Multiproduct Pricing and the Diamond Paradox

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January 2010
Think about shopping for basic items like groceries
Three basic observations

1. Consumers face shopping costs
   Visiting a store takes up time and effort

2. Consumers (usually) know what products a firm sells.....

3. But they only find out the prices of those products once in the store
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Even small shopping costs can have large consequences
The Diamond (1971) Paradox

- A monopolist sells a single product
- Consumers have unit demand, and know their valuation
- It costs $s > 0$ to visit the store and learn the price of the product
- Expecting a price $p^E$, consumers turn up iff valuation exceeds $p^E + s$
- So the firm charges more than $p^E$
- In equilibrium, $p^E$ must be so high that nobody visits the store
- Happens regardless of $s$, or the number of firms
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Shopping costs cause market breakdown
Several ways to resolve the Diamond Paradox

E.g. shoppers (Stahl 1989), repeated interaction (Bagwell and Ramey 1992), unknown match values and advertising (Anderson and Renault 1999, 2006)

This talk: multiproduct retailers also resolve the Paradox
Resolving the Paradox

Several ways to resolve the Diamond Paradox

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This talk: **multiproduct retailers also resolve the Paradox**

However logic behind the Paradox gives interesting predictions:

1. **A store with a larger product selection charges lower prices**
2. **Advertising selected products creates a ‘low-price image’**
3. **Prices are generally countercyclical**
Model

- A single firm costlessly produces $n$ goods
- Goods are independent in use. Consumers have unit demands
- A typical consumer’s valuations are denoted $(v_1, v_2, ..., v_n)$
- $v_j$ and $v_k$ are independent whenever $j \neq k$
- Each $v_j \sim [a_j, b_j]$ (where $b_j > 0$) using a distribution function $F_j(v_j)$

The corresponding density $f_j(v_j)$

- is strictly positive and continuously differentiable
- is logconcave and/or increasing

Note:

$$p^*_m = \arg \max p \left[ 1 - F_j(p) \right]$$ is the unique standard monopoly price
Model

- Consumers know their valuations **before** visiting the store.
- The firm can advertise the prices of specific products. Consumers learn these prices **before** visiting the store. Advertising must be truthful, but is costly, so not every price is advertised.
- Consumers learn the prices of unadvertised products only **after** visiting the store.
- To buy anything, consumers must visit the store. This costs $s > 0$.
Rational expectations equilibrium for unadvertised prices

Stage 1  The firm chooses which goods to advertise, and at what price

Stage 2  Consumers learn advertised prices, and form expectations about unadvertised prices; The monopolist sets actual unadvertised prices to maximise its profits

Stage 3  Consumers visit the store iff they expect a surplus above $s$
Once in the store, they learn actual prices, and make purchases

In equilibrium, consumer expectations about unadvertised prices are correct

Further notation:
$p_j^E$ is consumers’ expectation about the price of product $j$
$p_j$ is the actual price
Demand for unadvertised product 1 is

\[ D_1 (p_1; \cdot) = \int_{p_1}^{b_1} f_1 (v_1) \Pr \left( \sum_{j=1}^{n} \max (v_j - p_j^E, 0) \geq s \right) dv_1 \] (1)
Solving for Equilibrium

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Equilibrium first order condition

\[ D_1 \left( p_1^E; \cdot \right) - p_1^E f_1 \left( p_1^E \right) \Pr \left( \sum_{j=2}^{n} \max \left( v_j - p_j^E, 0 \right) \geq s \right) \leq 0 \]
Solving for Equilibrium
Shoppers versus Diamond Consumers

Let $t_j = \max \left( v_j - p_j^E, 0 \right)$. Rewrite the pricing condition as:

$$\Pr \left( \sum_{j=2}^{n} t_j \geq s \right) \left[ 1 - F_1 \left( p_1^E \right) - p_1^E f_1 \left( p_1^E \right) \right] + \Pr \left( \sum_{j=1}^{n} t_j \geq s > \sum_{j=2}^{n} t_j \right) \leq 0$$

(3)
Solving for Equilibrium

Shoppers versus Diamond Consumers

Let \( t_j = \max \left( v_j - p_j^E, 0 \right) \). Rewrite the pricing condition as:

\[
\Pr \left( \sum_{j=2}^{n} t_j \geq s \right) \left[ 1 - F_1 \left( p_1^E \right) - p_1^E f_1 \left( p_1^E \right) \right] + \Pr \left( \sum_{j=1}^{n} t_j \geq s > \sum_{j=2}^{n} t_j \right) \leq 0
\]

Visitors to the store divide into two groups

1. **Shoppers** for product 1 have \( \sum_{j=2}^{n} t_j \geq s \)
   Like a single-good standard monopoly problem with \( s = 0 \)

2. **Diamond consumers** for product 1 have \( \sum_{j=1}^{n} t_j \geq s > \sum_{j=2}^{n} t_j \)
   Like a single-good standard Diamond paradox with \( s > 0 \)
Suppose the firm does no advertising

$\implies$ Diamond equilibria exist
Suppose the firm does no advertising

$\implies$ Diamond equilibria exist, but:

**Lemma**

*Other (‘non-Diamond’) equilibria exist when n is sufficiently large*
Suppose the firm does no advertising

\[ \implies \text{Diamond equilibria exist, but:} \]

**Lemma**

Other (‘non-Diamond’) equilibria exist when \( n \) is sufficiently large

\[ \text{..... but } p_j^E \geq p_j^m \text{ for each unadvertised good } j \]
Comparative Statics

Three possible approaches:

1. **Look at small search costs**
2. Look at small numbers of products
3. Place some restrictions on the distribution of total surplus
Comparative Statics

Small s

Suppose that $s \to 0$ but remains strictly positive.

Interior equilibrium price of unadvertised good 1 limits to $p^E_1$, where

$$\frac{1 - F_1 \left( p^E_1 \right)}{p^E_1 f_1 \left( p^E_1 \right)} = 1 - \prod_{j=2}^{n} F_j \left( p^E_j \right)$$

Inverse elasticity
Suppose that $s \to 0$ but remains strictly positive
Interior equilibrium price of unadvertised good 1 limits to $p_1^E$, where

\[
\frac{1 - F_1 (p_1^E)}{p_1^E f_1 (p_1^E)} = 1 - \prod_{j=2}^{n} F_j (p_j^E)
\]

Inverse elasticity

Small shopping costs induce complementarities in prices

$\Rightarrow$ There always exists a Pareto dominant equilibrium
Hold fixed any advertised prices

**Lemma**

*When n increases, all unadvertised prices fall*

Larger stores charge less, but earn more profit per product

*Each product has more shoppers*

\[ \implies \] Firm faces a weaker sample selection problem

\[ \implies \] Demand is locally more elastic
Comparative Statics

Small $s$

Number of Products vs. Equilibrium Price

- Blue: Uniform $[0,1]$ Valuations
- Red: Uniform $[-2,1]$ Valuations
- Green: Uniform $[-4,1]$ Valuations
- Yellow: Uniform $[-9,1]$ Valuations

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The advertised price of one product is informative about the prices of other (completely unrelated) products. Low-price advertising on a few products *credibly commits* to a store-wide ‘low-price image’. Loss-leaders can profitably signal a low-price image.
Comparative Statics
Small s

Popularity/Demand shocks
Impose the following structure on valuations

\[ v_j : \begin{cases} 
= -\infty & \text{with probability } 1 - \alpha_j 
\sim [a, b] \text{ iid with } F(v) & \text{with probability } \alpha_j > 0
\end{cases} \]

- An increase in \( \alpha_1 \) reduces all unadvertised prices
  \( \implies \) Prices are countercyclical?
- Unadvertised products with higher \( \alpha_j \) are more expensive
  \( \implies \) More likely to advertise (and charge a low price on) a product when it is in high demand
Example with $n = 3$, iid $U \left[ \frac{1}{2}, 1 \right]$ and $\alpha_2 = \alpha_3 = \frac{2}{3}$
Summary
When the search cost is small, we can show that:

- A store with a broader product range charges lower prices, but earns more profit on each product
- A low advertised price on one product, enables the firm to credibly commit to charging low prices on other unrelated products
- When demand in the economy is high, prices are low
Three possible approaches:

1. Look at small search costs
2. Look at small numbers of products
3. **Place some restrictions on the distribution of total surplus**
   Let $T_{m,p}$ be the total surplus from $m$ products given an expected price vector $p = (p_1^E, \ldots, p_m^E)$
Complementarity

- When $p_k^E$ increases, the equilibrium $p_j^E$ should also increase.
- Equivalently when $p_k^E$ increases, the number of Diamond consumers for product $j$ increases relative to the number of shoppers.
Complementarity

- When $p^E_k$ increases, the equilibrium $p^E_j$ should also increase.
- Equivalently when $p^E_k$ increases, the number of Diamond consumers for product $j$ increases relative to the number of shoppers.
- A sufficient condition for this to happen is that

$$\frac{\Pr(T_{m,p} \geq z)}{\Pr(T_{m,q} \geq z)}$$

increases in $z$, $z \in (0, s)$ and $q > p$.

That is, expected surplus decreases in price in the sense of hazard rate dominance.
Search cost

- Suppose that when $s$ increases, the number of Diamond consumers increases relative to the number of shoppers
  $\implies$ Equilibrium prices increase in $s$, and decrease in both $n$ and $\alpha_j$
Search cost

- Suppose that when $s$ increases, the number of Diamond consumers increases relative to the number of shoppers.
  \[ \text{Equilibrium prices increase in } s, \text{ and decrease in both } n \text{ and } \alpha_j \]
- A sufficient condition for this to happen is that
  \[
  \frac{\Pr(T_{m,p} \geq z)}{\Pr(T_{m-1,p} \geq z)} \quad \text{increases in } z, \quad z \in (0, s)
  \]
  That is, expected surplus increases in $m$ in the sense of hazard rate dominance.
Other orderings

- $T_{m,p}$ always increases in $m$ and decreases in $p$ in the sense of first order stochastic dominance

- The above assumptions are stronger, and require that the conditional distribution also exhibits first order stochastic dominance

- But they are weaker than the Monotone Likelihood Ratio Properties
Product selection and price

- Anecdotal evidence that larger stores charge lower prices
  - Some evidence from surveys that consumers believe this too
  - But mixed evidence on whether large stores face more elastic demand
Discussion

Product selection and price

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  - Some evidence from surveys that consumers believe this too
  - But mixed evidence on whether large stores face more elastic demand

Countercyclical pricing

- Products are often advertised at a discount during periods when their demand is highest (e.g. Warner and Barsky 1995, MacDonald 2000)
Informativeness of selected price advertising

- Disagreement within the theoretical literature
  - Lal and Matutes (1994) say no relationship
  - Simester (1995) argues there may be

- In practice, supermarkets do sometimes try to claim a link between cheap advertised prices and store-wide ‘price image’

- Also evidence that advertised products have more elastic demand
Extensions

1. Relaxing independence of product valuations
2. Substitute products
3. Competition
4. Exogenous shoppers
5. Price learning by consumers
Shopping costs deter marginal consumers from visiting the store, which pushes up prices to the detriment of the firm. Stocking more products and using low-price advertising are both effective ways of (partially) overcoming this problem.