Open-Market Operations, Distributions and Market Segmentation

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Abstract

How can a Central Bank use nominal bonds as a tool for implementing monetary policy? What are the effects of open-market operations on the distribution of assets and prices in the economy? Why do we observe segmentation in the asset market? In order to answer these questions, I construct a model of monetary economy, in which the central bank implements policies by changing the supply of nominal bond and money. By using competitive search in the decentralized market for goods, I build a tractable model that can produce non-degenerate distributions of asset holding and price dispersion among submarkets. The model endogenously generates segmentation in the asset market. For high enough bond supply the equilibrium shows segmentation in the asset market. In an equilibrium with segmented asset market open-market operations affect the decision of the households and therefore has real effects on the economy. Numerical exercise shows that the central bank can make real changes in the economy and change the overall welfare by changing the supply of nominal bonds.

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1 Introduction

How can a Central Bank use nominal bonds as a tool for implementing monetary policy? What are the effects of open market operations on the distribution of assets and prices in the economy? Why do we observe segmentation in the asset market? In order to answer these questions, I construct a model of monetary economy, in which the Central Bank implements policies by changing the supply of nominal bonds and money. Households and firms trade goods in markets with and without frictions. The frictional markets are characterized by competitive search[1], where households face a trade-off between higher matching probability and better terms of trade. Households with different idiosyncratic labor cost shocks, choose to hold different amounts of assets. Despite a nontrivial distribution of money and bond across different agents, competitive search in the frictional markets makes this model highly tractable. I show that a segmented asset market arises under a specific parameters set. High enough bond supply generates segmentation in the asset market. In an equilibrium with segmented asset market, open-market operations affects the decision of the households and therefor has real effects on the economy.

Many papers in the monetary economics and monetary policy literature assume that central banks pursue monetary policy by increasing (decreasing) money supply through lump sum transfers (taxes). In this literature policy makers are mostly interested in controlling the volume and the of money growth rate, and there is no role for open-market purchase or sale of interest bearing assets by the central bank. Here, monetary policy is much more richer and closer to the real world. The central bank can use money and bond supply as tools for monetary policy, and affect the asset portfolio of households through open-market operations.

Following Wallace (1981) a branch of literature uses a Modigliani-Miller[2] argument to show that the size and the composition of the central bank balance sheet and thus open-market operations do not make any real effects in the economy. Williamson (2011) and Mahmoudi (2011) assume government bonds can provide partial liquidity services. In these models open-market operations change the overall liquidity in the economy. Because of partial liquidity, government bonds are not perfect substitutes for money and a Modigliani-Miller argument fails to hold. In this paper the same logic holds. Agents can only trade with money, thus government bond is completely illiquid in the market for goods. Government bond is an imperfect substitute for money, thus open-market operations can have real effects on the economy.

[1]Directed search

[2]As in Modigliani and Miller (1958)

This paper is related to the literature on the distribution of money and assets in the economy. In a micro-founded model of monetary economy, after each round of trading there would be agents that have been matched and have succeeded in trade and agents that have not traded. This would generate an evolving distribution of asset holding among agents which is a state variable. Camera and Corbae (1999) generates distribution of asset holdings among agents and price dispersion in equilibrium in a framework based on Kiyotaki and Wright (1989)[3]. The evolving distribution of asset holding makes their model highly intractable for policy analysis. A huge part of the monetary literature avoids the distribution of asset holding by simplifying assumptions. Lucas (1990) and Shi (1995) assume a large household structure and with this insurance mechanism agents within a household share consumption and asset holdings after each round of trading. After each round of trading the sharing mechanism collapses the distribution of asset holding to a single point. Lagos and Wright (2005) assumes a quasi-linear preference structure for the agents along with one round of centralized trading. These assumptions make the distribution of money holding degenerate and the model highly tractable. Chiu and Molico (2011) relax the assumption of quasi-linear preferences in a standard Lagos and Wright (2005) model, and this makes their model intractable. By using competitive search in the decentralized market for goods Menzio et al. (2011) is able to make the distribution of money holding non-degenerate. Sun (2011) adds a centralized market to Menzio et al. (2011) and makes the model much more tractable.

My paper closely follows Sun (2011) in using competitive search in the decentralized market together with a centralized market to adjust asset balances. Competitive search in the goods market makes the model highly tractable[4]. Agents with a high income shock choose a submarket with high price and low matching probability. Due to the competitive nature of the frictional goods market, households' decision does not affect matching probabilities and terms of trade in the submarkets. Households take the specification of the submarkets as given and choose which submarket to attend. Households only need to know the bond price and wage, and these prices contain all of the information about the distributions in the economy. Hence, the equilibrium is *partially block recursive*[5]. This makes households'

[3]Zhu (2003), Zhu (2005) and Green and Zhou (1998) use similar approach and have distribution of asset holding and price dispersion as an equilibrium object

[4]Aside from tractability, comparing to random search, competitive search is closer to the real world. e.g. as Howitt (2005) puts it:"In contrast to what happens in search models, exchanges in actual market economies are organized by specialist traders, who mitigate search costs by providing facilities that are easy to locate. Thus, when people wish to buy shoes they go to a shoe store; when hungry they go to a grocer; when desiring to sell their labor services they go to firms known to offer employment. Few people would think of planning their economic lives on the basis of random encounters with nonspecialists..."

[5]Similar to Shi (2009) and Menzio and Shi (2010)

decision independent of the distribution of asset holding in the economy.

My paper is also closely related to the papers with uninsured idiosyncratic shocks (e.g. Aiyagari (1994)). While I focus more on the role of central bank policies, Aiyagari (1994) studies a model with income shocks in an environment where households can borrow and lend. Here, households are not allowed to borrow/lend to each other. They can only save through government issued assets (i.e. money and bond). This gives the government more degrees of freedom in implementing monetary policy.

A branch of literature uses asset market segmentations to explain persistence responses to monetary shocks observed in data. In this literature with segmented asset markets, only the fraction of agents who are active in the asset markets immediately receive the monetary shocks. Therefore, it would take time for the monetary shocks to affect other agents in the economy. This literature explains real effect of money injection and open-market operations with the generated segmentation in the asset market. This literature uses two ways to generate the segmented asset market: models which assume agents must pay a fixed cost to enter the asset market; and limited participation models that assume only certain agents attend the asset market. In Alvarez et al. (2000), agents must pay a fixed cost to transfer money between the asset market and the goods market. In a similar fashion, Khan and Thomas (2010) assumes agents pay fixed costs to transfer wealth between interest-bearing assets and money. Chiu (2007) assumes that agents pay a fixed cost to attend the asset market and they choose the timing of money transfers. In Alvarez et al. (2001) only a fixed fraction of agents attend the asset market. In a micro-founded monetary framework, Williamson (2008) links the asset market segmentation to the goods market segmentation. Grossman and Weiss (1983) assume that only a fixed fraction of the population can withdraw funds from banks each period.

In this paper, I generate segmentation in the asset market without assuming any rigidities and frictions. All of the agents can attend asset market every period, and there is no transaction cost or any other frictions that prohibit agents from trading in the asset market. Segmentation in the asset market is generated endogenously. Agents hold different amounts of assets, and some agents choose to hold no bond in their asset portfolio. Here, the real and welfare effects of open-market operations and money injections are not caused by the segmentation in the asset market. However, open-market operations has real effects when the markets are segmented. With segmented asset market, agents at the margin of trading assets may change their decision with a marginal change in the bond supply. The results are robust to exogenous segmentation in the asset market.

2 Model environment

Time is discrete, and each period consists of three subperiods. The economy is populated by measure 1 of ex ante identical households. Each household consists of a worker and a buyer. There is a general good that can be produced and consumed by all of the households. There are also at least three types of special goods. Households are specialized in the production and consumption of the special goods, and there is no double coincidence of wants. The utility function of the household is

$$U(y, q, l) = U(y) + u(q) - \theta l$$

where y is the consumption of the general good, q is the consumption of the special goods, and l is the labor supply in a period of time. The parameter $\theta \in [\underline{\theta}, \overline{\theta}]$ is the random disutility of labor. It is iid across households and time, and it is drawn from the probability distribution $F(\theta)$ at the beginning of each period. $U()$ and $u()$ have all the usual properties of the utility functions. Goods are divisible and perishable. There are two fiat objects in the economy: money and nominal bond. They are supplied by the central bank. Nominal bond is supplied in a centralized market after the utility shocks has been realized. Agents redeem each unit of bond from last period for 1 unit of money at the beginning of each period. Agents can trade with money, but they are prohibited from trading with bond. This assumption has been discussed in the literature. Shi (2008) shows that legal restriction on trade with bonds can improve welfare. Mahmoudi (2011) extends Shi (2008) to a more general framework and shows that prohibiting trade with bonds can improve welfare. Kocherlakota (2003) shows that in a centralized market, agents use illiquid bonds to smooth consumption.

Agents can trade general goods in a perfectly competitive market, called frictionless market. There are search frictions in the markets for special goods. Following Moen (1997) and Peters (1991), I assume a competitive search environment where agents choose to search in submarkets indexed by terms of trade and matching probability. Agents are randomly matched, and only matched agents can trade goods. There is a measure one of competitive firms, who hire workers from the households at the beginning of a period in a competitive labor market. Households own equal shares in these firms. Firms need labor for production of the general good and one type of special goods. These firms are destroyed at the end of each period and new firms are formed in the second subperiod of each period. I assume free entry for the firms, therefor the number of firms follows a zero profit condition.

In the frictional market there exists a continuum of submarkets that have specific characteristics in terms of trade and matching probabilities. Firms choose the measure of shops to operate in each submarket. There is free entry in these submarkets. The fixed cost of operating a shop in a submarket is $k > 0$ units of labor. In producing q units of special goods, firms incur $\psi(q)$ units of labor in production cost. Where $\psi()$ is twice continousely differentiable and $\psi' > 0$, $\psi'' > 0$ and $\psi(0) = 0$.

Trading in these submarkets is characterized by competitive search. Each submarket is a particular set of terms of trade (q: amount of special goods and x: money to be paid) and matching probabilities (b:matching probability for buyers and s: matching probability for sellers/shops). Firms and households take terms of trade and matching frictions as given and decide which submarket to attend. In each submarket buyers and shops randomly match according to the respective matching probabilities. Households and firms decide which submarket to enter, therefor matching probabilities are a function of terms of trade (x, q). Each submarket is indexed by the respective terms of trade. I assume that matching probability is characterized by a constant return to scale matching function ($s = \mu(b)$), which has the standard characteristics of a matching function.

At the beginning of the period, government prints money at rate γ, redeems last period one-period nominal bonds (A_{-1}) for 1 unit of money, and issues and sells bonds (A) for the current period at price s and balances budget by a lump sum tax/transfers (T). Lets define

$$\lambda = \frac{A_{-1}}{M}$$

as the ratio of bonds to money in the economy. Government imposes policy by either changing the inflation rate (γ) or changing the relative supply of bond (λ). I assume that the government runs a balanced budget. I study the steady state equilibrium, and I will use labor as the numeraire of the model. Figure 1 shows the timing of events.

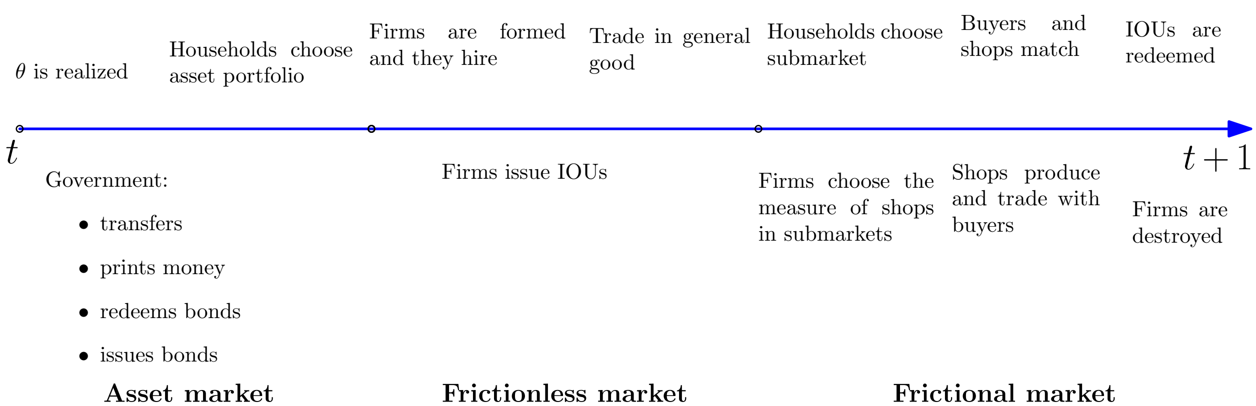

Figure 1: Timing

2.1 Firms' decision

Firms have access to a linear production technology. For each unit of labor input they produce a unit of output. Firms decide how much to produce in the frictionless market (Y) and the measure of shops in each submarket ($dN(x,q)$). They sell the produced general good at the given market price P. In each submarket the matching probability for each shop is $s(x,q)$. Shops sell the produced special goods to matched buyers at price x. In the production process, firms incur k units of labor in fixed cost, and $\psi(q)$ units of labor in variable costs. Firms maximize the following profit function

$$\begin{aligned} \pi = \quad & \max_{Y}\{pY - Y\} + \\ & \max_{dN(x,q)}\{\int\{\underbrace{s(x,q)x - [k + s(x,q)\psi(q)]}_{\text{Expected profit of a shop}}\}dN(x,q) \end{aligned} \tag{1}$$

If the expected profit in a submarket is strictly positive, firms will choose $dN(x,q) = \infty$. If the expected profit is strictly negative firms will choose $dN(x,q) = 0$. Therefor, the optimal $dN(x,q)$ satisfies the following inequalities with complementary slackness

$$s(x,q)[x - \psi(q)] \leq k \quad dN(x,q) \geq 0 \tag{2}$$

As is standard in the competitive search literature, I assume that the profit maximizing condition holds for the submarkets that are not visited by any buyers and firms. For all submarkets where $k < x - \psi(q)$ we have

$$s(x,q)[x - \psi(q)] = k$$

$$dN(x,q) = 0$$

For the submarkets where $k \geq x - \psi(q)$ we have $dN(x,q) = 0$, and I assume $s = 1$ and $b = 0$. I can write these two cases as

$$s(x,q) = \begin{cases} \frac{k}{x-\psi(q)} & k \leq x - \psi(q) \\ 1 & k > x - \psi(q) \end{cases} \tag{3}$$

2.2 Households' decision

2.2.1 Decision in the frictionless market

In the beginning of each period a centralized asset market opens. Households redeem each unit of their nominal bonds from previous period for 1 unit of money. Government prints and injects money at rate γ. Government supplies one period nominal bonds in a centralized market at the competitive price s. The asset market closes until the next period.

Let $W(m, a_{-1}, \theta)$ be the value function of a representative household at the beginning of a period. The representative household holds m units of money and a_{-1} units of nominal bonds from the last period. Given the prices (p, s) and transfers (T), the household decides on how much to consume in the frictionless market $(y \geq 0)$, labor supply $(l \geq 0)$, money balances (z) and bond holdings (a). Let $V(z, a)$ be the value of the representative household at the start of the frictional market. The households solves the following optimization problem subject to a standard budget constraint.

$$W(m, a_{-1}, \theta) = \max_{y,l,z,h,a} U(y) - \theta l + V(z, h, a)$$

$$st. \;\; py + z + h + sa \leq m + a_{-1} + l + T$$

Lets assume that $V(z, h, a)$ is differentiable and the choice of l is an interior solution (I will prove these later). As U is positively sloped the budget constraint is binding. I use the binding budget constraint to eliminate l from the optimization problem. Using the equilibrium condition $p = 1$, the value function of the representative household can be written as

$$W(m, a_{-1}, \theta) = \theta(m + T + a_{-1}) + \max_{y \geq 0} \{U(y) - \theta py\} + \max_{z,a,h} \{-\theta(z + sa + h) + V(z, h, a)\}$$

The optimal choices of y must satisfy

$$U'(y) = \theta \tag{4}$$

Similarly z, h and a satisfy

$$V_z(z, h, a) \begin{cases} \leq \theta & z \geq 0 \\ \geq \theta & z \leq \overline{m} - sa - h \end{cases} \tag{5}$$

$$V_h(z, h, a) \begin{cases} \leq \theta & h \geq 0 \\ \geq \theta & h \leq \overline{m} - sa - z \end{cases} \tag{6}$$

$$V_a(z,h,a)\begin{cases} \leq \theta s & a \geq 0 \\ \geq \theta s & sa \leq \overline{m} - z - h \end{cases} \tag{7}$$

Where the inequalities hold with complimentary slackness. Clearly households money balance (m), and bond holding (a_{-1}) does not affect the choices of y, z, h and a. Using the optimization problem of the household, I can write the value function as a linear function of m and a_{-1}

$$W(m, a_{-1}, \theta) = W(0, 0, \theta) + \theta m + \theta a_{-1} \tag{8}$$

Where

$$W(0, 0, \theta) = U(y(\theta)) - \theta y(\theta) + V(z(\theta), h(\theta), a(\theta)) - \theta(z(\theta) + h(\theta) + sa(\theta)) \tag{9}$$

It is clear that the value function is continuous and differentiable. The following lemma summarizes these findings.

Lemma 1. *The value function $W(m, a_{-1}, \theta)$ is continuous and differentiable in (m, a_{-1}, θ). It is also affine in m and a_{-1}.*

2.2.2 Decision in the frictional market

The representative household's decision in the frictional market is similar to Sun (2011). The representative household chooses which submarket to attend. As I can index the submarkets by the respective terms of trade, the household chooses x, and q to maximize expected value of attending the respective submarket. In a submarket the household matches with probability $b(x, q)$, and trades according to the stated terms of trade. In this match the representative household spends x, and consumes q. With probability $1 - b(x, q)$ there is no match and the representative household exits the frictional market with the starting portfolio of assets. I assume $b(x, q)$ is nonincreasing. The representative household solves the following optimization problem

$$\max_{x \leq z, q} \left\{ b(x,q) \left[u(q) + \beta E[W(\frac{z - x + h}{\gamma}, a_{-1}, \theta)] \right] + [1 - b(x,q)]\beta E \left[W(\frac{z+h}{\gamma}, a_{-1}, \theta) \right] \right\} \tag{10}$$

Using the linearity of $W(.)$ (8) and condition 1, I can eliminate q and the problem becomes

$$\max_{x \leq z, b} \left\{ b \left[u(\psi^{-1}(x - \frac{k}{\mu(b)})) - \beta E(\theta)\frac{x}{\gamma} \right] + \beta E \left[W(\frac{z+h}{\gamma}, a_{-1}, \theta) \right] \right\} \tag{11}$$

The optimal choices satisfy the following first-order conditions

$$\frac{u'\left(\psi^{-1}(x-\frac{k}{\mu(b)})\right)}{\psi'\left(\psi^{-1}(x-\frac{k}{\mu(b)})\right)}-\frac{\beta E(\theta)}{\gamma}\geq 0,\quad x\leq z \tag{12}$$

$$u\left(\psi^{-1}(x-\frac{k}{\mu(b)})\right)-\frac{\beta E(\theta)x}{\gamma}+\frac{u'\left(\psi^{-1}(x-\frac{k}{\mu(b)})\right)}{\psi'\left(\psi^{-1}(x-\frac{k}{\mu(b)})\right)}\frac{kb\mu'(b)}{[\mu(b)]^2}\leq 0,\quad b\geq 0 \tag{13}$$

where the two sets of inequality hold with complementary slackness. Note that $b=1$ cannot be an equilibrium outcome[6]. For $b(z)=0$ I assume $x(z)=z$.

Define $\phi(q)=\frac{u'(q)}{\psi'(q)}$. As is shown in Sun (2011), without loss of generality, I can focus on the case $x(z)=z$. If the following condition holds[7]

$$u\left(\phi^{-1}\left[\frac{\beta E(\theta)}{\gamma}\right]\right)-\frac{\beta E(\theta)}{\gamma}\left(\psi\left(\phi^{-1}\left[\frac{\beta E(\theta)}{\gamma}\right]\right)+k\right)>0 \tag{14}$$

Then the household's problem becomes

$$B(z)+\beta E\left[W(\frac{z+h}{\gamma},a_{-1},\theta)\right] \tag{15}$$

where

$$B(z)=\max_{b\in[0,1]} b\left[u(\psi^{-1}(z-\frac{k}{\mu(b)}))-\beta\frac{z}{\gamma}E(\theta)\right] \tag{16}$$

The value function $B(z)$ may not be concave in z. Equation 16 is the product of the choice variable b and a function of b. This product may not be concave. Following Menzio et al. (2011) and Sun (2011), I introduce lotteries to make the households' value function concave. A lottery is a choice of probabilities (π_1,π_2) and respective payments (L_1,L_2) that

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[6] $b=1$ implies $s=0$, $dN(z,q)=\infty$, and positive profits for the firms. This violates free entry.

[7] Lets assume for $b(z)>$, $x(z)<z$. Then 11 is independent of z. 13 holds with equality and can be written as:

$$q^*=\phi^{-1}[\frac{\beta E(\theta)}{\gamma}]$$

Given q^*, 13 can be written as:

$$u(q^*)-\frac{\beta E(\theta)}{\gamma}\left[\psi(q^*)+\frac{k}{\mu b^*}\right]+\left[\frac{u'(q^*)}{\psi'(q^*)}\right]\frac{kb^*\mu'(b^*)}{[\mu(b^*)]^2}=0$$

The left-hand side of the above equation is strictly increasing in b^*, and b^* exists and is unique if $E(\theta)$ satisfies:

$$u(q^*)-\beta E(\theta)[\psi(q^*)+k]>0$$

For all $z<x^*=\psi(q^*)+\frac{k}{\mu(b^*)}$, $x(z)=z$. For $z\geq x^*$, $x(z)=x^*$.

solves the following problem

$$\widetilde{V}(z) = \max_{L_1, L_2, \pi_1, \pi_2} [\pi_1 B(L_1) + \pi_2 B(L_2)] \tag{17}$$

Subject to

$$\pi_1 L_1 + \pi_2 L_2 = z; \quad L_2 \geq L_1 \geq 0$$

$$\pi_1 + \pi_2 = 1; \quad \pi_i \in [0, 1]$$

Note that the agent's policy functions for the lottery choices are: $L_{i\in\{1,2\}}(z)$ and $\pi_{i\in\{1,2\}}(z)$.

2.3 Properties of value and policy functions

Here I characterize policy functions and value functions. As shown in the previous section, the choice of bond holdings and bond prices does not directly affect households' decision in the frictional market. Therefor, the properties of value functions and policy functions are the same as in Sun (2011)

Lemma 2. *The following statements about the value functions and policy functions are true*

1. *The value function $B(z)$ is continuous and increasing in $z \in [0, \widehat{z}]$*
2. *The value function $\widetilde{V}(z)$ is continuous, differentiable, increasing and concave in $z \in [0, \widehat{z}]$.*
3. *For z such that $b(z) = 0$, the value function $B(z) = 0$ and the choice of q is irrelevant.*
4. *If and only if there exists a $q > 0$ that satisfies*

$$u(q) - \frac{\beta E(\theta)}{\gamma}[\psi(q) + k]$$

   There exists a $z > 0$ such that $b(z) > 0$

5. *For z such that $b(z) > 0$, the value function $B(z)$ is differentiable, $B(z) > 0$ and $B'(z) > 0$.*
6. *$b(z)$ and $q(z)$ are unique and strictly increasing in z.*
7. *$b(z)$ solves*

$$\max_{b\in[0,1]} \left\{ u\left(q(z)\right) - \frac{\beta E(\theta) z}{\gamma} + \frac{u'\left(q(z)\right)}{\psi'\left(q(z)\right)} \frac{kb\mu'(b)}{[\mu(b)]^2} \right\} \tag{18}$$

where:

$$q(z) = \psi^{-1}\left(z - \frac{k}{\mu(b(z))}\right) \tag{19}$$

8. *$b(z)$ strictly decreases with $E(\theta)$, and $q(z)$ strictly increases in $E(\theta)$*

9. *There exists $z_1 > k$ such that $b(z) = 0$ for all $z \in [0, z_1]$ and $b(z) > 0$ for all $z \in (z_1, \widehat{z}]$*

10. *There exists $z_0 > z_1$ such that a household with $z < z_0$ will play the lottery with the prize z_0.*

Since the choice of bond holdings and bond prices does not directly affect households' decision in the frictional market the proof of 2 is exactly similar to Sun (2011). Lemma 2 summarizes the characteristics of the value functions and policy functions. According to part 6 households with higher money balances choose to trade in submarkets with higher matching probabilities and higher terms of trade.

Equations 8, 10, 16 and 17 give

$$\begin{aligned} V(z,h,a) \quad &= \widetilde{V}(z) + \beta E\left[W(\frac{z+h}{\gamma}, a, \theta)\right] \\ &= \widetilde{V}(z) + \beta E\left[W(0,0,\theta)\right] + \frac{\beta E(\theta) z}{\gamma} + \frac{\beta E(\theta) h}{\gamma} + \beta E(\theta) a \end{aligned} \tag{20}$$

Equation 20 shows that $V(z, h, a)$ is linear in a and h

$$V_a(z, h, a) = \beta E(\theta) \tag{21}$$

$$V_h(z, h, a) = \frac{\beta E(\theta)}{\gamma} \tag{22}$$

Using conditions 7, 6, 21 and 22 , I can write the household's choice of bond holding and precautionary saving in money as follows

$$\begin{cases} a(\theta) \geq 0 & \theta \geq \frac{\beta E(\theta)}{s} \\ a(\theta) \leq \overline{m} - z(\theta) - h(\theta) & \theta \leq \frac{\beta E(\theta)}{s} \end{cases} \tag{23}$$

$$\begin{cases} h(\theta) \geq 0 & \theta \geq \frac{\beta E(\theta)}{\gamma} \\ h(\theta) \leq \overline{m} - z(\theta) - a(\theta) & \theta \leq \frac{\beta E(\theta)}{\gamma} \end{cases} \tag{24}$$

where the inequalities hold with complementary slackness. Using lemma 2, equations 21 and 22 and policy functions 23 and 24, I can conclude the following lemma

Lemma 3. *The value function V is continuous and differentiable in (z, h, a). $V(z, h, a)$ is increasing and concave in $z \in [0, \widehat{z}]$. $V(z, h, a) \geq \beta E[W(0, 0, \theta)] > 0$ for all z.*

Lemma 4 shows the properties of the policy functions of the households.

Lemma 4. *$a(\theta)$, $h(\theta)$, $z(\theta)$ and $l(m, a_{-1}, \theta)$ follow the following rules:*

Case I: $s < \gamma$

$$\begin{cases} \theta < \frac{\beta E(\theta)}{s} & \begin{cases} h(\theta) = 0 \\ a(\theta) = \overline{m} - z(\theta) \\ V_z = \widetilde{V}_z(z) + \beta E(\theta)(\frac{1}{\gamma} - 1) \\ l(m, a_{-1}, \theta) = py(\theta) + z(\theta)(1 - s) + s\overline{m} - a_{-1} - T \end{cases} \\ \theta \geq \frac{\beta E(\theta)}{s} & \begin{cases} h(\theta) = 0 \\ a(\theta) = 0 \\ V_z = \widetilde{V}_z(z) + \frac{\beta E(\theta)}{\gamma} \\ l(m, a_{-1}, \theta) = py(\theta) + z(\theta) - m - a_{-1} - T \end{cases} \end{cases} \tag{25}$$

Case II: $s \geq \gamma$

$$\begin{cases} \theta < \frac{\beta E(\theta)}{\gamma} & \begin{cases} h(\theta) = \overline{m} - z(\theta) \\ a(\theta) = 0 \\ V_z = \widetilde{V}_z(z) \\ l(m, a_{-1}, \theta) = py(\theta) + \overline{m} - a_{-1} - T \end{cases} \\ \theta \geq \frac{\beta E(\theta)}{\gamma} & \begin{cases} h(\theta) = 0 \\ a(\theta) = 0 \\ V_z = \widetilde{V}_z(z) + \frac{\beta E(\theta)}{\gamma} \\ l(m, a_{-1}, \theta) = py(\theta) - m - a_{-1} - T \end{cases} \end{cases} \tag{26}$$

Lemma 4 and equation 5 fully characterize the properties of the policy functions in two cases. When real interest rate is positive (25), and when it is negative (26). In an equilibrium with negative real interest rate households choose to hold all of their portfolio in terms of money. Higher amount of portfolio from previous period (m, a_{-1}) reduces $l(m, a_{-1}, \theta)$. In the next section I show that we cannot have negative real interest rate in the stationary equilibrium.

3 Stationary Equilibrium

Here I characterize the stationary equilibrium.

Definition 1. *A stationary equilibrium is the set of households' value functions* $(W, B, V, \widetilde{V})$*; household choices* $(y, l, z, a, h, q, b, L_1, L_2, \pi_1, \pi_2)$*; firm choices* $(Y, dN(q, b))$*; prices* (p, s, w)*; which satisfy the following conditions:*

1. *Given the prices* (p, s, w)*, realization of shocks* (θ)*, asset balances and terms of trade in all submarkets* (q, x)*, household choices solve 25 and 26*
2. *Given prices and the terms of trade in all submarkets, firms maximize profit (1)*
3. *Free entry condition (3)*
4. *Stationarity*
5. *Symmetry*
6. *Bond market clears (27), and labor market clears (28)*

In the bond market the total amount of bonds supplied equals the sum of demanded bonds by households of different type. Thus, the market clearing for bonds gives

$$\frac{A}{wM} = \int \int \int_{\underline{\theta}}^{\overline{\theta}} a(\theta) dF(\theta) dG(m) dH(a_{-1}) \tag{27}$$

Lemma 5. *No positive bond supply* $(\lambda > 0)$ *can support an equilibrium with negative real interest rate* $(s > \gamma)$*. Household's choose to hold bonds as precautionary saving and they only choose money for transaction purposes:*

- $h(\theta) = 0$
- $z(\theta) \geq 0$

From bond market clearing condition (27) and condition 26, its straightforward to show that positive amounts of bond supply would not clear the market when $s > \gamma$.

From lemma 5 and equations 25, 17 and 5 I can show that the general shape for $z(\theta)$ and $a(\theta)$ is similar to figure 2. There are two cases for the equilibrium. When $\underline{\theta} < \frac{\beta E(\theta)}{s} < \overline{\theta}$, households with low enough θ choose to hold positive amount of bonds. Changes in bond supply (λ) would only change the threshold $(\frac{\beta E(\theta)}{s})$. In the case where $\overline{\theta} < \frac{\beta E(\theta)}{s}$ all of the households hold a portfolio of bonds and money[8]. Figure 2 shows that an equilibrium with segmented asset market arise when $\underline{\theta} < \frac{\beta E(\theta)}{s} < \overline{\theta}$. In an equilibrium with segmented asset market open-market operations affects the decision of the households and therefor has

[8]Note that with positive bond supply, we cannot have the case in which $\frac{\beta E(\theta)}{s} < \underline{\theta}$

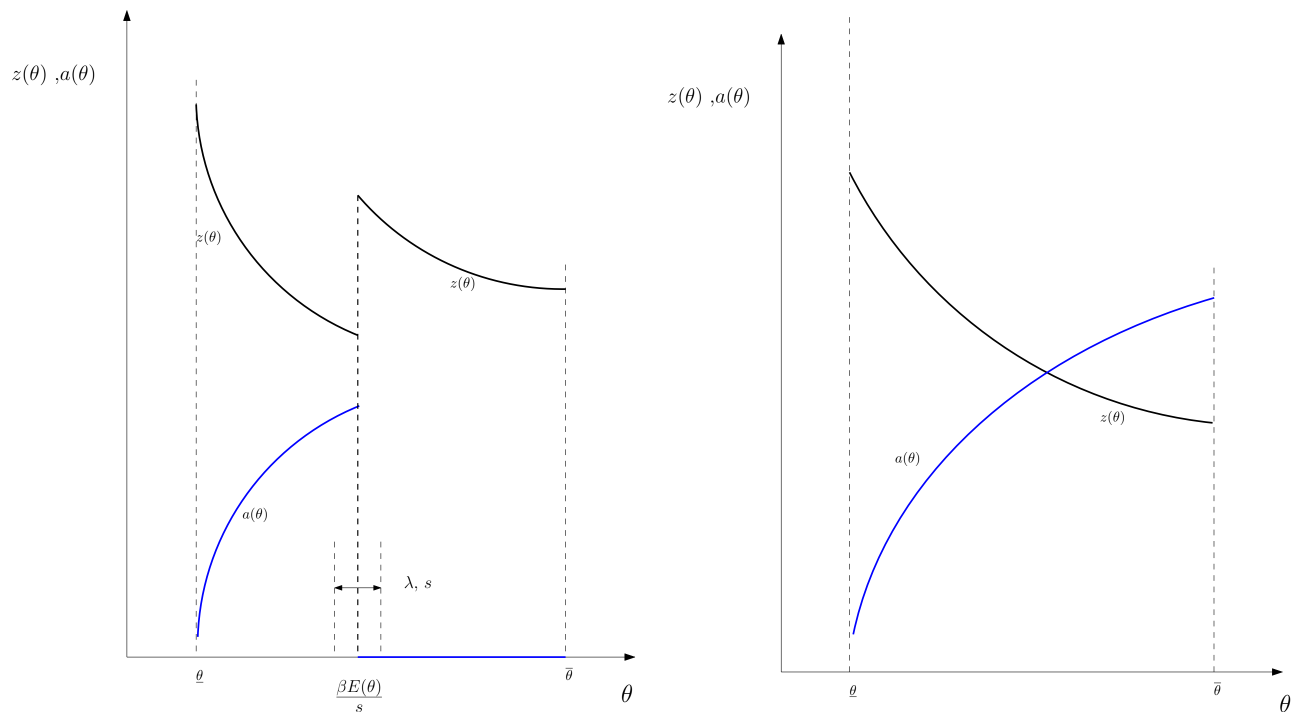

Figure 2: Policy functions for bond and money holding for $\underline{\theta} < \frac{\beta E(\theta)}{s} < \overline{\theta}$ and $\overline{\theta} < \frac{\beta E(\theta)}{s}$

real effects on the economy. This property of the equilibrium is completely endogenous. In section 5 I will impose exogenously segmented asset market, and show that most of the results hold under this assumption.

As shown in the appendix, the labor market clearing condition is

$$\begin{aligned}
&\frac{1}{w^*\gamma}[\gamma - 1 - \lambda + s\lambda] = \\
&\quad (2-s^*)\int_{\underline{\theta}}^{\overline{\theta}} a(\theta)dF(\theta) + (1-\frac{1}{\gamma})\int_{\underline{\theta}}^{\overline{\theta}} \pi_1(z(\theta))(1-b(L_1(z(\theta))))L_1(z(\theta))dF(\theta) \\
&\quad +(1-\frac{1}{\gamma})\int_{\underline{\theta}}^{\overline{\theta}} \pi_2(z(\theta))(1-b(L_2(z(\theta))))L_2(z(\theta))dF(\theta) - \frac{1}{\gamma}\int_{\underline{\theta}}^{\overline{\theta}} h(\theta)dF(\theta) \qquad (28)
\end{aligned}$$

Equations 27 and 28 show that the equilibrium is *partially block recursive*. Households do not need to know the distribution of the asset holding for their decision problems and prices (s,w) contain all the information they need about the distributions and the aggregate economy.

Using lemmas 4 and 5 , and equations 27 and 28, I can summarize the market clearing conditions to a single equation that could be solved for bond price s

$$\frac{\gamma - 1 - \lambda + s(\lambda + \gamma\lambda) - 2\gamma\lambda}{(1-\gamma)\lambda} \int_{\underline{\theta}}^{\frac{\beta E(\theta)}{s}} (\overline{m} - z(\theta))dF(\theta) =$$
$$\int_{\underline{\theta}}^{\overline{\theta}} \pi_1(z(\theta))(1 - b(L_1(z(\theta))))L_1(z(\theta))dF(\theta)$$
$$+ \int_{\underline{\theta}}^{\overline{\theta}} \pi_2(z(\theta))(1 - b(L_2(z(\theta))))L_2(z(\theta))dF(\theta) \tag{29}$$

Note that equation 29 cannot solely be used for numerical computations, and we need to compute wage (28) and check for positive wages. From equations 25, 27 and 29 I can characterize the set of prices in equilibrium. Let price be in the range: $\overline{s} = \frac{\beta E(\theta)}{\underline{\theta}} \leq s$. The left hand side of 29 is 0 while the right hand side is a positive number. In this case there is no equilibrium. I have shown that $s < \gamma$ in equilibrium. Therefor, the market clears at a price in the range $s < \min\{\gamma, \overline{s}\}$.

Lets define $\zeta(\gamma, \lambda)$ as

$$\zeta(\gamma, \lambda) =$$
$$\int_{\underline{\theta}}^{\overline{\theta}} \pi_1(z(\theta))(1 - b(L_1(z(\theta))))L_1(z(\theta))dF(\theta)$$
$$+ \int_{\underline{\theta}}^{\overline{\theta}} \pi_2(z(\theta))(1 - b(L_2(z(\theta))))L_2(z(\theta))dF(\theta) \tag{30}$$

For the case where $s < \underline{s} = \frac{\beta E(\theta)}{\overline{\theta}}$, the right graph on figure 2 shows the policy functions. $\zeta(\gamma, \lambda)$ is independent of λ, and I show it by $\zeta_1(\gamma)$. Equation 29 can be writen as

$$\frac{\gamma - 1}{\lambda} + (\gamma + 1)s =$$
$$1 + 2\gamma - \frac{(\gamma - 1)}{\int_{\underline{\theta}}^{\overline{\theta}} (\overline{m} - z(\theta))dF(\theta)} \zeta_1(\gamma) \tag{31}$$

The only policy variable on the right hand side of 31 is γ. For a constant rate of inflation the left hand side shows a positive relationship between bond price and bond supply. From 31 and 30 theorem 2 follows

Theorem 2. *There exists a threshold for bond supply*

$$\overline{\lambda}(\gamma) = \frac{\gamma - 1}{1 + 2\gamma - \frac{(\gamma-1)}{\int_{\underline{\theta}}^{\overline{\theta}}(\overline{m} - z(\theta))dF(\theta)}\zeta_1(\gamma) - (\gamma+1)\underline{s}} \tag{32}$$

1. *For $\lambda < \overline{\lambda}(\gamma)$ the policy functions are similar to the right graph in figure 2. Open market operations have no effect on the real economy and only change the return on bonds.*

2. *For $\lambda \geq \overline{\lambda}(\gamma)$ the policy functions are similar to the left graph on figure 2. Open market operations affect the policy functions and have effects on the real economy.*

Theorem 2 shows an important property of the equilibrium. For high enough bond supply asset market is segmented and pure open-market operations has real effects on the economy. In this case supplying more bonds will increase (decrease) the price (yield) of bonds. Fewer households decide to participate in the asset market due to lower return on bonds. They choose to hold more money for transaction purposes. This effect happens only when bond supply is high enough ($\lambda > \lambda(\gamma)$).

Lets define $\lambda_{min}(\gamma)$, $\lambda_{max}(\gamma)$ and $\widetilde{s}$ as in 33 and 34

$$\frac{1}{\lambda_{min}(\gamma)} = \frac{1+2\gamma}{\gamma-1} - \frac{\zeta_1(\gamma)}{\int_{\underline{\theta}}^{\overline{\theta}}(\overline{m} - z(\theta))dF(\theta)} \tag{33}$$

$$\frac{1}{\lambda_{max}(\gamma)} = \frac{1+\gamma+\gamma^2}{\gamma-1} - \frac{\zeta_1(\gamma)}{\int_{\underline{\theta}}^{\frac{\beta E(\theta)}{\gamma}}(\overline{m} - z(\theta))dF(\theta)} \tag{34}$$

where $\zeta(\gamma)$ is defined as 30, $z(\theta)$ solves 25 and $\overline{\lambda}(\gamma)$ is defined as 32. Lemma 6 shows the existance of the equilibrium with no segmentation in the asset market.

Lemma 6. *If $\gamma < \underline{s}$, for $\lambda \in (\lambda_{min}(\gamma), \lambda_{max}(\gamma))$ a stationary equilibrium exists. The equilibrium shows no segmentation in the asset market.*

The proof is in the appendix.

3.1 Welfare analysis

I have shown in the appendix that the steady state welfare can be calculated using the following expression

$$
\begin{aligned}
\varpi =& \int [U(y(\theta)) - \theta y(\theta) + u(q(z(\theta))) - \theta z(\theta) - \theta h(\theta) - s\theta a(\theta)] dF(\theta) \\
&+ \left[\int_{\underline{\theta}}^{\bar{\theta}} \pi_1(z(\theta))[1 - b(L_1(z(\theta)))] \frac{L_1(z(\theta))}{\gamma} dF(\theta) \right] \int \theta dF(\theta) \\
&+ \left[\int_{\underline{\theta}}^{\bar{\theta}} \pi_2(z(\theta))[1 - b(L_2(z(\theta)))] \frac{L_2(z(\theta))}{\gamma} dF(\theta) \right] \int \theta dF(\theta) \\
&+ (1 + \frac{1}{\gamma}) \left[\int a_{-1} dH_{a_{-1}} \right] \int \theta dF(\theta) + \frac{1}{\gamma} \left[\int h_{-1} dJ_{h_{-1}} \right] \int \theta dF(\theta) \\
&+ \frac{1}{w^* \gamma} [\gamma - 1 - \lambda + s\lambda] \int \theta dF(\theta)
\end{aligned}
$$

Using lemma 5 and equations 28 and 30 the measure of welfare can be written as

$$
\begin{aligned}
\varpi =& \int [U(y(\theta)) - \theta y(\theta)] dF(\theta) + \int [u(q(z(\theta))) - \theta z(\theta) - s\theta a(\theta)] dF(\theta) \\
&+ \left[\zeta(\gamma, \lambda) + (2 - s + 1 + \frac{1}{\gamma}) \int a(\theta) dF(\theta) \right] \int \theta dF(\theta)
\end{aligned} \tag{35}
$$

The following lemma shows the welfare effects of open-market operations

Lemma 7. *For $\gamma < \underline{s}$ and $\lambda \in (\lambda_{min}(\gamma), \lambda_{max}(\gamma))$, marginal open-market operations do not change the overall welfare.*

With $\gamma < \underline{s}$ and $\lambda \in (\lambda_{min}(\gamma), \lambda_{max}(\gamma))$ we can see that none of the policy functions are affected by open-market operations (change in λ). The welfare measure can be stated as

$$
\begin{aligned}
\varpi =& \int [U(y(\theta)) - \theta y(\theta)] dF(\theta) + \int [u(q(z(\theta))) - \theta z(\theta) - s\theta a(\theta)] dF(\theta) \\
&+ \left[\zeta_1(\gamma) + (2 - s + 1 + \frac{1}{\gamma}) \int a(\theta) dF(\theta) \right] \int \theta dF(\theta)
\end{aligned} \tag{36}
$$

Therefor, it is straightforward that changes in λ do not affect welfare.

4 Numerical Example

In order to simulate the economy, I use the following algorithm:

1. For given supply of bonds (λ) and inflation (γ), and an arbitrary bond price (s) calculate policy functions $(a(\theta), h(\theta), z(\theta), y(\theta), l(\theta), b(z(\theta)), q(z(\theta)))$ (4) and lottery choices

$(\pi_1(z(\theta)), \pi_2(z(\theta)), L_1(z(\theta)), L_2(z(\theta)))$ (17)

2. Calculate the value functions $(B(z(\theta)), \widetilde{V}(z(\theta)))$

3. Calcualte wage (w) using labor market clearing condition 28

4. If $w < 0$ change s and start from 1.

5. Check bond market clearing condition 27, adjust bond price and start from 1. until bond market clears.

I simulate the economy using the following functional forms:

$$u(c) = u_0 \frac{(c+a)^{1-\sigma} - a^{1-\sigma}}{1-\sigma}; U(c) = U_0 \frac{(c+a)^{1-\sigma_u} - a^{1-\sigma_u}}{1-\sigma_u}$$

$$\psi(q) = \psi_0 q^{\psi}; \mu(b) = 1 - b; F(\theta) \text{ is continuous uniform on } [\underline{\theta}, \overline{\theta}]$$

I use the following parameter values:

$\beta = 0.997$	$u_0 = 1$	$U_0 = 1000$	$a = 0.001$
$\sigma = 2$	$\sigma_u = 2$	$\phi = 2$	$\psi_0 = 1$
$k = 0.2$	$\overline{m} = 17$	$\theta \in [1, 2]$	

Figure 3 shows equilibrium bond price (s) for different amounts of bond supply (λ) and different inflation rates (γ). At each level of inflation bond price increases with higher supply of bonds[9].

Figure 4 shows equilibrium wage (w) for different amounts of bond supply (λ).

Figures 6 shows welfare for different bond supply and inflation rates.

5 Exogenously segmented asset market

Following Alvarez et al. (2001), I assume only a fixed fraction of households attend the asset markets (traders), and the remaining never has access to the asset market (non-traders). This extension allows me to compare the results of this paper to the literature that assumes the asset markets are exogenously segmented[10]. Theorem 3 shows that the same logic from the case with endogenous asset market segmentation applies and asset market traders and non-traders solve optimization problems similar to the problem in the previous sections. The households' decisions are only linked through the market clearing conditions and prices.

[9] Note that the price of bond is the inverse of return on bonds

[10] e.g. Alvarez et al. (2001), Khan and Thomas (2010) and Chiu (2007)

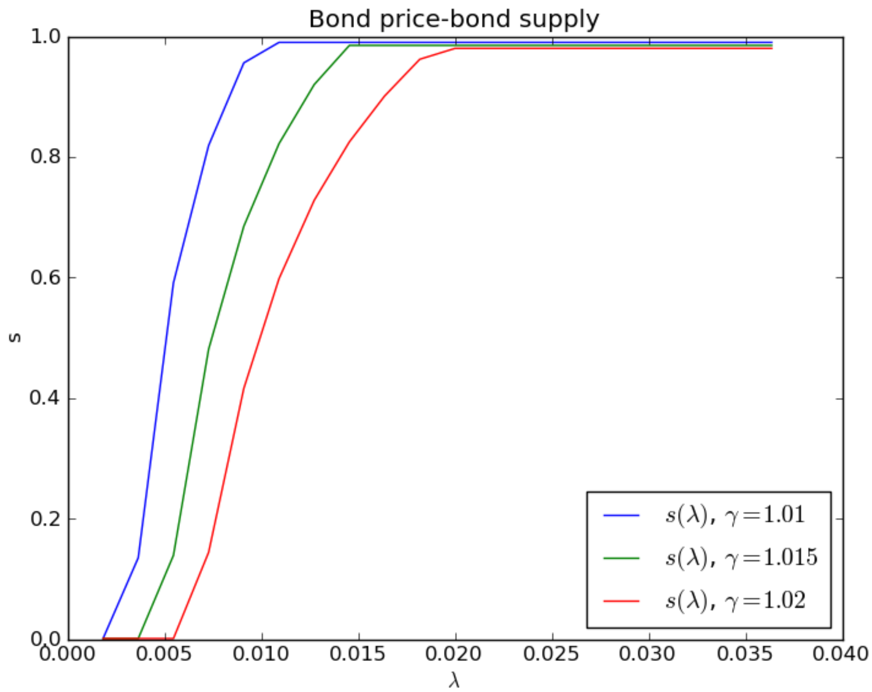

Figure 3: Bond prices

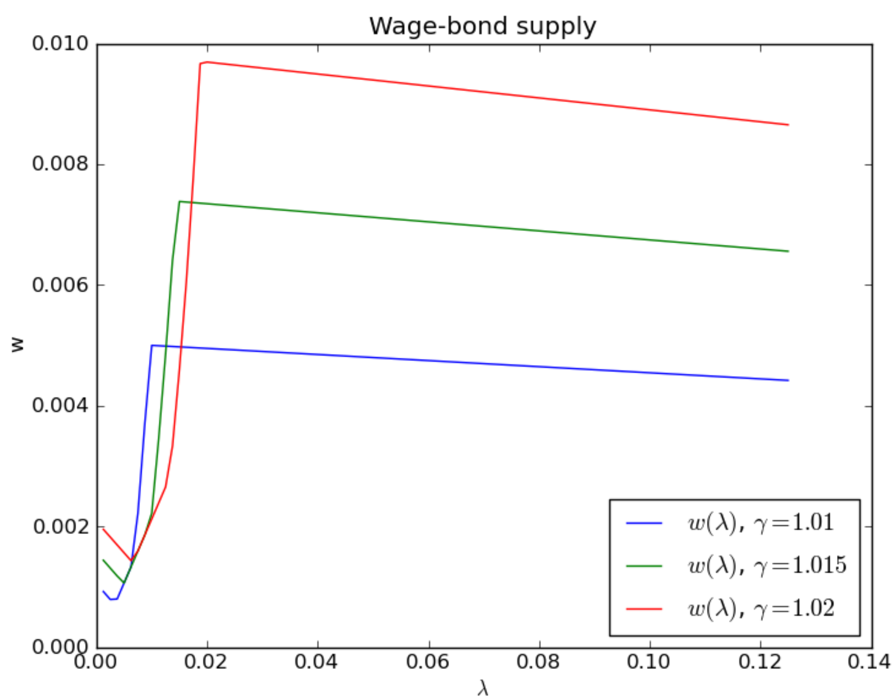

Figure 4: Wage

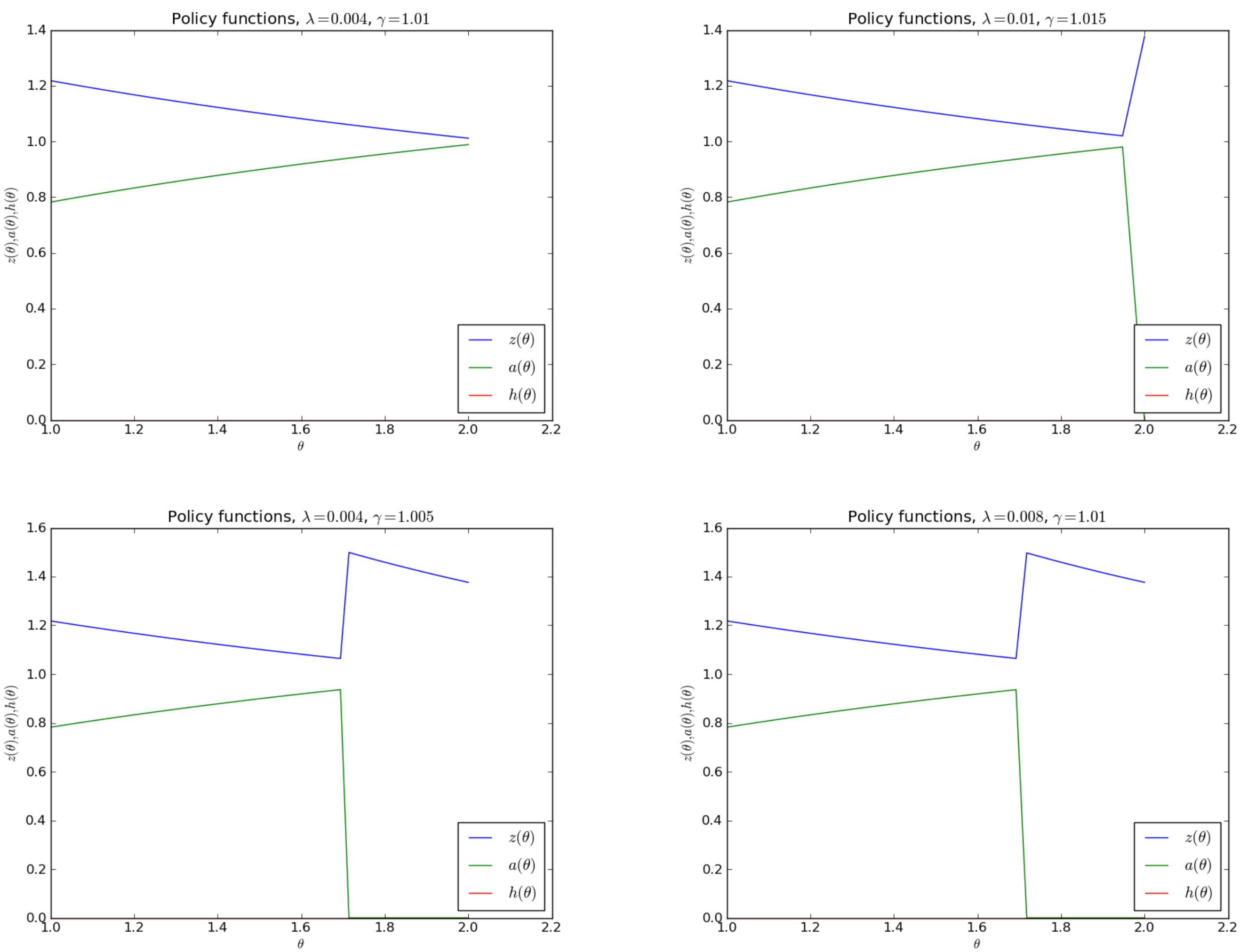

Figure 5: Policy functions

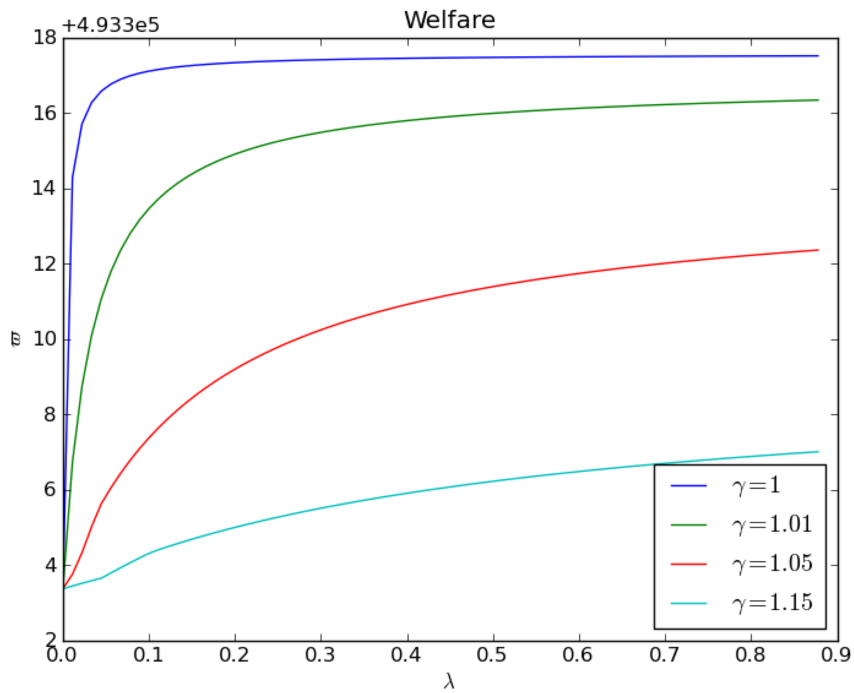

Figure 6: Welfare

Households do not take in to account the distribution of asset holdings among traders and non-traders. The following theorem shows that the main results in the previous sections are robust to adding exogenously segmented asset market.

Theorem 3. *With exogenously segmented asset markets, value functions, policy functions and labor choices are the same as the case without segmented asset market.*

The formal proof is in the appendix.

6 Concluding Remarks and Possible Extensions

This paper has studied central bank's open-market operations in a model with heterogenous agents. Using competitive search in the frictional market for goods allowed me to study the distribution of asset holding in a tractable model. Agents with good income shock choose submarkets with high price and low matching probability. They do not need to know the characteristic of the entire distribution for their decision.

Central bank can implement monetary policy by supplying money and trading bond in the asset market. There are two types of equilibrium. In equilibria with low bond supply the asset market is not segmented. All of the agents attend the asset market and hold positive portfolio of bond and money. In an equilibrium with high bond supply segmentation is generated endogenously. Households with good income shock attend the asset market and hold positive portfolio of bond and money. Household with low income only hold money in their portfolio.

In an equiloibrium with no segmentation, open-market operations have no real effects on the economy. in an equilibrium with segmented asset market open-market operations change the decision of of a subset of households and have real effects on the economy. The main results are robust to exogenously segmented asset market.

One possible extension of the model is to relax the loglinear preference of the households to a more general preference structure. Some of the properties of the equilibrium cannot be shown analytically and computational excercise is more critical.

By adding aggregate shocks to the economy, one can do an analysis similar to Krusell and Smith (1998) with the model. As the distribution of asset holding do not affect the decision of households, the model should be fairly tractable. The equilibrium of the model shows the proporties of a block recursive equilibrium similar to Shi (2009) and Menzio and Shi (2010). Therefore, the fact that agents only care about the average of the aggregate shocks, can be shown analytically. In a model with aggregate shocks the distributional effects of following a Taylor rule can be studied.

Appendix

A Market clearing conditions

I can find the cumulative distribution of money before lotteries by:

$$G(m) = \int \int_{z^{-1}(m)}^{\overline{\theta}} dF(\theta) dH \tag{37}$$

and similarly the distribution of bond before lotteries follows:

$$H(a_{-1}) = \int \int_{a^{-1}(a_{-1})}^{\overline{\theta}} dF(\theta) dG \tag{38}$$

I assume a balanced budget for government at each period of time. The total real transfer that a household receives is the sum of transfers from printing money and the transfers received from bond market:

$$T^* = \frac{\gamma - 1}{w^* \gamma} + \frac{s^* A - A_{-1}}{w^* M'} \tag{39}$$

In the bond market the total amount of bonds supplied equals the sum of demanded bonds by households of different type. Thus, the market clearing for bonds gives:

$$\frac{A}{wM} = \int \int \int_{\underline{\theta}}^{\overline{\theta}} a(\theta) dF(\theta) dG(m) dH(a_{-1}) \tag{40}$$

In the general-good market, the market clearing condition is:

$$Y = \int_{\underline{\theta}}^{\overline{\theta}} y(\theta) dF(\theta) \tag{41}$$

LD is the same as Sun (2011):

$$\begin{aligned} LD = \quad & Y + \int_{\underline{\theta}}^{\overline{\theta}} \frac{\pi_1(z(\theta)) b(L_1(z(\theta)))}{\mu(b(L_1(z(\theta))))} \left[k + \psi(q(L_1(z(\theta))))\mu(b(L_1(z(\theta))))\right] dF(\theta) \\ & + \int_{\underline{\theta}}^{\overline{\theta}} \frac{\pi_2(z(\theta)) b(L_2(z(\theta)))}{\mu(b(L_2(z(\theta))))} \left[k + \psi(q(L_2(z(\theta))))\mu(b(L_2(z(\theta))))\right] dF(\theta) \end{aligned} \tag{42}$$

The firms zero-profit condition gives:

$$k + \psi\left(q(L_i(z(\theta)))\right)\mu(b(L_i(z(\theta)))) = L_i(z(\theta))$$

Then LD becomes:

$$\begin{aligned} LD = \quad & \int_{\underline{\theta}}^{\overline{\theta}} y(\theta)dF(\theta) + \int_{\underline{\theta}}^{\overline{\theta}} \pi_1(z(\theta))b\left(L_1(z(\theta))\right)L_1(z(\theta))dF(\theta) \\ & + \int_{\underline{\theta}}^{\overline{\theta}} \pi_2(z(\theta))b\left(L_2(z(\theta))\right)L_2(z(\theta))dF(\theta) \end{aligned} \tag{43}$$

Labor supply is the sum of households labor supply:

$$LS = \int_{\underline{\theta}}^{\overline{\theta}} \int \int l(m, a, \theta)dF(\theta)dG_a(m)dH(a_{-1})$$

Substituting for l:

$$LS = \int_{\underline{\theta}}^{\overline{\theta}} \int \int \left[py(\theta) + z(\theta) + s^* a(\theta) - m - a_{-1} - T\right] dF(\theta)dG_a(m)dH(a_{-1}) \tag{44}$$

Substituting for T:

$$\begin{aligned} LS = \quad & \int_{\underline{\theta}}^{\overline{\theta}} \int \int [py(\theta) + z(\theta) + s^* a(\theta) - m - a_{-1} - \frac{\gamma - 1}{w^* \gamma} \\ & - \frac{s^* A}{w^* M'} + \frac{A_{-1}}{w^* \gamma M}]dF(\theta)dG_a(m)dH(a_{-1}) \end{aligned} \tag{45}$$

LS becomes:

$$\begin{aligned} LS = \quad & \frac{A_{-1}}{w^* \gamma M} - \frac{s^* A}{w^* M'} - \frac{\gamma - 1}{w^* \gamma} - \int m dG_a(m) - \int a_{-1} dH(a_{-1}) \\ & + \int_{\underline{\theta}}^{\overline{\theta}} [y(\theta) + z(\theta) + s^* a(\theta)]dF(\theta) \end{aligned} \tag{46}$$

Labor market clearing condition gives:

$$\int_{\underline{\theta}}^{\overline{\theta}} s^* a(\theta) dF(\theta) + \int_{\underline{\theta}}^{\overline{\theta}} \pi_1(z(\theta))(1 - b(L_1(z(\theta)))) L_1(z(\theta)) dF(\theta)$$
$$+ \int_{\underline{\theta}}^{\overline{\theta}} \pi_2(z(\theta))(1 - b(L_2(z(\theta)))) L_2(z(\theta)) dF(\theta)$$
$$= \frac{s^* A}{w^* M'} + \frac{\gamma - 1}{w^* \gamma} - \frac{A_{-1}}{w^* \gamma M} + \int m dG_a(m) + \int a_{-1} dH(a_{-1})$$

m is the distribution of money at the beginning of the period. Therefor, it consists of balances that are not spent plus the payments on nominal bonds:

$$\int m dG_a(m) =$$
$$\int_{\underline{\theta}}^{\overline{\theta}} \pi_1(z(\theta))[1 - b(L_1(z(\theta)))] \frac{L_1(z(\theta))}{\gamma} dF(\theta)$$
$$+ \int_{\underline{\theta}}^{\overline{\theta}} \pi_2(z(\theta))[1 - b(L_2(z(\theta)))] \frac{L_2(z(\theta))}{\gamma} dF(\theta) + \int \frac{a_{-1}}{\gamma} dH(a_{-1}) + \int \frac{h_{-1}}{\gamma} dJ_{h_{-1}}$$

Plug in the labor market clearing condition:

$$\frac{s^* A}{w^* M'} + \frac{\gamma - 1}{w^* \gamma} - \frac{A_{-1}}{w^* \gamma M} =$$
$$\int_{\underline{\theta}}^{\overline{\theta}} s^* a(\theta) dF(\theta) + (1 - \frac{1}{\gamma}) \int_{\underline{\theta}}^{\overline{\theta}} \pi_1(z(\theta))(1 - b(L_1(z(\theta)))) L_1(z(\theta)) dF(\theta)$$
$$+(1 - \frac{1}{\gamma}) \int_{\underline{\theta}}^{\overline{\theta}} \pi_2(z(\theta))(1 - b(L_2(z(\theta)))) L_2(z(\theta)) dF(\theta) - \int_{\underline{\theta}}^{\overline{\theta}} a_{-1}(1 + \frac{1}{\gamma}) dH(a_{-1}) - \int \frac{h_{-1}}{\gamma} dJ_{h_{-1}}$$

The labor-market-clearing can be written as:

$$\frac{1}{w^* \gamma}[\gamma - 1 - \lambda + s\lambda] =$$
$$(2 - s^*) \int_{\underline{\theta}}^{\overline{\theta}} a(\theta) dF(\theta) + (1 - \frac{1}{\gamma}) \int_{\underline{\theta}}^{\overline{\theta}} \pi_1(z(\theta))(1 - b(L_1(z(\theta)))) L_1(z(\theta)) dF(\theta)$$
$$+(1 - \frac{1}{\gamma}) \int_{\underline{\theta}}^{\overline{\theta}} \pi_2(z(\theta))(1 - b(L_2(z(\theta)))) L_2(z(\theta)) dF(\theta) - \int \frac{h(\theta)}{\gamma} dF(\theta) \quad (47)$$

B Proof of lemma 6

For $\gamma < \underline{s}$, in the following equation $\zeta_1(\gamma)$ is only a function of γ.

$$\frac{\gamma-1}{\lambda} + (\gamma+1)s = \\ 1 + 2\gamma - \frac{(\gamma-1)}{\int_{\underline{\theta}}^{\frac{\beta E(\theta)}{s}} (\overline{m} - z(\theta))dF(\theta)} \zeta_1(\gamma) \tag{48}$$

As s increases with λ, I can solve for the limits of λ by inserting the limits of s $(0, \gamma)$.

$$\frac{1}{\lambda_{min}(\gamma)} = \frac{1+2\gamma}{\gamma-1} - \frac{\zeta_1(\gamma)}{\int_{\underline{\theta}}^{\overline{\theta}} (\overline{m} - z(\theta))dF(\theta)} \tag{49}$$

$$\frac{1}{\lambda_{max}(\gamma)} = \frac{1+\gamma+\gamma^2}{\gamma-1} - \frac{\zeta_1(\gamma)}{\int_{\underline{\theta}}^{\frac{\beta E(\theta)}{\gamma}} (\overline{m} - z(\theta))dF(\theta)} \tag{50}$$

With $\lambda \in (\lambda_{min}(\gamma), \lambda_{max}(\gamma))$ there exists $s^* \in (0, \gamma)$ that satisfies bond market clearing condition(27). The right hand side of 28 is independent of w. A $w^* > 0$ exisits that clears the labor market(28). The equlibrium is characterized by w^* and s^*, and it is unique if and only if lottery choices in 17 $(\{L_1(\theta), L_2(\theta), \pi_1(\theta), \pi_2(\theta)\})$ are unique for all $z(\theta)$ and $a(\theta)$.

C Welfare Analysis

I use the household's utility function to calculate welfare:

$$\begin{aligned} \varpi =& \int\int\int \{U(y) + u(q) - \theta l\} dF(\theta)dG(m)dH(a_{-1}) \\ =& \int U(y(\theta))dF(\theta) + \int u(q(z(\theta)))dF(\theta) - \int\int\int \{\theta l\} dF(\theta)dG(m)dH(a_{-1}) \end{aligned}$$

I can write the last integral as:

$$\int\int\int (\theta l) dF(\theta)dG(m)dH(a_{-1}) = \\ \int \left[\theta(y(\theta) + z(\theta) + h(\theta) + sa(\theta))\right] dF(\theta) \\ - \int \theta \left(\int m dG_a\right) dF(\theta) - \int \theta \left(\int a_{-1} dH(a_{-1})\right) dF(\theta) - T\int \theta dF(\theta)$$

I have shown in the market clearing appendix the distribution of money before the lotteris is:

$$\int m dG_a(m) =$$
$$\int_{\underline{\theta}}^{\overline{\theta}} \pi_1(z(\theta))[1 - b(L_1(z(\theta)))] \frac{L_1(z(\theta))}{\gamma} dF(\theta)$$
$$+ \int_{\underline{\theta}}^{\overline{\theta}} \pi_2(z(\theta))[1 - b(L_2(z(\theta)))] \frac{L_2(z(\theta))}{\gamma} dF(\theta) + \int \frac{a_{-1}}{\gamma} dH(a_{-1}) + \int \frac{h_{-1}}{\gamma} dJ_{h_{-1}}$$

I can substitute for the distribution of money (mdG_a) and labor supply (l) from the above equations, and for government transfers (T) from the the market clearing appendix to simplify the equation for welfare:

$$\begin{aligned}
\varpi =& \int [U(y(\theta)) - \theta y(\theta) + u(q(z(\theta))) - \theta z(\theta) - \theta h(\theta) - s\theta a(\theta)] dF(\theta) \\
&+ \left[\int_{\underline{\theta}}^{\overline{\theta}} \pi_1(z(\theta))[1 - b(L_1(z(\theta)))] \frac{L_1(z(\theta))}{\gamma} dF(\theta) \right] \int \theta dF(\theta) \\
&+ \left[\int_{\underline{\theta}}^{\overline{\theta}} \pi_2(z(\theta))[1 - b(L_2(z(\theta)))] \frac{L_2(z(\theta))}{\gamma} dF(\theta) \right] \int \theta dF(\theta) \\
&+ (1 + \frac{1}{\gamma}) \left[\int a_{-1} dH_{a_{-1}} \right] \int \theta dF(\theta) + \frac{1}{\gamma} \left[\int h_{-1} dJ_{h_{-1}} \right] \int \theta dF(\theta) \\
&+ \frac{1}{w^* \gamma} [\gamma - 1 - \lambda + s\lambda] \int \theta dF(\theta)
\end{aligned}$$

D Proof of theorem 3

There are two types of agents in the economy, traders in the asset market (denoted by subscript T) and non-traders (denoted by subscript N).

Value function of a trader:

$$W_T(m_T, a_{-1}, \theta) = \max_{y_T, l_T, z_T, a} U(y_T) - \theta l_T + V_T(z_T, h_T, a)$$

$$st. \;\; py_T + z_T + sa \leq m_T + a_{-1} + l_T + T$$

Value function of a non-trader:

$$W_N(m_N, \theta) = \max_{y_N, l_N, z_N} U(y_N) - \theta l_N + V_N(z_N, h_N)$$

$$st. \quad py_N + z_N \leq m_N + l_N + T$$

Using the budget constraint to eliminate l:

$$W_T(m_T, a_{-1}, \theta) = \theta(m_T+T+a_{-1}) + \max_{y_T \geq 0}\{U(y_T) - \theta p y_T\} + \max_{z_T, a, h_T}\{-\theta(z_T + sa + h_T) + V_T(z_T, h_T, a)\}$$

$$W_N(m_N, \theta) = \theta(m_N + T) + \max_{y_N \geq 0}\{U(y_N) - \theta p y_N\} + \max_{z_N, h_N}\{-\theta(z_N + h_N) + V_N(z_N, h_N)\}$$

The optimal choices of $y_{i\in\{T,N\}}$, $z_{i\in\{T,N\}}$ and a must satisfy:

$$U'(y_T) = U'(y_N) = \theta \tag{51}$$

The above expression shows that a trader and a non-trader choose the same amount of consumption in the centralized market:

$$y_T(\theta) = y_N(\theta) = y(\theta)$$

$$\frac{\partial V_T(z_T, h_T, a)}{\partial z_T} \begin{cases} \leq \theta & z_T \geq 0 \\ \geq \theta & z_T \leq \overline{m} - sa - h_T \end{cases} \tag{52}$$

$$\frac{\partial V_T(z_T, h_T, a)}{\partial h_T} \begin{cases} \leq \theta & h_T \geq 0 \\ \geq \theta & h_T \leq \overline{m} - sa - z_T \end{cases} \tag{53}$$

$$\frac{\partial V_T(z_T, h_T, a)}{\partial a} \begin{cases} \leq \theta s & a \geq 0 \\ \geq \theta s & sa \leq \overline{m} - z_T - h_T \end{cases} \tag{54}$$

$$\frac{\partial V_N(z_N, h_N)}{\partial z_N} \begin{cases} \leq \theta & z_N \geq 0 \\ \geq \theta & z_N \leq \overline{m} - h_N \end{cases} \tag{55}$$

$$\frac{\partial V_N(z_N, h_N)}{\partial h_N} \begin{cases} \leq \theta & h_N \geq 0 \\ \geq \theta & h_N \leq \overline{m} - z_N \end{cases} \tag{56}$$

The value functions can be written as:

$$W_T(m_T, a_{-1}, \theta) = W_T(0, 0, \theta) + \theta m_T + \theta a_{-1} \tag{57}$$

Where:

$$W_T(0,0,\theta) = U(y(\theta)) - \theta y(\theta) + V_T(z_T(\theta), h_T(\theta), a(\theta)) - \theta(z_T(\theta) + h_T(\theta) + sa(\theta)) \quad (58)$$

$$W_N(m_T, \theta) = W_T(0,\theta) + \theta m_N \quad (59)$$

Where:

$$W_N(0,\theta) = U(y(\theta)) - \theta y(\theta) + V_N(z_N(\theta), h_N(\theta)) - \theta(z_N(\theta) + h_N(\theta)) \quad (60)$$

We can see that the value function $W()$ is linear in household's asset holdings for both traders and non-traders. Agents problem in the frictional market for traders and non-traders are similar. The difference comes from their value function which has 3 state variables for traders and 2 state variables for non-traders. After simplification and applying the lotteries as the previous section I can write agents value function as:

$$\begin{aligned} V_T(z_T, h_T, a) \quad &= \widetilde{V_T}(z) + \beta E\left[W_T(\frac{z_T + h_T}{\gamma}, a, \theta)\right] \\ &= \widetilde{V_T}(z_T) + \beta E\left[W_T(0,0,\theta)\right] + \frac{\beta E(\theta) z_T}{\gamma} + \frac{\beta E(\theta) h_T}{\gamma} + \beta E(\theta) a \quad (61) \end{aligned}$$

$$\begin{aligned} V_N(z_N, h_N) \quad &= \widetilde{V_N}(z) + \beta E\left[W_N(\frac{z_N + h_N}{\gamma}, \theta)\right] \\ &= \widetilde{V_N}(z_N) + \beta E\left[W_N(0,\theta)\right] + \frac{\beta E(\theta) z_N}{\gamma} + \frac{\beta E(\theta) h_N}{\gamma} \quad (62) \end{aligned}$$

Trader's and non-rader's choice of bond holding follows the following condition with complementary slackness:

$$\begin{cases} a(\theta) \geq 0 & \theta \geq \frac{\beta E(\theta)}{s} \\ a(\theta) \leq \overline{m_T} - z_T(\theta) - h_T(\theta) & \theta \leq \frac{\beta E(\theta)}{s} \end{cases} \quad (63)$$

$$\begin{cases} h_T(\theta) \geq 0 & \theta \geq \frac{\beta E(\theta)}{\gamma} \\ h_T(\theta) \leq \overline{m_T} - z_T(\theta) - a'(\theta) & \theta \leq \frac{\beta E(\theta)}{\gamma} \end{cases} \quad (64)$$

$$\begin{cases} h_N(\theta) \geq 0 & \theta \geq \frac{\beta E(\theta)}{\gamma} \\ h_N(\theta) \leq \overline{m_N} - z_N(\theta) & \theta \leq \frac{\beta E(\theta)}{\gamma} \end{cases} \quad (65)$$

The labor choices of traders are the same as the labor choices in equations 25 and 26. Labor choices of non-traders are as 66:

$$l_N(m,\theta) = \begin{cases} py(\theta) + z_N(\theta) - m_N - T_N & \theta > \frac{\beta E(\theta)}{\gamma} \\ py(\theta) + z_N(\theta) - m_N - T_N & \theta = \frac{\beta E(\theta)}{\gamma} \\ py(\theta) + \overline{m} - m_N - T_N & \theta < \frac{\beta E(\theta)}{\gamma} \end{cases} \tag{66}$$

D.1 Market clearing condition and welfare measure

Similar to the case where all of the agents trade in the asset market the real transfer is:

$$T^* = \frac{\gamma - 1}{w^*\gamma} + \frac{s^*A - A_{-1}}{w^*M'} \tag{67}$$

The markt clearing condition for the bond market and the general good market is the same as 27 and 41.

Similar to the case with only one type of agent, the labor demand can be written as:

$$\begin{aligned} LD = \quad & \int_{\underline{\theta}}^{\overline{\theta}} y(\theta)dF(\theta) + \int_{\underline{\theta}}^{\overline{\theta}} \pi_1(z_T(\theta))b\left(L_1(z_T(\theta))\right)L_1(z_T(\theta))dF_T(\theta) \\ & + \int_{\underline{\theta}}^{\overline{\theta}} \pi_1(z_N(\theta))b\left(L_1(z_N(\theta))\right)L_1(z_N(\theta))dF_N(\theta) \\ & + \int_{\underline{\theta}}^{\overline{\theta}} \pi_2(z_T(\theta))b\left(L_2(z_T(\theta))\right)L_2(z_T(\theta))dF_T(\theta) \\ & + \int_{\underline{\theta}}^{\overline{\theta}} \pi_2(z_N(\theta))b\left(L_2(z_N(\theta))\right)L_2(z_N(\theta))dF_N(\theta) \end{aligned} \tag{68}$$

Labor supply is the sum of households labor supply:

$$LS = \int_{\underline{\theta}}^{\overline{\theta}} \int \int l_T(m_T, a, \theta)dF_T(\theta)dG_a(m)dH(a_{-1}) + \int_{\underline{\theta}}^{\overline{\theta}} \int l_N(m_N, \theta)dF_N(\theta)dG_a(m)$$

Substituting for labor choices and transfers:

$$\begin{aligned}
Ls = \quad & \int_{\underline{\theta}}^{\overline{\theta}} \int \int \left[py(\theta) + z_T(\theta) + s^* a(\theta) - m_T - a_{-1} \right] dF_T(\theta) dG_a(m_T) dH(a_{-1}) \\
& + \int_{\underline{\theta}}^{\overline{\theta}} \int \left[py(\theta) + z_N(\theta) - m_N \right] dF_N(\theta) dG_a(m_N) \\
& - \frac{\gamma - 1}{w^* \gamma} - \frac{s^* A}{w^* M'} + \frac{A_{-1}}{w^* \gamma M}
\end{aligned} \tag{69}$$

$$\begin{aligned}
LS = \quad & \frac{A_{-1}}{w^* \gamma M} - \frac{s^* A}{w^* M'} - \frac{\gamma - 1}{w^* \gamma} - \int m_T dG_a(m_T) - - \int m_N dG_a(m_N) - \int a_{-1} dH(a_{-1}) \\
& + \int_{\underline{\theta}}^{\overline{\theta}} [y(\theta) + z_T(\theta) + s^* a(\theta)] dF_T(\theta) + \int_{\underline{\theta}}^{\overline{\theta}} [y(\theta) + z_N(\theta)] dF_N(\theta)
\end{aligned} \tag{70}$$

Labor market clearing condition gives:

$$\begin{aligned}
& \frac{A_{-1}}{w^* \gamma M} - \frac{s^* A}{w^* M'} - \frac{\gamma - 1}{w^* \gamma} - \int m_T dG_a(m_T) - \int m_N dG_a(m_N) \\
& - \int a_{-1} dH(a_{-1}) + \int_{\underline{\theta}}^{\overline{\theta}} [z_T(\theta) + s^* a(\theta)] dF_T(\theta) + \int_{\underline{\theta}}^{\overline{\theta}} [z_N(\theta)] dF_N(\theta) \\
= & \int_{\underline{\theta}}^{\overline{\theta}} \pi_1(z_T(\theta)) b\left(L_1(z_T(\theta))\right) L_1(z_T(\theta)) dF_T(\theta) + \int_{\underline{\theta}}^{\overline{\theta}} \pi_1(z_N(\theta)) b\left(L_1(z_N(\theta))\right) L_1(z_N(\theta)) dF_N(\theta) \\
& + \int_{\underline{\theta}}^{\overline{\theta}} \pi_2(z_T(\theta)) b\left(L_2(z_T(\theta))\right) L_2(z_T(\theta)) dF_T(\theta) + \int_{\underline{\theta}}^{\overline{\theta}} \pi_2(z_N(\theta)) b\left(L_2(z_N(\theta))\right) L_2(z_N(\theta)) dF_N(\theta)
\end{aligned}$$

Similar to the appendix A:

$$\begin{aligned}
& \int m_T dG_a(m_T) = \\
& \quad \int_{\underline{\theta}}^{\overline{\theta}} \pi_1(z_T(\theta)) [1 - b(L_1(z_T(\theta)))] \frac{L_1(z_T(\theta))}{\gamma} dF_T(\theta) \\
& \quad + \int_{\underline{\theta}}^{\overline{\theta}} \pi_2(z_T(\theta)) [1 - b(L_2(z_T(\theta)))] \frac{L_2(z_T(\theta))}{\gamma} dF_T(\theta) + \int \frac{a_{-1}}{\gamma} dH(a_{-1}) + \int \frac{h_{-1T}}{\gamma} dJ_{h_{-1T}}
\end{aligned}$$

and

$$\int m_N dG_a(m_N) =$$

$$\int_{\underline{\theta}}^{\overline{\theta}} \pi_1(z_N(\theta))[1-b(L_1(z_N(\theta)))]\frac{L_1(z_N(\theta))}{\gamma}dF_N(\theta)$$
$$+\int_{\underline{\theta}}^{\overline{\theta}} \pi_2(z_N(\theta))[1-b(L_2(z_N(\theta)))]\frac{L_2(z_N(\theta))}{\gamma}dF_N(\theta)+\int \frac{h_{-1N}}{\gamma}dJ_{h_{-1N}}$$

Plug in the labor market clearing condition:

$$\frac{1}{w^*\gamma}[\gamma-1-\lambda+s\lambda]=$$
$$(2-s^*)\int_{\underline{\theta}}^{\overline{\theta}} a(\theta)dF_T(\theta)+(1-\frac{1}{\gamma})\int_{\underline{\theta}}^{\overline{\theta}} \pi_1(z_T(\theta))(1-b(L_1(z_T(\theta))))L_1(z_T(\theta))dF_T(\theta)$$
$$+(1-\frac{1}{\gamma})\int_{\underline{\theta}}^{\overline{\theta}} \pi_2(z_T(\theta))(1-b(L_2(z_T(\theta))))L_2(z_T(\theta))dF_T(\theta)-\int \frac{h_T(\theta)}{\gamma}dF_T(\theta)$$
$$+(1-\frac{1}{\gamma})\int_{\underline{\theta}}^{\overline{\theta}} \pi_1(z_N(\theta))(1-b(L_1(z_N(\theta))))L_1(z_N(\theta))dF_N(\theta)-\int \frac{h_N(\theta)}{\gamma}dF_N(\theta)$$
$$+(1-\frac{1}{\gamma})\int_{\underline{\theta}}^{\overline{\theta}} \pi_2(z_N(\theta))(1-b(L_2(z_N(\theta))))L_2(z_N(\theta))dF_N(\theta)$$

It can be shown as in appendix for the benchmark model that the measure of welfare is:

$$
\begin{aligned}
\varpi =& \int [U(y(\theta)) + \theta y(\theta) + u(q(z_T(\theta))) + \theta z_T(\theta) + s\theta a(\theta)] dF_T(\theta) \\
& - \left[\int_{\underline{\theta}}^{\overline{\theta}} \pi_1(z_T(\theta))[1 - b(L_1(z_T(\theta)))] \frac{L_1(z_T(\theta))}{\gamma} dF_T(\theta) \right] \int \theta dF_T(\theta) \\
& - \left[\int_{\underline{\theta}}^{\overline{\theta}} \pi_2(z_T(\theta))[1 - b(L_2(z_T(\theta)))] \frac{L_2(z_T(\theta))}{\gamma} dF_T(\theta) \right] \int \theta dF_T(\theta) \\
& -(1 + \frac{1}{\gamma}) \left[\int a_{-1} dH_{a_{-1}} \right] \int \theta dF_T(\theta) - \frac{1}{\gamma} \left[\int h_{-1T} dJ_{h_{-1T}} \right] \int \theta dF_T(\theta) \\
& - \frac{1}{w^* \gamma} [\gamma - 1 - \lambda + s\lambda] \int \theta dF_T(\theta) \\
& \int [U(y(\theta)) + \theta y(\theta) + u(q(z_N(\theta))) + \theta z_N(\theta)] dF_N(\theta) \\
& - \left[\int_{\underline{\theta}}^{\overline{\theta}} \pi_1(z_N(\theta))[1 - b(L_1(z_N(\theta)))] \frac{L_1(z_N(\theta))}{\gamma} dF_N(\theta) \right] \int \theta dF_N(\theta) \\
& - \left[\int_{\underline{\theta}}^{\overline{\theta}} \pi_2(z_N(\theta))[1 - b(L_2(z_N(\theta)))] \frac{L_2(z_N(\theta))}{\gamma} dF_N(\theta) \right] \int \theta dF_N(\theta) \\
& - \frac{1}{\gamma} \left[\int h_{-1N} dJ_{h_{-1N}} \right] \int \theta dF_N(\theta) - \frac{1}{w^* \gamma} [\gamma - 1 - \lambda + s\lambda] \int \theta dF_N(\theta)
\end{aligned}
$$

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