A Dynamic Model of Altruistically-Linked Agents

Daniel Barczyk and Matthias Kredler

December 13, 2008
Motivation

Our goal: A dynamic model of agents with two-sided altruism

- Modeling agents with altruism natural in many contexts
- Want **building block** to study:
  - Social Insurance (e.g. long-term care)
  - Fiscal policy (e.g. Ricardian Equivalence)
  - Informal insurance (e.g. co-residence and time assistance)
- Theory for *timing* of inter-vivos transfers in strategic setting
Setting

- Two infinitely lived agents in continuous time: she \((x)\) and he \((x')\)
- No borrowing: \(k_t \geq 0, k'_t \geq 0\)
- Safe asset: pays at rate \(r\)
- Exogenous (stochastic) endowment stream: \(\{y, y'\}_{t_0}^{\infty}\)
- Non-negative transfers: \(g_t \geq 0, g'_t \geq 0\).

Laws of motion for wealth:

\[
\begin{align*}
\dot{k}_t &= rk_t + y_t - c_t - g_t + g'_t \\
\dot{k}'_t &= rk'_t + y'_t - c'_t - g'_t + g_t
\end{align*}
\]
Altruistic preferences

- She ranks allocations according to
  \[ V_0 \equiv E_{\{y,y'\}} \int_0^\infty e^{-\rho t} \left[ u(c_t) + \alpha u(c'_t) \right] dt \]

- He ranks allocations according to
  \[ W_0 \equiv E_{\{y,y'\}} \int_0^\infty e^{-\rho t} \left[ u(c'_t) + \alpha' u(c_t) \right] dt \]

- \( \alpha, \alpha' \in [0, 1] \) and \( \rho > r \)

Formulation encompasses:
- \( \alpha = \alpha' = 0 \): Selfishness
- \( \alpha = \alpha' = 1 \): Perfect altruism
- \( \alpha > 0, \alpha' = 0 \): One-sided altruism
Pareto frontier

\[
\max_{\{c_t,c'_t\}_0^\infty} V_0 + \eta W_0
\]

1. Intratemporal optimality:

\[
(1 + \eta \alpha') u'(c_t) = (\eta + \alpha) u'(c'_t), \quad \forall t.
\]

2. Intertemporal optimality:

\[
\frac{d}{dt} u'(c_t) = (\rho - r) u'(c_t).
\]
Markov-perfect equilibrium

State: $x \equiv (k, k'; y, y')$

**Definition:** A *Markov-perfect equilibrium* consists of feasible stationary policy functions \( \{c(x), g(x)\} \) and \( \{c'(x), g'(x)\} \) such that

- Her policy maximizes \( V_0 \) given his strategy for any initial \( x \), and
- His policy maximizes \( W_0 \) given her strategy for any initial \( x \)

$\Rightarrow$ Implies subgame perfection
For given strategies for him, at any point \( k, k' \), her HJB is given by:

\[
0 = -\rho V(k, k') + \alpha u(c') + \\
+ (rk + y + g')V_k(k, k') + (rk' + y' - c')V_{k'}(k, k') + \\
+ \max_{0 \leq g} \left\{ g\left[ V_{k'}(k, k') - V_k(k, k') \right] \right\} + \max_{0 \leq c} \left\{ u(c) - cV_k(k, k') \right\} \\
\equiv \mu(k, k')
\]
Euler equation for consumption

\[
\frac{d}{dt} u'(c_t) = (\rho - r) u'(c_t) + \left[ V_{k'} - \alpha u'(c') \right] \frac{\partial c'}{\partial k}
\]

<table>
<thead>
<tr>
<th>period 1</th>
<th>period 2</th>
<th>period 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-u'(c_1)$</td>
<td>$+ \beta Ru'(c_2)$</td>
<td>$- \beta^2 V_{k'}(k_3, k'_3) R \frac{\partial c'_2}{\partial k_2}$</td>
</tr>
</tbody>
</table>

$\Rightarrow$ Typically: inefficiency/overconsumption
Theoretical results

- If $\alpha + \alpha' > 0$, then there is **no fully autarkic equilibrium** (i.e. $g > 0$ or $g' > 0$ for some $(k, k')$).

- If $\alpha \alpha' < 1$, then there is **no wealth-pooling equilibrium**: $c(k, k') = c(k + k')$ and $c'(k, k') = c'(k + k')$.
  At $k' = 0$, *she would keep him shorter than wealth-pooling prescribes.*

- **Party theorem**: On paths into bankruptcy, $c_t$ exhibits downward jump at $k = 0$.
  *Recipient is not spending his own money.*

- Homogeneous case ($y = y' = 0$, $u'(c) = c^{-\gamma}$): ODE characterizing no-transfer regions.
$c$ and $c'$
Asset evolution

\[ y = 30.00 \]

Her k (\( \alpha = 0.4 \))

His k' (\( \alpha' = 0.4 \))
Conclusions

1. Theory consistent with following stylized facts on inter-vivos transfers:
   - Transfers likely when recipient is liquidity-constrained
   - Transfers tend to go from wealthier to cash-strapped households

2. This is a theory of dynamic partial insurance:
   - Transfers flow even if recipient will not reciprocate
   - Big role for liquidity constraints

3. Advantages of continuous-time setting:
   - Simplification: Drop second-order effects (HJB)
   - Characterization of paths into bankruptcy