UNCERTAINTY AND THE PRICE OF RISK IN A NOMINAL CONVERGENCE PROCESS

Ricardo Gimeno

José Manuel Marqués

Simposio de Análisis Económico 2008
DECOMPOSITION OF NOMINAL INTEREST RATE ON:

\[ Y_t^k = r_t + E_t[\pi_{t,t+k}] + \gamma_{t,t+k} \]

- Real risk-free rate
- Expected inflation
- Risk premium

Several fields:

- Monetary Policy
- Bond Pricing
- Portfolio decision
Different approach to decomposition

Split One observed variable in three unobserved components!!!

Macro Approach:
- Do not care about term premium (usually modelled one or two maturities)
- Look for macro interpretation of movements
- Usually introduce some assumption for $E[\Pi]$ and risk premium


Finance Approach:
- Term premium is important (Replicate the yield curve)
- Statistical approach, do not care about macro interpretation
- Risk premium use to be endogenous
- Usually forecasting purpose


Macro Finance Approach:
- Reply the yield curve using an affine model and introduce as factors some macro variables.

Macro Finance approach

The same model for several purposes:

• **Good estimation properties:** Unobserved components allows for flexible estimation
  

• **Macro interpretation:** By adding macro variables you can link their evolution to that of the interest rates.
  

• **Forecasting Purpose:** Financial market expectation would be embedded along the term structure
  
  Lee (2007), Garcia and Werner (2008)
What is new in this paper?

- We will use as factors the term structure components as factors
- We estimate the risk premia (instead of an inflation risk premia)

What do we obtain?

Easy to obtain the decomposition and more robust results.

The model provides some acceptable results for real interest rate in a country involved in a convergence process

The inflation expectations evolution and the risk premia evolution seems to be compatible with other variables
Basis of the Macro – Finance Models

Affine Model with factors

\[ P_t^k = e^{-k} Y_{t,t+k} \]

\[ kY_{t,t+k} = A_k + B'k X_t \]

\[ X_t = \mu + \varphi X_{t-1} + \varepsilon_t \]

Non arbitrage opportunities

produce recursive form for \( A_k, B_k \)

Risk aversion

generates risk premium

Discrete form:

\[ (1+Y_{t,t+k}) = (1+Y_{t,t+1})(1+E_t(Y_{t+1,t+k-1})) \]

\[ P_t \neq E_t^Q \left[ e^{-Y_{t,t+1}} \right] \]
The affine model

Interest rate evolution

1. Nominal Interest rates are affine to some factors $X_t$

$$y_{t,t+k} = \frac{1}{k} (A_k + B_k' X_t) + u_{t,t+k}$$

$u_t \sim N(0, \sigma^2 I)$

Changes along the term structure

Adjustment Term

Term

Changes in time

2. Factors dynamics follows a VAR:

$$X_t = \mu + \Phi X_{t-1} + \varepsilon_t$$

$\varepsilon_t \sim N(0, \Sigma)$

Factors can be forecasted

Uncertainty Term
The affine model

Bond Valuation (under risk aversion)

There is no arbitrage opportunity:

\[ P_t = E_t \left[ e^{-y_{t,t+1}} e^{\left(-\frac{1}{2} \lambda_t' \lambda_t - \lambda_t \epsilon_{t+1}\right)} \right] \]

Price of risk assumed to be time-variant and affine

\[ \lambda_t = \lambda_0 + \lambda_1 X_t \]

Risk compensation

Jensen Inequality

Price of risk

\[ A_{k+1} = A_1 + A_k + B_k' \mu - B_k' \Sigma \lambda_0 + \frac{1}{2} B_k' \Sigma \Sigma' B_k' \]

\[ B_{k+1}' = B_1' + B_k' \Phi - B_k' \Sigma \lambda_1 \]
Decomposition of nominal interest rates

Nominal interest rates can be decomposed in:

\[ \hat{y}_{t,t+k} = r_t + E_t[\pi_{t,t+k}] + \gamma_{t,t+k} \]

1. Risk premium

\[ \gamma_{t,t+k} = \hat{y}_{t,t+k} - \gamma(\lambda_t = 0)_{t,t+k} \]

2. Expected Inflation

\[ E_t[\pi_{t,t+k}] = \prod_{h=0}^{k-1} (1 + E_t[\pi_{t+h,t+h+1}])^{-1} \]

3. Real risk-free rate

\[ r_{t,t+k} = \hat{y}_{t,t+k} - E_t[\pi_{t,t+k}] - \gamma_{t,t+k} \]
Determination of affine factors

Differences among models will be given by the $X_t$ factors used

Standard approach in the decomposition literature (i.e. ABW, 2006):

- Latent factors + Inflation rate

\[
\begin{pmatrix}
q_{t+1} \\
\mu q \\
f_{t+1} \\
\mu f \\
\pi_{t+1} \\
\mu \pi
\end{pmatrix}
= \begin{pmatrix}
\Phi qq & 0 & 0 \\
\Phi qf & \Phi ff & 0 \\
\Phi q\pi & \Phi f\pi & \Phi \pi\pi
\end{pmatrix}
\begin{pmatrix}
q_t \\
f_t \\
\pi_t
\end{pmatrix}
+ \begin{pmatrix}
\sigma q & 0 & 0 \\
0 & \sigma f & 0 \\
0 & 0 & \sigma \pi
\end{pmatrix}
\begin{pmatrix}
u_q \\
u_f \\
u \pi
\end{pmatrix} \]
Traditional estimation based on latent factors

**Latent (endogenous) factor** model implies some estimation problems:

- Results are sensitive to the terms observed without error

  3 months and 5 years interest rates allows factors to a better approach to the term structure

- Optimization results are extremely sensitive to the parameter’s initial values

  Genetic algorithm allows to partially overcome initial values dependence

- Restrictions have to be added to the general affine model

  Loss of flexibility in the model
Traditional estimation based on latent factors

Estimation results:

• **Expected inflation** rate temporally over observed one

• Descending **risk premium**

• Less volatile **real interest rates**

…but

• Incomplete adjustment of the term structure

• Extremely computationally intensive

• No robust estimations
Exogenous- observed factors

Our proposal: exogenous determined factors

- Adapt Diebold and Li (2006) observed factors model (Nelson and Siegel, 1986)

$$y_{t,t+k} = L_t + S_t \frac{1-e^{-k/\tau}}{k/\tau} + C_t \left( \frac{1-e^{-k/\tau}}{k/\tau} - e^{-k/\tau} \right)$$

- Long term interest rates
- Slope
- Curvature

- Add inflation rate in order to decompose (Diebold and Li model aim was forecasting)

$$X_t = \begin{bmatrix} L_t \\ S_t \\ C_t \\ \pi_t \end{bmatrix}$$

(Figure)
Exogenous Factor model

Advantages:

• Better fitting of the whole term structure and inflation rate
  Better performance than latent factors
• Initial values easily recovered via OLS regressions
  No need of genetic algorithm
• No parameter restriction required
  More flexibility
• No problems of robustness on the estimation methodology
  Less computational intensive
Ex-post interest rates

Ex post interest rate decomposition

Ex post real interest rates
Observed inflation
Ex-ante interest rates

Interest rates decomposition

- Real risk-free rate
- Risk premium
- Inflation expectations

Reduction in inflation expectations
Volatility driven by risk premium
2.5%
Risk premium is related with interest rate differentials

![Risk premium chart](chart.png)
Risk premium is related with interest rate differentials
Inflation expectations

In a convergence process is usual to find that during some periods inflation expectations are persistently above realized one.
Inflation expectations

During this episodes there are some concerns about the success of the convergence process
Inflation expectations close to the monetary policy targets during this period
Conclusions

- We propose an affine methodology based on observed components to decompose nominal interest rate that was applied to a convergence episode.

- Traditional macro models did not offer an evolution for real risk free rate interest rate in a convergence country compatible with other macro figures (inflation rate, productivity, housing price, stock exchange index...)

- Our results point out that most of the variation of nominal interest rate was capture by risk premia and that during several years inflation expectation was clearly above the observed one.
Follow up

- Use this methodology for European and US interest rate decomposition and compare with other alternatives

- Analyze the forecast properties of the model

- Policy implications working with the VAR estimation

- Apply this methodology to other countries:
  - Countries currently involved in a convergence process
  - Countries with currency crisis and strong changes in inflation expectations and risk premia
Euro area decomposition

Nominal interest rate decomposition. Euro area
2 years

3 years

Nominal interest rate decomposition. Euro area
5 Years

Inflation expectation  Prima de riesgo  Tipo real ex-ante

2018  2019  2020  2021  2022  2023  2024  2025  2026  2027

0.0%  1.0%  2.0%  3.0%  4.0%  5.0%  6.0%
Euro area decomposition

2 Years. Euro Area
- SPF euro area
- Observed Inflation 2 years ahead
- Inflation expectations

5 Years. Euro Area
- SPF
- Expectations
- Consensus
- Observed inflation 5 years ahead
US decomposition

Nominal interest rate decomposition: US. 2 Years
- Expected Inflation
- Risk Premia
- Ex-ante Real interest rate

Nominal interest rate decomposition: US. 5 Years
- Expected Inflation
- Risk Premia
- Ex-ante Real interest rate
US decomposition

**Expected Inflation.- US 1 year**
- SPF US
- Inflation expectations

**Expected Inflation.- US 10 year**
- SPF US
- Inflation expectations
Emerging countries decomposition. Poland

Nominal interest rate decomposition. Poland
2 years

Nominal interest rate decomposition. Poland
5 Years
Other countries. Canada

Nominal interest rate decomposition. Canada
2 years

Expectativas de inflación

- Inflation expectation
- Prima de riesgo
- Ex ante real rate

- Inflation (Two years ahead)
- Inflation expectation
- Observed Inflation
THANK YOU FOR YOUR ATTENTION
Latent Factors: Expected inflation

Inflation Expectations

- Average Observed inflation (5 Years)
- Inflation expectations.

(dic-90 dic-91 dic-92 dic-93 dic-94 dic-95 dic-96 dic-97 dic-98)

(back)
Latent Factors: Risk premium

Risk Premium

Risk Premium

0.0% 0.2% 0.4% 0.6% 0.8% 1.0% 1.2%
dic-90 dic-91 dic-92 dic-93 dic-94 dic-95 dic-96 dic-97 dic-98
Latent Factors: Real interest rates

Real interest rate. Unobserved Components

Ex-Post Real Interest rates
Real Interest Rates

(dic-90 dic-91 dic-92 dic-93 dic-94 dic-95 dic-96 dic-97 dic-98)
Observed Factors

Observed factors

- Level
- Slope
- Curvature
- CPI

(Latent)

(back)
Latent factors

Unobserved factors (2 regimes)

CPI  f factor  q factor (Right)

(back)
Latent vs. observed factors

Curvature factors

- Exogenous model (Curvature factor)
- Endogenous model (q factor) (inverted, right scale)
Latent vs. observed factors

Level factors

- Exogenous model (Level+Slope factors)
- Endogenous model (f factor)(right scale)
### Table 1: Mean absolute error in the estimated models

#### Inflation rate (mean absolute error)

<table>
<thead>
<tr>
<th>1 month</th>
<th>3 months</th>
<th>1 year</th>
<th>5 years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Endogenous model</td>
<td>0.016%</td>
<td>0.158%</td>
<td>0.378%</td>
</tr>
<tr>
<td>Exogenous model</td>
<td>0.050%</td>
<td>0.161%</td>
<td>0.371%</td>
</tr>
<tr>
<td>Univariate ARIMA</td>
<td>0.075%</td>
<td>0.277%</td>
<td>0.462%</td>
</tr>
</tbody>
</table>

#### Nominal interest rates (mean absolute error)

<table>
<thead>
<tr>
<th>3 months</th>
<th>1 year</th>
<th>3 years</th>
<th>5 years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Endogenous model</td>
<td>0.000%</td>
<td>0.169%</td>
<td>0.132%</td>
</tr>
<tr>
<td>Exogenous model</td>
<td>0.019%</td>
<td>0.023%</td>
<td>0.014%</td>
</tr>
</tbody>
</table>
Term structure fitting

Nominal Interest Rates (1 year)
- Observed Nominal interest rates
- Exogenous model
- Endogenous model

Nominal Interest Rates (3 years)
- Observed Nominal interest rates
- Exogenous model
- Endogenous model

(back)
Term structure fitting

Nominal Term structure (June 1994)

Nominal Term structure (August 1995)

(back)
Term structure fitting

Inflation rate forecast 1 month ahead
- Observed CPI
- Univariate ARIMA
- Endogenous model
- Exogenous model

Inflation rate forecast 1 year ahead
- Observed CPI
- Univariate ARIMA
- Endogenous model
- Exogenous model

Inflation rate forecast 3 months ahead
- Observed CPI
- Univariate ARIMA
- Endogenous model
- Exogenous model

Inflation rate forecast 5 years ahead
- Observed CPI
- Univariate ARIMA
- Endogenous model
- Exogenous model

Observed CPI
Univariate ARIMA
Endogenous model
Exogenous model

(back)
Exogenous factor model (regime switching)

Real interest rate

- ExPost Real Interest Rate
- Real Interest Rate. 1 Regime
- Real Interest Rate. 2 Regime

 dic-90 dic-91 dic-92 dic-93 dic-94 dic-95 dic-96 dic-97 dic-98
Observed factor model (regime switching)

**Inflation Expectations**
- Average Inflation Rate (5 Years)
- Inflation Expectation. 1 regime
- Inflation Expectation. 2 Regimes

**Risk Premia**
- Risk Premia. 2 Regimes
- Risk Premia 1 Regime
Euro area interest rates

Euro area nominal rates decomposition

- Real risk free rate
- Risk premium
- Inflation expectations
Euro area interest rates

Inflation expectations

- Observed inflation rates
- Expected inflation

Graph showing observed and expected inflation rates from 1998 to 2006.
Estimated model

VAR equation:

\[
\begin{pmatrix}
L_{t+1} \\
S_{t+1} \\
C_{t+1} \\
\pi_{t+1}
\end{pmatrix} = \begin{pmatrix}
1.6 \times 10^{-4} \\
9.1 \times 10^{-6} \\
-1.5 \times 10^{-4} \\
-1.34 \times 10^{-5}
\end{pmatrix} + \begin{pmatrix}
1.006 & 0.048 & 0.045 & -0.039 \\
-0.0183 & 0.932 & 0.013 & 0.027 \\
-0.347 & -0.355 & 0.845 & 0.659 \\
0.024 & 0.014 & 0.008 & 0.944
\end{pmatrix}\begin{pmatrix}
L_t \\
S_t \\
C_t \\
\pi_t
\end{pmatrix} + \begin{pmatrix}
4.3 \times 10^{-4} \\
4.1 \times 10^{-4} \\
11.8 \times 10^{-4} \\
2.3 \times 10^{-4}
\end{pmatrix} + \begin{pmatrix}
\varepsilon_{1t+1}^1 \\
\varepsilon_{2t+1}^2 \\
\varepsilon_{3t+1}^3 \\
\varepsilon_{4t+1}^4
\end{pmatrix}
\]

Price of Risk equation:

\[\lambda_t = \begin{pmatrix}
-0.07 \\
0.28 \\
0.23 \\
-54.81
\end{pmatrix} + \begin{pmatrix}
66.75 & 105.96 & 45.64 & -91.49 \\
-79.59 & -64.24 & -40.81 & 66.82 \\
-336.92 & -295.64 & -36.01 & 556.27 \\
-1.6 \times 10^{-6} & -5.3 \times 10^{-5} & 62400.62 & 469.53
\end{pmatrix}\begin{pmatrix}
L_t \\
S_t \\
C_t \\
\pi_t
\end{pmatrix}\]

Short term interest rates:

\[y_{t,t+1} = -1.02 \times 10^{-4} + \begin{pmatrix}
1.014 & 0.979 & 0.005 & -0.0004
\end{pmatrix}\begin{pmatrix}
L_t \\
S_t \\
C_t \\
\pi_t
\end{pmatrix}\]