Lower Volatility & Higher Inequality: Are They Related?

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Job Market Paper

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Volatility and Income Inequality in the US (Motivation)

Volatility: Standard Deviation of Per Capita Real GDP Growth

Income Inequality: Variance of Log Incomes
Changes in volatility and inequality
Happen at the same time; Across countries and periods

Calls attention to the link between variables: individuals’ incomes

Individual Income Process: \( y_{it} = \alpha_i + y_{it-1} + \nu_{it} \)
where \( y_{it} \) is log real income of individual \( i \) at time \( t \)

- If \( \nu_{it} \) is omitted, this process implies no volatility and constant inequality

\[ \text{Var}_{ts} (\nu_{it}) \quad \text{Cov}_{cs} (\nu_{it}, \nu_{jt}) \]

- The effect of shocks (\( \nu_{it} \)):
  - Volatility: +
  - Inequality: +

Recurring pattern in the data across industrialized countries a reduction in volatility and an increase in inequality; point towards reduction in correlation of income shocks
Outline

1. International Evidence: Volatility and inequality across countries and time

2. Relating Volatility and Inequality

   Model: Deriving volatility to inequality from individual income model

   Data and Structural Break Estimation: To estimate the timing of the changes in the variables

3. Analyzing the Changes in the Variables

   Decomposing the Changes: Estimating variance and correlation from volatility and inequality data for the US, and comparing the results with findings from micro income data for this country

   Explaining the Changes: Empirical evidence for globalization as a source of change in the variables
International Evidence

Volatility and inequality across countries and time; observable simultaneity in the changes in variables

*Countries are chosen on the basis of availability of inequality data*
\[ y_{it} = \alpha_i + y_{it-1} + v_{it} \]

where shocks are

i.i.d.: \( E_{i,t} (v_{it}) = 0 \) and \( \text{Var}_i (v_{it}) = \sigma_t^2 \)

uncorrelated over time: \( \text{Corr}_{ts} (v_{it}, v_{it-1}) = 0 \)

but correlated across individuals: \( \text{Corr}_{cs} (v_{it}, v_{jt}) = \rho_t \)
Volatility arises from the deviation of the mean individual income growth, $\Delta y_t$, from its long term mean:

$$Volatility = \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} (\Delta y_t - \bar{E}_t(\Delta y_t))^2 = \rho_t \sigma_t^2$$

The change in inequality arises due to the difference in income gains of individuals:

$$Change \ In \ Inequality = \Delta \text{Var}_{cs}(y_{it}) = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} (\Delta y_{it} - \bar{E}(\Delta y_t))^2 = (1 - \rho_t)\sigma_t^2$$
I use data for Sweden, Canada, Denmark, Finland, Italy, Netherlands, Norway, the United Kingdom and the United States.

I collected data for the Gini index, then I mapped this data to the variance of log income by following the work of van Rens and Teulings (2008).

The Gini index itself is "constructed" from the World Income Inequality database (WIID), which summarizes the index data supplied by different sources (urban or rural area, uses household or individuals, before or after tax, ...).

My approach is, for each country, to use the Gini index data source which covers the longest time period, and to extend it by using the yearly growth rate of the data belonging to other sources.

Wage differential (W90/W10) from OECD Employment Statistics database.

The rest of the data (exports, imports, cross border assets and liabilities, GDP, population) are collected from International Financial Statistics database of IMF.
Testing for Breaks (Structural Break Estimation)

- Testing for a break in volatility

  **Theory**: $\frac{1}{T} \sum_{t=1}^{T} \left( \Delta y_t - E_t(\Delta y_t) \right)^2 = \rho_t \sigma_t^2$

  **Data**: $(\Delta y_{pc}^t - \Delta y_{pc}^t)^2 = c^n + \varepsilon_{1t}$

  Mean income growth is proxied by per capita GDP growth

  Considerations: Is $\Delta y_{pc}^t$ stationary? Is there a break in $\Delta y_{pc}^t$?

- Testing for a break in inequality

  **Theory**: $\Delta \text{Var}_{cs} (y_{it}) = (1 - \rho_t) \sigma_t^2$

  **Data**: $\text{Var}_{cs} (y_{it}) = c^n + \beta^n t + \varepsilon_{2t}$

  I do not take the difference of the data, but regress it on a time trend


- I use Perron and Qu (2007) to estimate for breaks (Estimates multiple breaks at a time, using multiple equations, similar to using SUR over OLS)
Volatility: $10.000 \times \text{Squared Deviations from the Mean GDP Growth}$

Income Inequality: $100 \times \text{Variance of Log Incomes}$

Break dates significant at 5% together with their confidence intervals:

<table>
<thead>
<tr>
<th>Inequality</th>
<th>Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>1967</td>
<td>1982</td>
</tr>
<tr>
<td>1984</td>
<td>1984</td>
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</tbody>
</table>

$(80, 86)$
Now I assume that there exist two components of income

Permanent component: \( y_{it}^p = \alpha + y_{it-1}^p + v_{it} \)

Transitory component: \( y_{it} = y_{it}^p + u_{it} \)

Together they form: \( y_{it} = \alpha + y_{it-1} + \Delta u_{it} + v_{it} \)

It means that today’s error includes past shocks as well.

If this is the true income process generating the data, then structural break estimation applied on previous model cannot accurately find the exact timing of the breaks (\( \Delta y_{it} = \alpha + \Delta u_{it} + v_{it} \)).

In order to dismiss past shocks so as to find precise results regarding the date of breaks, I regressed variables on their lags and obtain residuals from those regressions, on which I apply structural break estimation.
Coincidence of Breaks (& Causality Inferences)

Model only with permanent shocks

Model with transitory and permanent shocks

\[ T_{ineq} - T_{vol} \quad (at \%5\, sig.) \]

Horizontal Axis: Time difference between the break dates found in series
Columns: Number of countries with the same order of breaks

First histogram shows that 11 out of 13 breaks in volatility coincide with 11 out of 17 breaks in inequality.
Why This Income Process?

- Why do I use transitory and permanent decomposition for error terms?
- It is consistent with the literature (Moffitt and Gottschalk 1998)
- It nestles alternative decompositions like:
  - Splitting error terms to business cycle (aggregate) and idiosyncratic components (this is because I defined correlation between the individuals’ error terms, which corresponds to the effect of the business cycle shock)
  - The use of an error term with MA or AR behaviour (this is they both require us to apply structural break estimation in the same way, regressing the variables on their lags and obtaining today’s shocks in the residuals)
Volatility: 10,000 * Squared Deviations from the Mean GDP Growth
Income Inequality: 100 * Variance of Log Incomes
Income Dynamics (Decomposing the Changes)

Income Inequality Dynamics: Age Effect on Inequality

The Effect of Decline in Correlation at Time t-1
Estimating Variance and Correlation in the US

Volatility: 10.000 * Squared Deviations from the Mean GDP Growth
Income Inequality: 100 * Variance of Log Incomes

Theory:

Volatility = \frac{1}{T} \sum_{t=1}^{T} \left( \Delta y_t - E_t(\Delta y_t) \right)^2 = \rho \sigma^2

Data:

Volatility = (\Delta y_{tpc} - \Delta y_{tpc})^2

Theory:

Change in Within Cohort Inequality = (1 - \rho)\sigma^2

Data:

Within Country Inequality = \text{Var}_t(y_{it})
Deriving Inequality (Decomposing the Changes)

→ Variance Decomposition

(A) Variance of log incomes =
(B) Mean of the variances of within cohort log incomes +
(C) Variance of the means of within cohort log incomes

→ Theoretical measure of (B) can be calculated via the fact that each year within cohort inequality rises by an amount equal to \((1 - \rho_{pre})\sigma_{pre}^2\).

Inequality within the youngest cohort is \((1 - \rho_{pre})\sigma_{pre}^2\)

... 

Inequality within the oldest cohort is \(n(1 - \rho_{pre})\sigma_{pre}^2\)

\[ B = \frac{n+1}{2} \times (1 - \rho_{pre})\sigma_{pre}^2 \]

→ We have data for (A) & (C), and (B) can be found from their difference
Results (Decomposing the Changes)

- Estimation results for two different numbers of cohorts

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>$\rho_{pre}$</td>
<td>0.043</td>
<td>0.007</td>
</tr>
<tr>
<td>$\sigma^2_{pre}$</td>
<td>0.015</td>
<td>0.021</td>
</tr>
<tr>
<td>$\rho_{post}$</td>
<td>0.007</td>
<td>0.061</td>
</tr>
<tr>
<td>$\sigma^2_{post}$</td>
<td>0.021</td>
<td>0.061</td>
</tr>
</tbody>
</table>

US (n=45) 0.043 0.015 0.007 0.021 %-83 %48

US (n=15) 0.015 0.041 0.003 0.061 %-83 %51

- Findings from micro data

Gorbachev, Olga (2007)

Nichols Austin (2008)
Trade Globalization (Explaining the Changes)

Inequality and Trade Share in GDP

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Volatility & Inequality

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Breaks in the Trade Share in GDP and in Volatility:

Coincidence of Breaks and Causality Inferences

(Difference Between the Break Dates)

\[ T_{trsh} - T_{vol} \enspace (at \enspace %5 \enspace sig.) \]

Horizontal Axis: Time difference between the break dates found in series
Columns: Number of countries with the same order of breaks
Age Effect on Inequality

Relative Incomes

Cohort 3, t-1
Cohort 2, t-1
Cohort 1, t-1
Cohort 2, t
Cohort 3, t
Cohort 1, t

I (t) = I(t-1)

UNITED STATES

UNITED KINGDOM

USVLI
USTRSH

UKVLI
UKTRSH

Ozan Eksi (Universitat Pompeu Fabra)
Inequality and volatility are driven by the variance and correlation of individual income shocks.

The decline in the correlation is responsible for the changes in data in the recent decades (what matters is the percentage change in the correlation). This result limits to explanations for the changes in inequality and volatility.

Globalization may explain the changes in variance and correlation of income shocks. This gives interesting insights for the effect of globalization on inequality and volatility.

Finding the true source of the changes in the data is important since each possible source would require different precautionary policies to implement.

Cohort structure of the population is crucial on the time the inequality responds the changes in other variables. This finding questions time series regression estimates that only analyzes the simultaneous link between the inequality and any other variable of interest.
Income Inequality

Deaton and Paxson (1994) relates inequality to second moments of individuals’ income shocks through cohort structure, here I formalize this relation

Revealing the inequality dynamics, I question time series OLS estimates of inequality

For the US, I find that the decline in the correlation of income shocks outweighs the increase in their variance, this finding complements the literature on inequality (Moffitt & Gottschalk 1998, 2008; Primiceri & van Rens 2002)

GDP Volatility

I formalize the relation between real GDP volatility and second moments of individuals’ income shocks


Gorbachev (2007) informally discusses issue
Relation and Contributions to the Literature

- **Income Inequality and GDP Volatility**

  I relate inequality and volatility; both theoretically and empirically
  Uren (2008) points out for the relation, but does not document the fact
  My paper shows how to use inequality and volatility data to find variance and correlation in micro income data

- **The Effects of Globalization on Income Inequality and GDP Volatility**

  I link the changes in data to Globalization by structural break estimation
  By decomposing the changes in volatility and inequality, I show that empirical evidence is consistent with trade theory, which cannot be explained by time series OLS