Credit Constraints, Entrepreneurial Risk, and Aggregate Liquidity

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Motivation and Questions

- Financial market turbulence in 2007/2008
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  - How does public supply of liquidity affect private creation of liquidity by firms (inside liquidity)?
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  - How does public supply of liquidity affect private creation of liquidity by firms (inside liquidity)?
  - How does this interact with firms' demand for liquidity to influence investment?
Empirical Motivation

Debt-to-GDP Data

- Significant variation in U.S. debt-to-GDP during post-war era.

Figure: Gross National Public Debt as % of GDP for the United States. (Source: Office of Management and Budget, White House, 2008)
• Aggregate supply of liquidity and investment
Literature

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• Contribution
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Contribution

- analyze effects of public supply of liquidity on investment in a model with financial constraints, corporate demand for liquidity, and endogenous private supply of aggregate liquidity
Main results

- Conditions under which government debt may boost or reduce private investment:
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  - depend on three channels: (1) crowding-in, (2) crowding-out, and (3) redistributive.
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  - crowding-in dominates crowding-out with severe financial frictions
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  - redistribute positive in general, negative only if severe financial frictions.
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- Business cycle properties
Main results

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• Business cycle properties
  • Response of economy sensitive to liquidity conditions.
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- Business cycle properties
  - Response of economy sensitive to liquidity conditions.

- Asset pricing
  - Negative relationship between the debt-to-GDP ratio and the equity premium, explanation for reduction in the equity premium?
Talk

01 Benchmark model
02 Steady-State Analysis of Variations in Public Debt
03 Extension: time-varying liquidity of equity
01 Dynamics
04 Conclusions
Infinite-horizon, stochastic, discrete time economy
Model

- Infinite-horizon, stochastic, discrete time economy
- 3 agents
Model

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  - entrepreneurs
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    - All: can produce C good using L from workers and K
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2 goods: consumption goods and capital
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2 goods: consumption goods and capital
Important ingredient:
Model

- Infinite-horizon, stochastic, discrete time economy
- 3 agents
  - entrepreneurs
    - All: can produce $C$ good using $L$ from workers and $K$
    - Some: can also produce $K$
  - workers
  - government
- 2 goods: consumption goods and capital
- Important ingredient:
  - financial constraints $+$ stochastic arrival of investment opportunities $\implies$ demand for liquidity.
Entrepreneurs (I)

- Maximize $E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$, where $u(c) = \ln c$. 
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- *Occasionally* have access to capital-producing technology:
  $$k_{t+1} = (1 - \delta)k_t + i_t.$$
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- **INVESTING** entrepreneurs
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  \[ k_{t+1} = (1 - \delta)k_t + i_t. \]
- arrives only with prob $\pi$ in any given period
- arrival is i.i.d.
- **SAVING** entrepreneurs
Entrepreneurs (II)
Financial Frictions and Demand for Liquidity

- Entrepreneurs finance themselves by issuing equity $e_t$. 

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Entrepreneurs (II)
Financial Frictions and Demand for Liquidity

- Entrepreneurs finance themselves by issuing equity $e_t$.
  - All equity fully collateralized using capital $\Rightarrow$ same return as capital:

$$R_{t+1}^e = \frac{a_{t+1}^\alpha k_{t+1}^{\alpha-1} l_{t+1}^{1-\alpha} + (1 - \delta) q_{t+1}}{q_t}$$
Entrepreneurs (II)
Financial Frictions and Demand for Liquidity

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  - All equity fully collateralized using capital \( \rightarrow \) same return as capital:
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    R_{t+1}^e = \frac{a_{t+1}^\alpha k_{t+1}^{\alpha-1} l_{t+1}^{1-\alpha} + (1 - \delta) q_{t+1}}{q_t}
    \]
  - From now on: capital and outside equity as the same, denote it $n_t$ and call it "equity".
Entrepreneurs (II)

Financial Frictions and Demand for Liquidity

- Entrepreneurs finance themselves by issuing equity $e_t$.
  - All equity fully collateralized using capital $\implies$ same return as capital:
    $$R_{t+1}^e = \frac{a_{t+1}k_{t+1}^{\alpha-1}l_{t+1}^{1-\alpha} + (1 - \delta)q_{t+1}}{qt}$$
  - From now on: capital and *outside* equity as the same, denote it $n_t$ and call it "equity".

- Assumption
Entrepreneurs (II)

Financial Frictions and Demand for Liquidity

- Entrepreneurs finance themselves by issuing equity $e_t$.
  - All equity fully collateralized using capital $\rightarrow$ same return as capital:
    \[
    R_{t+1}^e = \frac{a_{t+1} \alpha k_{t+1}^{\alpha-1} I_t^{1-\alpha} + (1 - \delta) q_{t+1}}{q_t}
    \]
  - From now on: capital and outside equity as the same, denote it $n_t$ and call it "equity".

- Assumption
  - entrepreneurs can only pledge fraction $\theta$ of the returns to investment.
    \[
    e_t \leq \theta i_t, \quad (1)
    \]
    \[
    b_{t+1}^i \geq 0. \quad (2)
    \]
Entrepreneurs (III)
Optimization of an Investing Entrepreneur

• When $q_t > 1$ (Tobin’s q) invest as much as possible to produce capital:

$$c^i_t + (1 - \theta q_t) i_t + \tau^i_t = r_t n_t + q_t (1 - \delta) n_t + b_t.$$  (3)
Entrepreneurs (III)

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- When $q_t > 1$ (Tobin's $q$) invest as much as possible to produce capital:

$$c_t^i + (1 - \theta q_t)i_t + \tau_t^i = r_t n_t + q_t (1 - \delta) n_t + b_t.$$  \hspace{1cm} (3)

- Optimal consumption and investment choices:

$$c_t = (1 - \beta) \left\{ r_t n_t + q_t (1 - \delta) n_t + b_t - \tau_t^i \right\},$$ \hspace{1cm} (4)

$$i_t = \frac{[r_t n_t + q_t (1 - \delta) n_t + b_t - \tau_t^i] - c_t^i}{1 - \theta q_t}.$$ \hspace{1cm} (5)
Entrepreneurs (IV)
Optimization of a Saving Entrepreneur

• Budget constraint for a saver (superscript $s$):

$$c^s_t + \tau^s_t = r_t n_t + q_t \left[(1 - \delta)n_t - n^s_{t+1} - e^s_t\right]$$

$$+ \left(b_t - \frac{b^s_{t+1}}{1 + r^g_{t+1}}\right) + q_t e^s_t.$$
Entrepreneurs (IV)
Optimization of a Saving Entrepreneur

- Budget constraint for a saver (superscript $s$):

\[
c_t^s + \tau_t^s = r_t n_t + q_t \left[ (1 - \delta) n_t - n_{t+1}^s - e_t^s \right] + \left( b_t - \frac{b_{t+1}^s}{1 + r_{t+1}^g} \right) + q_t e_t^s.
\]

- Expression (6) can be simplified given that equity owned and equity issued pay the holder the same return:

\[
c_t^s + \tau_t^s + \frac{b_{t+1}^s}{1 + r_{t+1}^g} + q_t n_{t+1}^s = r_t n_t + q_t (1 - \delta) n_t + b_t
\]
Entrepreneurs (V)
Optimization of a Saving Entrepreneur

- Consumption/savings choice:

\[ c_t^s = (1 - \beta) \left[ r_t n_t + q_t (1 - \delta) n_t + b_t - \tau_t^s \right] \] (7)
Entrepreneurs (V)

Optimization of a Saving Entrepreneur

- Consumption/savings choice:

\[ c_t^s = (1 - \beta) \left[ r_t n_t + q_t (1 - \delta) n_t + b_t - \tau_t^s \right] \]  

- Portfolio:

\[ u'(c_t^s) = E_t \left\{ \frac{r_{t+1} + q_{t+1} (1 - \delta)}{q_t} \left[ \pi u'(c_{t+1}^i) + (1 - \pi) u'(c_{t+1}^s) \right] \right\} \]

and

\[ u'(c_t^s) = E_t \left\{ (1 + r_{t+1}^g) \left[ \pi u'(c_{t+1}^i) + (1 - \pi) u'(c_{t+1}^s) \right] \right\} \]
Workers

- Continuum of workers that maximize

\[
E_0 \sum_{t=0}^{\infty} \beta^t u \left[ c_t^w - \frac{\omega}{1 + \nu} (l_t^w)^{1+v} \right],
\] (10)

subject to:

\[
c_t^w + q_t n_{t+1} + \frac{b_t^{w}}{1 + r_{t+1}} + \tau_t^w
= w_t l_t^w + r_t n_t^w + q_t (1 - \delta) n_t^w + b_t^w.
\] (11)
Workers

- Continuum of workers that maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t u \left[ c_t^w - \frac{\omega}{1+\nu} (l_t^w)^{1+\nu} \right], \quad (10)$$

subject to:

$$c_t^w + q_t n_{t+1}^w + \frac{b_t^w}{1 + r_{t+1}^g} + \tau_t^w = w_t l_t^w + r_t n_t^w + q_t (1 - \delta) n_t^w + b_t^w. \quad (11)$$

- Optimization, and certain assumptions, deliver:

$$l_t^w = \left( \frac{w_t}{\omega} \right)^{\frac{1}{\nu}} \quad (12)$$

$$c_t^w = w_t l_t^w - \tau_t^w. \quad (13)$$
Government

- Target amount of one-period debt $B$, rolled over every period:

$$B = \int b_{t+1}^s d\Phi_t(s), \quad (14)$$

where $\Phi_t(s)$ is the distribution function of saving entrepreneurs.
Government

- Target amount of one-period debt $B$, rolled over every period:
  
  \[ B = \int b^s_{t+1} d\Phi_t(s), \]  
  \[ (14) \]

  where $\Phi_t(s)$ is the distribution function of saving entrepreneurs.

- Balances budget by financing interest cost with lump-sum taxation:
  
  \[ \left(1 - \frac{1}{1 + r^g_{t+1}}\right)B = \frac{r^g_t}{1 + r^g_{t+1}}B = \pi \tau^i_t + (1 - \pi)\tau^s_t + \tau^w_t, \]  
  \[ (15) \]

  where $\tau^i_t$, $\tau^s_t$ and $\tau^w_t$ are lump-sum taxes on investing entrepreneurs, saving entrepreneurs, and workers.
Government

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$$\left(1 - \frac{1}{1 + r_{t+1}^g}\right) B = \frac{r_t^g}{1 + r_{t+1}^g} B = \pi \tau_t^i + (1 - \pi)\tau_t^s + \tau_t^w, \quad (15)$$

where $\tau_t^i, \tau_t^s$ and $\tau_t^w$ are lump-sum taxes on investing entrepreneurs, saving entrepreneurs, and workers.

- Assumption of how the tax-burden is distributed important for some later results.
Competitive Equilibrium (I)

AGGREGATE RESOURCE CONSTRAINT

- Aggregate resource constraint:

\[ Y_t = C_t^i + C_t^s + C_t^w + I_t, \]

which becomes, after rearranging:

\[ \alpha a_t K_t^\alpha + \frac{r_t^g}{1 + r_t^g} B - \left[ \pi \tau_t^i + (1 - \pi) \tau_t^s + \tau_t^w \right] \]

\[ = I_t + (1 - \beta) \left\{ r_t K_t + q_t (1 - \delta) K_t + B_t - \left[ \pi \tau_t^i + (1 - \pi) \tau_t^s \right] \right\} \]

\[ + (w_t - \tau_t^w) \]

(16)
The expression for investment is:

\[ I_t = \frac{\pi \beta [r_t K_t + q_t (1 - \delta) K_t + B_t - \tau^i_t]}{1 - \theta q_t}. \]  

(17)

Competitive Equilibrium (II)

INVESTMENT EQUATION
Competitive Equilibrium (III)

AGGREGATE PORTFOLIO CHOICES

- The expression for the aggregate portfolio choices of the saving entrepreneurs is:

\[
E_t \left\{ \frac{[r_{t+1} + q_{t+1}(1 - \delta)]/q_t}{[r_{t+1} + q_{t+1}(1 - \delta)]N_{t+1}^S + B_{t+1}} \right\} = E_t \left\{ \frac{(1 + r_{t+1}^g)}{[r_{t+1} + q_{t+1}(1 - \delta)]N_{t+1}^S + B_{t+1}} \right\}. \tag{18}
\]
Steady State

• Resource constraint:

\[
\alpha aK^\alpha + \frac{rg}{1 + rg} B - \left[ \pi \tau^i + (1 - \pi) \tau^s + \tau^w \right] = \delta K \\
\]

\[
+(1 - \beta) \left\{ \alpha aK^\alpha + q(1 - \delta)K + B - \left[ \pi \tau^i + (1 - \pi) \tau^s \right] \right\}
\]
Steady State

- **Resource constraint:**

\[
\alpha aK^\alpha + \frac{r^g}{1 + r^g} B - \left[ \pi \tau^i + (1 - \pi) \tau^s + \tau^w \right] = \delta K \]

\[
(1 - \beta) \left\{ \alpha aK^\alpha + q(1 - \delta)K + B - \left[ \pi \tau^i + (1 - \pi) \tau^s \right] \right\}
\]

- **Investment equation**

\[
(1 - \theta q)\delta K = \pi \beta [\alpha aK^\alpha + q(1 - \delta)K + B - \tau^i]
\]
Steady State

- Resource constraint:
  \[
  \alpha aK^\alpha + \frac{r^g}{1 + r^g} B - \left[ \pi \tau^i + (1 - \pi) \tau^s + \tau^w \right] = \delta K \tag{19}
  \]
  \[
  + (1 - \beta) \left\{ \alpha aK^\alpha + q(1 - \delta) K + B - \left[ \pi \tau^i + (1 - \pi) \tau^s \right] \right\}
  \]

- Investment equation
  \[
  (1 - \theta q) \delta K = \pi \beta [\alpha aK^\alpha + q(1 - \delta) K + B - \tau^i] \tag{20}
  \]

- Portfolio equation
  \[
  \frac{[\alpha aK^{\alpha - 1} + q(1 - \delta)]}{q} = (1 + r^g) \tag{21}
  \]
Crowding-in vs Crowding-out

- Isolate two effects \( \implies \) assume all taxes paid for by saver-entrepreneurs:

\[
(1 - \pi)\tau^s = \frac{r^g}{1 + r^g}B \hspace{1cm} (22)
\]

\[
\tau^i = 0 \hspace{1cm} (23)
\]

\[
\tau^w = 0. \hspace{1cm} (24)
\]

**Proposition 1**: When the redistributive effects of government debt variations are ignored, the effects of variations in government debt on aggregate investment in the steady state are such that the crowding-in(out) effect dominates the crowding-out(in) effect for low(high) values of \( \theta \) (borrowing constraints are tight(loose)).
Crowding-in, crowding-out and the redistributive channel (I)

- We add the *inter-sectorial redistributive* effect by assuming that $\tau^s = 0$, $\tau^i = 0$, and:

$$
\tau^w = \frac{rg}{1 + rg} B.
$$
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  - Transfers between workers and entrepreneurs induced by government debt and taxation $z(B)$:

$$z(B) = \frac{r^g}{1 + r^g} B,$$
Crowding-in, crowding-out and the redistributive channel (II)

- Sensitivity of transfers to variations in the amount of $B$ is

$$z'(\cdot) = \frac{1}{1 + r_g} \left[ r_g + \frac{dr_g}{dB} \frac{B}{1 + r_g} \right].$$
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\[ \text{Taking the three channels into account, if financial frictions are...:} \]

- very severe: crowding-in effect will tend to dominate any potential negative distributional effect.
- moderate: positive net effect.
- very mild: ambiguous.
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- Sensitivity of transfers to variations in the amount of $B$ is

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Stochastic Liquidity (I)

- Assume equity has limited resaleability, and only a fraction $\phi_t$ of equity can be sold or re-mortgaged each period

$$(1 - \delta) n_t - [n^i_{t+1} - (i_t - e^i_t)] \leq \phi_t (1 - \delta) n_t.$$
Assume equity has limited resaleability, and only a fraction $\phi_t$ of equity can be sold or re-mortgaged each period

$$(1 - \delta)n_t - [n_{t+1}^i - (i_t - e_t^i)] \leq \phi_t (1 - \delta)n_t.$$ 

The budget constraint for an investing entrepreneur becomes:

$$c_t^i + q_t^R n_{t+1}^i + \tau_t^i = r_t n_t + [\phi_t q_t + (1 - \phi_t)q_t^R](1 - \delta)n_t + b_t,$$

where

$$q_t^R \equiv \frac{1 - \theta q_t}{1 - \theta}.$$
Stochastic Liquidity (II)

- Investment will be equal to:

\[
\begin{align*}
    i_t &= \frac{[r_t n_t + \phi_t q_t (1 - \delta) n_t + b_t - \tau^i_t] - c^i_t}{1 - \theta q_t} \\
    &= \beta \left[ r_t n_t + \phi_t q_t (1 - \delta) n_t + b_t - \tau^i_t \right] \\
    &\quad \times \frac{1}{1 - \theta q_t} \\
    &- \left(1 - \beta\right) (1 - \phi_t) q_t R_t (1 - \delta) n_t \\
    &\quad \times \frac{1}{1 - \theta q_t}
\end{align*}
\]

(25)
Demand for entrepreneurial equity and government debt given by:

\[ u'(c^s_t) = \pi E_t \left\{ u'(c^i_{t+1}) \frac{r_{t+1} + \phi_t q_{t+1} + (1 - \phi_t) q^R_{t+1}}{q_t} (1 - \delta) \right\} \]

and

\[ u'(c^S_t) = E_t \left\{ (1 + r^g_{t+1}) [\pi u'(c^i_{t+1}) + (1 - \pi) u'(c^S_{t+1})] \right\} \]
Competitive Equilibrium with Stochastic Liquidity (I)

AGGREGATE INVESTMENT EQUATION

\[ I_t = \frac{\pi \beta [r_t K_t + \phi_t q_t (1 - \delta) K_t + B_t - \tau_t^i]}{1 - \theta q_t} \]

\[ \pi (1 - \beta) (1 - \phi_t) q_t^R (1 - \delta) K_t \]

\[ 1 - \theta q_t \]
Competitive Equilibrium with Stochastic Liquidity (II)

AGGREGATE PORTFOLIO EQUATION

\[
(1 - \pi) E_t \left\{ \frac{[r_{t+1} + q_{t+1}(1 - \delta)] / q_t - (1 + r_{t+1}^g)}{[r_{t+1} + q_{t+1}(1 - \delta)] N_{t+1}^S + B_{t+1}} \right\} \\
= \pi E_t \left\{ \frac{(1 + r_{t+1}^g)}{[r_{t+1} + [\phi_t q_{t+1} + (1 - \phi_t) q_{t+1}^R](1 - \delta)] N_{t+1}^S + B_{t+1}} \right\} \\
- \pi E_t \left\{ \frac{[r_{t+1} + [\phi_t q_{t+1} + (1 - \phi_t) q_{t+1}^R](1 - \delta)] / q_t}{[r_{t+1} + [\phi_t q_{t+1} + (1 - \phi_t) q_{t+1}^R](1 - \delta)] N_{t+1}^S + B_{t+1}} \right\}
\]

(29)
Crowding-in versus crowding-out

Again, we assume $\tau^i = 0$ and $\tau^w = 0$, and that

$$(1 - \pi)\tau^s = \frac{rg}{1 + rg} B. \quad (30)$$

**Proposition 2**  When the redistributive effects of government debt variations are ignored, the effects of variations in government debt on aggregate investment in the steady state are such that the crowding-in(out) effect dominates the crowding-out(in) effect for (1) low(high) values of $\theta$ (borrowing constraints are tight(loose)), and (2) low(high) values of $\phi$ (equity has a limited(ample) liquidity).
Dynamics - Productivity Shock

**Figure:** Impulse Response of Key Variables to Productivity Shock - Analysis for Different Levels of Government Debt (periods = quarters). Responses are the percentage deviation of a variable from its steady-state value.
Dynamics - Liquidity Shock

Figure: Impulse Response of Key Variables to Liquidity Shock - Analysis for Different Levels of Government Debt (periods = quarters). Responses are the percentage deviation of a variable from its steady-state value.
Conclusion

- Analysis of effects of public supply of liquidity on investment in a model with *financial constraints*, *corporate* demand for liquidity, and *endogenous private supply* of aggregate liquidity
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