Abstract

This paper studies the Ramsey optimal monetary policy in the baseline New Keynesian model with capital accumulation and overlapping generations. With balanced-budget fiscal policy, the generational turnover effect reduces the aggregate capital stock, consumption, and social welfare. The traditional trade-off between nominal price stickiness and monopolistic competition becomes more intense, so that the monetary authority seems to be motivated to close the gap between the inefficient steady state and the modified golden rule. Optimal monetary policy obtains a stabilizing rather than an expansionary nature however, because the Ramsey planner is concerned with both current and future generations. Optimal policy becomes countercyclical, but deviations from strict price stability entail a social welfare cost below the lower bound described by Lucas (2003).

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†Department of Economics, University of Athens, 14 Evripidou Street, Athens 105 59, Greece, E-mail: kkatirt@econ.uoa.gr
1 Introduction

The optimal nature of monetary policy constitutes a central research issue within the context of the New Neoclassical Synthesis (NNS). The standard New Keynesian (NK) model with two imperfections, i.e., nominal price stickiness and monopolistic competition, indicates the optimality of price stability due to the divine coincidence property (Blanchard and Gali 2007). The so-called neutral monetary policy (Goodfriend and King 1997, 2001, Goodfriend 2002) may not always be the optimal policy choice in practice, however, as monetary authorities encounter wedges that prevent the inflation and output gap stabilization with inflation targeting policies. To provide a rationale behind the deviation of practical decision making from strict price stability, researchers revised the baseline NK model in many respects, such as the labor market set up and the inclusion of additional nominal or real distortions. All normative analyses assumed family dynasties and retained the Ricardian behavior of individuals, apart from Amato and Laubach (2003) who introduced a rule-of-thumb decision making in the demand or supply side of the model economy.

The present paper extends the normative analyses in the NK framework with non-Ricardian aspects by revisiting the infinite horizon assumption referring to the lifetime expectancy of representative agent. For this purpose, we employ the baseline NK model with overlapping generations à la Yaari (1965) and Blanchard (1985) where individuals have finite lifetimes attributed either to the absence of bequest motives or to their decision shortsightedness. In this framework, the non-Ricardian behavior is not interpreted as a limited participation in capital or bond markets, but rather as a myopia of agents in an otherwise rational decision-making, in the sense that individuals’ decisions concern a shorter time horizon than their lifetime expectancy. This behavior is captured under particular parameterization of the model (Leith and von Thadden 2008, Devereux 2011) so that finite lives introduce a non-Ricardian aspect in the NK framework rather than refer to a strict demographic measure.

While in the perpetual youth approach of the NK framework there is extensive research on the positive implications of fiscal expansion (Andrés et al. 2006, Annicchiarico et al. 2011), the interactions between monetary and fiscal policy, and the determinacy properties of fiscal and monetary policy rules as well, there is almost no attention on the consequences of this kind of modeling specification on monetary policy aspects per se. Researchers take advantage of this non-Ricardian framework to examine those issues, as money seignorage and government debt exercise a direct influence in the aggregate demand due to the collapse of the Ricardian equivalence.

The intention of the present paper is to disentangle the consequences of the finite horizon assumption on monetary policy implications from the rest imperfections of the NK model. For this purpose, the analysis abstains from fiscal policy issues, i.e., we assume a balanced-budget (Ricardian) fiscal policy. In a similar spirit, employing a money-in-utility NK model, Piergallini (2006) considered a balanced budget constraint for the government to make the implications of the finite horizons for the design of monetary policy as transparent as possible. The purpose of the present paper is to provide a bridge between the normative analysis of the Ramsey approach with models of the NK literature where the Ricardian equivalence collapses due to the presence of finite expectancy.

From normative perspective, the research question can be simplified on whether and to what extent the finite horizon assumption alters the traditional nature of optimal monetary policy in the baseline NK model. Insofar as this issue is closely associated with the number and kind of imperfections present in the model economy, we use the standard NK version of the Yaari (1965) and Blanchard (1985) discrete time model, where monopolistic competition and nominal price stickiness co-exist with overlapping generations. This will allow to isolate the consequences of the finite horizon assumption from any other distortion that would render the economy more complicated. We begin with the influence of the finite horizon specification on each imperfection of the model in the long run. Specifically, we test the relation between inflation and average markup at steady-state for alternative degrees of lifetime expectancy.

Then, we look for the separate consequences of finite lives on capital accumulation and the inefficiency of price dispersion. In a business cycle frequency, we derive the Ramsey optimal plan to demand and supply side disturbances and compare the Ramsey dynamics of the model—with and without finite lives—with the dynamic adjustment of the Pareto optimal RBC model, which constitutes the benchmark for monetary policy evaluation by Goodfriend and King (1997), King and Wolman (1999), Goodfriend (2002) and Khan et al. (2003). We complete the normative analysis by deriving and comparing the optimal Taylor rules with those associated with the infinite horizon specification of the same model (Faia 2008b).

The perpetual youth literature stresses that finite horizons alter the aggregate demand side of the economy. The contemporaneous non-human wealth, accumulated by households, enters the aggregate Euler equation and influences the intertemporal allocation of aggregate consumption. The so-called generational turnover effect (Ascari and Rankin 2010) implies in turn that the dynamic IS equation sustains amplitude effects attributed to changes in the financial wealth of households. Hence, we expect the assumption of finite lives may be a source of additional volatility in the demand side of the economy to exogenous driving shocks or, in contrast, it may implicitly generate endogenous stabilization forces during the adjustment process over the business cycle. Thus, the finite lives within the NK framework may constitute an additional motive for stabilization policy or may reduce the necessity for monetary policy intervention as an endogenous stabilization aspect of the model.

Insofar as the wealth effects on the demand-side of the economy are absent in the infinite horizon approach of the Synthesis, the above research question can be reduced on whether the contemporaneous non-human wealth accumulated to households can augment the traditional trade-off between the standard two imperfections of the NK economy. The non-human wealth may have consequences on the normative implications of the model if it transforms the standard trade-off between imperfect competition and nominal price stickiness to a tripodal one. Monopolistic competition calls for a deviation from price stability, for it implies an inefficiently low level of output. In contrast, nominal price stickiness necessitates price stability, because inflation volatility generates a welfare detrimental price dispersion, which leads to inefficient allocation of resources. In the standard NK model, the simple trade-off between those two inefficiencies is largely resolved in favor of price stability. If the finite horizon assumption influences the trade-off between those imperfections and indeed introduces an additional distortion, the baseline optimal monetary policy will be affected accordingly.

The finite horizon assumption reduces the capital-labor ratio below the modified golden rule level in the stylized Ramsey-Cass-Koopmans growth model, as Blanchard (1985) originally described. In the NK environment, this result indicates that the steady-state equilibrium of the economy becomes inefficiently low, which in turn calls for a deviation from price stability. The higher is the probability of death parameter, the higher is the gap between the modified golden rule and the competitive equilibrium steady-state. In other words, the more intense is the generational turnover effect, the stronger is the motivation for monetary authority to undertake active monetary policy. This is revealed by the influence of the finite horizons on the relation between the average markup and the inflation rate. The average markup remains a convex function of the inflation rate, as in the baseline NK model (King and Wolman 1999, Goodfriend 2002), but agents’ myopia affects the relation quantitatively. Under particular parameterization, finite lives magnify the monopolistic competition distortion and entail a higher steady-state inflation rate for average markup minimization. In contrast, the finite horizon assumption has no consequence on the price dispersion measure and the optimal price determined by price-setters. Hence, the OLG specification has no influence on the distortion of nominal price stickiness which always calls for price stability. Overall, the monetary authority encounters a magnified trade-off between price stickiness and an inefficiently low output. The Ramsey optimal steady-state, however, is associated with zero inflation rate as the monopolistic competition distortion, intensified by agents’ shortsightedness, can only be eliminated in the long run with fiscal policy measures, i.e., fiscal subsidies on production.

Over the business cycle, the Ramsey optimal plan indicates that the monetary authority with infinite planning
horizon deviates from strict inflation targeting to stabilize rather than augment the real economy. In the baseline NK model with endogenous capital accumulation and family dynasties, the Ramsey planner undertakes procyclical monetary policy in response to technology improvements by increasing the inflation rate, reducing the average markup, and expanding the aggregate demand over the business cycle (see Faia (2008b)). The procyclical policy reveals the concern of monetary authority to close the gap between the inefficiently low competitive equilibrium allocation and the modified golden rule. Faia (2008b) described the higher is the monopolistic competition inefficiency, the higher is the degree of procyclical policy over the business cycle. In the finite horizon specification, the Ramsey optimal plan delivers opposite results. The optimal inflation response becomes countercyclical to technology improvements, so that the policymaker undertakes a stabilizing rather than an expansionary monetary policy.

It seems the intuition behind this result comes from the policymaker’s concern for both current and future generations. Procyclical monetary policy that expands the business cycle to temporarily technological improvements boosts aggregate demand, augments the capital stock, and increases the welfare of currently alive generations. The generational turnover effect, however, indicates that households’ consumption is positively influenced by the contemporaneous non-human wealth. Expectations for increased financial wealth generates a positive income effect on current consumption that crowds out private investment. This in turn undermines the future productive capabilities of the macro-economy, and the welfare for next generations. Apart from monopoly power and price stickiness, the policymaker encounters the non zero financial wealth in the demand side of the economy that threatens the welfare prospects of future and yet unborn generations. The monetary authority stabilizes the macro-economy around the sub-optimal steady-state rather than undertakes expansionary short-run policies that contribute positively to financial no-human wealth of currently alive agents.

The rest of the paper proceeds as follows. Section 2 describes the perpetual youth version of the standard NK model with endogenous capital accumulation. Section 3 provides the calibration of the model. Section 4 presents the normative analysis; it discusses the long-run implications of finite lives on each distortion of the NK framework and compares the Ramsey optimal plan with the optimal dynamic adjustment of the infinite horizon counterpart and the dynamics of the Pareto optimal RBC model. Section 5 evaluates the welfare performance of simple and implementable Taylor-type rules, and section 6 concludes.

2 The Model

We use a discrete time approach (Frenkel and Razin 1986) of the perpetual youth model á la Yaari (1965) and Blanchard (1985) which incorporates the standard inefficiencies of the baseline NK framework: monopolistic competition and nominal price stickiness. We introduce capital accumulation to generate non-human wealth effects in the demand side, as we abstain from fiscal policy considerations and money seignorage.

2.1 Representative Agent

The representative agent \( j \in [0, 1] \) that belongs to generation born at time \( s \leq 0 \) derives utility from consumption \( c_{s,t}(j) \) and disutility from hours of work \( h_{s,t}(j) \). The agent maximizes her lifetime utility subject to a sequence of flow budget constraints, the capital accumulation equation, and a transversality condition that prevents no-Ponzi games. The lifetime utility function is log-separable on its arguments\(^2\), i.e.

\(^2\)The standard log-separable momentary utility function was also employed by similar studies within the perpetual youth framework such as Smets and Wouters (2002), Leith and von Thadden (2008). Also the preferences of Andrés et al. (2006), Piergallini (2006), Leith and Wren-Lewis (2006), Chadha and Nolan (2007), Ganelli (2007), Leith and Wren-Lewis (2008), Annicchiarico et al. (2008) with no real money balances are identical with the utility specification of the present analysis. The log-separable utility function is also employed by Faia (2008b), which is considered the benchmark for the present analysis.
where $E_0$ denotes the rational expectations operator, $\beta(0,1)$ is the subjective discount factor, $\pi_{t}\epsilon(0,1)$ is the probability of death parameter, and $c_{s,t}(j), h_{s,t}(j)$ denote consumption spending and hours of work, respectively.

The representative agent works $h_{s,t}(j)$ hours for a real wage rate $w_t$, earns real dividend payments $d_{s,t}(j)$ from the monopolistic sector due to firm ownership, and pays lump-sum taxes $\tau_{s,t}(j)$ to the government. The agent invests in capital stock $k_{s,t}(j)$ and in contingent bonds $b_{s,t}(j)$, which pay the rental price $r^k_t$ and a gross nominal interest rate $r_t$, respectively. The time period budget constraint of representative agent of cohort $s \leq 0$ is defined by

\[
(1 - \pi_d)k_{s,t+1}(j) = (1 - \delta)k_{s,t}(j) + x_{s,t}(j)
\]

where parameter $\delta t(0,1)$ denotes the depreciation rate.

The first order conditions of representative agent’s problem belonging to cohort $s \leq 0$ are given by,

\[
1 = \beta r_t E_t \left[ \frac{c_{s,t}(j)}{c_{s,t+1}(j)} \left( \frac{P_t}{P_{t+1}} \right) \right] 
\]

\[
1 = \beta E_t \left[ \frac{c_{s,t}(j)}{c_{s,t+1}(j)} (1 + r^k_{t+1} - \delta) \right] 
\]

\[
\chi \frac{c_{s,t}(j)}{1 - h_{s,t}(j)} = w_t 
\]

along with the time period budget constraint (2.1) with strict equality, the law of motion for capital (2.2), and the transversality condition,

\[
\lim_{T \to \infty} E_t (1 - \pi_d)^{T-t} \Lambda_{s,T}(s,j) = 0 
\]

Variables $\Lambda_{s,T}(s,j)$ and $\alpha_{s,t}(j)$ denote the stochastic discount factor and the non-human wealth of the representative agent $j\epsilon[0,1]$ belonging to cohort $s \leq 0$, defined by,

\[
\Lambda_{s,T}(s,j) = \beta^{T-1} (c_{s,t}(j)/c_{s,T}(j)) 
\]

and

\[
\alpha_{s,t}(j) = (1 + r^k_t - \delta) k_{s,t}(j) + \frac{b_{s,t}(j)}{P_t} 
\]

Conditions (2.3) and (2.4) are the Euler equations for bonds and capital stock, which deliver the no-arbitrage condition

\[
r_t^k = E_t \left[ (1 + r^k_{t+1} - \delta) \right] 
\]

and condition (2.5) describes the labor supply of individual agent. By employing the definition

\[
\text{We may set aside the criticism exercised by Casares and McCallum (2000) on models that undertake this simplified specification of capital formation, because we abstain from investment adjustment costs for analytical tractability and comparison purposes with Faia (2008b).} 
\]
we aggregate the optimality conditions of the demand size across cohorts \( s \leq 0 \) and obtain the flow and intertemporal budget constraints

\[
x_t = \sum_{s=-\infty}^{t} \left( \int_{0}^{\sigma_{d}(1-\pi_d)^{s-t}} x_{s,t}(j) \, dj \right)
\]

\[
c_t + E_t [\Lambda_{t,t+1} \alpha_{t+1}] = \omega_t + \alpha_t
\]  

(2.7)

and

\[
c_t = \zeta (\alpha_t + H_t) \quad \text{with} \quad \zeta = 1 - \beta(1-\pi_d)
\]  

(2.8)

where \( \alpha_t \) and \( H_t \) denote non-human (financial) and human wealth in aggregate terms, defined as follows:

\[
\alpha_t = \frac{b_t}{P_t} + \left( 1 + r_k^t - \delta \right) k_t
\]  

(2.9)

\[
H_t = E_t \sum_{T=t}^{\infty} (1-\pi_d)^{T-t} \Lambda_{t,T} \omega_T \quad \text{with} \quad \omega_T = w_T h_T + d_T - \tau_T
\]  

(2.10)

By combining the flow and intertemporal budget constraints (2.7) and (2.8) with the definitions of financial and human wealth (2.9) and (2.10), we obtain the dynamic equation for aggregate consumption, given by,

\[
c_t = \left( \frac{1}{\beta r_k^t} \right) E_t [\pi_{t+1} c_{t+1}] + \left( \frac{1}{r_k^t} \right) \xi E_t [\pi_{t+1} \alpha_{t+1}]
\]  

(2.11)

where \( \xi = \pi_d [1 - \beta(1-\pi_d)] / (\beta (1-\pi_d)) \). Condition (2.11) replaces the standard Euler equation of the infinite horizon model and indicates the intertemporal allocation of consumption depends not only on the opportunity cost of spending, i.e., the nominal interest rate, but also on the non-human wealth of agents. If the probability of death becomes zero (\( \pi_d = 0 \)), agents obtain an infinite life expectancy and the dynamic equation of consumption becomes the standard Euler equation.

The aggregate labor supply and capital accumulation equation across cohorts \( s \leq 0 \) are given by

\[
\chi \left( \frac{c_t}{1-h_t} \right) = w_t
\]  

(2.12)

\[
k_{t+1} = (1 - \delta) k_t + x_t
\]  

(2.13)

### 2.2 Firms

The supply side of the economy consists of intermediate good-producing firms and final good firms. Intermediate firms \( i \in [0, 1] \) produce and sell intermediate product varieties \( y_t(i) \) to final good-producing firms at a price formed in monopolistically competitive product markets with nominal price stickiness á la Calvo (1983). Final good firms transform the product varieties \( y_t(i) \) to homogeneous final good \( y_t \) which is sold accordingly to households in a perfectly competitive market.

#### 2.2.1 Final goods firms

The representative final good firm transforms \( y_t(i) \) units of the intermediate good \( i \in [0, 1] \) into homogeneous final good \( y_t \) according to the constant returns to scale production technology defined by Dixit and Stiglitz (1977) and given by,

\[
y_t \leq \left[ \int_{0}^{1} y_t(i)^{\frac{1}{2+\frac{1}{2}}} \, di \right]^{\frac{2}{2+\frac{1}{2}}}
\]
where \( \varepsilon > 1 \) denotes the elasticity of substitution among intermediate products \( i \in [0,1] \). Operating in a perfectly competitive market, the final good firm takes the price of the homogenous output as given and minimizes its production cost. The cost minimization problem subject to the Dixit and Stiglitz (1977) aggregator delivers the demand for each intermediate good \( i \in [0,1] \) by the final good firm,

\[
y_t(i) = \left[ \frac{P_t(i)}{P_t} \right]^{-\varepsilon} y_t
\]

for all \( i \in [0,1] \) and \( t \geq 0 \). Due to perfect competition in the final good market, the zero profit condition delivers the following aggregate price index:

\[
P_t = \left( \int_0^1 P_t(i)^{1-\varepsilon} \, di \right)^{\frac{1}{1-\varepsilon}}
\]

where \( P_t \) and \( P_t(i) \) denote the aggregate price level and the price of each intermediate good \( i \in [0,1] \), respectively.

### 2.2.2 Intermediate-goods firms

Each intermediate firm \( i \in [0,1] \) hires labor and rents capital to produce and sell the product variety \( y_t(i) \) at price \( P_t(i) \) in monopolistically competitive market. Nominal price rigidities are introduced by assuming that prices are set at a staggered fashion à la Calvo (1983).

First, each intermediate firm \( i \in [0,1] \) chooses capital and labor to minimize the total cost of production by taking the real wage and the rental price of capital as given. Namely, each firm minimizes

\[
\min_{\{h_t(i),k_t(i)\}} \text{tc}_t(i) = w_t h_t(i) + r_t^k k_t(i)
\]

subject to a standard, constant returns to scale production technology

\[
y_t(i) = z_t k(i)^a h_t(i)^{1-a}
\]

where \( k_t(i) \) and \( h_t(i) \) denote the capital stock and hours of labor employed by firm \( i \in [0,1] \), respectively. Variable \( z_t \) gives the exogenous neutral technology, which follows a stationary AR(1) process of the form,

\[
\ln(z_t) = \rho z \ln(z_{t-1}) + \varepsilon_{z,t}
\]

with \( |\rho_z| < 1 \) and \( \varepsilon_{z,t} \sim N(0, \sigma_z^2) \). The first order necessary conditions give the demand for labor and capital,

\[
w_t = z_t (1 - a) \, m_c(i) \left[ \frac{k_t(i)}{h_t(i)} \right]^a \tag{2.16}
\]

\[
r_t^k = z_t a \, m_c(i) \left[ \frac{h_t(i)}{k_t(i)} \right]^{1-a} \tag{2.17}
\]

where \( m_c(i) \) denotes the marginal cost of firm \( i \in [0,1] \).

The market power \( \varepsilon > 1 \) enables the intermediate good-producing firm to choose a price \( P_t(i) \) for its differentiated product. The pricing decision problem is subject to nominal price rigidities à la Calvo (1983). According to Calvo (1983) price setting, each firm resets its price \( P_t(i) \) with probability \( 1 - \theta \), which is independent from the time elapsed since the last adjustment and the pricing decisions of the other firms. Hence, in every period \( t \geq 0 \), a fraction \( 1 - \theta \) of firms re-optimizes the price \( P_t(i) \), while the remaining one \( \theta \in (0,1) \) set a price equal to the aggregate price index of the previous period according to the rule \( P_t(i) = \pi^\psi P_{t-1} \), with \( \psi = 0 \) (no price indexation condition).

\[^4\]According to Schmitt-Grohé and Uribe (2007b, pg. 1709), there is available empirical evidence which supports the no-price indexation for non-optimized prices.
The intermediate good producing firm chooses the price $P_t(i)$ to maximize the expected sum of (nominal) future profits, discounted by the pricing kernel $(\Lambda_{t,T})$, and the probability that the optimal price will remain fixed for $1/(1 - \theta)$ future periods,

$$\max_{\{P_t(i)\}} E_t \sum_{T=t}^{\infty} \theta^{T-t} \Lambda_{t,T} \left[ \left( \frac{P_t(i)}{P_t} \right) y_t(i) - mc_t(i) y_t(i) \right]$$

subject to the demand for the intermediate product $y_t(i)$, i.e. condition (2.14), by the final good firm. The first order necessary condition of the pricing decision problem is given by,

$$E_t \sum_{T=t}^{\infty} \theta^{T-t} \Lambda_{t,T} \left[ \left( \frac{P_t(i)}{P_T} \right)^{-1-\varepsilon} y_T \left\{ mc_T - \left( \frac{\varepsilon - 1}{\varepsilon} \right) \frac{P_t(i)}{P_T} \right\} = 0$$

The pricing decision condition (2.18) shows that firm $i \epsilon [0,1]$ chooses a price equal to a markup over a weighted sum of current and expected nominal marginal costs.

We follow Schmitt-Grohé and Uribe (2004b, 2006, 2007a)\footnote{In the context of normative analyses, Schmitt-Grohé and Uribe (2004b, 2006, 2007a) point out the necessity to retain the non-linear nature of the competitive equilibrium conditions, especially of the pricing decision one.} and re-write condition (2.18) in recursive form by defining the auxiliary variables $x_t^1$ and $x_t^2$ as the expected discounted future costs and revenues of firms. The pricing decision condition (2.18) becomes

$$x_t^1 = \left( \frac{\varepsilon - 1}{\varepsilon} \right) x_t^2$$

where

$$x_t^1 = (p_t^*)^{-\varepsilon - 1} y_t mc_t + \theta E_t \left( \frac{\pi_{t+1}}{r_t} \right) \left( \frac{p_t^*}{p_t^{t+1}} \right)^{-\varepsilon} \pi_{t+1} x_{t+1}^1$$

and

$$x_t^2 = (p_t^*)^{-\varepsilon} y_t + \theta E_t \left( \frac{\pi_{t+1}}{r_t} \right) \left( \frac{p_t^*}{p_t^{t+1}} \right)^{-\varepsilon} \pi_{t+1} x_{t+1}^2$$

and $p_t^* = P_t^*/P_t$ is the optimal price set by firms relatively to the aggregate price index $P_t$. The price index definition (2.15) along with the no-price indexation rule $P_t(i) = P_{t-1}$ (Schmitt-Grohé and Uribe 2007b) delivers the law of motion for the aggregate price level:

$$1 = \theta (\pi_t)^{\varepsilon - 1} + (1 - \theta) (p_t^*)^{1 - \varepsilon}$$

### 2.3 Aggregation and market clearing

In symmetric equilibrium all firms take identical decisions and all product markets clear. Intermediate goods firms $i \epsilon [0,1]$ satisfy the demand (2.14) for product varieties $y_t(i)$ by the final goods firms, i.e.

$$y_t = \left( \frac{1}{s_t} \right) z_t k_t^a h_t^{1-a}$$

where $h_t = \int_0^1 h_t(i) \, di$ and $s_t = \int_0^1 \left[ \frac{P_t(i)}{P_t} \right]^{-\varepsilon} \, di \geq 1$ denote the aggregate labor and the price dispersion measure. Following Schmitt-Grohé and Uribe (2007a), the price dispersion definition can be written in recursive form as

$$s_t = (1 - \theta) (p_t^*)^{-\varepsilon} + \theta (\pi_t)^{\varepsilon} s_{t-1}$$

Price dispersion captures the deadweight losses of the inefficient allocation of resources attributed to the nominal price stickiness imperfection (see Woodford (2003)). In flexible-price (natural) equilibrium, there is no price dispersion and thus no misallocation of resources, i.e., $s_t = 1$. 


We abstain from fiscal policy considerations by assuming zero net supply of bonds in equilibrium, i.e., \( b_t = b_{t-1} = 0 \). In this case, government expenditure \( (g_t) \) is financed by lump sum taxes \( (\tau_t) \) paid by agents, and the government budget constraint is always balanced, i.e., \( g_t = \tau_t \) for every period \( t \geq 0 \). The aggregate resource constraint of the model economy is given by,

\[
y_t = c_t + x_t + g_t
\]

(2.25)

where government spending \( g_t \) follows an exogenous stationary AR(1) process

\[
\ln (g_t) = (1 - \rho_r) \ln (\overline{g}) + \rho_r \ln (g_{t-1}) + \varepsilon_{g,t}
\]

(2.26)

with \(|\rho_g| < 1\) and \( \varepsilon_{g,t} \sim N \left(0, \sigma_g^2\right) \).

2.4 Monetary policy

Monetary policy takes the form of a simple, contemporaneous Taylor-type rule of the form,

\[
r_t = \left(\frac{r_{t-1}}{\overline{r}}\right)^{\alpha_r} \left(\frac{\pi_t}{\overline{\pi}}\right)^{\alpha_\pi} \left(\frac{y_t}{\overline{y}}\right)^{\alpha_y}
\]

(2.27)

where \( \{\overline{y}, \overline{\pi}\} \) denote the Ramsey steady-state levels of output and inflation rate, i.e., the measures associated with the steady-state that maximizes the social welfare definition of the Ramsey planner. The inflation and output response coefficients \( \{\alpha_\pi, \alpha_y\} \) lie within the intuitive plausible intervals \( \alpha_\pi \in (1, 3) \) and \( \alpha_y \in [-0.5, 0.5] \), which guarantee the determinacy of the rational expectations equilibrium. The interest rate inertia parameter \( (\alpha_r) \) lies within \( \alpha_r \in [0, 2] \), so that the rule can be non-inertial \((\alpha_r = 0)\), inertial \((0 < \alpha_r < 1)\), or superinertial \((1 \leq \alpha_r \leq 2)\). The above interest rate rule is simple and implementable in the notion described by Schmitt-Grohé and Uribe (2007b). That is, it involves few and readily available macroeconomic measures; it guarantees the local uniqueness of the rational expectations equilibrium under plausible parameterization of its policy response coefficients, and it may prevent non-negative equilibrium dynamics for the nominal interest rate \((r_t > 1)\).

The determinacy properties of the rule (2.27) remain robust both in the baseline NK model with family dynasties and in the perpetual youth counterpart. The determinacy areas associated with alternative specifications of the contemporaneous Taylor rule (2.27) are depicted in figure 7.

3 Calibration

We calibrate the model in quarterly frequency by setting empirically plausible values for the deep parameters gathered in table 1.

We consider a distorted steady-state with positive inflation rate equal to 4.2% annually (Schmitt-Grohé and Uribe 2007b). The subjective discount factor is set equal to the standard value of 0.99. The weight on the disutility of labor parameter (i.e., the free nuisance parameter \( \chi > 0 \)) is set so that the representative agent spends 0.3 of the total time endowment to labor activities (Faia 2008b).

The elasticity of substitution among intermediate products and the Calvo (1983) price rigidity parameter are set equal to \( \varepsilon = 6 \) and \( \theta = 0.75 \). The price elasticity \( \varepsilon = 6 \) implies an inefficient static markup of prices over marginal cost equal to \( \mu^p = 1.2 \). The nominal price stickiness parameter \( \theta = 0.75 \) implies that prices remain sticky for four quarters. These values are consistent with the estimates reported by Smets and Wouters (2007), Justiniano et al. (2010). Also, the share of capital in the production function and the quarterly depreciation rate are set equal to \( \alpha = 0.35 \), and \( \delta = 0.025 \) (Faia 2008b).

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6A determinacy analysis of the Taylor rule in a discrete time version of the NK model with endogenous capital accumulation is provided by Carlstrom and Fuerst (2005).
In the baseline calibration, we interpret the probability of death as a demographic parameter equal to \( \pi_d = 0.00357 \), which implies an average lifetime expectancy equal to 70 years (Chadha and Nolan 2007). As the probability of death determines the sensitivity of aggregate consumption spending to non-human wealth effects, we conduct a sensitivity analysis by using the values calibrated by studies within the perpetual youth framework. For example, in quarterly frequency, Smets and Wouters (2002) and Annicchiarico et al. (2011) set the probability of death equal to 0.01 and 0.015, respectively. According to Devereux (2011), the upper bound of the probability is equal to 0.05 or 0.2 annually, which implies a 5 year horizon for consumers’ decision making. Devereux (2011) rationalizes this value by citing the empirical study of Bayoumi and Sgherri (2008), who estimate the deep parameters of the Blanchard-Yaari framework directly from a reduced form consumption function. Devereux (2011, pg.400) interprets "...the probability of death in a broader manner than that implied by straightforward demographic data". Under this parameterization, the probability of death is rather interpreted as the degree of agents’ shortsightedness or myopia rather than a strict demographic measure (see also Leith and von Thadden (2008, pg.280)). Table 2 reports the alternative values of this parameter calibrated by the perpetual youth literature and employed in sensitivity analysis.

The exogenous AR(1) processes associated with technology \((z_t)\) and government spending \((g_t)\) variables are parameterized in conformity with the available empirical evidence, so that \((\rho_z, \sigma_z) = (0.95, 0.008)\) and \((\rho_g, \sigma_g) = (0.9, 0.0074)\) (see Faia (2008a,b, 2009)). Also, the steady-state value of the neutral technology variable is equal to unity \((\bar{z} = 1)\), and the government to output ratio is equal to \(\bar{g}/\bar{y} = 0.25\) (see Faia (2008a,b, 2009)).

The above calibration gives plausible values for the main macro-ratios, reported in table 3.

4 Ramsey Optimal Plan

The Ramsey optimal policy is derived by maximizing the lifetime utility of the representative agent subject to the complete set of the competitive equilibrium conditions. In the finite horizon version of the model, the lifetime utility function is associated with a representative agent belonging to cohort \( s \leq 0 \), so that the problem for the social planner becomes rather complicated (Calvo and Obstfeld 1988). In the OLG specification of the NK model, the maximization of the social welfare obtains an intratemporal and an intertemporal dimension. First, the policymaker has to decide on the optimal allocation of consumption and labor supply across generations \( s \leq 0 \) for every period \( t > 0 \). Second, the social planner has to determine the optimal allocation of the same arguments across time, as it is the case in the infinite horizon model.

We follow Leith et al. (2011) and abstain from the intratemporal aspect of the social welfare maximization, i.e., the distribution of aggregate per capita variables across cohorts. For that purpose, we use the log-separable utility function that describes the representative agent’s preferences, where the utility arguments refer to the per capita aggregate variables rather than to consumption and labor supply referring to a specific cohort \( s \leq 0 \). Hence, the welfare maximization problem of the policymaker is to choose a sequence of the endogenous, aggregate variables of the model \( \{ y_t, c_t, n_t, u_t, w_t, v_t, x_t^1, x_t^2, p_t^1, s_{t+1}, m_t, \pi_t, r_t \}_{t=0}^{\infty} \) and the Lagrange multipliers associated with the competitive equilibrium conditions to maximize the social welfare function defined by

\[
W_t \equiv E_0 \sum_{t=0}^{\infty} \rho^t \left[ \ln(c_t) + \chi \ln(1 - n_t) \right]
\]

\footnote{Leith et al. (2011) define a welfare function which is identical with the representative agents’ one, where cohort variables are replaced with the per capita aggregate ones. They employ this welfare metric because they abstract from the intratemporal, inter-generational aspects of the social welfare maximization problem. The focus on the intertemporal aspect of the welfare problem is justified by the authors’ interest to study the macroeconomic effects of fiscal adjustment rather than the consequences of fiscal policy on each cohort \( s \leq 0 \).}
subject to the complete set of the competitive equilibrium conditions of the model economy, i.e. (2.11), (2.13), (2.12), (2.6), (2.17), (2.16), (2.24), (2.25), (2.19), (2.20), (2.21), and (2.22), the non-negativity constraint for the policy instrument \( r_t \geq 1 \), and the exogenous processes (2.2.2), (2.26) pertaining to the technology and government spending driving processes \( \{ z_t, g_t \} \). The discount factor associated with the social welfare function is set equal to the subjective discount factor of agents, i.e., we set \( \rho = \beta \), because the social planner has an infinite lifetime expectancy.

The optimal monetary policy is described by the policy functions of the above welfare maximization problem. As we abstain from fiscal policy issues, the inflation rate is the available policy instrument for policymaker. Optimal monetary policy is the process of the nominal interest rate \( \{ r_t \} \) associated with the competitive equilibrium that maximizes the above social welfare definition. We take second order approximations of the first order conditions of the above problem around the distorted Ramsey steady state, using the perturbation method described by Schmitt-Grohé and Uribe (2004c).

4.1 Long Run Policy

In steady-state analysis, we study the impact of the perpetual youth assumption on each distortion of the NK framework. Then we derive the Ramsey steady-state where all imperfections of the model are taken into account. This will reveal to what extent the finite horizons’ assumption influences the traditional trade-off of the baseline NK model and in turn the long run decisions of the Ramsey planner.

4.1.1 Finite Lives and Monopolistic Competition

We ignore at the moment the nominal price stickiness and focus on the influence of the finite horizon assumption on the core of the NK framework, i.e., the RBC model with monopolistic competition. Figure 1 plots the phase diagram of Blanchard (1985), where the aggregate resource constraint and the consumption loci are derived under the assumption of perfectly flexible prices and monopolistic competition. The intersection of the perfectly inelastic consumption locus with the aggregate resource constraint gives the competitive equilibrium allocation of the flexible price economy with monopolistic competition and family dynasties. This allocation is Pareto suboptimal, as the monopolistic competition distortion reduces aggregate output below the efficient (or the modified golden rule) level due to positive markup. In the NK framework, the monopolistic competition inefficiency calls for an active monetary policy, i.e., a deviation from strict price stability, as initially described by Schmitt-Grohé and Uribe (2004a).

The convex curves in the phase diagram represent the consumption loci of the perpetual youth version of the same model. By increasing the probability of death, the convex consumption locus shifts leftwards, and the natural equilibrium corresponds to lower capital stock, consumption, and output. Each competitive equilibrium is associated with the same degree of monopolistic competition which renders the allocation inefficient.

The phase diagram shows the gap between the monopolistically distorted allocation of the infinite horizon model with the natural equilibrium where agents have finite lives is positively related with the probability of death. As the distance between the natural allocation of the infinite horizon model and the modified golden rule is constant and solely determined by the market power of firms, the probability of death increases the gap between the competitive equilibrium of the perpetual youth model with the efficient allocation as well. The higher is the probability of death, the higher is the gap between the efficient allocation with the natural equilibrium of the perpetual youth model. If the probability of death is calibrated as a demographic parameter, the convex consumption locus is close to the inelastic one, so that the gap between the competitive allocations is rather small. If the probability of death is calibrated to reflect a non-Ricardian or myopic aspect of individuals, the gap between the finite and infinite horizon allocations becomes more intense.

This is also noticeable by figure 2, which compares the steady-state equilibrium of three alternative versions of the RBC model. The solid lines plot the steady-state of the perpetual youth version of the RBC economy with
monopolistic competition, i.e., the natural allocation of the NK framework for alternative values of the probability of death. The dots and star-points represent the steady-states of the infinite-horizon RBC model (called, the core RBC), and the Pareto optimal counterpart, i.e., the RBC economy with perfectly competitive goods market, respectively. The distance between dots and star-points indicates the inefficiency of the monopolistic competition distortion, which reduces the real macro-aggregates (output, consumption, investment, capital) below the Pareto optimal level. The finite horizon or agents’ shortsightedness assumption intensifies this inefficiency. The solid lines of the main macro-variables are linearly and downward sloping, so that shorter time horizons increase the gap between the competitive and the Pareto optimal steady-state.

This entails in turn that agents’ myopia contributes to monopolistic competition consequences on real economy. As the monopolistic competition inefficiency induces policymakers to undertake active monetary policy (Schmitt-Grohé and Uribe 2004a, Faia 2008b), agents’ shortsightedness may constitute an additional motive for policymakers to deviate from neutral monetary policy and close the gap with the efficient allocation. This is revealed by the left panel of figure 3, which plots the relation between the average markup and the inflation rate, for a sensitivity analysis with respect to the probability of death. In the NK model, the average markup differs from the static one due to sticky prices. In steady-state, the average markup becomes a convex function of the inflation rate, which reveals the ability of the monetary authority to influence monopolistic competition inefficiency by employing the inflation rate as a policy instrument (see King and Wolman (1999) and Goodfriend and King (1997, 2001)). The sensitivity analysis indicates there are quantitative effects of agents’ shortsightedness on the average markup function: the probability of death shifts the convex relation downwards and rightwards. In the infinite horizon NK model, the average markup is minimized for close to zero inflation rate. In the perpetual youth version of the NK model, the probability of death shifts the convex relation rightwards, so that the average markup is minimized for higher inflation rate. If the probability of death is interpreted as a strict demographic parameter, there is no difference with the infinite horizon approach: the average markup is minimized for almost zero inflation rate, as the solid and the star-solid convex curves almost coincide. If the probability of death increases, the average markup is minimized for a positive and non trivial gross inflation rate.

According to table 4, the average markup is minimized for 0.4% annual inflation rate in the standard NK model with family dynasties, while for 1.61% (in annual frequency) in the perpetual youth counterpart with myopic agents, i.e., for $\pi_d = 0.05$ (Devereux 2011). This result strengthens the above intuition: under particular parameterization, in the perpetual youth version of the standard NK model, agents’ shortsightedness magnifies the effects of monopolistic competition on real economy, as it reduces the competitive equilibrium allocation below the distorted one with infinite horizons. This in turn calls for deviations from neutral monetary policy.

4.1.2 Finite Lives and Price Stickiness

The right panel of 3 depicts the optimal price set by intermediate goods firms relative to the aggregate price index and the price dispersion measure, as functions of the (gross) inflation rate, for alternative parameterizations of the probability of death. The panel shows the non Ricardian aspect of finite-lived agents has no consequence on the nominal price rigidity. This distortion is irrelevant to agents’ shortsightedness and always calls for strict price stability. The price dispersion attributed to sticky prices is always eliminated for zero inflation rate. The minimum of the convex price dispersion curve is unity and is obtained for a gross inflation rate equal to one. Accordingly, for each value of the probability of death, the zero inflation rate corresponds to an optimal relative price charged by firms equal to unity.

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8The average markup is the inverse of the real marginal cost, and it is determined by the optimal price equation (pricing decision condition) along with the aggregate price index.
4.1.3 Ramsey Steady-State

The finite horizon assumption has no consequence on nominal price stickiness but, under particular calibration, magnifies the consequences of monopolistic competition inefficiency on real economy and calls for an active monetary policy. The Ramsey steady-state however is associated with zero inflation rate for any parameterization of the probability of death. Independently of the degree of agents’ shortsightedness and, hence, of the gap between the competitive allocation with the efficient one, neutral monetary policy remains optimal in the long run.

Figure 4 plots the competitive (solid line) and the Ramsey (dashed line) steady-state of selected macro-variables for alternative parameterizations of the probability of death. This figure is in line with the phase diagram of figure 1 and the steady-state analysis of the RBC paradigm plotted in figure 2: the higher is the probability of death, the lower is the real allocation of the model. Each relation between the real macro-variables (output, consumption, investment, and capital) and the probability of death is linearly negative. From normative perspective, the difference between the competitive and the distorted Ramsey steady-state appears in terms of the gross inflation rate: for each value of the probability of death, there is a constant gap between the Ramsey optimal and the competitive equilibrium inflation rate. The policymaker eliminates the distortion of price dispersion by setting the inflation rate equal to zero. Hence, the Ramsey planner chooses a neutral monetary policy even if the consequences of monopolistic competition inefficiency are magnified due to agents’ shortsightedness: the Ramsey steady-state value of inflation rate is always zero while there is almost no difference between the distorted Ramsey allocation with the competitive equilibrium one in terms of real macro-variables. The difference between the Ramsey and the competitive equilibrium inflation rate explains the constant gap between the nominal interest rate under the two allocations. The nominal interest rate is linearly upward sloping with respect to the probability of death, because higher values of the probability parameter are associated with higher real interest rate.

Overall, the Ramsey planner cannot influence the allocation of the real economy in the long run with monetary policy tools. Agents’ shortsightedness magnify the consequences of monopolistic competition distortion which can be resolved in the long-run with fiscal policy measures, i.e., fiscal subsidies on production, as pointed out by Rotemberg and Woodford (1997), Woodford (2003, 2010). The inefficiency attributed to agents’ myopia should call for fiscal subsidies higher than the inverse of the net static markup ($\tau = 1/\varepsilon$), as the gap between the competitive and the efficient allocation is positively related to the degree of agents’ myopia. The policymaker finds it optimal to set the inflation rate zero in the long run to eliminate resource misallocations attributed to price distortion and increase the end-use value of output. Inflation stability remains optimal in the long-run in the OLG version of the NK model, which is in line with the long-run optimal policy derived under alternative versions of the standard NK model with additional frictions.

4.2 Optimal Adjustment

Over the business cycle, driven by neutral technology and government expenditure shocks, the zero inflation policy becomes suboptimal: under the Ramsey optimal adjustment the inflation rate deviates from steady-state. The sub-optimality of strict inflation targeting over the business cycle was described by Faia (2008b) in the standard NK model with infinite lives. The perpetual youth approach of the NK model replicates this result, but inflation responds in an opposite direction for each exogenous shock.

To evaluate the Ramsey optimal plan, we use as a benchmark the dynamics of the RBC model with flexible prices and perfectly competitive goods markets. According to Goodfriend (2002), inflation targeting constitutes the optimal monetary policy, for it makes the business cycles of the standard NK model to exogenous disturbances resemble the dynamic adjustment of the Pareto optimal RBC economy. Price stability accounts for markup stabilization and makes employment and output respond at least as in the second-best (or core) RBC model with flexible prices and monopolistic competition. By comparing the Ramsey optimal plan with the adjustment of the core RBC model (or the
first-best if we eliminate the monopolistic competition inefficiency), we can observe whether price stability remains optimal and indeed whether optimal monetary policy obtains a procyclical or countercyclical nature.  

4.2.1 Technology Shock

Faia (2008b) described the Ramsey optimal plan within the baseline NK model with family dynasties and capital accumulation. In response to 1% increase of technology shock, there is a small but positive response of inflation rate within a period of around five quarters. The inflation increase is associated with an inverse response of the average markup, which boosts real economic activity. The policymaker uses inflation as a tax on monopolistic profits. Monetary authority becomes procyclical, as it magnifies the business cycle to eliminate within the business cycle frequency the inefficiency of monopolistic competition and close the gap with the efficient allocation. The upper panel of figure 5 shows there is positive increase of inflation rate at around 2 basis points in response to 1% increase of technology shock which is associated with a reduction of the average markup at around 4 basis points within the same period. The positive response of inflation rate explains the gap between the marginal product of labor and the real wage. The neutral technology shock raises the labor productivity and the demand for labor. The marginal product of labor increases on impact at around 81 basis points and follows a hump-shaped response with a peak increase around the fifth quarter. The real wage follows a similar response with an impact increase at around 85 basis points. During the early five quarters, there is a positive gap between the real wage and the marginal product of labor, justified by the impact increase of inflation rate by the Ramsey planner within the same period.

In the perpetual youth approach of the NK model, the Ramsey planner encounters as a competitive equilibrium constraint the dynamic equation for consumption (2.11), which contains the financial, non-human wealth, rather than the standard Euler equation which determines the intertemporal decision making of households belonging to each generation ($s < 0$). According to the dynamic equation for consumption, the contemporaneous non-human wealth affects the current consumption positively. The expected capital stock earnings generate positive wealth effects on current consumption. The technology shock raises the current non-human wealth and aggregate consumption. The non-human wealth adds to consumption and reduces the available resources for investment spending, which undermires future capital stock, the expected productive capacity of the macro-economy and thus the consumption spending of future and yet unborn generations. The Ramsey planner however is concerned with welfare maximization of all households independently of the cohort to which they belong to. A procyclical monetary policy in the finite horizon approach of the model would expand the current aggregate demand, the financial wealth of currently alive generations, and thus their consumption spending, at the cost of future capital stock, consumption of next generations, and hence future social welfare. The Ramsey planner avoids this self-fulfilling relation and stabilizes the economy around the second-best steady-state with monopolistic competition.

In policymaking terminology, a procyclical economic policy magnifies economic fluctuations; by contrast, a countercyclical policy lessens business cycle fluctuations. Hence, a monetary policy that expands the real economy in response to a positive productivity shock is called procyclical or expansionary. A countercyclical policy stabilizes rather than expands the real economy around the steady-state equilibrium.
The dashed and dashed-dotted lines show that optimal monetary policy is countercyclical in the OLG specification of the NK model. The Ramsey planner reduces inflation rate in the short-run to increase the average markup and tax the contemporaneous non-human wealth that adds to currently alive households’ income. Monetary policy obtains a stabilization nature in this case. Within the first 5 quarters, the output response falls short of the Pareto optimal one associated with the RBC paradigm.

4.2.2 Government Spending Shock

In response to government expenditure shock, the Ramsey optimal plan in the baseline NK model with family dynasties gives an infinitesimal but countercyclical inflation response (bottom panel of figure 5). The policymaker increases the average markup to reduce consumption and satisfy the implementability constraint (see Khan et al. (2003), Faia (2008b)). The deviation from price stability, however, is trivial on average: the inflation rate reduces on impact at only around 1 basis point.

In the perpetual youth approach of the same model (bottom panel of figure 6), the Ramsey plan gives a positive response of inflation rate, which entails in turn a procyclical nature of monetary policy. The positive government expenditure shock creates a negative wealth effect on private sector. Households expect that government spending will exhaust future capital stock, and hence the contemporaneous non-human wealth declines. The negative wealth effect overcomes the substitution effect of real wages on labor supply. In response to government spending shock, there is more work and less consumption spending. The economy falls below the second-best steady state and the Ramsey planner finds it optimal to stabilize consumption by lowering the average markup. The reduction of average markup counterbalances the negative effect of government spending on private sector and offsets the negative influence of contemporaneous non-human wealth on consumption.

5 Welfare Analysis

The Ramsey plan determines the optimal allocation of the model both in the long-run and over the business cycle, but provides no feedback about how the monetary authority can implement it. As monetary policy takes the form of simple and implementable Taylor rules, the Ramsey optimal plan could be replicated by an appropriate parameterization of the Taylor rule (2.27) (see Schmitt-Grohé and Uribe (2004b, 2006, 2007a,b), Faia (2008a), Levin et al. (2005)).

Towards this direction, we start with the welfare evaluation of standard (ad hoc) Taylor rules by computing the conditional welfare cost if the associated policy response coefficients vary within intuitively plausible and appropriately specified intervals that guarantee the uniqueness of the rational expectations equilibrium. We take two alternative versions of the Taylor rule (2.27): first, an acyclical rule where inflation rate is the only endogenous policy argument (i.e., \( \alpha_y = 0 \)); second, a cyclical rule with both inflation and output as endogenous policy arguments. Each specification of the feedback rule may be inertial (\( \alpha_r = 0.8 \)) or non-inertial (\( \alpha_r = 0 \)).

Then, we search numerically for the optimal values of the policy response coefficients for the above specifications of the feedback rule (2.27). Namely, we search for the policy coefficients \( \{ \alpha_\pi, \alpha_y \} \) and the interest rate inertia parameter \( \alpha_r \in [0, 2] \) that close the gap between the social welfare associated with the Ramsey optimal plan and the Taylor-type policy (see Schmitt-Grohé and Uribe (2007b)).

To assess the performance of simple Taylor rule in replicating the Ramsey optimal plan, we compute to what extent the social welfare under a Taylor-type policy falls short of the maximum social welfare associated with the Ramsey optimal plan. This measure is expressed by the conditional welfare cost \( (\lambda_c) \), defined as the fraction of

\[\lambda_c = \frac{\lambda_{\text{Ramsey}} - \lambda_{\text{Taylor}}}{\lambda_{\text{Ramsey}}},\]

10The interest rate inertia \( \alpha_r = 0.8 \) is plausible, because it lies within the interval [0.6, 0.9] estimated by Galí and Gertler (2007) in quarterly frequency.
consumption that each representative household is willing to give up to be as well off under the allocation attributed to Taylor-type policy as under the Ramsey optimal one. If we define the conditional social welfare under the Ramsey optimal plan as

\[ W_{0,t}^* = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t^*) \]

where \( c_t^* \) is the Ramsey level of macro-aggregated consumption, the conditional welfare cost (\( \lambda_c \)) associated with the simple Taylor rule satisfies the condition

\[ W_{0,t}^T = E_0 \sum_{t=0}^{\infty} \beta^t u((1 - \lambda_c)c_t^*) \]

where \( W_{0,t}^T \) denotes the conditional social welfare if monetary authority commits to Taylor rule. As the momentary utility function is additively separable and logarithmic in consumption, the solution of the above condition with respect to the conditional welfare cost (\( \lambda_c \)) is given by,

\[ \lambda_c = 1 - \exp\left\{ [1 - \beta] (W_{0,t}^T - W_{0,t}^*) \right\} \]  

(5.1)

We focus on the computation of conditional welfare and thus conditional welfare cost (\( \lambda_c \)) rather than unconditional counterparts to take into account the transitional welfare effects, as explained by Woodford (2002) and Kim et al. (2005) (see also Faia (2008a)). We also use the second order perturbation method described by Schmitt-Grohé and Uribe (2004c) to obtain accurate approximations of the social welfare metric and thus correct welfare rankings of the Taylor rules (Kim and Kim 2003, Schmitt-Grohé and Uribe 2004c).

5.1 Simple Taylor rules

Table 5 reports the conditional welfare costs associated with each specification of the contemporaneous Taylor rule (2.27) in the baseline NK model with family dynasties (Faia 2008b) and the perpetual youth counterpart. Inertial or non-inertial but cyclical Taylor rules (\( \alpha_y = 0.5/4 \)) have better performance in the perpetual youth version of the NK model rather than in the standard framework: the conditional welfare cost of the cyclical rules in the perpetual youth model is always lower than the one computed in the infinite horizon NK model. By contrast, acyclical Taylor rules (\( \alpha_y = 0 \)), with or without interest rate inertia, generate higher welfare losses in the OLG specification of the model than in the standard infinite-horizon approach. This comes as a direct consequence of the difference between the Ramsey optimal plans in the two versions of the otherwise standard NK framework. In the baseline approach (Faia 2008b), optimal monetary policy is procyclical, while in the perpetual youth version, the Ramsey optimal plan generates a countercyclical inflation response which reveals a stabilizing rather than an expansionary nature of optimal monetary policy.

The differences, however, between the welfare losses of the same rule under the two policy frameworks signal only a qualitative meaning, because they remain quantitatively trivial. The gap between the conditional welfare loss of the same feedback rules under the two policy environments lies below 0.01 percent of consumption, which is quite below the threshold value of 0.05 percent described by Lucas (2003).

We also observe that the welfare ranking of the Taylor rules remains identical in both models. Acyclical Taylor rules are associated with conditional welfare costs lower than the welfare losses of the cyclical rules. Strict inflation targeting rules, i.e., feedback rules with significant policy response coefficient on inflation (\( \alpha_\pi = 3.0 \)), perform better that Taylor rules with lower inflation response coefficient \( \alpha_\pi \in [1.5, 3.0] \). Interest rate inertia improves the performance of both cyclical and acyclical Taylor rules.
Those properties appear as a direct consequence of the Ramsey optimal plan. In both versions of the NK model, the deviation of optimal monetary policy from strict inflation targeting is rather low, both in terms of impact response and persistence. The optimal inflation response deviates from steady-state for approximately 5 quarters, while the average duration of the business cycle, attributed to either technology or government expenditure shock, remains above 20 quarters. Hence, monetary policy prescription for strict inflation targeting remains on average optimal.

### 5.2 Sensitivity Analysis

To test the robustness of the above results, we compute the conditional welfare cost if policy response coefficients range within plausible intervals that guarantee the determinacy of the rational expectations equilibrium. Figures 8 - 11 plot the conditional welfare loss surfaces and contour lines as a function of the policy response coefficients for each specification of the Taylor rule (2.27) in the perpetual youth NK model.

Figure 8 compares the welfare loss surfaces of the contemporaneous cyclical Taylor rule (2.27) with and without interest rate inertia ($r_\pi = 0$ or $r_\pi = 0.8$) if inflation and output response coefficients lie within $\alpha_\pi \in [1.5, 3.0]$ and $\alpha_y \in [-0.05, 0.2]$. Pro-cyclical ($\alpha_y < 0$) or countercyclical ($\alpha_y > 0$) Taylor rules with non-trivial output response coefficient perform worse than Taylor rules with a mute response to output. Welfare loss surfaces become flat for $\alpha_y$ close to zero, so that the welfare cost remains negligible for any value of $\alpha_\pi \in [1.5, 3.0]$. Also, the inertial Taylor rule always performs better than the non-inertial counterpart. For all pairs of the policy response coefficients $\{\alpha_y, \alpha_\pi\}$, the welfare loss surface of the inertial Taylor rule remains below the one associated with the non-inertial feedback rule. The welfare cost surface of the inertial rule is also flatter, so that deviations from mute response on output are less penalized.

Figure 9 plots the welfare loss surface and contour lines associated with the inflation targeting oriented Taylor rule ($\alpha_\pi = 3.0$). The welfare loss surface is convex with respect to output response coefficient and reaches a minimum for an output response coefficient close to zero. Indeed, significant deviations from mute response on output are associated with welfare losses above the threshold value of 0.05% of consumption. The social welfare surface remains almost insensitive to the inertial coefficient $\alpha_\pi \in [0, 2]$, which indicates that interest rate smoothing has a complementary importance to the welfare performance of the rule. Interest rate inertia improves the properties of the rule, but the mute response on output remains the most important aspect. This is also revealed by the contour lines which are almost vertical to $\alpha_y \in [-0.25, 0.25]$. For any degree of interest inertia, the cyclical but inflation targeting oriented Taylor rule generates almost the same welfare loss. Interest rate inertia becomes important whenever the inflation response coefficient declines.

This is noticeable from the welfare cost surface and contour lines of an acyclical Taylor rule when the inflation response coefficient and interest rate inertia vary within the intervals $\alpha_\pi \in [1.5, 3.0]$ and $\alpha_y \in [0, 2]$ (figure 10). Interest rate inertia becomes essential if the Taylor rule is not inflation targeting oriented. If the inflation response coefficient lies within [1.5, 2.0] and the rule is non-inertial, the social welfare cost rises significantly. For the same interval, the welfare loss is minimized once the Taylor rule becomes inertial with coefficient $\alpha_y$ around unity. Interest rate inertia increases the aggressiveness of monetary policy to inflation and allows for a lower inflation response coefficient (Schmitt-Grohé and Uribe 2007b). If the Taylor rule obtains an inflation targeting nature, i.e., the inflation response coefficient is close to the upper bound of $\alpha_\pi = 3.0$, the interest rate inertia becomes irrelevant. According to contour lines, the social welfare cost remains insensitive to interest rate inertia if the inflation response coefficient is significant.

Figure 11 closes the robustness analysis by comparing the welfare loss surfaces of the main specifications of Taylor rule (2.27) for the lower and upper bound value of the probability of death parameter, i.e., for $\pi_d = 0.00357$ (Chadha and Nolan 2007) and $\pi_d = 0.05$ (Devereux 2011). If the probability of death measures the degree of agents’ myopia with $\pi_d = 0.05$, the welfare loss surface of the acyclical Taylor rule shifts upwards. For any pair of the policy response coefficients $\{\alpha_y, \alpha_\pi\}$, the simple Taylor rule generates higher social welfare cost in the perpetual youth.
than in the baseline specification of the NK model. This indicates that acyclical Taylor rules become less efficient in the NK model with myopic agents. The bottom panel of the same figure signals the same result. For $\pi_d = 0.05$, the welfare loss surfaces associated with the non-inertial and inertial cyclical Taylor rules rotate rightwards. This means that positive values of output response coefficient deliver lower welfare losses in comparison to the baseline calibration of $\pi_d = 0.00357$. Similarly, negative values of the output response coefficient deliver lower welfare cost in the standard than in the OLG specification of the same framework. Sensitivity analysis, however, obtains only a qualitative significance as the differences between the welfare loss surfaces are quantitatively trivial. Namely, the difference in the conditional welfare cost is less than 0.0025, which is far below the threshold value of 0.05 mentioned by Lucas (2003). Indeed, the welfare loss surfaces retain their shapes for $\pi_d \epsilon [0, 1)$ so that the welfare properties of the Taylor rules described above remain robust on average.

5.3 Optimal Taylor rules

In this section, we use the optimization algorithm of Schmitt-Grohé and Uribe (2007b) to search for the policy response coefficients that minimize the difference between the social welfare under the Ramsey plan and the one associated with the competitive equilibrium allocation in both versions of the NK model: with family dynasties (Faia 2008b) and overlapping generations. The optimal policy response coefficients reported in table 6 conform to the above welfare analysis.

Whenever we allow for a cyclical argument in the Taylor rule, the optimization algorithm computes a low, though negative output response coefficient in the infinite horizon NK model. In the OLG version of the model, the optimal output response coefficient remains low in absolute terms, but becomes positive. Specifically, in the standard NK model, the output response coefficient is equal to $\alpha_y = -0.03/4$ and $\alpha_y = -0.018/4$, for the non-inertial and inertial cyclical Taylor rule, respectively. In the perpetual youth framework, the output response coefficient becomes equal to $\alpha_y = 0.0648/4$ and $\alpha_y = 0.0688/4$, for the non-inertial and inertial specifications of the Taylor rule, respectively. The output response coefficient is mute in both frameworks, but signals the procyclical and countercyclical nature of monetary policy in the corresponding versions of the NK model.

Indeed, the conditional welfare loss between the cyclical and acyclical Taylor rules is almost equal. In the standard NK model, the procyclical and non-inertial Taylor rule generates a conditional welfare cost equal to $\lambda^c = 0.0015$ percentage points of consumption, which is almost equal to the welfare loss of $\lambda^c = 0.0011$ associated with the acyclical and non-inertial Taylor rule within the same framework. Also, the procyclical and inertial Taylor rule with $\alpha_r = 0.8079$ and $\alpha_y = -0.0045$ generates a conditional welfare loss equal to $\lambda^c = 0.00001$, which is identical to the welfare cost associated with the acyclical and inertial Taylor rule within the same framework. In the OLG version of the NK model, the conditional welfare cost of the countercyclical and non-inertial Taylor rule is equal to $\lambda^c = 0.0022$ percentage points of macro-aggregated consumption, which differs from the conditional welfare cost of the acyclical and non-inertial feedback rule at only 0.0001 percentage points. Similarly, the welfare cost difference between the cyclical and superinertial Taylor with the acyclical counterpart is only 0.0002 percentage points.

Overall, in both versions of the NK model, the contribution of the cyclical policy argument in the welfare performance of the Taylor rule is rather trivial quantitatively. Either with or without a cyclical policy argument, the welfare performance of the simple Taylor rule is rather similar. The role of the cyclical policy argument is limited to the signal of the procyclical or countercyclical nature of monetary policy, as determined by the Ramsey optimal plan.

Finally, according to table 6 a significant response to inflation rate is always optimal. In both versions of the NK model, the optimization algorithm computes an inflation response coefficient close to the upper bound so that the interest rate feedback rules are inflation targeting oriented. This implies in turn that price stability constitutes the primary objective of the policymaker, for it eliminates the welfare detrimental price dispersion which appears in both specifications of the NK model. The optimal inflation response coefficient is almost $\alpha_r = 3.0$ for each type of the...
interest-rate rule, cyclical or acyclical, with or without interest rate inertia, in both NK frameworks. Also, interest rate inertia contributes to the welfare performance of the Taylor rule either with or without the cyclical policy argument in both versions of the model. In all cases, the interest rate inertia coefficient is above or close to $\rho_r = 0.8$.

6 Conclusion

We revised the infinite horizon assumption of the standard NK framework to study the influence of the perpetual youth specification on normative implications of the Synthesis. The assumption of finite horizons can be attributed to the absence of bequest motives of agents or to their shortsightedness within an otherwise rational decision making, which introduces a non-Ricardian aspect in the demand side of the standard NK framework.

From normative perspective, the finite horizons under particular calibration increase the gap between the competitive equilibrium and the efficient allocation. In this sense, agents shortsightedness has similar to monopolistic competition inefficiency effects on real economy. The average markup is minimized for higher inflation rate if the probability of death increases. In the sticky price framework, however, the monetary authority finds it optimal to retain stable prices in the long run because the gap between the inefficiently low competitive equilibrium and the modified golden rule can only be eliminated with fiscal policy measures, i.e., fiscal subsidies on production. In business cycle frequency, there is deviation from strict inflation targeting in the short run. Optimal policy becomes countercyclical, revealing a stabilization rather than an expansionary motive for monetary policy. Apart from monopoly power and sticky prices, the Ramsey planner encounters the non-zero contemporaneous financial wealth that adds to households’ income. The non-human wealth accrued to currently alive agents undermines future productive capacity and hence the welfare of next generations, which influences the policy decisions of the benevolent Ramsey planner with infinite horizons.
References


Faia, E.: 2008b, Ramsey monetary policy with nominal rigidities and capital accumulation, Macroeconomic Dynamics 12, 90–99.


Table 1: Deep Parameters (Quarterly Frequency)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Baseline Value</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A. Demand side</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Discount factor</td>
<td>$\beta$</td>
<td>0.99</td>
</tr>
<tr>
<td>(Gross) inflation rate</td>
<td>$\pi$</td>
<td>1.042^{1/4}</td>
</tr>
<tr>
<td>Probability of death</td>
<td>$\pi_d$</td>
<td>1/(70 \cdot 4) [0.00357, 0.05]</td>
</tr>
<tr>
<td>Government spending to output</td>
<td>$g/y$</td>
<td>0.25</td>
</tr>
<tr>
<td><strong>B. Supply side</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elasticity of intermediate products</td>
<td>$\varepsilon$</td>
<td>6</td>
</tr>
<tr>
<td>Calvo price rigidity</td>
<td>$\theta$</td>
<td>0.75</td>
</tr>
<tr>
<td>Capital share in output</td>
<td>$\alpha$</td>
<td>0.35</td>
</tr>
<tr>
<td>Capital depreciation rate</td>
<td>$\delta$</td>
<td>0.025</td>
</tr>
<tr>
<td><strong>C. Exogenous Shocks</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Technology shock persistence</td>
<td>$\rho_z$</td>
<td>0.95</td>
</tr>
<tr>
<td>Technology shock std.dev.</td>
<td>$\sigma_z$</td>
<td>0.008</td>
</tr>
<tr>
<td>Government spending persistence</td>
<td>$\rho_g$</td>
<td>0.9</td>
</tr>
<tr>
<td>Government spending std.dev.</td>
<td>$\sigma_g$</td>
<td>0.0074</td>
</tr>
<tr>
<td><strong>D. Policy Rule</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inflation response coefficient</td>
<td>$\alpha_\pi$</td>
<td>1.5</td>
</tr>
<tr>
<td>Output response coefficient</td>
<td>$\alpha_y$</td>
<td>0.5/4</td>
</tr>
<tr>
<td>Interest rate inertia</td>
<td>$\alpha_r$</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2: Probability of Death Parameter ($\pi_d$) Values

<table>
<thead>
<tr>
<th>Quarterly Value</th>
<th>Expected Working Life (years)</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>-</td>
<td>Devereux (2011)</td>
</tr>
<tr>
<td>0.015</td>
<td>-</td>
<td>Ganelli (2007), Aninicaiaro et al. (2011)</td>
</tr>
<tr>
<td>0.01</td>
<td>-</td>
<td>Smets and Wouters (2002), Ganelli (2007)</td>
</tr>
<tr>
<td>0.00926</td>
<td>27</td>
<td>Leith and Wren-Lewis (2008)</td>
</tr>
<tr>
<td>0.00833</td>
<td>30</td>
<td>Leith and Wren-Lewis (2006), Ascari and Rankin (2010)</td>
</tr>
<tr>
<td>0.00625</td>
<td>40</td>
<td>Galí (1990)</td>
</tr>
<tr>
<td>0.005</td>
<td>50</td>
<td>Andrés et al. (2006), Leith et al. (2011)</td>
</tr>
<tr>
<td>0.004167</td>
<td>60</td>
<td>Galí (1990)</td>
</tr>
<tr>
<td>0.00357</td>
<td>70</td>
<td>Chadha and Nolan (2007)</td>
</tr>
</tbody>
</table>

Note: The values of the probability of death above 0.01 in quarterly frequency can be interpreted as reflecting the degree of agents’ shortsightedness or myopia (non-Ricardian behavior).
Table 3: Steady-State Macro Ratios

<table>
<thead>
<tr>
<th>y/c</th>
<th>y/h</th>
<th>x/y</th>
<th>k/y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.8346</td>
<td>3.0801</td>
<td>0.20492</td>
<td>8.1969</td>
</tr>
</tbody>
</table>

Baseline calibration ($\pi_d = 0.00357$)

Table 4: Minimum Average Markup and Inflation Rate

<table>
<thead>
<tr>
<th>Probability of Death ($\pi_d$)</th>
<th>0.00357</th>
<th>0.015</th>
<th>0.02</th>
<th>0.025</th>
<th>0.0333</th>
<th>0.05</th>
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<tr>
<td>Annual $\pi$ (%)</td>
<td>0.4</td>
<td>0.4</td>
<td>0.8</td>
<td>1.21</td>
<td>1.21</td>
<td>1.61 – 2.02</td>
</tr>
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</table>

Table 5: Welfare Evaluation of Simple (Contemporaneous) Taylor Rules

<table>
<thead>
<tr>
<th>Arguments</th>
<th>Infinite Horizon NK Model (Faia, 2008)</th>
<th>Perpetual Youth NK Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_r$</td>
<td>$\alpha_x$</td>
<td>$\alpha_y$</td>
</tr>
<tr>
<td>1.5</td>
<td>0.5/4</td>
<td>-154.7181</td>
</tr>
<tr>
<td>0.8</td>
<td>1.5</td>
<td>0.5/4</td>
</tr>
<tr>
<td>3.0</td>
<td>0.5/4</td>
<td>-154.3100</td>
</tr>
<tr>
<td>0.8</td>
<td>3.0</td>
<td>0.5/4</td>
</tr>
<tr>
<td>1.5</td>
<td>-154.2960</td>
<td>0.0187</td>
</tr>
<tr>
<td>0.8</td>
<td>1.5</td>
<td>-154.2775</td>
</tr>
<tr>
<td>$\pi_t = \pi$</td>
<td>-154.2768</td>
<td>0.0000</td>
</tr>
<tr>
<td>3.0</td>
<td>-154.2780</td>
<td>0.0011</td>
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<tr>
<td>0.8</td>
<td>3.0</td>
<td>-154.2769</td>
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</tbody>
</table>

Table 6: Optimized (Contemporaneous) Interest Rate Rules

<table>
<thead>
<tr>
<th>Arguments</th>
<th>Infinite NK (Faia, 2008)</th>
<th>OLG NK ($\pi_d = 0.00357$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_r$</td>
<td>$\alpha_x$</td>
<td>$\alpha_y$</td>
</tr>
<tr>
<td>$\pi_t$</td>
<td>$y_t$</td>
<td>- 2.8824</td>
</tr>
<tr>
<td>$r_{t-1}$</td>
<td>$\pi_t$</td>
<td>$y_t$</td>
</tr>
<tr>
<td>$\pi_t$</td>
<td>- 2.9856</td>
<td>- 154.2780</td>
</tr>
<tr>
<td>$r_{t-1}$</td>
<td>$\pi_t$</td>
<td>0.874</td>
</tr>
</tbody>
</table>

Note: The conditional social welfare ($W_{0,t}^T$) and the conditional welfare cost ($\lambda \times 100$) associated with each specified Taylor rule in comparison with the unconstrained Ramsey optimal plan are computed using second order perturbation method and simulating the model economy to both technology and government spending shocks.
Figure 1: Natural Equilibrium and Phase Diagram.

Notes: Solid concave line: Aggregate resource constraint; Solid vertical line: Consumption locus under infinite horizons; Solid, dashed, dashed-dotted, and dotted convex lines: Consumption loci under alternative values of probability of death $\pi_d(0, 1)$ parameter.
Figure 2: Steady-State of the baseline and the OLG version of the RBC model.
Figure 3: Steady-state relations: average markup ($\mu^p$), optimal relative price ($\tilde{p}$), and price dispersion measure ($s$), as functions of inflation rate.
Figure 4: Competitive vs. Ramsey steady-state for alternative values of the probability of death $\pi_d \epsilon (0, 1)$
Figure 5: Baseline NK Model (Faia, 2008): Impulse responses to neutral technology ($z_t$) and government spending ($g_t$) shock under the Ramsey Optimal Plan.

Notes: All variables are expressed in percentage deviations from Ramsey steady-state. Upper subplot: IRs to $z_t$ shock. Bottom subplot: IRs to $g_t$ shock.
Figure 6: Impulse responses to neutral technology ($z_t$) and government spending ($g_t$) shock under the Ramsey Optimal Plan.

Notes: All variables are expressed in percentage deviations from Ramsey steady-state. Solid line: Infinite horizon NK model; Dashed-line: Finite horizon with 70 years life-expectancy; Dashed-dotted line: Finite horizon where $\pi_d = 0.05$ reflects agents’ shortsightedness. Dotted-Solid line: Standard RBC model with Pareto efficient allocation. Upper subplot: IRs to $z_t$ shock. Bottom subplot: IRs to $g_t$ shock.
Figure 7: Determinacy Regions for Taylor Rules in OLG NK model with $\pi_d = 0.00357$.

Determinacy Region for $r = 0.8 r_{t-1} + \alpha_r \pi_t + \alpha_y y_t$

Determinacy Region for $r = \alpha_r r_{t-1} + \alpha_\pi \pi_t + \alpha_y y_t$

Determinacy Region for $r = 0.8 r_{t-1} + \alpha_\pi \pi_t + \alpha_y y_t$

Determinacy Region for $r = \alpha_r r_{t-1} + 3.0 \pi_t + \alpha_y y_t$
Figure 8: Finite Horizon NK model ($\pi_d = 0.00357$): Welfare Evaluation of Cyclical Taylor Rules $\hat{r}_t = \alpha_x \hat{\pi}_t + \alpha_y \hat{y}_t$ (Tpy) vs. $\hat{r}_t = 0.8 \hat{r}_{t-1} + \alpha_x \hat{\pi}_t + \alpha_y \hat{y}_t$ (Tpyi)
Figure 9: Finite Horizon NK model ($\pi_d = 0.00357$): Welfare Evaluation of $\hat{r}_t = \alpha_r \hat{r}_{t-1} + 3 \hat{\pi}_t + \alpha_y \hat{y}_t$
Figure 10: Finite Horizon NK model ($\pi_d = 0.00357$): Welfare Evaluation of $\hat{r}_t = \alpha_r \hat{r}_{t-1} + \alpha_\pi \hat{\pi}_t$
Figure 11: Finite Horizon NK model: Welfare Evaluation of Taylor rules.

Welfare Loss Surface (Tpi)

Welfare Loss Surface (Tpy)

Welfare Loss Surface (Tpyi)