
Yanren Zhang

Department of Economics, University of Aberdeen

Firms and Markets Workshop
November, 2009
Motivation

I try to incorporates an overlapping generation structure in a tournament model.

- Developing a dynamic framework to analyze the problems of lifetime incentives in hierarchical organizations.
- Providing some insights on group norm among the agents within organization.
- Reconsidering moral hazard from firm’s side when contestants are heterogeneous.
Why is lifetime incentive in tournament important?

- Empirical evidence show the existence of internal labor market and long-term attachments between workers and firms. So, it's interesting to know how to motivate agents over lifetime by tournaments.

- Long-term attachment plays crucial roles in many powerful theories, such as deferred compensation, relational contract and firm-specific human capital investment. But no theory investigated the long-term features and life-cycle characteristics in tournament.
In order to model OLG tournament,

- I treat senior position as a kind of assets which generates income flow permanently. Thus, the value of promotion differs across contestants with different ages.
- The state of contestant pool is determined by the distribution of agents.
- The optimal policy for the firm must be contingent on the state.
A firm maximizing its value sequentially has incentive to:

- balance the expected profits across states.
- distort the resource allocation in heterogeneous state.

Contestant heterogeneity in life stage is important to understand group norm among agents.

- Heterogeneity makes group norm as an informal agreement more self-enforceable.
- Heterogeneity generates a possibility that collusion among agents benefits principal.

The advantage of tournament in eliminating moral hazard for principal’s side should be reconsidered since the property of fixed wage bill fails in OLG tournament.
Related Literatures

- Tournament
  - Single-stage contest scheme.
  - Multi-stage contest scheme (round-robin tournament and elimination tournament).
  - Heterogeneity in tournament.

- Wage policy within firm: wage increases are serially correlated; and that promotions and wage growth are significantly related.

- Collusion: collusion with side-payments causes efficiency losses, and collusion with implicit side-contracts are more likely based on long-term relationships through a reputation mechanism.
Setup: Workers

- Time is discrete, \( t = 1, 2, 3, \ldots \) and only steady states are considered.

- Workers live for two periods and then be out of model. In each period \( i \), the worker born in period \( j \) earns income \( I^j_i \) but exerts effort \( e^j_i \). Hence, in time \( t \) there are two generations of workers alive: one young worker born in \( t \) and one old worker born in \( t - 1 \).

- Young and old workers’ preference over income and effort allocations are represented by

\[
E_t U^t_t = E_t \left\{ \left[ I^t_t - c \left( e^t_t \right) \right] + \beta \left[ I^t_{t+1} - c \left( e^{t+1}_t \right) \right] \right\} \quad \text{and} \\
E_t U^{t-1}_t = E_t \left[ I^{t-1}_t - c \left( e^{t-1}_t \right) \right],
\]

respectively.
Setup: Firm

- Firms live forever and make wage policies to maximize their values in each period.
- Each firm has two levels: junior workers and senior workers, and only contains two junior positions (the scale of firm is fixed).
- Each firm has access to a technology:
  \[ y_{i,t} = y(e_{i,t}, \varepsilon_{i,t}) , \]
  where \( \varepsilon_{i,t} \) is a random shock.
- Workers’ effort cannot be observed or verified.
- Firms hire new workers from the young generations to fill vacant junior position due to workers’ death or promotion.
- Normalize the wage of junior worker to zero.
Setup: Tournament Rule

- If someone wins the tournaments among junior workers in period \( t \), he is promoted to senior position in the same period and will hold the position in the next period if still alive (rule out demotion).

- At the beginning of each period \( t \), firms commit themselves to pay senior workers \( w_t \) each period.

That is to say, if a young junior worker is promoted to senior position in period \( t \), he will get \( w_t \) in period \( t \) and period \( t + 1 \) \(((1 + \beta)w_t \) in lifetime). Instead, if an old junior worker is promoted, he will get \( w_t \) immediately and be out of model.
The timing of the model can be summarized as follows:

- **Period $t$**
  - Firm sets Promotion and wage policy
  - Workers decide whether to participate the tournament
  - Workers choose effort level
  - The outcome of the tournament is realized and reported

- **To period $t + 1$**
  - Firm executes promotion and wage policy

**Figure 1: Timing of model**
One-shot Case

- If junior workers cannot be promoted to senior position when young they will be fired, which means the contestant pool must be homogeneous in each period.

- In this case, contests are independent cross periods. Maximizing the value of firm $\Pi = \sum_{t=1}^{\infty} \pi_t$ is equivalent to maximizing every single period profit.
Two-shot Case

In this case, if a young worker fails to be promoted in the first round he can still participate in the next round when old. Actually, there exist two states in tournaments:

- **State 1:** Young worker versus young worker;
- **State 2:** Old worker versus young worker.

- If current state is state 1, the next must be state 2;
- If current state is state 2 and the young worker wins, the next must be state 1;
- If current state is state 2 and the old worker wins, the next must still be state 2.
Each firm has to recruit two new young workers in period $t$ if the worker born in $t - 2$ fails to be promoted again in period $t - 1$, otherwise, just recruit one.

Assume the firm use Markov strategies—firms’ wage policy is contingent on current state:

- $w$ for state 1;
- $\bar{w}$ for state 2.

Workers choose optimal effort level:

- $e^s$ for state 1;
- $e^o, e^y$ for state 2.
Proposition 1: (Fast Track Effect)

- For any nondiscriminatory wage policy in heterogeneous state, the young contestants must choose higher effort level and win with higher probability relative to the old.
- Then, The loser in last round is more likely to lose in current round.
- Some empirical works show the evidence of promotion fast track in reality, that is, those promoted quickly at one level are promoted more quickly at the next level. Such as Baker, Gibbs and Holmstrom (1994) and John Treble, Edwin van Gameren, Sarah Bridges, Tim Barmby (2001).
Proposition 2

- In sufficiently regular case, the effort level of contestant is an increasing function of $\bar{w}$ if the marginal probability of win is increasing with opponents effort level.

- The intuition is the effort level of workers depend not only on the net value of promotion but on the distribution function of random shock.
Proposition 3

- For any wage policy change in state 2, the expected period profit in state 1 decreases with expected utility of old contestants.
- The intuition is increased expected utility of old junior compromises the value of promotion for young workers and thus increases the cost of incentive in homogeneous state.
- So, wage policy in heterogeneous state $\bar{w}$ affects the expected period profit in homogeneous state.
Intertemporal Incentive

Denote the maximum of firm value in state 1 and 2 by $\Pi^*$ and $\bar{\Pi}^*$, respectively. Given optimal wage policy $w^*$, $\bar{w}^*$, the value of firm in state 1 and 2 have the following recursive structure:

\begin{align}
\Pi^* &= E\pi (w^*, \bar{w}^*) + \beta \bar{\Pi}^*, \\
\bar{\Pi}^* &= E\bar{\pi} (\bar{w}^*) \\
&\quad + \beta \left[ P(e^o, e^y)|_{\bar{w}=\bar{w}^*} \bar{\Pi}^* + P(e^y, e^o)|_{\bar{w}=\bar{w}^*} \Pi^* \right].
\end{align}

where $P(e^i, e^j)$ is the winning probability of contestant $i$. 

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Seniority, Loyalty and Factionalism
Intertemporal Incentive

Then, the value of firm in state 1 is

$$\Pi^* = \left[1 - \beta P(e^y, e^o)|\bar{w} = \bar{w}^*\right] E\pi(w^*, \bar{w}^*) + \beta E\bar{\pi}(\bar{w}^*) \frac{1 - \beta P(e^o, e^y)|\bar{w} = \bar{w}^* - \beta^2 P(e^y, e^o)|\bar{w} = \bar{w}^*}{1 - \beta P(e^o, e^y)|\bar{w} = \bar{w}^*}.$$  

It’s easy to get FOC and observe the fact

$$\frac{\partial \Pi^*}{\partial w^*} = 0 \Leftrightarrow \frac{\partial E\pi}{\partial w^*} = 0,$$

which means firm can just consider maximizing current profit in state 1. Here, the key point is that the wage policy taken in state 1 cannot affect either the expected period profit in state 2 $E\bar{\pi}$ or the transition probability $P(e^y, e^o)$.
Intertemporal Incentive

Define

$$\lambda (\bar{w}^*) = \frac{1}{1 + \beta \, P (e^y, e^o)|_{\bar{w} = \bar{w}^*}}$$

The value of firm in state 2 is

$$\bar{\Pi}^* = \frac{E \bar{\pi} (\bar{w}^*) + \beta \, P (e^y, e^o)|_{\bar{w} = \bar{w}^*} \, E \pi (w^*, \bar{w}^*)}{1 - \beta \, P (e^o, e^y)|_{\bar{w} = \bar{w}^*} - \beta^2 \, P (e^y, e^o)|_{\bar{w} = \bar{w}^*}}$$

$$= \frac{1}{1 - \beta} \left\{ \lambda (\bar{w}^*) \, E \bar{\pi} (\bar{w}^*) + [1 - \lambda (\bar{w}^*)] \, E \pi (w^*, \bar{w}^*) \right\}.$$  

In state 2, maximizing the value of firm is equivalent to maximizing a weighted average of expected period profit across states.
Intertemporal Incentive

Firm in state 2 has to care about two things when making decision.

- One is balancing the expected profits between two states given transition probability across states since \( \bar{w} \) determines both \( E\pi \) and \( E\bar{\pi} \).

- The other is controlling the heterogeneity of contestant pool by affecting the transition probabilities across the states.

- In state 2, static profit-maximizing condition cannot be satisfied in equilibrium. Firm will distort induced effort of junior workers in state 2 to reduce incentive cost in tournament as a whole.
Proposition 4

- In homogeneous state, firm maximizing its value can just set wage policy to reach maximum of expected period profit like static case.
- In heterogeneous state, the firm instead has to determine the heterogeneity of contestant pool and balance expected period profits across states.
In this section, I will consider the side contracts among agents.

- Junior workers have incentives to make side deals given the wage policy of firm if they can observe others’ action ex post.
- I will use “self-enforcing side contracts” approach instead of presuming that any gain from side contract can be realized—any side contract should be a subgame perfect equilibrium given the incentive mechanism of the firm.
Proposition 5

- In collusive case, young contestants choose lower effort level relative to non-collusive case, and the expected utility of old contestants must be improved. In addition, old contestants choose lower effort level if and only if the marginal probability of win for old contestants declines.

- Intuitively, collusion reduces the chance of promotion as a young worker but improve the chance of promotion as an old worker if they fail to be promoted when young, which makes the young better off. But old workers have no incentive to be cooperative and their effort depends on the distribution function of random shock.
Proposition 6

- Conditions ensuring self-enforcing collusion in homogeneous state are more restrictive than those in heterogeneous state, and then collusive behaviors among agents will be more pervasive in tournaments with heterogeneous contestant pool.

- Intuitively, in state 1, OLG tournament is similar to static case, which means agents have more incentive to deviate side agreement. In contrast, in state 2, OLG tournament contains dynamic feature in infinite horizon, which makes side contract self-enforceable.
What is the Role of Group Norm within Organization?

Unlike other models of collusion, we cannot say collusion is necessarily harmful to principal in OLG tournament.

For example, even in the case that both old and young contestants choose lower effort level in collusion which causes the expected output decline, the direction of change of firm value is ambiguous because the expected wage cost will decline as well if $P(e^o_c, e^y_c) > P(e^o, e^y)$ (firm has to pay $(1 + \beta)\bar{w}$ to the young winners but only $\bar{w}$ to the old). So, it is still possible in some case that collusion among agents even benefits organization.
Why is collusion beneficial to organization?

- The logic is side deals in heterogenous contestant pool make some effort allocation feasible, which is beneficial to firm but cannot realize economically through formal institution.

- In the OLG tournament, side contracts reallocate agent’s effort and thus at least partially offsets the distortion caused by formal institution.

- Wage policy determines not only the effort but also the collusive behavior within organization. As a result, firm has to decide whether to eliminate collusion by imposing collusion-proof condition or just keep it when contestant’s effort can be observed by his opponent.
Moral Hazard from Firm’s Side: Proposition 7

- For any nondiscriminatory wage policy in state 2, the property of fixing wage bill fails in the OLG tournament and firm have no incentive to report the rank of performance truly.

- For example, suppose the work is done and the young worker performed better in state 2.
Proposition 7

Given workers’ strategies,

➢ if the performance ranking is reported truly, firm has to pay young worker $(1 + \beta)\bar{w}^*$ and the tournament will turn to state 1 in the next period.

➢ if firm manipulates the outcome of tournament—insists the old worker performed better, only $\bar{w}^*$ is paid, but the next round must still be in state 2.

The firm choose to report performance ranking honestly if and only if

$$-\bar{w}^* + \beta \bar{\Pi}^* \leq -(1 + \beta) \bar{w}^* + \beta \Pi^*. \quad (3)$$
Proposition 7

Similarly, in the case that the old worker performed better, the firm has no incentive to deviate from truth-telling if and only if

\[-(1 + \beta) \bar{w}^* + \beta \Pi^* \leq -\bar{w}^* + \beta \bar{\Pi}^*.\]  (4)

The above two inequations show, in every period, truth-telling is optimal for the firm only when

\[\bar{w}^* = \Pi^* - \bar{\Pi}^*,\]  (5)

which is almost impossible. Such kind of moral hazard in tournament arises from the heterogeneity of contestant pool rather than influence activities discussed in Milgrom (1988) and Milgrom and Roberts (1988), or favorites in Prendergast and Tope1 (1996). in organization. So relational contracts still matters even when tournament is applied in organization.
Conclusion

So far,

- I have shown a dynamic framework to analyze the problems of lifetime incentives in hierarchical organizations.
- This paper have studied the group norm as a self-enforcing collusion among contestants.
- My analysis also provide some clue to reconsider moral hazard from principal’s side in tournament.
Thanks for listening!