Targeted Competition: Choosing Your Enemies in Multiplayer Games.

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Examples of Targeted Competition
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- Competition in product space and location space
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- Multiproduct firms
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- Comparative advertisement
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- Unethical competition practices
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- Political competition
Examples of Targeted Competition

- Competition in product space and location space
- Multiproduct firms
- Comparative advertisement
- Unethical competition practices
- Political competition
- Warfare
Research Question

Strategic considerations

Potential outcomes of targeted competition
Research Question

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- Instantaneous payoffs from competition

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- Instantaneous payoffs from competition
- Balance of powers among the rivals

Potential outcomes of targeted competition
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- Instantaneous payoffs from competition
- Balance of powers among the rivals

Potential outcomes of targeted competition

- The weaker players lose to the strongest player
Research Question

Strategic considerations

- Instantaneous payoffs from competition
- Balance of powers among the rivals

Potential outcomes of targeted competition

- The weaker players lose to the strongest player
- The weaker players coordinate against the strongest and, consequently, all the players converge in their powers
Related Literature
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- Colonel Blotto games – see, e.g. Roberson (2006).
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State Space
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- There are 3 players
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- $x_i$ – power of player $i$; $x = (x_1, x_2, x_3)$
State Space

- There are 3 players
- \( x_i \) – power of player \( i \); \( x = (x_1, x_2, x_3) \)
- \( X \) – state space

\[
X = \left\{ x \in \mathbb{R}^3 \left| \sum_{i} x_i = 1, \; 0 \leq x_i < \frac{2}{3} \sum_{j \neq i} x_j \; \forall i \right. \right\}
\]
Actions, Strategies and Payoffs I

\[ y_{ij} \geq 0 \] – amount of power player \( i \) uses to compete against player \( j \),

\[ \sum_{j \neq i} y_{ij} \leq x_i \]

\[ \phi(y_{ij}, y_{ji}) \] – instantaneous payoff that player \( i \) gets from competition with player \( j \),

\[ \phi(y_{ij}, y_{ji}) = (a - b(y_{ij} + y_{ji}))y_{ij} \]

\[ \pi_i(y) \] – total instantaneous payoff for player \( i \),

\[ \pi_i(y) = \sum_{j \neq i} \phi(y_{ij}, y_{ji}) \]
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Actions, Strategies and Payoffs I

- $y_{ij} \geq 0$ – amount of power player $i$ uses to compete against player $j$, $\sum_{j \neq i} y_{ij} \leq x_i$
- $\varphi(y_{ij}, y_{ji})$ – instantaneous payoff that player $i$ gets from competition with player $j$

$$\varphi(y_{ij}, y_{ji}) = (a - b(y_{ij} + y_{ji}))y_{ij}$$

- $\pi_i(y)$ – total instantaneous payoff for player $i$

$$\pi_i(y) = \sum_{j \neq i} \varphi(y_{ij}, y_{ji})$$
Actions, Strategies and Payoffs II

- $T$ – game duration, $T = \infty$ if the game never ends
- $S_i(x)$ – terminal payoff for player $i$

$$S_i(x) = \begin{cases} M & \text{if } x_i > x_j \forall j \neq i \\ 0 & \text{otherwise} \end{cases}$$
Actions, Strategies and Payoffs II

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- $y(x)$ – Markovian strategies

$U_i$ – payoff for player $i$ (for the whole game)
\[ U_i = \int_0^T e^{-\delta t} \pi_i(y(x(t))) \, dt + e^{-\delta T} S_i(x(T)) \]
**Actions, Strategies and Payoffs II**

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Power shifts from the player who fights less to the player who fights more

\[ \dot{x}_i(t) = f_i(y(x(t))) \]
\[ f_i(y) = \sum_{j \neq i} (y_{ij} - y_{ji}) k \]

\( k \) – power shift intensity
Myopic Players

Proposition
Suppose, without a loss of generality, that \( x_1(0) > x_2(0), \)
\( x_1(0) > x_3(0) \). Then there exists a unique MPE. Moreover, the
equilibrium dynamics are such that the game ends and the
strongest player wins, i.e. \( T < \infty \) and \( x_1(T) > x_2(T), \)
\( x_1(T) > x_3(T) \)

Proof.
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Proof.

▶ Best responses: \( \hat{y}_{ij}(x) = \frac{x_i}{2} + \frac{x_k-x_j}{10} \)
**Myopic Players**

**Proposition**

*Suppose, without a loss of generality, that* $x_1(0) > x_2(0)$, $x_1(0) > x_3(0)$. *Then there exists a unique MPE. Moreover, the equilibrium dynamics are such that the game ends and the strongest player wins, i.e. $T < \infty$ and* $x_1(T) > x_2(T)$, $x_1(T) > x_3(T)$.

**Proof.**

- Best responses: $\hat{y}_{ij}(x) = \frac{x_i}{2} + \frac{x_k - x_j}{10}$
- Equilibrium dynamics: $\dot{x}_i(t) = \frac{9k}{5} \left( x_i(t) - \frac{1}{3} \right)$
The Main Result

Proposition

If $\delta < \frac{4k}{3}$, then there exists an MPE such that for all $i$

$x_i(t) \to \frac{1}{3}$ as $t \to \infty$.

The idea is to prove the proposition by construction. Let

$$\hat{y}_{ij}(x) = \frac{x_i + c(x_k - x_j)}{2}$$

$$c = \frac{5\delta - 14k - \sqrt{(25\delta - 76k)(\delta - 4k)}}{18k}$$

$$\dot{x}_i(t) = \frac{3k(c + 1)}{2} \left(x_i(t) - \frac{1}{3}\right)$$

If $\delta < \frac{4k}{3}$, then $c < -1$. Consequently, $x_i(t) \to \frac{1}{3}$ as $t \to \infty$. 
Conclusions and further research

we defined the problem on $X$ and found an equilibrium;
we showed, that myopic players die out in the game, while if players are patient enough the game continues infinitely, thus the option to target competitors stabilizes competition;
we demonstrated that “collusion-like” behaviour can arise from Markov strategies.
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Thanks! Questions?..