Optimal Disability Insurance with Informal Child Care

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Abstract

We analyse optimal disability insurance in the presence of unobserved health shocks and informal child care activities. We show how a combination of lump sum taxation and child care subsidies can implement the constrained efficient allocation with unobservable health shocks only. In the case of multiple health types, the optimal subsidies follow a sliding scale with increasing subsidies for lower family types. We then calibrate an overlapping generations model with child care needs to features of the US economy. Optimal benefits are higher for families with younger children and with higher child care needs. Optimal average tax rate is lower for healthy families as compared to the US average tax rate and vice versa for disabled families. We also find that adopting the optimal scheme can lead to sizeable cost savings.

**JEL Codes:** H21, H24, H31, J14, J22, J26

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1 Introduction

With increasing life expectancies and declining fertility rates comes concerns associated with an ageing population. The Social Security Disability Insurance (SSDI) program is a major program for individuals under the full retirement age who can no longer work due to a disability expected to last at least 12 months. The number of beneficiaries for SSDI increased\(^1\) from 2.6 million with an average monthly benefit of $470 in 1984 to 7.1 million with an average benefit of $1,004 in 2007. There has also been a rapid increase in female labour force participation thereby making more women eligible for SSDI benefits. The proportion of awards made to women increased from 26% in 1960 to 48% in 2007. As of 2007, 3.3 million women were receiving SSDI benefits.

**Figure 1: Trends in Disability Insurance**

The past few decades have also witnessed another major demographic trend. Data from the US Census Bureau show that there has been nearly a double increase in the number of children under 18 living in grandparent headed households\(^2\), from 2 million or 3.2% in 1970 to 4.5 million or 6.3% in 2000. There has also been an increase in the proportion of children of pre-primary school age with a working mother, who are being cared for primarily by their grandparents during the day; from 13.9% in 1988 to 19.6% in 2005. On average, preschoolers with employed mothers were spending 24 hours per week in grandparent care\(^3\).

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\(^1\)The increasing trends in SSDI participation have been attributed to two main factors in the literature. Firstly, in 1984 congressional reforms redefined disability as the inability to engage in substantial gainful activity due to a physical or mental impairment and placed the controlling evidentiary weight on a disability applicant’s own health practitioner thereby making the disability screening process easier for applicants [See Hu Lahi, Vaughan and Wixon (2001), Autor and Duggan (2006) for more detail]. Secondly, there has been an increase in the disability replacement rate since the 1970’s. Haveman, Jong and Wolfe (1991) and Gruber (2000) find that increased generosity in the disability system lead to lower labour force participation by older workers while Black and Sanders (2002) and Autor and Duggan (2003) find evidence that adverse economic conditions increase participation in disability programs in the USA.

\(^2\)The median age of US grandparent caregivers is 57. The majority, 68% of grandparent caregivers are White while 29% are African-American.

\(^3\)Source: US Census Bureau Reports “Who’s Minding the Kids? Child Care Arrangements” based on data from the Survey of Income and Program Participation (SIPP)
Evidence of intergenerational interaction in the USA is provided in McGarry and Schoeni (1995) and Soldo and Hill (1995) who document time and money transfer measures in the Health and Retirement Study (HRS). Ying Wang and Marcott (2007) use Panel Study of Income Dynamics (PSID) data to show that taking in a grandchild in the household leads to greater labour supply of the grandparents and conjecture that this is the result of increased money needs. Ho (2008) uses a general collective framework of intergenerational family decision making to show how grandchild care needs can affect intergenerational resource allocations and finds that higher formal child care cost leads to higher grandmother provided child care and money transfers. Using data from the HRS, Ho (2009) evaluates the indirect impacts of the 1996 Personal Responsibilities and Work Opportunities Reconciliation Act in the USA on elderly women’s time and money transfers to the low income young mothers being targeted by the welfare-to-work reform. She finds that the reform decreased grandchild care by about 1.5 hours a week. The impact was even higher for families living together with a decrease in grandchild care of around 5 hours a week. This tends to suggest that child care subsidies offered to the young generation may have quantitatively important impacts on the elder generation.

A challenge for most authorities concerns the provision of social insurance to families while preserving work incentives. This paper analyses optimal disability insurance in the presence of unobserved health shocks and informal child care activities. Agents are subject to unobserved health shocks which may leave them unable to work as in the standard asymmetric information framework. Moreover, they are able to engage in unobserved informal child care activities which exacerbates their work incentives and encourage them to claim social security benefits. Our contribution is threefold: (i) we provide an efficiency case for child care subsidies, (ii) we demonstrate implementation of the optimal insurance scheme via a combination of lump sum transfers and child care subsidies, and (iii) we provide quantitative illustrations of the optimal scheme and compare it to the US welfare system.

We use a static model à la Diamond and Mirrlees (1978) to illustrate the exacerbation of work incentives
caused by unobserved informal child care activities and present an efficiency case for child care subsidies. We then show how a combination of lump sum taxation and child care subsidies can implement the optimum where families use the optimal level of formal child care. The intuition behind this result is that families which plan to falsely claim disability benefits also have an incentive to engage in informal child care activities. However, child care subsidies put downward pressure on informal child care so that families who want to claim the subsidies will use the optimal level of formal child care. We also show that in the presence of multiple health types, optimal child care subsidies will follow a sliding scale where the subsidies increase the lower the family type. Incentive compatibility requires benefits to decrease the lower the family type. However, this increases the marginal utility from informal child care so that high type families pretending to be of lower types have higher incentives to engage in child care activities. Thus, higher subsidies need to be offered to lower family types so as to discourage informal child care activities and encourage work on the market.

In the second part of the paper, we develop an overlapping generations model with child care needs and calibrate the model to features of the US economy. The qualitative results from the static model extend to the dynamic environment because of the static nature of child care decisions. We find that optimal benefits are higher for families with young children in need of child care as compared to families with older children. Optimal average tax rate is lower for healthy families as compared to the US average tax rate and vice versa for disabled families. We also find that adopting the optimal scheme with lump sum taxation can lead to sizeable cost savings of around 3.4%. On the other hand, the additional use of child care subsidies as policy instrument leads to additional cost savings of small magnitude of around 0.52% suggesting that the informational costs of informal grandchild care on the US disability insurance program are relatively small.

Our paper is related to several papers in the optimal policy literature with the presence of non market activities. Beaudry, Blackorby, and Szalay (2009) analyse optimal taxes and employment subsidies in a multidimensional screening framework where agents have different unobserved values for both market and non market activities and find there there is an optimal cut off wage where those above the cut off wage are taxed and those below are subsidised. Chone and Laroque (2009) argue for subsidisation of low skilled agents in a discrete labour supply framework where individuals have different productivities and work opportunity costs. Kleven, Richter, and Sorensen (2000) analyse optimal commodity taxation in the presence of household production and find that it may be optimal to impose a lower tax on market services which are complementary to leisure.

Blau (2000) surveys the literature on child care subsidy programs and documents three main arguments in support of child care subsidies. (i) Promotion of economic self-sufficiency where subsidies are given to low income families to get them off welfare and into work. The argument is that this will lead to human

\footnote{Data from the HRS panels 1992 to 2002 show that out of the grandmothers (aged below 62 who is the early retirement age in the US) who reported receiving Social Security Disability benefits, 23% looked after grandchildren for more than 2 hours a week and 8% looked after grandchildren for more than 20 hours a week.}

\footnote{Many states in the USA have sliding scale child care subsidies which decrease with income of young families. Our model provides a potential justification for such subsidies in the presence of unobserved informal child care.}
capital accumulation thereby leading to future cost savings for the government. (ii) Market imperfections where parents are uninformed about quality care providers. In this case, the subsidies are targeted towards high quality child care providers. (iii) Distributional issues where quality child care is viewed as a merit good. In this case, families do not consider the positive externalities of quality care and underinvest in child care. To our knowledge our paper is the first to analyse optimal disability insurance in the presence of child care needs thereby offering an efficiency argument for child care subsidies payable to disabled families. Moreover, we are the first to apply this framework to the intergenerational family and quantify the optimal disability insurance scheme for the intergenerational family.

In Section 2, we use a static model to illustrate how informal child care activities can exacerbate work incentives. In Section 3, we present a case for child care subsidies and discuss implementation. In Section 4, we model the overlapping generations model with child care needs. We calibrate the model to features of the US economy in Section 5 and present optimal policy rules for disability insurance benefits and child care subsidies in Section 6. We conclude in Section 7.

2 Static Model

In this section, we use a simple static model to illustrate how unobserved informal child care can exacerbate work incentives and encourage an agent to claim disability benefits.

2.1 Framework

2.1.1 Agent

Suppose that we have a continuum of agents with one child. The child is passive and requires one unit of child care time. Family preferences are separable in consumption and leisure

\[ u(c) + v(l) \]

where \( c \) is total household consumption and \( l \) is leisure. We assume that \( u(.) \) and \( v(.) \) are strictly concave twice differentiable functions and that \( v(0) = 0 \).

The agent is endowed with one unit of time which can be spent on either work, leisure, or child care. We consider discrete labour supply decisions. If the agent works, she works for 1 unit of time and produces \( w \). On the other hand, if the agent does not work, she can spend time either on child care or leisure so that \( l = 1 - h \) where \( h \) is informal child care. The agent can receive health shocks which leaves her healthy with probability \( p \) and disabled with probability \( (1 - p) \). If the agent is healthy, she can either work or provide child care. On the other hand, if she is disabled she can neither work nor provide child care nor enjoy leisure so that \( l = 0 \). One can view disabled individuals as having zero amount of healthy time available. The family can hire formal child care at cost \( w^F \) per unit of formal care time allocated to the child \( h^F = 1 - h \). We assume that \( w > w^F \) so that the value of production by the agent is higher than
the value of formal child care $w^F$.

2.1.2 Planner

We have a risk neutral social planner who provides disability insurance to families at a minimum cost and guarantees families an expected utility level of at least $V$. The planner observes employment status of the agent but only knows the distribution of health shocks and does not observe actual health shocks of the agent nor informal child care. Thus, we have information asymmetry in the sense that if an agent does not work, the planner would not be able to distinguish whether it is due to (i) disability, (ii) leisure enjoyment, or (iii) involvement in child care. We write the problem as though the planner takes all market production and allocates benefits $b = \{b_H, b_D\}$ to the family according to whether the agent works and is therefore healthy $H$ or whether the agent claimed disabled $D$. This is equivalent to assuming that the planner imposes lump sum taxes such that $\tau_H = w - b_H$ when the agent is working and $\tau_D = -b_D$ when the agent is disabled and not working. Thus, family consumption is made up of benefits from the planner minus formal child care cost: $c_z = b_z - w^F h_z^F$ when the agent is in state $z \in \{H, D\}$.

2.2 Planner’s Problem

The planner minimises cost subject to participation constraint and incentive compatibility constraints

$$P = Min_{\{b_H, b_D, h_D\}} p(b_H - w) + (1 - p) (b_D)$$ (1)

s.t.

$$pu \left( b_H - w^F \right) + (1 - p) u \left( b_D - w^F \right) = V$$ (2)

$$u \left( b_H - w^F \right) \geq u \left( b_D - w^F \left( 1 - h_D \right) \right) + v \left( 1 - h_D \right)$$ (3)

$$\hat{h}_D = argmax \{ h \} u \left( b_D - w^F \left( 1 - h \right) \right) + v \left( 1 - h \right)$$ (4)

where equation (2) is the participation constraint stating that the planner guarantees the family an expected utility $V$. Inequality (3) is the incentive compatibility constraint for work which states that the utility that healthy individuals receive when working should be at least as high as the utility that they would get if they falsely claim disabled and engage in unobserved informal child care activities $\hat{h}_D$. A healthy family chooses $\hat{h}_D$ such that it maximises utility when pretending to be disabled as given by equation (4).

2.3 Informal Child Care and Work Incentives

In this section, we illustrate how the introduction of unobserved informal child care in the standard Diamond and Mirrlees (1978) framework exacerbates the usual incentive compatibility constraint. We compare three situations: (i) FI - full information case where the planner can observe health shocks and
informal child care, (ii) AIO - asymmetric information case where the planner cannot observe health shocks but can observe informal child care, and (iii) AIU - asymmetric information case where the planner cannot observe health shocks nor informal child care.

2.3.1 Full Information

In this situation, there is perfect information and the planner maximises (1) s.t. (2). First order conditions yield \( u'(b_H^* - w^F) = u'(b_D^* - w^F) \) where * denotes the full information (FI) optimal allocation. This implies perfect insurance with \( b_H^* = b_D^* \). This is illustrated by point A in Figure 3 which is the tangency between the planner’s cost function \( BC_1 \) and individual’s indifference curve \( EU \).

Figure 3: Optimal Allocations under FI, AIO and AIU

![Figure 3: Optimal Allocations under FI, AIO and AIU](image)

2.3.2 AIO Unobservable Health Shocks and Observable Informal Child Care

In this situation, the planner cannot observe health shocks but can observe informal child care. Since the planner can observe informal child care, the planner will recommend zero informal child care on agents who claim disability benefits since disabled individuals have zero healthy time available. The planner maximises (1) s.t. (2) and incentive compatibility constraint for agent’s work:

\[
 u\left(b_H - w^F\right) \geq u\left(b_D - w^F\right) + v \tag{1}
\]

The full information optimal allocations are not implementable when the planner cannot observe health shocks since under FI allocations, \( u\left(b_H^* - w^F\right) < u\left(b_D^* - w^F\right) + v \) which would violate (5). Diagrammatically, the incentive constraint can be illustrated by the curve \( IC_2 \) in Figure 3. An agent is willing...
to work only in the region below $IC_2$ where benefits when she is healthy and working are high enough to compensate for her loss of leisure time\(^6\). The AIO optimum is at point $B$ where in order to provide the same expected utility\(^7\) level as in the full information case, the planner will have to face higher costs\(^8\) represented by $BC_2$.

### 2.3.3 AIU Unobservable Health Shocks and Unobservable Informal Child Care

The previous subsection illustrated the typical asymmetric information problem. We now consider the case where the planner can observe neither health shocks nor informal child care. Since the family now has the additional option of engaging in unobserved informal child care, the utility from claiming disability benefits and engaging in unobserved informal child care is at least as high as the utility from claiming disabled and not engaging in informal child care activities.

\[
u \left( b_D - w^F \right) + v \left( 1 \right) \leq \max_{\{h\}} \left[ u \left( b_D - w^F \left( 1 - h \right) \right) + v \left( 1 - h \right) \right]
\]

Thus, at the AIO optimal allocations, healthy agents who want to claim disability will also have an incentive to engage in informal child care activities so that the incentive compatibility constraint for work is exacerbated. Diagrammatically, the presence of unobserved informal child care would shift the incentive compatibility constraint down\(^9\) to $IC_3$ as illustrated in Figure 3. Expected utility of an agent who does not intend to work when healthy is now given by $p \left[ u \left( b_D - w^F \left( 1 - \hat{h}_D \right) \right) + v \left( 1 - \hat{h}_D \right) \right] + (1 - p) u \left( b_D - w^F \right)$. The AIU optimum is illustrated by point $C$. To deliver the same expected utility as in the full information case, the planner will have to incur even greater cost represented by $BC_3$.

### 3 Policy Implications

In the previous section, we illustrated how the presence of unobserved informal child care exacerbates the incentive constraint for work so that the AIO optimal allocations may not be implementable when only lump sum transfers are used. In this section, we present a case for child care subsidies and show how a combination of lump sum transfers and child care subsidies can implement the AIO optimal allocations.

\(^6\)Under the assumption that utility is concave, slope of the incentive constraint must be less than 1. To see this, slope of the incentive constraint is $\frac{\partial u}{\partial h} = \frac{\partial v}{\partial \left( b_D \right)}$ which is less than one since $c_H > c_D$ and $u \left( . \right)$ is concave.

\(^7\)Expected utility of a honest agent who works when healthy is given by $pu \left( b_H - w^F \right) + (1 - p) u \left( b_D - w^F \right)$ while expected utility of an agent who does not intend to work when healthy is given by $p \left[ u \left( b_D - w^F \right) + v \left( 1 \right) \right] + (1 - p) u \left( b_D - w^F \right)$. The indifference curve along which expected utility of the latter is constant will thus be a horizontal line up to point $B$ where it intersects the incentive compatibility constraint.

\(^8\)This stems from the assumption of concavity of the utility function and therefore diminishing marginal utility of consumption. To preserve work incentives, the planner will have to increase benefits to working agents by relatively more than the decrease in benefits to disabled agents.

\(^9\) $IC_2$ converges to $IC_3$ because at high $b_D$, the incentive to provide child care becomes lower. This can be seen from the family’s first order condition for child care which requires that $u' \left( b_D - w^F \left( 1 - \hat{h}_D \right) \right) w^F \leq u' \left( 1 - \hat{h}_D \right) w^F$. Thus, if $b_D$ is very high such that $b_D^{high} > u' \left( \frac{\hat{h}_D}{1 - \hat{h}_D} \right) + w^F$ we get zero informal child care and $IC_3$ will coincide with $IC_2$. 

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We also show that in the case of multiple health types, the child care subsidies will need to follow a sliding scale with higher subsidies paid to lower type families.

3.1 An Efficiency Case for Child Care Subsidies

A healthy family which claims disabled has an incentive to engage in unobserved informal child care activities as well. In this case, the social planner can make use of child care subsidies to decrease the relative cost of formal child care and thereby discourage informal child care activities. This is shown in the following proposition.

Proposition 1: There is a positive wedge between the social planner’s desired first order condition for informal child care and the family’s first order condition for informal child care.

\textbf{Proof:} From a healthy family’s problem, first order condition with respect to informal child care is

\[ u' \left( b_D - w^F \left( 1 - \hat{h}_D \right) \right) w^F \leq u' \left( 1 - \hat{h}_D \right) , \hat{h}_D \geq 0 \]

where the first inequality is satisfied with strict equality if \( \hat{h}_D > 0 \). Thus, if informal child care is unobservable, a healthy family claiming disabled has an incentive to engage in informal child care activities. On the other hand, in the AIO case with unobserved health shocks and observable informal child care, the planner recommends zero informal child care on families claiming disabled since disabled individuals have zero healthy time available. The planner therefore recommends that the family’s first order condition be satisfied at zero informal child care \( \hat{h}_D = 0 \). QED

Since the family problem is concave, subsidies \( s > 0 \) on formal child care cost \( w^F \) can help decrease the family’s marginal value from informal child care activities and put downward pressure on informal care thereby implementing the optimum with no informal child care as shown in the next subsection.

3.2 Implementation of the AIO Optimum

In this section, we show how a combination of lump sum transfers and subsidies can implement the AIO optimum with unobservable health shocks only.

\textbf{Proposition 2:} A schedule of lump sum transfers and child care subsidies such that

\[ c^*_H = b^*_H - w^F \quad ; \quad c^*_D = b^*_D - (1 - s) w^F \quad ; \quad s = 1 - \frac{w' (1)}{w' (c^*_D)} \frac{1}{w^F} \]

implements the AIO optimum. \( * \) and \( ** \) represent the AIO optimal allocations, \( b^*_D \) lump sum transfers to disabled families and \( s \) child care subsidies that disabled families are eligible for.

\textbf{Proof:} See Appendix A1
The intuition is as follows: design lump sum transfers and child care subsidies such that families still get the same consumption levels as in the AIO optimum with unobserved health shocks only. The child care subsidies are obtained from the family’s first order condition at zero informal child care. Since the family problem is concave, the subsidies will induce the family to provide zero informal care. The way the scheme would work would be to allocate working families lump sum benefits $b^*_H$, which is the same as in the AIO optimum and allocate families declaring disabled lump sum benefits $b^*_D$. In addition, those declaring disabled can claim child care subsidies $s$ per unit of formal child care cost used. While a healthy family declaring disabled has an incentive to engage in informal child care activities, the subsidies put downward pressure on informal child care and ensure that the family chooses the optimal level of formal child care.

Note that in this particular setting with two health types and discrete labour supply, the subsidies could also be universal in the sense that they could be paid directly to day care centres. The universal child care subsidies result does not hold in a setting with multiple health types though since in this case the subsidies will have to vary with health type as shown in the next sub-section.

3.3 A Case for Sliding Scale Child Care Subsidies

We show in this section how the previous results extend to the case with $N$ possible health types and continuous labour supply choices. Suppose that we now have $N$ possible health shocks: $n \in \{1, \ldots, N\}$ with 1 being the worst state where the agent is fully disabled and has 0 unit of healthy time and $N$ being the best health state with the agent having 1 unit of healthy time. Each possible health status $n$ has time cost of $d_n \in \{d_1, \ldots, d_N\}$ with $1 = d_1 > \ldots > d_n > \ldots > d_N = 0$. This accounts for potentially varying degrees of disability levels. For instance, individuals suffering from chronic back pain may not be able to work for as long as perfectly healthy individuals. The amount of healthy time available for each individual may therefore be different according to the disability level. Output produced is a function of wages $w$ and market work $e_n$ with $n \in \{1, \ldots, N\}$. Let $y_n$ be observed output produced by a agent of health type $n$. $y_n = w e_n$ and let $p_n$ denote the probability of getting health shock $n$. We show in Appendix A.2 that the results presented in this section also extend to the general Mirrleesian framework with multiple market productivity types.

The planner minimises cost subject to participation constraint (7) and incentive compatibility constraints (8)

$$P = \min_{\{b_n, y_n, \hat{h}^*_n\}_{n=1}^N} \sum_{n=1}^N p_n (b_n - y_n)$$

s.t.

$$\sum_{n=1}^N p_n \left[ u \left( b_n - w^F (1 - \hat{h}^*_n) \right) + v \left( 1 - d_n - \frac{y_n}{w} - \hat{h}^*_n \right) \right] = V$$
\[ \forall m,n, \ u \left( b_n - w^F \left( 1 - \hat{h}_n^m \right) \right) + v \left( 1 - d_n - \frac{y_n}{w} - \hat{h}_n^m \right) \geq u \left( b_m - w^F \left( 1 - \hat{h}_m^m \right) \right) + v \left( 1 - d_n - \frac{y_m}{w} - \hat{h}_n^m \right) \]

\[ \forall m,n, \ \hat{h}_n^m = \arg \max \{h\} \ u \left( b_m - w^F \left( 1 - h \right) \right) + v \left( 1 - d_n - \frac{y_m}{w} - h \right) \]

where \( \hat{h}_n^m \) is child care that a family of type \( n \) would choose if claiming that they are of type \( m \).

### 3.3.1 AIO Unobservable Health Shocks and Observable Informal Child Care

With observable informal child care, the planner will recommend zero informal child care on families. This stems from our assumption that agents are more productive on the market \( w > w^F \) than at home. In this case, the planner minimises (6) s.t. (7) and incentive compatibility constraints for work

\[ \forall m,n, \ u \left( b_n - w^F \right) + v \left( 1 - d_n - \frac{y_n}{w} \right) \geq u \left( b_m - w^F \right) + v \left( 1 - d_n - \frac{y_m}{w} \right) \]

In the following lemma, we show that optimal benefits must be non-decreasing in types so that incentive compatibility constraints are satisfied.

**Lemma 1:** Let \( n > m \). Then \( b_n \geq b_m \).

**Proof:** See Appendix A.1

### 3.3.2 AIU Unobservable Health Shocks and Unobservable Informal Child Care

With unobservable health shocks and unobservable informal child care, the planner minimises (6) s.t. (7), (8) and (9). In the following Lemma, we show that if a high type family who pretends to be of type \( n \) does not have an incentive to provide informal child care, then a lower type family who pretends to be of type \( n \) will also not have any incentive to provide informal child care. We then proceed to show how sliding scale child care subsidies where the subsidies increase the lower the family type can implement the AIO optimum in Proposition 3 below.

**Lemma 2:** Let \( k < m \). If \( u' \left( b_n - w^F \right) w^F \leq v' \left( 1 - d_m - \frac{y_m}{w} \right) \) at \( \hat{h}_m^m = 0 \), then \( u' \left( b_n - w^F \right) w^F < v' \left( 1 - d_k - \frac{y_m}{w} \right) \) at \( \hat{h}_k^k = 0 \)

**Proof:** See Appendix A.1

This lemma holds because healthier type families have lower marginal disutility of engaging in informal child care activities. Thus, if they are unwilling to participate in informal child care, lower type families who have higher marginal disutility will also not participate in informal child care. In this case, designing...
child care subsidies such that the highest type family will not have an incentive to engage in informal child care will also work for lower type families.

**Proposition 3**: Lump sum transfers and sliding scale child care subsidies which decrease the healthier the family type such that

$$
\forall n, c_n^{**} = b_n^{##} - (1 - s_n) w^F ; \quad y_n^{**} ; \quad s_n = 1 - \frac{v'(1 - \frac{w_n^{**}}{w})}{u'(c_n^{**})} \frac{1}{w^F}
$$

implements the AIO optimum. ** denotes the AIO optimal allocations, $b_n^{##}$ are lump sum transfers given to type $n$ family which produces output $y_n^{**}$, and $s_n$ are child care subsidies that type $n$ family is eligible for.

**Proof**: See Appendix A.1

The intuition behind the sliding scale child care subsidies is that incentive compatibility requires benefits to be non decreasing in types. However, since low type families get lower benefits, the marginal utility from informal child care activities when a high type pretends to be of lower type increases so that there are higher incentives to engage in informal child care. Thus, higher child care subsidies need to be offered to lower family types so as to discourage unobserved informal child care and encourage work on the market.

## 4 Overlapping Generations Model

In this section we extend the simple static model and present an overlapping generations model with informal child care. The qualitative results from the static model extend to the dynamic environment because of the static nature of child care decisions. We calibrate the model to the US economy taking into account the US welfare system. We then quantify the optimal benefits and child care subsidies, and the social planner’s cost savings from adopting the optimal policy.

### 4.1 Framework

#### 4.1.1 Agents

**Life Cycle** Each dynasty lives for twelve periods of 6.5 years each and experiences four periods as: child (C), parent (M), and grandparent (G). The child is passive and resource consuming requiring child care time which declines as the child gets older. A parent is not subject to health shocks and is always employed. A grandparent $G$ experiences health shocks at the start of each period which can either leave her healthy $H$ or disabled $D$. Disability shocks are absorbing so that if she experienced a bad health shock, she remains disabled in future periods until death at age 78. The life cycle of an agent is illustrated in Figure 4.
**Overlapping Generations**  Each generation takes decisions jointly with the next who does the same with the subsequent generation and so on. Thus, each generation indirectly cares for all of its descendants. One can view the problem as having a continuum of grandparents, parents, and children with constant population over time. Every four periods, we have an Age I household with a grandparent, a young parent, and a very young child requiring full time child care. We then have an Age II household with a grandparent, a young parent, and a young child requiring child care during half of the time\(^\text{10}\). As the household gets older, we have an Age III household with a retired grandparent, a middle aged parent and a teenager requiring zero child care time. Finally, we have an Age IV household with a retired grandparent, a middle aged parent and an older teenager requiring zero child care time.

We choose a life cycle of twelve periods to keep the model simple while conforming both with the realism of declining child care needs over the life cycle, and with the US welfare system for the calibration exercise. Age I and Age II households have children under 13 and are therefore eligible for Dependent Care Tax Credit (DCTC). Moreover, the grandparent is below 65, the usual retirement age, and is therefore eligible for Social Security Disability Insurance (SSDI) benefits. On the other hand, Age III and Age IV households have a grandparent above age 65 and are therefore eligible for social security retirement benefits.

\(^{10}\) According to US Census Bureau reports "Who’s Minding the Kids? Child Care Arrangements Fall 1995", preschoolers were spending between 31.2 to 39 hours per week in formal care while children aged between 5 to 14 were spending between 15.1 to 25.5 hours per week in formal care arrangements, therefore approximately half time.
benefits.

**Time Constraints** Each agent has one unit of time at their disposal. When an agent is employed, she devotes the full unit of time to work and produces output worth $w^G$ if a grandparent and $w^M$ if a young parent. Employed agents cannot engage in informal child care. A healthy unemployed grandparent can allocate her time between leisure $l^G$ or child care $h^G$

$$l^G + h^G = 1$$

while a disabled grandparent can neither work nor provide child care nor enjoy leisure.

**Child Care** The child generation requires child care time in the first two periods of its life which can be provided by the formal child care market at cost $w^F$ per unit or by informal child care by the grandparent. Formal child care time employed is decreased by the amount of informal child care time allocated by the grandparent

$$h^F_1 = 1 - h^G_1 \text{ if Age I}$$
$$h^F_2 = \frac{1}{2} - h^G_2 \text{ if Age II}$$

**Preferences** Family preferences are separable over time and intra period preferences are separable in consumption and leisure of each agent

$$u(c) + v^M(l_M) + v^G(l^G)$$

where $c$ is household consumption, $l^M$ leisure of the young parent, and $l^G$ leisure of the grandparent. $c$ is a public good comprising of consumption of both generations. We assume that $u(\cdot), v^M(\cdot)$ and $v^G(\cdot)$ are strictly concave and twice differentiable functions and $v^M(0) = v^G(0) = 0$. Let $\beta \in (0, 1)$ be the discount factor for the family between periods. Since the child eventually becomes a young parent who then becomes a grandparent and so on, the intergenerational family automatically cares for future generations. In this case, it is as though we have an infinitely lived dynastic family.

### 4.1.2 Planner

We have a risk neutral social planner who provides disability insurance to families at minimum cost and guarantees families an expected utility level of at least $V$. The planner knows the distribution of health shocks and can observe employment status of the agents but does not observe actual health shocks nor informal child care. Thus, we have information asymmetry in the sense that if a grandparent does not work, the planner would not be able to distinguish whether it is due to (i) disability, (ii) leisure enjoyment, or (iii) involvement in child care. On the other hand, the planner knows that the parent is not subject to health shocks and can constrain her to work. For Age I and Age II households, we model the problem as though the planner takes all market production and allocates benefits $b = \{b_H, b_D\}$ to the
family according to whether the grandparent works and is therefore healthy $H$ or whether the grandparent claimed disabled $D$. This is equivalent to assuming that the planner imposes lump sum taxes such that $\tau_H = w^M + w^G - b_H$ when the grandparent is working and $\tau_D = w^M - b_D$ when the grandparent is disabled and not working. Thus, family consumption is made up of benefits from the planner minus formal child care cost: $c_z = b_z - w^F h^F_z$ when the grandparent is in state $z \in \{H, D\}$. On the other hand, the grandparent is retired in Age III and Age IV households. In this case, the planner will allocate benefits $b = \{b_H, b_D\}$ according to the reported health of the grandparent. By the revelation principle, we can restrict ourselves to such direct mechanism. Let $q \in (0, 1)$ be the discount factor for the social planner between periods.

4.1.3 Timing

The first period starts with a healthy grandparent $G$ who then experiences health shocks which leaves her healthy with probability $p_1$ and disabled with probability $1 - p_1$. The grandparent then works or declares disabled and the planner allocates welfare payments accordingly. Given the allocated welfare payments, the family chooses consumption and informal child care. At the beginning of the second period a previously disabled grandparent is still disabled while a healthy grandparent receives health shocks which leaves her healthy with probability $p_2$ and disabled with probability $1 - p_2$. The grandparent then works or declares disabled and the planner allocates welfare payments accordingly. In the third period, the grandparent is retired and receives social security retirement benefits. If the grandparent is healthy, she stays healthy with probability $p_3$. Otherwise, she becomes disabled with probability $1 - p_3$. In the fourth period, the grandparent is retired and receives social security benefits. If she is healthy, she stays healthy with probability $p_4$ and becomes disabled with probability $1 - p_4$. The grandparent then dies and the parent becomes a grandparent in the following period. From an intergenerational point of view, each four periods are therefore a repetition of the previous four.

4.2 Recursive Formulation

Since each four periods is a repetition of the previous four, we can write the model in recursive form with the planner solving a similar problem every four periods. We split the planner’s problem according to whether we have an Age I, II, III or IV household.

4.2.1 Persistence in Privately Observed Shocks and Work Incentives

In the static problem outlined in the previous section, we had the typical incentive compatibility constraint for a healthy grandparent’s work, equation (3), which stated that utility from working needs to be at least as great as utility from falsely reporting disabled. In an intertemporal model where there is persistence in privately observed shocks, current shocks influence not only current probability distributions

\[11\text{Note that a family who reported disabled in the previous period will report disabled in the following period since if otherwise, the planner would know that the family lied in the previous period as disability is an absorbing state.}\]
but also future probability distributions so that the modeling of the incentive compatibility constraint is less straightforward.

In our model, the grandparent’s absorbing disability shocks is problematic for Ages I and II households where the grandparent has not yet reached retirement age and therefore needs to be incentivated to work. The absorbing nature of disability shocks creates a link between periods which leads to next period’s family preferences not being common knowledge anymore since the planner does not observe actual disability shocks: a truly disabled grandparent has probability one of being disabled hereafter while a falsely disabled grandparent still has a positive probability of staying healthy.

To see this, consider a planner who faces two exante identical Age I intergenerational families A and B in period t. Suppose family A gets the disability shock while family B gets the good health shock. The following period, t + 1, family A has probability 1 of having a disabled grandparent while family B has probability $p_2$ of the grandparent staying healthy and probability $1 - p_2$ of the grandparent being disabled. In period $t + 3$, a family B grandparent who got the good health shock at $t + 2$ still has probability $p_3$ of staying healthy and so on. Thus, even though both families were exante identical at $t$, they have different expected utilities in the future since they now have different health shock distributions. This persistence in privately observed shocks is problematic since the planner does not observe disability shocks and therefore does not know the actual probability distribution of health shocks that each family faces in the future. For instance, if family B decides to claim disability in period $t$, the planner will not be able to differentiate it from the truly disabled family A in future periods since the grandparents in both families would not be working from period $t$ onwards.

We follow Fernandes and Phelan (2000) recursive formulation with history dependence in privately observed shocks. In this case, the planner will offer the family welfare benefits $b_D$ payable to a grandparent who claims disability in period $t$. The planner also offers continuation value of utility such that if a grandparent is truly disabled, the family gets $V_D$, and if the grandparent is falsely disabled, the family gets $\tilde{V}_D$. $V_D$ is the usual promised utility that the planner implements the next period via the promise keeping constraint. On the other hand, $\tilde{V}_D$ the “threatened” utility that the that the planner implements next period via a threat keeping constraint. In period $t + 1$, the planner chooses welfare payments for $t + 1$ and continuation value of utility for $t + 2$ such that families who were truly disabled in the previous period get promised utility $V_D$ in $t + 1$ and such that families who were not truly disabled in the previous period get promised utility $\tilde{V}_D$ according to their respective probability distributions. The planner will need to keep track of the threat keeping constraint until the grandparent’s death.

4.2.2 Planner’s Problem

Let $P^I (V)$ be the net expected cost for the planner when facing an Age I family and having committed to deliver a dynamic utility level $V$ to the family from now onwards. $P^{Age}_H (V)$ is the net expected cost when facing an Age $\in \{II, III, IV\}$ family starting the period with a healthy $H$ grandparent. $P^{Age}_D (V, \tilde{V})$ is the net expected cost for the social planner when facing an Age $\in \{II, III, IV\}$ family starting the
period with a grandparent who previously claimed disabled. $V$ is the promised utility for a truly disabled grandparent while $\hat{V}$ is the threatened utility for a falsely disabled grandparent.

**Age I** The social planner chooses benefits, continuation utilities, and informal child care so as to minimise expected cost subject to promised keeping constraint (10), incentive compatibility constraint for work (11), and informal child care constraint for a falsely disabled grandparent (12).

$$P^I(V) = \min_{\{b_H, b_D, V_H, V_D, \hat{V}_D, \hat{h}^G\}} p_1 \left[ b_H - \left( w^M + w^G \right) + qP^I_H(V_H) \right] + (1 - p_1) \left[ b_D - w^M + qP^I_D(V_D, \hat{V}_D) \right]$$

s.t.

$$p_1 \left[ u \left( b_H - w^F \right) + \beta V_H \right] + (1 - p_1) \left[ u \left( b_D - w^F \right) + \beta V_D \right] = V$$

(10)

$$u \left( b_H - w^F \right) + \beta V_H \geq u \left( b_D - w^F \left( 1 - \hat{h}^G \right) \right) + v^G \left( 1 - \hat{h}^G \right) + \beta \hat{V}_D$$

(11)

$$\hat{h}^G = \arg\max_{\{h\}} u \left( b_D - w^F \left( 1 - h \right) \right) + v^G \left( 1 - h \right)$$

(12)

Equation (10) is the promise keeping constraint where the social planner guarantees expected dynamic utility $V$ to the family. Inequality (11) is the incentive compatibility constraint which states that healthy families utility when working need to be at least as high as utility from falsely claiming disabled and engaging in informal child care activities $\hat{h}^G$. $\hat{V}_D$ is the continuation utility level that the planner gives to a false disabled family next period via threat keeping constraint (17) below.

**Age II Family** Age II families have child care needs requiring half of the time. For an Age II family starting the period with a previously working grandparent, the planner minimises expected cost subject to promise keeping constraint (13), incentive compatibility constraint for work (14), and informal child care constraint for a falsely disabled grandparent (15).

$$P^{II}_H(V) = \min_{\{b_H, b_D, V_H, V_D, \hat{V}_D, \hat{h}^G\}} p_2 \left[ b_H - \left( w^M + w^G \right) + qP^{II}_H(V_H) \right] + (1 - p_2) \left[ b_D - w^M + qP^{II}_D(V_D, \hat{V}_D) \right]$$

s.t.

$$p_2 \left[ u \left( b_H - w^F \frac{1}{2} \right) + \beta V_H \right] + (1 - p_2) \left[ u \left( b_D - w^F \frac{1}{2} \right) + \beta V_D \right] = V$$

(13)

$$u \left( b_H - w^F \frac{1}{2} \right) + \beta V_H \geq u \left( b_D - w^F \left( \frac{1}{2} - \hat{h}^G \right) \right) + v^G \left( 1 - \hat{h}^G \right) + \beta \hat{V}_D$$

(14)

$$\hat{h}^G = \arg\max_{\{h\}} u \left( b_D - w^F \left( \frac{1}{2} - h \right) \right) + v^G \left( 1 - h \right)$$

(15)

For an Age II family starting the period with a grandparent who claimed disabled in the previous period, the planner minimises expected cost subject to promise keeping constraint (16), threat keeping constraint

17
(17), and informal child care constraint for a falsely disabled grandparent (18).

\[ P_D^H (V, \hat{V}) = \min_{\{b_D, V_D, \hat{V}_D, \hat{h}^G\}} b_D - w^M + qP_D^{HI} (V_D, \hat{V}_D) \]

s.t.

\[ u \left( b_D - w^{F \frac{1}{2}} \right) + \beta V_D = V \quad (16) \]

\[ p_2 \left[ u \left( b_D - w^{F \frac{1}{2}} \right) + v^G \left( 1 - \hat{h}^G \right) + \beta \hat{V}_D \right] + (1 - p_2) \left[ u \left( b_D - w^{F \frac{1}{2}} \right) + \beta V_D \right] = \hat{V} \quad (17) \]

\[ \hat{h}^G = \arg\max_{h} u \left( b_D - w^{F \frac{1}{2}} \right) + v^G (1 - h) \quad (18) \]

Equation (17) is the threat keeping constraint allocating threatened utility \( \hat{V} \) to a family which falsely claimed disabled at Age I.

**Age III Family** The grandparent in an Age III family is retired. For an Age III family starting the period with a previously working grandparent, the planner minimises expected cost subject to promise keeping constraint (19) and incentive compatibility constraints for truthful reporting of health shock (20) and (21).

\[ P_H^{III} (V) = \min_{\{b_H, b_D, V_H, \hat{V}_H, V_D, \hat{V}_D\}} p_3 \left[ b_H - w^M + qP_H^{IV} (V_H, \hat{V}_H) \right] + (1 - p_3) \left[ b_D - w^M + qP_D^{IV} (V_D, \hat{V}_D) \right] \]

s.t.

\[ p_3 \left[ u (b_H) + v^G (1) + \beta V_H \right] + (1 - p_3) \left[ u (b_D) + \beta V_D \right] = V \quad (19) \]

\[ u (b_H) + v^G (1) + \beta V_H \geq u (b_D) + v^G (1) + \beta \hat{V}_D \quad (20) \]

\[ u (b_D) + \beta V_D \geq u (b_H) + \beta \hat{V}_H \quad (21) \]

Inequality (20) is the incentive compatibility constraint for a healthy family to truly report healthy while inequality (21) is the incentive compatibility constraint of the disabled so that they truthfully report their disability status. We need to consider the latter constraint here since healthy individuals are not required to work after the retirement age and the disabled can therefore pretend to be healthy. \( \hat{V}_D \) is the continuation utility that a planner allocates to a healthy family claiming disabled and corresponds to threat keeping constraint (29) below while \( \hat{V}_H \) is the continuation utility that a planner allocates to a disabled family claiming to be healthy and corresponds to threat keeping constraint (25) below.

For an Age III family starting the period with a grandparent who previously claimed disabled, the planner minimises expected cost subject to promise keeping constraint (22) and threat keeping constraint (23).

\[ P_D^{III} (V, \hat{V}) = \min_{\{b_D, V_D, \hat{V}_D\}} b_D - w^M + qP_D^{IV} (V_D, \hat{V}_D) \]
s.t.

\[ u(b_D) + \beta V_D = V \]  \hspace{1cm} (22)

\[ p_3 \left[ u(b_D) + v^G(1) + \beta V_D \right] + (1 - p_3) \left[ u(b_D) + \beta V_D \right] = \hat{V} \]  \hspace{1cm} (23)

**Age IV Family**  The Age IV family does not have any child care needs and the grandparent is retired. For an Age IV family with a grandparent who previously claimed healthy, the planner minimises cost subject to promise keeping (24), threat keeping (25), and incentive compatibility constraints (26) and (27).

\[ P_{IV}^H (V, \hat{V}) = \min \{b_H, b_D, V_H, V_D\} \ p_4 \left[ b_H - w^M + qP_I (V_H) \right] + (1 - p_4) \left[ b_D - w^M + qP_I (V_D) \right] \]

s.t.

\[ p_4 \left[ u(b_H) + v^G(1) + \beta V_H \right] + (1 - p_4) \left[ u(b_D) + \beta V_D \right] = V \]  \hspace{1cm} (24)

\[ \max \{ u(b_H) + \beta V_H, u(b_D) + \beta V_D \} = \hat{V} \]  \hspace{1cm} (25)

\[ u(b_H) + v^G(1) + \beta V_H \geq u(b_D) + v^G(1) + \beta V_D \]  \hspace{1cm} (26)

\[ u(b_D) + \beta V_D \geq u(b_H) + \beta V_H \]  \hspace{1cm} (27)

The threat keeping constraint (25) now needs to consider the fact that a grandparent who got the disability shock at Age III but who declared healthy, can still pretend to be healthy at Age IV or alternatively she can declare disabled. Moreover, since the grandparent dies next period, there is no persistence in health shock between an Age IV and a future Age I family so that we do not need to keep track of the threatened utilities anymore. The incentive compatibility constraints for truthful telling of health shocks (26) and (27) are therefore modelled as the typical temporary incentive compatibility constraints from recursive problems without persistence in unobserved shocks.

For an Age IV family starting the period with a grandparent who previously claimed disabled, the planner minimises cost subject to promise keeping (28) and threat keeping constraints (29).

\[ P_{IV}^D (V, \hat{V}) = \min \{b_D, V_D\} \ b_D - w^M + qP_I (V_D) \]

s.t.

\[ u(b_D) + \beta V_D = V \]  \hspace{1cm} (28)

\[ p_4 \left[ u(b_D) + v^G(1) + \beta V_D \right] + (1 - p_4) \left[ u(b_D) + \beta V_D \right] = \hat{V} \]  \hspace{1cm} (29)
5 Calibration

5.1 Baseline Parameters

Each period in our model consists of 6.5 years. We set the discount factors for the planner and for the family \( q = \beta \frac{1}{1 + R} = 0.94 \) corresponding to an annual interest rate of 1%. Since women are more likely to be involved in child care, we calibrate the model according to women’s disability probabilities. We choose the probability of being disabled according to McNeil (2001) “Americans with Disabilities: 1997” based on the SIPP (Survey of Income and Program Participation). Proportion of disabled grandparents in an Age I, Age II, Age III and Age IV households are respectively \( 1 - p_I = 0.24, 1 - p_{II} = 0.37, 1 - p_{III} = 0.48, \) and \( 1 - p_{IV} = 0.48. \) Proportion of healthy grandparents is one minus the proportion of disabled grandparents.

We compute the probability of being healthy conditional on age group and conditional on being healthy in the previous period so that \( p_1 = 0.76, p_2 = 0.83, p_3 = 0.84 \) and \( p_4 = 1. \) We assume that preferences take the form \( \ln (c + 1) + \alpha_M \ln \left( t^M + 1 \right) + \alpha_G \ln \left( t^G + 1 \right) \) so that \( u(0) = v^M(0) = v^G(0) = 0. \) We calibrate \( \alpha_M = 0.48 \) and \( \alpha_G = 0.42 \) so that Age I families with working parents and grandparents devote approximately \( \frac{1}{3} \) of total time endowment to work given the US tax system outlined in the next section.

We set the wage of young parents \( w^M \) and wage of grandparents \( w^G \) according to median usual earnings from “Highlights of Women’s Earnings in 1998” report based on CPS (Current Population Survey) data [US Department of Labour]. Median hourly earnings of young parents aged 25 to 54 were between $8.81 to $9.79, so on average $9.3 in 1998. We normalise wage of young parents to one \( w^M = 1. \) Median hourly earnings of older women aged 55 and over were between $7.22 to $8.86, so on average $8.04 yielding a relative wage of \( w^G = 0.86 \) for grandparents. Median hourly earnings of child care workers were on average $6.91 in the period 1997 to 1999 based on the “1999 National Occupational Employment and Wage Estimates Personal Care and Service Occupations” [US Department of Labour, Bureau of Labour Statistics] yielding a relative child care cost of \( w^F = 0.74. \) A summary of the parameters is available in Table 1.

<table>
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<th>Table 1: Baseline Parameters</th>
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5.2 Computation of Initial Promised Utility

We compute initial promised utility according to the 1997 US welfare system. In particular, we take into account (i) Federal Taxes that working individuals have to pay, (ii) Earned Income Tax Credit (EITC), and Dependent Care Tax Credit (DCTC) that working young parents with children below 13 are eligible for, (iii) Social Security Disability Insurance (SSDI) for disabled individuals under 65, and (iv) Social Security Retirement Benefits for retired individuals aged 65 and above.

Federal taxes are progressive in the United States and allow for deductions such as the EITC and DCTC. The EITC is a refundable tax credit that supplements the earnings of low income workers. There are three phases: (i) a subsidy range where each additional earned dollar increases the credit by 40 cents up to a maximum of $2,210 for a family with one child in 1997, (ii) a flat range where workers receive the maximum credit, and (iii) a phase-out range where credit is gradually reduced as workers’ earnings increase. The DCTC is available to tax payers with children under 13 and can be used to offset 30% of child care expenses up to $2,400 per year in 1997 for a family of one child and up to $4,800 per year for a family with two or more children. An agent is eligible for SSDI if (i) they are below the full retirement age of 65, (ii) have worked for at least 5 out of the 10 most recent years, (iii) have a physical or mental impairment preventing them from performing work related activities. This includes backpain and mental illnesses. Once an agent reaches the retirement age, the benefits are automatically converted into retirement benefits. Detailed computation of taxes and benefits are outlined in Appendix B1.

The US welfare system does not consider intergenerational linkages and therefore promised utility to an Age I intergenerational family is the same irrespective of the dynasty they belong to. From our overlapping generations framework, each four period is a repetition of each other so that an Age I intergenerational family utility is

\[ V_{USA} = u^I + \beta u^{II} + \beta^2 u^{III} + \beta^3 u^{IV} + \beta^4 V_{USA} \]

where \( u^{Age} \) is expected instantaneous utility of an \( Age \in \{I, II, III, IV\} \) family. The intergenerational family maximises \( u^I + \beta u^{II} + \beta^2 u^{III} + \beta^3 u^{IV} \) by choosing when to claim disability benefits and informal child care in the case of a false claim. Since disabled grandparents cannot work, the family will always claim disability benefits if the grandparent is truly disabled. On the other hand, healthy families have three options: (i) claim only when truly disabled, (ii) claim from Age I onwards, or (iii) claim from Age II onwards. At Ages III and IV the grandparent is retired and receives retirement benefits.

According to the parameters of our model, our representative family would maximise utility by claiming disability benefits from Age I onwards under the US welfare system. Moreover, a healthy Age I grandparent would on average provide 24 hours of child care per week which corresponds to the average hours of grandparent provided child care received by pre-school children with employed mothers in the USA\(^{12}\).

\(^{12}\)US Census Bureau Reports “Who’s Minding the Kids? Child Care Arrangements” based on data from the Survey of Income and Program Participation (SIPP)
6 Optimal Policy

We compute the optimal policy using parametric dynamic programming. The numerical algorithm is outlined in Appendix B2. In this section, we first analyze the optimal policy in the case where the planner can only use lump sum transfers. We then compute the optimal subsidies required to implement the AIO optimum with unobservable health shocks only. We contrast the optimal average tax rate with that of the US system and compute the potential cost savings from implementing the optimal policy.

6.1 Optimal Disability Insurance with Lump Sum Transfers

We illustrate the optimal policy in the AIU case of asymmetric information with unobserved health shocks and unobserved informal child care when the social planner uses lump sum transfers only. Figure 5 illustrates the optimal benefits and consumption in two scenarios. The dashed lines represent optimal benefits and consumption when the family is faced with a series of good shocks so that the grandparent is healthy and working prior to retirement age 65. The solid lines represent a family with a disability shock at Age I so that the grandparent does not work.

![Figure 5: Optimal Lump Sum Benefits and Consumption](image)

As can be seen from Figure 5(a), optimal benefits decline as the family gets older. Recall from our life cycle diagram 4 that a grandparent in an Age I family is aged between 52 and 58.5 while the child is less than 6.5 years old thereby requiring full time child care. On the other hand, a grandparent in an Age II family is aged between 58.5 and 65 while the child is aged between 6.5 and 13 thereby requiring half of child care time. Age III and IV families have retired grandparents above 65 and no child care needs. Thus, as the child gets older, thereby requiring lower child care time, optimal benefits decline.

Benefits and consumption given to a family with a healthy working grandparent are higher compared to those given to a disabled family in order to maintain work incentives. As can be seen from Figure 5(b) for a family experiencing the good health shock, consumption increases at Age II when the grandparent is still working. From there onwards, consumption is perfectly smoothed since the grandparent will be retired at age 65 and there is no more need to provide work incentives. For a disabled family, consumption is perfectly smoothed once the grandparent is disabled.
6.2 Child Care Subsidies

Figure 6(a) illustrates the level of informal child care that a falsely disabled family would engage in should the planner use only lump sum transfers. Since optimal lump sum benefits decline with child care needs, the family maintains the same level of informal child care to smooth consumption. Informal child care in this case corresponds to 14.4 hours per week until the child is in no more need of child care. In this case, the optimal child care subsidies are also smoothed across Ages I and II as shown by the solid line in Figure 6(b). Optimal child care subsidies corresponds to approximately 26% of formal child care cost. To contrast with the US child care subsidies, we compute the subsidy rate for the USA from the DCTC amount that our representative family would receive: \( \text{Subsidy}_{USA} = \frac{\text{Total DCTC Benefit}}{\text{Formal Child Care Cost}} \). This yields subsidies of 4% of formal child care cost when the child is below 6.5 and 10% of cost when the child is between 6.5 and 13. The subsidy rate increases as the child gets older because the DCTC total benefit is capped above formal child care cost of $2,400 per year for a family with one child. On the other hand, formal child care cost required by our representative family is halved as the child gets older, thereby the increase in the subsidy rate. Thus, the subsidy rate from our optimal program is higher than the current US subsidies and does not impose an upper limit on the total subsidy level\(^\text{13}\).

\[\text{Figure 6: Optimal Child Care Subsidies}\]

\[\text{(a) Child Care provided by a Falsely Disabled Grandparent} \]

\[\text{(b) Child Care Subsidy} \]

6.3 Average Tax Rate

We now contrast the average tax rate implied by our optimal program with that of the US system. Average tax rate is compute as follows: \( \text{ATR} = 1 - \frac{\text{Family Consumption}}{\text{Family Current Income}} \). Optimal average tax rate is represented by the solid lines in Figure 7 while the dashed lines represent average tax rate in the US. Panel (a) shows the ATR for families with a healthy grandparent who works prior to age 65 while panel (b) shows the ATR for families with a disabled grandparent who does not work. ATR in the US decreases as the family gets older due to the declining child care needs thereby leading to an increase in family consumption. The

\(^\text{13}\)Of course other informational problems not considered in this paper may enter the equation so that an upper limit might be desirable to prevent families from using overpriced formal child care. In this case, one can think of imposing an upper limit that varies with the age of the child since smaller children have higher child care needs. This might help in smoothing the subsidy rate across period.
A small jump between Age III and Age IV is due to the fact that once the child is above 18, the family loses EITC benefits. Moreover, when the grandparent is 65, she is retired and receives retirement benefits, thereby leading to a negative ATR. Our optimal program recommends a lower ATR compared to the US for a healthy family and vice versa for a disabled family. This stems from the fact that higher consumption needs to be provided for healthy families and lower consumption to disabled families in order to maintain work incentives. Thus, while the optimal child care subsidies are higher compared to the US subsidies so as to discourage involvement in informal child care, disabled families also receive lower consumption and therefore lower lump-sum benefits so as to preserve work incentives on the market.

Figure 7: Average Tax Rate

Figure 8 illustrates how the ATR path varies if one claims disability at Age I as compared to Age II. While the grandparent is aged below 58.5, average tax rate in the USA is higher for families with a working grandparent as compared to disabled families as illustrated respectively by the dashed and solid lines in panel (a). This illustrates the potential disincentive effects of the disability insurance system [Haveman & al (1991), Gruber (2000)]. The ATR also declines with child care needs as the family gets older. Under the SSDI, disability benefits are permanent and increasing in the age at which one declares disabled thereby explaining the permanent gap in ATR between the two families even after the grandparent reaches retirement age of 65.

Figure 8: Average Tax Rate by Disability Claiming Age

The optimal scheme recommends a slightly higher ATR for disabled families as opposed to working
families as illustrated in panel (b). Families who become disabled at Age II enjoy much lower ATR compared to those who were already disabled at Age I. Once retirement age is reached, ATR is nearly the same for the two families. The intuition behind this result is that while the planner needs to maintain work incentives by imposing a higher average tax rate on families becoming disabled early on, once the retirement age of 65 is reached, there are no work incentive problems anymore so that consumption can be smoothed out.

6.4 Cost Savings

The cost of the US welfare system is computed based on delivering the initial promised utility computed in section 5.2. Since the current system does not take intergenerational linkages into account, under the assumption of stationarity of the US welfare system, each new family is expected to receive the same utility so that

\[ P_{USA}(V_{USA}) = \frac{1}{1 - q} \sum_{t=0}^{3} q^t Cost_t \]

where \( Cost_t \) are expected net taxes and benefits given to a family in period \( t \). We then compute the cost of providing the same initial promised utility under the optimal policy rules in the AIU world where the planner can only use lump sum transfers

\[ P_{AIU}^I (V_{USA}) = p_1 \left[ b_H^{**} - \left( w^M + w^G \right) + qP^I_H (V_H^{**}) \right] + (1 - p_1) \left[ b_D^{**} - w^M + qP^I_D \left( V_D^{**}, \hat{V}_D^{**} \right) \right] \]

where \( ** \) denotes optimal AIU allocations. On the other hand, if child care subsidies can be used in addition to the lump sum transfers, the planner would face costs similar to the AIO level

\[ P_{AIO}^I (V_{USA}) = p_1 \left[ b_H^{**} - \left( w^M + w^G \right) + qP^I_H (V_H^{**}) \right] + (1 - p_1) \left[ b_D^# + sw^F - w^M + qP^I_D \left( V_D^{**}, \hat{V}_D^{**} \right) \right] \]

where \( # \) denote AIO benefit level and \( \# \) AIU while \( s \) denote child care subsidies given to disabled families. We also compute the cost savings for the full information case. The cost savings with respect to the US system for families of one and two children are as follows:

<table>
<thead>
<tr>
<th>( \frac{P_{USA} - P_{Optimal}}{P_{USA}} \times 100 )</th>
<th>AIU</th>
<th>AIO</th>
<th>FI</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.371%</td>
<td>3.389%</td>
<td>3.571%</td>
<td></td>
</tr>
</tbody>
</table>

Adopting the optimal scheme with lump sum taxation can lead to sizeable cost savings of around 3.4% for our representative family with a grandmother, a young working mother, and one child. On the other hand, the additional use of child care subsidies as policy instrument leads to additional cost savings of small magnitude relative to the AIU optimum (approximately 0.52%), suggesting that the informational costs of informal grandchild care on the US disability program are relatively small.
7 Conclusion and Discussion

An ageing population poses increasing strains on the welfare system as witnessed by the rising trends in health expenditures in the US. The current welfare system focuses on the nuclear family and does not take intergenerational linkages into account. Yet, with increased life expectancy and declining fertility rates, comes a verticalisation of families so that intergenerational help and interaction is not uncommon as witnessed by the rising trends in grandparent provided child care in the US. In this paper, we analyse the implications of elderly grandparents engaging in unobserved informal child care on the work incentives of those grandparents and propose an optimal benefit scheme consisting of lump sum taxes and child care subsidies. Our main findings are that while unobserved informal child care exacerbates work incentives, the use of child care subsidies can help remedy the problem, thereby offering an efficiency case for child care subsidies. We find that the optimal scheme can potentially lead to sizeable cost savings relative to the US welfare system.

We now discuss some potential limitations and extensions. Firstly, implementing the optimal child care subsidies may be tricky. Due to the discrete nature of market work in our model, child care subsidies given to all families can implement the AIO optimum. One can then think of implementing the scheme via subsidised day care where the subsidy is given to formal care providers. However, should there be multiple possible health types thereby requiring a sliding scale for child care subsidies, the subsidies paid to the young parents would be a function of the reported health of the grandparent. In this case, the subsidy will have to be paid directly to the family. This raises the issue that not all intergenerational families are as cooperative as the one presented in this paper so that the intergenerational link gets broken down. The presence of different types of families introduces another informational asymmetry between the social planner and the family. A possible proxy to distinguish between the different types of families could be living arrangements with those living together most likely to cooperate and get involved in child care. Moreover, our results for the sliding scale child care subsidies are so far based on the assumption of separability between consumption and leisure of the grandparent. The child care subsidies may not be monotonically increasing in disability type should there be non separability in preferences. Future research could involve estimating intergenerational household preferences although current lack of detailed information on all members in intergenerational households make this empirically challenging. Furthermore, there is the additional issue that young parents could also be subject to employment shocks whose distribution is influenced by unobserved job search effort thereby introducing another information asymmetry between the planner and the intergenerational family [Ho and Pavon work-in-progress]. Both grandparent and young parent could also have different market as well as home productivities which would make the design of the optimal contract much more complicated since it would bring us in a multidimensional screening context [Chone and Larroque (2009), Beaudry and al (2009)]]. Considering the ageing population problem as well as the increased grandparent involvement in grandchild care, we believe that designing policies targeted at the intergenerational families is an important issue and that further research in this direction would be interesting.
References


Appendix A: Static Model

Appendix A.1: Proofs

Proof of Proposition 2

Consider the AIO problem where the planner does not observe health shocks but can observe informal child care. The planner minimises (1) s.t. (2) and (5). This yields AIO allocations

\[ c_H^* = b_H^* - w^F \]
\[ c_D^* = b_D^* - w^F \]

where \( c_z^* \) is optimal AIO consumption in state \( z \in \{ H, D \} \) and \( ^* \) represent AIO.

Now, consider the AUI problem where the planner can neither observe health shocks nor informal child care. A tax system that implements the AIO allocation would be to choose benefits and child care subsidy such that \( b_H^*, b_D^* \) and \( s \) satisfy

\[ c_H^* = b_H^* - w^F \]
\[ c_D^* = b_D^* - (1 - s) w^F \]

and where child care subsidies are such that given \( c_D^* \), the a healthy family pretending to be disabled would choose zero child care. Since the family problem is concave, we can get the optimal subsidies from the family’s first order condition for informal child care at \( \hat{h}_D = 0 \)

\[
\begin{align*}
    u' \left( b_D^* - (1 - s) w^F \right) (1 - s) w^F &= v' (1) \\
    u' (c_D^*) (1 - s) w^F &= v' (1) \\
    s &= 1 - \frac{v' (1)}{w (c_D^*)} \frac{1}{w^F}
\end{align*}
\]

We now proceed to show that this schedule of lump sum transfers and child care subsidies is incentive compatible, yields the same expected utility to families and costs the same to the social planner as under the AIO optimum. Healthy families now have three choices: (i) work and receive \( b_H^* \), (ii) declare disabled and receive \( b_D^* \) but do not claim the subsidies, or (iii) declare disabled and receive \( b_D^* \) and claim the subsidies and receive \( sw^F \) for each unit of formal child care employed. Thus, given the planner’s benefit and child care subsidies schedule, healthy families decide

\[
\max \{ \text{Work, No Sub, No Work} \} \{ v_{\text{Work}}, v_{\text{No Sub}}, v_{\text{No Work}} \}
\]

where

\[
v_{\text{Work}} = u (b_H^* - w^F) \\
v_{\text{No Sub}} = u \left( b_D^* - w^F \left( 1 - \hat{h} \right) \right) + v \left( 1 - \hat{h} \right) , \hat{h} = \arg \max_{\{ h \}} u \left( b_D^* - w^F \left( 1 - h \right) \right) + v \left( 1 - h \right) \\
v_{\text{No Work}} = u \left( b_D^* - (1 - s) w^F \right) + v (1)
\]

\( V_{\text{Work}} \) is the utility of a healthy family that works. \( V_{\text{No Sub}} \) is the utility of a family that declares disabled but does not claim the child care subsidies. In this case, the family can engage in unobserved informal child care where \( \hat{h} \) is the family’s utility maximising level of informal child care given benefits \( b_D^* \). \( V_{\text{No Work}} \) is the utility of a family that declares disabled and claims the child care subsidies as well. We now proceed to show that the healthy family would prefer to work.

Child care subsidies \( s > 0 \) ensure that

\[
u \left( b_D^* - (1 - s) w^F \left( 1 - \hat{h} \right) \right) + v \left( 1 - \hat{h} \right) > u \left( b_D^* - w^F \left( 1 - \hat{h} \right) \right) + v \left( 1 - \hat{h} \right) = v_{\text{No Sub}}
\]
Thus, if the family declares disabled, it would also prefer to claim the subsidies. We also know that the subsidies are designed such that from family’s first order condition for child care

\[ u' \left( b^\#_D - (1 - s) w_F \right) (1 - s) w_F = v' (1) \]

implying that if \( \hat{h} > 0 \)

\[ u' \left( b^\#_D - (1 - s) w_F \left( 1 - \hat{h} \right) \right) (1 - s) w_F < v' \left( 1 - \hat{h} \right) \]

from concavity of \( u(.) \) and \( v(.) \). Thus if the family claims the subsidies, it will be optimal to not provide informal child care

\[ V^{NoWork} = u \left( b^\#_D - (1 - s) w_F \right) + v (1) > u \left( b^\#_D - (1 - s) w_F \left( 1 - \hat{h} \right) \right) + v \left( 1 - \hat{h} \right) \]

Thus, it must be that \( V^{NoWork} > V^{NoSub} \). Also, from incentive compatibility constraint for work (5) in the AIO problem,

\[ u \left( b^*_{H} - w_F \right) \geq u \left( b^\#_D - w_F \right) + v (1) \]

\[ u \left( b^*_{H} - w_F \right) \geq u \left( b^\#_D - (1 - s) w_F \right) + v (1) \]

Thus, it must be that \( V^{Work} \geq V^{NoWork} > V^{NoSub} \) so that incentive constraints are satisfied and healthy families would prefer to work.

Moreover, since families are getting the same consumption as in AIO, their expected utility is the same as in AIO.

\[ p u \left( b^*_{H} - w_F \right) + (1 - p) u \left( b^\#_D - (1 - s) w_F \right) = p u \left( b^*_{H} - w_F \right) + (1 - p) u \left( b^\#_D - w_F \right) = V \]

Planner’s cost is also the same since under this tax system

\[ P = p \left( b^*_{H} - w \right) + (1 - p) \left( b^\#_D + sw_F \right) \]

\[ = p \left( c^*_{H} + w_F - w \right) + (1 - p) \left( c^\#_D + w_F \right) \]

\[ = p \left( b^*_{H} - w \right) + (1 - p) \left( b^\#_D \right) \]

which is the same value as under the AIO optimum. QED

**Proof of Lemma 1**

Let \( n > m \). Consider the incentive constraints for type \( n \) trying to mimic type \( m \) and for type \( m \) trying to mimic type \( n \) respectively

\[ u \left( b_n - w_F \right) + v \left( 1 - d_n - \frac{y_n}{w} \right) \geq u \left( b_m - w_F \right) + v \left( 1 - d_n - \frac{y_n}{w} \right) \]

\[ u \left( b_m - w_F \right) + v \left( 1 - d_m - \frac{y_m}{w} \right) \geq u \left( b_n - w_F \right) + v \left( 1 - d_m - \frac{y_m}{w} \right) \]

Adding them together and rearranging, we have

\[ v \left( 1 - d_n - \frac{y_n}{w} \right) - v \left( 1 - d_m - \frac{y_m}{w} \right) = \int_m^n v' \left( 1 - d_i - \frac{y_n}{w} \right) di \]

\[ \geq \int_m^n v' \left( 1 - d_i - \frac{y_m}{w} \right) di = v \left( 1 - d_n - \frac{y_m}{w} \right) - v \left( 1 - d_m - \frac{y_m}{w} \right) \]

and from concavity of \( v(.) \), it must be that

\[ y_n \geq y_m \]
Thus, for the incentive constraints to be satisfied, if $n > m$ it must be that
\[ b_n \geq b_m \]
so that benefits are non-decreasing in types. QED

Proof of Lemma 2

Consider the first order condition for child care of a type $m$ family claiming to be of type $n$ and the first order condition for child care of a type $k < m$ family claiming to be of type $n$ respectively
\[
u' \left( b_n - w^F \left( 1 - \hat{h}_m^n \right) \right) w^F \leq \nu' \left( 1 - d_m - \frac{y_n}{w} - \hat{h}_m^n \right), \quad \hat{h}_m^n \geq 0
\]
\[
u' \left( b_n - w^F \left( 1 - \hat{h}_k^n \right) \right) w^F \leq \nu' \left( 1 - d_k - \frac{y_n}{w} - \hat{h}_k^n \right), \quad \hat{h}_k^n \geq 0
\]
Suppose that it is optimal to provide zero child care for the $m$ type family so that their first order condition for child care is satisfied at zero child care
\[
u' \left( b_n - w^F \right) w^F \leq \nu' \left( 1 - d_m - \frac{y_n}{w} \right), \quad \hat{h}_m^n = 0
\]
Since $d_m < d_k$, by concavity of $\nu()$, we have $\nu' \left( 1 - d_k - \frac{y_n}{w} \right) > \nu' \left( 1 - d_m - \frac{y_n}{w} \right)$, so that first order condition for child care at zero child care for family of type $k < m$ is automatically satisfied with strict inequality
\[
u' \left( b_n - w^F \right) w^F < \nu' \left( 1 - d_k - \frac{y_n}{w} \right), \quad \hat{h}_k^n = 0
\]
Thus, it would be optimal for family of type $k < m$ to provide zero child care when claiming to be of type $n$ if this is optimal for family of type $m$\textsuperscript{14}. QED

Proof of Proposition 3

Consider the AIO problem where the planner does not observe health shocks but can observe informal child care. In this case, the planner minimises (6) s.t. (7) and incentive compatibility constraints for work
\[
\forall m > n, \ u \left( b_n - w^F \right) + v \left( 1 - d_n - \frac{y_n}{w} \right) \geq u \left( b_m - w^F \right) + v \left( 1 - d_n - \frac{y_m}{w} \right)
\]
This yields AIO allocations
\[
\forall n, \ c_n^{**} = b_n^{**} - w^F; \quad y_n^{**}
\]
Now consider the AIU problem where the planner can neither observe health shocks nor informal child care. A tax system that implements the AIO allocation would be to choose benefits and child care subsidies such that
\[
\forall n, \ c_n^{**} = b_n^{**} \left( 1 - s_n \right) w^F; \quad y_n^{**}
\]
and where child care subsidies are such that given $c_n^{**}$, an $N$ type family who reports to be of type $n$ would choose zero child care. Consider the first order condition for child care of the highest/healthiest family type $N$ with $d_N = 0$ claiming to be of type $n < N$.
\[
u' \left( b_n - w^F \left( 1 - \hat{h}_N^n \right) \right) w^F \leq \nu' \left( 1 - \frac{y_n}{w} - \hat{h}_N^n \right), \quad \hat{h}_N^n \geq 0
\]
\textsuperscript{14}Note that the above result depends on the assumption of separability of preferences between consumption and leisure. Should leisure be a substitute to consumption, so that lower leisure leads to higher marginal utility of consumption, the result may not hold. In this case, the $k$ type family might choose to increase child care so as to increase consumption.
To induce this family to provide zero child care when reporting that they are of type \( n \), the planner needs to set subsidies such that given the AIO consumption level, at zero child care the family’s first order condition is

\[
u' \left( b_n^# - (1 - s_n) w^F \right) (1 - s_n) w^F = v' \left( 1 - \frac{y_n^*}{w} \right), \quad \hat{h}_n^* = 0\]

By Lemma 2 above, if this condition holds for family of type \( N \) with, it will also be optimal for all families of type \( k < N \) to provide zero child care when claiming to be of type \( n \). First order conditions for all families of type \( k < N \) at zero child care

\[
u' \left( b_n^# - (1 - s_n) w^F \right) (1 - s_n) w^F < v' \left( 1 - d_k - \frac{y_n^*}{w} \right), \quad \hat{h}_k^* = 0\]

Thus, by giving child care subsidies equal to

\[s_n = 1 - \frac{v' \left( 1 - \frac{y_n^*}{w} \right)}{u' \left( c_n^* \right)} \frac{1}{w^F}\]

when a family produces \( y_n^* \), the planner will induce zero child care production from the dishonest family. Moreover, for families of type \( m > n \), since \( y_m \geq y_n \) and from Lemma 1 \( c_m^* \geq c_n^* \), by concavity of \( v(.) \) and \( u(.) \), \( s_n \geq s_m \) so that those who are unhealthier get higher subsidies, thereby leading to a sliding scale for child care subsidies.

To verify that this tax and benefit system implements the AIO optimum, we check whether incentive compatibility constraints are satisfied, families get the same expected utility as in AIO and planner’s value is the same as in AIO. The child care subsidies ensure that there is no unobserved informal child care when dishonest and incentive compatibility constraints for work are also satisfied

\[
\forall m, n, \quad u(\hat{b}_n^* - w^F) + v \left( 1 - d_n - \frac{y_n^*}{w} \right) \geq u(\hat{b}_m^* - w^F) + v \left( 1 - d_n - \frac{y_m^*}{w} \right)
\]

\[
\forall m, n, \quad u(\hat{b}_n^# - (1 - s_n) w^F) + v \left( 1 - d_n - \frac{y_n^*}{w} \right) \geq u(\hat{b}_m^# - (1 - s_m) w^F) + v \left( 1 - d_n - \frac{y_m^*}{w} \right)
\]

Families get the same expected utility since they get the same consumption levels as in AIO

\[
\sum_{n=1}^{N} p_n \left[ u(\hat{b}_n^# - (1 - s_n) w^F) + v \left( 1 - d_n - \frac{y_n^*}{w} \right) \right] = \sum_{n=1}^{N} p_n \left[ u(\hat{b}_n^* - w^F) + v \left( 1 - d_n - \frac{y_n^*}{w} \right) \right]
\]

Moreover, planner’s cost is

\[
P = \sum_{n=1}^{N} p_n \left( \hat{b}_n^# + s_n w^F - y_n^* \right)
\]

\[
= \sum_{n=1}^{N} p_n \left( c_n^* + w^F - y_n^* \right)
\]

\[
= \sum_{n=1}^{N} p_n \left( b_n^* - y_n^* \right)
\]

which is the same as under the AIO optimum. QED
Appendix A.2: Multiple Productivity Types with Continuous Labour Supply Choices and Unobserved informal Child Care

Framework

Consider an economy where we have $N$ type individuals with market wages $w_1 < \ldots < w_N$. Individuals also have child care needs requiring one unit of time and costing $w^F$ per unit of formal child care time employed. Each individual $n$ is endowed with one unit of time which they can allocate between labour supply $e_n$ and informal child care $h_n$. Informal child care and formal child care are perfect substitutes so that informal child care time decreases formal child care needs. Suppose that we have a risk neutral planner who provides disability insurance to families at a minimum cost and guarantees families an expected utility level of at least $V$. The planner does not observe wages, labour supply or informal child care but observes total market output of the individual $y_n = w_ne_n$. The planner gives benefits to individuals $b_n$ based on observed market output of that individual. This is equivalent to assuming that the planner sets taxes $\tau_n = y_n - b_n$.

Full Information

In the full information world, the planner can observe wages, hours work on the market and informal child care. In this case, the planner chooses benefit, market output and informal child care levels so as to minimise costs subject to delivering a given utility level to the family.

$$\min_{y_n, b_n, h_n} \sum_n p_n (b_n - y_n)$$

s.t.

$$\sum_n p_n \left[ u (b_n - w^F (1 - h_n)) + v \left(1 - h_n - \frac{y_n}{w_n}\right)\right] = V$$

Since market work and informal child care are perfect substitutes, if $w_n \geq w^F$, individual $n$ will only work on the market and have zero informal child care. On the other hand, if $w_n < w^F$, individual $n$ only work at home and not on the market. Moreover, from first order condition with respect to $b_n$,

$$u'(c_n) = \frac{1}{\lambda}$$

where $\lambda$ is the Lagrange multiplier on the participation constraint. Since their marginal utilities of consumption are equalised, consumption is also equalised for everyone. Moreover, if $w_n \geq w^F$,

$$u' (b_n - w^F) = u' \left(1 - \frac{y_n}{w_n}\right) \frac{1}{w_n} = \frac{1}{\lambda} \Rightarrow u' (b_n - w^F) w_n = u' \left(1 - \frac{y_n}{w_n}\right)$$

Since $b_n - w^F$ is constant, an increase in $w_n$ means an increase in $y_n$, and in $c_n$. Thus, higher ability types work more and produce more than lower ability types in the full information case: $y_F \leq \ldots \leq y_N$. Moreover, if $w_n < w^F$, individuals work at home only and informal child care is determined by first order condition

$$u' (b_n - w^F (1 - h_n)) w^F = u' (1 - h_n)$$

Since from first order condition for $b_n$, we know that $u' (b_n - w^F (1 - h_n)) = \frac{1}{\lambda}$ so that marginal utility of consumption is equalised for everyone and since $w^F$ is the same for everyone, it must be that $h_n = \frac{w^F}{\lambda} = e^*_F = h^*$ is also the same for everyone working at home. Thus, the difference between market types is irrelevant once one works at home since everyone has the same productivity $w^F$ once they work at home.
Full information allocations are

\[ h_n = \begin{cases} 0 & \text{if } w_n \leq w^F \\ h^* & \text{if } w_n > w^F \end{cases} \]
\[ c_n = \begin{cases} e_n^* & \text{if } w_n \geq w^F \\ 0 & \text{if } w_n < w^F \end{cases} \]
\[ b_n = \begin{cases} b^* + w^F h^* & \text{if } w_n \geq w^F \\ b^* & \text{if } w_n < w^F \end{cases} \]
\[ e_n = b^* - w^F (1 - h^*) \]
\[ c_n = b^* - w^F (1 - h^*) \]

AIO with Observable informal Child Care

In the AIO world, the planner cannot observe wage and labour supply separately but only output produced. In this case, full information allocations are not implementable. Since everyone gets the same consumption level irrespective of whether they work or not, individuals would choose labour supply and informal child care so as to maximise utility. Under full information allocations, those with \( w_n < w^F \) get utility \( u(b^* - w^F (1 - h^*)) \) and invest \( h_n = e^*_n = h^* \) into informal child care. They would therefore have no incentive to deviate since deviating would mean reporting a type higher than \( w^F \) and having to work more but for the same amount of consumption. On the other hand, those with \( w_n > w^F \) would have an incentive to declare that they are of a lower type, more particularly, type \( w^F \) and work on the market so as to produce \( y_F = w_F e^*_n = w_n c^*_n < w_n e^*_n \) which requires lower effort than the full information effort level \( e^*_n \) since in the full information case, \( y_F < \ldots < y_N \).

Thus, the planner has to take the work incentive constraints into account. In the following analysis, we proceed under the assumption that \( w_n \geq w^F \), \( \forall n \) for now so that the planner wishes for everyone to work on the market and not engage in informal child care. Thus, if informal child care is observable, the planner will recommend zero informal child care on all individuals.

The planner solves

\[
\min_{\{y_n, b_n\}} \sum_n p_n (b_n - y_n)
\]

s.t.
\[
\sum_n p_n \left[ u(b_n - w^F) + v \left( 1 - \frac{y_n}{w_n} \right) \right] = V
\]
\[
\forall m, n \quad u(b_n - w^F) + v \left( 1 - \frac{y_n}{w_n} \right) \geq \quad u(b_m - w^F) + v \left( 1 - \frac{y_m}{w_m} \right)
\]

**Lemma A1:** If \( n > m \), then \( b_n \geq b_m \)

**Proof:** Let \( n > m \). Consider the incentive constraints for type \( n \) trying to mimic type \( m \) and for type \( m \) trying to mimic type \( n \) respectively

\[
u(b_n - w^F) + v \left( 1 - \frac{y_n}{w_n} \right) \geq u(b_m - w^F) + v \left( 1 - \frac{y_n}{w_m} \right)
\]
\[
u(b_m - w^F) + v \left( 1 - \frac{y_m}{w_m} \right) \geq u(b_n - w^F) + v \left( 1 - \frac{y_m}{w_n} \right)
\]

Adding them together and rearranging, we have
\[
v \left( 1 - \frac{y_n}{w_n} \right) - v \left( 1 - \frac{y_m}{w_m} \right) = \int_{w_m}^{w_n} v' \left( 1 - \frac{y_n}{w} \right) dw \geq \int_{w_m}^{w_n} v' \left( 1 - \frac{y_m}{w} \right) dw = v \left( 1 - \frac{y_m}{w_n} \right) - v \left( 1 - \frac{y_m}{w_m} \right)
\]
and from concavity of \( v \), it must be that
\[
y_n \geq y_m
\]
Thus, for the incentive constraints to be satisfied, if \( n > m \) it must be that
\[
b_n \geq b_m
\]

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so that benefits are non decreasing in types. QED

**AIU with Unobservable informal Child Care**

With unobservable informal child care, the AIO optimal allocations may not be implementable since at the AIO consumption levels, individuals may choose to adjust informal child care and increase their utility when they lie. In this case, the incentive compatibility constraint would be violated. The planner’s problem now becomes

$$\min_{(y_n, b_n)} \sum_n p_n (b_n - y_n)$$

s.t.

$$\sum_n p_n \left[ u \left( b_n - w^F \left( 1 - \hat{h}_n^m \right) \right) + v \left( 1 - \frac{y_n}{w_n} - \hat{h}_n^m \right) \right] = V$$

$$\forall m, n \quad u \left( b_n - w^F \left( 1 - \hat{h}_n^m \right) \right) + v \left( 1 - \frac{y_n}{w_n} - \hat{h}_n^m \right) \geq u \left( b_m - w^F \left( 1 - \hat{h}_m^m \right) \right) + v \left( 1 - \hat{h}_n^m - \frac{y_m}{w_n} \right)$$

$$\forall m, n \quad u' \left( b_n - w^F \left( 1 - \hat{h}_n^m \right) \right) w^F \leq v' \left( 1 - \hat{h}_n^m - \frac{y_m}{w_n} \right), \hat{h}_n^m \geq 0$$

were $\hat{h}_n^m$ is informal child care by an agent of type $n$ claiming to be type $m$.

**Lemma A2**: Given planner’s benefit and tax schedule, if it is optimal for an $m$ type family to not provide child care when claiming to be an $n$ type, it will also be optimal for a $k < m$ type family to not provide child care when claiming to be an $n$ type.

**Proof**: Consider the first order condition for child care of a dishonest type $m$ family claiming to be of type $n$ and the first order condition for child care of a dishonest type $k < m$ family claiming to be of type $n$ respectively

$$u' \left( b_n - w^F \left( 1 - \hat{h}_n^m \right) \right) w^F \leq v' \left( 1 - \hat{h}_m^m - \frac{y_m}{w_m} \right), \hat{h}_m^m \geq 0$$

$$u' \left( b_n - w^F \left( 1 - \hat{h}_n^k \right) \right) w^F \leq v' \left( 1 - \hat{h}_k^m - \frac{y_m}{w_m} \right), \hat{h}_k^m \geq 0$$

Suppose that it is optimal to provide zero child care for the $m$ type family so that their first order condition for child care is satisfied at zero child care

$$u' \left( b_n - w^F \right) w^F \leq v' \left( 1 - \frac{y_n}{w_m} \right), \hat{h}_m^n = 0$$

Since $w_k < w_m$, by concavity of $v(\cdot)$, we have $v' \left( 1 - \frac{w_k}{w_m} \right) > v' \left( 1 - \frac{w_m}{w_m} \right)$, so that first order condition for child care at zero child care for family of type $k < m$ is also satisfied with strict inequality at zero child care

$$u' \left( b_n - w^F \right) w^F < v' \left( 1 - \frac{y_n}{w_k} \right), \hat{h}_k^n = 0$$

Thus, it would be optimal for family of type $k < m$ to provide zero child care when claiming to be of type $n$ if this is optimal for family of type $m$. QED

**Proposition A1**: There exists a schedule of lump sum taxes and sliding scale child care subsidies that can implement the AIO optimum.

**Proof**: Denote AIO consumption and production levels by

$$\forall n \quad c_n^{**} = b_n^{**} - w^F$$

$$y_n^{**} = y_n^{**}$$

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Now consider the AIU problem where the planner cannot observe health shocks nor informal child care. A tax system that implements the AIO allocation would be to choose benefits and child care subsidies such that

\[
\forall n, \quad c_n^{**} = \frac{b_n^# - (1 - s_n) w^F}{y_n^{**}},
\]

and where child care subsidies are such that given c_n^{**}, the dishonest family would choose zero child care. Consider the first order condition for child care of the highest/highest family type N claiming to be of type n.

\[
u' \left( b_n - w^F(1 - \bar{h}_N^N) - (1 - s_n) w^F \right) \leq v' \left(1 - \frac{y_n^{**}}{w_N} - \bar{h}_N^N \right)
\]

To induce this family to provide zero child care when reporting that they are of type n, the planner needs to set subsidies such that given the AIO consumption level, at zero child care the family’s first order condition is

\[
u' \left( b_n^# - (1 - s_n) w^F \right) (1 - s_n) w^F = v' \left(1 - \frac{y_n^{**}}{w_N} \right)
\]

By Lemma A2 above, if this condition holds for family of type N, it will also be optimal for all families of type \(k < N\) to provide zero child care when claiming to be of type \(n\). Thus, by giving child care subsidies equal to

\[
s_n = 1 - \frac{v' \left(1 - \frac{y_n^{**}}{w_n} \right)}{u' \left( c_n^{**} \right) \frac{1}{w^F}}
\]

when a family produces \(y_n\), the planner will induce zero child care production. Moreover, for families of type \(m > n\), since \(\frac{b_m^#}{w_N} \leq \frac{b_n^#}{w_N}\) by Lemma A1 and since from incentive compatibility constraint for work \(c_m^{**} \geq c_n^{**}\), by concavity of \(v(.)\) and \(u(.)\), \(s_n \geq s_m\) so that lower types get higher subsidies, thereby leading to a sliding scale for child care subsidies.

To verify that this tax and benefit system implements the AIO optimum, we firstly check whether families get the same expected utility as in AIO, incentive compatibility constraints are satisfied and planner’s value is the same as in AIO. Families get the same expected utility since they get the same consumption levels as in AIO

\[
\sum_{n=1}^{N} \left[ u \left( b_n^# - (1 - s_n) w^F \right) + v \left(1 - \frac{y_n^{**}}{w_n} \right) \right] = \sum_{n=1}^{N} \left[ u \left( b_n^{**} - w^F \right) + v \left(1 - \frac{y_n^{**}}{w_n} \right) \right]
\]

The child care subsidies ensure that there is no unobserved informal child care when dishonest and incentive compatibility constraints for work are also satisfied

\[
\forall m, n, \quad u \left( b_m^{**} - w^F \right) + v \left(1 - \frac{y_m^{**}}{w_m} \right) \geq u \left( b_n^{**} - w^F \right) + v \left(1 - \frac{y_n^{**}}{w_m} \right)
\]

\[
\forall m, n, \quad u \left( b_m^# - (1 - s_n) w^F \right) + v \left(1 - \frac{y_m^{**}}{w_m} \right) \geq u \left( b_n^# - (1 - s_n) w^F \right) + v \left(1 - \frac{y_n^{**}}{w_m} \right)
\]

Moreover, planner’s cost is

\[
P = \sum_{n=1}^{N} p_n \left( b_n^# - s_n w^F - y_n^{**} \right) = \sum_{n=1}^{N} p_n \left( c_n^{**} - w^F - y_n^{**} \right) = \sum_{n=1}^{N} p_n \left( b_n^{**} - y_n^{**} \right)
\]

which is the same as under the AIO optimum. QED
Appendix B: Numerical Analysis

Appendix B1: Taxes and Benefits

We compute federal taxes, EITC benefits, and DCTC over the life cycle of our representative family using the NBER TAXSIM calculator\(^5\). Taxes and benefits for the young working parent are computed under the assumption that (i) she works 40 hours a week at an hourly wage of $9.3 for 50 weeks a year, and (ii) employs formal care at $6.91 per hour. The taxes and benefits are then deflated to 1997 dollars and normalised according to the normalisation employed above with wage of young parent \(w^M = 1\). Net annual benefits for a working young parent are as follows

<table>
<thead>
<tr>
<th>Age</th>
<th>I</th>
<th>1 Kid</th>
<th>2 Kids</th>
<th>Age</th>
<th>II</th>
<th>1 Kid</th>
<th>2 Kids</th>
<th>Age</th>
<th>III</th>
<th>1 Kid</th>
<th>2 Kids</th>
<th>Age</th>
<th>IV</th>
<th>1 Kid</th>
<th>2 Kids</th>
</tr>
</thead>
<tbody>
<tr>
<td>Federal Tax</td>
<td>-1372</td>
<td>-975</td>
<td>-1372</td>
<td>975</td>
<td>-1372</td>
<td>-975</td>
<td>1770</td>
<td>-1770</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EITC</td>
<td>1143</td>
<td>2251</td>
<td>1143</td>
<td>2251</td>
<td>1143</td>
<td>2251</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DCTC</td>
<td>604</td>
<td>1209</td>
<td>604</td>
<td>1209</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Normalised</td>
<td>0.02</td>
<td>0.13</td>
<td>0.02</td>
<td>0.12</td>
<td>-0.01</td>
<td>0.07</td>
<td>-0.1</td>
<td>-0.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

According to our model, a grandparent only has adult children and therefore is not eligible for EITC or DCTC. A working grandparent still has to pay federal taxes. We compute taxes based on the assumption that she works 40 hours a week at an hourly wage of $8.04 for 50 weeks a year. We compute SSDI and Retirement benefits according to the Social Security Benefits calculator\(^6\). We compute benefits based on the assumption that (i) the grandparent worked as a young parent, (ii) the grandparent becomes disabled at the start of the period. Benefits level depend on the age at which one becomes disabled and Average Indexed Monthly Earnings (AIME) which is a function of monthly earnings, work experience, and average wage growth in the economy. The lowest earning years are excluded from the formula\(^7\). Disability and retirement benefits are given below

<table>
<thead>
<tr>
<th>Working Grandparent</th>
<th>Ages I and II</th>
<th>Disabled at 52</th>
<th>Age I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual Federal Tax</td>
<td>-1638</td>
<td>Monthly Disability Benefits</td>
<td>1105</td>
</tr>
<tr>
<td>Normalised</td>
<td>-0.08</td>
<td>Normalised</td>
<td>0.71</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Disabled at 58.5</th>
<th>Age II</th>
<th>Retired at 65</th>
<th>Age III and IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monthly Disability Benefits</td>
<td>1359</td>
<td>Monthly Retirement Benefits</td>
<td>990</td>
</tr>
<tr>
<td>Normalised</td>
<td>0.88</td>
<td>Normalised</td>
<td>0.64</td>
</tr>
</tbody>
</table>

\(^5\)The National Bureau of Economic Research (NBER) TAXSIM calculator is a FORTRAN program for calculating liabilities under US Federal and State income tax laws from individual data and is available at http://www.nber.org/taxsim/.


\(^7\)AIME is computed as

\[
AIME = \frac{1}{N} \sum_{a=21}^{A} Y_{a} \cdot \text{Max} \left( 1, \frac{Y_{a}}{Y_{a-1}} \right) I_{a}
\]

where \(A\) is age at which one becomes disabled, \(Y_{a}\) is average monthly earnings that were subject to Social Security taxes, \(\bar{Y}_{a}\) is average economy wide earnings, \(I_{a}\) is an indicator of whether earnings at age \(a\) included, Past wages indexed using an inflator equal to average wage growth in the economy. Years with the lowest earnings are excluded from the average. Disability benefits are computed as follows

\[
d = \begin{cases} 0.9AIME & \text{if } AIME \in [0, a_1] \\ 0.9a_1 + 0.32(AIME - a_1) & \text{if } AIME \in [a_1, a_2] \\ 0.9a_1 + 0.32(a_2 - a_1) + 0.15(AIME - b_2) & \text{if } AIME > a_2 \end{cases}
\]

where thresholds \(a_1\) and \(a_2\) are scaled each year by average nominal wage growth in the economy.
Appendix B2: Numerical Algorithm

We use parametric dynamic programming method and approximate the planner’s value by Chebyshev polynomials. We have two state variables: V promised utility and ̃V threatened utility. Note that from threat keeping constraints ̃V is a function of V and ̃h. Thus, in the case of no child care ̃V is just a linear function of V. ̃V(V) can then be substituted into the previous period’s incentive compatibility constraints. We pool Age I and Age II problems together and make use of the linear relationship between ̃V and V for Age III and Age IV problems to simplify the state space to one dimension only. The algorithm is as follows

1. Define model parameters
2. Define grid over state variable V
   (a) Set nbk the number of grid points
   (b) Set n the order of polynomials
   (c) Construct interpolating nodes
       \[ r_k = -\cos \left( \frac{2j - 1}{2\text{nbk}} \pi \right) \]
       where \( j \in [0, \text{nbk}] \)
   (d) Construct collocation points for promised utility
       \[ V = V_{\text{min}} + (rk + 1) \frac{(V_{\text{max}} - V_{\text{min}})}{2} \]
3. Initial Guess
   (a) Approximate the value function
       \[ P^I_t (V) = \sum_{l=1}^{n} \theta_l T_l (t(V)) \]
       where \( t(V) \) is a function which converts a value \( V \) to the corresponding value \( rk \)
       \[ rk_j = t(V_j) = 2 \frac{V_j - V_{\text{min}}}{V_{\text{max}} - V_{\text{min}}} - 1 \]
       from formula in 2(d) and \( \{T_i\}_{i=1}^{n} \) is a set of Chebyshev polynomials
       • To do so let
         \[ X = T (t(V)) = T (rk, n) \]
         the \( nbk \times (n + 1) \) matrix of Chebyshev polynomials approximating the value function with each row corresponding to a point on the state variable grid and each column corresponding to a polynomial with the first column being a column of ones
   (b) Initial guess for the value function \( P^I_0 (V) \) according to full information planner’s cost function
(c) Get the corresponding initial guess for the Chebyshev parameters

\[ \theta_0 = (X'X)^{-1} X' P_0^D (V) \]

4. Solve Age IV Family Problems

(a) Solve \( P_{IV}^D \left( V, \hat{V} (V) \right) \)

i. For each point \( i \) in the state variable grid

A. Use guess of \( P_0^D (V) \) and \( \theta_0 \) to approximate approximate the planner’s cost function in the following way

- Find the interpolation node corresponding to the optimal \( V_D \)
  \[ j_D = 2 \frac{V_D - V_{min}}{V_{max} - V_{min}} - 1 \]

- Approximate the corresponding value function using Chebyshev polynomials
  \[ P_0^D (V_D) = T(j_D, n) \cdot \theta_0 \]

- Construct \( P_{IV}^D \left( V_i, \hat{V} (V_i) \right) \)

B. Maximise \( P_{IV}^D \left( V_i, \hat{V} (V_i) \right) \) w.r.t. \( c_D, V_D \) and subject to promise keeping constraint

ii. Get \( P_{IV}^D \left( V, \hat{V} (V) \right) \)

(b) Solve \( P_{IV}^H \left( V, \hat{V} (V) \right) \)

i. For each point \( i \) in the state variable grid

A. Use guess of \( P_0^H (V) \) and \( \theta_0 \) to approximate approximate the planner’s cost function in the following way

- Find the interpolation nodes corresponding to the optimal \( V_H \) and \( V_D \)
  \[ j_H = 2 \frac{V_H - V_{min}}{V_{max} - V_{min}} - 1 \]
  \[ j_D = 2 \frac{V_D - V_{min}}{V_{max} - V_{min}} - 1 \]
- Approximate the corresponding value function using Chebyshev polynomials
  \[ P_0^I(V_H) = T(j_H, n) \cdot \theta_0 \]
  \[ P_0^D(V_D) = T(j_D, n) \cdot \theta_0 \]
- Construct \( P_{HI}^{IV} \left( V_i, \hat{V}(V) \right) \)

B. Maximise \( P_{HI}^{IV} \left( V_i, \hat{V}(V) \right) \) w.r.t. \( c_H, c_D, V_H, V_D \) and subject to promise keeping constraint and incentive compatibility constraint

ii. Get \( P_{HI}^{IV} \left( V, \hat{V}(V) \right) \)

5. Solve Age III Family Problems

(a) Solve \( P_{III}^{IIV} \left( V, \hat{V}(V) \right) \)

i. For each point \( i \) in the state variable grid
  - Use the estimate of \( P_{III}^{IIV} \left( V, \hat{V}(V) \right) \) obtained in step 4
  - Interpolate \( P_{III}^{IIV} \left( V, \hat{V}(V) \right) \) to get their values at the optimal \( V_D \) using the spline method
  - Construct \( P_{III}^{IIV} \left( V, \hat{V}(V) \right) \)
  - Maximise \( P_{III}^{IIV} \left( V_i, \hat{V}(V) \right) \) w.r.t. \( c_D, V_D \) and subject to promise keeping constraint

ii. Get \( P_{III}^{IIV} \left( V, \hat{V}(V) \right) \)

(b) Solve \( P_{HI}^{III} \left( V \right) \)

i. For each point \( i \) in the state variable grid
  - Use the estimate of \( P_{III}^{IIV} \left( V, \hat{V}(V) \right) \) and \( P_{III}^{IV} \left( V \right) \) obtained in step 4
  - Interpolate \( P_{III}^{IIV} \left( V, \hat{V}(V) \right) \) to get their values at the optimal \( V_H \) using the spline method
  - Interpolate \( P_{III}^{IV} \left( V, \hat{V}(V) \right) \) to get their values at the optimal \( V_D \) using the spline method
  - Construct \( P_{III}^{III} \left( V_i \right) \)
  - Maximise \( P_{III}^{III} \left( V_i \right) \) w.r.t. \( c_H, c_D, V_H, V_D \) and subject to promise keeping constraint and incentive compatibility constraint
ii. Get $P_{H}^{III}(V)$

6. Solve Age I and Age II Family Problem

(a) Solve $P^I(V), P_H^{II}(V), P_D^{II}\left(V, \hat{V}(V)\right)$

i. For each point $i$ in the state variable grid

- Use the estimates of $P_{H}^{III}(V)$ and $P_{D}^{III}\left(V, \hat{V}(V)\right)$ obtained in step 5
- Interpolate $P_{H}^{III}(V)$ to get their values at the optimal $V_{HH}$ using the spline method
- Interpolate $P_{D}^{III}\left(V, \hat{V}(V)\right)$ to get their values at the optimal $V_{HD}$ and $V_{DD}$ using the spline method
- Construct $P^I(V)$
- Write informal child care as functions of benefits when claim disabled from household first order condition w.r.t. informal child care

\[
\hat{h}^I = \max \left\{ 1, \min \left\{ 0, 2 \cdot \frac{1}{1+\alpha_G} - \frac{\alpha}{1+\alpha_G} \frac{1}{w_F} \left( b^I_D - w^F + 1 \right) \right\} \right\}
\]

\[
\hat{h}_{HD}^{II} = \max \left\{ 0.5, \min \left\{ 0, 2 \cdot \frac{1}{1+\alpha_G} - \frac{\alpha}{1+\alpha_G} \frac{1}{w_F} \left( b_{HD}^I - w^F 0.5 + 1 \right) \right\} \right\}
\]

\[
\hat{h}_{DD}^{II} = \max \left\{ 0.5, \min \left\{ 0, 2 \cdot \frac{1}{1+\alpha_G} - \frac{\alpha}{1+\alpha_G} \frac{1}{w_F} \left( b_{DD}^I - w^F 0.5 + 1 \right) \right\} \right\}
\]

- Maximise $P^I(V_i)$ w.r.t. $b^I_H, b^I_D, b^I_{HH}, b^I_{HD}, b^I_{DD}, V_{HH}, V_{HD}, V_{DD}$ and subject to promise keeping and incentive compatibility constraints

ii. Get $P^I(V)$

7. Update guess

8. Check convergence of Chebyshev parameters

9. Iterate until convergence