Annuity Decisions with Systematic Longevity Risk

Ralph Stevens
Netspar, CentER, Tilburg University
The Netherlands
Goal: obtain the optimal annuity decisions;

Setting:
- Normative approach;
- Life Cycle model;

Contribution to existing literature:
- Allow for systematic longevity risk;
- Allow for deferred annuities.

Conclusions:
- Systematic longevity risk makes annuities less attractive;
- Postponing annuity purchase not optimal;
- Utility loss of deferred annuities is small.
Annuity: Product which provides a yearly income stream until death;

Consider two types of annuities:
- Immediate annuities;
- Deferred annuities.

Timing decision:
- Purchasing at retirement;
- Postponing the purchase of annuities.
Yaari (1965): Full annuitization is optimal;

Empirical results: few voluntary purchase annuities.

Extensive literature with possible explanations:
- Actuarially unfairness;
- Equity risk premium.

Milevsky (1998): postpone until mortality credit = equity risk premium;

Milevsky and Young (2002): real option to annuitize is still valuable until the mid-70s or mid-80s;

Blake, Cairns, and Dowd (2003): Optimal annuitization age in the range of 65 to 80.
Literature on deferred annuities

- Idea: Milevsky (2005, NAAJ): Buy annuities with long deferral period at retirement;

- Deferred annuities:
  - Cheap;
  - Provide “real” longevity insurance;

- Research on deferral period:
  - Evidence that it may not reduce lifetime ruin probability (Bayaraktar and Young, 2009, NAAJ);
  - Deferred annuity (20 years) compared with immediate is utility increasing when loading factor is high enough (Horneff and Maurer, 2008).
Systematic longevity risk

Model:
- Life Cycle Model
- Quantification of the risks
- Annuities

Results: optimal annuity choices:
- Deferred annuities
- Immediate annuities

Conclusions
Longevity forecast UK

Actual and projected male period life expectancy at birth, UK, 1966-2031
Life cycle model
CRRA intertemporally separable expected lifetime utility function:

\[ \mathbb{E}_t \left[ \sum_{\tau \geq 0} \tau p_{x,t} \cdot \beta^\tau \cdot \frac{C^1 - \gamma}{1 - \gamma} \right]. \]

Control variables:
- Annuity;
- Consumption;
- Fraction liquid wealth in risky asset.

Constraints:
- Wealth dynamics;
- Short-selling constraint;
- Positive wealth level.
Dynamic optimization problem.

Without annuity decision:


Maximize utility conditional on:

- Timing of purchasing of annuities;
- Type of annuity (deferral period).

Assumptions:

- Purchase only once annuities;
- Only one type (deferral period) of annuities.

Compare utility in the different sub-optima.
Quantification of the risks
Sources of risk:
- Investment risk;
- Idiosyncratic (non-systematic) longevity risk;
- Systematic longevity risk.

Financial market consists of
- **Riskfree asset**: Yearly time-independent return $r_{rf}$.
- **Risky asset**: Stock price $S_t$ follows a Brownian motion with drift:
  \[ dS_t = \mu S_t dt + \sigma S_t dZ_t, \]
  where $\mu = r_{rf} + \lambda \sigma$.
  \( \lambda \): Equity risk premium; \( \sigma \): Volatility parameter.
- **Annuities**.
Cairns-Blake-Dowd-model is given by:

$$\log\left(\frac{q_{x,t}}{1 - q_{x,t}}\right) = k^{(1)}_t + x \cdot k^{(2)}_t + \epsilon_{x,t},$$

where $q_{x,t}$ is the time-$t$ mortality probability for a $x$-year old.

- $k^{(1)}_t$: General level of mortality (generally decreasing over time)
- $k^{(2)}_t$: General level of increase in mortality by age (generally increasing over time)

Forecast processes using random walk with drift;
Allow for parameter risk using Jeffreys prior and Bayesian updating.
Annuities

Pricing annuities
Price of deferred annuities
Price of immediate annuities

Results

Conclusions

Actuarially unfairness due to:
- Price of systematic longevity risk.

When:
- Efficient market;
- No arbitrage.

The time-$t$ value of an annuity with a deferral period of $d$ years:

$$V_t(A^{(d)}_{x,t}) = \sum_{\tau \geq d} \mathbb{E}_t^Q [\tau p_{x,t}] \cdot \left(\frac{1}{1 + rf}\right)\tau.$$
Price of deferred annuities

- Deferred annuities are much cheaper;
- Loading factor increases with deferral period.
Generally cheaper to postpone the purchase of annuities;

Future prices of an annuities are currently stochastic.
Results
Advantage longer deferral period:
- Lower longevity risk premium;
- Cheaper → more liquid wealth when MC is low;
→ Allows for more capital gains from equity risk premium;

Disadvantage longer deferral period:
- Lower gains from mortality credit;
- Fewer periods with income guarantee;
→ More uncertainty in consumption level;
→ Less consumption smoothing.
The utility is quantified by:

\[ CEC: \text{Certainty Equivalent Consumption}: \]
How much deterministic yearly consumption do I need in order to be equally well off as in the optimal strategy;

The CEC is normalized by:
The CEC in case of fully immediate annuitization \( CEC_{fa} \);

Hence, \( \frac{CEC}{CEC_{fa}} \) relative utility gain by optimal choices instead of currently fully annuitizing.
Solid and dashed curves: Pricing using risk-adjusted probabilities; Dashed-dotted curve: Pricing using (7.3%) loading factor; Dotted curve: without systematic longevity risk.
- First: increasing function of $d$;
- Then: decreasing function of $d$. 
Postponing the annuity decision → allows for consumption smoothing;

- It also allows for capital gains from equity risk premium;

- Postponing → less longevity risk conditional on realized survival probabilities;
  Reduces longevity risk premium.

- Future annuity prices are stochastic.
i) Equity risk premium;
ii) Mortality credit;
iii) Conversion rate risk.
- **Effect systematic longevity risk on consumption level:**
  - Survival probabilities low → more consumption;
  - Survival probabilities high → less consumption;

- **Effect including systematic longevity risk on annuity choice:**
  - Immediate annuities becomes less attractive → postponing more favorable; 
    Fraction annuities wealth with systematic longevity risk: 84%;  
    without systematic longevity risk: 90%;  
    Due to: uncertainty in the value of annuities.
  - Annuities prices are stochastic → postponing less favorable.
Systematic longevity risk → large confidence intervals of optimal fraction annuitized wealth.
Conclusions
Conclusions

- There exists systematic longevity risk; Systematic longevity risk has a non-zero market price;

- Systematic longevity risk affects annuity decision;
  - Annuities provide lower utility;

- Postponing annuity purchase:
  - Systematic longevity risk → uncertain prices;
  - Postponing not utility increasing;

- Type of annuity:
  - Deferred annuities are preferable;
  - Optimal deferral period is short.