = \text{sgn} \left( (1 - 2\alpha) (1 - |\varepsilon_M (t_M)|) - 1 \right) < 0. \text{ Similarly for } t_R:
\]

\[
\frac{\partial t_R}{\partial f_R} = - \left( \frac{\partial \Phi}{\partial f_R} (1 - |\varepsilon_R (t_R)|) x_R (t_M, t_R) - 1 \right) / D
\]

\[
\text{sgn} \left( \frac{\partial t_R}{\partial f_R} \right) = \text{sgn} \left( (\alpha (1 - |\varepsilon_R (t_R)|) - 1 \right) < 0
\]

The degree of progressivity decreases with \(f_R\), since \(\frac{\partial \Phi}{\partial f_R} = \frac{\alpha}{x_R (t_M, t_R)} > 0\) and more generally \(\text{sgn} \left( \frac{\partial t_j}{\partial f_j} \right) = \text{sgn} \left( \frac{\partial \Phi}{\partial f_j} \right) > 0, \ j = P, R. \text{ Finally the redistribution level } G (t_M, t_R) \text{ is increasing in } f_P, \text{ the political power of the group whose preferred tax schedule is the}
peak of the Laffer curve.

$$\frac{\partial G}{\partial f_P} = \frac{\partial G}{\partial t_M} \frac{\partial t_M}{\partial f_P} + \frac{\partial G}{\partial t_R} \frac{\partial t_R}{\partial f_P} > 0$$

If we think of \((t_M^*, t_R^*)\) as the solution to a bargaining between groups \(P, M\) and \(R\) with the no-agreement option settled at zero utility, with \(\alpha f_P\) the bargaining power of group \(P\), \(\alpha f_R\) the bargaining power of \(R\) and \((1 - 2\alpha) f_M\) the bargaining power of group \(M\). The previous result states that the degree of progressivity -the type of tax schedule most preferred by the middle class- is lower the lower is the bargaining power of the \(M\)–group. This is the case in the CN and the LW model. Finally in the LW model redistribution is unambiguously increasing in the political power of the \(P\)–group.

6 Conclusion

With this simple model we wanted to show that even with the chosen specific preferences, with only substitution effects from labor supply, only marginal rate progressive taxes will constitute a political equilibrium in the CN model. Our assumption on the elasticity of labor supply (that \(\varepsilon_l\) is decreasing in marginal wage) that is met by some familiar utility specifications ease the implementation of a marginal rate progressive tax, it gives an additional dimension for the middle class \((M)\) to be more swing. We could say that provided that \(\varepsilon_l\) is decreasing in marginal wage, it makes the richer group cheaper to tax compared to the fixed income case. At the fixed or exogenous income case we have that a tax schedule will be marginal rate progressive as long as \(\frac{x_M(t,t)}{x_R(t,t)} < \frac{I_M}{I_R}\) (condition in Proposition 1 for progressivity at \(\varepsilon_j = 0, j = M, R\)). This condition is stronger or harder to satisfy than the general condition stated in Proposition 1.
The second point to highlight is that inequality is not one dimensional. We have to think also about polarization, of population groups or political power and on differences in wages. The tax progression is decreasing in polarization in the CN game and it will be by an analogous analysis decreasing in the political power of the middle class in the LW game.

Bibliographical references


