Monetary Policy Regimes and the Volatility of Long-Term Interest Rates

Job Market Paper

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Abstract

This paper addresses two important questions that have, so far, been studied separately in the literature. On the one hand, the paper aims to explain the excess volatility of long-term interest rates observed in the data, which is hard to replicate using standard macro models. By building a small macroeconomic model, I show empirically that the policy responses of a central bank that is uncertain about the natural rate of unemployment can explain this volatility puzzle. On the other hand, the paper aims to shed new light on the distinction between rules and discretion in monetary policy. I show that using yield curve data may facilitate the empirical discrimination between different monetary policy regimes.

Keywords: long-term interest rates, optimal monetary policy, discretion, commitment, Bayesian estimation

JEL-classification: C11, C13, C15, E32, E42, E43, E47, E50

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1 Introduction

This paper addresses two important questions that have, so far, been studied separately in the literature. First, the paper aims to explain the excess volatility of long-term interest rates observed in the data, which is hard to replicate using standard macro models with a deterministic steady state. I show that the policy responses of a central bank that is uncertain about the natural rate of unemployment can help explain this volatility puzzle. Second, the paper aims to shed some new light on the distinction between discretion and rules in monetary policy. Despite a great deal of theoretical work, there are few clear-cut empirical results regarding the real-word prevalence of alternative policy regimes. I show that including yield curve data may make it possible to empirically distinguish between different monetary policy regimes.

The model in the paper is a forward-looking model where the private sector has full information but the central bank cannot observe the shocks affecting the economy, in particular the natural rate of unemployment. Following the results in Orphanides and Williams (2002), I model policymakers' misperceptions of the natural rate of unemployment (the rate of unemployment consistent with stable inflation) as an autoregressive process. This has important implications for inflation, which are amplified in the case of discretionary monetary policy. When the policymaker cannot commit, he loses control over inflation expectations, and inflation and interest rate volatility are higher than when the policymaker can commit. The intuition is that when the monetary authority underestimates the natural rate of unemployment, it sets a higher inflation than its own target in order to reduce the perceived unemployment gap. Since misperceptions about the natural rate of unemployment are persistent, this raises expectations of future inflation and future short-term interest rates. Once we augment the model with the expectation hypothesis of interest rates, which establishes a relationship between long-term and short-term interest rates, a discretionary regime can also explain the volatility puzzle.
In contrast to other papers that combine a macro model with no-arbitrage models of the term structure,\textsuperscript{1} I focus on a simple macro model and then explore the implications of different monetary regimes for long-term interest rates.\textsuperscript{2} This allows me to analyze the unresolved issue on how monetary policy has been conducted in the last 45 years. Certainly, the goal of the paper is not to construct a very precise model of the yield curve, but to find some linkages between macroeconomic fundamentals, monetary policy, and the behavior of long-term rates. In particular, I want to calculate how much of the total volatility of long-term interest rates is explained by macro variables as opposed to financial risks. For this purpose, long-term interest rates are modelled by the expectation hypothesis.

To investigate the implications of the model, I estimate it using Bayesian methods. To the best of my knowledge, this is the first paper to rely on a structural estimation of a macromodel to distinguish between different monetary policy regimes or to explain the volatility puzzle.

A large literature has documented a decline in business cycle volatility in the U.S. in the mid 1980s.\textsuperscript{3} Based on this evidence, I divide the data into two periods: 1960Q1-1978Q4 and 1983Q1-2005Q4 (excluding the four years at the beginning of the Volcker period when the Fed targeted nonborrowed reserves and the volatility of interest rates of all maturities increased dramatically). Despite the lower volatility of the macro fundamentals, the second period shows higher interest rate volatilities than the first period. In my model, this is attributed to a slightly larger estimated persistence in policymakers’

\textsuperscript{1} See, for instance, Bekaert, Cho, and Moreno (2005), Hördal, Tristani, and Vestin (2006) and Rudebusch and Wu (2004).

\textsuperscript{2} Diebold, Rudebusch, and Arouba (2006) find that the effects of the yield curve on macro variables are less important than the effects of macro variables on the yield curve. They also find the short rate to be a sufficient statistic for interest rate effects in macro dynamics.

\textsuperscript{3} There has been a growing debate over whether the decline in volatility is a consequence of smaller economic disturbances ("good luck?") or better monetary policy. Among many others, Gordon (2005), Justiniano and Primiceri (2006), Sims and Zha (2005) and Stock and Watson (2002) mainly attribute this decline to smaller shocks in the economy, while Boivin and Giannoni (2005), Clarida, Gali, and Gertler (2000) and Cogley and Sargent (2005) also stress the importance of changes in monetary policy.
misperceptions which translates into a more persistence inflation behavior. Moreover, to explain the volatility of long-term interest rates in both periods, we need lack of commitment from the monetary authority. Thus, the results indicate that U.S. monetary policy is best understood as originating from a discretionary regime.

To analyze the role of different institutions in monetary policy, the paper also estimates the same model for two periods in the U.K., namely 1983-1997 and 1998-2005. In the latter, but not the former period, the Bank of England was operationally independent. This exercise attempts to address the importance of central bank independence in the design of monetary policy. The U.K. evidence is different than the U.S. evidence. If anything, the post-independence monetary policy of the Bank of England has been closer to rules than discretion.

The rest of the paper is organized as follows. Section 2 reviews previous related literature. Section 3 describes the model. Sections 4 and 5 present the empirical evidence for the U.S. and the U.K. respectively. Section 6 concludes.

2 Literature Review

2.1 Long-Term Interest Rates

Three facts in the data on interest rates are hard to replicate using standard macro models. First, short- and long-term interest rates are strongly positively correlated (e.g., Cook and Hahn (1989)) As shown in Table 1, the correlations are positive and above 0.75 for all subperiods and all maturities. Second, as stressed in Shiller (1979), long-term rates present excess volatility: the volatility of long-term rates is higher than predicted by expectation models of the term structure. Long-term interest rates should be expected to be much smoother than short-term rates, given that we can consider long rates as an average of expected short-term interest rates plus a premium term. However, the data in Table 1 shows that long-term interest rates are about as volatile as short rates. Third,
as shown by Gürkaynak, Sack, and Swanson (2005), long-term forward rates exhibit excess sensitivity to monetary policy announcements and macroeconomic news.\footnote{A forward rate is the rate of return that an investor demand today to commit to lending money in the future.} These three facts cannot easily be explained by standard macro models where the long-term properties of the model are given, say, by a deterministic steady state.

A number of papers have tried to model the behavior of long-term interest rates. Ellingsen and Söderström (2001) and Ellingsen and Söderström (2005) argue that a rise in the short-term interest rate perceived as a response to shocks to inflation or output will lead to higher inflation expectations and increases in long-term interest rates. On the other hand, a rise in the interest rate perceived to be triggered by a change in the preferences of the monetary policymaker towards lower inflation, will reduce inflation expectations and long-term interest rates. Ellingsen and Söderström obtain these results in models with high output and inflation inertia or a very persistent inflation target.\footnote{For instance, in Ellingsen and Söderström (2001) inflation is determined by an accelerationist Phillips curve: $\pi_{t+1} = \pi_t + \alpha y_t + \varepsilon_{t+1}$. This type of relation is not microfounded and implies highly persistent inflation, which in their model translates into responses of long-term interest rates. In Ellingsen and Söderström (2005), when the autoregressive coefficient of the inflation target process is less than 0.80, the results do not hold.} They empirically test their model and find that in general, long-term interest rates move in the same direction as short-term rates, except on days where market participants see movements in short rates as a change in policy preferences.

Using the same idea, other authors have explained the response of long-term interest rates to the central bank policy instrument using time-varying inflation targets. Shocks to the central bank inflation target change future expected inflation and thereby nominal long-term rates. In the extreme case of a random walk inflation target, an increase in the central bank inflation objective will trigger an equal size increase in long-term rates. Gürkaynak, Sack, and Swanson (2005) and Beechey (2005) develop calibrated models with a variable inflation target and imperfect information which generate long-term rate
volatility since expected inflation is not anchored.\textsuperscript{6} In both models, inflation and the short-term interest rate have a different steady state value after a shock.\textsuperscript{7}

Similarly, Hördal, Tristani, and Vestin (2005) explain the volatility of long-term interest rates using a second-order approximation of a standard DSGE model with a variable inflation target, where they calibrate the autocorrelation coefficient of the inflation target to be 0.99.

In all these papers, the high persistence of inflation and thus, the volatility of long-term rates, arises either from an accelerationist Phillips curve (where inflation is highly persistent by definition) or from very persistent inflation target shocks. In my model, on the other hand, inflation persistence is estimated rather than imposed and intrinsic to the model. Inflation persistence arises because of central bank misperceptions about the natural rate of unemployment, which are empirically very persistent. Moreover, in all the above mentioned papers, except Ellingsen and Söderström (2001), the volatility of long-term rates is explained by a shock to a policy objective, namely the inflation target. In my model, the policymaker’s objectives are stable and long-term rates mainly move due to his misperception shocks.

Alexius and Welz (2005) resort to a time-varying natural real interest rate to explain the behavior of long-term yields. Given empirical evidence\textsuperscript{8} showing that changes in long-term yields on U.S. Treasury bonds are mostly due to changes in long-term inflationary expectations, real forward interest rates are quite stable and the term premium is small, I abstract from variations in the real interest rate and explain long-term rate volatility through inflation expectations. Nevertheless, it would be interesting to bring together

\footnotetext[6]{Using a macro-finance model, Rudebusch and Wu (2004) also introduce time variation in the inflation target to generate responses of long rates to macro shocks.}

\footnotetext[7]{While Gürkaynak, Sack, and Swanson (2005) introduce an ad-hoc equation to specify the evolution of the inflation target, Beechey (2005) uses a random walk inflation target. In the first case, any shock affecting inflation will generate a new steady state level for inflation and interest rates, while in the second case, only inflation target shocks will have this effect.}

\footnotetext[8]{See, for instance, Ireland (1996), Gürkaynak, Sack, and Swanson (2003), Rudebusch and Wu (2004) and Diebold, Rudebusch, and Arouba (2006).}
the nominal and real channel to explain the volatility puzzle.

Baxter (1989) tries to explain the high volatility of long- and short-term interest rates during the 1979-1982 period with a Bayesian learning model, where the response to shocks is largest in the initial stages of a new policy. However, she does not find empirical support for her model.

In a robust control framework, where the policymaker adopts a min-max approach, Giordani and Söderlind (2004) show in a calibrated model that robustness leads to higher and more persistent reactions of inflation and the nominal interest rate after a shock. This feature of the robust solution implies that robustness makes long-term interest rates more volatile than in the standard rational expectation case. In their paper, they show this for one-year interest rates and assuming a discretionary monetary policy. In my paper, a discretionary regime is able to explain the high volatility of long-term rates even with model certainty.

2.2 Rules versus Discretion

A large theoretical literature analyzes the properties of monetary policy under discretion and commitment. In general, this literature considers the qualitative and not the quantitative implications of both regimes and, to my knowledge, no paper has explicitly analyzed the implication of these regimes for long-term interest rates. As pointed out by Baxter (1988) almost 20 years ago, it is important to use established statistical procedures for selecting among alternative models of policymaking. However, very little has been achieved on this empirical agenda and most current papers model monetary policy using a Taylor-type interest rate rule.

A first generation of theoretical papers studying the differences between commitment and discretion in monetary policy focuses on the time-consistency problem described in Kydland and Prescott (1977) and Barro and Gordon (1983). The main assumption of the so-called Barro-Gordon model is that a central bank lacking commitment will pursue an
accommodative monetary policy, (unsuccessfully) trying to push unemployment below its natural rate. As a result, a discretionary regime gives rise to an inflation bias, where inflation is higher than the target.\footnote{In dynamic models, average inflation is larger than the inflation target.}

More recently, a second generation of papers, including Clarida, Gali, and Gertler (1999), Svensson (1997) and Woodford (1999) among others, stresses the fact that in forward-looking models, a discretionary regime generates a dynamic loss, even if the central bank targets the natural rate of unemployment.\footnote{McCallum and Nelson (2004) find the magnitude of these losses to be significant, and depending on the parameters, greater than the losses arising from the inflation bias.} In these models, a discretionary monetary policy causes a suboptimal response to shocks given that the central bank cannot affect the private sector’s expectations. In the commitment case, the monetary authority can effectively control private expectations about future inflation and thus, the behavior of the private sector today.

Empirical papers addressing the inflation bias problem of the Barro-Gordon model include Christiano and Fitzgerald (2003), Ireland (1999) and Ruge-Murcia (2003). The two first papers argue that their results support the Barro-Gordon model as an explanation for U.S. inflation since 1960. However, neither paper estimates the model nor considers the counterfactual of monetary policy under commitment. On the other hand, Ruge-Murcia (2003) uses full information maximum likelihood to test the predictions of the Barro-Gordon model against an alternative model where the central bank gives different weights to upward and downward deviations of unemployment from its target. The problem is solved under a discretionary regime. Reduced-form estimates indicate that the Fed targeted the natural rate of unemployment, but gave more weight to positive than to negative unemployment deviations between 1960 and 1999.

Unalike these previous papers, I look at the problem from a different perspective and use data on long-term interest rates to distinguish between monetary policy regimes. Moreover, I assume that the monetary authority targets the natural rate of unemploy-
ment, which eliminates the Barro-Gordon type of inflation bias. In this sense, my model is closer to the second generation of papers described above. Given the volatility of long-term interest rates and their correlation with the short rate, my results show that a monetary regime under discretion is more likely to have prevailed in the U.S. since 1960.

Another related paper is Bikbov (2005). Like I do, he stresses the importance of including term structure data to identify different monetary policy regimes. Bikbov models monetary policy as a forward-looking interest rate rule with monetary policy shocks. Allowing for switches in the parameters, he interprets periods with high variance in the monetary policy shock as discretionary regimes and periods with low variance as commitment regimes.\(^{11}\) Bikbov’s results indicate that since the 1970s, monetary policy in the U.S. has continuously alternated between "active" versus "passive" policy regimes\(^{12}\) and between high versus low volatility monetary policy shocks. While Bikbov’s results are suggestive, they are hard to interpret since he does not include optimal monetary policies of any kind in his analysis.

3 The Model

The model in this paper is a new Keynesian forward-looking model where firms have market power and get to adjust their prices with a fixed probability in each period (Calvo (1983)).\(^{13}\)

The loglinearized version of the Phillips curve and the expectations based IS curve are given by

\[
\pi_t = \beta E_t \pi_{t+1} - \theta (u_t - u^N_t) + \varepsilon_t
\]

\(^{11}\) He argues that more volatile monetary shocks can be seen as the Fed is more willing to deviate from the systematic rule.

\(^{12}\) An "active" policy regime aggressively stabilizes inflation, while a "passive" one reacts to expected inflation less strongly.

\(^{13}\) See, for instance, Clarida, Gali, and Gertler (1999) and Woodford (2003).
and
\[ u_t = E_t u_{t+1} + \delta E_t (i_t - \pi_{t+1}) + \eta_t, \]  
(2)
where \( \pi_t \) is the rate of inflation, \( u_t \) the unemployment rate, and \( u_t^N \) the natural rate of unemployment. The nominal interest rate, \( i_t \), is the return on a short-term instrument from period \( t \) to \( t+1 \), \( \eta_t \) is an exogenous demand shock assumed to be i.i.d. \( N (0, \sigma^2_\eta) \), e.g. government expenditures, while \( \varepsilon_t \) can be considered as an i.i.d. \( N (0, \sigma^2_\varepsilon) \) markup shock. \( E_t (\cdot) \) denotes the rational expectations operator given the private sector information in period \( t \).

In the standard new Keynesian literature, equations (1) and (2) are expressed in terms of the output gap rather than the unemployment gap.\(^{14}\) However, by reference to Okun’s law, we can express the output gap as a monotonic function of the unemployment gap.\(^{15}\)

I assume that the natural rate of unemployment follows a first-order autoregressive process:
\[ u_t^N = \gamma u_{t-1}^N + \chi_t, \]  
(3)
where \( \chi_t \) is i.i.d. \( N (0, \sigma^2_\chi) \) and the unconditional mean of \( u_t^N \) is zero.\(^{16}\)

A time-varying natural rate of unemployment is consistent with the substantial changes observed in U.S. unemployment in the last decades. Staiger, Stock, and Watson (1997) find that the natural rate has fluctuated during the last 30 years in the U.S., and decreased by one percentage point between the 1980s and mid 1990s. Shocks to the natural rate of unemployment could, e.g., be associated with exogenous changes in productivity or labor force demographics which affect the labor supply.

\(^{14}\) This is due to the fact that employment variations only occur in the intensive margin and unemployment is always zero.

\(^{15}\) Supply equations using unemployment gap instead of output gap have been used, for instance, in Blanchard and Gali (2006a), Blanchard and Gali (2006b), Primiceri (2006) and Reis (2003). One way of deriving the Okun’s gap relation from first principles is to formulate a model of search and matching in the labor market.

\(^{16}\) In practice, I work with demeaned data, so all the variables have an unconditional mean of zero in the model.
3.1 Information and the Natural Rate of Unemployment

I assume that the private sector has complete information about the current state of the economy, while the policymaker knows the structural relations of the economy (equations (1) and (2)) and the true parameter values, but conducts monetary policy under uncertainty about the shocks affecting the economy and, in particular, about $u_t^N$. This type of asymmetries in information has been used in Svensson and Woodford (2004), Aoki (2003) and Primiceri (2006). Svensson and Woodford (2004) argue that

"(this) is the only case in which it is intellectually coherent to assume a common information set for all members of the private sector, so that the model’s equations can be expressed in terms of aggregate equations that refer to only a single ‘private information set’, while at the same time these model equations are treated as structural, and hence invariant under the alternative policies that are considered in the central bank’s optimization problem... But if all private agents are to have a common information set, they must then have full information about the relevant variables."

The importance of the natural rate of unemployment in choosing monetary policy follows from the effect on inflation of deviations of unemployment from its natural rate in equation (1). If the policymaker is unable to observe this gap, it may set interest rates higher or lower than optimal. As a result, misperceptions about the natural rate of unemployment can be costly in terms of stabilization performance. The private sector understands this fact when forming expectations about future inflation, and these inflationary expectations influence long-term interest rates.

Since this paper is a positive study aiming at explaining the high volatility of long-term interest rates, I abstract from any kind of optimal filtering by the monetary au-

\footnote{Policymakers are uncertain about equation (3).}
I assume that the gap between the actual natural rate of unemployment, \( u_t^N \), and the central bank estimate of the natural rate at time \( t \), \( \tilde{u}_t^N \), evolves according to

\[
(u_t^N - \tilde{u}_t^N) = \rho (u_{t-1}^N - \tilde{u}_{t-1}^N) + \xi_t,
\]

where \( \xi_t \) is assumed to be an i.i.d. \( N(0, \sigma^2) \) misperception shock.

Orphanides and Williams (2002) empirically estimate the relationship in (4) and find that natural rate misperceptions are very persistent, independent of the filtering method. They calculate the gap as the difference between the retrospective estimates of the natural rate of unemployment (two-sided estimates) and the real time estimates (one-sided estimates) for six different estimation methods (four univariate filters and two multivariate unobserved-components models) which together give 36 alternative measures of natural rate misperceptions. They document a frequency distribution for \( \rho \) with median 0.96 and a fifty percent confidence interval \((0.95, 0.97)\), where the estimate of \( \rho \) using the Kalman filter is 0.95. They point out that equation (4) approximates several filtering methods and that the persistence in misperceptions is related to the nature of the filtering problem and does not necessarily imply that real time estimates are inefficient.

In particular, equation (4) encompasses different filtering methods. In Appendix A, I show for a calibrated example that when the central bank learns about the state of the natural rate of unemployment using a constant-gain learning rule\(^{19}\) or an optimal filter, the simulated value of \( \rho \) is around 0.95. In that appendix, I also show that the main results of the paper hold up if the central bank uses those types of updating rules.

Figure 1 shows the path of unemployment in the U.S. between 1965 and 2005. At the same time, one- and two-sided estimates of the natural rate of unemployment are

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\(^{18}\) For instance, Svensson and Woodford (2003) and Svensson and Woodford (2004) derive the optimal weights on indicator variables in models with partial information.

\(^{19}\) This kind of learning rule about the natural rate of unemployment has been used, for instance, in Primiceri (2006).
plotted using the Hodrick-Prescott filter with a smoother parameter of 1,600 and the band-pass filter with an eight-year window. The estimated autoregressive parameter of the difference between the one- and two-sided filter is 0.97 in the first case and 0.94 in the latter.

3.2 Optimal Monetary Policy

To close the model, I study optimal monetary policy under discretion or commitment\(^{20}\), where the instrument of monetary policy is the nominal interest rate, \(i_t\). In each period, policymakers set the optimal policy after forming their beliefs about the natural rate of unemployment according to equation (4).\(^{21}\)

Under discretion, the central bank chooses the optimal nominal interest rate in each period, without any binding commitment to future actions. The private sector is aware that the monetary authority cannot resist the temptation to exploit the short-run trade off between inflation and unemployment and hence, the central bank cannot influence private sector expectations. When maximizing, the monetary authority therefore takes future expectations as given. Under commitment, the central bank has the ability to bind its future actions to follow an optimal state-contingent rule for the nominal interest rate conditional upon the shocks arising in any period. In this case, the central bank can exploit its influence on private sector expectations for the entire future to stabilize the economy.

A well known result in the literature is that the two regimes differ in their credibility properties. Under discretion, the rational expectations equilibrium is "time consistent": conditional on the state of the economy as described by a set of shocks, the central bank chooses the same policy in any period, even though it has the discretion to change it,\(^{20}\)

\(^{20}\) Even though the commitment solution is unrealistic in the absence of a commitment mechanism, it is a useful benchmark and closely related to other types of rules, or institutions, often used in the literature.

\(^{21}\) In Appendix A, I show the case when policymakers update their beliefs about the natural rate of unemployment using a constant-gain learning rule or an optimal filter.
implying an equilibrium state-contingent policy rule. Under commitment, the optimal state-contingent rule is credible by assumption, although the same policy rule would not be credible in a discretionary policy regime.

The central bank sets its policy instrument \( i_t \), to minimize

\[
\tilde{E}_t \sum_{i=0}^{\infty} \beta^i \left[ \pi_{t+i}^2 + \lambda (u_{t+i} - u^N_{t+i})^2 \right],
\]

subject to equations (1)-(2) describing the economy, and where \( \tilde{E}_t(\cdot) \) denotes the expectation operator given the central bank information set in period \( t \). In particular, \( \tilde{E}_t u^N_t = \tilde{u}^N_t \) given that the central bank cannot observe \( u^N_t \). This loss function penalizes deviations of inflation and unemployment from their targets, where the inflation target is normalized to zero.

The first-order conditions of this problem under discretion imply

\[
\pi_t = \frac{\lambda}{\theta} (u_t - \tilde{u}^N_t).
\]

Using this result in equation (1) and performing repeated substitutions, the equilibrium outcome for inflation in the discretionary case is

\[
\pi_t = \frac{\lambda \theta}{\lambda + \theta^2 - \beta \lambda \rho} (u^N_t - \tilde{u}^N_t) + \frac{\lambda}{\lambda + \theta^2} \epsilon_t.
\]

This last equation shows that when the central bank estimate of the natural rate of unemployment differs from the real value, there is an inflation bias, only in the sense that inflation will be different from its target. Note that the model does not have a conventional (Kydland and Prescott (1977), Barro and Gordon (1983)) inflation (level) bias. The existence of such a bias is not essential for the argument in this paper.

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22 Since I do not specify the filtering method of the central bank, \( \tilde{E}_t(\cdot) \) does not imply fully rational expectations. Only in the case of optimal filters, \( \tilde{E}_t(\cdot) \) is the rational expectation operator given the information set of the central bank in period \( t \).

23 The rationales for these costs are that inflation volatility is costly because it induces an inefficient allocation of resources, while unemployment volatility is costly for risk averse households.

24 In practice, since I work with demeaned data, the inflation target is equal to the mean of inflation in each period.

25 Moreover, inflation is zero in steady state.
What is essential, however, is the fact that a discretionary policy regime implies that the policymaker loses control over private expectations.

Doing some algebra, it is easily shown that inflation expectations evolve as

\[ E_t \pi_{t+1} = \frac{\lambda \theta}{\lambda + \theta^2 - \beta \lambda \rho} \rho^i (u^N_t - \bar{u}^N_t). \]

As a result, when the natural rate of unemployment is higher (lower) than the central bank’s estimate, there is a persistent rise (fall) in inflation.\(^ {26} \) The intuition is that when the monetary authority underestimates the natural rate of unemployment, it sets the interest rate so as to achieve a higher inflation than the target in order to reduce the perceived unemployment gap. Since misperceptions about the natural rate of unemployment are persistent, this raises expectations of future inflation (and thereby long-term interest rates).

For the commitment case, the first-order conditions of the central bank imply\(^ {27} \)

\[ \pi_t = \frac{\lambda}{\theta} (u_t - \bar{u}^N_t) - \frac{\lambda}{\theta} (u_{t-1} - \bar{u}^N_{t-1}). \]  

(6)

It can be shown that for given parameters, inflation reacts less to supply and misperception shocks in the commitment case. The reason is that the monetary authority can control future expectations under commitment and thus, the behavior of inflation today: lower expected future inflation implies lower inflation today.

For example, after a positive supply shock, the commitment solution implies periods of deflation after the initial positive impact on inflation. This is the case because lower inflation is achieved with the promise of having positive unemployment gaps in the future. In the full information case, the dynamic feature of the model introduces a

\(^{26}\) Some authors have used this argument to explain the stagflation episode in the 1970s. See, for instance, Orphanides and Williams (2002), Primiceri (2006) and Reis (2003).

\(^{27}\) I assume optimal monetary policy under commitment to be a timeless perspective policy. Moreover, I assume that the central bank does not revise its estimates of the natural rate of unemployment in the next period, and \( \hat{E}_t u^N_t = \hat{E}_{t+1} u^N_t = \bar{u}^N_t \). In the data, the difference between \( \hat{E}_t u^N_t \) and \( \hat{E}_{t+1} u^N_t \) has a standard deviation of 0.16 in the case of the Hodrick-Prescott filter and 0.05 for the band-pass filter. However, since univariate filters are excessively sensitive to final observations, this difference could be expected to be even smaller in multivariate filters.
stabilization bias, in that unemployment is overstabilized and inflation volatility is higher under discretion than under commitment. However, in my model, the volatility of unemployment turns out to be similar in both regimes. But the presence of the second term in equation (6) makes the inflation rate less autoregressive than in equation (5).

Svensson and Woodford (2004) show that equations (5) and (6) follow the principle of certainty equivalence, where the optimal response is the same as if the central bank had full information, except that it responds to an estimate of the state of the economy rather than to the actual values.

Orphanides and Williams (2002) show that when the policymaker adopts policy rules ignoring the misperceptions regarding the natural rate of unemployment, this is costly in terms of inflation and unemployment stabilization. In my model, misperceptions also translate into long-term interest rate volatility.

3.3 Expectation Hypothesis of the Yield Curve

To calculate long-term interest rates, I use the expectation hypothesis of interest rates, which establishes a relationship between long-term interest rates and short rates. The interest rate on a discount bond of maturity \( m \) at time \( t \) should be equal to the expected average of future short interest rates over the same period, plus a term premium:

\[
i_t^m = \frac{1}{m} \left[ i_t + i_{t+1}|_t + i_{t+2}|_t + \ldots + i_{t+m-1}|_t \right] + \tau_t^m,
\]

where \( i_{t+m}|_t = E_t(i_{t+m}) \) and term premium shocks are assumed to be i.i.d. \( N(0, \sigma_m^2). \)

Even though the empirical evidence on the relevance of the expectation hypothesis is mixed, it is often used in formal macroeconomic analysis. Fuhrer (1996) finds that changes in monetary policy regimes can account for most of the empirical failure of the expectation hypothesis. Given that I study two periods when monetary policy may have been stable, the use of the expectation hypothesis may be a good approximation.

\footnote{28 According to the expectation hypothesis, the term premium varies with maturity \( m \) but not with time. That is, \( \tau_t^m = \tau^m \).}
Moreover, among the papers rejecting the expectation hypothesis, some fail to reject it at the long end of the yield curve, which is the main focus in this paper.\textsuperscript{29} Since I am not interested in constructing a very precise model of the yield curve, but in finding some macroeconomic fundamentals that potentially affect long-term interest rates, I assume that the expectation hypothesis holds if one adds time-varying term premium shocks.

3.4 Solution Method

Given the asymmetry in the information set of the central bank and the private sector, usual optimal control methods, such as those described in Söderlind (1999), cannot be applied here. However, equations (1)-(4) and the first-order condition of the monetary authority (equation (5) or (6)) form a system of difference equations that can be solved using the methods described in Sims (2002). Moreover, since \( i_t^m \) does not enter the first five equations of the model, the model is solved recursively as described in Appendix B. Once the model is solved and expressed in state-space form, I can estimate it using the Kalman filter.

4 Empirical Evidence for the U.S.

The model is estimated using Bayesian methods. The advantage of Bayesian estimation over maximum likelihood estimation is that the solution of the model implies many restrictions and boundary values for the parameters, which are difficult to impose using maximum likelihood. Moreover, using Bayesian methods makes it possible to formally incorporate prior beliefs about the parameters and obtain posterior moments of the variables in the model, which is of great importance in this paper.

Five quarterly macro data series are used in the estimation: U.S. unemployment, inflation, short-term nominal interest rate and U.S. Treasury securities at five and ten

\textsuperscript{29} See, for instance, Campbell and Shiller (1991) and Sarno, Thornton, and Valente (2005).
years between 1960Q1-2005Q4. All series were demeaned.

As mentioned earlier, I divide the data into two periods, from 1960Q1 to 1978Q4 and 1983Q1 to 2005Q4, excluding the Volcker nonborrowed reserves target period when the volatility of interest rates at all maturities increased dramatically. Many studies have pointed out that these two periods have different characteristics in monetary policy and/or business cycles volatility. Figure 2 clearly shows there to be a break in volatility in the early 1980s. Table 1 shows the standard deviation of the data, where we can see that the volatility of inflation and unemployment has indeed decreased in the second period. Even though inflation volatility is lower in the second period, interest rates at all maturities are more volatile.

As far as I know, no one has structurally estimated this kind of model, neither to distinguish between different monetary policy regimes nor to explain the excess volatility puzzle. An important element of the paper is that estimating the full structural model for each policy regime separately overcomes the problem of unstable nonpolicy parameters across different regimes. In other words, if one thinks that monetary policy has changed across the two subperiods and affected private sector behavior, this is not a major problem because I assume the parameters to be constant only within each subperiod.

The prior distributions of the parameters are presented in Table 2. All standard deviations have a gamma distribution with mode 0.10 and a standard error of 0.05, which implies a diffuse variance given the lack of knowledge about these parameters. The persistence in the natural rate of unemployment, $\gamma$, is beta distributed with mode 0.95 and a standard error of 0.02. In general, there is agreement among economists.

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30 The data on unemployment is seasonally adjusted data from the Bureau of Labor Statistics (BLS). Nominal interest rate is the quarterly Federal Funds Rate, and inflation is calculated as the change in the seasonally adjusted GDP deflator obtained from the Bureau of Economic Analysis (BEA). Long-term interest rates are quarterly market yields on U.S. Treasury securities at five and ten years constant maturity obtained from the Federal Reserve Board.


32 Since the paper does not include any counterfactual analysis, it is immune to the Lucas critique.
that the natural rate of unemployment is highly persistent, close to a unit root process. The weight on output gap in the central bank loss function, $\lambda$, is normal distributed with mode 1 and standard error 0.20. The slope coefficient in the Phillips curve, $\theta$, is gamma distributed with mode 0.10 and standard error 0.02. This is approximately the value estimated by Orphanides and Williams (2002) and Rudebusch (2002) using survey data as proxies for inflation expectations.

One prior that deserves special attention is the persistence in misperceptions, $\rho$, which is beta distributed with mode 0.95 and standard error 0.005. I set a very tight prior on this parameter to rule out cases where $\rho$ is close to one, meaning that misperceptions never die out. Naturally, misperceptions can still be very persistent. In particular, a value of $\rho$ equal to 0.95 implies that the half-life of a shock (the time it takes for the shock to dissipate by 50%) is three years and one quarter. As mentioned before, the high persistence in misperceptions is documented in Orphanides and Williams (2002). I could alternatively have fixed this parameter, but allowing for some flexibility seems a better solution.

As is common practice, I fix the value of the discount factor, $\beta$, at 0.99, which corresponds to an annual steady state real rate of four percent. Finally, the value of the slope parameter in the IS-curve, $\delta$, was pre-set at 0.5, corresponding to a degree of risk aversion equal to one, and an output gap approximately two times the unemployment gap.

To obtain the joint posterior distribution of the parameters for each model, I start by finding the posterior mode and Hessian matrix evaluated at the mode. Next, I generate draws from the posterior distribution using Markov Chain Monte Carlo (MCMC)

---

33 The prior for $\lambda$ is higher than values commonly used in the literature. However, when I estimate the model with a flat prior for $\lambda$, the model prefers values of $\lambda$ greater than one (or around one). This is robust to different priors for the shocks and estimating the model without long-term rates.

34 Since they use annual data on inflation, their results must be interpreted as four times $\theta$. Moreover, Rudebusch uses the output gap instead of the unemployment gap, which should also be transformed in terms of the unemployment gap.

35 The half-life of an AR(1) process is $-\log(2)/\log(\rho)$. 

18
methods. For each model, two MCMC chains were simulated with 50,000 draws each and a burn-in period of 20%.

4.1 Estimation Results

Before going into the main topics of the paper, I first discuss the general properties of my estimation results. Tables 2 and 3 report the mean and the 5th and 95th percentile of the posterior distribution of the parameters under alternative monetary policy regimes.\(^{36}\)

A first thing to notice is that most of the estimates are robust to the monetary policy regime. However, the posterior mean of the standard deviation of misperception shocks, \(\sigma_\xi\), and the weight on the unemployment gap in the central bank loss function, \(\lambda\), are larger in the commitment case. Higher values of these parameters imply a larger impact of misperceptions and thus, higher volatility in the data (specially long-term rates). This is important because, as discussed below, the commitment regime has difficulties in replicating the volatility of long-term rates observed in the data. In the same way, the variances of term premium shocks are larger in the commitment case.

It is worth mentioning that under discretion, the weights on inflation and unemployment are similar to each other and stable across the two subperiods. This implies that the Fed gave equal importance to both variables during the whole post-war period.

In accordance with most estimates in the literature, both the natural rate of unemployment and misperceptions about this variable exhibit a high degree of persistence in both regimes. The slope coefficient in the Phillips curve, \(\theta\), is stable across time and also similar to other estimates in the literature, although considerably lower in the commitment case.

One slightly puzzling result is that the variance of supply shocks across regimes is larger in the second period. This result is opposite to the common perception that

---

\(^{36}\) Convergence to a stationary distribution was monitored computing the potential scale reduction for all the parameters, as described in Gelman, Carlin, Stern, and Rubin (2004), and plotting the path of the different parameters along the chain.
certain supply shocks, e.g. oil shocks, were larger in the 1970s. The estimates also show that the variance of shocks to the natural rate of unemployment has been lower in the second period. One explanation for this time pattern is the productivity slowdown. Last, the estimates of $\sigma_n$, the variability of demand shocks, are also lower in the second period. This result is in line with Gordon (2005) who provides some evidence for smaller demand shocks after 1984 due to a reduced volatility of Federal government spending, residential housing and inventory change.\textsuperscript{37}

4.2 Macroeconomic Variables and Monetary Policy Regimes

Figures 3 and 4 show the posterior predictive distribution of the standard deviation of unemployment, inflation and the short-term interest rate.\textsuperscript{38} A first look at the graphs indicates that in the first period, both regimes replicate the observed volatility in the data reasonably well.

In the second period, however, both regimes have problems replicating the volatility of unemployment and inflation, while a discretionary regime matches the volatility of the short-term interest rate much better. The model’s inability to match the volatility of inflation in the second period is related to the high estimates of the variance of supply shocks, which seem at odds with the data.

Overall, and in line with the common view in the literature, it is not possible to distinguish between alternative monetary policy regimes by only looking at the volatility of the macro variables.

\textsuperscript{37} Gordon attributes these changes respectively to "the reduced share of military spending in GDP, banking and financial market reforms, and information technology".

\textsuperscript{38} The posterior density was computed using a kernel smoothing method, for a sample of 200 simulations for 75 periods from 500 draws of the posterior. To avoid autocorrelation, the draws from the posterior were picked in fixed intervals.
4.3 Long-Term Rates and Monetary Policy Regimes

4.3.1 Variance Decomposition

Both monetary policy regimes can explain a large part of the volatility of long-term interest rates, since the term premium shock, the residual in equation (7), will capture a great deal of the variation not explained by the macro model.\textsuperscript{39} However, the sources of interest rate volatility differ across monetary regimes.

The variance decomposition of inflation, the short interest rate and long-term interest rates at different horizons are shown in Tables 4 and 5. Misperception shocks that feed into monetary policy account for a great deal of the variation in long-term rates in a model under discretion. After a period of ten years, misperception shocks explain 87% of the variation in long-term rates in the first sub-sample and 96% in the second.

In the commitment regime, the variation in ten-year interest rates is instead predominantly explained by term premium shocks. After ten years, term premium shocks explain 45% of the variation in long-term rates in the first sub-sample and 88% in the second. Hence, if we want to attribute some of the variation in long-term rates to macroeconomic fundamentals, rather than to residual variation in time-varying term premiums, a monetary policy regime under discretion provides a better explanation for the volatility puzzle. Moreover, this implies that the expectation hypothesis of interest rates allows us to account for most of the observed long-term interest rate volatility when the central bank acts under discretion.

4.3.2 Switching off Term Premium Shocks

To further investigate how much of the total volatility of long-term interest rates is explained by macro variables as opposed to financial risks, I once more simulate the model, but switching off the term premium shocks. This allows me to isolate the effect

\textsuperscript{39} Notice that the model is estimated with quarterly data, implying that to annualize the standard deviation of the term premiums, they should be multiplied by four.
of macro variables in explaining the volatility of long-term rates. Figure 5 shows the posterior predictive distribution of the standard deviation of the ten-year long-term interest rate implied by the model, both with and without time-varying term premiums. Even in the case with term premium shocks, the model underpredicts the volatility in the data. The reason why the model does not exactly match the interest rate volatility in the data is that other moments in the data can be pulling the estimates of the term premiums down.\footnote{When simulating the posterior distribution, all moments in the data and not specific ones, such as the variance of long-term rates, are considered.}

The left-hand panel in the figure shows that the model under discretion is much closer to explaining the volatility of the long-term interest rate and can replicate a large part of the volatility observed in the data, especially during the second period.\footnote{However, other factors absent in the model, such as a time-varying inflation target or real interest rate, could also potentially add some extra volatility to my results.} The main features of the model driving this result are policymakers’ autocorrelated misperceptions about the natural rate of unemployment and a discretionary monetary policy. Together, these translate into a very persistent inflation response.

The figure also shows that U.S. interest rates were more volatile in the second period than in the first. In the model, there is also an increase in bonds volatility; there is a shift to the right in the posterior predictive distribution of long rates in the second period. This is due to a slightly larger estimate of the persistence in misperceptions. Interestingly, when I calculate the difference between one- and two-sided estimates of the natural rate of unemployment using the same univariate filters as those described in Section 3.1, I also find an increase in the autocorrelation coefficient in the second period.

Moreover, the model is also able to explain bond returns volatility. Table 6 reports the simulated volatility in bond returns implied by the model when term premiums are switched off, and where the volatility of bond returns is defined as the standard deviation of the quarter-to-quarter change in long-term interest rates.\footnote{The return on a bond of maturity $m$ is $\ln\left(\frac{P_{t+m}^m}{P_{t+1}^m}\right) \simeq -m \left(\bar{i}_t^m - i_{t+1}^m\right)$, where $P_t^m = \exp(-i_t^m m)$ is} Once more, a monetary...
regime under discretion appears to more closely replicate the data. The table also shows an increase in bond returns volatility in the second period, both in the model and in the data. As mentioned before, in the model, this is caused by a slightly higher persistence in policymakers’ misperceptions.

4.3.3 Correlations with Short-term Interest Rate

As shown in Table 1, on average there is a positive relationship between short- and long-term interest rates. Figures 6 and 7 show posterior predictive distributions of the correlation coefficient between the short-term interest rate and the other variables in the model. As in the case of volatility, the model solved under discretion fits the data in a better way. In particular, the discretionary regime can replicate the high positive correlation between short- and long-term interest rates observed in the data, while the commitment regime fails miserably in this regard.

The model can also explain another puzzling observation. In the real world, long-term interest rates typically move in the same direction as the short rate. However, during certain episodes, they move in the opposite direction. In the model, this can happen when the economy is simultaneously hit by a negative demand shock and a positive misperception shock. In that particular case, nominal long-term rates move up because of the positive misperception shock, since this will have a positive effect on future inflation. On the other hand, the movement in the short rate is determined by the relative size of the two shocks. When demand shocks are sufficiently large to offset misperception shocks, the short rate goes down to prevent a higher unemployment rate. This mechanism can clearly be seen from the impulse response functions plotted in Figure 8.

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the price of the zero coupon bond.
4.3.4 Monetary Policy Regimes

Finally, let us explicitly consider the second issue motivating the paper, namely the debate about monetary policy regimes. The model I estimate uses long-term interest rate data to empirically distinguish between different monetary policy regimes. Given the results already discussed in this section, it should be clear that a monetary regime under discretion is more likely to have prevailed in the U.S. In the data, we observe long rates to be highly volatile and correlated with the short rate. The results generated by the model seem to preclude a regime where the central bank can commit to future actions and stabilize inflation expectations. It seems that market participants believed and behaved as if the monetary policy followed by the Fed was discretionary during the whole sample. Moreover, the different chairmen of the Fed do not seem to have influenced those beliefs. This way, long-term interest rates can help us to understand how monetary policy has been conducted in the last 45 years.

This result is formally confirmed if we use the Bayes factor to compare the two policy regimes.\(^{43}\) Table 7 shows that the Bayes factor favors the discretionary regime in both periods.\(^{44}\) Similarly to my previous observations, this result is starker in the second period.

A discretionary monetary policy implies a more volatile process for inflation and the short-term interest rate than a commitment regime. In the model, this translates into larger movements in long-term interest rates which are strongly correlated with movements in the short rate. A central bank that can credibly commit does not need to move its instrument so much to control inflation, since it can effectively control the path of inflation by managing inflation expectations. In that sense, policymakers’ misperceptions about the natural rate of unemployment are less important in the commitment regime.

\(^{43}\) The Bayes factor is the ratio of the marginal data densities between model \(i\) and \(j\). Values of the log Bayes factor greater than five are considered as decisive evidence against model \(j\). To calculate the marginal likelihood, I use the modified harmonic mean.

\(^{44}\) The Bayes factor also favors the discretionary regime compared to a model with a Taylor-type interest rate rule.
5 The Case of the U.K.

A large literature has studied the relation between monetary institutions and credibility. In particular, many papers stress the fact that independent central banks with price stability as their main objective, will increase credibility and stabilize inflation without much effect on output or unemployment.

In May of 1997 the Bank of England was officially granted operational independence. Since then, the bank is committed to "promoting and maintaining monetary and financial stability as its contribution to a healthy economy". Given the specific inflation target objective of the bank, one may think that to achieve this goal monetary policy can be approximated by a commitment regime.

A first look at the data shows that indeed, after the Bank of England achieved independence, U.K. data is less volatile both when it comes to inflation and unemployment. Table 8 shows that the volatility of short and long rates in the U.K. during this period has been lower than in earlier periods. The table also includes data for the U.S. over the same two periods. Clearly, the volatility in long-term rates fell proportionally more in the U.K. than in the U.S. To investigate whether this can be attributed to a change of monetary regime, I estimate the model under discretion and commitment for U.K. data during the periods 1983-1997 and 1998-2005.

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45 See Persson and Tabellini (2000) for a review.
46 Alesina and Summers (1993), among others, find that a more independent central bank reduces the level and variability of inflation, but has not impact on real activity.
47 Quotation from the Bank of England’s home page.
48 Sholtes (2002) document that inflation expectations have fallen and that U.K. monetary policy credibility is stronger since the Bank of England gained independence.
49 The data was obtained from the OECD database on unemployment, short-term interest rate (3 months Treasury bill), GDP deflator and 10 year government bond yields. All series were demeaned.
5.1 Estimation Results for the U.K.

In the estimation, I use the same priors as those used above for the U.S. Table 9 reports the estimated mean of the parameters for the U.K. The results are in general similar to those in the U.S. reported in Section 4: high persistence of the natural rate of unemployment and central bank’s misperceptions, a weight on the unemployment gap in the central bank loss function greater than one, and a response of inflation to the unemployment gap close to 0.08.

5.2 Implications of Different Monetary Policy Regimes

Results not reported here show that both regimes replicate the observed volatility in the macro data reasonably well in both periods. One interesting issue is that after 1997 the correlation between inflation and the short-term interest rate becomes negative in the U.K. (see Table 8). Figure 9 shows that this can be replicated only by the commitment regime.\(^{50}\) However, the discretionary regime does better in replicating the correlation of the short- and long-term rate.

Figure 10 reports the posterior predictive distribution of the standard deviation of the ten-year long-term interest rate implied by the model, both with and without time-varying term premiums, before and after 1998. The figure shows that the discretionary regime does better in replicating the volatility of long-term rates before 1998 even if we add term premium shocks. However, after 1998 the commitment regime is closer to replicating the observed volatility. Moreover, variance decomposition analysis for the U.K. after 1998 show that term premium shocks have a large role in explaining the volatility of long-term rates in both regimes.\(^{51}\) Rephrasing, term premium shocks are now an important component of long-term rates volatility in the U.K., independently of

\(^{50}\) For the U.S., none of the regimes replicates the negative correlation between inflation and the short-term rate.

\(^{51}\) After a period of ten years, term premium shocks explain one fourth of the variation in long-term rates in the discretionary regime and three fourths in the commitment case.
the monetary policy regime. This indicates that we only need to add a small amount of variable term premiums to the model, for the commitment regime to do well in replicating the volatility of long-term rates.\footnote{Alternatively, one could introduce some kind of monetary policy shock to explain long-term volatility. For instance, we can think of control errors as introducing higher volatility.}

Last, Table 10 formally shows that the Bayes factor decisively prefers the discretionary regime before 1998. However, after 1998 we can no longer reject the commitment regime in favor of the discretionary regime; in fact, there is slight evidence in favor of the former.\footnote{In the case of the U.S. during the same period, and despite the lower volatility of long-term interest rates, the Bayes factor still favors the discretionary regime after 1998.} If anything, the evidence suggests that after the Bank of England has gained independence, its monetary policy regime is closer to rules than discretion.

## 6 Conclusion

This paper attempts to explain the behavior of long-term U.S. interest rates from a macroeconomic perspective in the last 45 years. Most papers in the literature rely on a time-varying inflation target to explain the volatility of long-term rates. I propose an alternative explanation and show that the high volatility observed in long-term yields and their correlation with the short rate may be due to a combination of quite persistent misperceptions about the natural rate of unemployment and discretionary monetary policy. In a discretionary regime, the policymaker loses control over inflation expectations and actual inflation. Persistent misperceptions that feed into policy make inflationary expectations quite volatile, which has an effect on the volatility of long-term rates and their correlation with the short rate. Because of this, incorporating yield curve data in the analysis makes it possible to empirically distinguish between different monetary policy regimes.

To further analyze the role of different institutions in monetary policy, the paper estimates the same model with U.K. data during 1983-1997 and 1998-2005, the latter...
being a period during which the Bank of England was operationally independent. The evidence suggests that during the independence period, the policy pursued by the Bank of England can equally well be classified as a commitment regime or a discretionary regime.

If there are benefits in stabilizing inflation expectations and bonds volatility, the paper has some normative implications. In particular, providing a commitment technology for the monetary authority can reduce the costs of a discretionary regime. Persson and Tabellini (2000) survey the institutional reforms suggested in the literature to enhance the credibility of policymakers, such as the appointment of an independent (conservative) central bank, rigid monetary rules with escape clauses, and explicit inflation targets and contracts to stabilize inflation expectations. Moreover, reaction functions for the central bank that do not respond to the natural rate of unemployment will avoid the problem of policymakers’ misperceptions.\textsuperscript{54} Although all these extensions are interesting and relevant matters for the conduct of monetary policy, they are beyond the scope of this paper and left to future research.

\textsuperscript{54} Orphanides and Williams (2002) suggest, for instance, that the monetary authority could react to unemployment growth instead of the unemployment gap.
References


A Central Bank Updating Rules

This appendix investigates the behavior of the model when the central bank updates its estimate about the natural rate of unemployment using a constant-gain learning rule or an optimal filter.

A.1 Constant-Gain Learning Rules

This is the same learning mechanism for the natural rate as the one used, for instance, in Primiceri (2006). Primiceri assumes that policymakers form their estimates about the natural rate using univariate methods: the monetary authority updates its beliefs of the natural rate only looking at the behavior of unemployment. This is consistent with results in Staiger, Stock, and Watson (2001) who show that the natural rate estimated on macro data and the univariate trend in unemployment track each other very closely.

Assuming that, on average, unemployment is equal to its natural rate, the algorithm for updating $\widetilde{u}_t^N$ is

$$\widetilde{u}_t^N = \widetilde{u}_{t-1}^N + \psi R_{t-1}^{-1} (u_{t-1} - \widetilde{u}_{t-1}^N) ,$$  

(A1)

$$R_t = R_{t-1} + \psi (1 - R_{t-1})$$  

(A2)

where $\psi$ is the gain parameter and $R_t$ is the variance of the regressor in equation (A1). For this particular problem, this is equivalent to the adaptive expectations formula, since the regressor in the first equation is one. Evans and Honkapohja (2001) show that in general, constant-gain learning rules do not converge to rational expectations. In this model, this will have the effect of producing very little volatility in unemployment.

To solve the model, I calibrate the parameters of a model with commitment and discretion using the estimated posterior means between 1960-1978 reported in Table 2. For the constant gain parameter, $\psi$, I set it equal to 0.50.\textsuperscript{55} Using these calibrated values,

\textsuperscript{55} This is a higher value than the one generally used in the literature. In another calibration exercise, I set $\psi = 0.03$ which is the same value used in Primiceri (2006), and adjust the variance of the shocks to match the observed volatility in inflation and the short-term interest rate. In that case, the discretionary
one can simulate implied values for $\rho$ in equation (4), which are shown in the last row of Table 11. As in my model, misperceptions about the natural rate of unemployment are very persistent. The table also shows the simulated standard deviation and correlation of the variables in the model. The table shows that in the case of constant-gain learning, the discretionary regime better replicates both the volatility and the correlation of long-term interest rates. This is in line with the main results in the paper. However, both regimes have problems replicating the behavior of unemployment.

A.2 Optimal Filter

Next, I follow the work of Svensson and Woodford (2003) who derive the optimal weights on indicators in models with symmetric partial information.\footnote{The case of asymmetric partial information is more complicated and does not add much for the purpose of this exercise.} The structure of the model is similar to the one in Section 3, but now the central bank uses an optimal filter to infer $u^N_t$ and the other shocks affecting the economy.\footnote{For a detailed description of the solution see Svensson and Woodford (2003).} To generate a more well-defined signal extraction problem for the bank, I assume that the supply shock in equation (1) follows a first-order autoregressive process

$$\varepsilon_t = \omega \varepsilon_{t-1} + \varphi_t,$$

where $\varphi_t$ is assumed to be i.i.d. $N(0, \sigma^2_{\varphi})$.

Once again, I calibrate the model for the commitment and discretionary case using the estimated posterior means between 1960-1978 reported in Table 2. I set $\omega$ equal to 0.85 and $\sigma_{\varphi}$ equal to 0.16. These values imply an unconditional standard deviation for $\varepsilon$ of 0.30, which is approximately the estimated mean value reported in Table 2.

Table 11 shows that when the central bank updates its estimates optimally, the discretionary regime better replicates the volatility of long-term interest rates. This regime does even better matching the volatility of long-term rates.
example shows that the main results of my paper still holds in the extreme case of optimal filtering.

Last, the last row in Table 11 reports the simulated implied values for \( \rho \) in equation (4). As Orphanides and Williams (2002) show, misperceptions about the natural rate of unemployment are very persistent even when the monetary authority uses an optimal filter.

B State-Space Representation of the Model

This appendix shows the state-space representation of the model to be estimated with the Kalman filter. I solve the model recursively. First, I solve for equations (1)-(4) and the corresponding first-order condition, equation (5) or (6). Second, I use this solution to solve for long-term interest rates using equation (7).

B.1 Macro Variables

To solve the model numerically, I follow the method described in Sims (2002). Let us define a 7x1 vector of variables

\[
Y_t = (\tilde{u}_t^N, u_t^N, u_t, \pi_t, i_t, E_t u_{t+1}, E_t \pi_{t+1})',
\]

a 4x1 vector of exogenous shocks

\[
Z_t = (\eta_t, \varepsilon_t, \chi_t, \xi_t)',
\]

and a 2x1 vector of expectational errors

\[
X_t = (e_t^\pi, e_t^\nu)'.
\]

We can then write the structural model in compact form as:

\[
\Gamma_0 Y_t = \Gamma_1 Y_{t-1} + \Psi Z_t + \Pi X_t, \quad i = D, C
\]
where

\[
\Gamma_0 = \begin{pmatrix}
0 & \theta & -\theta & -1 & 0 & 0 & \beta \\
0 & 0 & 1 & 0 & -\delta & -1 & \delta \\
-\frac{1}{\delta} & 0 & \frac{1}{\delta} & -1 & 0 & 0 & 0 \\
1 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 \\
\end{pmatrix},
\]

\[
\Gamma_1^D = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{pmatrix},
\]

in the discretionary case and

\[
\Gamma_1^C = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{1}{\delta} & 0 & \frac{1}{\delta} & 0 & 0 & 0 & 0 \\
\rho & -\rho & 0 & 0 & 0 & 0 & 0 \\
0 & \gamma & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{pmatrix},
\]

in the commitment case,

\[
\Psi = \begin{pmatrix}
0 & 1 & 0 & 0 \\
-1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{pmatrix}
\quad \text{and} \quad
\Pi = \begin{pmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 1 \\
\end{pmatrix}.
\]

Using Sims matlab code gensys.m, the system can be expressed in standard state-space form

\[
Y_t = MY_{t-1} + QZ_t.
\] (B1)
B.2 Long-term Interest Rates

Using the previous solution and equation (7), we can solve for long-term interest rates as:

\[
    i_{t}^{m} = \frac{1}{m} (M_{5}Y_{t-1} + Q_{5}Z_{t}) + \frac{M_{5}}{m} (I + M + ..., M^{m-2}) (M\bar{Y}_{t-1} + Q\bar{Z}_{t}) + \tau_{t}^{m}, \quad (B2)
\]

where \( M_{5} = M(5,:) \) is the fifth row of matrix \( M \) and \( Q_{5} = Q(5,:) \) is the fifth row of matrix \( Q \). Equations (B1) and (B2) form the state-space representation of the whole system.
### Table 1: U.S. Data

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<td>1.48</td>
<td>1.27</td>
</tr>
<tr>
<td>Short-term interest rate</td>
<td>3.32</td>
<td>2.33</td>
<td>3.38</td>
<td>2.54</td>
</tr>
<tr>
<td>5 year bonds</td>
<td>2.65</td>
<td>1.57</td>
<td>2.44</td>
<td>2.38</td>
</tr>
<tr>
<td>10 year bonds</td>
<td>2.55</td>
<td>1.56</td>
<td>2.30</td>
<td>2.26</td>
</tr>
<tr>
<td><strong>Correlation with short-term rate</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inflation</td>
<td>0.67</td>
<td>0.82</td>
<td>0.23</td>
<td>0.45</td>
</tr>
<tr>
<td>5 year bonds</td>
<td>0.91</td>
<td>0.82</td>
<td>0.27</td>
<td>0.92</td>
</tr>
<tr>
<td>10 year bonds</td>
<td>0.87</td>
<td>0.76</td>
<td>0.84</td>
<td>0.87</td>
</tr>
</tbody>
</table>

*Note: Annualized data*
Table 2: Distribution of the Parameters for the U.S. between 1960-1978

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior Density</th>
<th>Mode</th>
<th>St. Error</th>
<th>Posterior Discretion</th>
<th>5% Mean</th>
<th>95% Mean</th>
<th>Posterior Commitment</th>
<th>5% Mean</th>
<th>95% Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_e$ std. dev. supply shock</td>
<td>Gamma 0.10</td>
<td>0.05</td>
<td></td>
<td>0.284</td>
<td>0.325</td>
<td>0.372</td>
<td>0.264</td>
<td>0.305</td>
<td>0.351</td>
</tr>
<tr>
<td>$\sigma_x$ std. dev. natural rate of unemployment</td>
<td>Gamma 0.10</td>
<td>0.05</td>
<td></td>
<td>0.270</td>
<td>0.309</td>
<td>0.351</td>
<td>0.267</td>
<td>0.309</td>
<td>0.355</td>
</tr>
<tr>
<td>$\sigma_y$ std. dev. demand shock</td>
<td>Gamma 0.10</td>
<td>0.05</td>
<td></td>
<td>0.150</td>
<td>0.173</td>
<td>0.200</td>
<td>0.155</td>
<td>0.180</td>
<td>0.208</td>
</tr>
<tr>
<td>$\sigma_\xi$ std. dev. misperceptions</td>
<td>Gamma 0.10</td>
<td>0.05</td>
<td></td>
<td>0.066</td>
<td>0.092</td>
<td>0.124</td>
<td>0.206</td>
<td>0.252</td>
<td>0.302</td>
</tr>
<tr>
<td>$\rho$ autocor. coef. misperceptions</td>
<td>Beta 0.95</td>
<td>0.005</td>
<td></td>
<td>0.959</td>
<td>0.964</td>
<td>0.970</td>
<td>0.974</td>
<td>0.977</td>
<td>0.981</td>
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<tr>
<td>$\gamma$ autocor. coef. natural rate of unemployment</td>
<td>Beta 0.95</td>
<td>0.02</td>
<td></td>
<td>0.974</td>
<td>0.982</td>
<td>0.967</td>
<td>0.976</td>
<td>0.984</td>
<td></td>
</tr>
<tr>
<td>$\lambda$ weight on unemployment gap in loss function</td>
<td>Normal 1.00</td>
<td>0.20</td>
<td></td>
<td>0.716</td>
<td>1.015</td>
<td>1.331</td>
<td>1.342</td>
<td>1.611</td>
<td>1.884</td>
</tr>
<tr>
<td>$\theta$ response of $\pi$ to unemployment gap</td>
<td>Gamma 0.10</td>
<td>0.02</td>
<td></td>
<td>0.065</td>
<td>0.091</td>
<td>0.121</td>
<td>0.018</td>
<td>0.023</td>
<td>0.029</td>
</tr>
<tr>
<td>$\sigma_5$ std. dev. 5 year term premium shock</td>
<td>Gamma 0.10</td>
<td>0.05</td>
<td></td>
<td>0.020</td>
<td>0.038</td>
<td>0.059</td>
<td>0.041</td>
<td>0.059</td>
<td>0.078</td>
</tr>
<tr>
<td>$\sigma_{10}$ std. dev. 10 year term premium shock</td>
<td>Gamma 0.10</td>
<td>0.05</td>
<td></td>
<td>0.085</td>
<td>0.103</td>
<td>0.122</td>
<td>0.112</td>
<td>0.136</td>
<td>0.163</td>
</tr>
</tbody>
</table>

Table 3: Distribution of the Parameters for the U.S. between 1983-2005

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior Density</th>
<th>Mode</th>
<th>St. Error</th>
<th>Posterior Discretion</th>
<th>5% Mean</th>
<th>95% Mean</th>
<th>Posterior Commitment</th>
<th>5% Mean</th>
<th>95% Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_e$ std. dev. supply shock</td>
<td>Gamma 0.10</td>
<td>0.05</td>
<td></td>
<td>0.438</td>
<td>0.491</td>
<td>0.551</td>
<td>0.456</td>
<td>0.512</td>
<td>0.576</td>
</tr>
<tr>
<td>$\sigma_x$ std. dev. natural rate of unemployment</td>
<td>Gamma 0.10</td>
<td>0.05</td>
<td></td>
<td>0.166</td>
<td>0.187</td>
<td>0.212</td>
<td>0.126</td>
<td>0.146</td>
<td>0.169</td>
</tr>
<tr>
<td>$\sigma_y$ std. dev. demand shock</td>
<td>Gamma 0.10</td>
<td>0.05</td>
<td></td>
<td>0.115</td>
<td>0.131</td>
<td>0.149</td>
<td>0.008</td>
<td>0.016</td>
<td>0.025</td>
</tr>
<tr>
<td>$\sigma_\xi$ std. dev. misperceptions</td>
<td>Gamma 0.10</td>
<td>0.05</td>
<td></td>
<td>0.063</td>
<td>0.081</td>
<td>0.102</td>
<td>0.172</td>
<td>0.201</td>
<td>0.234</td>
</tr>
<tr>
<td>$\rho$ autocor. coef. misperceptions</td>
<td>Beta 0.95</td>
<td>0.005</td>
<td></td>
<td>0.973</td>
<td>0.976</td>
<td>0.980</td>
<td>0.975</td>
<td>0.979</td>
<td>0.982</td>
</tr>
<tr>
<td>$\gamma$ autocor. coef. natural rate of unemployment</td>
<td>Beta 0.95</td>
<td>0.02</td>
<td></td>
<td>0.968</td>
<td>0.977</td>
<td>0.986</td>
<td>0.949</td>
<td>0.959</td>
<td>0.969</td>
</tr>
<tr>
<td>$\lambda$ weight on unemployment gap in loss function</td>
<td>Normal 1.00</td>
<td>0.20</td>
<td></td>
<td>0.875</td>
<td>1.141</td>
<td>1.411</td>
<td>1.479</td>
<td>1.732</td>
<td>1.992</td>
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<tr>
<td>$\theta$ response of $\pi$ to unemployment gap</td>
<td>Gamma 0.10</td>
<td>0.02</td>
<td></td>
<td>0.062</td>
<td>0.083</td>
<td>0.107</td>
<td>0.027</td>
<td>0.032</td>
<td>0.039</td>
</tr>
<tr>
<td>$\sigma_5$ std. dev. 5 year term premium shock</td>
<td>Gamma 0.10</td>
<td>0.05</td>
<td></td>
<td>0.013</td>
<td>0.025</td>
<td>0.039</td>
<td>0.233</td>
<td>0.265</td>
<td>0.301</td>
</tr>
<tr>
<td>$\sigma_{10}$ std. dev. 10 year term premium shock</td>
<td>Gamma 0.10</td>
<td>0.05</td>
<td></td>
<td>0.077</td>
<td>0.090</td>
<td>0.105</td>
<td>0.303</td>
<td>0.345</td>
<td>0.391</td>
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<tr>
<td>Shock</td>
<td>Inflation</td>
<td>Int. Rate</td>
<td>5 year bonds</td>
<td>10 year bonds</td>
<td>Inflation</td>
<td>Int. Rate</td>
<td>5 year bonds</td>
<td>10 year bonds</td>
<td></td>
</tr>
<tr>
<td>-----------------------------</td>
<td>-----------</td>
<td>-----------</td>
<td>--------------</td>
<td>---------------</td>
<td>-----------</td>
<td>-----------</td>
<td>--------------</td>
<td>---------------</td>
<td></td>
</tr>
<tr>
<td>After 1 year</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Demand</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0.40</td>
<td>0</td>
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</tr>
<tr>
<td>Supply</td>
<td>0.54</td>
<td>0.02</td>
<td>0</td>
<td>0</td>
<td>0.69</td>
<td>0.03</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Natural Rate of Unem.</td>
<td>0</td>
<td>0</td>
<td>0.01</td>
<td>0.01</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Misperceptions</td>
<td>0.46</td>
<td>0.38</td>
<td>0.96</td>
<td>0.67</td>
<td>0.31</td>
<td>0.57</td>
<td>0.99</td>
<td>0.84</td>
<td></td>
</tr>
<tr>
<td>5 year term premium</td>
<td>-</td>
<td>-</td>
<td>0.04</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.01</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>10 year term premium</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>0.16</td>
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<td>After 10 years</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Demand</td>
<td>0</td>
<td>0.29</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.12</td>
<td>0</td>
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<td></td>
</tr>
<tr>
<td>Supply</td>
<td>0.24</td>
<td>0.01</td>
<td>0</td>
<td>0</td>
<td>0.31</td>
<td>0.01</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Natural Rate of Unem.</td>
<td>0</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Misperceptions</td>
<td>0.76</td>
<td>0.69</td>
<td>0.98</td>
<td>0.87</td>
<td>0.69</td>
<td>0.87</td>
<td>1</td>
<td>0.96</td>
<td></td>
</tr>
<tr>
<td>5 year term premium</td>
<td>-</td>
<td>-</td>
<td>0.01</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>10 year term premium</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.11</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.04</td>
<td></td>
</tr>
</tbody>
</table>

Note: Calculated using the posterior mean of the parameters estimated in a model with optimal monetary policy under discretion.
### Table 5: U.S. Variance Decomposition Under Commitment

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Inflation</td>
<td>Int. Rate</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>After 1 year</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Demand</td>
<td>0</td>
<td>0.76</td>
</tr>
<tr>
<td>Supply</td>
<td>0.61</td>
<td>0</td>
</tr>
<tr>
<td>Natural Rate of Unem.</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Misperceptions</td>
<td>0.39</td>
<td>0.24</td>
</tr>
<tr>
<td>5 year term premium</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>10 year term premium</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>After 10 years</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Demand</td>
<td>0</td>
<td>0.49</td>
</tr>
<tr>
<td>Supply</td>
<td>0.32</td>
<td>0</td>
</tr>
<tr>
<td>Natural Rate of Unem.</td>
<td>0</td>
<td>0.02</td>
</tr>
<tr>
<td>Misperceptions</td>
<td>0.68</td>
<td>0.49</td>
</tr>
<tr>
<td>5 year term premium</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>10 year term premium</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Note: Calculated using the posterior mean of the parameters estimated in a model with optimal monetary policy under commitment.

### Table 6: Simulated Bond Returns Volatility for the U.S.

<table>
<thead>
<tr>
<th></th>
<th>1960-1978</th>
<th>1983-2005</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Discretion</td>
</tr>
<tr>
<td>5 year returns</td>
<td>0.39</td>
<td>0.44</td>
</tr>
<tr>
<td>10 year returns</td>
<td>0.28</td>
<td>0.32</td>
</tr>
</tbody>
</table>

Note: Bond returns volatility is calculated as \( Std \left( i_t^m - i_{t-1}^m \right) \), performing 1,000 simulations for 75 periods using the posterior mean of the parameters. Annualized data.
Table 7: Model Comparison for the U.S.

<table>
<thead>
<tr>
<th></th>
<th>1960-1978</th>
<th></th>
<th>1983-2005</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Discretion</td>
<td>Commitment</td>
<td>Discretion</td>
<td>Commitment</td>
</tr>
<tr>
<td>log marginal likelihood</td>
<td>315.2</td>
<td>241.9</td>
<td>409.4</td>
<td>263.0</td>
</tr>
<tr>
<td>log Bayes factor: $\log(\pi_{D,T}/\pi_{C,T})$</td>
<td>-</td>
<td>73.3</td>
<td>-</td>
<td>146.4</td>
</tr>
</tbody>
</table>

Note: The marginal likelihood is approximated by the modified harmonic mean. Values of the log Bayes factor greater than 5 are considered as decisive evidence against the commitment regime.

Table 8: U.K. and U.S. Data before and after 1998

<table>
<thead>
<tr>
<th></th>
<th>U.K.</th>
<th></th>
<th>U.S.</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard deviation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inflation</td>
<td>2.96</td>
<td>1.32</td>
<td>0.98</td>
<td>0.83</td>
</tr>
<tr>
<td>Unemployment</td>
<td>1.94</td>
<td>0.53</td>
<td>1.21</td>
<td>0.73</td>
</tr>
<tr>
<td>Short-term interest rate</td>
<td>2.93</td>
<td>1.17</td>
<td>2.13</td>
<td>1.94</td>
</tr>
<tr>
<td>10 year bonds</td>
<td>1.51</td>
<td>0.44</td>
<td>1.92</td>
<td>0.76</td>
</tr>
<tr>
<td>Correlation with short-term rate</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inflation</td>
<td>0.51</td>
<td>-0.15</td>
<td>0.58</td>
<td>-0.21</td>
</tr>
<tr>
<td>10 year bonds</td>
<td>0.83</td>
<td>0.71</td>
<td>0.85</td>
<td>0.82</td>
</tr>
</tbody>
</table>

Note: Annualized data
Table 9: Distribution of the Parameters for the U.K.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>std. dev. supply shock</td>
<td>Gamma</td>
<td>0.10</td>
<td>0.05</td>
<td>0.549 0.569</td>
<td>0.314 0.359</td>
</tr>
<tr>
<td>std. dev. natural rate of unemployment</td>
<td>Gamma</td>
<td>0.10</td>
<td>0.05</td>
<td>0.325 0.301</td>
<td>0.119 0.123</td>
</tr>
<tr>
<td>std. dev. demand shock</td>
<td>Gamma</td>
<td>0.10</td>
<td>0.05</td>
<td>0.121 0.070</td>
<td>0.085 0.033</td>
</tr>
<tr>
<td>std. dev. misperceptions</td>
<td>Gamma</td>
<td>0.10</td>
<td>0.05</td>
<td>0.171 0.311</td>
<td>0.064 0.103</td>
</tr>
<tr>
<td>autocor. coef. misperceptions</td>
<td>Beta</td>
<td>0.95</td>
<td>0.005</td>
<td>0.954 0.968</td>
<td>0.950 0.953</td>
</tr>
<tr>
<td>autocor. coef. natural rate of unemployment</td>
<td>Beta</td>
<td>0.95</td>
<td>0.02</td>
<td>0.967 0.946</td>
<td>0.940 0.915</td>
</tr>
<tr>
<td>weight on unemployment gap in loss function</td>
<td>Normal</td>
<td>1.00</td>
<td>0.20</td>
<td>1.269 1.519</td>
<td>1.182 1.190</td>
</tr>
<tr>
<td>response of ( \pi ) to unemployment gap</td>
<td>Gamma</td>
<td>0.10</td>
<td>0.02</td>
<td>0.077 0.043</td>
<td>0.085 0.096</td>
</tr>
<tr>
<td>std. dev. 10 year term premium shock</td>
<td>Gamma</td>
<td>0.10</td>
<td>0.05</td>
<td>0.129 0.236</td>
<td>0.059 0.080</td>
</tr>
</tbody>
</table>

Table 10: Model Comparison for the U.K.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Discretion</td>
<td>Commitment</td>
</tr>
<tr>
<td>log marginal likelihood</td>
<td>91.0</td>
<td>50.8</td>
</tr>
<tr>
<td>log Bayes factor: ( \log(\pi_{D,T}/\pi_{C,T}) )</td>
<td>-</td>
<td>40.2</td>
</tr>
</tbody>
</table>

Note: The marginal likelihood is approximated by the modified harmonic mean. Values of the log Bayes factor greater than 5 are considered as decisive evidence against the commitment regime. Negative values indicate support for the commitment regime. In particular, a value of -1.6 is slight evidence against the discretionary regime.
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflation</td>
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<td>10 year interest rate</td>
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Note: Performing 1,000 simulations for 75 periods. Calculated for the case of constant term premiums.
Figure 2: 10 years rolling standard deviation (centered)
Figure 5: Posterior predictive distribution of the standard deviation for U.S. 10 year interest rates. Solid line distribution: optimal monetary policy under discretion. Dashed line distribution: optimal monetary policy under commitment. Bar: actual data.
Figure 8: Impulse response functions using the posterior mean of the model under discretion estimated with U.S. data from 1983-2005.
Figure 10: Posterior predictive distribution of the standard deviation for U.K. 10 year interest rates. Solid line distribution: optimal monetary policy under discretion. Dashed line distribution: optimal monetary policy under commitment. Bar: actual data.