Introduction

- Most of Political Economics is about distribution or ideology (Downs 1957, Black 1948)
- Voters have divergent interests
- Elections decide which preferences prevail
- Alternative: Elections as information aggregation
- Voters want the same but have limited information
- This approach goes back to Condorcet
The swing voter’s curse

- Feddersen and Pesendorfer discovered "The Swing Voter’s Curse" (1996)
- Rational voters have to condition on being pivotal
- Similar to the winner’s curse in auction theory
- Rational voting is not necessary the same as sincere voting (Austen-Smith and Banks 1996)
Introducing endogenous policy alternatives

- Electoral competition is about ideology as well as about information
- Feddersen and Pesendorfer (1999) relax the assumption of interest convergence among voters
- New in my paper: Competition by the candidates who offer policy alternatives
- The policies on offer are endogenously determined
The Swing Voters' Blessing

Jan Klingelhöfer

Introduction

Preview of the results

I show that ignoring information about policy positions is optimal for uninformed voters

Partisan voting is entirely rational for them

An uninformed conservative should vote for McCain even if she thinks that she might prefer Obama if she had full information

Rational voting leads to the full-information outcome
Preview of the results II

- Introducing swing voters increases electoral control
- Swing voters are boundedly rational
- Therefore they can play a strategy that would otherwise not be credible
- Candidates for office now about the swing voters and adjust their positions accordingly
- I find the "swing voters" blessing" instead of a curse
Some more related literature

- There are several more papers that deal with voting under uncertainty
- McKelvey and Ordeshook (1985; 1986) have uncertainty about policy positions
- Cukierman (1991) is in their tradition and has a setup very similar to mine
- In his paper voters are uncertain about the quality of politicians
- However, none of these papers formulates the voters problem as a game
Some more related literature II

- A very similar idea to mine is Bond and Eraslan (2009)
- They endogenize proposals in a Feddersen-Pesendorfer setup
- However, they do not model political competition, but decision making within a committee
- They also find that endogenizing offers makes a qualitative difference
The model

Policy dimensions

- There is a polity with two candidates and a one-and-a-half dimensional policy space
- The ideological policy space is the interval $[0, 1]$
- The quality "half-space" (Groseclose 2007) is on the real line
Candidate $j$ has the utility function:

$$U_j = -(p - b_J)^2,$$

with $J = L, R$

- where $p$ is implemented policy
- $b_J$ is the preferred policy of candidate $J$
- $b_L < b_R$. 

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Introduction
Preview of the results
Model
The candidates
Voter utility function
Uninformed and informed voters
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Solving the model
Uninformed voters
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Generalizing the utility function
The case of an uninformed median
Introducing swing voters
Example with swing voters
Conclusion
Additional material
Candidates' strategies
Proof welfare
Uninformed median
Proof cutoff point
The voters

- There is an odd number $N$ of voters
- Voters care about the quality of the candidates
- The difference in quality is denoted by the variable $\nu = q_R - q_L$
- $q_R$ and $q_L$ are drawn from random distribution functions
- $\nu$ is the valence advantage of the right candidate
- $\nu$ has support everywhere on the real line
Voter utility function

The utility of a voter is given by:

\[ U(b) = - (b - p)^2 + q, \]

- It depends on her bliss point \( b \),
- implemented policy \( p \),
- and the quality of the candidate in power \( q \).
- All bliss points are distinct
Uninformed and informed voters

- Informed voters observe the quality of the candidates before the elections
- Uninformed voters do not
- By assumption the informed voters are in the majority
- For the moment I assume that the median voter is informed
The elections

- The elections take place and the candidate with the majority of votes wins
- His announced policy is implemented

The elections take place and the candidate with the majority of votes wins
His announced policy is implemented
1. Nature choses v

2. The candidates announce their binding policy positions after observing v

3. Elections take place:
   Informed voters vote after observing v as well as the announced policy positions, and uninformed voters vote after only observing the announced policy positions

4. The candidate with the majority of votes wins, and his announced policy position is implemented
Solving the model backwards

- I solve backwards and begin with the voters.
- The problem of the informed voters is straightforward.
- There is a cutoff point $b_i^*(p_L, p_R, \nu)$.
- Informed voters with a $b < b_i^*$ will prefer to vote for the left policy position.
- Informed voters with a $b > b_i^*$ will prefer to vote for the right policy position.
The cutoff point

- The cutoff point can be found by solving:

$$-(b_i^* - p_L)^2 + (b_i^* - p_R)^2 - \nu = 0$$

$$\Rightarrow b_i^*(p_L, p_R, \nu) = \frac{p_L + p_R}{2} - \frac{\nu}{2(p_R - p_L)} \text{ for } p_L \neq p_R$$

- if $p_L < p_R$ informed voters with $b < b_i^*$ vote for candidate $L$

- This describes a weakly dominating strategy to vote in favor of the preferred policy position

- I assume that informed voters vote for the candidate with valence advantage if they are indifferent
Uninformed voters

- The problem of an uninformed voter is less straightforward
- Again I try to find a cutoff point
- However, this time it can only depend on the announced policy positions: \( b^*_U(p_L, p_R) \)
- What is important is to support the preferred (given \( v \)) candidate when a voter is pivotal
The problem of an uninformed voter with bliss point $b$ is that she does not know for sure which candidate she favors.

- $F_I(b)$ is the number of informed voters with bliss point smaller than or equal to $b$.
- $F_I^{-1}(x)$ is the bliss point of the informed voter with the $x_{th}$ lowest $b$ among the informed voters bliss points.
The decisive informed voter

Define:

$$b_i^d(l_U) = F_i^{-1}\left(\frac{N+1}{2} - l_U\right),$$

- where $b_i^d(l_U)$ is the bliss point of the decisive informed voter given the number of votes by uninformed voters for the left policy position denoted by $l_U$.
- For a majority, $\frac{N+1}{2}$ votes are necessary and therefore $(\frac{N+1}{2} - l_U)$ left votes by informed voters for the left position to win.
- If the decisive informed voter votes left, all the informed voters with bliss point to the left of her vote left.
- This is sufficient for a left majority.
The decisive informed voter II

- If the decisive informed voter votes right, all the informed voters with bliss point to the right of her vote right
- This is sufficient for a right majority
- Therefore the candidate with the support of the decisive informed voter wins the elections
- The decisive voter is decisive in the same sense as the median voter in standard models
- Note that I make a distinction between "decisive" and "pivotal"
Equilibrium conditions

- In an equilibrium none of the voters has an incentive to deviate.
- This means that none of the uninformed voters would prefer to change the position of the decisive informed voter by changing her vote.
- A simple strategy fulfills this condition:

\[ b_m = b_U^*(p_L, p_R), \]

- \( b_m \) is the cutoff point between voting for the left policy position and voting for the right position.
- It is independent of the policy platforms that the candidates announce.
The median voter is decisive

- The median voter turns out to be decisive:

\[
\begin{align*}
\quad b^d_l\left(F_U(b_m)\right) \\
= \quad b^d_l\left(\frac{N + 1}{2} - F_I(b_m)\right) \\
= \quad F^{-1}_I(F_I(b_m)) \\
= \quad b_m,
\end{align*}
\]

- The first equality comes from the implicit definition of the median voter: \( \frac{N + 1}{2} = F_I(b_m) + F_U(b_m) \).

- The second equality uses the definition of the decisive voter: \( b^d_l(I_U) = F^{-1}_I\left(\frac{N + 1}{2} - I_U\right) \).
The median voter is decisive

The intuition is simple

In a model with full information the median voter is decisive because:

Whenever she votes left everybody to the left of her votes left

Whenever she votes right everybody to the right of her votes right

But exactly the same is the case with uninformed voter in the model given their cutoff point $b_m$
The problem of an uninformed voter

Consider the problem of an uninformed voter with bliss point \( b < b_m \): \( b^* < b_m \) : Right wins independently of the vote of the uninformed voter

\[
b^* = \frac{p_L + p_R}{2} - \frac{v}{2(p_R - p_L)}
\]

\[
b_r = F_I^{-1}(F_I(b_m) + 1)
\]
The problem of an uninformed voter

Consider the problem of an uninformed voter with bliss point $b < b_m: b^* > b_r$: Left wins independently of the vote of the uninformed voter.

\[
b^* = \frac{p_L + p_R}{2} - \frac{v}{2(p_R - p_L)}
\]

\[
b_r = F_l^{-1}(F_l(b_m) + 1)
\]
The problem of an uninformed voter

Consider the problem of an uninformed voter with bliss point \( b < b_m \): 
\( b_m < b^* < b_r \): The uninformed voter is pivotal

\[
b^* = \frac{p_L + p_R}{2} - \frac{v}{2(p_r - p_L)}
\]

\[
b_r = F^{-1}_I(F_I(b_m) + 1)
\]
The problem of an uninformed voter

Consider the problem of an uninformed voter with bliss point $b < b_m$:
$b_m < b^* < b_r$ : The uninformed voter is pivotal

- She prefers left because by assumption $b < b_m$
- $\Rightarrow b < b^*$
- Whenever she is pivotal the uninformed voter prefers left
- So voting left must be optimal
Intuition for the uninformed voters’ strategy

- It is helpful to reinterpret the voting strategy of the uninformed as a way to appoint the decisive informed voter.
- All uninformed voters try to get a decisive informed voter with preferences as close as possible to their own.
- Voters attempt to pull the position of the decisive voter closer to their own bliss point.
- This way uninformed voters make sure that they vote for their favorite whenever they are pivotal.
Which candidate is voted into office?

- The candidate who is supported by the median voter wins the elections
- Elections have the same outcome as if all voters were informed
The strategies of the candidates

- Candidates are only interested in ideology, not in winning as such
- Because there is no uncertainty the candidate with valence advantage can win for sure by playing the median position
The strategies of the candidates II

- However, he can do better
- He moves as close as possible to his own bliss point
- The maximum amount he can go is where median voter becomes indifferent between the candidates
- The candidate with valence disadvantage offers the median position
The strategies of the candidates

The equilibrium policy platforms of the candidates are:

\[
\begin{align*}
p_R^* &= \min \{b_R, b_m + (v)^{0.5}\}
\quad \text{if } v > 0, \\
p_L^* &= b_m
\quad \text{if } v \leq 0. \\
p_R^* &= b_m \\
p_L^* &= \max\{b_L, b_m - (-v)^{0.5}\}
\end{align*}
\]

And implemented policy is:

\[
p = \begin{cases} 
p_R^* & \text{if } v > 0 \\
p_L^* & \text{if } v \leq 0
\end{cases}
\]
Beliefs of the uninformed voters

\[ v(p_L, p_R) = \begin{cases} 
(p_R - b_m)^2 & \text{if } p_L = b_m, \ p_R \in [b_m, b_R) \\
-(p_L - b_m)^2 & \text{if } p_R = b_m, \ p_L \in (b_L, b_m], \\
g(v) & \text{if } p_L = b_m, \ p_R = b_R \\
1 - G((b_m - b_R)^2) & \text{for } v \geq (b_m - b_R)^2 \\
0 & \text{for } v < (b_m - b_R)^2 \\
g(v) & \text{for } v \leq -(b_m - b_L)^2 \\
G(-(b_m - b_L)^2) & \text{for } v > -(b_m - b_R)^2 \\
0 & \text{for } v \geq -(b_m - b_R)^2 \\
\end{cases} \]

where \( v(p_L, p_R) \) is a value of that leads to the combination of \( p_L \) and \( p_R \) in equilibrium.

For all other (out of equilibrium) combinations of policy offers I assume:

\[ g(v|p_L, p_R) = g(v). \]
Generalizing the utility function

- In the main model the utility function are chosen to be as simple as possible.
- However, it should be clear from the proof that the exist that more than one cutoff point would constitute a problem.
- On the other hand it should be clear all the proofs go through without any further modification if their is indeed only cutoff point.
Generalizing the utility function II

A more general utility function depending only on distance and quality of politicians is:

\[ U_i(p, b_i) = u(d_i, q), \text{ with } d = |b_i - p|. \]

Sufficient to ensure that there is at most one cutoff point are the following restrictions on its derivatives:

\[ u_d(d, q) \leq 0, \]
\[ u_{dd}(d, q) < 0, \]
\[ u_q(d, q) \geq 0, \]
\[ u_{qd}(d, q) \leq 0. \]
The case of an uninformed median

- The median "punishes" the candidate who is further away
- She is actually better off compared to the case when she is informed
- If she were informed this strategy would not be credible
The strategies of the candidates and policy

The strategies of the candidates are given by:

\[
\begin{align*}
    p_L^* &= \max(b_m - \frac{\nu}{4(b_m - b_l)}, b_l) \\
    p_R^* &= \min(b_R, b_l + (\nu + (b_l - p_L^*)^2)^{0.5}) \\
    p_R^* &= \min(b_m + \frac{-\nu}{4(b_r - b_m)}, b_r) \\
    p_L^* &= \max(b_L, b_r - (-\nu + (b_r - p_R^*)^2)^{0.5})
\end{align*}
\]

if \( \nu > 0 \)

where \( b_l \) denote the bliss point of the informed voter who is located closest to the median of all informed voters with a smaller bliss point and \( b_r \) denotes the bliss point of the informed voter located closest to the right of the median. Implemented policy is:

\[
p^* = \begin{cases} 
p_R^* & \text{if } \nu > 0 \\
p_L^* & \text{if } \nu \leq 0 \end{cases}
\]
What if uninformed voters vote "naively"?

- What if some uninformed voters just vote for whomever they prefer ideologically?
- \[ b_{UU} = \frac{(p_L + p_R)}{2} \]
- This is what they do if they do not understand what it means to be pivotal
- It is also what is assumed (without further discussion) in lots of the PolScience literature
- To ensure the existence of an equilibrium, I assume that a voter at position \( b_{UU}^*(p_L, p_R) \) supports left if \( b_{UU}^*(p_L, p_R) \leq b_m \) and right if \( b_{UU}^*(p_L, p_R) > b_m \).
The decisive informed voter

The decisive informed voter is now:

\[ b^d_I(p_L, p_R) = F^{-1}_i \left( \frac{N + 1}{2} - l_{SU}(p_L, p_R) - l_{UU}(p_L, p_R) \right), \]

- We have an equilibrium if all sophisticated uninformed voters vote left if their bliss point is smaller than \( b^d_I \) and vote right if their bliss point is larger than \( b^d_I \).
- However, this might not be possible
The cutoff point for sophisticated uninformed voters

The cutoff point for sophisticated uninformed voters is:

\[ b_{SU}^c(p_L, p_R) = F_{S}^{-1} \left( \frac{N + 1}{2} - l_{UU} \left( \frac{p_L + p_R}{2} \right) \right) \]

- where \( F_{S}^{-1}(x) \) gives the voter with the \( x_{th} \) smallest \( b \) of the combined set \( B_S = B_I \cup B_{SU} \)
- \( b_{SU}^c(p_L, p_R) \) is the bliss point of the voter who would be decisive if all of them were informed.
The decisive uninformed voter

- A sophisticated uninformed voter at the cutoff point has to decide between voting left and right.
- I assume that he follows the strategy of voting for the candidate whose position is closer to her own bliss point.
- From this we find the decisive voter dependent on $p_L$ and $p_R$:

$$b^d_L(p_L, p_R) = \begin{cases} 
    b_{SU}^*(p_L, p_R) & \text{if } b_{SU}^*(p_L, p_R) \in B_I \\
    F_I^{-1}(F_I(b_{SU}^*(p_L, p_R))) & \text{if } b_{SU}^* \notin B_I \\
    F_I^{-1}(F_I(b_{SU}^*(p_L, p_R) + 1)) & \text{if } b_{SU}^* \notin B_I \\
    F_I^{-1}(F_I(b_{SU}^*(p_L, p_R) + 1)) & \text{if } b_{SU}^* \notin B_I \\
\end{cases}$$

where $b_{SU}^*(p_L, p_R)$ is the uninformed voter's bliss point, $F_I$ is a function that maps the bliss point to the candidate's position, and $B_I$ is the cutoff point where the voter decides to vote.

If

- $b_{SU}^*(p_L, p_R) \in B_I$
- and $\frac{p_L + p_R}{2} > b_{SU}^*$
- and $\frac{p_L + p_R}{2} = b_{SU}^*$ and $\frac{p_L + p_R}{2} > b_m$
- and $\frac{p_L + p_R}{2} = b_{SU}^* < b_{SU}^*$ and $\frac{p_L + p_R}{2} < b_m$

then

- $b_{SU}^*(p_L, p_R) \in B_I$
- and $\frac{p_L + p_R}{2} > b_{SU}^*$
- and $\frac{p_L + p_R}{2} = b_{SU}^*$ and $\frac{p_L + p_R}{2} > b_m$
- and $\frac{p_L + p_R}{2} = b_{SU}^* < b_{SU}^*$ and $\frac{p_L + p_R}{2} < b_m$

The decisive uninformed voter

- A sophisticated uninformed voter at the cutoff point has to decide between voting left and right.
- I assume that he follows the strategy of voting for the candidate whose position is closer to her own bliss point.
- From this we find the decisive voter dependent on $p_L$ and $p_R$:

$$b^d_L(p_L, p_R) = \begin{cases} 
    b_{SU}^*(p_L, p_R) & \text{if } b_{SU}^*(p_L, p_R) \in B_I \\
    F_I^{-1}(F_I(b_{SU}^*(p_L, p_R))) & \text{if } b_{SU}^* \notin B_I \\
    F_I^{-1}(F_I(b_{SU}^*(p_L, p_R) + 1)) & \text{if } b_{SU}^* \notin B_I \\
    F_I^{-1}(F_I(b_{SU}^*(p_L, p_R) + 1)) & \text{if } b_{SU}^* \notin B_I \\
\end{cases}$$

where $b_{SU}^*(p_L, p_R)$ is the uninformed voter's bliss point, $F_I$ is a function that maps the bliss point to the candidate's position, and $B_I$ is the cutoff point where the voter decides to vote.

If

- $b_{SU}^*(p_L, p_R) \in B_I$
- and $\frac{p_L + p_R}{2} > b_{SU}^*$
- and $\frac{p_L + p_R}{2} = b_{SU}^*$ and $\frac{p_L + p_R}{2} > b_m$
- and $\frac{p_L + p_R}{2} = b_{SU}^* < b_{SU}^*$ and $\frac{p_L + p_R}{2} < b_m$

then

- $b_{SU}^*(p_L, p_R) \in B_I$
- and $\frac{p_L + p_R}{2} > b_{SU}^*$
- and $\frac{p_L + p_R}{2} = b_{SU}^*$ and $\frac{p_L + p_R}{2} > b_m$
- and $\frac{p_L + p_R}{2} = b_{SU}^* < b_{SU}^*$ and $\frac{p_L + p_R}{2} < b_m$
Equilibrium policy

- Assume right has a valence advantage.
- He can play the median position and win for sure against any $p_L < b_m$ because he wins all unsophisticated uninformed voters to the right of the median.
- However, his best reply to $p_L$ is given by:
Equilibrium policies are given by:

\[
\begin{align*}
  p^*_R &= \min_{p_L \in [0,1]} p^b_R(p_L, v) \\
  p^*_L &= \arg\min_{p_L \in [0,1]} p^b_R(p_L, v) \\
  p^*_R &= \max_{p_R \in [0,1]} p^b_L(p_R, v) \\
  p^*_L &= \arg\max_{p_R \in [0,1]} p^b_L(p_R, v)
\end{align*}
\]

if \( v > 0 \),

\[
\begin{align*}
  p^*_R &= \min_{p_L \in [0,1]} p^b_R(p_L, v) \\
  p^*_L &= \arg\min_{p_L \in [0,1]} p^b_R(p_L, v) \\
  p^*_R &= \max_{p_R \in [0,1]} p^b_L(p_R, v) \\
  p^*_L &= \arg\max_{p_R \in [0,1]} p^b_L(p_R, v)
\end{align*}
\]

if \( v \leq 0 \),

and the implemented policy is:

\[
p = \begin{cases} 
  p^*_R & \text{if } v > 0 \\
  p^*_L & \text{if } v \leq 0 
\end{cases}.
\]  

(1)
Lemma

Taking the overall set of bliss points $B$ as given, having sophisticated uninformed voters (case 'I') instead of informed voters (case II') at some bliss points ($B' = B''$, $B'_I \subsetneq B''_I$, $B''_S \subsetneq B''_S$, $B'_I = B''_I$, $B'_U = B''_U$) leads to equilibrium policies as close or closer to the median bliss point for all values of $v$ ($|p^{*'}(v) - b_m| \leq |p^{*''}(v) - b_m|$).

Lemma

Taking the overall set of bliss points $B$ as given, having unsophisticated uninformed voters (case 'I') instead of sophisticated uninformed voters (case II') at some bliss points ($B' = B''$, $B'_I = B''_I$, $B'_S \subsetneq B''_S$, $B'_U \subsetneq B''_U$) leads to equilibrium policies as close or closer to the median bliss point for all values of $v$ ($|p^{*'}(v) - b_m| \leq |p^{*''}(v) - b_m|$).
Welfare analysis

- Take $g(v)$ as given.
- Let $p''(v)$ an equilibrium policy and $p'(v)$ a different one with stronger constraints on the candidate with valence advantage.
- We know that $(p''(v) - b_m)^2 \geq (p'(v) - b_m)^2$ for any difference in quality $v$.

The difference in utility for a voter with arbitrary $b$ is:

$$\Delta E(U, b) = E(U(p'', b)) - E(U(p', b))$$
$$= \int_{-\infty}^{\infty} \left(-p''(v)^2 + p'(v)^2 + 2b(p''(v) - p'(v))\right)g(v).$$
Welfare analysis II

We know that difference is weakly negative for the median voter:

\[ \Delta E(U, b_m) = \int_{-\infty}^{\infty} (-p''(v)^2 + p'(v)^2 + 2b_m(p''(v) - p'(v)))g(v) \leq 0 \]

- It is easy to see that because the median is better off with \( p'(v) \) a majority of voter must be better off in expectations
Who is better off?

- If the expected value of $p'(v)$ is the same as the expected value of $p(v)$ all voters without exception are better off.

- The cutoff point between voters who are better and who are worse off is given by:

$$
\Delta E(U, b_{cut}) = \int_{-\infty}^{\infty} (-p''(v)^2 + p'(v)^2 + 2b_{cut}(p''(v) - p'(v)))g(v) = 0
$$

$$
\Rightarrow b_{cut} = \frac{\int_{-\infty}^{\infty} (p''(v)^2 - p'(v)^2)g(v)}{2\int_{-\infty}^{\infty} (p''(v) - p'(v))g(v)} \text{ for } \int_{-\infty}^{\infty} (p''(v) - p'(v))g(v) \neq 0.
$$
An example

- Mass 1 of voters
- $1 - \alpha - \beta > 0.5$ are informed, the rest is uninformed
- $\alpha$ are uninformed but sophisticated as before
- $\beta$ follow the behavioral strategy given above and have $b_{UU} = \frac{(p_L + p_R)}{2}$
- All three groups of voters are uniformly distributed in the policy space $[0, 1]$
- The candidates have the bliss points with the same distance to the median
- CDF of $\nu$ is $G(\nu)$ and the density $g(\nu) = g(-\nu)$
What if uninformed voters vote "naively"?

The new equation for the decisive informed voter is:

\[
\begin{align*}
    b_i^d(p_L, p_R) &= F_I^{-1}(0.5 - l_{SU}(p_L, p_R) - l_{UU}(p_L, p_R)) \\
    &= F_I^{-1}(0.5 - l_{SU}(p_L, p_R) - \beta \frac{(p_L + p_R)}{2}) \\
    &= 0.5 - l_{SU}(p_L, p_R) - \beta \frac{(p_L + p_R)}{2} \\
    &= \frac{0.5 - l_{SU}(p_L, p_R) - \beta \frac{(p_L + p_R)}{2}}{1 - \alpha - \beta}
\end{align*}
\]
What if uninformed voters vote "naively"?

If all sophisticated uninformed voters make the "correct" decision:

\[ F_{SU}(b^d_i(p_L, p_R, l_U)) = \alpha b^d_i(p_L, p_R, l_U) = l_{SU}(p_L, p_R) \]

Taking both conditions together I get:

\[ b^d_i(p_L, p_R) = \frac{0.5 - \alpha b^d_i - \beta \frac{(p_L + p_R)}{2}}{1 - \alpha - \beta} = \frac{0.5 - \beta \frac{(p_L + p_R)}{2}}{1 - \beta} \]

\( \alpha \) drops out!
What if some uninformed voters vote "naively"?

- So I focus on just on

\[ I = (1 - \beta) \left( \frac{p_L + p_R}{2} \right) - \frac{v}{2(p_R - p_L)} + \beta \frac{p_L + p_R}{2} \]

- This time the position of the "weak" candidate is not trivial

- trade-off between getting support from both groups of voters
The candidate strategies

\[ p_R = \min((1 - \beta)^{0.5}v^{0.5} + 0.5, p^*_R) \]

\[ p_L = \max(p_R - (1 - \beta)^{0.5}v^{0.5}, 0) \]

\[ p = p_R \]

if \( v > 0 \),

(2)

\[ p_L = \max(-(1 - \beta)^{0.5}(-v)^{0.5} + 0.5, p^*_L) \]

\[ p_R = \min(p_L + (1 - \beta)^{0.5}(-v)^{0.5}, 1) \]

\[ p = p_L \]

if \( v \leq 0 \).

(3)
And implemented policy is:

\[ p(v) = \begin{cases} 
\max((1 - \beta)^{0.5}v^{0.5} + 0.5, p^*_R) & \text{if } v \geq 0 \\
\min(-(1 - \beta)^{0.5}(-v)^{0.5} + 0.5, p^*_L) & \text{if } v < 0.
\end{cases} \]
Expected utility

The ex-ante expected utility of a voter with bliss point $b$ is:

$$E(U(b)) = \int_{v=-\infty}^{v=\infty} -(p(v) - b)^2 g(v) dv + E(\max(q_R, q_L))$$

The derivative with respect to uninformed unsophisticated voters:

$$\frac{dE(U(b))}{d\beta} = \int_{v=-\infty}^{v=\infty} v \left( \frac{(p^*_R - 0.5)^2}{1-\beta} - \frac{(p^*_L - 0.5)^2}{1-\beta} \right) g(v) dv > 0$$
Conclusion

- If the electorate is better informed this might decrease electoral control
- There is no curse of the swing voter
- On the contrary one might even talk about "the swing voters’ blessing"
- Swing voters increase electoral control and force the winning politician closer to the median position
- No rational for abstention
Finding the candidates’ strategies

Define:

\[ p^\text{bestchance}_L(p_R, \nu > 0) = \arg\max_{p_L \in [0, p_R]} \beta \left( \frac{p_L + p_R}{2} \right) + (1 - \beta) \left( \frac{p_L + p_R}{2} - \frac{\nu}{2(p_R - p_L)} \right) \]

\[ \implies p^\text{bestchance}_L(p_R, \nu) = \max(p_R - (1 - \beta)^{0.5} \nu^{0.5}, 0) \]
Finding the candidates’ strategies II

\[ p_R(v; v \geq 0) = \arg \max_{p_R \in [0.5, p^*_R]} p \]

\[ \left( p_L^{\text{bestchance}}(p_R, v) + p_R \right) \frac{v}{2} - (1 - \beta) \frac{v}{2(p_R - p_L)} \leq 0.5 \]

Clearly this is globally increasing in \( p_L \) so the solution is either \( p^*_R \) or the solution to:

\[ \frac{2p_R - (1 - \beta)^{0.5}v^{0.5}}{2} - (1 - \beta) \frac{v}{2((1 - \beta)^{0.5}v^{0.5})} = 0.5 \]

And from this I find:

\[ p_R(v; v \geq 0) = \max((1 - \beta)^{0.5}v^{0.5} + 0.5, p^*_R) \]

And therefore also:

\[ p_L(v; v \geq 0) = \max(p_R(v; v \geq 0) - (1 - \beta)^{0.5}v^{0.5}, 0) \]
Finding the candidates’ strategies III

It is easy to do the same calculations for $v \leq 0$ and to find:

$$p_L(v; v < 0) = \min(-(1 - \beta)^{0.5}(-v)^{0.5} + 0.5, p_L^*).$$

And:

$$p_R(v; v < 0) = \min(p_L(v; v \geq 0) + (1 - \beta)^{0.5}(-v)^{0.5}, 1).$$
\[
E(U(b)) = \int_{-\infty}^{\infty} -(p(v) - b)^2 g(v) dv + E(\max(q_R, q_L))
\]
\[
= \int_{-\infty}^{\infty} -p(v)^2 g(v) dv + b - b^2 + E(\max(q_R, q_L))
\]
\[
= -(1 - G) \left( \frac{(p_R^* - 0.5)^2}{1 - \beta} \right) p_R^2 - G \left( \frac{-(p_L^* - 0.5)^2}{1 - \beta} \right) p_L^2
\]
\[
- \int_{-\infty}^{\infty} \frac{(p_R^* - 0.5)^2}{1 - \beta} ((1 - \beta)|v| + \text{sign}(v)((1 - \beta)|v|)^{0.5} + \frac{1}{4}) g(v) dv
\]
\[
+ b - b^2 + E(\max(q_R, q_L))
\]

Where the third equality is due the fact that the expected implemented policy \(\int_{v=-\infty}^{\infty} p(v)g(v) = 0.5\) and the fourth equality the fact that equilibrium policy is the bliss point of the voter with valence advantage if \(v \geq \frac{(p_R^* - 0.5)^2}{1 - \beta}\) or \(v \leq -\frac{(p_L^* - 0.5)^2}{1 - \beta}\). The last equality follows from substituting in the equilibrium policies.
Taking the derivative with respect to $\beta$ by applying Leibniz’s rule gives:

\[
\frac{dE(U(b))}{d\beta} = g \left( \frac{(p_R^* - 0.5)^2}{(1 - \beta)} \right) \frac{(p_R^* - 0.5)^2}{(1 - \beta)^2} p_R^2 + g \left( -\frac{(p_L^* - 0.5)^2}{(1 - \beta)} \right) \frac{(p_L^* - 0.5)^2}{(1 - \beta)^2} p_L^2 \\
+ \int_{v=\frac{(p_R^*-0.5)^2}{(1-\beta)} (p_L^*-0.5)^2}{v=\frac{(p_R^*-0.5)^2}{(1-\beta)}} |v| g(v) dv \\
- g \left( \frac{(p_R^* - 0.5)^2}{(1 - \beta)} \right) \frac{(p_R^* - 0.5)^2}{(1 - \beta)^2} \left( (p_L^* - 0.5)^2 + p_R^* - \frac{1}{4} \right) \\
- g \left( -\frac{(p_L^* - 0.5)^2}{(1 - \beta)} \right) \frac{(p_L^* - 0.5)^2}{(1 - \beta)^2} \left( (p_L^* - 0.5)^2 + p_L^* - \frac{1}{4} \right) \\
= \int_{v=\frac{(p_R^*-0.5)^2}{(1-\beta)} (p_L^*-0.5)^2}{v=\frac{(p_R^*-0.5)^2}{(1-\beta)}} |v| g(v) dv > 0.
\]
Equilibrium with uninformed median voter

1. The informed voters vote for their favorite candidate as in the main text.
2. All uninformed voters with the exception of the median follow the same strategies as in the main text and vote left if their bliss point is left of the median bliss point and right if their bliss point is right of the median bliss point.
3. The uninformed median voter votes left if \(|p_L - b_m| \leq |p_R - b_m|\) and right if \(|p_L - b_m| > |p_R - b_m|\).
4. The strategies of the candidates are given by:

   \[
   \begin{align*}
   p_L &= \max(b_m - \frac{v}{4(b_m - b_l)}, b_l) \\
   p_R &= \min(p_R^*, b_l + (b_l - p_L)^2)^{0.5}) \quad \text{if } v > 0
   \\
   p_R &= \min(b_m + \frac{-v}{4(b_m - b_l)}, b_r) \\
   p_L &= \max(p_L^*, b_R - (-v + (b_R - p_R)^2)^{0.5}) \quad \text{if } v \leq 0
   \end{align*}
   \]

   where \(b_l\) denote the bliss point of the informed voter who is located closest to the median of all informed voters with a smaller bliss point and \(b_r\) denotes the bliss point of the informed voter located closest to the right of the median.
5. Implemented policy is:

   \[
   p = \begin{cases} 
   p_R & \text{if } v > 0 \\
   p_L & \text{if } v \leq 0
   \end{cases}
   \]
\( V(p_L, p_R) \) denotes the set of all values of \( v \) that would lead to the equilibrium policy positions \( p_L \) and \( p_R \).

The following is a consistent belief system for the uninformed voters given the equilibrium strategies of the candidates:

\[
\Pr(v|p_L, p_R) = \begin{cases} 
\frac{g(v)}{\sum_{v \in V(p_L, p_R)} g(v)} & \text{if } V(p_L, p_R) \text{ is countable}, \\
\frac{g(v)}{\int_{V(p_L, p_R)} g(v) \, dv} & \text{if } V(p_L, p_R) \text{ is not countable},
\end{cases}
\]

\[
g(v|p_L, p_R) = \begin{cases} 
\frac{g(v)}{\int_0^\infty g(v) \, dv} & \text{for } v > 0 \\
0 & \text{for } v \leq 0
\end{cases}
\]

where \( \Pr(v|p_L, p_R) \) is the probability of \( v \) conditioning on the policy positions and \( g(v|p_L, p_R) \) the density.
Proof cutoff point

Proposition

The cutoff point \( b_m = b^*_U(p_L, p_R) \) describes an equilibrium strategy for uninformed voters given that informed voters play the weakly dominating strategy described by the cutoff point \( b_I(p_L, p_R, v) \). As in standard models with full information the preferences of the median voter decide the election result.

Proof.

For showing optimality of the strategy of an individual voter it is sufficient to show that by changing her decision she cannot be better off given the strategies of the other voters. Consider the case of a voter with her bliss point to the left of \( b_m \). Because an uninformed voter with bliss point to the left of \( b_m \) votes left her alternative is voting right. This shifts the bliss point of the decisive voter from \( b_m \) to 
\[
b_I^d(F_U(b_m) - 1) = b_I^d\left(\frac{N+1}{2} - F_I(b_m) - 1\right) = F_I^{-1}(F_I(b_m) + 1) > b_m.
\]
This can only make a difference for the election results if \( b^*(p_L, p_R, v) \) takes a value such that \( b_m \leq b^*(p_L, p_R, v) \leq F_I^{-1}(F_I(b_m) + 1) \) and the right candidate wins instead of the left one. Because the voter’s bliss point is to the left of the median bliss point and consequently also left of the cutoff point \( b^*(p_L, p_R, v) \), this would make her worse off. The same kind of argument can be applied to show that a voter whose bliss point is to the right of \( b_m \) can never be better off voting left given the strategies of the other voters. In the case of \( p_L = p_R \) no cutoff point for informed voters exists, but because all of them vote for the candidate with the valence advantage this candidate wins independently of the decision of the uninformed voters.