"E-stability of Monetary Policy when the Cost Channel Matters"*

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Abstract

We study how expectational stability (E-stability) of rational expectations equilibrium may be affected by monetary policy when the cost channel of monetary policy matters. We focus on both instrumental taylor-type rules and optimal rules. We show analytically that standard instrument rules -contemporaneous and forecast based rules - can easily induce indeterminacy and expectational instability when the cost channel is present. Overall, a naive application of the Taylor principle in this setting could be misleading. Regarding optimal rules, we find that "expectational-based rules", under discretion and commitment, do not always induce determinate and E-stable equilibrium. This result stands in contrast to the findings of Evans and Honkapohja (RES, 2003) for the baseline "New Keynesian" model.

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1 Introduction

There is growing and recent empirical evidence showing that the supply side effect of monetary policy has important implications in both inflation dynamics and the design of optimal monetary policy. For example, Barth and Ramey (2001) find a significant cost channel effect on U.S. data at industry level[1]. Similarly, Ravenna and Walsh (2006, RW) and Shabert et. al. (2006) provides empirical evidence for the cost channel in the U.S. and the Euro Area, respectively[2]. From the normative point of view, RW (2006) show the implications in terms of monetary policy of the cost channel and they find that a trade-off between stabilizing inflation and output arises endogenously as a consequence of the presence of supply side effects of monetary policy.

At the same time, a recent ongoing literature has begun to evaluate stability under adaptive learning (also referred as learnability or expectational stability) of the rational expectations equilibrium (REE) in New Keynesian Models[3]. Most contributions of this literature has focused on the standard New Keynesian Framework (or baseline model) in which the monetary transmission mechanism is demand driven and thus the cost channel is absent. For example, Bullard and Mitra (2002, BM hereafter) examine determinacy and stability under adaptive learning dynamics of the REE under a variety of instrumental rules. Using the same framework, Evans and Honkapohja (2003, 2006, hereafter EH) perform the analysis of stability under adaptive learning when the policy rule of the central bank is optimally derived.

On one hand, BM find that if the monetary authority is able to commit to Taylor type interest rate rule the associated REE is stable under learning as long as the Taylor principle is satisfied[4]. On the other hand, EH (2003, 2006) show that under both discretion and commitment the REE is always unstable under learning when the optimal policy rule is derived based on the assumption that agents have rational expectations (they refer to this rule as "fundamental-based rule"). EH proposed an alternative implementation of the optimal rule by relaxing the assumption of rational expectations on private agents, referred as "expectation-based optimal rule", and find that these type of rules can always induce determinate and expectational stable REE under both discretion and commitment, see EH (2003, 2006).

In this paper, we study the implications of the cost channel to the learnability conditions of a New Keynesian baseline model. Our learnability criterion is the one proposed by EH (2001), i.e. the stability under adaptive learning is governed by E-stability conditions[5]. We also check whether E-stability is related to determinacy, i.e. to the possibility of a unique REE

[1]Barth and Ramey (2001), using data for trade credit from the US Flow of Funds, report that over the period 1995 to 2000 net working capital (inventories plus trade receivables, net of trade payables) averaged 11 months of sales, an amount comparable to the investment in fixed capital.

[2]Christiano, Eichenbaum and Evans (2005) estimate a DSGE model of the U.S. economy and find that monetary policy operates also through the supply side. Instead, Rabanal (2006) using Bayesian methods and U.S data finds no role for the cost channel of monetary policy.

[3]As Bullard (2006) pointed out, since learnability is a "minimal deviation from rational expectations", the learnability criterium should be viewed as an additional minimal stability condition that a REE equilibrium should meet.

[4]See Llosa and Tuesta (2006) for the analysis of E-stability in a small open economy. The authors find that the naive taylor principle is not a sufficient condition for E-stability.

[5]Therefore, we assume a particular way of adaptive learning called least-squares learning. See EH (2001) for further details.

solution path. We analyze both issues under two types of rules: a) instrument rules and b) optimal target rules. Regarding the first type of rules and following BM (2002), we evaluate standard Taylor-rules under two specifications: (i) the monetary authority reacts to current values and it is called contemporaneous data specification and (ii) it reacts to expectations and it is called forward looking rules. In the case of optimal rules and in the same fashion of EH (2003, 2006), we analyze "fundamental-based" and "expectations-based" rules under both discretion and commitment.

In general, our analytical findings show that the cost channel influences both determinacy and learnability conditions, making them more stringent with respect to the baseline new Keynesian model. Under instrumental rules, the so-called Taylor principle or active policy may fail in generating a determinate and learnable REE. On one hand, under contemporaneous Taylor rules determinacy requires an active but moderate reaction to inflation and hence the Taylor principle is not a sufficient condition for determinacy. E-stability, however, is always assured when the Taylor principle holds. On the other hand, under forecast-based rules the Taylor principle may promote both indeterminacy and expectational instability. In particular, provided a null response to the output gap, if the nominal interest rate is adjusted positively and more than one-for-one to inflation expectations, a determinate and E-stable REE is not attainable. This remarkable result rises doubts about the validity of the Taylor principle as a useful guideline for forward-looking instrument rules when the cost channel is present. This result stands in contrast to the celebrated idea that the Taylor principle or "active" policy guarantees both determinacy and E-stability. The possibility that the cost channel creates an unstable equilibrium is striking and simple in this framework.

Under optimal policy rules, our results can be summarized as follows. First, under discretion, we find that a "fundamental-based" optimal policy rule, that is, an interest rate rule that reacts only to fundamental shocks, implies that the equilibrium is indeterminate and unstable in the learning dynamics, results that coincide with those of EH (2003). We also derive an "expectations-based" rule that might, allegedly according to EH (2003), perform well in both grounds. However, we show that the implied "expectations-based" rule does not always lead to stability under learning because of the cost channel. Second, under optimal commitment, we find that the "fundamental-based" policy can be E-stable for a given parameter arrangement, contrasting with EH (2006) who show that this class of optimal rules is never stable under learning. Hence, to the extent that the cost channel is present, the policymaker´s ability to commit to an optimal policy when the policy reaction function depends on fundamentals and lagged endogenous values, could be sufficient to stabilize the economy[6]. In addition, in contrast with EH (2006) the implementable "expectations-based" reaction function does not always induce a determinate an E-stable equilibrium.

In summary, BM and EH´s proposed resolutions to the monetary policy instability problem (i.e. the Taylor principle or conditioning policy on private sector expectations) might *always* perform well only in the baseline new Keynesian model. However, a very simple modification of the baseline model, namely the cost channel, can lead to very different conclusions. Indeed,

[6]This finding concurs with those of Duffy and Xiao (2005). They find that if one includes the interest rate deviations in the central bank's loss function, E-stability can be achieved without requiring the central bank to react to private sector expectations.

we argue that the presence of cost channel imposes some difficulties to the central bank in achieving a determinate and learnable equilibrium even if the Taylor principle holds or the monetary authority reacts to private sector expectations.

This paper shed lights to the effects of the cost channel to the stability of the economy. The issue was first analyzed by Brückner and Schabert (2003) who studied determinacy and pointed out that the cost channel introduces an additional upper bound to the inflation reaction in a contemporaneous Taylor rule. Using RW's model, Surico (2006) performs determinacy analysis of Taylor rules with smoothness in the interest rate. The author focuses on two specifications of Taylor rules, namely contemporaneous and lagged data, and finds that if a central bank assigns positive weight to output fluctuations the economy is more prone to multiple equilibria relative to the standard model. Thus, our paper contribution is twofold. First, we analyze a broader set of policy rules including not only other specifications of Taylor-type rules (e.g. forecast-based) but also optimal ones. Second, based on these rules, we also check whether there is a link between determinacy and learnability conditions. Our paper also contributes to another strand of the literature that has been studying determinacy and E-stability in models in which monetary policy has supply-side effects. For example, Benhabib et. al. (2003) show that the conditions for determinacy depends on the way in which money is assumed to enter in the model. Particularly, in a sticky price environment, they find that some forms of active monetary policy bring about indeterminacy whereas passive monetary policy gives rise to equilibrium uniqueness. Recently, Kurozomi (2004) addresses determinacy and E-stability under alternative timing assumptions in a money-in-utility-function model with sluggish adjustments in prices. His analysis shows that the determinacy and stability conditions under recursive learning are altered depending on the degree of non-separability of the utility function between consumption and real balances. Specifically, he proves that even a small degree of non-separability causes the Taylor rule to be much more likely to induce indeterminacy or E-instability. Our paper finds additional support to the detrimental effect of the supply-side monetary mechanism on the determinacy and E-stability of the REE.

The rest of the paper is organized as follows. Section 2 outlines RW's model and discusses its main differences with respect to the baseline model. Section 3 describes the analysis of determinacy and learning under instrumental and target rules. Section 4 concludes.

2 The Simple Environment

In this section we summarize the log-linearized version of the model presented in RW . The model can be summarized by the following equations (equations 27 and 28 in RW´s paper):

$$\pi_t = \kappa_x x_t + \beta \widehat{E}_t \pi_{t+1} + \delta \kappa i_t + \mu_t \quad (1)$$

$$x_t = \widehat{E}_t x_{t+1} - \frac{1}{\sigma}\left(i_t - \widehat{E}_t \pi_{t+1}\right) \quad (2)$$

$\kappa \equiv [(1-\theta\beta)(1-\theta)/\theta]$ and $\kappa_x \equiv \kappa(\eta+\sigma)$.

where x_t is the output gap, π_t inflation and i_t is the percentage point deviation of the nominal interest rate around its steady state value. The notation of $\widehat{E}_t$ is meant to indicate a possibly non-rational expectation using information available at date t, so that E_t, without the hat, is the normal expectations operator[7]. Equation (1) is a short run aggregate supply (AS) curve that relates inflation with the output gap and the nominal interest rate. The parameter β denotes the agent´s discount factor and κ_x captures the sensitivity of inflation to movements in the output gap that depends on deep parameters such as the degree of price stickiness captured by θ and the inverse of the elasticity of the labor supply η. Equation (2) is an IS curve that relates the output gap inversely to the domestic interest rate and positively with the expected future output gap. In this equation $\frac{1}{\sigma}$ represents the intertemporal elasticity of substitution.

Note that the previous two-equation system differs from the standard new Keynesian model (see Woodford 2003) due to the presence of the nominal interest i_t in the staggered price equation, i.e. the cost channel of monetary policy. The existence of the cost channel is justified if firms must borrow working capital from intermediaries (for further details see RW). Just for comparison we define δ which is a discrete variable that takes the value of 1 when there is cost channel and 0 where there is not (baseline model). Note that μ_t represents the traditional cost-push shock. We will assume that μ_t evolves according to an exogenous first order autoregressive process

$$\mu_t = \rho\mu_t + \varepsilon_t \tag{3}$$

where ε_t is an i.i.d noise with variances σ_ε^2 . Finally, $0 \leq \rho < 1$ is the correlation parameter.

3 Determinacy and E-stability

We supplement equations (1) through (2) with a policy rule for the interest rate i_t that represents the behavior of the monetary authority. Here we study interest rate policy rules which have been extensively studied in the literature: instrumental and target rules. The main question is whether such rules lead to stability under adaptive learning. We also check the link between learnability and determinacy. The bottom line is that either the Taylor principle or EH´s proposal to the resolution of the instability under learning cannot be taken for granted when the cost channel matters.

3.1 Instrumental Rules

In this subsection, we consider Taylor type of rules being evaluated with both *contemporaneous data* and *forward expectations* as in BM (2002)[8].

[7]We implicity rely our analysis of learning and monetary policy based on "Euler Equation" approach as it is suggested in Honkapohja et al. (2003). Recently, Preston (2002) has proposed an interesting reformulation of the intertemporal behavior under learning in which agents are assumed to incorporate a "subjective version" of their intertemporal budget constraint in their behavior under learning.

[8]BM (2002) also consider the case of rules with lagged data and contemporaneous expectations. Results based on these kind of rules for our model are available upon request.

3.1.1 Contemporaneous data in the Taylor Rule

We first assume a simple Taylor type rule (see Taylor 1993) in which the central bank reacts to price inflation and the output gap

$$i_t = \phi_\pi \pi_t + \phi_x x_t \tag{4}$$

where ϕ_π and ϕ_x are non-negative which measure the degree of responsiveness of the policy interest rate to deviations of the inflation and output gap, respectively.

Substituting the policy rule (4) into (1) and (2), we can write the model involving the two endogenous variables x_t and π_t

$$y_t = \Gamma + \Omega \widehat{E}_t y_{t+1} + k\mu_t \tag{5}$$

where $y_t = [\pi_t, x_t]', \Gamma = 0$, and

$$\Omega = \psi \begin{bmatrix} \sigma\beta + \kappa_x + \beta\phi_x + \delta\kappa\phi_x & \sigma\left(\kappa_x + \delta\kappa\phi_x\right) \\ 1 - \beta\phi_\pi - \kappa\phi_\pi & \sigma\left(1 - \delta\kappa\phi_\pi\right) \end{bmatrix} \tag{6}$$

with $\psi = \left(\sigma + \phi_x + \kappa_x\phi_\pi - \delta\kappa\sigma\phi_\pi\right)^{-1}$

We omit k since it is not important for the analysis of both determinacy and E-stability.[9] To have a determinate equilibrium, we need both of the eigenvalues of Ω to be inside the unit circle (see the methodological appendix). The necessary and sufficient conditions for determinacy are summarized in Proposition 1.

Proposition 1. *Under contemporaneous data interest rate rules the necessary and sufficient conditions for a rational expectations equilibrium to be unique are that*

$$\left(1 - \beta - \delta\kappa\right)\phi_x + \kappa_x\left(\phi_\pi - 1\right) > 0 \tag{7}$$

$$2\sigma\left(1+\beta\right) + \left(1 + \beta + \delta\kappa\right)\phi_x + \left(\kappa_x - 2\delta\kappa\sigma\right)\phi_\pi + \kappa_x > 0 \tag{8}$$

Proof. See appendix A. □

Following McCallum (1983), the Minimal State Variable (MSV) solution in this case takes the form $y_t = \overline{a} + \overline{c}\mu_t$ with $\overline{a} = 0$ and $\overline{c} = (1 - \rho\Omega)^{-1}k$. To find E-stability conditions we assume that agents utilize the *Perceived Law of Motion* (PLM) which corresponds to the MSV solution[10]

$$y_t = a + c\mu_t \tag{9}$$

Then, we obtain the *Actual Law of Motion* (ALM)

$$y_t = \Omega a + \left(\Omega c\rho + k\right)\mu_t \tag{10}$$

[9] For the sake of simplicity here forth we purposely omit matrices that are not relevant for either determinacy or E-stability analyses.

[10] Note that we include an intercept vector although the MSV solution does not have it. However, in practice agents will need to estimate intercept as well as slope parameters.

This allows us to define the T-mapping from the PLM to the ALM as

$$T\left(a,c\right)=\left(\Omega a,\Omega c\rho+k\right) \tag{11}$$

E-stability conditions are discussed in the methodological appendix. The next proposition shows the necessary and sufficient conditions that guarantee an E-stable equilibrium.

Proposition 2. *Suppose the time t information set is $(1,\mu_t)'$. Under contemporaneous data interest rate rules, the necessary and sufficient condition for an MSV solution $(0,\overline{c})$ to be E-stable is that*

$$\left(1-\beta-\delta\kappa\right)\phi_x+\kappa_x\left(\phi_\pi-1\right)>0 \tag{12}$$

Proof. See appendix B. □

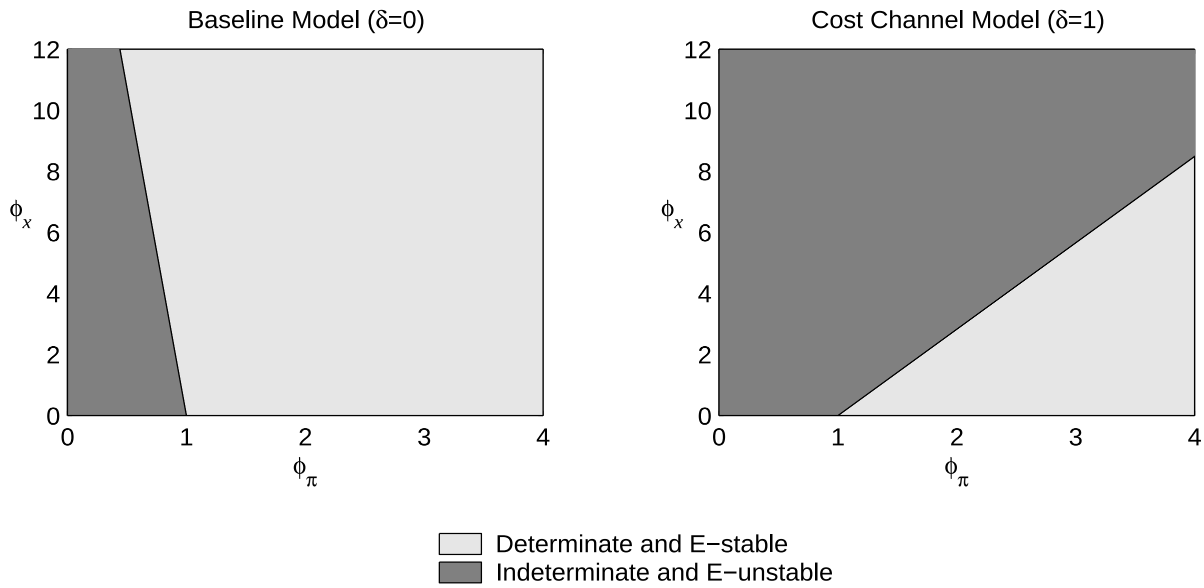

Figure 1: Regions of determinacy and expectational estability for contemporaneous data policy rules. Left panel corresponds to the baseline model ($\delta=0$). Right panel corresponds to the cost channel model ($\delta=1$).

Proposition 1 and 2 show how the cost channel modifies the conditions for both determinacy and learnability relative to the baseline case. In the baseline model ($\delta=0$),while inequality (8) always holds, conditions (7) and (12) impose the restrictions for a determinate and E-stable REE, respectively. In fact, the latter conditions coincide with the Taylor principle and imply that: facing inflationary pressures, the central bank increases its interest rate by more than the rise in inflation, which raises real interest rates until inflation returns to the target. When there is no response to the output gap, $\phi_\pi>1$ or active reaction is sufficient for both determinacy and E-stability. But even for $\phi_\pi<1$ or a passive policy, the monetary authority can compensate for a relatively low value of ϕ_π by choosing a sufficiently large value of ϕ_x in such a way as to still satisfy (7) and (12).

In the cost channel model ($\delta = 1$) there are some novel differences with respect to the standard case. First, the Taylor principle may not be a sufficient and necessary condition for determinacy. Note that condition (8) binds if the inverse of the intertemporal elasticity of the substitution is greater than the inverse of the elasticity of the labor supply, $\eta < \sigma$. This result arises from the term $\kappa_x - 2\delta\kappa\sigma$ in (8) which equals to $\eta - \sigma$ after replacing for κ_x and δ. For example, if we assume $\phi_x = 0$, ϕ_π is two-sided constrained and therefore the Taylor principle is not a sufficient condition for determinacy.

$$1 < \phi_\pi < \frac{2\sigma(1+\beta) + \kappa(\eta+\sigma)}{\kappa(\sigma-\eta)} \tag{13}$$

Another key difference is the presence of the parameter κ in the term accompanying ϕ_x in (7) and (12). Note that as κ enters in the latter conditions, it restricts relatively more the sensitiveness of the interest rate to the output gap. In fact, provided that ϕ_π is one, if $1 - \beta - \delta\kappa < 0$, both conditions (7) and (12) fail for any positive value of ϕ_x.

To gain more insights about the effect of cost channel we illustrate the results by using a calibrated case. We set a quarterly discount factor, β, equal to 0.99, which implies an annualized rate of interest of 4%. The coefficient of risk aversion parameter, σ, the inverse of the elasticity of labor supply, η, are calibrated according to RW (2004) and are set equal to 1.5 and 1, respectively. As it is common in the literature on the Calvo (1983) pricing technology, we let the probability of not adjusting prices, $\theta = 0.75$. The serial correlation for the exogenous cost push shock is assumed 0.3. In addition, as in BM (2002), we calibrate the policy reactions parameters for the values $0 \leq \phi_\pi \leq 12$ and $0 \leq \phi_x \leq 4$. We apply this calibration to the rest of numerical analyses.

Figure (1) depicts determinacy and E-stable regions as functions of ϕ_π and ϕ_x, with the rest of the parameters set at their baseline values. The figure of the left side depicts the baseline case ($\delta = 0$) whereas the figure of the right shows the cost channel case ($\delta = 1$). Compared to the baseline case, under the cost channel a central bank must be more cautious in reacting to output and inflation. For example, provided a null response to the output gap, in the cost channel case ϕ_π must lie between 1 and 144 (not shown in the graph) to guarantee a determinate equilibrium while in the standard case a ϕ_π bigger than 1 is sufficient for determinacy. Further, under the cost channel model a passive reaction to inflation, i.e. $\phi_\pi < 1$, (even if it is accompanied by an output gap reaction) never generates determinacy and E-stability. Overall, a naive application of the Taylor principle in cost channel setting could be misleading.

3.1.2 Forward data in the Taylor rule

Following BM we denote these type of rule as "inflation-forecast-based rules". These rules, in general, adopt the following form

$$i_t = \phi_\pi \widehat{E}_t \pi_{t+1} + \phi_x \widehat{E}_t x_{t+1} \tag{14}$$

where ϕ_π and ϕ_x are non-negative.

We reduce the system of equations (1), (2) and (14) to two equations involving the endogenous variables x_t and π_t

$$y_t = \Gamma + \Omega \widehat{E}_t y_{t+1} + \Theta \mu_t \tag{15}$$

where $y_t = [\pi_t, x_t]', \Gamma = 0$, and Ω is as defined by

$$\Omega = \frac{1}{\sigma} \begin{bmatrix} \delta\kappa\sigma\phi_\pi - \kappa_x (\phi_\pi - 1) + \sigma\beta & \kappa_x\sigma - \kappa_x\phi_x + \delta\kappa\sigma\phi_x \\ -(\phi_\pi - 1) & \sigma - \phi_x \end{bmatrix} \tag{16}$$

The following proposition summarizes the necessary and sufficient conditions for a rational expectations equilibrium to be unique.

Proposition 3. *Under interest rate rules with forward expectations the necessary and sufficient conditions for determinacy are that*

$$(\beta + \delta\kappa)\phi_x - \delta\kappa\sigma\phi_\pi < \sigma(1+\beta) \tag{17}$$

$$\delta\kappa\sigma\phi_\pi - (\beta + \delta\kappa)\phi_x < \sigma(1-\beta) \tag{18}$$

$$(1+\beta+\delta\kappa)\phi_x + (\kappa_x - 2\delta\kappa\sigma)\phi_\pi < 2\sigma(1+\beta) + \kappa_x \tag{19}$$

$$(1-\beta-\delta\kappa)\phi_x + \kappa_x(\phi_\pi - 1) > 0 \tag{20}$$

Proof. See appendix C. □

For t dating of expectations, the MSV solution takes the form of $y_t = \overline{a} + \overline{c}\mu_t$ with $\overline{a} = 0$, and $\overline{c} = (I - \rho\Omega)^{-1}k$. The analysis of E-stability is analogous to section 3.1. The following proposition provides the condition for E-stability of the MSV solution.

Proposition 4. *Suppose the time t information set is $(1, \mu_t)'$. Under interest rate rules with forward expectations, the necessary and sufficient conditions for an MSV solution $(0, \overline{c})$ to be E-stable are that*

$$\delta\kappa\sigma\phi_\pi - (\beta + \delta\kappa)\phi_x < \sigma(1-\beta) \tag{21}$$

$$(1-\beta-\delta\kappa)\phi_x + \kappa_x(\phi_\pi - 1) > 0 \tag{22}$$

Proof. See appendix D. □

Proposition 3 and 4 show again that the cost channel alters the conditions for both determinacy and learnability relative to the baseline model. For example, to the extent that $\phi_x = 0$, conditions (18) and (21) imply the following inequality

$$\phi_\pi < \frac{(1-\beta)}{\delta\kappa} \tag{23}$$

whereas conditions (19) and (22) imply the traditional restriction: the Taylor principle

$$\phi_\pi > 1 \tag{24}$$

Notice that under the baseline case ($\delta = 0$), the first inequality (23) goes to infinite and hence ϕ_π greater than 1 is a necessary and sufficient condition for both determinacy and E-stability.

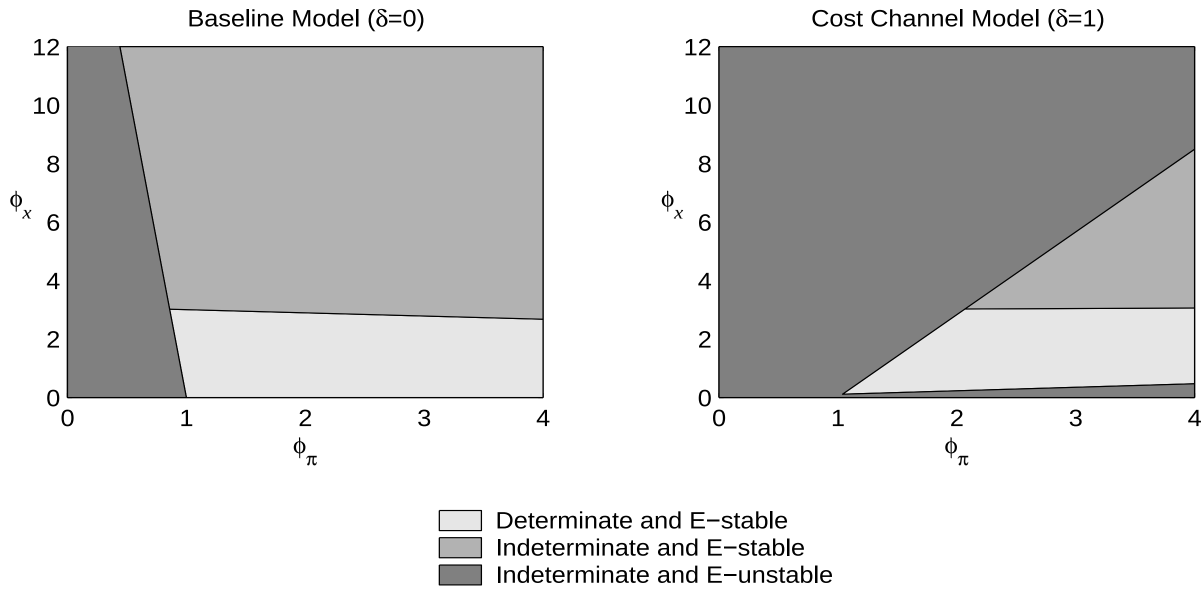

Figure 2: Regions of determinacy and expectational estability for forward expectations policy rules. Left panel corresponds to the baseline model ($\delta = 0$). Right panel corresponds to the cost channel model ($\delta = 1$).

In contrast, under the cost channel ($\delta = 1$), the limit given by (23) can be higher or lower than 1, implying that the Taylor principle could no longer lead to stability. Moreover, as discussed earlier, given that under the cost channel the parameter κ appears in (19) and (22), the range of ϕ_x that induce determinacy and learnability shrinks.

To illustrate these findings, figure (2) plots the intersections of the regions of determinacy and learnability. The figure of the left side depicts the baseline case ($\delta = 0$) whereas the figure of the right shows the cost channel case ($\delta = 1$). The figure of the left indicate that a forecast-based Taylor rule described by $\phi_\pi > 1$ and a relatively small response to output gap guarantees a determinate and learnable equilibrium in the standard model ($\delta = 0$). Moreover, a passive reaction to inflation may also promote stability if it is accompanied by a sufficient reaction to the output gap. The figure of the right show the results under the cost channel ($\delta = 1$). Contrary to the standard case, the Taylor Principle (i.e. $\phi_\pi > 1$) *may not guarantee a determinate and E-stable equilibrium* even if ϕ_x is positive[11]. This is a remarkable result since the idea that the Taylor principle or "active" policy is a matter of changing nominal interest rates more than one-for-one with inflation is a celebrated result in the literature.

[11] This result holds for the alternative calibrations as suggested by Woodford (1999), McCallum and Nelson (1999) and Clarida et. al. (2000).

3.2 Target Rules

In this section we study those rules that can be implemented optimally under both discretion and commitment as in EH (2003, 2006). For each case, we studied two forms of implementing n optimal rule through a linear reaction function for the interest rate. The first form is called "fundamental based" rule and is derived under the assumption of rational expectations. The second form is called "expectations-based" rule and is derived under the assumption that agents do not posses (initially) rational expectations.

3.2.1 Discretionary Policy

We now take the standard formulation similar to the one derived in RW from first principles

$$L_0 = -(1/2)E_0 \sum_{i=0}^{i=\infty} \beta^i \left[\lambda x_{t+i}^2 + \pi_{t+i}^2\right] \tag{25}$$

where λ is the relative weight of output deviations. Optimal monetary policy under discretion implies to minimize (25) subject to versions of (1) and (2) equations modified to take account of the central bank's lack of commitment. It is straightforward to obtain the optimal condition that shows the trade-off between stabilizing domestic inflation and output gap, which reads:

$$\pi_t = -\frac{\lambda}{\kappa_o} x_t \tag{26}$$

where $\kappa_o \equiv \kappa_x - \delta\kappa\sigma \equiv \kappa\left[\eta - (\delta - 1)\sigma\right]$. Note that when $\delta = 0$, $\kappa_o = \kappa_x = \kappa(\eta + \sigma)$ and we get back to the standard trade-off found in Clarida et. al. (2000, hereafter CGG), whereas when $\delta = 1$ we have that $\kappa_o = \kappa\eta$. Note that the cost channel entails larger volatility of inflation since $\kappa(\eta + \sigma) > \kappa\eta$ (see RW 2004 for further details).

EH (2003) discussed several forms of implementing the optimal plan given by (26). A first form is called "fundamental based" rule and implies that the central bank assumes that private agents have perfectly RE and that the REE takes the form of the Minimum State Variable (MSV) solution. Under such assumptions, the "fundamental based" rule for the interest rate reacts only to fundamental shocks[12].

$$i_t = \phi_\mu \mu_t \tag{27}$$

We reduce the system of equations (1), (2) and (27) to the following matrix system involving the endogenous variables x_t and π_t

$$y_t = \Gamma + \Omega \widehat{E}_t y_{t+1} + \Theta \mu_t \tag{28}$$

where $y_t = [\pi_t, x_t]'$, $\Gamma = 0$, and Ω is as defined by

$$\Omega = \begin{bmatrix} \beta + \frac{1}{\sigma}\kappa_x & \kappa_x \\ \frac{1}{\sigma} & 1 \end{bmatrix} \tag{29}$$

[12]EH (2003) consider a more general case in which fiscal shocks appear in the fundamental-based reaction function. However, it is straightforward to show that the result of this section apply to the general case.

Note that the matrix (29) is independent from δ and therefore it is exactly the same matrix analyzed by EH (2003). Thus, irrespective of whether the cost channel is present or not, the "fundamental-based" optimal interest rate rule *always* leads to indeterminacy and instability under learning. In fact, as EH (2003) prove that any linear policy rule of the form of (27) induces both indeterminacy and E-instability.

EH (2003) propose a second form, referred as "expectations-based" rule, which is derived under the assumption that agents do not posses RE and that their expectations can be observed by the central bank. The "expectations-based" optimal rule under discretion is obtained by solving i_t from the structural equations (1) and (2) and the optimal condition (26)

$$i_t = \phi_\pi \widehat{E}_t \pi_{t+1} + \phi_x \widehat{E}_t x_{t+1} + \phi_\mu \mu_t \tag{30}$$

where the coefficients are

$$\begin{aligned} \phi_\pi &= \frac{(\lambda + \sigma\kappa_o\beta + \kappa_o\kappa_x)}{(\lambda + \kappa_o^2)} \\ \phi_x &= \frac{\sigma(\lambda + \kappa_o\kappa_x)}{(\lambda + \kappa_o^2)} \\ \phi_\mu &= \frac{(\sigma\kappa_o)}{(\lambda + \kappa_o^2)} \end{aligned}$$

Notice that when $\delta = 0$ the "expectations-based" rule collapses to the one proposed by EH(2003).

$$\begin{aligned} \phi_\pi^s &= \frac{\left(\lambda + \sigma\kappa_x\beta + \kappa_x^2\right)}{(\lambda + \kappa_x^2)} \\ \phi_x^s &= \sigma \\ \phi_\mu^s &= \frac{\sigma\kappa_x}{(\lambda + \kappa_x^2)} \end{aligned}$$

where s refers to the "standard" case.

We highlight that, regardless of whether the cost channel is active or not, the Taylor principle holds under the "expectations-based" rule. That is, not only $\phi_\pi^s > 1$ as EH (2003) point out, but also $\phi_\pi > 1$ (the latter derives from $\kappa_o < \kappa_x$). Besides, numerical results show the optimal reaction under the cost channel model (ϕ_π) is lower than the one under the standard case (ϕ_π^s)[13]. In adittion, given that $\kappa_o < \kappa_x$, the optimal reaction to output gap expectations under the cost channel is bigger than the one under the standard case.

The reduce form of the model under (30) is,

$$y_t = \Gamma + \Omega \widehat{E}_t y_{t+1} + \Theta \mu_t \tag{31}$$

where $y_t = [\pi_t, x_t]', \Gamma = 0$, and Ω is as defined by

$$\Omega = \psi \begin{bmatrix} (\beta + \kappa\delta)\lambda & \delta\sigma\lambda\kappa \\ -\kappa_o(\beta + \delta\kappa) & -\delta\sigma\kappa\kappa_o \end{bmatrix} \tag{32}$$

[13]The same result hold for different calibrations and different values of λ. The results are available upon request.

with $\psi = (\lambda + \kappa_o \kappa_x - \delta\kappa\sigma\kappa_o)^{-1}$.

The system is determinate if and only if matrix Ω has both eigenvalues inside de unit circle and E-stable if all eigenvalues of $\Omega - I$ have negative real parts. One of the roots of Ω is zero and the other one is given by

$$r = \frac{\lambda(\beta + \kappa\delta) - \kappa\sigma\delta\kappa_o}{\lambda + \kappa_o\kappa_x - \kappa\sigma\delta\kappa_o} \tag{33}$$

Under the standard case, i.e. $\delta = 0$, r is positive and lower that 1, thus determinacy and E-stability immediately follows (see EH 2003). Under the cost channel, however, we found that only a subset of the parameter space is related to either determinacy or E-stability (see propositions 5 and 6). Therefore, the "expectations-based" optimal rule does not always lead to determinacy and learnability for all parameter values as in the standard case.

Proposition 5. *Under the expectations-based optimal rule derived under discretion the necessary and sufficient condition for a rational expectations equilibrium to be unique is that*

$$\frac{\kappa^2\eta(\sigma - \eta)}{(\beta + \kappa + 1)} < \lambda < \frac{\kappa^2\eta(\eta + \sigma)}{(\beta + \kappa - 1)} \tag{34}$$

Proof. See appendix E. □

Proposition 6. *Under the expectations-based optimal rule derived under discretion the necessary and sufficient condition for a MSV solution $(0, \bar{c})$ to be E-stable is that*

$$\lambda < \frac{\kappa^2\eta(\eta + \sigma)}{(\beta + \kappa - 1)} \tag{35}$$

Proof. See appendix F. □

Using our baseline calibration (see section 3.1), Proposition 5 implies that the "expectations-based" optimal rule leads to a unique REE as long as λ ranges between 0.05 and 0.24. Following Proposition 6, E-stability follows if λ is lower than 0.24. Alternative calibrations (see Table 1) bring about either wider or narrower ranges. Under Woodford (1999, W) parametrization determinacy and E-stability require $\lambda < 0.05$ (the lower bound for determinacy is negative). For CGG (2000) calibration determinacy and learnability need $\lambda < 0.32$. For McCallum and Nelson (1999, MN) calibration uniqueness of the REE is guaranteed if $0.28 < \lambda < 0.39$ and E-stability is guaranteed if $\lambda < 0.39$. Note that under Woodford´s parametrization the slope coefficient of the Phillips curve is ten times smaller than the ones of CGG and MN (0.024 compared to 0.3), hence the effect of the output gap over inflation through the Phillips curve will be significantly smaller and the cost channel effect will become relative more important. This explains why the permissible parameter range for λ to guarantee both determinacy and E-stability shrinks.

Table 1

	Baseline	W(1999)	CGG(2000)	MN(1999)
σ	1.5	0.157	1	1/0.164
κ_x	0.21	0.024	0.3	0.3

EH (2003) provide a intuition about why the “expectations-based” reaction function always leads to stability under both determinacy and learning. They argue that under such policy rule the Taylor principle always holds, i.e. $\phi_\pi > 1$, and thus, the central bank succeeds in stabilizing the economy towards the optimal REE. In contrast, in a model in which monetary policy works also through the cost channel this is no longer true. As we showed earlier, the Taylor principle does not guarantee determinacy and learnability of the forward data Taylor rule (14), which has the same form of the “expectations-based” rule (30). Furthermore, under the forward data Taylor rule, the central bank sensitiveness towards the output gap is constrained even if the reaction to expected inflation deviations is greater than 1. As a consequence, when the cost channel matters and under discretionary policy, there are some parameter values under which the economy displays indeterminacy and/or expectational instability.

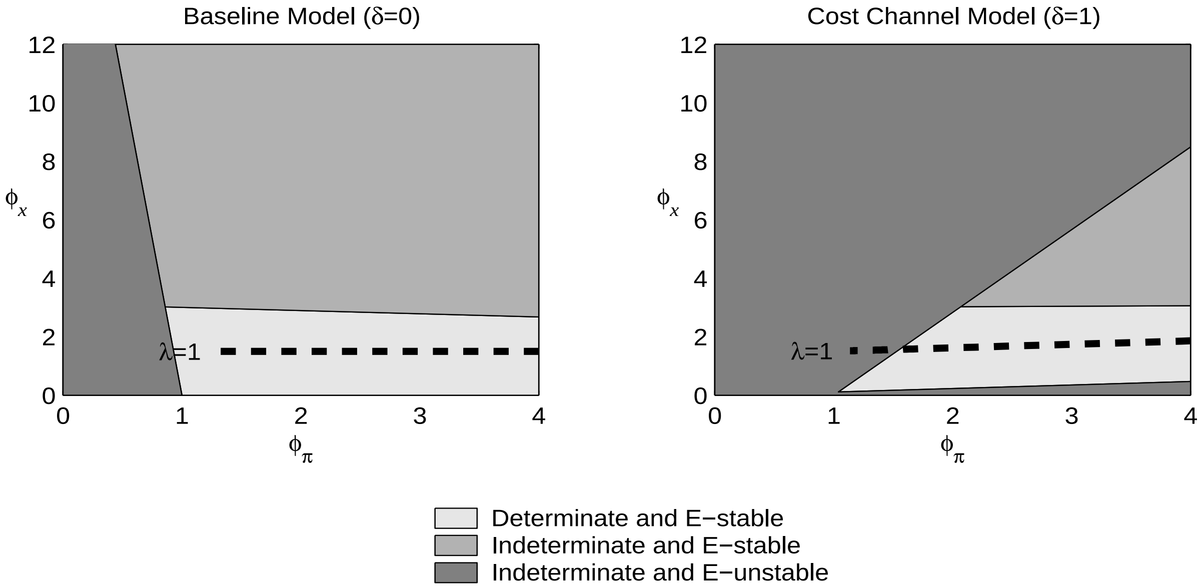

Figure 3: Regions of determinacy and expectational estability for forward expectations policy rules. Left panel corresponds to the baseline model ($\delta = 0$). Right panel corresponds to the cost channel model ($\delta = 1$). The dotted lines plot the optimal parameters ϕ_π and ϕ_x under the “expectations-based” rule derived under discretion.

A key question is which conditions of Proposition 3 and 4 do not hold under the “expectations-based” rule and hence generate the indeterminacy and E-instability result. Figure (3) plots the determinacy and E-stable regions of the Taylor rule with forward data (14) as in figure (2). The dotted lines correspond to the rays $\{\phi_\pi^s, \phi_x^s\}$ and $\{\phi_\pi, \phi_x\}$ given by the “expectations-based” rule (30). The rays plot the combination of optimal parameters under different values of λ (ranging from 0 to 1). Under the baseline model, the ray always stays in the determinate and E-stable area. Under the cost channel model, the numerical results show that as λ both ϕ_π and ϕ_x decrease and the ray crosses the condition (22) for E-stability, or equivalently (20) for determinacy. We check this result by plugging the optimal parameters ϕ_π and ϕ_x into (22). The resulting expression collapses to (36) confirming that E-stability requires Proposition 5.

$$\frac{\sigma\left(\kappa^2\eta\left(\eta+\sigma\right)-\left(\beta+\kappa-1\right)\lambda\right)}{\left(\lambda+\kappa^2\eta^2\right)}>0 \tag{36}$$

Another interesting implication of the previous analysis is that there exists a conflict between the desirable properties of an optimal discretionary rule in terms of the volatility that it entails and the learnability and determinacy criteria. Indeed, note that optimal condition (26) shows that the cost channel increases the trade-off between stabilizing inflation and the output gap ($\kappa_0<\kappa_x$) and simultaneously it implies smaller optimal reaction to expected inflation with respect to the baseline model ($\phi_\pi<\phi_\pi^s$). Yet, as shown in figure (3), the latter implication might induce undesirable properties in terms of both learnability and determinacy. Hence, the achievement of determinacy and learnability under the cost channel would need a bigger reaction to inflation expectations - as the baseline model suggests (ϕ_π^s) - at the cost of larger macroeconomic volatility.

To sum up, our results suggest that EH´s proposal to solve the instability of "fundamental-based" rules by conditioning optimally on private sector expectations, can be misleading when the cost channel matters. As it was shown, the possible solution to this issue, as suggested by EH (2003), does not always provide stability under learning. As the next section will prove, this conclusion emerges also when the policymaker commits to an optimal policy rule.

3.2.2 Commitment Policy

The policy problem is the following: Let's write the Lagrangian

$$L_0=E_0\sum_{t=0}^{\infty}\beta^t\left\{\begin{array}{c}\frac{1}{2}\left[\lambda x_t^2+\pi_t^2\right]+\varphi_{1,t}\left[x_t-x_{t+1}+\frac{1}{\sigma}(i_t-\pi_{t+1})\right]\\ +\varphi_{2,t}\left[\pi_t-\beta\pi_{t+1}-\kappa_x x_t-\delta\kappa i_t\right]\end{array}\right\}-\varphi_{1,0}\pi_0 \tag{37}$$

the first order conditions with respect to π_t, x_t and i_t are respectively:

$$\pi_t-\frac{1}{\sigma\beta}\varphi_{1,t-1}+\varphi_{2,t}-\varphi_{2,t-1} = 0 \tag{38}$$

$$\lambda x_t+\varphi_{1,t}-\frac{1}{\beta}\varphi_{1,t-1}-\kappa_x\varphi_{2,t} = 0 \tag{39}$$

$$\frac{1}{\sigma}\varphi_{1,t}-\delta\kappa\varphi_{2,t} = 0 \tag{40}$$

and $\pi_0=\overline{\pi}_0$.

Combining (38), (39), (40) we get the following set of equations[14],

$$\varphi_{2,t} = (1+\beta^{-1}\delta\kappa)\varphi_{2,t-1}-\pi_t \tag{41}$$

$$x_t = \lambda^{-1}\beta^{-1}\sigma\delta\kappa\varphi_{2,t-1}+\lambda^{-1}\left(\kappa_x-\sigma\delta\kappa\right)\varphi_{2,t} \tag{42}$$

[14]Note that different from the standard case, analized by EH (2006), we cannot eliminate the lagrange mutipliers in order to get a tractable system.

Combining equations (1), (2) with (41) and (42), we can obtain the following reduced form,

$$y_t = A + B\widehat{E}_t y_{t+1} + C y_{t-1} + D\mu_t \tag{43}$$

where $y_t = [\pi_t, \varphi_{2,t}]'$, $A = 0$, and

$$\begin{aligned} B &= \vartheta \begin{bmatrix} \beta\lambda(\beta+\delta_o) & \sigma\beta\delta_o\kappa_o \\ -\beta\lambda(\beta+\delta_o) & -\sigma\beta\delta_o\kappa_o \end{bmatrix} \\ C &= \vartheta \begin{bmatrix} 0 & \sigma\kappa_o\delta_o + \left(\sigma^2\delta_o^2 + \beta\kappa_o^2\right)\left(\beta^{-1}\delta_o + 1\right) \\ 0 & \beta\lambda\left(\beta^{-1}\delta_o + 1\right) - \sigma\kappa_o\delta_o \end{bmatrix} \\ D &= \vartheta \begin{bmatrix} \lambda\beta \\ -\lambda\beta \end{bmatrix} \end{aligned}$$

with $\vartheta = \left(\beta\lambda + \sigma^2\delta_o^2 + \beta\kappa_o^2\right)^{-1}$, $\delta_o = \delta\kappa$ and $\kappa_o \equiv \kappa_x - \delta\kappa\sigma$.

The Minimal State Variable (see McCallum 1983) solution of (43) can be written as a function of the lagrange multiplier, $\varphi_{2,t-1}$, and the fundamental shock, μ_t,

$$\pi_t = b_\pi \varphi_{2,t-1} + c_\pi \mu_t \tag{44}$$

$$\varphi_{2,t} = b_\varphi \varphi_{2,t-1} + c_\varphi \mu_t \tag{45}$$

After replacing (44) and (45) (and their respective expected values) into (43), we obtain the following polynomial characterizing b_φ

$$\beta b_\varphi^2 - \gamma b_\varphi + 1 = 0 \tag{46}$$

where $\gamma = \frac{\left(\sigma^2\delta_o^2 + \beta\kappa_o^2 + \lambda\beta\right) + \lambda(\beta+\delta_o)^2}{(\lambda(\beta+\delta_o) - \sigma\kappa_o\delta_o)}$. Unlike the standard model, both roots of (46) are not necessarily positive. To have positive roots we need γ to be positive, which translates to the following condition $\lambda > \sigma\kappa^2\eta/(\beta+\kappa)$. Under such condition, the stable root of (46) is given by,

$$\bar{b}_\varphi = (2\beta)^{-1}\left[\gamma - \left(\gamma^2 - 4\beta\right)^{1/2}\right]$$

This root delivers a stationary REE, since $0 < \bar{b}_\varphi < 1$. The rest of coefficients are given:

$$\begin{aligned} \bar{b}_\pi &= \beta^{-1}\kappa\delta + \left(1 - \bar{b}_\varphi\right) \\ \bar{c}_\pi &= -\bar{c}_\varphi \\ \bar{c}_\varphi &= -\beta\lambda\left[\left(\sigma^2\delta_o^2 + \beta\kappa_o^2 + \beta\lambda\right) + \sigma\kappa_o\delta_o(\delta_o+\beta) - (\lambda(\beta+\delta_o) - \sigma\kappa_o\delta_o)\beta\left(\rho - \bar{b}_\pi\right)\right]^{-1} \end{aligned}$$

We refer to this REE as the optimal REE. The next proposition summarizes this result.

Proposition 7. *The optimal REE under a commitment policy is characterized by* $0 < \bar{b}_\varphi < 1$ *and* $\bar{b}_\pi > 0$ *iff*

$$\frac{\sigma\kappa^2\eta}{(\beta+\kappa)} < \lambda$$

Fundamental-based reaction function Taking determinacy for granted, we replace the RRE solution of the form of (44) and (45), and their respective expectations into the structural relationships (1) and (2). Then, we solve for i_t and the resulting equation is called "fundamental-based" optimal rule,

$$i_t = \phi_\varphi \varphi_{2,t-1} + \phi_\mu \mu_t \tag{47}$$

where ϕ_φ and ϕ_μ are given by

$$\phi_\varphi = (\beta\Delta)^{-1} \begin{pmatrix} \bar{b}_\pi^2 \beta^2 \lambda^2 - \delta_o \kappa_x \sigma^2 \left(\bar{b}_\varphi \beta \kappa_o + \delta_o \sigma\right) \\ +\bar{b}_\pi \beta \lambda \sigma \left(\left(-1 + \bar{b}_\varphi\right) \beta \kappa_o - \beta \delta_o \sigma + \delta_o \left(-\kappa_x + \sigma\right)\right) \end{pmatrix}$$

$$\phi_\mu = \Delta^{-1} \begin{pmatrix} \beta \bar{c}_\pi \kappa_o \kappa_x \lambda \rho - \beta \kappa_o \left(-\bar{c}_\varphi \kappa_o \kappa_x \rho + \left(-1 + \bar{b}_\varphi\right) \lambda \left(1 - \bar{c}_\pi + \beta \bar{c}_\pi \rho\right)\right) \sigma \\ -\delta_o \lambda \left(1 + \bar{c}_\pi \left(-1 + \beta \rho\right) \sigma^2 + \bar{b}_\pi \beta \lambda \left(\left(-1 + \bar{c}_\pi\right) \lambda + \beta \bar{c}_\varphi \kappa_o \rho \sigma\right)\right) \end{pmatrix}$$

with $\Delta = \lambda \left(\beta \kappa_o \kappa_x + \bar{b}_\pi \beta \left(\beta + \delta_o\right) \lambda + \left(-1 + \bar{b}_\varphi\right) \beta \delta_o \kappa_o \sigma + \delta_o^2 \sigma^2\right)$.

Combining the "fundamental-based" reaction function (47) with (1), (2) and (42) we collapse to the following system,

$$y_t = \Gamma + \Omega \widehat{E}_t y_{t+1} + \Phi y_{t-1} + \Theta \mu_t \tag{48}$$

where $y_t = [\pi_t, \varphi_{2,t}]', \Gamma = 0$, and

$$\begin{aligned} \Omega &= \psi \begin{bmatrix} \sigma^{-1} \left(\beta \kappa_o \left(\beta \sigma + \kappa_x\right) - \beta \sigma^2 \delta_o\right) & \beta \kappa_o^2 \kappa_x \lambda^{-1} \\ \beta \lambda \sigma^{-1} & \beta \kappa_o \end{bmatrix} \\ \Phi &= \psi \begin{bmatrix} 0 & (\beta \lambda \sigma)^{-1} \left(\delta_o \sigma \left(\left(\beta \lambda \phi_\varphi + \sigma \kappa_x\right) \left(\beta \kappa_o - \sigma \delta_o\right) - \beta \kappa_o \kappa_x \sigma\right) - \beta^2 \kappa_o \kappa_x \lambda \phi_\varphi\right) \\ 0 & -\sigma^{-1} \left(\beta \lambda \phi_\varphi + \sigma^2 \delta_o\right) \end{bmatrix} \end{aligned}$$

with $\psi = (\beta \kappa_o - \sigma \delta_o)^{-1}$, $\delta_o \equiv \delta \kappa$ and $\kappa_o \equiv \kappa_x - \delta \kappa \sigma$.

Determinacy is evaluated numerically using the benchmark calibration of section 3.1. We treat λ as a free policy parameter ranging from 0 to 100. Numerical results reveal that for the benchmark calibration determinacy is guaranteed if $\lambda > 0.05$. For alternative calibrations we obtain different threshold values. With W(1999), CCG (2000) and MN (1999) the lower bounds are 0.44, 0.02 and 0.11, respectively.

Next, we analyze whether the "fundamental-based" optimal policy guarantees a learnable REE. We study learnability by evaluating E-stability conditions (64) through (66). Under the standard new Keynesian model, EH (2006) show that the fundamental reaction function leads to instability under learning. Remarkably, our results stand in contrast with those of EH(2006). In fact, the fundamental-based reaction function (48) can induce a E-stable under a specific parametrization. In particular, provided proposition 7, if the inverse of the intertemporal elasticity of substitution (σ) is approximately greater than the inverse of the intertemporal elasticity of labor supply (η) the REE is E-stable.

Proposition 8. *Provided proposition 7 and under the fundamental-based reaction function derived under commitment, the necessary and sufficient conditions for the optimal REE to be E-stable is that $\sigma > \beta\eta$*

Proof. See appendix G. □

For the baseline calibration, the last proposition implies E-stability. We also check whether alternative calibrations yield stability under learning. For the majority of alternative calibrations, i.e. CGG (2000) or MN (1999), $\sigma > \beta\eta$ and thus E-stability holds. Under W(1999) calibration the "fundamental-based" reaction function induces expectational instability.

Expectations-based reaction function Following EH (2006) we also study the so-called "expectations-based" optimal rule under commitment. After plugging (41) and (42) into the aggregate supply (1), we can rewrite it as,

$$\begin{aligned} \varphi_{2,t} &= \left(\kappa_x\lambda^{-1}\kappa_o+1\right)^{-1}(1+\beta^{-1}\delta_o+\kappa_x\lambda^{-1}\beta^{-1}\sigma\delta_o)\varphi_{2,t-1} \\ &\quad -\left(\kappa_x\lambda^{-1}\kappa_o+1\right)^{-1}\beta\widehat{E}_t\pi_{t+1}-\left(\kappa_x\lambda^{-1}\kappa_o+1\right)^{-1}\delta_o i_t-\left(\kappa_x\lambda^{-1}\kappa_o+1\right)^{-1}\mu_t \end{aligned} \tag{49}$$

Then, we express the aggregate demand (2) in terms of $\varphi_{2,t}$ by using equation (42),

$$\begin{aligned} \varphi_{2,t} &= \left(\kappa_o-\beta^{-1}\sigma\delta_o\right)^{-1}\kappa_o\widehat{E}_t\varphi_{2,t+1}-\left(\kappa_o-\beta^{-1}\sigma\delta_o\right)^{-1}\beta^{-1}\sigma\delta_o\varphi_{2,t-1} \\ &\quad -\lambda\left(\kappa_o-\beta^{-1}\sigma\delta_o\right)^{-1}\sigma^{-1}\left(i_t-\widehat{E}_t\pi_{t+1}\right) \end{aligned} \tag{50}$$

By solving i_t from the last equations (49) and (50), we get the following expression for the "expected-based" optimal rule,

$$i_t = \phi_L\varphi_{2,t-1}+\phi_\pi\widehat{E}_t\pi_{t+1}+\phi_\varphi\widehat{E}_t\varphi_{2,t+1}+\phi_\mu\mu_t \tag{51}$$

where,

$$\begin{aligned} \phi_L &= -\frac{\delta_o\left(\kappa_o-\beta^{-1}\sigma\delta_o\right)^{-1}\beta^{-1}\sigma+\left(\kappa_x\lambda^{-1}\kappa_o+1\right)^{-1}(1+\beta^{-1}\delta_o+\kappa_x\lambda^{-1}\beta^{-1}\sigma\delta_o)}{\lambda\left(\kappa_o-\beta^{-1}\sigma\delta_o\right)^{-1}\sigma^{-1}-\delta_o\left(\kappa_x\lambda^{-1}\kappa_o+1\right)^{-1}} \\ \phi_\pi &= \frac{\left(\kappa_x\lambda^{-1}\kappa_o+1\right)^{-1}\beta+\lambda\left(\kappa_o-\beta^{-1}\sigma\delta_o\right)^{-1}\sigma^{-1}}{\lambda\left(\kappa_o-\beta^{-1}\sigma\delta_o\right)^{-1}\sigma^{-1}-\delta_o\left(\kappa_x\lambda^{-1}\kappa_o+1\right)^{-1}} \\ \phi_\varphi &= \frac{\left(\kappa_o-\beta^{-1}\sigma\delta_o\right)^{-1}\kappa_o}{\lambda\left(\kappa_o-\beta^{-1}\sigma\delta_o\right)^{-1}\sigma^{-1}-\delta_o\left(\kappa_x\lambda^{-1}\kappa_o+1\right)^{-1}} \\ \phi_\mu &= \frac{\left(\kappa_x\lambda^{-1}\kappa_o+1\right)^{-1}}{\lambda\left(\kappa_o-\beta^{-1}\sigma\delta_o\right)^{-1}\sigma^{-1}-\delta_o\left(\kappa_x\lambda^{-1}\kappa_o+1\right)^{-1}} \end{aligned}$$

The reduced form dynamics under the optimal rule (51) is given by,

$$y_t = \Gamma+\Omega\widehat{E}_t y_{t+1}+\Phi y_{t-1}+\Theta\mu_t \tag{52}$$

where $y_t = [\pi_t, \varphi_{2,t}]', \Gamma = 0$, and

$$\begin{aligned} \Omega &= \begin{bmatrix} \frac{\lambda(\phi_\pi-1)}{\sigma(\kappa_o-\beta^{-1}\sigma\delta_o)} & -\frac{\left(\kappa_o-\lambda\sigma^{-1}\phi_\varphi\right)}{(\kappa_o-\beta^{-1}\sigma\delta_o)} \\ -\frac{(\beta+\delta_o\phi_\pi)}{(\kappa_x\lambda^{-1}\kappa_o+1)} & -\frac{\delta_o\phi_\varphi}{(\kappa_x\lambda^{-1}\kappa_o+1)} \end{bmatrix} \\ \Phi &= \begin{bmatrix} 0 & \frac{\left(\beta^{-1}\sigma\delta_o+\lambda\sigma^{-1}\phi_L\right)+\left(1+\beta^{-1}\delta_o\right)\left(\kappa_o-\beta^{-1}\sigma\delta_o\right)}{(\kappa_o-\beta^{-1}\sigma\delta_o)} \\ 0 & \frac{\left(1+\beta^{-1}\delta_o+\kappa_x\lambda^{-1}\beta^{-1}\sigma\delta_o-\delta_o\phi_L\right)}{(\kappa_x\lambda^{-1}\kappa_o+1)} \end{bmatrix} \end{aligned}$$

In appendix H it is shown that a necessary condition for determinacy is the same to the one characterizing the optimal REE (see proposition 7).

Proposition 9. *Under the expected-based reaction function derived under commitment, the necessary and sufficient condition for a rational expectations equilibrium to be unique is that*

$$\frac{\sigma\kappa^2\eta}{(\beta+\kappa)} < \lambda$$

Proof. See appendix H. □

We evaluate numerically the threshold given in Proposition 9. According to the benchmark calibration, a value of λ greater than 0.01 delivers a determinate REE. For other parameterizations such as W(1999), CCG (2000) and MN (1999) the lower bounds are 0, 0.02 and 0.11, respectively.

Now we turn to the analysis of E-stability by evaluating (64) through (66). In the workhorse sticky price model, EH (2006) prove that the "expectations-based" reaction function guarantees stability under learning for all parameter configurations. In contrast, when the economy features the cost channel, the "expectations-based" policy rule given by (51), may or may not guarantee E-stability of the optimal REE.

Proposition 10. *Under the expected-based reaction function derived under commitment, the necessary and sufficient conditions for the optimal REE* $(0, \bar{b}, \bar{c})$ *to be E-stable is that*

$$\frac{\kappa\eta\sigma}{\beta+\kappa} < \lambda < \frac{\kappa^2\eta\,(\eta+\sigma)}{(\beta+\kappa-1)} \tag{53}$$

Proof. See appendix I. □

We calculate the thresholds given by the last proposition. According to our benchmark calibration, E-stability requires $0.12 < \lambda < 0.24$. W(1999), CCG (2000) and MN (1999) parameterizations imply the following ranges respectively: $0 < \lambda < 0.05$, $0.13 < \lambda < 0.32$ and $0.25 < \lambda < 0.39$. Notice that W(1999) calibration has yielded a tighter space for λ compared to the rest. This is because in Woodford´s parametrization the effect of the output gap over inflation through the aggregate supply is significantly smaller than the cost channel effect and therefore it is harder to induce E-stability.

4 Conclusions

In this paper we have studied determinacy and E-stability of different monetary policy rules when the cost channel matters. Particularly, we have extended Bullard and Mitra (2002) and Evans and Honkapohja (2003, 2006) analysis in the presence of supply side effects of monetary policy through the cost channel. Our results show that the cost channel modifies the standard conditions for both determinacy and learnability when the central banks operates with either instrument or target rules. In general, the presence of the cost channel threatens the determinacy and learnability of the rational expectations equilibrium. Moreover, popular policies to counteract instability, like the Taylor principle or "expectations-based" reaction functions, may not be effective or could even be counterproductive. Interestingly, a policy that it is a source of instability under learning without the cost channel, i.e. "fundamental rule" under commitment, is a possible antidote when the cost channel is active. The bottom line is that either the Taylor principle or EH´s proposal to the resolution of the instability under learning cannot be taken for granted when the cost channel matters.

5 Appendices

5.1 Methodology

In this appendix we summarize the conditions for determinacy and E-stability. The readers are referred to EH (2001) for further details. Consider a model given by the general form,

$$
\begin{aligned}
y_t &= \Gamma + \Omega E_t y_{t+1} + \Phi y_{t-1} + \Theta w_t && (54)\\
w_t &= \rho w_{t-1} + \varepsilon_t && (55)
\end{aligned}
$$

where y_t is an $nx1$ vector of endogenous variables, Γ is an $nx1$ vector of constants, Ω, Φ, Θ and ρ are nxn matrices of coefficients, and w_t is an $nx1$ vector of exogenous variables which is assumed to follow a stationary VAR, so that ε_t is an $nx1$ vector of white noise errors. In particular, throughout the main text $n = 2$, $\Gamma = 0$, $w_t = \mu_t$.

5.1.1 Determinacy

Suppose that $y_t = [z_t^1, z_t^2]'$ and one column of Φ is filled with zeros, i.e. only one of the endogenous variables is predetermined (e.g. z_t^1). Then, in general, the matrices of the system can be written as,

$$
\begin{aligned}
\Omega &= \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{bmatrix}\\
\Phi &= \begin{bmatrix} \Phi_{11} & 0 \\ \Phi_{21} & 0 \end{bmatrix}
\end{aligned}
$$

Rewriting the system by introducing a new variable $z_t^L = z_{t-1}^1$ we have

$$
\begin{bmatrix} 1 & 0 & -\Phi_{11} \\ 0 & 1 & -\Phi_{21} \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} z_t^1 \\ z_t^2 \\ z_t^L \end{bmatrix} = \begin{bmatrix} \Omega_{11} & \Omega_{12} & 0 \\ \Omega_{21} & \Omega_{22} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} E_t z_t^1 \\ E_t z_t^2 \\ E_t z_{t+1}^L \end{bmatrix} + other \qquad (56)
$$

where "other" includes terms that are not relevant in assessing determinacy. This system can be re-expressed as (provided that $\Phi_{11} \neq 0$),

$$
\begin{bmatrix} z_t^1 \\ z_t^2 \\ z_t^L \end{bmatrix} = J \begin{bmatrix} E_t z_t^1 \\ E_t z_t^2 \\ E_t z_{t+1}^L \end{bmatrix} + other \qquad (57)
$$

where

$$J = \begin{bmatrix} 1 & 0 & -\Phi_{11} \\ 0 & 1 & -\Phi_{21} \\ 1 & 0 & 0 \end{bmatrix}^{-1} \begin{bmatrix} \Omega_{11} & \Omega_{12} & 0 \\ \Omega_{21} & \Omega_{22} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since the model has one predetermined value, i.e. z_t^L, determinacy requires that one of the eigenvalues of J to lie outside and the other two eigenvalues outside. If just one or no roots lie inside the unit circle (with the other roots outside), then the model is indeterminate. For models with $\Phi = 0$, i.e. there are not predetermined variables in the system, the determinacy condition is that Ω has both eigenvalues inside the unit circle. If at least one of the roots is outside the unit circle, the model is indeterminate.

5.1.2 Stability conditions under learning

Here, we follow closely the criterion of *Expectational Stability* (or E-stability) developed by Evans and Honkapohja (2001). Under learning, the agents do not have rational expectations; instead agents form their expected values with adaptive rules which are updated as data is produced by the system.

Given the general form (54) with $\Phi \neq 0$, an MSV REE (see McCallum 1983) takes the following form,

$$y_t = \overline{a} + \overline{b} y_{t-1} + \overline{c} w_t \tag{58}$$

To define E-stability we consider *Perceived Law of Motion* (PLM) of the same form of the MSV,

$$y_t = a + b y_{t-1} + c w_t \tag{59}$$

EH (2001, 2003) analyze different information assumption about how agents update their PLM. The first information assumption treats expectations as determined before the current values of endogenous variables are realized. Under such assumption, the next period expectation is,

$$E_t y_{t+1} = a + b\left(a + b y_{t-1} + c w_t\right) + c w_t \tag{60}$$

Plugging it into (54), we can compute the *Actual Law of Motion* (ALM),

$$y_t = \Gamma + (\Omega(I + b))a + (\Omega b^2 + \Phi) y_{t-1} + (\Omega(bc + c\rho) + \Theta) w_t. \tag{61}$$

To analyze the E-stability conditions, we have to check the stability of the mapping T from the PLM to ALM,

$$T(a, b, c) = \left(\Gamma + (\Omega(I + b))a, \Omega b^2 + \Phi, \Omega(bc + c\rho) + \Theta\right). \tag{62}$$

The answer of the question of whether the system is stable under learning is given by the principle of E-stability, which comes from analyzing the following matrix differential equation,

$$\frac{\partial T(a, b, c)}{\partial \tau} = T(a, b, c) - (a, b, c) \tag{63}$$

where τ is a notional time. The E-stability conditions are shown in EH (2003),

$$DT_a = \Omega(1+\bar{b}) \tag{64}$$
$$DT_b = \bar{b}' \otimes \Omega + I \otimes \Omega\bar{b} \tag{65}$$
$$DT_c = \rho' \otimes \Omega + I \otimes \Omega\bar{b} \tag{66}$$

The REE $(\bar{a}, \bar{b}, \bar{c})$ is *E-stable* or *learnable* if all real parts of the eigenvalues of DT_a, DT_b and DT_c are lower than 1. The solution is *E-unstable* if any of them have a real part higher than 1. Alternatively, E-stability holds if all eigenvalues of $DT_a - I, DT_b - I$ and $DT_c - I$ have negative real parts.

Under the second information assumption expectations and current values are determined simultaneously, i.e. current values are used to compute expectations,

$$E_t y_{t+1} = a + by_t + cw_t \tag{67}$$

The E-stability conditions are derived in Evans and Honkapohja (2001, p. 238), proposition 10.3,

$$DT_a = (I - \Omega\bar{b})^{-1}\Omega \tag{68}$$
$$DT_b = \left[(I - \Omega\bar{b})^{-1}\Phi\right]' \otimes (I - \Omega\bar{b})^{-1}\Omega. \tag{69}$$
$$DT_c = \rho' \otimes \left[(I - \Omega\bar{b})^{-1}\Omega\right] \tag{70}$$

As in the first information assumption, E-stability holds if all eigenvalues of $DT_a - I, DT_b - I$ and $DT_c - I$ have negative real parts.

5.2 Appendix A: Proof of Proposition 1

The characteristic polynomial of Ω (given by 6) is $P(\xi) = \xi^2 + A_1\xi + A_0$ where

$$A_0 = \frac{\sigma\beta}{\sigma + \phi_x + (\kappa_x - \kappa\delta\sigma)\,\phi_\pi} \tag{A1}$$

$$A_1 = \frac{-\sigma(1+\beta) - \beta\phi_x + \kappa_x(\phi_\pi - 1) - [\kappa\delta\phi_x + (\kappa_x - \kappa\delta\sigma)\,\phi_\pi]}{\sigma + \phi_x + (\kappa_x - \kappa\delta\sigma)\,\phi_\pi} \tag{A2}$$

Both eigenvalues of Ω are inside the unit circle if and only if both of the following conditions hold

$$|A_0| < 1 \tag{A3}$$

$$|A_1| < 1 + A_0. \tag{A4}$$

We can note that condition (A3) is always true, whereas condition (A4) implies (7) and (8).

5.3 Appendix B: Proof of Proposition 2

Using results of Evans and Honkapohja (2001), E-stability needs that the eigenvalues of $\rho\Omega$ (Ω is given by equation. 6) to have real parts less than one. The eigenvalues of $\rho\Omega$ are given by the product of the eigenvalues of Ω and ρ, and since $0 < \rho < 1$, it suffices that eigenvalues of B to have real parts less than 1. On the other hand, the MSV solution will not be E-stable if any eigenvalue of Ω has a real part greater than 1. The characteristic polynomial of $\Omega - I$ (where I is a corresponding identity matrix) given by $P(\xi) = \xi^2 + A_1\xi + A_0$ where

$$A_1 = \frac{\sigma(1-\beta) + (\kappa_x - \delta\kappa\sigma)\phi_\pi + \phi_x + \kappa_x(\phi_\pi - 1) + (1-\beta-\delta\kappa)\phi_x}{\sigma + \phi_x + (\kappa_x - \delta\kappa\sigma)\phi_\pi} \tag{B1}$$

$$A_0 = \frac{(1-\beta-\delta\kappa)\phi_x + \kappa_x(\phi_\pi - 1)}{\sigma + \phi_x + (\kappa_x - \delta\kappa\sigma)\phi_\pi} \tag{B2}$$

It is necessary for both eigenvalues of $\Omega - I$ to have negative real parts. According to the Routh Theorem, that condition holds if and only if $A_1 > 0$ and $A_0 > 0$. We can note that

$$A_1 = A_0 + \frac{\sigma(1-\beta) + (\kappa_x - \delta\kappa\sigma)\phi_\pi + \phi_x}{\sigma + \phi_x + (\kappa_x - \delta\kappa\sigma)\phi_\pi} \tag{B3}$$

Thus, given that $\kappa_x - \delta\kappa\sigma$ equals $\kappa\eta$ when $\delta = 1$, $A_0 > 0$ implies $A_1 > 0$. Hence, the E-stability condition, given by (12), is derived from $A_0 > 0$.

5.4 Appendix C: Proof of Proposition 3

The characteristic polynomial of Ω (given by 16) is $P(\xi) = \xi^2 + A_1\xi + A_0$ where

$$A_0 = \frac{\beta(\sigma - \phi_x) + \delta\kappa(\sigma\phi_\pi - \phi_x)}{\sigma} \tag{C1}$$

$$A_1 = \frac{\phi_x + \kappa_x(\phi_\pi - 1) - \sigma(\beta + 1) - \delta\kappa\sigma\phi_\pi}{\sigma} \tag{C2}$$

Both eigenvalues of Ω are inside the unit circle if and only if conditions (A3) and (A4) hold. We can note that condition (A3) implies (17) and (18), whereas condition (A4) implies (19) and (20).

5.5 Appendix D: Proof of Proposition 4

As in the previous case, E-stability conditions are given by analyzing the characteristic polynomial of $\Omega - I$ (where Ω is given by 16) given by $P(\xi) = \xi^2 + A_1\xi + A_0$ where

$$A_1 = \frac{(\kappa_x - \delta\kappa\sigma)\phi_\pi + \phi_x + \sigma(1-\beta) - \kappa_x}{\sigma} \tag{D1}$$

$$A_0 = \frac{\phi_x(1-\beta-\delta\kappa) + \kappa_x(\phi_\pi - 1)}{\sigma} \tag{D2}$$

It is necessary for both eigenvalues of $\Omega - I$ to have negative real parts. According to the Routh Theorem, that condition holds if and only if $A_1 > 0$ and $A_0 > 0$. We can note

$$A_1 = A_0 + \frac{\sigma(1-\beta) + (\beta + \kappa\delta)\phi_x - \kappa\delta\sigma\phi_\pi}{\sigma} \quad \text{D3}$$

Different from the case of contemporaneous data, $A_0 > 0$ does not imply $A_1 > 0$. In this case, the first E-stability condition, given by (21), is derived from $A_0 > 0$. The second E-stability condition, given by (22), is derived from $A_1 > 0$ (using equation D3), provided that $A_0 > 0$.

5.6 Appendix E: Proof of Proposition 5

The characteristic polynomial of Ω (given by 32) has one eigenvalue equal to zero and the other equal to (after replacing κ_x and κ_o and making $\delta = 1$)

$$r = \frac{\lambda\beta + \lambda\kappa - \kappa^2\sigma\eta}{\lambda + \kappa^2\eta^2} \quad \text{E1}$$

Determinacy requires that $|r| < 1$ and thus the latter condition implies

$$\frac{\kappa^2\eta(\sigma - \eta)}{(\beta + \kappa + 1)} < \lambda < \frac{\kappa^2\eta(\eta + \sigma)}{(\beta + \kappa - 1)} \quad \text{E2}$$

5.7 Appendix F: Proof of Proposition 6

E-stability is guaranteed if and only if all of the eigenvalues of $\Omega - I$ (where Ω is given by 32) have negative real parts. The characteristic polynomial of $\Omega - I$ is given by $\rho(\xi) = \xi^2 + A_1\xi + A_0$ where

$$A_1 = 1 + \frac{\lambda + \kappa^2\eta^2 + \kappa^2\eta\sigma - \lambda\beta - \lambda\kappa}{\lambda + \kappa^2\eta^2} \quad \text{F1}$$

$$A_0 = \frac{\lambda + \kappa^2\eta^2 + \kappa^2\eta\sigma - \lambda\beta - \lambda\kappa}{\lambda + \kappa^2\eta^2} \quad \text{F2}$$

It is necessary for both eigenvalues of $\Omega - I$ to have negative real parts. According to the Routh Theorem, that condition holds if and only if $A_1 > 0$ and $A_0 > 0$. We can note

$$A_1 = A_0 + 1 \quad \text{F3}$$

Thus, a sufficient condition for E-stability is that $A_0 > 0$ implying

$$\lambda < \frac{\kappa^2\eta(\eta + \sigma)}{(\beta + \kappa - 1)} \quad \text{F4}$$

5.8 Appendix G: Proof of Proposition 8

We apply the E-stability conditions (64) through (66) to the model given by (48). A first necessary condition is that $DT_a - I$ has eigenvalues with negative real parts, which is equivalent to $\mathrm{tr}(DT_a - I) < 0$ and $\det(DT_a - I) > 0$.

$$DT_a - I = \begin{bmatrix} -\frac{\sigma^{-1}\left(\sigma^2\beta\delta_o - \beta\kappa_o(\kappa_x+\sigma\beta)\right)+(\beta\kappa_o-\sigma\delta_o)}{(\beta\kappa_o-\sigma\delta_o)} & \frac{\beta\lambda^{-1}\kappa_o^2\kappa_x\left(\bar{b}_\varphi+1\right)-\sigma^{-1}\bar{b}_\pi\left(\sigma^2\beta\delta_o-\beta\kappa_o(\kappa_x+\sigma\beta)\right)}{(\beta\kappa_o-\sigma\delta_o)} \\ \frac{\sigma^{-1}\beta\lambda}{(\beta\kappa_o-\sigma\delta_o)} & \frac{\beta\kappa_o\left(\bar{b}_\varphi+1\right)+\sigma^{-1}\beta\lambda\bar{b}_\pi-(\beta\kappa_o-\sigma\delta_o)}{(\beta\kappa_o-\sigma\delta_o)} \end{bmatrix}$$

where $\bar{b}_\pi$ and $\bar{b}_\varphi$ are the coefficients of the MSV at the REE. The determinant of $DT_a - I$ is given by the following expression

$$\det(DT_a - I) = \frac{1}{\sigma^2\delta\kappa - \sigma\beta\kappa\eta}\left(\sigma^2\kappa(1-\beta) + \beta\kappa\eta\kappa_x + \beta\lambda\bar{b}_\pi + \sigma\beta\kappa\eta\bar{b}_\varphi(1-\beta)\right)$$

Which is positive iff $(\delta = 1)$ $\sigma > \beta\eta$ since $0 < \beta < 1$, $\bar{b}_\pi > 0$ and $\bar{b}_\varphi > 0$. Notice that without the cost channel $(\delta = 0)$ the expression is always negative and therefore $DT_a - I$ is an unstable matrix (see EH 2006). The trace of $DT_a - I$ can be written as

$$\mathrm{tr}(DT_a - I) = \frac{1}{(\beta\eta - \sigma\delta)}\left(\sigma(2-\beta) + \beta\eta\bar{b}_\varphi + \sigma^{-1}\kappa^{-1}\beta\lambda\bar{b}_\pi + \sigma^{-1}\beta\kappa\eta^2 + \beta\eta(\beta+\kappa-1)\right)$$

and is always negative if and only if $(\delta = 1)$ $\sigma > \beta\eta$. When the cost channel is absent $(\delta = 0)$, the trace is always positive and $DT_a - I$ is an unstable matrix (see EH 2006).

We now turn to analyze the rest of matrices,

$$DT_b - I = \begin{bmatrix} \Delta_{11} & 0 \\ \Delta_{21} & \Delta_{22} \end{bmatrix}$$

where:

$$\Delta_{11} = \begin{bmatrix} -1 & \frac{\beta\left(\bar{b}_\pi(\beta+\delta_o)\lambda+\bar{b}_\varphi\delta_o\kappa_o\sigma\right)}{\beta(\kappa_o^2+\lambda)+\delta_o^2\sigma^2} \\ 0 & -\frac{\bar{b}_\pi\beta^2\lambda+\delta_o^2\sigma^2+\beta\left(\kappa_o^2+\lambda+\bar{b}_\pi\delta_o\lambda+\bar{b}_\varphi\delta_o\kappa_o\sigma\right)}{\beta(\kappa_o^2+\lambda)+\delta_o^2\sigma^2} \end{bmatrix}$$

$$\Delta_{21} = \begin{bmatrix} \frac{\beta\bar{b}_\pi(\beta+\delta_o)\lambda}{\beta(\kappa_o^2+\lambda)+\delta_o^2\sigma^2} & \frac{\beta\bar{b}_\pi\delta_o\kappa_o\sigma}{\beta(\kappa_o^2+\lambda)+\delta_o^2\sigma^2} \\ \frac{-\beta\bar{b}_\pi(\beta+\delta_o)\lambda}{\beta(\kappa_o^2+\lambda)+\delta_o^2\sigma^2} & \frac{-\beta\bar{b}_\pi\delta_o\kappa_o\sigma}{\beta(\kappa_o^2+\lambda)+\delta_o^2\sigma^2} \end{bmatrix}$$

$$\Delta_{22} = \begin{bmatrix} -1 + \frac{\beta(\beta+\delta_o)\lambda\rho}{\beta(\kappa_o^2+\lambda)+\delta_o^2\sigma^2} & \frac{\beta\left(\bar{b}_\pi(\beta+\delta_o)\lambda+\left(\bar{b}_\varphi+\rho\right)\delta_o\kappa_o\sigma\right)}{\beta(\kappa_o^2+\lambda)+\delta_o^2\sigma^2} \\ \frac{-\beta(\beta+\delta_o)\lambda\rho}{\beta(\kappa_o^2+\lambda)+\delta_o^2\sigma^2} & -\frac{\bar{b}_\pi\beta^2\lambda+\delta_o^2\sigma^2+\beta\left(\kappa_o^2+\lambda+\bar{b}_\pi\delta_o\lambda+\left(\bar{b}_\varphi+\rho\right)\delta_o\kappa_o\sigma\right)}{\beta(\kappa_o^2+\lambda)+\delta_o^2\sigma^2} \end{bmatrix}$$

The characteristic polynomial of $DT_b - I$ is given by $P(\xi) = \xi^4 + A_3\xi^3 + A_2\xi^2 + A_1\xi + A_0$. It can be shown that two of roots (e.g. ξ_1 and ξ_2) are equal to -1. The rest of roots are given by the following expressions,

$$\xi_3 = \frac{-\beta\kappa_o^2 - \beta^2\lambda\left(1-\bar{b}_\varphi\right) - \beta\lambda\left(1-\bar{b}_\varphi\beta\right) - \bar{b}_\pi\beta\delta_o\lambda - \delta_o\lambda\beta\left(1-\bar{b}_\varphi\right) - 2\bar{b}_\varphi\beta\delta_o\kappa_o\sigma - \delta_o^2\sigma^2}{\beta\kappa_o^2 + \beta\lambda + \delta_o^2\sigma^2}$$

$$\xi_4 = \frac{-\beta\kappa_o^2 - \beta\lambda - \bar{b}_\pi\beta^2\lambda - \bar{b}_\pi\beta\delta_o\lambda - \bar{b}_\varphi\beta\delta_o\kappa_o\sigma - \delta_o^2\sigma^2}{\beta\kappa_o^2 + \beta\lambda + \delta_o^2\sigma^2}$$

and are always negative since $0 < \bar{b}_\varphi < 1$, $0 < \bar{b}_\pi$ and $0 < \beta < 1$. It follows the $DT_b - I$ is a stable matrix.

Finally, we analyze the stability of $DT_c - I$.

$$DT_c - I = \begin{bmatrix} \Delta_{11} & 0 \\ 0 & \Delta_{22} \end{bmatrix}$$

where:

$$\begin{aligned}
\Delta_{11} &= \begin{bmatrix} -1 & \frac{\beta\left(\bar{b}_\pi(\beta+\delta_o)\lambda+\bar{b}_\varphi\delta_o\kappa_o\sigma\right)}{\beta(\kappa_o^2+\lambda)+\delta_o^2\sigma^2} \\ 0 & -\frac{\bar{b}_\pi\beta^2\lambda+\delta_o^2\sigma^2+\beta\left(\kappa_o^2+\lambda+\bar{b}_\pi\delta_o\lambda+\bar{b}_\varphi\delta_o\kappa_o\sigma\right)}{\beta(\kappa_o^2+\lambda)+\delta_o^2\sigma^2} \end{bmatrix} \\
\Delta_{22} &= \begin{bmatrix} -1+\frac{\beta(\beta+\delta_o)\lambda\rho}{\beta(\kappa_o^2+\lambda)+\delta_o^2\sigma^2} & \frac{\beta\left(\bar{b}_\pi(\beta+\delta_o)\lambda+\left(\bar{b}_\varphi+\rho\right)\delta_o\kappa_o\sigma\right)}{\beta(\kappa_o^2+\lambda)+\delta_o^2\sigma^2} \\ \frac{-\beta(\beta+\delta_o)\lambda\rho}{\beta(\kappa_o^2+\lambda)+\delta_o^2\sigma^2} & -\frac{\bar{b}_\pi\beta^2\lambda+\delta_o^2\sigma^2+\beta\left(\kappa_o^2+\lambda+\bar{b}_\pi\delta_o\lambda+\left(\bar{b}_\varphi+\rho\right)\delta_o\kappa_o\sigma\right)}{\beta(\kappa_o^2+\lambda)+\delta_o^2\sigma^2} \end{bmatrix}
\end{aligned}$$

The characteristic polynomial $DT_c - I$ has the form of $P(\xi) = \xi^4 + A_3\xi^3 + A_2\xi^2 + A_1\xi + A_0$. It can be shown that two of roots (e.g. ξ_1 and ξ_2) are equal to -1. The rest of roots are given by the following expressions,

$$\begin{aligned}
\xi_3 &= \frac{-\beta\kappa_o^2 - \beta\lambda(1-\rho\beta) - \beta^2\lambda\left(1-\bar{b}_\varphi\right) - \bar{b}_\pi\beta\delta_o\lambda - \delta_o\beta\lambda(1-\rho) - \bar{b}_\varphi\beta\delta_o\kappa_o\sigma - \rho\beta\delta_o\kappa_o\sigma - \delta_o^2\sigma^2}{\beta\kappa_o^2 + \beta\lambda + \delta_o^2\sigma^2} \\
\xi_4 &= \frac{-\beta\kappa_o^2 - \beta\lambda - \bar{b}_\pi\beta^2\lambda - \bar{b}_\pi\beta\delta_o\lambda - \bar{b}_\varphi\beta\delta_o\kappa_o\sigma - \delta_o^2\sigma^2}{\beta\kappa_o^2 + \beta\lambda + \delta_o^2\sigma^2}
\end{aligned}$$

and are always negative since $0 < \bar{b}_\varphi < 1$, $0 < \bar{b}_\pi$, $0 < \beta < 1$ and $0 < \rho < 1$. It follows the $DT_c - I$ is a stable matrix.

5.9 Appendix H: Proof of Proposition 9

Unlike EH (2006), the determinacy conditions of the reduced system under the expected-based reaction function cannot be studied by using the tools provided in the methodological appendix. We use the approach of Woodford (2003, chapter 4) to perform the determinacy analysis. First, the reduced form (52) is rewritten as,

$$\begin{bmatrix} E_t\varphi_{2,t+1} \\ E_t\pi_{t+1} \\ \varphi_{2L,t+1} \end{bmatrix} = \Upsilon \begin{bmatrix} \varphi_{2,t} \\ \pi_t \\ \varphi_{2L,t} \end{bmatrix} + other \qquad (71)$$

The system is determinate iff exactly one root lie inside the unit circle and the rest outside the unit circle. As shown in Woodford (2003) the following cases guarantee determinacy:

either (Case I)

$$\begin{aligned}
1 + A_2 + A_1 + A_0 &< 0 \\
-1 + A_2 - A_1 + A_0 &> 0
\end{aligned}$$

or (Case II):

$$\begin{aligned} 1 + A_2 + A_1 + A_0 &> 0 \\ -1 + A_2 - A_1 + A_0 &< 0 \\ A_0^2 - A_0 A_2 + A_1 - 1 &> 0 \end{aligned}$$

(Case III):

$$\begin{aligned} 1 + A_2 + A_1 + A_0 &> 0 \\ -1 + A_2 - A_1 + A_0 &< 0 \\ A_0^2 - A_0 A_2 + A_1 - 1 &< 0 \\ |A_2| &> 3 \end{aligned}$$

Let the characteristic equation of the matrix Υ be written in the form

$$P(\xi) = \xi^3 + A_2 \xi^2 + A_1 \xi + A_0$$

where

$$A_2 = -\frac{\beta (\beta\lambda + \delta_o \lambda + \delta_o \kappa_o \sigma) \left(\beta \kappa_o \kappa_x + \beta\lambda + \beta \delta_o \kappa_o \sigma + \delta_o^2 \sigma^2\right)}{\psi}$$

$$A_1 = \frac{\left(\beta \kappa_o \kappa_x + \beta\lambda + \beta \delta_o \kappa_o \sigma + \delta_o^2 \sigma^2\right) \left(\beta \kappa_o \kappa_x + \beta\lambda + \beta \delta_o \kappa_o \sigma + \delta_o^2 \sigma^2 + \beta^2 \lambda + 2\beta \delta_o \lambda + \delta_o^2 \lambda\right)}{\psi}$$

$$A_0 = -\frac{\left(\beta\lambda + \delta_o \lambda - \delta_o \kappa_x \sigma + \delta_o^2 \sigma^2\right) \left(\beta \kappa_o \kappa_x + \beta\lambda + \beta \delta_o \kappa_o \sigma + \delta_o^2 \sigma^2\right)}{\psi}$$

where $\psi = 2\beta^2 \delta_o (\beta + \delta_o) \kappa_o \lambda \sigma$

In the polynomial $P(\xi)$, $A_1$and A_2 are always positive and negative, respectively. Note A_0 is negative iff $\beta\lambda + \delta_o\lambda - \delta_o\kappa_x\sigma + \delta_o^2\sigma^2 > 0$. After replacing δ_o and κ_x, it turns out that $\lambda > \kappa^2\sigma\eta/(\beta+\kappa)$ guarantees A_0's negativity. Note that such condition for λ characterizes the optimal REE (see proposition 7 in the main text). Under such assumption, $-1+A_2-A_1+A_0 < 0$ and then Case I is ruled out.

5.10 Appendix I: Proof of Proposition 10

We apply the E-stability conditions (64) through (66) to model given by (52). A first necessary condition is that $DT_a - I$ has eigenvalues with negative real parts, which is equivalent to $\text{tr}(DT_a - I) < 0$ and $\det(DT_a - I) > 0$.

$$DT_a - I = \begin{bmatrix} -1 + \frac{\beta(\beta+\delta_o)\lambda}{\delta_o^2\sigma^2+\beta(\lambda+\kappa_o(\kappa_x+\delta_o\sigma))} & \frac{\beta\left(\bar{b}_\pi(\beta+\delta_o)\lambda-\left(1+\bar{b}_\varphi\right)\delta_o\kappa_o\sigma\right)}{\delta_o^2\sigma^2+\beta(\lambda+\kappa_o(\kappa_x+\delta_o\sigma))} \\ \frac{-\beta(\beta+\delta_o)\lambda}{\delta_o^2\sigma^2+\beta(\lambda+\kappa_o(\kappa_x+\delta_o\sigma))} & -\frac{\bar{b}_\pi\beta^2\lambda+\delta_o^2\sigma^2+\beta\left(\lambda+\bar{b}_\pi\delta_o\lambda+\kappa_o\left(\kappa_x-\bar{b}_\varphi\delta_o\sigma\right)\right)}{\delta_o^2\sigma^2+\beta(\lambda+\kappa_o(\kappa_x+\delta_o\sigma))} \end{bmatrix}$$

where $\bar{b}_\pi$ and $\bar{b}_\varphi$ are the coefficients of the MSV at the REE. The determinant of $DT_a - I$ is given by the following expression

$$\det(DT_a - I) = \frac{\beta\left[\kappa_o\kappa_x - (\beta+\delta_o-1)\lambda\right] + \delta_o\left[\lambda(\beta+\delta_o) - \kappa_o\sigma\right]}{\delta_o^2\sigma^2 + \beta\left(\lambda + \kappa_o\left(\kappa_x + \delta_o\sigma\right)\right)} + \frac{\lambda\beta(\beta+\delta_o)\left(1-\bar{b}_\varphi\right) + \delta_o\kappa_o\sigma\left(1-\beta\bar{b}_\varphi\right) + \delta_o^2\sigma^2}{\delta_o^2\sigma^2 + \beta\left(\lambda + \kappa_o\left(\kappa_x + \delta_o\sigma\right)\right)}$$

There two necessary and sufficient conditions for $\det(DT_a - I)$ to be positive.

$$\kappa_o\kappa_x - (\beta+\delta_o-1)\lambda > 0 \rightarrow \lambda < \frac{\kappa^2\eta(\eta+\sigma)}{(\beta+\kappa-1)}$$
$$\lambda(\beta+\delta_o) - \kappa_o\sigma > 0 \rightarrow \lambda > \frac{\kappa\eta\sigma}{\beta+\kappa}$$

Notice that under the standard new Keynesian framework δ_o is zero and $\det(DT_a - I)$ is always positive for all parameters value (see EH 2006).

The trace of $DT_a - I$ can be written as

$$\operatorname{tr}(DT_a - I) = \frac{-\beta\left[\kappa_o\kappa_x - (\beta+\delta_o-1)\lambda\right] - \bar{b}_\pi\beta(\beta+\delta_o)\lambda - 2\delta_o^2\sigma^2 - \beta(\lambda + \kappa_o\kappa_x) - \beta\kappa_o\delta_o\sigma\left(1-\bar{b}_\varphi\right)}{\delta_o^2\sigma^2 + \beta\left(\lambda + \kappa_o\left(\kappa_x + \delta_o\sigma\right)\right)}$$

which is always negative iff

$$\kappa_o\kappa_x - (\beta+\delta_o-1)\lambda > 0 \rightarrow \lambda < \frac{\kappa^2\eta(\eta+\sigma)}{(\beta+\kappa-1)}$$

Again, under the standard new Keynesian framework δ_o is zero and $\operatorname{tr}(DT_a - I)$ is always negative regardless the parametrization (see EH 2006).

We now turn to the rest of matrices,

$$DT_b - I = \begin{bmatrix} \Delta_{11} & 0 \\ \Delta_{21} & \Delta_{22} \end{bmatrix}$$

where:

$$\Delta_{11} = \begin{bmatrix} -1 & \frac{\beta\left(\bar{b}_\pi(\beta+\delta_o)\lambda - \bar{b}_\varphi\delta_o\kappa_o\sigma\right)}{\delta_o^2\sigma^2+\beta(\lambda+\kappa_o(\kappa_x+\delta_o\sigma))} \\ 0 & -\frac{\bar{b}_\pi\beta^2\lambda + \delta_o^2\sigma^2 + \beta\left(\lambda + \bar{b}_\pi\delta_o\lambda + \kappa_o\left(\kappa_x + \delta_o\sigma - \bar{b}_\varphi\delta_o\sigma\right)\right)}{\delta_o^2\sigma^2+\beta(\lambda+\kappa_o(\kappa_x+\delta_o\sigma))} \end{bmatrix}$$

$$\Delta_{21} = \begin{bmatrix} \frac{\beta\bar{b}_\pi(\beta+\delta_o)\lambda}{\delta_o^2\sigma^2+\beta(\lambda+\kappa_o(\kappa_x+\delta_o\sigma))} & \frac{-\beta\bar{b}_\pi\delta_o\kappa_o\sigma}{\delta_o^2\sigma^2+\beta(\lambda+\kappa_o(\kappa_x+\delta_o\sigma))} \\ \frac{-\beta\bar{b}_\pi(\beta+\delta_o)\lambda}{\delta_o^2\sigma^2+\beta(\lambda+\kappa_o(\kappa_x+\delta_o\sigma))} & \frac{\beta\bar{b}_\pi\delta_o\kappa_o\sigma}{\delta_o^2\sigma^2+\beta(\lambda+\kappa_o(\kappa_x+\delta_o\sigma))} \end{bmatrix}$$

$$\Delta_{22} = \begin{bmatrix} -1 + \frac{\beta(\beta+\delta_o)\lambda\bar{b}_\varphi}{\delta_o^2\sigma^2+\beta(\lambda+\kappa_o(\kappa_x+\delta_o\sigma))} & \frac{\beta\left(\bar{b}_\pi(\beta+\delta_o)\lambda - 2\bar{b}_\varphi\delta_o\kappa_o\sigma\right)}{\delta_o^2\sigma^2+\beta(\lambda+\kappa_o(\kappa_x+\delta_o\sigma))} \\ \frac{-\beta(\beta+\delta_o)\lambda\bar{b}_\varphi}{\delta_o^2\sigma^2+\beta(\lambda+\kappa_o(\kappa_x+\delta_o\sigma))} & -\frac{\bar{b}_\pi\beta^2\lambda + \delta_o^2\sigma^2 + \beta\left(\lambda + \bar{b}_\pi\delta_o\lambda + \kappa_o\left(\kappa_x + \delta_o\sigma - 2\bar{b}_\varphi\delta_o\sigma\right)\right)}{\delta_o^2\sigma^2+\beta(\lambda+\kappa_o(\kappa_x+\delta_o\sigma))} \end{bmatrix}$$

The characteristic polynomial of $DT_b - I$ is given by $P(\xi) = \xi^4 + A_3\xi^3 + A_2\xi^2 + A_1\xi + A_0$. It can be shown that two of roots (e.g. ξ_1 and ξ_2) are equal to -1. The rest of roots are given by the following expressions,

$$\begin{aligned}
\xi_3 &= \frac{-\beta\kappa_o\kappa_x - \beta\lambda - \bar{b}_\pi\beta^2\lambda - \bar{b}_\pi\beta\delta_o\lambda - \beta\delta_o\kappa_o\sigma\left(1-\bar{b}_\varphi\right) - \delta_o^2\sigma^2}{\delta_o^2\sigma^2 + \beta\left(\lambda + \kappa_o\left(\kappa_x + \delta_o\sigma\right)\right)} \\
\xi_4 &= \frac{-\beta\left[\kappa^2\eta^2 + (1-\beta)\lambda\right] - \delta_o^2\lambda - 2\beta\left(\lambda\left(\beta+\delta_o\right) + \delta_o\kappa_o\sigma\right)\left(1-\bar{b}_\varphi\right) - \delta_o^2\sigma^2}{\delta_o^2\sigma^2 + \beta\left(\lambda + \kappa_o\left(\kappa_x + \delta_o\sigma\right)\right)}
\end{aligned}$$

and are always negative since $0 < \bar{b}_\varphi < 1$, $0 < \bar{b}_\pi$ and $0 < \beta < 1$. It follows the $DT_b - I$ is a stable matrix.

Finally, we analyze the stability of $DT_c - I$.

$$DT_c - I = \begin{bmatrix} \Delta_{11} & 0 \\ 0 & \Delta_{22} \end{bmatrix}$$

where:

$$\begin{aligned}
\Delta_{11} &= \begin{bmatrix} -1 & \frac{\beta\left(\bar{b}_\pi(\beta+\delta_o)\lambda - \bar{b}_\varphi\delta_o\kappa_o\sigma\right)}{\delta_o^2\sigma^2+\beta(\lambda+\kappa_o(\kappa_x+\delta_o\sigma))} \\ 0 & -\frac{\bar{b}_\pi\beta^2\lambda+\delta_o^2\sigma^2+\beta\left(\lambda+\bar{b}_\pi\delta_o\lambda+\kappa_o\left(\kappa_x+\delta_o\sigma-\bar{b}_\varphi\delta_o\sigma\right)\right)}{\delta_o^2\sigma^2+\beta(\lambda+\kappa_o(\kappa_x+\delta_o\sigma))} \end{bmatrix} \\
\Delta_{22} &= \begin{bmatrix} -1 + \frac{\beta(\beta+\delta_o)\lambda\rho}{\delta_o^2\sigma^2+\beta(\lambda+\kappa_o(\kappa_x+\delta_o\sigma))} & \frac{\beta\left(\bar{b}_\pi(\beta+\delta_o)\lambda - \left(\bar{b}_\varphi+\rho\right)\delta_o\kappa_o\sigma\right)}{\delta_o^2\sigma^2+\beta(\lambda+\kappa_o(\kappa_x+\delta_o\sigma))} \\ \frac{-\beta(\beta+\delta_o)\lambda\rho}{\delta_o^2\sigma^2+\beta(\lambda+\kappa_o(\kappa_x+\delta_o\sigma))} & -\frac{\bar{b}_\pi\beta^2\lambda+\delta_o^2\sigma^2+\beta\left(\lambda+\bar{b}_\pi\delta_o\lambda+\kappa_o\left(\kappa_x-\delta_o\sigma\left(-1+\bar{b}_\varphi+\rho\right)\right)\right)}{\delta_o^2\sigma^2+\beta(\lambda+\kappa_o(\kappa_x+\delta_o\sigma))} \end{bmatrix}
\end{aligned}$$

The characteristic polynomial $DT_c - I$ has the form of $P(\xi) = \xi^4 + A_3\xi^3 + A_2\xi^2 + A_1\xi + A_0$. It can be shown that two of roots (e.g. ξ_1 and ξ_2) are equal to -1. The rest of roots are given by the following expressions,

$$\begin{aligned}
\xi_3 &= \frac{-\beta\kappa_o\kappa_x - \beta\lambda - \bar{b}_\pi\beta^2\lambda - \bar{b}_\pi\beta\delta_o\lambda - \beta\delta_o\kappa_o\sigma\left(1-\bar{b}_\varphi\right) - \delta_o^2\sigma^2}{\delta_o^2\sigma^2 + \beta\left(\lambda + \kappa_o\left(\kappa_x + \delta_o\sigma\right)\right)} \\
\xi_4 &= \frac{-\beta\left[\kappa^2\eta^2 + (1-\rho\beta)\lambda + \delta_o\lambda(1-\rho)\right] - \delta_o^2\lambda - \beta\lambda\left(\beta+\delta_o\right)\left(1-\bar{b}_\varphi\right) - \beta\delta_o\kappa_o\sigma\left(2-\rho-\bar{b}_\varphi\right) - \delta_o^2\sigma^2}{\delta_o^2\sigma^2 + \beta\left(\lambda + \kappa_o\left(\kappa_x + \delta_o\sigma\right)\right)}
\end{aligned}$$

and are always negative since $0 < \bar{b}_\varphi < 1$, $0 < \bar{b}_\pi$, $0 < \beta < 1$and $0 < \rho < 1$. It follows the $DT_c - I$ is a stable matrix.

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