Aggregate productivity growth and the allocation of resources over the business cycle*

Sophie Osotimehin†

Paris School of Economics and CREST (ENSAE)

PRELIMINARY AND INCOMPLETE

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October 12, 2011

Abstract

This paper analyzes the consequences of the reallocation of resources between entering, exiting and incumbents firms for the dynamics of aggregate productivity. I propose a novel decomposition of aggregate productivity growth derived from the aggregation of firm-level production functions. This approach extends the Solow (1957) growth accounting exercise to a framework with heterogeneous firms and frictions in the allocation of resources across firms. Using firm-level data from the French manufacturing industry, I find that 1) entry and exit contribute very little to the dynamics of aggregate productivity growth. 2) movements in allocative efficiency are somewhat countercyclical, with a correlation to real value added growth of -0.25. 3) within-firm productivity changes are procyclical, with a correlation coefficient equal to 0.64.

Keywords: aggregate productivity, resource allocation, entry and exit, cleansing

JEL codes: E32, O47, D24,

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*I thank Guy Laroque, Jean-Olivier Hairault, Isabelle Méjean and Julien Martin for many helpful comments and suggestions.

†Address for correspondence: CREST INSEE, Timbre J360, 15 Bd Gabriel Péri, 92245 Malakoff Cedex, France.

1 Introduction

Recessions are often viewed as time where the economy is "cleansed": the least productive firms are forced to exit the market, allowing resources to be reallocated towards more productive uses. This Schumpeterian view of recessions, which has been emphasized by Caballero and Hammour (1994), suggests that the efficiency in the allocation of resources improves during recessions. While the theoretical literature has focused on the contribution of entry and exit, aggregate productivity could also be driven by changes in the efficiency of resource allocation across existing firms. Is allocative efficiency an important determinant of aggregate productivity changes over the business cycle? To answer this question, I propose a novel approach to separate out the variations in aggregate productivity which are due to within-firm productivity changes from those due to changes in the allocation of resources between incumbents, entering and exiting firms. To this end, I derive the link between micro and aggregate productivity dynamics in a framework where firms are heterogeneous and where market frictions distort the allocation of resources across firms. This approach extends the Solow (1957) growth accounting exercise to a framework with firm heterogeneity and allocative inefficiency.

The importance of resource reallocation for aggregate productivity growth has been documented in many empirical papers (Baily et al., 1992; Foster et al., 2001; Griliches and Regev, 1995).[1] However, all these papers focus on long run productivity changes and therefore provide little evidence on the contribution of resource reallocation at business cycle frequency. More importantly, the approach used in these papers is likely to give a biased measure of the role of allocative efficiency for aggregate productivity growth. First, these papers focus on *average* rather than on *aggregate* productivity: at the macro level, productivity is computed as the weighted average of firm-level total factor productivity (TFP). While the two concepts are equivalent when firms produce homogeneous goods using a linear technology with only one input, in the general case the consequences of input reallocation on average productivity are very different from that on aggregate productivity. Shifting resources towards high TFP firms mechanically raises average productivity but may reduce allocative efficiency and aggregate production if high TFP firms have a low marginal productivity. Though widely used in the literature, the correlation between changes in input shares and firm-level TFP does not capture the contribution of resource reallocation to aggregate productivity growth. Contrary to the existing literature, I analyze the contribution of resource reallocation on *aggregate* productivity which I derive from the aggregation of firm-level production functions. My decomposition also differs from the existing literature in its objective. While the existing literature studies the consequences of changes input shares, my objective is to assess the contribution of changes in allocative efficiency. Changes in input

[1] The decompositions found in this literature slightly differ from one another by the weights used (previous or/and current period, labor or market shares) and whether or not firm-level productivity is normalized (relative to the aggregate productivity index).

shares do not necessarily imply changes in allocative efficiency. In fact, allocative efficiency is likely to be modified if input shares remain constant while the firms' productivity fluctuate. In this paper, I isolate the changes in input shares that lead to changes in the level of allocative efficiency. The level of allocative inefficiency is measured as the dispersion in labor and capital marginal productivities. This corresponds to the distance with respect to the first-best benchmark in which the value of marginal productivities are equalized across firms.

The decomposition of aggregate productivity growth is derived in a model where frictions in output and input markets generate production inefficiencies. For a given level of aggregate inputs, a higher level of output could be reached if resources were more efficiently allocated. Following Restuccia and Rogerson (2008) and Chari et al. (2007), I do not specify the frictions that induce this resource misallocation. Rather, the frictions are modelled as wedges between the firms' marginal products. The model therefore encompasses various sources of distortions such as adjustment costs, search frictions, financial constraints or distortionary regulation. Within this framework, I show how to aggregate the heterogenous production functions into an aggregate production function. The aggregation of heterogeneous production functions is a classical problem in macroeconomics. It is well known that if one allows the individual firm inputs to take any values, the aggregation of firm-level technological constraints is impossible unless very restrictive conditions are imposed. The usual solution consists in defining the aggregate production function as the efficient frontier of the production possibilities set (e.g. Fisher (1969), Houthakker (1955)). As the focus is on misallocation and production inefficiencies, this is not the approach considered here. Following Malinvaud (1993), I define the aggregate production function as the relation between aggregate output and input for a given allocation of resources. Computed from this aggregate production function, aggregate productivity growth captures the variations in output that are due to changes in resource allocation, as well as those due to within-firm productivity change[2]. Then, using a method similar to the index number approach, I decompose aggregate productivity growth between productivity changes at the firm-level, changes in allocative efficiency, and changes in the pattern of entry and exit.

The decomposition of aggregate productivity growth is estimated from 1991 to 2006 on French firm-level data from the manufacturing sector.[3] I use a dataset collected annually by the tax administration and combined with survey data in the INSEE unified system of business statistics (SUSE). The empirical analysis leads to the following findings: 1) entry and exit contribute very

[2]I do not investigate the source of within-firm productivity change. Just as the aggregate productivity also depends on allocative efficiency, within-firm productivity could also depend on the allocation of resources within the firm. This dimension is left aside in the paper.

[3]Note that the link between micro and macro productivity is likely to be different if plant-level data were used instead. However, the availability of plan-level data is limited in France, and in particular does not allow to compute productivity.

little to the dynamics of aggregate productivity growth. 2) movements in allocative efficiency are somewhat countercyclical, with a correlation to real value added growth of -0.25. 3) within-firm productivity changes are procyclical, with a correlation coefficient equal to 0.64. These results differ substantially from those obtained when implementing the decomposition proposed by Foster et al. (2001). Using their decomposition, the extensive margin accounts for a larger share of aggregate productivity growth and the reallocation component appears procyclical.

The finding that entry and exit flows have a negligible role for aggregate productivity growth contrasts sharply with the literature on the cleansing effect of recession (Caballero and Hammour, 1994; Barlevy, 2003; Ouyang, 2009). While the cleansing effect would imply a countercyclical extensive margin component, I find that, not only is the contribution of entry and exit small, it is also positively correlated to real value added growth. In fact, it is the reallocation of resources between incumbents, and not that between entering and exiting firms that tends to raise aggregate productivity during recessions. Despite the heterogeneity between sectors, the countercyclicality of allocative efficiency also holds for most sectors. This finding suggests new directions for future theoretical work as very little is known on the mechanisms behind the cyclical patterns of allocative efficiency.

This paper is not the first to advocate the use of a well-defined measure of allocative efficiency. Several recent papers (Petrin and Levinsohn, 2005; Basu et al., 2009; Petrin et al., 2011) have emphasized that the reallocation component should capture changes in the allocation of inputs between firms with different marginal values. None of these papers investigate the role of the extensive margin. More importantly, the methods used, as well as the results obtained are substantially different from these papers. Their decompositions are based on the Solow residual measure of productivity at the firm-level. They all build on Basu and Fernald (2002)'s insight that, under some conditions, the Solow residual approximates the welfare change of a representative consumer even when the allocation of resources is distorted by imperfect competition. Their measure of aggregate productivity growth therefore includes the effects of changes in aggregate input reflecting the fact that, under imperfect competition, welfare increases with aggregate input use. Moreover, their measure of allocative efficiency only captures the consequences of reallocation across firms with different markups and therefore does not account for all changes in the dispersion of marginal products. Basu et al. (2009) compute this decomposition for several European countries over the period 1995-2005 and find that allocative efficiency is not an important component of aggregate productivity. In particular, for France, they find that within-firm productivity changes explain all the changes in the Solow residual. This contrasts with my results in which allocative efficiency reduces by 51% the volatility of sectoral productivity growth. Contrary to Basu and Fernald (2002), I explicitly address the aggregation issue to investigate the link between the Solow residual and firm-level productivity change. By taking explicit account of firm heterogeneity and microeconomic frictions, I provide a complementary analysis to

Hall (1991) who highlights the consequence of market imperfections for the measure of aggregate productivity growth in a representative firm framework. I show that the Solow residual gives a biased measure of productivity change when the heterogeneity in factor elasticities is accounted for, even when resources are efficiently allocated.

This paper is also closely related to Hsieh and Klenow (2009) who study the role of resource misallocation in explaining the TFP differential between China, India and the United States. As in their paper, resource misallocation is captured by the dispersion in the marginal products of capital and labor. However, both the objective and the decomposition differ from their paper. They analyze TFP differentials across countries, and quantify misallocation by measuring the distance between observed and first-best TFP levels. By contrast, I focus on TFP variation across time, and propose a decomposition of observed TFP using an approach similar to the index number theory. Furthermore, contrary to Hsieh and Klenow (2009), I use a unified approach at both the sectoral and aggregate levels. This allows me to provide an estimation of allocative efficiency not only within sectors but also between sectors.

The paper is organized as follows. Section 2 lays out the framework and shows how to aggregate heterogeneous production units to derive the aggregate productivity index. Section 3 presents the decomposition of aggregate productivity both within and between sectors. Section 4 presents the estimation method and the results obtained on French firm-level data. Section 5 concludes.

2 Aggregation of heterogenous production units

Aggregate productivity is a concept which is intrinsically related to the production function. Aggregate productivity is defined as the change in real output not accounted for by the change in real input. To analyze changes in aggregate productivity, it is therefore necessary to derive the link between aggregate inputs and aggregate output. This section lays out the setup and shows how to derive the aggregate production function in a framework where producers are heterogeneous and face allocation frictions.

2.1 Framework

Consider an economy with S sectors. In each sector $s = 1...S$, there are N_s potential firms indexed by $i = 1, ..., N_s$. Firms produce a differentiated good Y_{it} using a Cobb-Douglas technology[4]:

$$Y_{it} = z_{it} K_{it}^{\alpha_s} L_{it}^{\beta_s},$$

[4]To simplify notations, sector subscripts s are omitted for the firms's output and inputs.

where K_{it} denotes capital, L_{it} labor and z_{it} the firm-level total factor productivity. Depending on the value of $\gamma_s \equiv \alpha_s + \beta_s$, firms face either decreasing or constant returns to scale $\gamma_s \leq 1$, for all $s = 1, ..., S$. The factor elasticities are assumed to be identical within sectors, but may vary from one sector to another. Firms face a downward sloping demand curve. For simplicity let us assume that the demand functions are iso-elastic. The price demand for good i reads:

$$p_{it} = b_{it} Y_{it}^{\theta_s - 1},$$

where $0 < \theta_s \leq 1$. The parameter θ_s governs the price elasticity of demand in sector s and b_{it} is a firm-specific demand shock. It is important to note that the size of the firm is indeterminate if firms use a constant returns to scale technology and behave competitively ($\gamma_s = 1$ and $\theta_s = 1$) In this case, the allocation rule is not unique and the aggregate production function is then not defined. I therefore restrict the analysis to the case where $\theta_s \leq 1$ and $\gamma_s \leq 1$, with at least one strict inequality.

In this economy, the allocation of input across firms is distorted by market frictions. Following Restuccia and Rogerson (2008) and Chari et al. (2007), let us remain agnostic about the nature of the frictions. Consider a generic model in which distortions are captured by the presence of wedges $\tau_{it} = (\tau_{it}^K, \tau_{it}^L)$ that disrupt the firms' input decisions with respect to the frictionless first-order conditions:

$$\alpha_s \theta_s \frac{p_{it} Y_{it}}{K_{it}} = r_t (1 + \tau_{it}^K) \quad (1)$$

$$\beta_s \theta_s \frac{p_{it} Y_{it}}{L_{it}} = w_t (1 + \tau_{it}^L), \quad (2)$$

The distortions τ_{it} reduce the efficiency of aggregate production as they prevent the value of marginal products to be equalized across firms. The cross-sectional inefficiency may differ across inputs: τ_{it}^K denotes distortions that affect specifically the marginal product of capital, and τ_{it}^L, the marginal product of labor. As already mentioned, this model encompasses different types of frictions. Adjustment costs, search frictions, financial constraints and distortionary regulation all generate gaps between the firms' marginal products. Note that frictions that generate a wedge between the marginal product and the marginal cost of an input are not a source of allocative inefficiency if they affect all the firms in the same way. Hence, in this framework, imperfect competition does not distort the allocation of resources across firms within sectors as the markup $(1/\theta_s)$ is assumed to be identical within sectors. However, imperfect competition may reduce the efficiency of resource allocation across sectors. The allocation of resources across firms from different sectors is distorted both by firm-specific distortions and by sector-specific markups.

The firm does not always find it profitable to produce; depending on its level of productivity and distortions, the firm could decide to exit. The firm exits if $I_{it}(z_{it}, b_{it}, \tau_{it}) < 0$ and produces if $I_{it}(z_{it}, b_{it}, \tau_{it}) \geq 0$. The function I_{it} characterizes the participation decision of the firm and may vary across firms and across time as the decision to produce depends on factor prices, firm-specific fixed costs, and whether the firm consider to enter or exit. Combining the participation decision with equations (1) and (2), and with the demand curve, we can derive the input levels as a function of the firm's distortions, productivity and demand shock:

$$K_{it} = \begin{cases} \theta_s z_{it}^{\frac{\theta_s}{1-\gamma\theta_s}} b_{it}^{\frac{1}{1-\gamma\theta_s}} \left(\frac{r}{\alpha_s}(1+\tau_{it}^K)\right)^{-\frac{1-\beta_s\theta_s}{1-\gamma\theta_s}} \left(\frac{w}{\beta_s}(1+\tau_{it}^L)\right)^{-\frac{\beta_s\theta_s}{1-\gamma\theta_s}} & \text{if } I_{it}(z_{it}, b_{it}, \tau_{it}) \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

$$L_{it} = \begin{cases} \theta_s z_{it}^{\frac{\theta_s}{1-\gamma\theta_s}} b_{it}^{\frac{1}{1-\gamma\theta_s}} \left(\frac{r}{\alpha_s}(1+\tau_{it}^K)\right)^{\frac{-\alpha_s\theta_s}{1-\gamma\theta_s}} \left(\frac{w}{\beta_s}(1+\tau_{it}^L)\right)^{-\frac{1-\alpha_s\theta_s}{1-\gamma\theta_s}} & \text{if } I_{it}(z_{it}, b_{it}, \tau_{it}) \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

2.2 Aggregation

The aggregation of heterogeneous firm-level production functions is a classical problem in macroeconomics. The link between aggregate input and output is not straightforward to derive. In fact, changes in aggregate input may affect aggregate output differently, depending on the way the additional input is allocated across firms. It is well known that, if one allows the firms' inputs to take any values, the aggregation of firm-level technological constraints is possible only under very restrictive conditions.[5] In this paper, as the objective is to disentangle the effects of aggregate input from that of allocative efficiency and within-firm productivity changes, the aggregate production function need not be a pure technical relationship : we are looking for the relationship between aggregate output and input for a given allocation of resources. This is Malinvaud (1981)'s approach to the aggregate production function. With this definition, the aggregate production function becomes specific to the economic environment and the conditions for aggregation become less restrictive.

The aggregate production function is derived in three steps. First, we must take into account the heterogeneity in the goods produced. In particular, the impact of a reallocation of resources between two different goods on aggregate productivity should depend on the relative value of the goods produced. Then, I show how to derive the aggregate production function at the sectoral level, where factor and demand elasticities are homogeneous. Finally, I take into account the heterogeneity across sectors and aggregate the sectoral production functions. Deriving first the sectoral production functions allows us to disentangle the changes in the allocative efficiency within and between sectors.

[5] The individual production functions must be linear and have the same slopes (Nataf, 1948)

2.2.1 Accounting for goods heterogeneity

Since goods are heterogeneous, the relative value of the goods should be accounted for and the individual production functions described in section 2.1 should therefore not be aggregated directly. This is all the more important when studying the impact of reallocations on aggregate productivity. Suppose that all the firms use a constant returns to scale technology. If the impact of reallocation on the relative value of the goods is not accounted for, aggregate production is maximized when resources are reallocated towards the firm with the highest TFP level, leading the economy to produce only one type of good. However, this does not correspond to the optimal allocation of resources when goods are heterogeneous. The optimal allocation is the one that equalizes the *value* of marginal productivity across firms. Reallocating all the resources toward the production of one good would decrease the value of this good, and raise that of non-produced goods. To obtain a measure of aggregate productivity consistent with this optimal allocation rule, we must take into account the impact of input changes not only on production but also on relative prices. This leads to a modified production function, where the output of the firm depends on the value of the goods produced. The output of the firm is then $\frac{p_{it}}{P_{st}} Y_{it} = f_i(K_{it}, L_{it})$, where the production function is given by:

$$f_i(K_{it}, L_{it}) = A_{it} K_{it}^{\alpha_s \theta_s} L_{it}^{\beta_s \theta_s}, \tag{5}$$

with $A_{it} \equiv z_{it}^{\theta_s} b_{it} / P_{st}$ and P_{st} is the sectoral price index. When accounting for changes in the relative value of goods, the actual productivity of the firm combines both technical efficiency and demand factors. Note that A_{it} is also the usual measure of productivity at the firm level. Because firm-level prices are rarely available, the firm's output is usually measured by its nominal output divided by a sectoral deflator. Though it hinders the measure of technical efficiency at the firm-level[6], the absence of firm-level prices is not problematic here. What matters for aggregate productivity is the measured firm-level productivity A_{it}. In fact, the whole model can be rewritten as a function of measurable variables only. In particular, the input decision described in equations (3) and (4) can be rewritten as $K_i(P_{st}A_{it}, \tau_{it}, r_t, w_t)$ and $L_i(P_{st}A_{it}, \tau_{it}, r_t, w_t)$:

$$K_{it} = \begin{cases} \theta_s (P_{st} A_{it})^{\frac{1}{1-\gamma\theta_s}} \left(\frac{r}{\alpha_s}(1+\tau_{it}^K)\right)^{-\frac{1-\beta_s\theta_s}{1-\gamma\theta_s}} \left(\frac{w}{\beta_s}(1+\tau_{it}^L)\right)^{-\frac{\beta_s\theta_s}{1-\gamma\theta_s}} & \text{if } I_{it}(z_{it}, b_{it}, \tau_{it}) \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

$$L_{it} = \begin{cases} \theta_s (P_{st} A_{it})^{\frac{\theta_s}{1-\gamma\theta_s}} \left(\frac{r}{\alpha_s}(1+\tau_{it}^K)\right)^{\frac{-\alpha_s\theta_s}{1-\gamma\theta_s}} \left(\frac{w}{\beta_s}(1+\tau_{it}^L)\right)^{-\frac{1-\alpha_s\theta_s}{1-\gamma\theta_s}} & \text{if } I_{it}(z_{it}, b_{it}, \tau_{it}) \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

[6] The consequences of the absence of firm-level prices for the measure of firm-level TFP have been recently emphasized by Foster et al. (2008).

2.2.2 The sectoral production functions

Let us now aggregate the modified production functions described above. To find the relation between aggregate output ($Y = \sum_i \frac{p_i}{P} Y_i$) and aggregate inputs ($K = \sum_i K_i$ and $L = \sum_i L_i$), I use Malinvaud (1993)'s approach. The idea is to find the allocation rules, i.e how aggregate inputs are allocated between firms, and then aggregate output over firms using the allocation rules. More formally, once the allocation rules $K_i = k_i(K, L)$ and $L_i = l_i(K, L)$ are known, the aggregate production function is simply: $Y = \sum_i f_i(k_i(K, L), l_i(K, L))$.

The allocation rules can be derived from the equilibrium condition on the inputs markets. Specifically, they are obtained after inverting the aggregate input equations. At the sectoral level, total inputs are given by:

$$\begin{aligned} K_{st} &= \sum_{i=1}^{N_s} K_i(P_{st}A_{it}, \tau_{it}, r_t, w_t) \\ L_{st} &= \sum_{i=1}^{N_s} L_i(P_{st}A_{it}, \tau_{it}, r_t, w_t). \end{aligned}$$

Inverting this system of equations allows us to write the factor prices as a function of aggregate capital, labor input and the vector of firm-level productivity $\widetilde{A}_{st} = \{A_{1t}, ..., A_{N_s t}\}$ and distortions $\widetilde{\tau}_t = \{\tau_{1t}, ..., \tau_{N_s t}\}$.[7] The input levels can then be written as a function of aggregate inputs, firm-level productivities and distortions. The sectoral production function follows:

$$F_s(K_{st}, L_{st}, \widetilde{A}_{st}, \widetilde{\tau}_{st}) = \sum_{i=1}^{N_s} f_i(k_i(P_{st}\widetilde{A}_{st}, \widetilde{\tau}_{st}, K_{st}, L_{st}), l_i(P_{st}\widetilde{A}_{st}, \widetilde{\tau}_{st}, K_{st}, L_{st})).$$

In general, the aggregate production function does not share the same functional form as the individual production functions. However, when the factor and demand elasticities are identical across firms (as assumed within sectors), I show in Appendix A that the aggregate production function is also Cobb-Douglas. The sectoral production function is:

$$F_s(K_{st}, L_{st}, \widetilde{A}_{st}, \widetilde{\tau}_{st}) = TFP_{st} K_{st}^{\alpha_s \theta_s} L_{st}^{\beta_s \theta_s},$$

where TFP_{st} is a function of both firm-level productivity $\widetilde{A}_{st}$ and distortions $\widetilde{\tau}_{st}$.

And the change in sectoral productivity can be computed as:

$$\frac{dTFP_{st}}{TFP_{st}} = \frac{dY_{st}}{Y_{st}} - \alpha_s\theta_s \frac{dK_{st}}{K_{st}} - \beta_s\theta_s \frac{dL_{st}}{L_{st}} \qquad (8)$$

[7] This system is locally invertible if the Jacobian of the application $\{K_s(r, w), L_s(r, w)\}$ is non-zero.

2.2.3 From the sectoral to the aggregate production function

In order to aggregate the sectoral production functions, let us characterize each sector's representative firm.The sectoral input demand function are identical to the firm-level functions (equations (6) and (7) with the participation equation always satisfied). The sectoral input levels $K_s(P_{st}TFP_{st}, \omega_{st}, r_t, w_t)$ and $L_s(P_{st}TFP_{st}, \omega_{st}, r_t, w_t)$ are functions of sectoral level of distortions $\omega_{st} = (\omega^K_{st}, \omega^L_{st})$ which are given by:

$$1 + \omega^K_{st} = \sum_i \frac{K_{it}}{K_{st}}(1 + \tau^K_{it}) \tag{9}$$

$$1 + \omega^L_{st} = \sum_i \frac{L_{it}}{L_{st}}(1 + \tau^L_{it}). \tag{10}$$

Like at the sectoral level, the aggregate production function is obtained after inverting the aggregate input equations.[8] Since the factor and demand elasticities are allowed to be heterogenous across sectors, the aggregate production function is not Cobb-Douglas. Real aggregate output Y is given by $\sum_s Y_s = F(K_t, L_t, \widetilde{TFP}_t, \widetilde{\omega}_t, P_{st})$, where:

$$F(K_t, L_t, \widetilde{TFP}_t, \widetilde{\omega}_t, \widetilde{P}_t) = \sum_{s=1}^{S} TFP_{st} K_{st}^{\alpha_s\theta_s} L_{st}^{\beta_s\theta_s}$$

$$\begin{aligned} \text{with } K_{st} &= k_s(\widetilde{TFP}_t, \widetilde{\omega}_t, \widetilde{P}_t, K_t, L_t) \\ L_{st} &= l_s(\widetilde{TFP}_t, \widetilde{\omega}_t, \widetilde{P}_t, K_t, L_t). \end{aligned}$$

$\widetilde{TFP}_t = \{TFP_{1t}, ..., TFP_{St}\}$ is the vector of sectoral level productivity, $\widetilde{\omega}_t = \{\omega_{1t}, ..., \omega_{St}\}$ the vector of sectoral level distortions and $\widetilde{P}_t = \{P_{1t}, ..., P_{St}\}$ is the vector of sectoral price indexes.

Aggregate TFP growth is then:

$$\frac{dTFP_t}{TFP_t} = \frac{dY}{Y} - \varepsilon_K(\widetilde{\alpha_s}, \widetilde{\beta})\frac{dK}{K} - \varepsilon_L(\widetilde{\alpha_s}, \widetilde{\beta})\frac{dL}{L}, \tag{11}$$

with $\widetilde{\alpha} = \{\alpha_1\theta_1, ..., \alpha_S\theta_S\}$, $\widetilde{\beta} = \{\beta_1\theta_1, ..., \beta_S\theta_S\}$.

Note that the aggregate elasticity to capital ε_K is not a weighted average of sector-level elasticities to capital, and similarly, the aggregate elasticity to labor ε_L is not a weighted average of sector-level elasticities to labor. The aggregate elasticity of capital and labor both depend on the sector-level elasticities of labor and capital and on sectoral price elasticities. To see more explicitly this

[8] The aggregate input equations are:
$K = \sum_{s=1}^{S} K_s(P_{st}TFP_{st}, \omega_{st}, r_t, w_t)$
$L = \sum_{s=1}^{S} L_s(P_{st}TFP_{st}, \omega_{st}, r_t, w_t)$

point, let us assume that the decreasing returns to scale and the elasticity of demand parameters are identical across sectors $\gamma_s = \gamma$ and $\theta_s = \theta$, for all s. The change in aggregate TFP is then:

$$\frac{dTFP_t}{TFP_t} \equiv \frac{dY}{Y} - \left(\frac{\alpha^Y(1-\alpha^L) - \beta^Y\alpha^L}{1-\alpha^L-\beta^K}\right)\frac{dK}{K} - \left(\frac{\beta^Y(1-\beta^K) - \alpha^Y\beta^K}{1-\alpha^L-\beta^K}\right)\frac{dL}{L},$$

with $\alpha^X = \sum_s \frac{X_s}{X}\alpha_s\theta$ and $\beta^X = \sum_s \frac{X_s}{X}\beta_s\theta$ for $X = Y, K, L$.

As will be shown in section 4, the heterogeneity in factor elasticities may bias the estimate of aggregate factor elasticities and of aggregate productivity growth.

3 Accounting for changes in aggregate productivity

This section presents the decomposition of aggregate productivity growth into changes in firm-level productivity, changes in the efficiency of resource allocation within and between sectors, and changes in entry and exit patterns. I first decompose sectoral productivity growth. Then, I use the aggregate production function to aggregate the within-sector components and to derive the contribution of allocative efficiency between sectors.

3.1 Decomposition of sectoral productivity

As shown in section 2.2.2 and Appendix A, the sectoral production function is given by: $Y_{st} = TFP_{st}K_{st}^{\alpha_s\theta_s}L_t^{\beta_s\theta_s}$, where sectoral productivity is a function of firm-level productivity A_{it} and distortions $\tau_{it} = (\tau_{it}^K, \tau_{it}^L)$:[9]

$$TFP_{st} = \frac{\sum_{i=1}^{N_s} g_s^Y(A_{it},\tau_{it})}{\left(\sum_{i=1}^{N_s} g_s^K(A_{it},\tau_{it})\right)^{\alpha_s\theta_s}\left(\sum_{i=1}^{N_s} g_s^L(A_{it},\tau_{it})\right)^{\beta_s\theta_s}}. \quad (12)$$

To show more specifically how aggregate productivity depends on the cross-sectional distribution of firm-level productivity and distortions, let us assume that A_i, $(1+\tau_i^K)$ and $(1+\tau_i^L)$ are jointly lognormally distributed. Sectoral TFP can then be written as a function of the first two moments of the joint distribution:

$$\begin{aligned}\ln TFP_{st} \;=\; & (1-\gamma_s\theta_s)\ln N_{st} + E_t \ln A_i + \frac{1}{2}\frac{1}{1-\gamma_s\theta_s}V_t\ln A_i \\ & -\frac{1}{2}\frac{\alpha_s\beta_s\theta_s^2}{1-\gamma_s\theta_s}\left(\frac{1-\beta_s\theta_s}{\beta_s\theta_s}V_t\ln(1+\tau_i^K) + \frac{1-\alpha_s\theta_s}{\alpha_s\theta_s}V_t\ln(1+\tau_i^L) + Cov_t(\ln(1+\tau_i^K),\ln(1+\tau_i^L))\right),\end{aligned}$$

[9] The g functions are given by:
$g_s^Y(A_{it},\tau_{it}) = A_i^{\frac{1}{1-\gamma\theta_s}}(1+\tau_i^K)^{-\frac{\alpha_s\theta_s}{1-\gamma\theta_s}}(1+\tau_i^L)^{-\frac{\beta_s\theta_s}{1-\gamma\theta_s}}$
$g_s^K(A_{it},\tau_{it}) = A_i^{\frac{1}{1-\gamma\theta_s}}(1+\tau_i^K)^{-\frac{1-\beta_s\theta_s}{1-\gamma\theta_s}}(1+\tau_i^L)^{-\frac{\beta_s\theta_s}{1-\gamma\theta_s}}$
$g_s^L(A_{it},\tau_{it}) = A_i^{\frac{1}{1-\gamma\theta_s}}(1+\tau_i^K)^{-\frac{\alpha_s\theta_s}{1-\gamma\theta_s}}(1+\tau_i^L)^{-\frac{1-\alpha_s\theta_s}{1-\gamma\theta_s}}$

where E_t, V_t and Cov_t denote the period t cross-sectional expectation, variance and covariance. Sectoral TFP depends on the average and the dispersion of firm-level productivity, but also on the dispersion in the distortions, on the covariance between labor and capital distortions, and on the number of firms in the sector. Idiosyncratic (uncorrelated) shocks to either productivity or distortions have an impact on aggregate productivity only to the extent that these shocks also modify the cross-sectional variance. This expression also shows that average distortions, as well as the covariance between firm level productivity and distortion plays no role for aggregate productivity growth. Though the result on the covariance will probably not hold in a more general framework, the insensitivity of aggregate productivity to a common distortion factor holds beyond the case of the log-normal distribution.

Let us now decompose sectoral TFP in the general case, without specifying the joint distribution of A_i, $(1+\tau_i^K)$ and $(1+\tau_i^L)$. In appendix B, I show that aggregate productivity growth (ΔTFP_{st}) can be decomposed into changes in within-firm productivity ($\Delta\mathrm{TE}_s$), changes in allocative efficiency ($\Delta\mathrm{AE}_s$) and changes at the extensive margin ($\Delta\mathrm{EX}_s$)[10]:

$$\Delta TFP_{st} \approx \Delta\mathrm{TE}_s + \Delta\mathrm{AE}_s + \Delta\mathrm{EX}_s,$$

An approximation for each component is given below. The decomposition between changes in within-firm productivity and changes in allocative efficiency is similar to the decompositions we can find in the index number literature. To avoid any asymmetry induced by the functional form of the indexes, I measure in the data each component with a Fisher-like index, i.e. the geometric mean of the Laspeyres-like and Paasche-like indexes. The exact decomposition with the Fisher-like index is described in Appendix B. For simplicity, I present here an approximation in which the effects of within-firm productivity changes are measured with a Laspeyres-like index, while those of allocative efficiency are measured with a Paasche-like index.

The change in firm-level productivity can be approximated as a combination of weighted averages of the firm level productivity changes:[11]

$$\Delta TE_s \simeq \frac{1}{1-\gamma_s\theta_s} \sum_{i\in C_s} \frac{\Delta A_{it}}{A_{it-1}} \left[\frac{p_{it-1}Y_{it-1}}{\sum\limits_{i\in C_s} p_{it-1}Y_{it-1}} - \alpha_s\theta_s \frac{K_{it-1}}{\sum\limits_{i\in C_s} K_{it-1}} - \beta_s\theta_s \frac{L_{it-1}}{\sum\limits_{i\in C_s} L_{it-1}} \right], \qquad (13)$$

with C_s is the set of continuing firms ($I_{it-1} \geq 0$ and $I_{it} \geq 0$). Note that the impact of within-firm productivity change is measured for a given level of allocative efficiency. Therefore, this measure includes the effect of within-firm productivity changes for a given distribution of inputs, as well as the consequences of within-firm productivity changes on input shares for a given level of distortions. The level of distortions is measured here by the relative marginal productivities

[10] For convenience, we denote the aggregate productivity growth by $\Delta TFP_{st} \equiv TFP_{st}/TFP_{st-1} - 1$

[11] Note that the case of perfect competition and constant returns to scale is not determinate in this framework

of input across firms.[12] Therefore, the firm-level efficiency component captures the impact of within-firm productivity changes, had relative marginal productivities remained constant.

The change in allocative efficiency is a combination of weighted averages of firm-level changes in distortion:

$$\Delta AE_s \simeq \frac{\alpha_s\theta_s}{1-\gamma_s\theta_s}\sum_{i\in C_s}\frac{\Delta(1+\tau_{it}^K)}{1+\tau_{it-1}^K}\left[(1-\beta_s\theta_s)\frac{K_{it}}{\sum\limits_{i\in C_s}K_{it}}+\beta_s\theta_s\frac{L_{it}}{\sum\limits_{i\in C_s}L_{it}}-\frac{p_{it}Y_{it}}{\sum\limits_{i\in C_s}p_{it}Y_{it}}\right]$$
$$+\frac{\beta_s\theta_s}{1-\gamma_s\theta_s}\sum_{i\in C_s}\frac{\Delta(1+\tau_{it}^L)}{1+\tau_{it-1}^L}\left[\alpha_s\theta_s\frac{K_{it}}{\sum\limits_{i\in C_s}K_{it}}+(1-\alpha_s\theta_s)\frac{L_{it}}{\sum\limits_{i\in C_s}L_{it}}-\frac{p_{it}Y_{it}}{\sum\limits_{i\in C_s}p_{it}Y_{it}}\right] \quad (14)$$

The contribution of the extensive margin to aggregate productivity growth depends of the relative weights of entering and exiting firms:

$$\Delta EX_s \simeq \frac{\sum\limits_{i\in E_s}\frac{p_{it}Y_{it}}{P_{st}Y_t}-\sum\limits_{i\in X_s}\frac{P_{it-1}Y_{it-1}}{P_{st-1}Y_{t-1}}}{1-\sum\limits_{i\in E_s}\frac{p_{it}Y_{it}}{P_{st}Y_t}}-\alpha_s\theta_s\frac{\sum\limits_{i\in E_s}\frac{K_{it}}{K_t}-\sum\limits_{i\in X_s}\frac{K_{it-1}}{K_{t-1}}}{1-\sum\limits_{i\in E_s}\frac{K_{it}}{K_t}}-\beta_s\theta_s\frac{\sum\limits_{i\in E_s}\frac{L_{it}}{L_t}-\sum\limits_{i\in X_s}\frac{L_{it-1}}{L_{t-1}}}{1-\sum\limits_{i\in E_s}\frac{L_{it}}{L_t}} \quad (15)$$

with E_s the set of new entrants ($I_{it-1}<0$ and $I_{it}\geq 0$) and X_s the set of exiting firms ($I_{it-1}\geq 0$ and $I_{it}<0$) in sector s. The extensive margin contributes positively to aggregate productivity growth if entering firms are more productive and face less distortions than exiting firms. As long as there are decreasing returns to scale $\gamma_s\theta_s<1$ (in production or in demand), the positive effect of higher output shares is not offset by larger input shares. All else equal, a higher returns to scale parameter reduces the role of entry and exit for aggregate productivity growth. Note that with decreasing returns to scale, the efficiency of production increases with the number of firms.

When resources are perfectly allocated across firms ($1+\tau_i = 1+\tau$, for all $i=1,...,N_s$), the allocation of labor, capital and output depends only on the firms' relative productivity. Moreover, in this case, input and output shares are equal ($\frac{K_i}{K}=\frac{L_i}{L}=\frac{p_{it}Y_{it}}{P_{st}Y_t}$), and the components of aggregate productivity simplify to:

$$\Delta TE_s^{FL} \simeq \sum_{i\in C_s}\frac{\Delta A_{it}}{A_{it-1}}\frac{p_{it-1}Y_{it-1}}{\sum\limits_{i\in C_s}p_{it-1}Y_{it-1}}$$

[12] An alternative measure of distortions is the *difference* between marginal productivities (instead of the *ratio*). Note that in this case, the decomposition is identical to the case where elasticities are heterogeneous (equations (18) to (21)) but with a different values for aggregate elasticities ε_K and ε_L. In fact, the way distortions are measured affects not only the measure of changes in allocative efficiency, but also the impact of changes in aggregate inputs and within-firm productivity on aggregate output as these are computed holding fixed the level of distortions.

$$\Delta AE_s^{FL} = 0$$

$$\Delta EX_s^{FL} \simeq (1-\gamma_s\theta_s)\frac{\sum\limits_{i\in E_s}\frac{p_{it}Y_{it}}{P_{st}Y_t} - \sum\limits_{i\in X_s}\frac{p_{it-1}Y_{it-1}}{P_{st-1}Y_{t-1}}}{1-\sum\limits_{i\in E_s}\frac{p_{it}Y_{it}}{P_{st}Y_t}}$$

In the absence of allocation distortions, the impact of within-firm productivity changes can be measured by a simple weighted average. The allocative efficiency component is null as resources are always efficiently allocated, and the extensive margin component simplifies to the difference between the output shares of entering and exiting firms.

3.2 Decomposition across sectors

The aggregate production function, derived in section 2.3.3, is a function of the vector of sectoral productivity $\widetilde{TFP}_t = \{TFP_{1t}, ..., TFP_{St}\}$, and sectoral distortions $\widetilde{\omega}_t = \{\omega_{1t}, ..., \omega_{St}\}$:

$$\sum_s Y_s = F(K_t, L_t, \widetilde{TFP}_t, \widetilde{\omega}_t, P_{st}), \tag{16}$$

As defined in equation (11), aggregate productivity growth is the change in output that is not due to the change in aggregate inputs. Combining this equation with the total differential of equation (16), aggregate productivity can also be written as the change in aggregate output explained by changes in sectoral productivity and distortions. This leads to the following decomposition:[13]

$$\frac{dTFP_t}{TFP_t} = \underbrace{\sum_{s=1}^{S}\frac{\partial F}{\partial TFP_s}\frac{TFP_s}{Y}\frac{dTFP_s}{TFP_s}}_{\text{Within sectors}} + \underbrace{\sum_{s=1}^{S}\frac{\partial F}{\partial \omega_s^K}\frac{1+\omega_s^K}{Y}\frac{d\omega_s^K}{1+\omega_s^K} + \frac{\partial F}{\partial \omega_s^L}\frac{1+\omega_s^L}{Y}\frac{d\omega_s^L}{1+\omega_s^L}}_{\text{Between sectors}}.$$

The first term measures changes in within-sector productivity and the second term measures changes in the efficiency of resource allocation across sectors. Using the implicit function theorem to compute the derivative of F with respect to TFP_s and replacing the within-sector productivity change by the decomposition derived at the sectoral level yields the decomposition of aggregate productivity growth (ΔTFP_t) into changes in firm-level productivity $\Delta\overline{\text{TE}}$, changes in the allocation of resources between $\Delta\overline{\text{AE}}_{\text{between}}$ and within sectors $\Delta\overline{\text{AE}}_{\text{within}}$, as well as changes in entry and exit patterns $\Delta\overline{\text{EX}}$:

$$\Delta TFP_t = \Delta\overline{\text{TE}} + \Delta\overline{\text{AE}}_{\text{within}} + \Delta\overline{\text{EX}} + \Delta\overline{\text{AE}}_{\text{between}}. \tag{17}$$

[13] Note that the impact of changes in sectoral prices are neglected. In the empirical results we verify that the last term of the decomposition $\sum_{s=1}^{S}\frac{\partial F}{\partial P_s}\frac{P_s}{Y}\frac{dP_s}{P_s}$ is indeed negligible.

The details of the derivation are given in Appendix B.[14] The within-sector productivity decomposition is aggregated at the macro level using input and output sectoral shares and the elasticities of the aggregate production function with respect to aggregate inputs, ε_K and ε_L:

$$\Delta\overline{\text{TE}} = \sum_{s=1}^{S} \frac{1}{1-\gamma_s\theta_s} \left(\frac{Y_s}{Y} - \varepsilon_K \frac{K_s}{K} - \varepsilon_L \frac{L_s}{L} \right) \Delta\text{TE}_s \tag{18}$$

Similarly, the change in within-sector allocative efficiency and the aggregate impact of the extensive margin can be computed as:

$$\Delta\overline{\text{AE}}_{\text{within}} = \sum_{s=1}^{S} \frac{1}{1-\gamma_s\theta_s} \left(\frac{Y_s}{Y} - \varepsilon_K \frac{K_s}{K} - \varepsilon_L \frac{L_s}{L} \right) \Delta\text{AE}_s \tag{19}$$

$$\Delta\overline{\text{EX}} = \sum_{s=1}^{S} \frac{1}{1-\gamma_s\theta_s} \left(\frac{Y_s}{Y} - \varepsilon_K \frac{K_s}{K} - \varepsilon_L \frac{L_s}{L} \right) \Delta\text{EX}_s \tag{20}$$

Finally, the change in between-sector allocative efficiency is computed as follows:

$$\begin{aligned} \Delta\overline{\text{AE}}_{\text{between}} \quad = \quad & \sum_{s=1}^{S} \frac{1}{1-\gamma_s\theta_s} \left(\varepsilon_K (1-\beta_s\theta_s) \frac{K_s}{K} + \varepsilon_L \alpha_s\theta_s \frac{L_s}{L} - \alpha_s\theta_s \frac{Y_s}{Y} \right) \frac{d\omega_s^K}{1+\omega_s^K} \\ & + \sum_{s=1}^{S} \frac{1}{1-\gamma_s\theta_s} \left(\varepsilon_K \beta_s\theta_s \frac{K_s}{K} + \varepsilon_L (1-\alpha_s\theta_s) \frac{L_s}{L} - \beta_s\theta_s \frac{Y_s}{Y} \right) \frac{d\omega_s^L}{1+\omega_s^L} \end{aligned} \tag{21}$$

If factor elasticities were equal across sectors, all these expressions would be exactly equal to the decomposition derived at the sectoral level.[15]

4 Estimation

Aggregate and sectoral productivity growth can be computed directly from aggregate data. However, estimating the decomposition given by equation (17) requires the use of firm-level data. First, the firm-level efficiency, allocative efficiency and extensive margin components are computed at the sectoral level using equations (13) to (15). Then the within-sector components can be aggregated at the macro level as described by equation (18) to (20). The change in between-sector allocative efficiency is computed with equation (21). In this section, I present the data used, the estimation strategy and the results of this decomposition estimated on French firm-level data.

[14] Note that, contrary to the within-sector decomposition, the decomposition across sectors requires no specific assumptions about the individual production functions ; it requires though the existence of a unique allocation rule, necessary for the existence of the aggregate production function.

[15] We cannot, however, use the same method at the sectoral level. Because of the discontinuity introduced by the entry and exit decisions, we cannot take the total derivative of the sectoral production function.

4.1 Data description

Following the bulk of the literature, I investigate the dynamics of aggregate productivity in the manufacturing industry. The data used in this analysis are collected every year by the French tax administration, and combined with survey data in the INSEE unified system for business statistics (SUSE). I use the database of private businesses that declare their profits under the "normal" regime (*Bénéfice réel normal*) from 1989 to 2007. In 2003, the "normal" regime accounted for 24.4 % of firms and 94.3% of total sales. Firms are required to provide balance sheet data, which includes measures of the firms' value added, expenditures on capital and number of employees. I use sectoral value added price indexes to deflate firm-level value added and reconstruct the real capital stock using the perpetual inventory method.

Potential entries and exits are detected by following firms with at least one employee that appear in and disappear from the database.[16] Since firms are followed through their identification number (SIREN), they appear and disappear from the database not only when they actually open or shut down their businesses, but also in case of restructuring or takeover as this induces a change in their identification number. Entry and exit rates are therefore likely to be overestimated. I try to limit this bias by excluding firms that disappear or appear with a number of employees higher than the 99.8 percentile among exiting and entering firms[17]. Furthermore, to avoid spurious entry or exit flows induced by missing values, firms that temporarily move below the threshold of one employee, or temporarily disappear from the database are excluded. The procedure used for excluding temporary exits is likely to overestimate the entry rate in the first years of the sample, and overestimate the exit rate in the last years of the sample. Since about 75% of the firms that temporarily disappear from the database are absent only one year, dropping the first and the last year of the sample considerably reduces the amount of spurious entry and exit flows. I therefore remove the first two years (entry cannot be identified in the first year) and the last year of the sample, and undertake the analysis over the period 1991-2006. I also remove outliers as the decomposition is quite sensitive to extreme values[18]. After this trimming procedure, the dataset is constituted by about 126 000 observations each year. Figure A.1 in appendix C, shows

[16]Note that entering flows are likely to be overestimated as firms appear in the database when they cross the sales threshold of 763 000 euros above which the *Bénéfice réel normal* regime is mandatory. However, these spurious entry flows are likely to be limited as the "normal" tax regime is widely chosen by firms which are below the threshold: in 2003, 46% of the sample was below the sales threshold. Furthermore the dataset covers a lot of small firms, in 2003 71% of the sample (restricted to firms with at least one employee) had less than 20 employees

[17]This correspond to firms appearing in or disappearing from the database with about 1000 employees. Despite this correction, the entry and exit rates are likely to be overestimated since the threshold is quite high in light of the median size of target firms. Using French data over the period 2000-2004, Bunel et al. (2009) find a median size between 30 and 87 employees, depending on the exact nature of the restructuring operation. Note however that the average entry and exit rates displayed in Table A.1 are in line with what has been found for the US manufacturing sector (Dunne et al., 1989).

[18]Firms whose TFP is multiplied or divided by 2.5 are excluded from the analysis.

that the aggregate value added is not affected much by this trimming procedure and that it compares quite well with the national accounts data. Appendix C also provides a more detailed description of the data and the sources.

4.2 Estimation method

To implement the decomposition, I need estimates of firm-level and aggregate factor elasticities, firm-level and sectoral distortions, and firm-level productivity. In the following, I present the issues that arise and the assumptions required to estimate these parameters.

4.2.1 Estimation of production functions

The firm-level productivity estimates are likely to suffer from the traditional bias linked to unobserved factor utilization[19]. I abstract from this well-known measurement issue and focus here on the specific issues that arise in the presence of heterogeneity in factor elasticities and unobserved allocation frictions. First, I show that the standard growth accounting approach gives a biased measure of aggregate factor elasticities when the underlying factor elasticities are heterogeneous. Second, I investigate the biases induced by unobserved micro frictions.

Heterogeneity bias

To isolate the effects of the heterogeneity bias from the biases generated by market imperfections, let us assume that resources are perfectly allocated across firms ($\tau_{it}^K = \tau_{it}^L = 0$ for all i), and that firms behave competitively ($\theta_s = 1$, for all $s = 1, ...S$). In that case, as shown in section 2.2.3, the aggregate capital and labor elasticity are both a combination of capital and labor elasticities. In particular, when the decreasing to scale and the demand parameters are identical across sectors ($\gamma_s = \gamma$ for all $s = 1, ...S$), the aggregate elasticities are given by:

$$\varepsilon_K(\widetilde{\alpha}, \widetilde{\beta}) = \frac{\alpha^Y(1-\alpha^L) - \beta^Y \alpha^L}{1-\alpha^L-\beta^K} \text{ and } \varepsilon_L(\widetilde{\alpha}, \widetilde{\beta}) = \frac{\beta^Y(1-\beta^K) - \alpha^Y \beta^K}{1-\alpha^L-\beta^K}, \tag{22}$$

with $\alpha^X = \sum_s \frac{X_s}{X}\alpha_s$ and $\beta^X = \sum_s \frac{X_s}{X}\beta_s$ for $X = PY, K, L$.

Standard growth accounting method yield $\widehat{\varepsilon_K} = rK/PY$ and $\widehat{\varepsilon_L} = wL/PY$. Using equation (1) and (2), we find:

[19] As emphasized in section 2.2.1, another bias comes from the absence of firm-level data which leads firm-level productivity estimates to combine both demand and technical efficiency components.

$$\widehat{\varepsilon_K} = \alpha^Y \text{ and } \widehat{\varepsilon_L} = \beta^Y,$$

which is different from the actual elasticity. Though there are no market imperfections, the Solow residual does not properly measure aggregate productivity changes when factor elasticities are heterogeneous. Aggregate elasticities are therefore computed using equations (22).

Unobserved micro distortions

There is no heterogeneity bias when using the Solow residual at the firm-level. However, because of the presence of allocation distortions, the growth accounting approach cannot be used at the firm-level for it is not possible to identify separately the factor elasticities from the firm-level distortions (equations 1 and 2).[20] I use the assumption that factor elasticities are identical within sectors, and estimate the firms' production functions using growth accounting at the sectoral level. To derive an estimate for $\beta_s\theta_s$, I use equation (2) aggregated over firms and over time :

$$\beta_s\theta_s \frac{1}{T}\sum_t \frac{1}{(1+\omega_{st}^L)} = \frac{1}{T}\sum_t \frac{w_t L_{st}}{P_{st}Y_{st}}$$

In order to identify the factor elasticity, I assume that the average labor sectoral distortions are null over the period considered.[21] This assumption may bias the estimates of the factor elasticities but has no impact on the contribution of firm-level distortions to aggregate productivity. As shown in the next subsection, what matters for aggregate productivity growth is the change in the relative level of distortions and not in their absolute level. The elasticity of demand is pinned down using estimates of the elasticity of substitution between goods. For simplicity, I assume that the elasticity is identical across sectors. I set $\theta = 0.8$, which corresponds to an elasticity of substitution between goods of 5, in line with Broda and Weinstein (2006) estimates.[22] Finally, capital elasticities are derived by assuming constant return to scale in each sector $\alpha_s = 1 - \beta_s$. Once the factor elasticities are known, firm-level productivities can be estimated from output and input levels using equation (5).

[20] Note that the presence of unobserved allocation distortion also rules out the use of standard semi-parametric methods such as Olley Pakes or Levinshon Petrin. These methods, which consists in using a proxy for productivity, can accommodate only a unique unobservable state variable. In the present framework, firms decisions depend on productivity, but also on capital and labor distortions, which raise to three the number of unobservable state variables.

[21] More specifically, it is the average of the inverse of sector specific distortions that is assumed to be zero. Actually assuming an average of zero leads to estimate $\beta_s\theta_s$ as $1/(\frac{1}{T}\sum_t \frac{P_{st}Y_{st}}{w_t L_{st}})$. The results are not affected to this choice

[22] For the 3-digit aggregation level, they report a mean of 6.8 over the period 1972-1988 and of 4 over the period 1990-2001.

4.2.2 Estimation of firm-level distortions

Firm-level distortions (τ_i^K, τ_i^L) are computed as the wedge between the marginal productivity of labor and capital and factor prices (equations (1) and (2)). Firm-level distortions are therefore sensitive to factor prices, which are known to be difficult to measure. Luckily, both changes in sectoral and aggregate TFP do not depend on the absolute level of distortions, but only on changes in relative distortions. Hence, the value chosen for factor prices has no impact on aggregate TFP growth and on its decomposition. To illustrate this point, suppose that we use biased measures of factor prices ($\hat{r}_t \neq r_t$ and $\hat{w}_t \neq w_t$) to measure firm-level distortions. This leads to the following biased measures:

$$\begin{aligned} 1+\widehat{\tau_{it}^K} &= (1+\tau_{it}^K)\frac{r_t}{\widehat{r_t}} \\ 1+\widehat{\tau_{it}^L} &= (1+\tau_{it}^L)\frac{w_t}{\widehat{w_t}} \end{aligned}$$

Using equation (12), it can be shown that the obtained measure of sectoral TFP is unbiased despite the measurement errors on firm-level distortions.

$$\begin{aligned} \widehat{TFP_{st}} &= \frac{(r_t/\widehat{r_t})^{\frac{-\alpha_s\theta_s}{1-\gamma\theta_s}}(w_t/\widehat{w_t})^{\frac{-\beta_s\theta_s}{1-\gamma\theta_s}}}{\left((r_t/\widehat{r_t})^{\frac{-(1-\beta_s\theta_s)}{1-\gamma\theta_s}}(w_t/\widehat{w_t})^{\frac{-\beta_s\theta_s}{1-\gamma\theta_s}}\right)^{\alpha_s\theta_s}\left((r/\widehat{r_t})^{\frac{-\alpha_s\theta_s}{1-\gamma\theta_s}}(w_t/\widehat{w_t})^{\frac{-(1-\alpha_s\theta_s)}{1-\gamma\theta_s}}\right)^{\beta_s\theta_s}} TFP_{st} \\ &= TFP_{st} \end{aligned}$$

At the aggregate level, we can show that this measurement error in factor prices has no impact on changes in aggregate productivity if the returns to scale and demand parameters are homogenous across sectors ($\gamma_s = \gamma$ and $\theta_s = \theta$).The measured sectoral distortions read:

$$\begin{aligned} \frac{d\widehat{\omega_t^K}}{1+\widehat{\omega_t^K}} &= \frac{d\omega_t^K}{1+\omega_t^K} + \frac{d(r_t/\widehat{r_t})}{r_t/\widehat{r_t}} \\ \frac{d\widehat{\omega_t^L}}{1+\widehat{\omega_t^L}} &= \frac{d\omega_t^L}{1+\omega_t^L} + \frac{d(w_t/\widehat{w_t})}{w_t/\widehat{w_t}} \end{aligned}$$

Using equation (19) and replacing with the value of ε_K and ε_L, it comes:

$$\begin{aligned} \Delta\widehat{\text{AE}}_{\text{between}} &= \Delta\overline{\text{AE}}_{\text{between}} + \frac{1}{1-\gamma\theta}\left(\varepsilon_K(1-\beta^K)+\varepsilon_L\alpha^L-\alpha^Y\right)\frac{d(r_t/\widehat{r_t})}{r_t/\widehat{r_t}} \\ &\quad + \frac{1}{1-\gamma\theta}\left(\varepsilon_K\beta^K+\varepsilon_K(1-\alpha^L)-\beta^Y\right)\frac{d(w_t/\widehat{w_t})}{w_t/\widehat{w_t}} \\ &= \Delta\overline{\text{AE}}_{\text{between}} \end{aligned}$$

Therefore, the values chosen for factor prices have no impact on the measure of changes in aggregate and sectoral productivity.

4.3 Empirical results

In line with the rest of the literature, I implement the decomposition on the manufacturing industry.[23] First, I investigate the role of allocative efficiency, firm-level efficiency and the extensive margin in explaining the dynamics of productivity growth within sectors in the manufacturing industry. Then, I present the contribution of each component to the industry-wide productivity growth, and show the role of between-sector allocative efficiency. Finally, I compare these aggregate results to the standard decomposition used in the literature.

4.3.1 Decomposition at the sectoral and aggregate levels

Table 1 gives the average productivity growth for each sector as well as its decomposition in a firm-level efficiency, allocative efficiency and an extensive margin component. This decomposition is computed using the exact decomposition described in Appendix B. Over the period 1991-2006, average productivity grew by 2.3% in those sectors. Firm-level efficiency and allocative efficiency contribute both significantly to average TFP growth. While the contribution of firm-level efficiency is positive in virtually all sectors (+3.9 percentage points on average), changes in allocative efficiency tend to decrease sectoral productivity (-1.3 p.p.). By contrast, the contribution of the extensive margin appears to be negligible for most sectors. Entry and exit flows have increased sectoral productivity by an average of 0.1 percentage points. As shown in Appendix D, net entry rates are positive and quite large (average of 2 p.p.) for most years, which tends to enhance aggregate productivity as the latter increase with the number of firms in the industry. This effect is however partially offset by the relatively small size of entrants, suggesting that entrants face either more distortion or have lower TFP than exiting firms. Figure A.2, A.3 A.4 in Appendix D depicts the TFP and distortion distributions of exiting and entering firms. These figures indicate that the TFP of entrants is larger than that of exiting firms, but that entering firms face higher distortions especially in the labor market. This result is line with empirical studies that point to the existence of important financial constraints that hampers the development of new firms. These frictions limits the impact of entering firms on aggregate productivity. [24]

Let us analyze the role of each of these components for the volatility of sectoral productivity growth. The variance of sectoral productivity growth can be computed as follows:

$$V(\Delta TFP) = Cov(\Delta TFP, \Delta TE) + Cov(\Delta TFP, \Delta AE) + Cov(\Delta TFP, \Delta EX)$$

[23] I dropped the electronic components sector as the estimation led to $\alpha<0$.

[24] Note also that we measure here only the impact of the extensive margin on the short run productivity. On the long run, the impact of entering firms could be higher if entering firms are characterized by a higher productivity growth rate.

Table 1: Sectoral productivity decomposition, 1991-2006 average (in %)

Sector	ΔTFP_s	ΔTE_s	ΔAE_s	ΔEX_s
Food products*	0.54	0.09	1.01	-0.32
Wearing apparel and leather products	3.53	6.88	-3.97	1.10
Printing and reproduction	2.03	3.14	-0.84	0.15
Pharmaceutical and perfumes products	3.52	2.91	2.26	-0.11
Furniture	1.95	4.75	-2.74	0.14
Motor vehicle	0.29	2.59	-1.14	0.09
Other transportation equipment	2.97	8.33	-4.65	-0.06
Mechanical equipments*	2.94	4.08	-1.46	0.46
Mineral products	1.74	3.70	-1.75	0.10
Textiles	1.80	5.18	-3.64	0.58
Wood and paper products	1.79	2.77	-1.04	0.14
Rubber and plastics products*	4.95	5.60	-0.76	0.39
Fabricated metal products*	0.26	1.61	-1.35	0.15
Electronic products	6.26	8.54	-2.20	0.20
Weighted average	2.29	3.90	-1.33	0.13

Note: This table presents average sectoral productivity growth over the period 1991-2006, as well as the average level of its components. The first column gives the average sectoral productivity growth ΔTFP_s, the second column gives the average change in firm-level efficiency ΔTE_s, the third column gives the average change in allocative efficiency ΔAE_s, and the last column gives the average change in sectoral productivity due to the extensive margin ΔEX_s. Sectoral productivity has been computed using equation (12) and the components ΔTE_s , ΔAE_s and ΔEX_s have been computed using the formulas given in Appendix B. Because of approximation errors, the sum of the components is not exactly equal to sectoral productivity growth. Each sector denoted with * represents more than 10% of the total manufacturing industry value added.

The contribution of firm-level efficiency is therefore measured as $Cov(\Delta TFP, \Delta TE)/V(\Delta\Delta TFP)$[25]. It gives the variation in ΔTFP which is due to variations in within-firm productivity ΔTE, both directly and through its correlations with ΔEX and ΔAE. Correspondingly, the contribution of within-sector allocative efficiency and the extensive margin are measured as $Cov(\Delta TFP, \Delta AE)/V(\Delta TFP)$ and $Cov(\Delta TFP, \Delta EX)/V(\Delta TFP)$. Table 2 reports the contribution of each component to the volatility of sectoral TFP.

Firm-level efficiency appears to be the main driver of sectoral productivity dynamics. In fact, changes in firm-level efficiency tend to exacerbate the volatility of sectoral productivity growth in many sectors. On the contrary, the contribution of changes in allocative efficiency to the volatility of productivity is negative, and allocative efficiency component thus dampens the movement in sectoral productivity growth. In absolute terms, the contribution of allocative efficiency is smaller than firm-level efficiency but still large. The extensive margin component plays a negligible role

[25]Note that this is the parameter obtained when regressing ΔTE on ΔTFP. As noted by Fujita and Ramey (IER, 2009), this measure is conceptually equivalent to the beta used in finance.

Table 2: Contribution to sectoral volatility, 1991-2006

Sector	Δ TFP stand. dev.	contribution to ΔTFP volatility		
		ΔTE_s	ΔAE_s	ΔEX_s
Food products*	2.36 %	1.26	-0.30	0.04
Wearing apparel and leather products	4.34 %	1.21	-0.33	0.12
Printing and reproduction	2.69 %	1.98	-1.05	0.07
Pharmaceutical and perfumes products	2.85 %	2.26	-1.20	-0.06
Furniture	2.66 %	0.91	0.01	0.08
Motor vehicle	9.35 %	1.54	-0.54	0.00
Other transportation equipment	5.98 %	1.47	-0.48	0.00
Mechanical equipments*	4.16 %	1.34	-0.39	0.05
Mineral products	3.74 %	1.36	-0.37	0.02
Textiles	3.96 %	1.00	-0.03	0.03
Wood and paper products	5.04 %	1.10	-0.09	-0.01
Rubber and plastics products*	3.72 %	1.38	-0.30	-0.07
Fabricated metal products*	6.39 %	1.15	-0.19	0.04
Electronic products	5.22 %	1.02	-0.09	0.07

Note: This table presents the contribution of each components to the volatility of sectoral productivity growth. The first column gives the standard deviation of sectoral productivity growth. The second column gives the contribution of firm-level efficiency (ΔTE_s) to the volatility of sectoral productivity growth. The third column gives the contribution of allocative efficiency (ΔAE_s), and the last column gives the contribution of the extensive margin (ΔEX_s). The contributions are computed as described in the text. The sum of the contribution of ΔTE_s, ΔAE_s and ΔEX_s equals 1. To avoid the discrepancies caused by approximation errors, the contributions to volatility have been computed with respect to the volatility of ($\Delta TE_s + \Delta AE_s + \Delta EX_s$). Each sector denoted with * represents more than 10% of the total manufacturing industry value added.

in the volatility of sectoral productivity growth in virtually all sectors.

Table 3 reports the correlation of aggregate productivity growth and each of its components with the sector's real value added growth. Sectoral TFP growth is highly correlated with changes in the sector's activity. The firm-level efficiency component also exhibits a positive, though smaller, correlation. This result is not surprising, since in this framework firm-level productivity depends both on technical and demand factors, a higher sectoral demand then leads to a higher within-firm productivity component.[26] In most sectors, the extensive margin component also is procyclical. This procyclicality contrasts with theories on the cleansing effect of recessions, according to which the higher exit rate of low productivity firms in recessions would enhance aggregate productivity. The results suggest that it is the reallocation of resources between continuing firms, and not between entering and exiting firms, that tends to raise aggregate productivity during recessions. For most sectors, allocative efficiency is indeed countercyclical.

[26] Recall that firm-level productivity reads $A_{it} = z_{it}^{\theta_s} b_{it}^{1-\theta}/P_{st}$, where z_{it} is technical efficiency and b_{it} is the firm-specific demand shock.

Table 3: Correlation with sectoral value added growth, 1991-2006

Sector	ΔTFP_s	ΔTE_s	ΔAE_s	ΔEX_s
Food products*	0.84	0.30	0.04	0.40
Wearing apparel and leather products	0.93	0.58	-0.10	0.81
Printing and reproduction	0.70	0.38	-0.18	0.46
Pharmaceutical and perfumes products	0.92	0.64	-0.46	-0.03
Furniture	0.84	0.47	0.11	0.31
Motor vehicle	0.84	0.54	-0.21	-0.17
Other transportation equipment	0.94	0.89	-0.66	0.09
Mechanical equipments*	0.91	0.80	-0.41	0.48
Mineral products	0.97	0.73	-0.31	0.28
Textiles	0.86	0.49	0.23	-0.21
Wood and paper products	0.96	0.86	-0.11	-0.01
Rubber and plastics products*	0.90	0.58	-0.08	-0.26
Fabricated metal products*	0.97	0.90	-0.34	0.35
Electronic products	0.89	0.82	-0.18	0.36

Note: This table presents the correlation of sectoral productivity growth and its components with real value added growth. The first column gives the correlation of sectoral productivity growth (ΔTFP_s) to real value added growth. The second column gives the correlation of firm-level efficiency (ΔTE_s). The third column gives the contribution of allocative efficiency (ΔAE_s), and the last column that of the extensive margin (ΔEX_s). Each sector denoted with * represents more than 10% of the total manufacturing industry value added.

Figure 1 and 2 present the results at the aggregate level computed using equations (18) to (21) (the numbers are provided in Appendix C). These figures, which display the value added growth of the entire manufacturing industry, complete the picture given by sectoral data. They give the aggregation of the sectoral firm-level efficiency, allocative efficiency and extensive margin components, as well as the contribution of inter-sectoral allocative efficiency. Like at the sectoral level, changes in within-firm productivity growth and in allocative efficiency are the main determinants of aggregate productivity growth. Figure 1 and 2 show that the variation of aggregate productivity due to the extensive margin is small compared to the other components. While the firm-level efficiency component is procyclical, the overall allocative efficiency component is countercyclical. Their correlation with the manufacturing industry's value added is respectively 0.64 and -0.25. Note that the between-sector allocative efficiency is more procyclical (-0.55) than within-sector allocative efficiency (-0.17). Figure A.6 and A.7 in Appendix D show that the overall picture is not modified when a higher elasticity of substitution between goods is chosen (equal to 6).

Figure 1: Firm-level productivity and allocative efficiency (manufacturing industry)

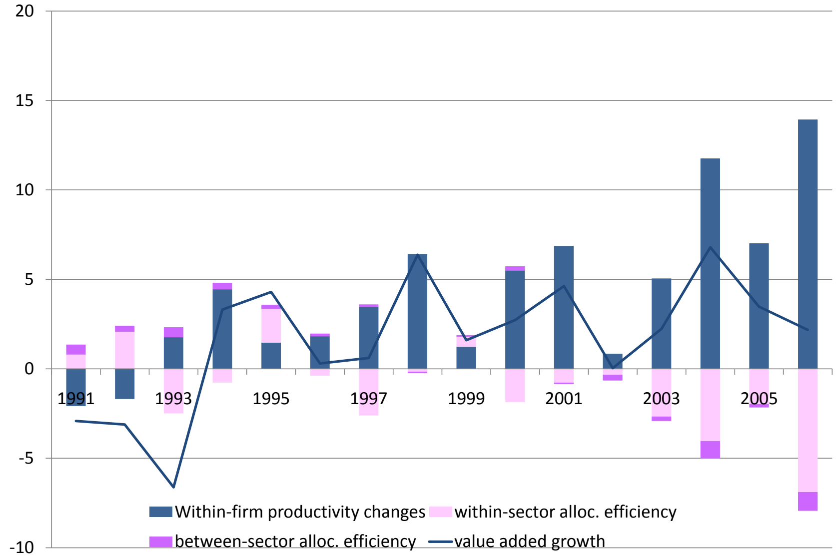
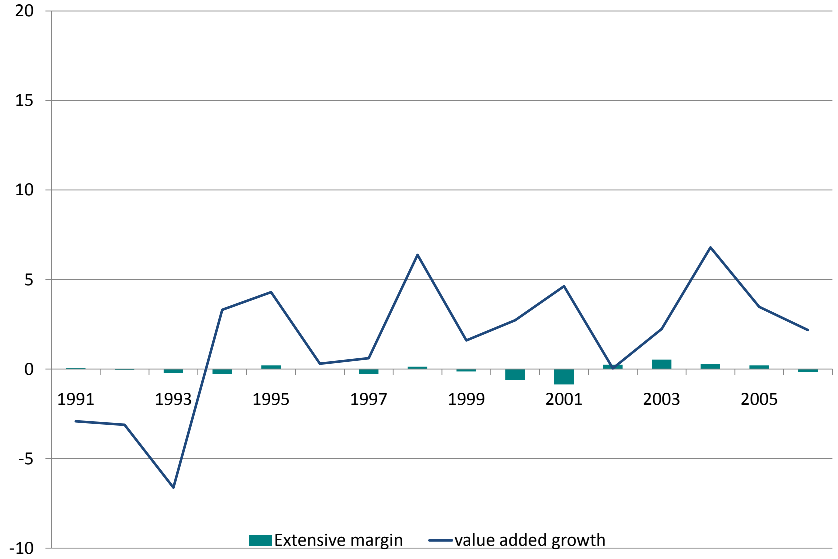


Figure 2: The extensive margin (manufacturing industry)



4.3.2 Comparison with the existing approach

I now show how these results differ from those obtained with the approach used in the applied literature. With this approach, aggregate productivity is not derived from an aggregate production function but is computed as the weighted average of firm-level TFP.

$$\ln TFP_t = \sum_i s_{it} \ln A_{it},$$

where $s_{it} = Y_{it}/Y_t$ is the output share of firm i.

Let us consider the decomposition proposed by Foster, Haltiwanger and Krizan (hereafter FHK, 2001):

$$\begin{aligned}\Delta \ln TFP_t \quad = \quad & \underbrace{\sum_{\text{stay}} s_{it-1} \Delta \ln A_{it}}_{\text{within}} + \underbrace{\sum_{\text{stay}} \Delta s_{it} (\ln A_{it-1} - \ln TFP_{t-1})}_{\text{reallocation}} \\ & + \underbrace{\sum_{\text{entry}} s_{it} (\ln A_{it} - \ln TFP_{t-1}) - \sum_{\text{exit}} s_{it-1} (\ln A_{it-1} - \ln TFP_{t-1})}_{\text{extensive margin}} \\ & + \underbrace{\sum_{\text{stay}} \Delta \ln A_{it} \Delta s_{it}}_{\text{cross term}}\end{aligned} \tag{23}$$

The first component captures changes in within-firm productivity holding fixed the market shares. The second component measures the impact of changes in market shares on aggregate productivity. The third component measures the contribution of entering and exiting firms, and the last component is a cross term that measures the correlation between changes in within-firm productivity and changes in market shares. This decomposition is completely different from the one derived from the aggregation of firm-level production functions (equations (13) to (15), and (18) to (21)). The main differences lie in the measure of the contribution of the extensive margin and that of cross-sectional efficiency. In the FHK decomposition, the contribution of entry and exit depends on the relative TFP of entering an exiting firms. Compared to equation (15), the FHK component does not take into account the impact of the distortions face by entering and exiting firms on aggregate productivity growth. Furthermore, this measure does capture the impact of entry and exit and the total number of firms in the industry. When firms use a decreasing returns to scale technology or when consumers have a love for variety, entry and exit flows affect aggregate productivity not only through a composition effect, but also trough their impact on the number of firms. The FHK measure of cross-sectional allocation is also completely different from my approach. Changes in cross-sectional allocation are measured by the correlation between changes in market shares and firm-level productivity. This measure captures changes in allocative efficiency when goods are homogeneous and produced using a technology with constant returns to scale and only one input. However, in the more general case where goods are heterogeneous or the marginal productivity of one input is decreasing, reallocating resources towards high TFP firms could decrease aggregate output. In this case, allocative efficiency depends on the dispersion of the value of marginal productivity and not on the correlation between market shares and firm-level productivity.

To illustrate these differences, I estimate the decomposition proposed by FHK and compare them with my results. Figure A.5 in Appendix C shows that the dynamics of *average* productivity given by the FHK measure is very close to that of *aggregate* productivity. The discrepancies are, however, substantial for the decomposition which are depicted in Figure 4.3.2 and 4.3.2 (numbers are provided in appendix D). The results are qualitatively similar to those obtained by FHK on the US manufacturing sector. The cross term is the main determinant of the mean and the volatility of aggregate productivity growth. On average, the net entry component contributes positively and reallocation negatively to aggregate productivity growth. When compared with Figure 1 and 2, the year-to-year changes in the contribution of the extensive margin, within-firm productivity and reallocations are considerably different. In particular, entry and exit play a relatively larger role for aggregate productivity growth in the FHK decomposition. Computed in absolute values, the contribution of entry exit is about 2 times lower than that of input reallocations in the FHK decomposition, and 7 times lower using my decomposition. Furthermore, the cyclical properties of the reallocation component are at odds with my findings. While my results indicate a countercyclical allocation efficiency, the reallocation component is here positively correlated with value added growth (correlation of 0.19). All in all, these results illustrate that the micro determinants of average and aggregate productivity are substantially different.

Figure 3: The FHK decomposition

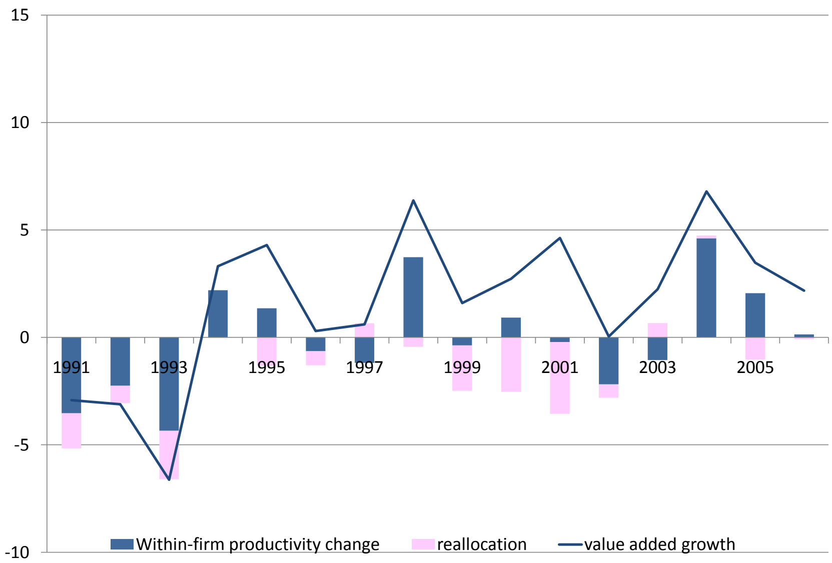
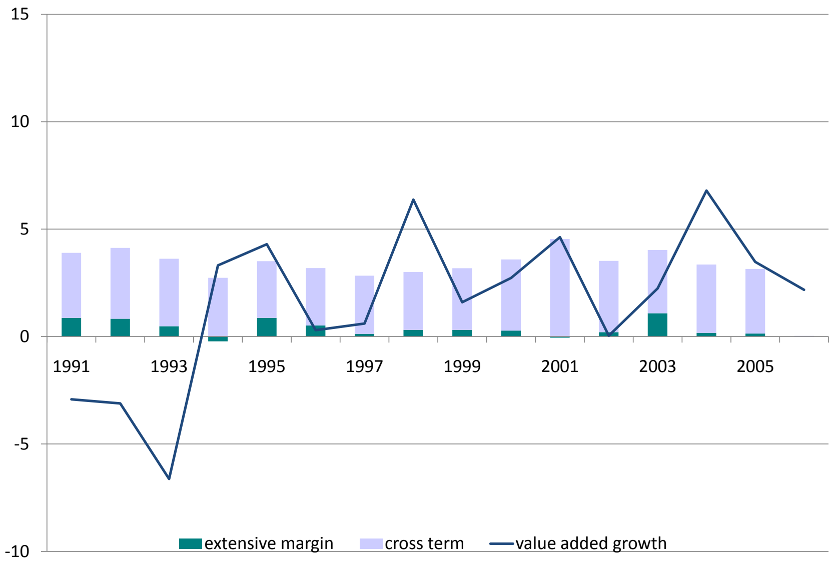


Figure 4: The FHK decomposition



5 Conclusion

This paper proposes a novel approach for analyzing the micro determinants of aggregate productivity dynamics. After deriving aggregate productivity from the aggregation of firm-level production functions, I show how to decompose aggregate productivity into changes in firm-level efficiency, changes in allocative efficiency and changes in entry and exit flows. Contrary to both existing empirical and theoretical works that emphasize the role of entry and exit for aggregate productivity growth, this paper shows that the extensive margin plays a negligible role in the dynamics of aggregate productivity growth and highlights the contribution of changes in the efficiency of resource allocation across firms. I show that changes in allocative efficiency are countercyclical and tend to stabilize the movements in aggregate productivity growth. Furthermore, I find no evidence in support of the cleansing effect of recession as the extensive margin component is not only small but also procyclical. However, it must be emphasized that these results give only the contemporaneous impact of entry and exit on aggregate productivity growth. The effects of entry and exit flows on long run productivity growth are not captured in this paper. New firms have a small contribution to aggregate productivity growth in the year they enter, but may have a large contribution on the long run if they have a higher productivity growth than exiting firms. Finally, the results calls for further research on the behavior of allocative efficiency over the business cycle. In fact, a deeper understanding of the dynamics of allocative efficiency would require a more precise modelling of frictions. Here, any distortion that generate a wedge between the firms' value of marginal productivity is captured in our measure of allocative efficiency, and we cannot therefore identify among the various sources of frictions those that are the more relevant.

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Appendix A: Deriving the sectoral aggregate production function

In this Appendix, we show how to derive the sectoral production functions. When factor and demand elasticities are identical across firms the allocation rules can be derive explicitly. Using equations (6) and (7), we can write:

$$
\begin{aligned}
K_{it} &= \left(\frac{A_i^{\frac{1}{1-\gamma\theta_s}} (1+\tau_i^K)^{-\frac{1-\beta_s\theta_s}{1-\gamma\theta_s}} (1+\tau_i^L)^{-\frac{\beta_s\theta_s}{1-\gamma\theta_s}}}{\sum A_i^{\frac{1}{1-\gamma\theta_s}} (1+\tau_i^K)^{-\frac{1-\beta_s\theta_s}{1-\gamma\theta_s}} (1+\tau_i^L)^{-\frac{\beta_s\theta_s}{1-\gamma\theta_s}}} \right) K_{st} \\
L_{it} &= \left(\frac{A_i^{\frac{1}{1-\gamma\theta_s}} (1+\tau_i^K)^{-\frac{\alpha_s\theta_s}{1-\gamma\theta_s}} (1+\tau_i^L)^{-\frac{1-\alpha_s\theta_s}{1-\gamma\theta_s}}}{\sum A_i^{\frac{1}{1-\gamma\theta_s}} (1+\tau_i^K)^{-\frac{\alpha_s\theta_s}{1-\gamma\theta_s}} (1+\tau_i^L)^{-\frac{1-\alpha_s\theta_s}{1-\gamma\theta_s}}} \right) L_{st}
\end{aligned}
$$

and the sectoral production function is then:

$$
F_s(K_{st}, L_{st}, \widetilde{A}_{st}, \widetilde{\tau}_{st}) = \sum_i A_{it} \left(\frac{g_s^K(A_{it}, \tau_{it})}{\sum_i g_s^K(A_{it}, \tau_{it}} K_{st} \right)^{\alpha_s\theta_s} \left(\frac{g_s^K(A_{it}, \tau_{it})}{\sum_i g_s^K(A_{it}, \tau_{it}} L_{st} \right)^{\beta_s\theta_s}
$$

which can be written

$$
F_s(K_{st}, L_{st}, \widetilde{A}_{st}, \widetilde{\tau}_{st}) = TFP_{st} K_{st}^{\alpha_s\theta_s} L_{st}^{\beta_s\theta_s}
$$

Hence, sectoral TFP is a function of both firm-level productivity $\widetilde{A}_{st}$ and distortions $\widetilde{\tau}_{st}$:

$$
TFP_{st} = \frac{\sum_{i=1}^{N_s} g_s^Y(A_{it}, \tau_{it})}{\left(\sum_{i=1}^{N_s} g_s^K(A_{it}, \tau_{it})\right)^{\alpha_s\theta_s} \left(\sum_{i=1}^{N_s} g_s^L(A_{it}, \tau_{it})\right)^{\beta_s\theta_s}}.
$$

where the g functions are given by:

$g_s^Y(A_{it}, \tau_{it}) = A_i^{\frac{1}{1-\gamma\theta_s}} (1+\tau_i^K)^{-\frac{\alpha_s\theta_s}{1-\gamma\theta}} (1+\tau_i^L)^{-\frac{\beta_s\theta_s}{1-\gamma\theta_s}}$

$g_s^K(A_{it}, \tau_{it}) = A_i^{\frac{1}{1-\gamma\theta_s}} (1+\tau_i^K)^{-\frac{1-\beta_s\theta_s}{1-\gamma\theta_s}} (1+\tau_i^L)^{-\frac{\beta_s\theta_s}{1-\gamma\theta_s}}$

$g_s^L(A_{it}, \tau_{it}) = A_i^{\frac{1}{1-\gamma\theta_s}} (1+\tau_i^K)^{-\frac{\alpha_s\theta_s}{1-\gamma\theta_s}} (1+\tau_i^L)^{-\frac{1-\alpha_s\theta_s}{1-\gamma\theta_s}}$

Appendix B: Decomposition of sectoral productivity: Exact and approximate formulas

This appendix presents the decomposition of sectoral TFP. First, I describe the method and show how changes in sectoral TFP can be approximated. Then, I give the formulas for the exact decomposition.

From equation (12), the change in sectoral aggregate productivity can be computed as:

$$\frac{\Delta TFP_{st}}{TFP_{st}} \simeq \ln\left(\frac{\sum g_s^Y(A_{it},\tau_{it})}{\sum g_s^Y(A_{it-1},\tau_{it-1})}\right) - \alpha_s\theta_s \ln\left(\frac{\sum g_s^K(A_{it},\tau_{it})}{\sum g_s^K(A_{it-1},\tau_{it-1})}\right) - \beta_s\theta_s \ln\left(\frac{\sum g_s^L(A_{it},\tau_{it})}{\sum g_s^L(A_{it-1},\tau_{it-1})}\right)$$

Sectoral TFP growth can then be decompose into changes in within-firm productivity ($\Delta \mathrm{TE}_s$), changes in allocative efficiency ($\Delta \mathrm{AE}_s$) and changes in the extensive margin ($\Delta \mathrm{EX}_s$), by decomposing each component as follows:

$$\ln \frac{\sum g_s^Y(A_{it},\tau_{it})}{\sum g_s^Y(A_{it-1},\tau_{it-1})} = \underbrace{\ln \frac{\sum_{i\in C_s} g_s^Y(A_{it},\tau_{it-1})}{\sum_{i\in C_s} g_s^Y(A_{it-1},\tau_{it-1})}}_{\Delta \mathrm{TE}_Y} + \underbrace{\ln \frac{\sum_{i\in C_s} g_s^Y(A_{it},\tau_{it})}{\sum_{i\in C_s} g_s^Y(A_{it},\tau_{it-1})}}_{\Delta \mathrm{AE}_Y} + \underbrace{\ln \frac{1-\frac{\sum_{i\in X_s} g_s^Y(A_{it-1},\tau_{it-1})}{\sum_{i=1}^{N_{t-1}} g_s^Y(A_{it-1},\tau_{it-1})}}{1-\frac{\sum_{i\in E_s} g_s^Y(A_{it},\tau_{it})}{\sum_{i=1}^{N_t} g_s^Y(A_{it},\tau_{it})}}}_{\Delta \mathrm{EX}_Y}$$

with C_s is the set of continuing firms ($I_{it-1} \geq 0$ and $I_{it} \geq 0$), E_s the set of new entrants ($I_{it-1} < 0$ and $I_{it} \geq 0$) and X_s the set of exiting firms ($I_{it-1} \geq 0$ and $I_{it} < 0$) in sector s. Aggregate productivity growth can then be expressed as:

$$\frac{\Delta TFP_{st}}{TFP_{st}} \simeq \Delta \mathrm{TE}_s + \Delta \mathrm{AE}_s + \Delta \mathrm{EX}_s,$$

where the within-firm productivity component is defined as $\Delta \mathrm{TE}_s = \Delta \mathrm{TE}_Y - \alpha_s\theta_s\Delta \mathrm{TE}_K - \beta_s\theta_s\Delta \mathrm{TE}_L$, and the allocative efficiency $\Delta \mathrm{AE}_s$ and the extensive margin $\Delta \mathrm{EX}_s$ components are symmetrically defined.

The decomposition between changes in within-firm productivity and changes in allocative efficiency is similar to the decompositions we can find in the index number literature. In the expression given above the effects of within-firm productivity changes are measured with a Laspeyres-like index, while those of allocative efficiency ($\Delta \mathrm{AE}_Y$) are measured with a Paasche-like index. Using the expressions for g_s^Y, g_s^K and g_s^L, together with the approximation $ln(1+x) \simeq x$ yield the expression given in equations (13) to (15).

For simplicity, the main text only provides approximations. The decomposition is computed from firm-level data using the exact formulas. The change in TFP can be broken down into three components.

$$\frac{TFP_{st}}{TFP_{st-1}} = ITE\ IAE\ IEX$$

The extensive margin component is measured as follows:

$$IEX = \frac{\frac{1-\sum_{X_s} \frac{P_{it-1}Y_{it-1}}{P_{st-1}Y_{t-1}}}{1-\sum_{E_s} \frac{P_{it}Y_{it}}{P_{st}Y_t}}}{\left(\frac{1-\sum_{X_s} \frac{K_{it-1}}{K_{t-1}}}{1-\sum_{E_s} \frac{K_{it}}{K_t}}\right)^{\alpha_s\theta_s} \left(\frac{1-\sum_{X_s} \frac{L_{it-1}}{L_{t-1}}}{1-\sum_{E_s} \frac{L_{it}}{L_t}}\right)^{\beta_s\theta_s}}$$

To avoid any asymmetry induced by the functional form of the indexes, I measure the firm-level and allocative efficiency component with a Fisher-like index, i.e. the geometric mean of the Laspeyres-like and Paasche-like indexes. The Fisher-like index for within-productivity changes is computed as follows:

$$ITE = ITE0^{0.5} ITE1^{0.5}$$

where

$$ITE0 = \frac{\sum \frac{A_{it}}{A_{it-1}}^{\frac{1}{1-\gamma\theta_s}} \frac{p_{it}Y_{it-1}}{\sum p_{it}Y_{it-1}}}{\left(\sum \frac{A_{it}}{A_{it-1}}^{\frac{1}{1-\gamma\theta_s}} \frac{K_{it-1}}{\sum K_{it-1}}\right)^{\alpha_s\theta_s} \left(\sum \frac{A_{it}}{A_{it-1}}^{\frac{1}{1-\gamma\theta_s}} \frac{L_{it-1}}{\sum L_{it-1}}\right)^{\beta_s\theta_s}}$$

$$ITE1 = \frac{\left(\sum \frac{A_{it-1}}{A_{it}}^{\frac{1}{1-\gamma\theta_s}} \frac{K_{it}}{\sum K_{it}}\right)^{\alpha_s\theta_s} \left(\sum \frac{A_{it-1}}{A_{it}}^{\frac{1}{1-\gamma\theta_s}} \frac{L_{it}}{\sum L_{it}}\right)^{\beta_s\theta_s}}{\sum \frac{A_{it-1}}{A_{it}}^{\frac{1}{1-\gamma\theta_s}} \frac{p_{it}Y_{it}}{\sum p_{it}Y_{it}}}$$

The change in allocative efficiency is similarly computed. Then, each component of aggregate productivity growth is computed as $\Delta TE = ITE - 1$, $\Delta AE = IAE - 1$ and $\Delta EX = IEX - 1$.

I now provide some indications for the decomposition of aggregate productivity growth across sectors. The heterogeneity in elasticities makes the calculation more tedious. For simplicity, I will describe the method in the case where there is only one input ($\beta = 0$). In this case $Y = \sum_s TFP_s K_s^{\alpha_s}$, with $K_s = K_s(P_s TFP_s, \omega_s, r)$

Taking the derivative with respect to TFP_s gives:

$$\frac{\partial F}{\partial TFP_s}\frac{TFP_s}{Y} = \sum_s \frac{Y_s}{Y}\frac{dTFP_s}{TFP_s} + \frac{\alpha_s}{1-\alpha_s}\frac{Y_s}{Y}\frac{dTFP_s}{TFP_s} + \alpha_s \frac{Y_s}{Y}\frac{\partial K_s}{\partial r}\frac{r}{K_s}\frac{dr}{r}$$

where dr/r is the change in interest rate induced by changes in firm-level efficiency holding fixed aggregate inputs and distortions. Using the implicit function theorem, this change can be computed as:

$$\begin{aligned} 0 &= -\sum_s \frac{1}{1-\alpha_s} K_s \frac{dr}{r} + \sum_s \frac{1}{1-\alpha_s} K_s \frac{dTFP_s}{TFP_s} \\ \frac{dr}{r} &= \frac{1}{\sum_s \frac{1}{1-\alpha_s} K_s} \sum_s \frac{1}{1-\alpha_s} K_s \frac{dTFP_s}{TFP_s} \end{aligned}$$

The derivative with respect to TFP_s can then be written:

$$\begin{aligned}
\frac{\partial F}{\partial TFP_s}\frac{TFP_s}{Y} &= \sum_s \frac{1}{1-\alpha_s}\frac{Y_s}{Y} - \left(\frac{\sum_s \frac{\alpha_s}{1-\alpha_s}\frac{Y_s}{Y}}{\sum_s \frac{1}{1-\alpha_s}\frac{K_s}{K}}\right)\frac{1}{1-\alpha_s}\frac{K_s}{K}\frac{dTFP_s}{TFP_s} \\
&= \sum_s \frac{1}{1-\alpha_s}\left(\frac{Y_s}{Y} - \varepsilon_K \frac{K_s}{K}\right)\frac{dTFP_s}{TFP_s}.
\end{aligned}$$

which gives the weights use in equations (18) to (20) with $\beta = \varepsilon_L = 0$. The changes in between-sector allocative efficiency are computed similarly.

Appendix C: Data

Data used are collected by the French tax administration and controlled for inconsistencies and combine with the responses to the annual business surveys in the INSEE unified system for business statistics (SUSE). I consider firms that declare under the "normal" regime (SUSE-BRN) from 1989 to 2007. This tax regime is mandatory for firms with sales above 763 000 euros (230 000 in the service sector), but is also widely chosen by firms which are below the threshold: in 2003 46% of the sample restricted to the manufacturing sector had sales below 763 000 euros. As described in section 4.2, this dataset is corrected for spurious entry and exit flows, as well as for outliers in terms of productivity growth.

Description of variables

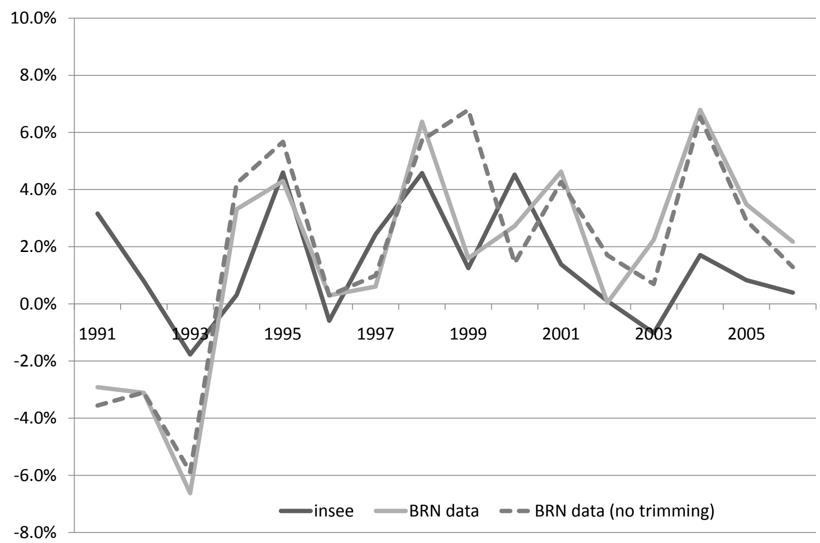
Output: gross value added less operating subsidies plus taxes, deflated by the NES36 sectoral price index published by the national accounts (INSEE, base 2000).

Labour: average number of employees over the year.

Capital: constructed using the perpetual inventory method. Real capital stock are computed from the previous period undepreciated stock and the period's real investment in tangible and intangible assets. I use a linear sector-specific depreciation rate, based on Sylvain (2003) estimates of equipments life span. The the NES36 sectoral investment deflator is derived from the national accounts (INSEE, base 1995 and 2000). As Dunne et al. (1989), the initial year capital stock is estimated by assuming that the firm's relative real capital stock is equal to its relative book value of assets.

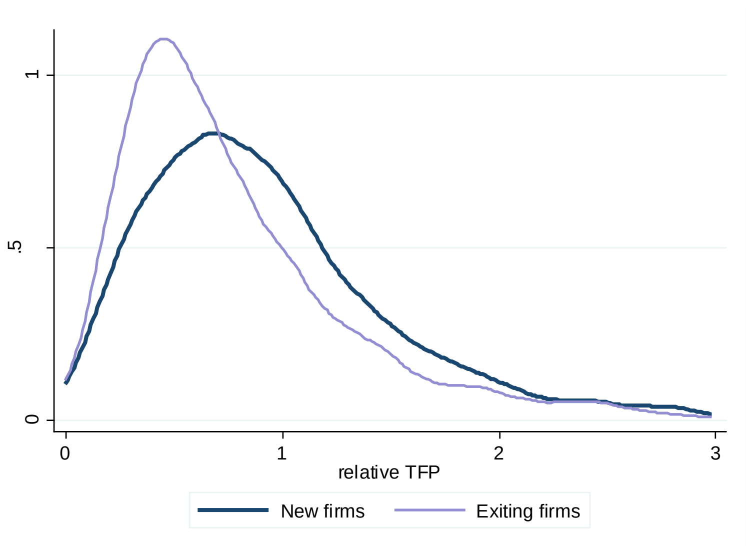
Sectoral labor share: computed at the N36 level from BRN data as 1- ratio of the sectoral gross operating surplus over value added.

Figure A.1: Growth of real value added in the manufacturing industry: national accounts vs SUSE-BRN



Appendix D: Additional tables and figures

Figure A.2: New firms vs exiting firms: relative TFP density



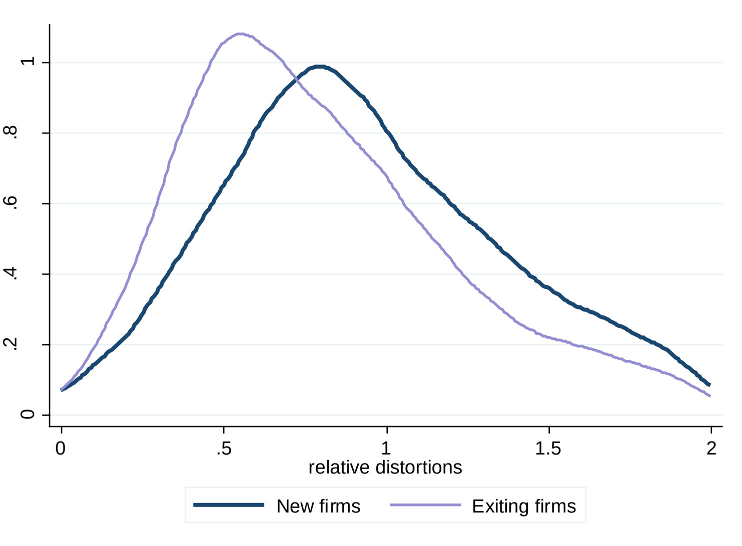
Note: computed in 2000 for the whole manufacturing industry, relative to the median TFP value

Table A.1: Entry and Exit rates in the manufacturing industry

	Entry rate	Exit rate	Entry rate (empl.weighted)	Exit rate (empl.weighted)
1991	0.125	0.158	0.044	0.057
1992	0.134	0.170	0.037	0.062
1993	0.212	0.267	0.052	0.085
1994	0.161	0.120	0.048	0.054
1995	0.106	0.114	0.041	0.046
1996	0.095	0.099	0.037	0.048
1997	0.115	0.088	0.041	0.040
1998	0.082	0.073	0.032	0.037
1999	0.080	0.076	0.031	0.039
2000	0.096	0.081	0.035	0.041
2001	0.099	0.099	0.042	0.043
2002	0.081	0.057	0.031	0.038
2003	0.077	0.069	0.030	0.036
2004	0.077	0.067	0.031	0.034
2005	0.080	0.063	0.027	0.034
2006	0.084	0.065	0.029	0.034
average	0.107	0.104	0.037	0.045

Note: This table presents entry and exit rate and their employment weighted counterparts. Note that the data are corrected for temporary exits and outliers as described in section 4.1

Figure A.3: New firms vs exiting firms: relative labor distortions density



Note: computed in 2000 for the whole manufacturing industry, relative to the median distortion value

Table A.2: Decomposition of the manufacturing industry TFP growth

	ΔTFP	$\Delta\overline{\text{TE}}$	$\Delta\overline{\text{AE}}_{\text{within}}$	$\Delta\overline{\text{EX}}$	$\Delta\overline{\text{AE}}_{\text{between}}$
1991	-0.92	-2.08	0.80	0.06	0.56
1992	0.32	-1.69	2.08	-0.06	0.33
1993	-1.92	1.77	-2.49	-0.23	0.56
1994	4.74	4.44	-0.77	-0.28	0.36
1995	4.01	1.46	1.88	0.21	0.24
1996	1.17	1.82	-0.39	0.00	0.15
1997	0.65	3.46	-2.61	-0.28	0.14
1998	6.24	6.42	-0.16	0.13	-0.08
1999	1.57	1.23	0.57	-0.13	0.09
2000	1.95	5.50	-1.86	-0.60	0.22
2001	2.97	6.87	-0.78	-0.86	-0.08
2002	0.13	0.85	-0.33	0.25	-0.32
2003	2.29	5.05	-2.67	0.53	-0.25
2004	7.28	11.76	-4.04	0.26	-0.98
2005	4.73	7.02	-1.96	0.21	-0.19
2006	2.90	13.94	-6.89	-0.17	-1.05

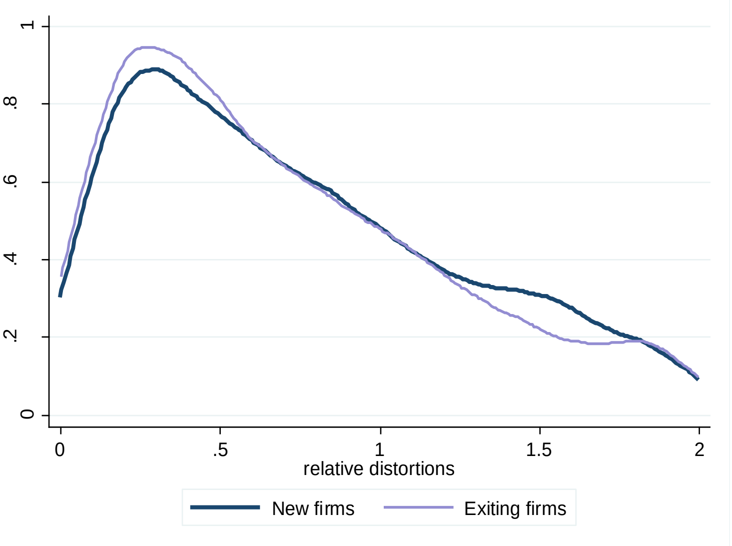
Note: This table presents the decomposition of productivity growth in the manufacturing industry (ΔTFP) into a aggregate firm-level efficiency component ($\Delta\overline{\text{TE}}$), an aggregate within-sector allocative efficiency component ($\Delta\overline{\text{AE}}_{\text{within}}$), an aggregate extensive margin component ($\Delta\overline{\text{EX}}$) and a between-sector allocative efficiency component ($\Delta\overline{\text{AE}}_{\text{between}}$). The aggregate productivity growth has been computed using equation (11) and its components have computed using equations (18) to (21). Because of approximation errors, the sum of the components does not exactly equal changes in aggregate TFP.

Table A.3: Decomposition of the manufacturing industry TFP growth (Foster Haltiwanger and Krizan's method)

	$\Delta \ln TFP$	within	reallocation	cross term	net entry
1991	-1.08	-3.54	-1.63	3.02	0.87
1992	0.99	-2.25	-0.80	3.30	0.83
1993	-3.01	-4.35	-2.25	3.14	0.48
1994	4.65	2.19	-0.03	2.73	-0.22
1995	3.60	1.35	-1.31	2.64	0.87
1996	1.90	-0.65	-0.65	2.67	0.52
1997	2.29	-1.19	0.66	2.70	0.13
1998	6.25	3.73	-0.44	2.69	0.31
1999	0.68	-0.38	-2.11	2.87	0.31
2000	1.93	0.92	-2.53	3.31	0.28
2001	0.92	-0.21	-3.35	4.54	-0.05
2002	0.69	-2.19	-0.62	3.32	0.20
2003	3.59	-1.06	0.67	2.94	1.08
2004	8.01	4.61	0.13	3.18	0.17
2005	4.27	2.06	-1.03	3.01	0.14
2006	3.96	0.14	-0.09	0.03	0.00

Note: This table presents the decomposition proposed by Foster et al. (2001). The components are computed as described in equation (23).

Figure A.4: New firms vs exiting firms: relative capital distortions density



Note: computed in 2000 for the whole manufacturing industry, relative to the median distortion value

Figure A.5: Aggregate productivity growth: comparison with FHK approach

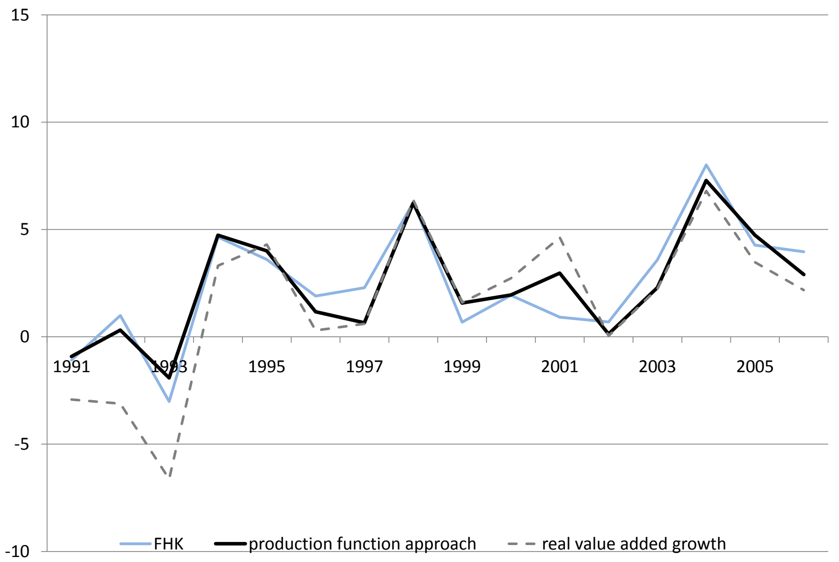


Figure A.6: Firm-level efficiency and allocative efficiency (elasticity of substitution equal to 6)

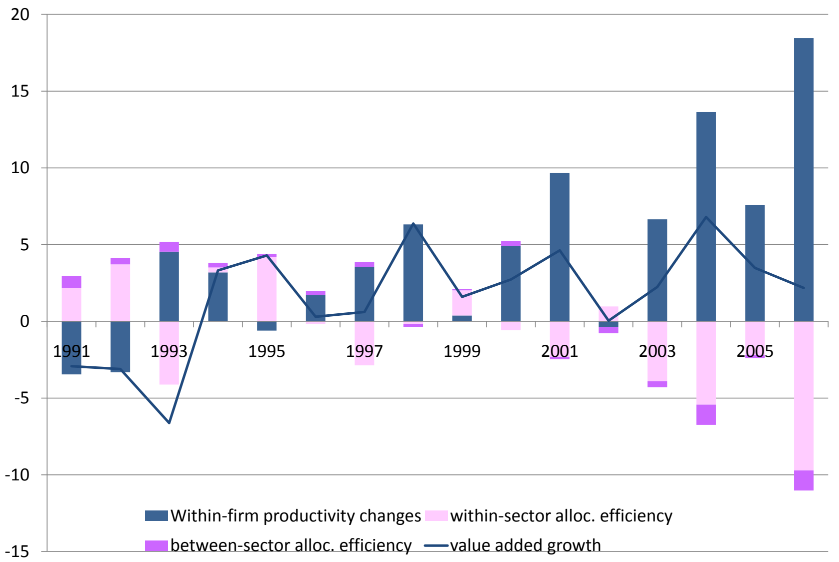


Figure A.7: The extensive margin (elasticity of substitution equal to 6)