Venture Capital, Patents and Innovation

Roberta Dessi[*], Nina YIN[†][‡]

Toulouse School of Economics (GREMAQ and IDEI)

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Abstract

This paper provides a new channel through which venture capital promote starts-ups in the sense that by gaining a private benefit from start-up financing without patent protection, the venture capital is more willing to finance entrepreneur at the beginning than other investors. Based on a double moral hazard model, we find that comparing to the non-VC investors, the willingness to invest is higher for venture capitalists, it mitigates the credit constraints of entrepreneur and thus facilitate the startup of entrepreneur and foster their productive innovation activities.

[*]IDEI, Toulouse School of Economics, Manufacture des Tabacs, Aile Jean-Jacques Laffont, 21 Allée de Brienne, 31000 Toulouse, France (dessi@cict.fr).

[†]Toulouse School of Economics, Manufacture des Tabacs, Aile Jean-Jacques Laffont, 21 Allée de Brienne, 31000 Toulouse, France (yinnina.ccer@gmail.com).

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1 Introduction

Does venture capital foster productive innovations? If so, how? Besides advising and management benefits, what's the advantage of venture capital financing in high-tech industries with innovation? Does patent protection always serve innovators positively? By answering these questions, our paper poses a novel channel through which venture capital promote starts-ups: without patent protection, venture capitalists could gain a private benefit from expropriation or knowledge tranmission from financed portforlio startups; thus it lowers down the willingness to fund of venture capitalists, softs the credit constraints of entrepreneurs, and therefore fosters entrepreneurs' innovation activities.

There has been substantial empirical literature exploring the impact of venture capital on innovation. By investigating 20 U.S. manufacturing industries between 1965 and 1992, Kortum and Lerner(2000) reveals that the venture capital is three times as potent as the corporate R&D and accouts for 8% of industrial innovations in the decade ending in 1992. However, limited by the industry data, Kortum and Lerner(2000) fails to distinguish the ex-ante selection effect from causal effect of venture capital on innovation. By employing individual firm data over different developed countries, Engel and Keilbach(2007), Caselli, Gatti and Perrini(2008), Haeussler, Harhoff and Muller(2009) and Peneder(2010) were able to find out that innovative firms have a higher probability of being venture funded. In other words, the selection effect does exist. Furthermore, Da Rin and Penas(2007) show that venture capital favors the buid-up of absorptive capacity and results in a more permanent in-house R&D efforts. Although not immediately overwelming, this evidence suggests that the causal effect of venture capital in innovation also exists. But overall, there is very little evidence on how venture capital affects innovation.

Besides considering the ex-ante screening effect, the theoretical literature on venture capital primarily illustrates two mechanisms whereby venture capitalists affect the performance of their portfolio firms: monitoring and intervention[1], which alleviates potential moral hazard problems

[1] See, for example, Dessi (2005) and Holmstrom and Tirole (1997).

on the side of entrepreneur and the provision of advice and support[2] which helps performance directly. Both mechanisms could apply to innovation. However, while there is growing evidence of the role played by venture capitalists in helping to commercialize innovations[3], as well as their role in helping to recruit key personnel and replace founders with new CEOs[4], there is very little direct evidence showing that they play an important role in fostering innovation. Our paper is going to fill this gap by providing a possible mechanism in which venture capital influence innovation.

The remaining part is organized as follows. Section 2 introduces the basic model setting. Section 3 simplifies the model as a benchmark. Section 4 and 5 consider the complete model and discusses the implications. Section 6 concludes.

2 The Model

2.1 Project

Consider an entrepreneur (start-up firm) endowed with an innovative investment project. The project requires a contractible initial investment I (money) and two unobservable (and *a fortiori* noncontractible) efforts by the entrepreneur: e_1 to transform an innovative idea into a valuable innovation ("innovative effort"); e_2 to make the project succeed ("commercialization effort"). The two efforts are sequential. The project is risky and the first, innovative effort may result in the development of a valuable innovation or it may fail. For simplicity, assume that the probability of a valuable innovation being developed is given by e_1. If there is such an innovation (and in the absence of expropriation, see below), the project will finally succeed with probability e_2, where e_2 captures the additional value of the entrepreneur's effort following the innovation.

[2] See, Bottazzi et al. (2005), Casamatta (2003), Cestone (2000), Cumming, Fleming and Suchard (2005), Dessi (2010), Hellman (1998), Jeng and Wells (2000), Kaplan et al. (2003), Lerner and Schoar (2005), Repullo and Suarez (2000, 2004), Riyanto and Schwienbacher (2006), and Schmidt (2003).

[3] Colombo, Grilli and Piva (2006), Gans, Hsu and Stern (2002), Hsu (2006).

[4] Hellmann and Puri (2002).

Thus to make things reasonable, there must be $e_1, e_2 < 1$. If there is no (valuable) innovation[5], the project's success probability is reduced; for simplicity, we assume it is equal to zero. If the project succeeds, it yields verifiable returns R; if it fails, it yields nothing ($R > 0$). All agents are risk neutral and protected by limited liability. For simplicity, R and I is normalized to be smaller than unit one[6].

Assume the entrepreneur has no initial monetary wealth, and needs to raise finance from outside investors. At date 0, assuming outside funding has been obtained and the project has been undertaken, the entrepreneur chooses his innovative effort level e_1, where $0 \leq e_1 < 1$. The cost of effort is given by $c(e_1) \equiv \frac{1}{2}e_1^2$.

For ease of analysis, for the moment, we don't consider the possibility that entrepreneur could apply for patent protection for his innovation, we will introduction patent protection in section 5. Without patent protection, anyone could potentially expropriate the innovation without infringement; Furthermore, the venture capitalist (henceforth VC) would find it much easier to expropriate the innovation than any other outsiders, since he interacted closely and repeatedly with the entrepreneur, and had privileged access to information, throughout the time in which the innovation was being developed. Therefore, for the moment, we don't consider the possibility that innovation is infringed or expropriated by other competitors except VC. We will introduce this situation in Section 5.

Expropriation by the venture capitalist may take different forms. The VC could sell knowledge to another start-up or established firm - although the difficulties of selling knowledge are well known (refs). Another form of expropriation seems more likely and easier for venture capitalists. The VC could communicate knowledge along with helpful advice to another firm it is already funding, so that the other firm's probability of success is increased while decreasing the chances of success of the expropriated entrepreneur.

[5] For ex-positional convenience, we will refer to this as the "no innovation" outcome, but it should be understood throughout as capturing also the possibility of a worthless innovation.

[6] Recalling that the entrepreneur's effort decision is $e^* = \arg\max_e eR - \frac{1}{2}e^2$ for the simplification case and from social optimal view. Therefore, the first order condition tells us $e^* = R$. According to our setting, the project will succeed with probability e, therefore, $e^* = R < 1$.

We therefore model expropriation very simply as follows. If the VC transfers knowledge about the innovation to another firm, the latter makes a gain with expected value G, while the innovating firm's probability of success decreases to ke_2 (where $1 > k > 0$). Expropriation by the VC cannot be the object of a straightforward sale. For this reason, we assume that the resulting benefit to the VC will be lower than the expected gain to the firm receiving the knowledge: denote it by λG, with $\lambda < 1$.

For simplicity, we also assume that except VC, there is no other firm that could develops an equivalent innovation independently or expropriates the entrepreneur's innovation. In Section 5, we will consider a more realistic situation that if the innovation hasn't been expropriated by VC, other firm could develops an equivalent innovation or expropriates the entrepreneur's innovation with probability α in the absence of patent.

After innovation has been developed, the entrepreneur chooses his second effort level e_2, where $0 \leq e_2 < 1$, and the cost of effort is given by $c(e_2) \equiv \frac{1}{2}e_2^2$. This second effort is needed to make the project succeed: the innovation has to be commercialized, key strategic decisions have to be made, new personnel may need to be recruited, and so on.

To summarize, expropriation by the VC means that the project will succeed with probability ke_2, and the VC enjoys a private benefit of value λG. We will assume that $\lambda G < I$, implying that the expected private benefit from expropriation on its own would never be sufficient to induce the venture capitalist to fund the project: he also needs a share of the project's financial returns. Also, we assume that $\lambda G + kR < 1$.The arguments for this assumption is the same as that for $R < 1$.For a social planner who decides to exert effort $e_1$to maximize the total revenue from the project and expropriation, i.e., $e_1^* = \arg\max_{e_1} e_1(\lambda G + e_2 kR) - \frac{1}{2}e_1^2$, the first order condition means that $e_1^* = \lambda G + e_2 kR$, it should be smaller than one since e_1^* is the probability as we assume it. Therefore, only when $\lambda G + kR < 1$, $e_1^* < 1$ no matter what e_2 is.

2.2 Investors

In our model the main difference between raising finance from a venture capitalist and raising it from other investors is due to the venture capitalist's connections with other firms, and in particular the fact that the VC can transfer knowledge relatively easily to another firm it is funding. As we shall see, this brings potential advantages and disadvantages. To focus on the trade-off between these costs and benefits, we abstract from other roles played by venture capitalists, such as monitoring or screening, which have been studied in a number of contributions to the literature on venture capital. For the same reason, we assume that venture capitalists, just like other investors, are competitive and do not earn any rents.

2.3 Contract Design

Contracts specify the investor's (venture capitalist's) financial contribution at the beginning (I), and the following contractual terms:

- a reward R_e for the entrepreneur at the intermediate stage, paid if, and only if, he has succeeded in developing a valuable innovation;
- a sharing rule for final returns, contingent on whether the innovation at the intermediate stage was patented. We denote these as $\left(R_e^P, R - R_e^P\right)$ and $\left(R_e^I, R - R_e^I\right)$.

2.4 Time line

The model has two periods and three dates, $0, 1, 2$. At the beginning of the first period (date 0), the contract is designed, the investment I is made to undertake the project, and the entrepreneur chooses effort level e_1 to develop the idea into an innovation. At the end of the first period (date 1), the innovation is generated with probability e_1. And if the project has been funded by a venture capitalist, the latter chooses whether to expropriate the innovation or not. The entrepreneur then chooses effort level e_2 to make the project succeed. At the end of the second period (date 2), the outcome of the project is realized.

3 Sequential efforts: complete information

Based on the framework we describe above, we begin our analysis of whether and when entrepreneurs prefer VC investors instead of other investors with the simplified version in which we assume that entrepreneurs will not apply the patent and there is no other possibility of expropriation. And in the next section, we will relax these assumptions and consider a more realistic situation.

To see why we consider the effort level of entrepreneur in two phase, a simple model without sequential efforts is provided in the appendix. The model proves that, without second-stage effort, the entrepreneur will always prefer to raise financing from VC rather than other investor with a contract which specifies all the project surplus to the entrepreneur before the decision made by VC about whether to expropriate. The VC's decision about expropriation fully internalizes the benefits and costs of expropriation and thus is socially optimal. And VC will be break even. This case is less realistic and not the situation we are interested in. Therefore, from now on, we will consider the case in which the entrepreneur's effort is required at the innovation stage and also at the further development and commercialization stage.

To build the intuition for the results, we first examine the first best case when all actions by the contracting parties are observable and contractible (efforts and expropriation by the VC). This will provide a useful benchmark for the analysis that follows. Since actions by the contracting parties (the entrepreneur and the investor or venture capitalist) are contractible, we can, without loss of generality, focus on the case where the project is funded by a venture capitalist.

- When expropriation of VC is not allowed, the objective function of this program becomes:

$$\max_{\hat{e}_1,\hat{e}_2} V(\hat{e}_1,\hat{e}_2) = \hat{e}_1 e_2 R - \frac{1}{2}\hat{e}_1^2 - \frac{1}{2}e_1 e_2^2$$

which gives us

$$\begin{aligned}\hat{e}_1^{FB} &= \frac{1}{2}R^2 \\ (\hat{e}_2)^{FB} &= R\end{aligned}$$

The overall revenue from this project is thus given by $V(\hat{e}_1, \hat{e}_2) = \frac{1}{2}(\hat{e}_1^{FB})^2$.

- When allowing venture capitalist to transfer knowledge to another firm, the objective function of this program becomes:

$$\max_{e_1, e_2} V(e_1, e_2) = e_1 [\lambda G + k e_2 R] - \frac{1}{2}e_1^2 - \frac{1}{2}e_1 e_2^2$$

which gives us

$$\begin{aligned}e_1^{FB} &= \lambda G + \frac{1}{2}k^2 R^2 \\ (e_2)^{FB} &= kR\end{aligned}$$

where e_1 is the first-stage effort level, e_2 is the second-stage effort level. Thus the overall revenue from this project can be deducted by plug effort levels in $V(e_1, e_2) = \frac{1}{2}(e_1^{FB})^2$.

- Comparing the case of expropriation with the case of no expropriation, we find that when $V(e_1, e_2) > V(\hat{e}_1, \hat{e}_2)$, expropriation by venture capitalists is favourable. This inequality refers to the threshold value for λG : as long as $\lambda G > (\lambda G)^*$, expropriation is of benefit for the project and should be encouraged, where $(\lambda G)^* \equiv \frac{1}{2}R^2(1 - k^2)$. Comparing to the case without second stage effort, where as long as $\lambda G > R(1-k)$, expropriation is favourable, in current situation, expropriation demands lower private benefit (i.e., $(\lambda G)^* < R(1-k)$). The reason is that VC is more willing to expropriate since the cost of expropriation is shared between VC and entrepreneur, while when there is no second-stage effort, entrepreneur extract all the surplus before expropriation take place, therefore VC bears the cost of expropriation by himself and thus will be reluctant to expropriate????.

4 Sequential efforts: Incomplete information

We now turn to the analysis of optimal contracts in the presence of asymmetric information. We begin by assuming that external finance is raised from investors and/or financial intermediaries with no opportunities for knowledge transfer.

4.1 Financing with knowledge transfer ruled out

Suppose knowledge transfer by investors is ruled out. If the entrepreneur succeeds in developing a valuable innovation, no expropriation is allowed. The model is illustrate in following Figure.The problem for entrepreneur is simply to design a contract $(R_e^0, R_e, R - R_e^0 - R_e)$ to induce the entrepreneur exert optimal effort while satisfying the investor's break even constraint, where R_e^0 is the claim go to the entrepreneur after entrepreneur develop a successful innovation, and R_e is the pay-off to entrepreneur when the project is finally successful. The second-stage effort level is

$$e_2^* = \arg\max_{e_2} e_2 R_e - \frac{1}{2} e_2^2 \Longrightarrow e_2^* = R_e \tag{1}$$

Given the second-effort level e_2^*, the first-stage effort level is

$$e_1^* = \arg\max_{e_1} e_1 R_e^0 + e_1 e_2 R_e - \frac{1}{2} e_1^2 - \frac{1}{2} e_1 e_2^2 \Longrightarrow e_1^* = R_e^0 + \frac{1}{2} R_e^2 \tag{2}$$

The optimal contract maximizes the entrepreneur's expected utility subject to the investor's participation constraint:

$$\max_{R_e^0, R_e} U = e_1^* R_e^0 + e_1^* e_2^* R_e - \frac{1}{2} e_1^{*2} - \frac{1}{2} e_1^* e_2^{*2} \tag{3}$$

$$s.t. \;\; e_1^* = R_e^0 + \frac{1}{2} R_e^2, \;\; e_2^* = R_e \;\; (IC_e) \tag{4}$$

$$e_1^* e_2^* (R - R_e) - e_1^* R_e^0 \geqslant I \;\; (PC_I) \tag{5}$$

$$0 \leqslant R_e, R_e^0 < R \;\; (LL_e) \tag{6}$$

The solution to this problem, $P1$, is described by the following result:

Lemma 1 *When the entrepreneur raises external finance from non-VC investors (no knowledge transfer), the optimal contract sets* $R_e^0 = 0$*,* $R_e \in [R_e^*, R)$*, where* $R_e^* = \frac{3}{4}R$*,* R_e *is a decreasing function of initial investment* I*, the effort level entrepreneur exerts is* $e_1 = \frac{1}{2}R_e^2, e_2 = R_e$ *which is lower than the social optimal level. The maximum investment entrepreneur could raise from non-VC investors is* $I_{\max}^i = \frac{27}{512}R^4$.

Proof. See the Appendix. ■

The solution to $P1$ can be illustrated in Figure 1:

Figure 1 plots the feasible pair of (R_e, I) which satisfy the non-VC investor's participation constraint PC_i. Since maximizing the objective function in $P1$ is equivalent to maximize R_e, thus the solution to $P1$ is to choose the maximum feasible contract R_e for given I. For example, the point $B(I^*, R_e^*)$ suggests the optimal contract is R_e^* when initial investment is I^*. Therefore, R_e locates in the interval $(\frac{3}{4}R, R)$ and it's a decreasing function of I. The maximum size of capital entrepreneur can raise from non-VC investor is $I_{\max}^i$,which is implementable when the project revenue claimed to the entrepreneur is $\frac{3}{4}R$.

With two-stage effort setting, the optimal contract to induce entrepreneur's second stage effort implies that the payment in intermediate stage should be postponed to the final stage, i.e., $R_e^0 = 0$. If part of the project revenue R rewards to entrepreneur before the second-stage effort

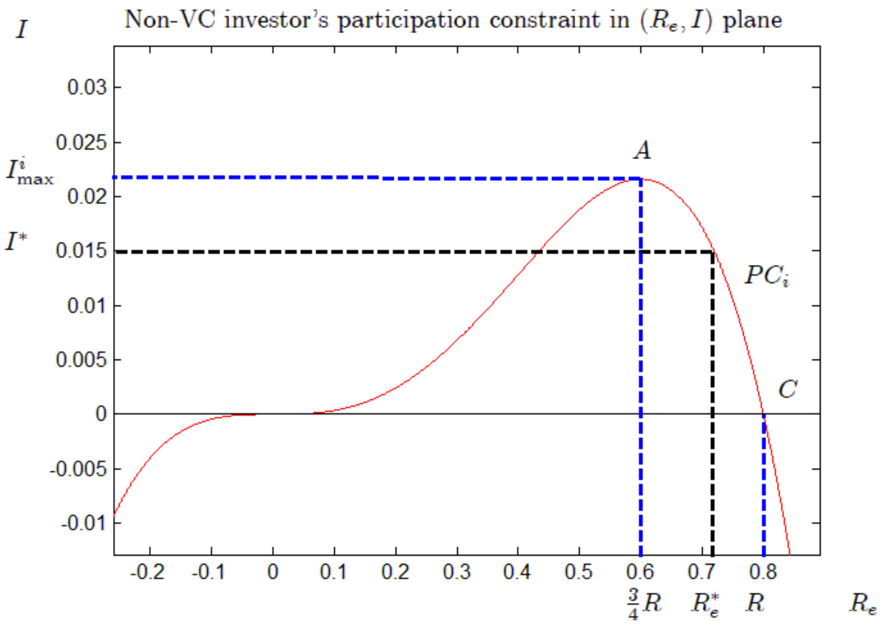

Figure 1:

exerting, it reduces the stake of second-stage effort, therefore decreases the incentive and cannot be optimal. Henceforth, in the following model, we will only consider the contract which rewards entrepreneur in the end.

4.2 Knowledge transfer: the role of venture capitalists

Now consider the case where external finance is provided by a venture capitalist, who can expropriate the innovation. If the VC chooses not to expropriate, this option cannot be better than funding by non-VC investors, from the entrepreneur's point of view. We can therefore focus on the case where the VC does expropriate the innovation. The second-stage effort level is

$$e_2^* = \arg\max_{e_2} ke_2\tilde{R}_e - \frac{1}{2}e_2^2 \Longrightarrow e_2^* = k\tilde{R}_e \tag{7}$$

Given the second-effort level e_2^*, the first-stage effort level is

$$e_1^* = \arg\max_{e_1} e_1 e_2 k\tilde{R}_e - \frac{1}{2}e_1^2 - \frac{1}{2}e_1 e_2^2 \Longrightarrow e_1^* = \frac{1}{2}k^2\tilde{R}_e^2 \tag{8}$$

The optimal contract maximizes the entrepreneur's expected utility subject to the investor's participation constraint, incentive compatibility constraint, the entrepreneur's incentive compatibility constraint and the feasible constraints:

$$\max_{R_e^0, R_e} U = e_1^* e_2^* k\tilde{R}_e - \frac{1}{2}e_1^{*2} - \frac{1}{2}e_1^* e_2^{*2} \tag{9}$$

$$s.t.\ \ e_1^* = \frac{1}{2}k^2\tilde{R}_e^2,\quad e_2^* = k\tilde{R}_e \quad (IC_e) \tag{10}$$

$$e_1^*[\lambda G + e_2^* k(R - \tilde{R}_e)] \geqslant I \quad (PC_{vc}) \tag{11}$$

$$e_1^*[\lambda G + e_2^* k(R - \tilde{R}_e)] \geqslant e_1^* e_2^*(R - \tilde{R}_e) \quad (IC_{vc}) \tag{12}$$

$$0 \leqslant \tilde{R}_e \quad (LL_e) \tag{13}$$

The solution to this problem, $P2$, is described by the following result:

Proposition 2 *When entrepreneur raises funding from Venture Capital, as long as*$\lambda G > B^*(I,k,R)$,*(where* B^* *is the cutoff value satisfing* $B^* = k(1-k)\tilde{R}_e(B^*)(R - \tilde{R}_e(B^*)))$*, the optimal contract* $\tilde{R}_e \in [\tilde{R}_e^*, \tilde{R}_e^{**})$ *is a decreasing function of initial investment* I*, and increases with* λG*, where* $\tilde{R}_e^* = \frac{\frac{3}{2}kR + \sqrt{\frac{9}{4}k^2R^2 + 8\lambda G}}{4k} > \frac{3}{4}R$, $\tilde{R}_e^{**} > R$ *is the maximum value of* $\tilde{R}_e$ *such that LHS of* PC_{VC} *equal to zero; the feasible pair of* (R_e, I) *is the curve DE as shown in Figure 2. The effort level entrepreneur exerts is* $e_1 = \frac{1}{2}k^2\tilde{R}_e^2, e_2 = k\tilde{R}_e$ *which might not be lower than the social optimal level if* $\tilde{R}_e > R$ *such that* $k\tilde{R}_e > R$.

Proof. See the Appendix. ■

The analysis of $P1$ and $P2$ can be illustrated in Figure 2.

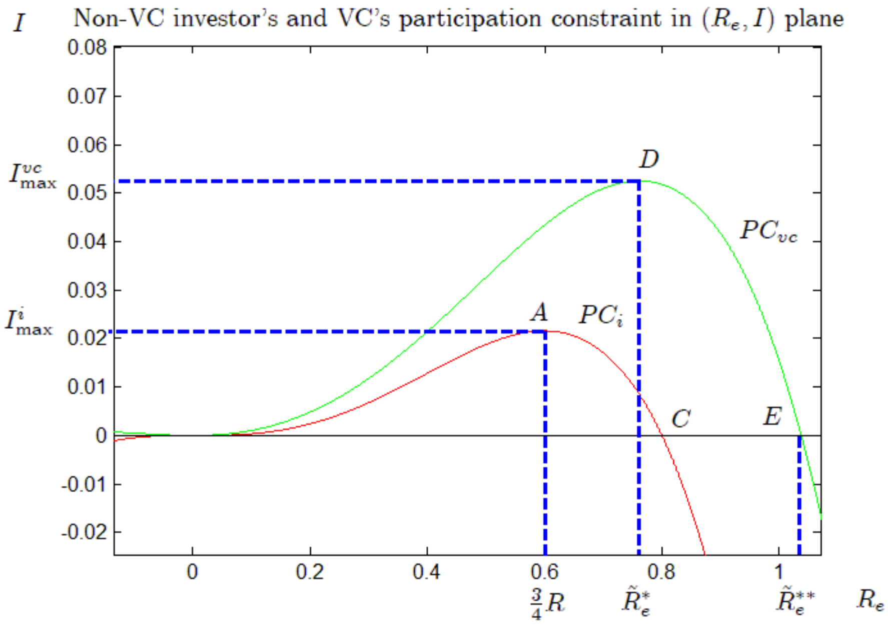

Figure 2:

Figure 2 plots the participation constraints of non-VC investor and VC. The red curve is non-VC investor's participation constraint(PC_i), while the green curve is the VC's participation constraint(PC_{vc}). Without loss of generality, we set $k = 0.9, \lambda G = 0.2, R = 0.8$ while plotting PC_{vc}. Similar to $P1$, the objective function of $P2$ is also equivalent to maximize the payment to the entrepreneur at the final stage $\tilde{R}_e$. However, besides to satisfying PC_{vc}, $P2$ is subjected to additional constraint: VC is willing to expropriate(IC_{vc}) .

$\tilde{R}_e^{**} > R$ implies that when λG is large enough such that even entrepreneur captures all the project revenue R and VC pays an extra benefit $\tilde{R}_e^{**} - R$ to entrepreneur, the project is still implementable without violating the participation constraint of VC.

From this proposition, we could find that the size of λG is critical in the capital financing of VC. It determines on what extent, it could relax the participation constraint of VC and benefit the entrepreneur who is subjeted to the reduces of project expected revenue due to expropriation. It's easy to see that as $\lambda G \to 0$,and $k \to 1$,the interval of $\tilde{R}_e$, $[\tilde{R}_e^*, \tilde{R}_e^{**})$will converge to $[R_e^*, R)$.

In this case, financing from venture capital makes no difference with financing from non-VC investors. Ituitively, if the private benefit of VC from expropriation is quite large (i.e., λG is far above 0), while the probability loss of project success is small (i.e., k is quite close to 1), then entrepreneur will always prefer to raise funding from VC; while, if λG is close to 0, while k is far lower than 1, then entrepreneur will prefer to acquire funding from non-VC investors.

4.3 Venture capital and innovation

When will the entrepreneur prefer to raise external finance from a venture capitalist? What are the implications for innovation? We can now address these questions. Comparing the entrepreneur's problem when raising external finance from VC and non-VC investors shows two key differences: when a valuable innovation is developed, the probability of success of the project decreases from e_2 to ke_2 under VC financing; at the same time, the VC (unlike non-VC investors) obtains a private benefit $B = \lambda G$. The effects on first-period effort work in opposite directions, as can be seen from problem $P2$ above. The decrease in the probability of success is equivalent to a reduction in k in problem $P2$: this reduces effort directly (as is clear from the entrepreneur's incentive compatibility condition) and indirectly, by making it more difficult to satisfy the participation constraint. The private benefit B, on the other hand, relaxes the participation constraint: a higher value of B therefore makes it possible to increase R_e without violating the participation constraint, thereby increasing effort.

A number of implications then follow from our analysis:

(i) venture capital can foster innovation by enabling capital-constrained innovative entrepreneurs essentially to "collateralize" potential knowledge transfers occurring in the absence of patent protection. Ex post, when a valuable innovation is developed but does not obtain patent protection, the venture capitalist can benefit by transferring knowledge to another firm it is funding. Ex ante, this relaxes the venture capitalist's participation constraint, making it possible to offer a higher share of success returns to the entrepreneur. This in turn elicits higher entrepreneurial effort, which increases the probability of developing a valuable innovation.

To our knowledge, this channel, through which venture capital can encourage innovation, has not been explored in the existing literature.

(ii) if entrepreneurs with potentially worthwhile innovative projects can all obtain the form of external finance they prefer, there is no reason to expect to find a positive association between venture backing and innovative success. However, if some of those that would prefer venture funding are unable to obtain it and secure non-venture funding instead, we would expect to observe a positive association between venture funding and innovation.

(iii) Venture capitalists tend to finance projects with higher potential innovation. λG which measures the expropriation value of venture capitalists also denotes the value of innovation. we can imagine that the more innovative idea leads to the higher expropriation value. And thus the entrepreneur with higher potentially innovative projects can be more easily financed by VC since they bring more private benefit to VC if expropriation.

This need not translate into a positive association between venture funding and ultimate project success, since this will also depend on development and commercialization efforts.

5 The possibility of patent and outsider's expropriation

In this section, we will introduce the patent and assume that after entrepreneur is successful in developing a valuable innovation, the entrepreneur can apply for patent protection. We assume that the patent protection will be approved with probability β, or rejected with probability $1-\beta$. The exogenous parameter β captures differences in the efficiency of the patenting process and the requirements of "novelty, nontrial, non-obviousness" for patenting across industries and/or countries, as well as differences in the potential for patenting products and processes with different characteristics. If the patent is granted, the innovation is under legal protection and no one can expropriate the idea and damage the commercial value of the project; however, if the patent is not granted, the entrepreneur faces the danger that the innovation might be expropriated by another firm, or that another firm might develop an equivalent innovation independently. We assume that this happens with probability α. If the entrepreneur obtains the funds to undertake the project from a venture capitalist, there is also a possibility of expropriation by the venture capitalist. Since VC would be easier to expropriate the innovation than any outsider, we assume that only when VC doesn't expropriate the innovation, the innovation will be expropriated by others with probability α; if VC expropriate it, then other outsiders will not expropriate again since there is no much surplus left for expropriation.

To analyze the advantage and disadvantage of VC-funding, we also comparing the case when raising funding from non-VC investors with the case when raising funding from VC. If the VC decides not to expropriate the innovation, it makes no difference between the non-VC investors, therefore, we restrict our attension to the situation that VC will expropriate given the condition that he prefers to expropriate is satisfied.

5.1 The non-VC investor case

If the entrepreneur succeeds in developing a valuable innovation but does not obtain a patent, there remains the possibility of expropriation by another firm, with probability α. The entrepreneur will take this into account in choosing his effort level in the second period. When the innovation is granted a patent, the second-period effort, e_2^P, is such that $e_2^P \in \arg\max_{e_2} e_2 R_e^P - \frac{1}{2}e_2^2$. The first order condition yields

$$e_2^P = R_e^P. \tag{14}$$

When the innovation is not granted as patent, on the other hand, second-period effort, e_2^I, is given by $e_2^I \in \arg\max_{e_2}(1-\alpha)e_2 R_e^I + \alpha k e_2 R_e^I - \frac{1}{2}e_2^2$. The first order condition yields

$$e_2^I = [1 - \alpha(1-k)]R_e^I \equiv z R_e^I. \tag{15}$$

The entrepreneur's effort in the first period, e_1, is determined by the incentive compatibility condition:

$$\begin{aligned} e_1 \in & \arg\max_{e_1} e_1\beta e_2^P R_e^P + e_1(1-\beta)ze_2^I R_e^I \\ & -\frac{1}{2}e_1^2 - \frac{1}{2}e_1[\beta(e_2^P)^2 + (1-\beta)(e_2^I)^2]|_{e_2^P = R_e^P, e_2^I = zR_e^I} \end{aligned} \quad (16)$$

yielding the first order condition:

$$e_1 = \frac{1}{2}[\beta(R_e^P)^2 + (1-\beta)(zR_e^I)^2] \quad (17)$$

The optimal contract maximizes the entrepreneur's expected utility subject to the investors' participation constraint, as well as the entrepreneur's incentive compatibility and limited liability constraints:

$$\begin{aligned} \max_{R_e, R_e^I, R_e^P} U &= e_1\beta e_2^P R_e^P + e_1(1-\beta)ze_2^I R_e^I - \frac{1}{2}e_1^2 - \frac{1}{2}e_1[\beta(e_2^P)^2 + (1-\beta)(e_2^I)^2] \\ s.t. \quad & e_2^P = R_e^P, e_2^I = zR_e^I, \\ e_1 &= \frac{1}{2}[\beta(R_e^P)^2 + (1-\beta)(zR_e^I)^2](IC_e) \\ & e_1[\beta e_2^P(R - R_e^P) + (1-\beta)e_2^I z(R - R_e^I)] \geq I, (PC_I) \\ R_e &\geqslant 0, R_e^I \geqslant 0, R_e^P \geqslant 0, (LL_e) \end{aligned}$$

The solution to this problem, $P3$, is described by the following result:

Proposition 3 *With the possibility of patent protection and the expropriation by others, when the entrepreneur raises external finance from non-VC investors (no knowledge transfer), the optimal contract sets* $R_e^P = R_e^I$ *which lies between* $\frac{3}{4}R$ *and* R*; and it's a decreasing function of* I *: as* I *increase from* 0 *to* $I_{\max}$, R_e^P *and* R_e^I *decrease from* R *to* $\frac{3}{4}R$*. At point* $R_e^P = R_e^I = \frac{3}{4}R$*, the investment level reaches the maximum* $\tilde{I}_{\max}^i = \frac{27c}{256}R \leqslant I_{\max}^i$*, where* $c = \frac{1}{2}(z^2 + \beta(1-z^2))^2 < \frac{1}{2}$

, $z = 1 - \alpha + \alpha k$.

Proof. *See the Appendix.* ■

Proposition 4 *As the probability of patent granting β or the parameter value k increases, the implementable investment level I increases, and the affordable payoff to entrepreneur R_e^P and R_e^I also increases. As the probability of being expropriated by others α increases, investment level I decreases, and the contract specified to entrepreneur R_e^P and R_e^I also decreases.*

Proof. See the Appendix. ■

It's easy to see that when there is possibility of being expropriated by others, $\tilde{I}^i_{\max} < I^i_{\max}$. And when $\alpha = 0, \tilde{I}^i_{\max} = I^i_{\max}$.

5.2 The VC investor case

Now consider the case where external finance is provided by a venture capitalist, who can expropriate the innovation ex post if it is not protected by a patent. If the VC chooses not to expropriate, this option cannot be better than funding by non-VC investors, from the entrepreneur's point of view. We can therefore focus on the case where the VC does expropriate the innovation in the absence of a patent. When the innovation is granted a patent, the second-period effort, e_2^P, is such that $e_2^P \in \arg\max_{e_2} e_2 R_e^P - \frac{1}{2}e_2^2$. The first order condition yields $e_2^P = R_e^P$.When the innovation is not granted as patent, on the other hand, second-period effort, e_2^I, is given by $e_2^I \in \arg\max_{e_2} k e_2 R_e^I - \frac{1}{2}e_2^2$. The first order condition yields $e_2^I = kR_e^I$. The entrepreneur's effort in the first period, e_1, is determined by the incentive compatibility condition:

$$\begin{aligned} e_1 \in\ & \arg\max_{e_1}\ e_1 \beta e_2^P R_e^P + e_1(1-\beta)k e_2^I R_e^I \\ & -\frac{1}{2}e_1^2 - \frac{1}{2}e_1[\beta(e_2^P)^2 + (1-\beta)(e_2^I)^2]|_{e_2^P = R_e^P, e_2^I = kR_e^I} \end{aligned} \quad (18)$$

yielding the first order condition: $e_1 = \frac{1}{2}[\beta(R_e^P)^2 + (1-\beta)(kR_e^I)^2]$.

The entrepreneur's problem becomes:

$$
\begin{aligned}
\max_{R_e, R_e^I, R_e^P} U &= e_1 \beta e_2^P R_e^P + e_1(1-\beta) k e_2^I R_e^I - \frac{1}{2}e_1^2 - \frac{1}{2}e_1\beta(e_2^P)^2 - \frac{1}{2}e_1 \\
s.t. \quad & e_2^P = R_e^P, e_2^I = kR_e^I, \\
e_1 &= \frac{1}{2}[\beta(R_e^P)^2 + (1-\beta)(kR_e^I)^2](IC_e) \\
e_1\beta e_2^P(R - R_e^P) + e_1(1-\beta)[\lambda G + k e_2^I(R - R_e^I)] &\geq I, (PC_{vc}^p) \\
\lambda G + k e_2^I (R - R_e^I) > [1 - \alpha(1-k)] e_2^I (R - R_e^I), (IC_{vc}^p) & \\
R_e &\geqslant 0, R_e^I \geqslant 0, R_e^P \geqslant 0, (LL_e)
\end{aligned}
$$

The solution to this problem, $P2$, is described by the following result:

Proposition 5 *With the possibility of patent protection and outsider's expropriation, when the entrepreneur raises external finance from non-VC investors (no knowledge transfer), as long as* $\lambda G > \bar{B}^*(I, k, R, \beta)$,*(where* $\bar{B}^*$ *is the cutoff value satisfing* $\bar{B}^* = k(z-k)\bar{R}_e(\bar{B}^*)(R - \bar{R}_e(\bar{B}^*))$*), the optimal contract sets* $R_e^P = R_e^I \equiv \bar{R}_e \in [\bar{R}_e^*, \bar{R}_e^{**})$ *, is a decreasing function of initial investment* I*, and increases with* λG*, where* $\bar{R}_e^* = \frac{\frac{3}{2}bR + \sqrt{\frac{9}{4}b^2R + 8b(1-\beta)\lambda G}}{4b} \in (\frac{3}{4}R, \tilde{R}_e^*], \bar{R}_e^{**} \in (R, \tilde{R}_e^{**})$ *is the maximum value of* $\bar{R}_e$ *such that LHS of* PC_{vc}^p *equal to zero. The effort level entrepreneur exerts is* $e_1 = \frac{1}{2}\left[\beta + (1-\beta)k^2\right]\bar{R}_e^2, e_2^p = \bar{R}_e, e_2^I = k\bar{R}_e$ *which might not be lower than the social optimal level if* $\bar{R}_e > R$ *such that* $k\bar{R}_e > R$*.*

Proof. *See the Appendix.* ■

5.3 Patent protection is good?

The parameter β which captures differences in the efficiency of the patenting process and the propensity of patent granting across industries and countries must closely related with the contract of our model and the behavior of agents. Investigation of β not only provides us policy insights about patenting process, but also make us more clear of the role of venture capitalists. The effect of β lies in two aspects: on the one hand, the entrepreneur tends to exert more effort at the first stage if the probability of being patented is high since it increase the probability that the final project succeeds; on the other hand, higher β makes the potential benefit of VC from financing less because that expropriation would be infringement under patent protection and will not happen. Therefore it reduces the project claims of entrepreneur comparing to the case without patent protection as long as in latter case expropriation is favorable by VC. This first effect is positive while the second one is negative. The overall effect of β depends on which effect dominates.

Proposition 6 *If* $\lambda G < \frac{R\bar{R}_e(1-k^2)}{2}$ & $\lambda G < \frac{b\bar{R}_e(2\bar{R}_e - \frac{3}{2}R)}{1-\beta}$ *or if* $\lambda G > \frac{R\bar{R}_e(1-k^2)}{2}$ & $\lambda G > \frac{b\bar{R}_e(2\bar{R}_e - \frac{3}{2}R)}{1-\beta}$ *then as* β *increases, the effort level* e_1 *increases and therefore the expected utility of entrepreneur increases; otherwise, if* $\frac{R\bar{R}_e(1-k^2)}{2} < \lambda G < \frac{b\bar{R}_e(2\bar{R}_e - \frac{3}{2}R)}{1-\beta}$*,then as* β *increases, the effort level* e_1 *decreases and the expected utility of entrepreneur decreases.*

Proof. See the appendix. ■

5.4 VC financing and outsiders' expropriation rate α

Proposition 7 *As the probability of being expropriated by outsiders* α *increases, the cutoff value of* λG*,* $\bar{B}^*$ *to satisfy the incentive participation constraint decreases.*

Proof. See the Appendix. ■

As we can see, when VC expropriate from entrepreneur's innovation, α plays no role directly in the return of entrepreneur and VC; however, it affects the incentive participation constraint of VC. It shifts the willingness of VC to expropriate the innovation in case of no patent protection. As the probability of being expropriated by outsiders α increases, the probability of successful project decrease when VC doesnot expropriate, which reduces the opportunity cost of VC to expropriate, therefore VC is more willing to expropriate the innovation in case of no patent protection.

6 Conclusion

This paper provides a new channel through which venture capital promote starts-ups in the sense that by gaining a private benefit from start-up financing without patent protection, the venture capital is more willing to finance entrepreneur at the beginning than other investors. Based on a double moral hazard model, we find that comparing to the non-VC investors, the willingness to invest is higher for venture capitalists, it mitigates the credit constraints of entrepreneur and thus facilitate the startup of entrepreneur and foster their productive innovation activities.

7 Reference

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8 Appendix

8.1 Appendix 1: Why sequential efforts matter: A simplified model

To see why the fact that the entrepreneur's effort is valuable in both stages matters, we can briefly consider a simplified model without second-stage effort. The picture of the model is in figure 3. The entrepreneur chooses innovative effort e, a valuable innovation is developed with probability e and the project succeeds with probability 1 in the most favourable case (innovation, no expropriation). If VC expropriate the innovation, VC will obtain a private benefit λG as usual and the project will be success with probability $k(0 < k < 1)$.

It is straightforward to verify that the optimal contract in this simplified setting rewards the entrepreneur at the intermediate stage if, and only if, he succeeds in developing a valuable innovation, while the VC receives the project's final returns. The intuition for this result is simple: since the entrepreneur only provides effort in the first stage, and his effort only affects the outcome at the intermediate stage, it is efficient to reward him on the basis of the intermediate outcome (innovation or no innovation). And since the VC receives the final project returns, he fully internalizes the costs and benefits for the project of patenting or expropriating any innovation, thereby making the efficient decisions at the second stage.

The result is summarized in the following:

Proposition 8 ***Corollary 9*** *By setting $R_e = kR + \lambda G - \frac{I}{kR+\lambda G}$, the entrepreneur will exert social optimal effort level as $e^{FB} = kR + \lambda G$. All the surplus $\frac{1}{2}e^2 - I$ goes to the entrepreneur; and the VC will transfer the knowledge to another firm if and only if $\lambda G > (1-k)R$, which is also social optimal.*

Proof. In the simplified model, without the second-stage effort, the incentive compatibility constraint of the entrepreneur is to choose the first-stage effort level e such that his expected net return is maximized, which is denoted as IC_e; the optimal contract problem is to set R_e such that maximize the revenue of the entrepreneur conditional on that the participation constraint of the venture capitalist and the incentive compatibility constraints of the entrepreneur and the VC is satisfied:

$$\begin{aligned}
\max_{R_e} U &= eR_e - \frac{1}{2}e^2 \\
s.t. \ e &\in \arg\max_{\tilde{e}} \tilde{e}R_e - \frac{1}{2}\tilde{e}^2 \ (IC_e); \\
e(kR+\lambda G) - eR_e &\geqslant I \ (PC_{VC}); \\
e(kR+\lambda G) - eR_e &\geqslant eR - eR_e \ (IC_{VC}).
\end{aligned}$$

The participation constraint of venture capitalist must be binding, thus plug it into the objective function and IC_e, we can find that the objective function of the entrepreneur is equivalent to the social value function,i.e., $e(kR+\lambda G) - I - \frac{1}{2}e^2$. From IC_e, we can get $e = kR + \lambda G$, and plug it into PC_{VC}, we have $R_e = kR + \lambda G - \frac{I}{kR+\lambda G}$. IC_{VC} implies that $\lambda G \geqslant (1-k)R$, therefore, $R_e \geqslant R - \frac{I}{R}$. As long as expropriation is efficient from the social point of view, which implies that IC_{VC} is satisfied, then the venture capitalist will transfer the knowledge to other firms, and the optimal contract for this model is to set $R_e = e = kR + \lambda G - \frac{I}{kR+\lambda G}$. The social surplus is $e(kR+\lambda G) - I - \frac{1}{2}e^2 = \frac{1}{2}(kR+\lambda G)^2 - I$. ■

An interesting feature of the simplified model without second-period effort is that the entrepreneur will always (weakly) prefer to raise financing from a venture capitalist rather than another investor. The reason is that the VC will always make the efficient decisions ex post, including the decision to transfer knowledge to another firm it is funding, when this is efficient. A non-VC investor, on the other hand, lacks the ability to transfer knowledge in this way.

However, with a second stage effort of entrepreneur, our problem becomes more realistic and meaningful since it's the entrepreneur who enforce the project till the final stage in reality. In this case, the entrepreneur cannot be rewarded right after the innovation stage, otherwise, he has no incentive to exert effort thereafter. Also by offering part of the final return to the entrepreneur, the venture capitalist cannot recover all the returns in the end from his decision of expropriation. Thus his decision will not be first-best. Even when the social optimal condition for expropriation is not met, as long as the venture capitalist's personal incentive constraint is satisfied, he will choose to expropriate. We will consider this problem step by step.

8.2 Proof of Lemma 1:

By substituting the incentive compatibility constraint of entrepreneur into the objective function, and simplifying it, the entrepreneur's problem (P1) can be rewritten as:

$$\max_{R_e, R_e^I, R_e^P} \ U = \frac{1}{2}(R_e^0 + \frac{1}{2}R_e^2)^2$$

$$s.t. \ \ (R_e^0 + \frac{1}{2}R_e^2)(RR_e - R_e^2 - R_e^0) \geqslant I(PC_I)$$

$$0 \leqslant R_e, R_e^0 < I(LL_e)$$

To implement the first-best effort levels in the second period, $e_2 = R$(as suggested in Section 3) would require setting $R_e = R$, which would violate the investors' participation constraint. Thus effort levels will be lower than the first-best levels, and the investors' participation constraint will hold as an equality.

Suppose $R_e^0 > 0$ at the optimum. Then it would be feasible to decrease R_e by dR_e without violating the entrepreneur's limited liability constraint. Consider corresponding increases in R_e, denoted by dR_e , that would keep first-period effort $R_e^0 + \frac{1}{2}R_e^2$ unchanged which implies $dR_e^0 + R_e dR_e = 0$.

Take the differential of the LHS of the participation constraint then gives

$$(dR_e^0 + R_e dR_e)(RR_e - R_e^2 - R_e^0) + (R_e^0 + \frac{1}{2}R_e^2)(RdR_e - 2R_e dR_e - dR_e^0)$$
$$= (dR_e^0 + R_e dR_e)(RR_e - R_e^2 - R_e^0) + (R_e^0 + \frac{1}{2}R_e^2)[(R - R_e)dR_e - (dR_e^0 + R_e dR_e)]$$

Since $dR_e^0 + R_e dR_e = 0$, then above formula can be reduced to $(R_e^0 + \frac{1}{2}R_e^2)(R - R_e)dR_e$. As we know, $R_e^0 + \frac{1}{2}R_e^2 > 0, R_e < R$, therefore, by decreasing dR_e^0 and increasing dR_e, the participation constraint of investor is relaxed without affecting the entrepreneur's expected utility. It implies we cannot be at the optimum if $R_e^0 > 0$. Hence at the optimum we must have $R_e^0 = 0$.

The entrepreneur's problem becomes:

$$\begin{aligned} \max_{R_e} U &= \frac{1}{2}(\frac{1}{2}R_e^2)^2 = \frac{1}{8}R_e^4 && (19) \\ s.t.\ \frac{1}{2}R_e^2(RR_e - R_e^2) &= I \ \ (PC_I) && (20) \\ 0 &\leqslant R_e < R \ \ (LL_e) && (21) \end{aligned}$$

The participation constraint implies $I = \frac{1}{2}R_e^4 + \frac{1}{2}RR_e^3 \equiv f(R_e)$, where $f(R_e)$ is a quatric function. The maximum value of this function is achieved at $R_e^* = \frac{3}{4}R$ (where $f'(R_e^*) = 0, f"(R_e^*) < 0$,it's easy to see that at R_e^* it's not only a local maximum but also a global maximum). Therefore, the maximum investment value the investor will provide is $I_{\max}^i = (-\frac{1}{2}R_e^4 + \frac{1}{2}RR_e^3)_{R_e=\frac{3}{4}R} = \frac{1}{8}(\frac{3}{4})^3R^4 = \frac{27}{512}R^4$. The objective function of entrepreneur is to choose R_e as large as possible while not violating PC_I. It implies that given value I, the contract is set in the interval $[\frac{3}{4}R, R)$.

8.3 Proof of proposition 2

By substituting the incentive compatibility constraint of entrepreneur into the objective function, and simplifying it, the entrepreneur's problem (P2) can be rewritten as:

$$\begin{aligned} \max_{R_e} U &= \frac{1}{8}k^4\tilde{R}_e^4 \\ s.t.\frac{1}{2}k^2\tilde{R}_e^2[\lambda G + k^2\tilde{R}_e(R-\tilde{R}_e)] &= I \ \ (PC_{VC}) \\ \lambda G &\geqslant k(1-k)\tilde{R}_e(R-\tilde{R}_e) \ \ (IC_{VC}) \\ 0 &\leqslant \tilde{R}_e \ \ (LL_e) \end{aligned}$$

We first assume IC_{VC} can be satisfied, and will check it later. From PC_{VC}, we have $I = -\frac{1}{2}k^4\tilde{R}_e^4 + \frac{1}{2}k^4R\tilde{R}_e^3 + \frac{1}{2}\lambda Gk^2\tilde{R}_e^2 \equiv g(\tilde{R}_e)$,where $g(\tilde{R}_e)$ is a quatric equation. We plot this quatric equation on $R_e - I$ plane in Figure 2. The maximum value of this function is achieved at

$\tilde{R}_e^* = \frac{\frac{3}{2}kR+\sqrt{\frac{9}{4}k^2R^2+8\lambda G}}{4k}$as shown in Figure 2 (where$g'(\tilde{R}_e^*) = 0, g"(\tilde{R}_e^*) < 0$,it's easy to see that at $\tilde{R}_e^*$ it's not only a local maximum but also a global maximum). Therefore, the maximum investment value the investor will provide is $I_{\max}^{vc} = g(\tilde{R}_e^*)|_{\tilde{R}_e^* = \frac{\frac{3}{2}kR+\sqrt{\frac{9}{4}k^2R^2+8\lambda G}}{4k}}$.

As in Figure 2, we can see that as long as we are choosing $\tilde{R}_e$ as large as possible to maximize the objective function, $\tilde{R}_e > 0$ can be easily satisfied. Given PC_{vc}, LL_e are all satisfied, we could obtain the optimal contract $\tilde{R}_e$(here, it's without constraint IC_{VC}, we will check IC_{VC} in the second step) ,which is a function of $I, \lambda G, k, R$, denoted as $h(I, \lambda G, k, R)$. Substitute it into the incentive compatibility constraint IC_{VC}, the constraint is essentially an inequality between I, k, R and λG,i.e., $\lambda G \geqslant k(1-k)h(\cdot)(R-h(\cdot))$. It's easy to see that: $h(\cdot)$ is increasing in λG(since as λG increase, $\tilde{R}_e^*, I_{\max}^{vc}, \tilde{R}_e^{**}$are all increase, (h, I) which are the curve between interval $(\tilde{R}_e^*, \tilde{R}_e^{**})$ moves upward and right, therefore for any give I, $h(\cdot)$ increases). Also the optimal contract $h(\cdot) \geqslant \tilde{R}_e^* > \frac{1}{2}R$, it implies $k(1-k)h(\cdot)(R-h(\cdot))$ is decreasing in $h(\cdot)$. Therefore, $k(1-k)h(\cdot)(R-h(\cdot))$ is decreasing in λG. Then there exists $B^*(I, k, R)$ such that when $\lambda G > B^*(I, k, R)$, IC_{VC} will be satisfied. And B^* is determined in the equation $B^* = k(1-k)h(B^*)(R-h(B^*))$. In short, when $B^*(I, k, R) < \lambda G < 1 - kR$, the contract is the curve between interval $(\tilde{R}_e^*, \tilde{R}_e^{**})$, the implicit condition is $B^*(I, k, R) < 1 - kR$, otherwise, there is no optimal contract available in this case.

8.4 Proof of proposition 3

We rewrite the entrepreneur's problem as:

$$\begin{aligned}
\max_{R_e^I, R_e^P} U &= e_1\beta R_e^P e_2^P + e_1(1-\beta)e_2^I z R_e^I - \frac{1}{2}e_1^2 - \frac{1}{2}e_1\beta(e_2^P)^2 - \frac{1}{2}e_1(1-\beta)(e_2^I)^2 \\
s.t. \quad & e_2^P = R_e^P, e_2^I = zR_e^I, \\
e_1 &= \frac{1}{2}[\beta(e_2^P)^2 + (1-\beta)(e_2^I)^2](IC_e) \\
& e_1\beta e_2^P(R - R_e^P) + e_1(1-\beta)e_2^I z(R - R_e^I) \geq I, (PC_I) \\
R_e &\geqslant 0, R_e^I \geqslant 0, R_e^P \geqslant 0, (LL_e)
\end{aligned}$$

Plug the IC_e into the objective function, and simplify it, we have $U = \frac{1}{2}e_1^2$.

The entrepreneur's problem becomes $(P3*)$:

$$\begin{aligned} \max_{R_e^I, R_e^P} U &= \frac{1}{2}e_1^2 &(22)\\ s.t. &: e_1 = \frac{1}{2}\beta x^2 + \frac{1}{2}(1-\beta)y^2 \ (IC_e) &(23)\\ & e_1\beta x(R-x) + e_1(1-\beta)y(zR-y) = I \ (PC_{VC}) &(24)\\ x &\geqslant 0, y \geqslant 0 &(25)\end{aligned}$$

where $x \equiv R_e^P$ and $y \equiv zR_e^I$.

The Lagrangian function for the problem can then be written as:

$$L = \frac{1}{2}e_1^2 + \mu\left[e_1\beta x(R-x) + e_1(1-\beta)y(zR-y) - I\right]$$

where μ is the lagrangian multiplier. Thus:

$$\begin{aligned} \frac{dL}{dx} &= e_1\frac{\partial e_1}{\partial x} + \mu\{\frac{\partial e_1}{\partial x}[\beta x(R-x) + (1-\beta)y(zR-y)] \\ &+e_1[\beta(R-2x]\} \end{aligned}$$

where

$$\frac{\partial e_1}{\partial x} = \beta x$$

$$\begin{aligned} \frac{\partial L}{\partial y} &= e_1\frac{\partial e_1}{\partial y} + \mu\frac{\partial e_1}{\partial y}[\beta x(R-x) + (1-\beta)y(zR-y)] \\ &+\mu e_1[(1-\beta)(zR-2y] \end{aligned}$$

where

$$\frac{\partial e_1}{\partial y} = (1-\beta)y$$

The first-order conditions are given by:

$$\begin{aligned}\frac{\partial L}{\partial x} &= e_1\beta x + \mu\beta x[\beta x(R-x) + (1-\beta)y(zR-y)] \\ &+\mu e_1[\beta(R-2x] = 0\end{aligned}$$

$$\begin{aligned}\frac{\partial L}{\partial y} &= e_1(1-\beta)y + \mu(1-\beta)y[\beta x(R-x) + (1-\beta)y(zR-y)] \\ &+\mu e_1[(1-\beta)(zR-2y)] = 0\end{aligned}$$

Let $w \equiv \beta x(R-x) + (1-\beta)y(zR-y)$. Then:

$$e_1\beta x = -\mu\{\beta xw + e_1[\beta(R-2x]\}$$

$$e_1(1-\beta)y = -\mu\{(1-\beta)yw + e_1[(1-\beta)(zR-2y)]\}$$

$$\frac{x}{y} = \frac{xw + e_1(R-2x)}{yw + e_1(zR-2y)}$$

$$xyw + xe_1(zR-2y) = yxw + ye_1(R-2x)$$

$$x(zR-2y) = y(R-2x)$$

$$xzR = yR$$

$$y = zx$$

where $x \equiv R_e^P$ and $y \equiv zR_e^I$, so that

$$R_e^P = R_e^I \tag{26}$$

Therefore, substitute $y = zx$ into program $(P3*)$, we have:

$$\max_x U = \frac{1}{2}e_1^2 \tag{27}$$

$$s.t.e_1 = \frac{1}{2}\beta x^2 + \frac{1}{2}(1-\beta)z^2x^2 \ (IC_e) \tag{28}$$

$$e_1\beta x(R-x) + e_1(1-\beta)zx(zR-zx) = I \ (PC_{VC}) \tag{29}$$

$$x \geqslant 0 \tag{30}$$

We substitute IC_e into the objective function, and get:

$$\max_x U = \frac{1}{8}x^4(z^2+\beta(1-z^2))^2 \tag{31}$$

$$s.t.\frac{1}{2}(z^2+\beta(1-z^2))^2x^3(R-x) = I \ (PC_i) \tag{32}$$

$$x \geqslant 0 \tag{33}$$

To implement the first-best effort levels in the second period, $e_2^P = R$ and $e_2^I = zR$, would require setting $R_e^P = R_e^I = R$, which would violate the investors' participation constraint. Thus effort levels will be lower than the first-best levels, and the investors' participation constraint will hold as an equality.

Maximizing the ojective function is equivalent to maximizing x, in this sense, the optimal contract is to choose the maximum value of x which satisfies the participation constraint of investors. Denote $c \equiv \frac{1}{2}(z^2 + \beta(1 - z^2))^2$, which is determined by exogenous parameters z and β. The participation constraint of investors can be rewritten as:

$$-cx^4 + cRx^3 = cx^3(R - x) = I$$

It's easy to see that: In the plane(x, I), the curve of PC_i across the point $(0, 0), (R, 0)$; and at point $x = \frac{3}{4}R$, I reaches the maximum value $I_{\max} = \frac{27c}{256}R^4$(since $I'(x = \frac{3}{4}R) = 0, I"(x = \frac{3}{4}R) < 0$, it's local maximum; and we can check that it's also a global maximum). When $x > R$, $I < 0$. Therefore, the maximum contract x must lie in the interval $[\frac{3}{4}R, R)$, and given the value I, the optimal contract to problem $(P3*)$ is determined by curve PC_i during the interval $[\frac{3}{4}R, R)$.

8.5 Proof of proposition 4

From the proof of proposition 3, it's easy to see that:

(1) As β increase, LHS of PC_i increase, the participation constraint will be relaxed; therefore, increasing a positive amount of I or x will not violate the participation constraint of investors and it increases the expected utility of entrepreneur. Even only increasing I without changing x to make the participation constraint binding, the objective function of entrepreneur also increases since β directly affect the utility function of entrepreneur.(2) Since $c = \frac{1}{2}(z^2 + \beta(1 - z^2))^2$,$\frac{\partial c}{\partial k} = \frac{\partial c}{\partial z}\frac{\partial z}{\partial k} = 2(z^2 + \beta(1 - z^2))(1 - \beta)z\alpha > 0$, $\frac{\partial c}{\partial \alpha} = \frac{\partial c}{\partial z}\frac{\partial z}{\partial \alpha} = -2(z^2 + \beta(1 - z^2))(1 - \beta)z(\& - k) < 0$,therefore as k increases or α decrease, c increases, it has the effect as β.

8.6 Proof of proposition 5

We rewrite the entrepreneur's problem as:

$$\begin{aligned}
\max_{R_e^I, R_e^P} U &= e_1 \beta e_2^P R_e^P + e_1(1-\beta) k e_2^I R_e^I - \frac{1}{2} e_1^2 - \frac{1}{2} e_1 \beta (e_2^P)^2 - \frac{1}{2} e_1 (1-\beta \\
s.t. & \quad e_2^P = R_e^P, e_2^I = k R_e^I, \\
e_1 &= \frac{1}{2}[\beta (R_e^P)^2 + (1-\beta)(k R_e^I)^2](IC_e) \\
e_1 \beta e_2^P (R - R_e^P) + e_1(1-\beta)[\lambda G + k e_2^I (R - R_e^I)] &\geq I, (PC_{VC}) \\
\lambda G + k e_2^I (R - R_e^I) > z e_2^I (R - R_e^I), (IC_{VC}) & \\
R_e &\geqslant 0, R_e^I \geqslant 0, R_e^P \geqslant 0, (LL_e)
\end{aligned}$$

By substitute IC_e into the objective function, we simplify it and get $U = \frac{1}{2} e_1^2$.

The entrepreneur's problem becomes:

$$\begin{aligned}
\max_{R_e^I, R_e^P} U &= \frac{1}{2} e_1^2 \\
s.t. &: \quad e_1 = \frac{1}{2} \beta (R_e^P)^2 + \frac{1}{2}(1-\beta)(k R_e^I)^2 \\
& \quad e_1 \beta R_e^P (R - R_e^P) + e_1 (1-\beta)[\lambda G + k^2 R_e^I (R - R_e^I)] = I \\
R_e &\geqslant 0, R_e^I \geqslant 0, R_e^P \geqslant 0 \\
\lambda G &> (z-k) k R_e^I (R - R_e^I)
\end{aligned} \tag{34}$$

The entrepreneur's problem becomes $(P4*)$:

$$\begin{aligned}
\max_{\tilde{x},\tilde{y}} U &= \frac{1}{2}e_1^2 \\
e_1 &= \frac{1}{2}\beta\tilde{x}^2 + \frac{1}{2}(1-\beta)\tilde{y}^2 \ (IC_e) && (35) \\
e_1[\beta\tilde{x}(R-\tilde{x}) + (1-\beta)(\lambda G + \tilde{y}(kR-\tilde{y}))] &= I \ (PC_{VC}) && (36) \\
\tilde{x} &\geqslant 0, \tilde{y} \geqslant 0 && (37) \\
\lambda G &> \frac{z-k}{k}\tilde{y}(kR-\tilde{y}) \ (IC_{VC}) && (38)
\end{aligned}$$

where $\tilde{x} \equiv R_e^P$ and $\tilde{y} \equiv kR_e^I$. To begin with, we ignore the feasible constraints (37) and check it afterwards.

The Lagrangian function for the problem can then be written as:

$$L = \frac{1}{2}e_1^2 + \mu\{e_1[\beta\tilde{x}(R-\tilde{x}) + (1-\beta)(\lambda G + \tilde{y}(kR-\tilde{y}))] - I\}$$

where μ is the lagrangian multiplier. Thus:

$$\begin{aligned}
\frac{\partial L}{\partial \tilde{x}} &= e_1\frac{\partial e_1}{\partial \tilde{x}} + \mu\{\frac{\partial e_1}{\partial \tilde{x}}[\beta\tilde{x}(R-\tilde{x}) + (1-\beta)(\lambda G + \tilde{y}(kR-\tilde{y}))] \\
&\quad + e_1[\beta(R-2\tilde{x}]\}
\end{aligned}$$

where

$$\frac{\partial e_1}{\partial \tilde{x}} = \beta\tilde{x}$$

$$\begin{aligned}
\frac{\partial L}{\partial \tilde{y}} &= e_1\frac{\partial e_1}{\partial \tilde{y}} + \mu\{\frac{\partial e_1}{\partial \tilde{y}}[\beta\tilde{x}(R-\tilde{x}) + (1-\beta)(\lambda G + \tilde{y}(kR-\tilde{y}))] \\
&\quad + e_1[(1-\beta)(kR-2\tilde{y}]\}
\end{aligned}$$

where

$$\frac{\partial e_1}{\partial \tilde{y}} = (1-\beta)\tilde{y}$$

The first-order conditions are given by:

$$\begin{aligned}\frac{\partial L}{\partial x} &= e_1\beta\tilde{x} + \mu\{\beta\tilde{x}[\beta\tilde{x}(R-x) + (1-\beta)(\lambda G + \tilde{y}(kR-\tilde{y}))] \\ &\quad + e_1[\beta(R-2\tilde{x}]\} = 0\end{aligned}$$

$$\begin{aligned}\frac{\partial L}{\partial y} &= e_1(1-\beta)\tilde{y} + \mu\{(1-\beta)\tilde{y}[\beta\tilde{x}(R-\tilde{x}) + (1-\beta)(\lambda G + \tilde{y}(kR-\tilde{y}))] \\ &\quad + e_1[(1-\beta)(kR-2\tilde{y})]\} = 0\end{aligned}$$

Let $w \equiv \beta\tilde{x}(R-\tilde{x}) + (1-\beta)(\lambda G + \tilde{y}(kR-\tilde{y}))$. Then:

$$e_1\beta\tilde{x} = -\mu\{\beta\tilde{x}w + e_1[\beta(R-2\tilde{x}]\}$$

$$e_1(1-\beta)\tilde{y} = -\mu\{(1-\beta)\tilde{y}w + e_1[(1-\beta)(kR-2\tilde{y})]\}$$

$$\frac{\tilde{x}}{\tilde{y}} = \frac{\tilde{x}w + e_1(R-2\tilde{x})}{\tilde{y}w + e_1(kR-2\tilde{y})}$$

$$\tilde{x}\tilde{y}w + \tilde{x}e_1(kR-2\tilde{y}) = \tilde{y}\tilde{x}w + \tilde{y}e_1(R-2\tilde{x})$$

$$\tilde{x}(kR-2\tilde{y}) = \tilde{y}(R-2\tilde{x})$$

$$\tilde{x}kR = \tilde{y}R$$

$$k\tilde{x} = \tilde{y}$$

where $x \equiv R_e^P$ and $y \equiv kR_e^I$, so that

$$R_e^P = R_e^I \tag{39}$$

Therefore, the Program $(P4*)$ could be rewritten as:

$$\begin{aligned}
\max_{\tilde{x}} \ U &= \frac{1}{2}e_1^2 \\
e_1 &= \frac{1}{2}[\beta + (1-\beta)k^2]\tilde{x}^2 \ (IC_e) && (40) \\
e_1[\beta + (1-\beta)k^2]\tilde{x}(R-\tilde{x}) + e_1(1-\beta)\lambda G &= I \ (PC_{VC}) && (41) \\
\lambda G &\geqslant (z-k)k\tilde{x}(R-\tilde{x}) \ (IC_{VC}) && (42) \\
\tilde{x} &\geqslant 0(LL_e) && (43)
\end{aligned}$$

Let $b = \beta + (1-\beta)k^2 = k^2 + \beta(1-k^2)$, and substitute IC_e into the objective function and PC_{vc}, IC_{vc},we have:

$$\begin{aligned}
\max_{\tilde{x}} \ U &= \frac{1}{8}b^2\tilde{x}^4 && (44) \\
-\frac{1}{2}b^2\tilde{x}^4 + \frac{1}{2}b^2R\tilde{x}^3 + \frac{1}{2}b(1-\beta)\lambda G\tilde{x}^2 &= I \ (PC_{VC}^p) && (45) \\
\lambda G &\geqslant (z-k)k(R\tilde{x} - \tilde{x}^2) \ (IC_{VC}^p) && (46) \\
\tilde{x} &\geqslant 0(LL_e) && (47)
\end{aligned}$$

Consider curve PC_{vc}^p in plane$(\tilde{x}, I)$, where $\tilde{x} = R_e^P = R_e^I$, the first derivative of I on $\tilde{x}$ is $I'(\tilde{x}) = -2b^2\tilde{x}^3 + \frac{3}{2}b^2R\tilde{x}^2 + b(1-\beta)\lambda G\tilde{x}$. At points $\tilde{x}_1 = 0, \tilde{x}_2 = \frac{\frac{3}{2}bR - \sqrt{\frac{9}{4}b^2R + 8b(1-\beta)\lambda G}}{4b}$,and $\tilde{x}_3 = \frac{\frac{3}{2}bR + \sqrt{\frac{9}{4}b^2R + 8b(1-\beta)\lambda G}}{4b}$, $I'(\tilde{x}) = 0$. It's easy to see that $\tilde{x}_2 < 0$ and $\tilde{x}_3 > \frac{3}{4}R > 0$. It's easy to prove that $I"(\tilde{x}_3) < 0$(since $I"(\tilde{x}) = -6b^2\tilde{x}^2 + 3b^2R\tilde{x} + b(1-\beta)\lambda G$ is a parabola, is decreasing if $\tilde{x} > \frac{R}{4}$and when $\tilde{x} > \tilde{x}_4 = \frac{R}{4} + \frac{\sqrt{9b^2R^2 + 24b(1-\beta)\lambda G}}{12b}$, $I"(\tilde{x}) < 0$, and we can check $\tilde{x}_3 > \tilde{x}_4$), therefore $\tilde{x}_3$ is a local maximum of curve PC_{vc}. It's easy to see that it's also a global maximum. Denote $\tilde{x}_3 \equiv \bar{R}_e^*$, therefore, the optimal contract should lie in the interval between $[\bar{R}_e^*, \bar{R}_e^{**})$, where $\bar{R}_e^{**}$ is the maximum point that makes I equal to zero. It's easy to see that as $\beta \to 0, PC_{vc}^p \to PC_{vc}$, the problem $(P4)$ reduces to problem $(P2)$. However, as long as $\beta > 0, \bar{R}_e^* < \tilde{R}_e^*, \bar{R}_e^{**} < \tilde{R}_e^{**}$. Also from $I(R) = -\frac{1}{2}b^2R^4 + \frac{1}{2}b^2R^4 + \frac{1}{2}b(1-\beta)\lambda GR^2 = \frac{1}{2}b(1-\beta)\lambda GR^2 > 0$, we get that $\bar{R}_e^{**} > R$, since $I(\bar{R}_e^{**}) = 0$.

We can see that as long as we are choosing $\bar{R}_e$ as large as possible to maximize the objective function, $\bar{R}_e > 0$ can be easily satisfied. Given PC_{vc}^p, LL_e are all satisfied, we could obtain the optimal contract $\bar{R}_e$(here, it's without constraint IC_{VC}^p, we will check IC_{VC}^p in the second step) ,which is a function of $I, \lambda G, k, R, \beta$, denoted as $\bar{h}(I, \lambda G, k, R, \beta)$. Substitute it into the incentive compatibility constraint IC_{VC}^p, the constraint is essentially an inequality between I, k, R, β and λG,i.e., $\lambda G \geqslant k(z-k)\bar{h}(\cdot)(R - \bar{h}(\cdot))$. It's easy to see that: $\bar{h}(\cdot)$ is increasing in λG(since as λG increase, $\bar{R}_e^*, I_{\max}^{vc}, \bar{R}_e^{**}$are all increase, $(\bar{h}, I)$ which are the curve between interval $(\bar{R}_e^*, \bar{R}_e^{**})$ moves upward and right, therefore for any give I, $\bar{h}(\cdot)$ increases). Also the optimal contract $\bar{h}(\cdot) \geqslant \bar{R}_e^* > \frac{1}{2}R$, it implies $k(z-k)\bar{h}(\cdot)(R - \bar{h}(\cdot))$ is decreasing in $\bar{h}(\cdot)$. Therefore, $k(z-k)\bar{h}(\cdot)(R - \bar{h}(\cdot))$ is decreasing in λG. Then there exists $\bar{B}^*(I, k, R, \beta)$ such that when $\lambda G > \bar{B}^*(I, k, R, \beta)$, IC_{VC}^p will be satisfied. And $\bar{B}^*$ is determined in the equation $\bar{B}^* = k(z-k)\bar{h}(\bar{B}^*)(R - \bar{h}(\bar{B}^*))$. In short, when $\bar{B}^*(I, k, R, \beta) < \lambda G < 1 - kR$, the contract is the curve between interval $(\bar{R}_e^*, \bar{R}_e^{**})$, the implicit condition is $\bar{B}^*(I, k, R, \beta) < 1 - kR$, otherwise, there is no optimal contract available in this case. Notice that since $\bar{B}^* \geqslant 0, z > k$, therefore, $\bar{h}(\bar{B}^*) \leqslant R$.

8.7 Proof of proposition 6

We know that $\frac{dU}{d\beta}=\frac{\partial U}{\partial e_1}\frac{de_1}{d\beta}=e_1\cdot\frac{de_1}{d\beta}$, therefore to obtain the derivative of U on β, is equivalent to see the derivative of e_1 on β. From $e_1=\frac{1}{2}\beta(R_e^P)^2+\frac{1}{2}(1-\beta)(kR_e^I)^2=\frac{1}{2}\left[\beta+(1-\beta)k^2\right]\bar{R}_e^2$, we know that $\frac{de_1}{d\beta}=\frac{\partial e_1}{\partial\beta}+\frac{\partial e_1}{\partial\bar{R}_e}\frac{\partial\bar{R}_e}{\partial\beta}$.

Take the partial derivative of $e_1$over β, we get that $\frac{\partial e_1}{\partial\beta}=\frac{1}{2}(1-k^2)\bar{R}_e^2$. Similarly, we have $\frac{\partial e_1}{\partial\bar{R}_e}=\left[\beta+(1-\beta)k^2\right]\bar{R}_e$.

To derive $\frac{\partial\bar{R}_e}{\partial\beta}$, notice that from proposition 5, we get that as long as the IC_{vc}^p,i.e., $\lambda G>\bar{B}^*(I,k,R,\beta)$ is satisfied, $\bar{R}_e$ is determined by $PC_{vc}^p:-\frac{1}{2}b^2\bar{R}_e^4+\frac{1}{2}b^2R\bar{R}_e^3+\frac{1}{2}b(1-\beta)\lambda G\bar{R}_e^2=I$. Take a full differentiation of this equation over b, β and $\bar{R}_e$, we have:

$$-b\bar{R}_e^4db-2b^2\bar{R}_e^3d\bar{R}_e+bR\bar{R}_e^3db+\frac{3}{2}b^2R\bar{R}_e^2d\bar{R}_e+\frac{1}{2}(1-\beta)\lambda G\bar{R}_e^2db-\frac{1}{2}b\lambda G\bar{R}_e^2d\beta+b(1-\beta)\lambda G\bar{R}_ed\bar{R}_e=0$$

Since $db=(1-k^2)d\beta$, substitute it into the above equation and rearrange it, we get that

$$\left\{\left[-b\bar{R}_e^4+bR\bar{R}_e^3+\frac{1}{2}(1-\beta)\lambda G\bar{R}_e^2\right](1-k^2)-\frac{1}{2}b\lambda G\bar{R}_e^2\right\}d\beta=\left[2b^2\bar{R}_e^3-\frac{3}{2}b^2R\bar{R}_e^2-b(1-\beta)\lambda G\bar{R}_e\right]d\bar{R}_e$$

Therefore,

$$\frac{d\bar{R}_e}{d\beta}=\frac{\left[-b\bar{R}_e^4+bR\bar{R}_e^3+\frac{1}{2}(1-\beta)\lambda G\bar{R}_e^2-\frac{\frac{1}{2}b\lambda G\bar{R}_e^2}{1-k^2}\right](1-k^2)}{2b^2\bar{R}_e^3-\frac{3}{2}b^2R\bar{R}_e^2-b(1-\beta)\lambda G\bar{R}_e}$$

Substitute the above expression into $\frac{de_1}{d\beta}=\frac{\partial e_1}{\partial\beta}+\frac{\partial e_1}{\partial\bar{R}_e}\frac{\partial\bar{R}_e}{\partial\beta}$, we have

$$\frac{de_1}{d\beta}=\frac{1}{2}(1-k^2)\bar{R}_e^2+\left[\beta+(1-\beta)k^2\right]\bar{R}_e\cdot\frac{\left[-b\bar{R}_e^4+bR\bar{R}_e^3+\frac{1}{2}(1-\beta)\lambda G\bar{R}_e^2-\frac{\frac{1}{2}b\lambda G\bar{R}_e^2}{1-k^2}\right](1-k^2)}{2b^2\bar{R}_e^3-\frac{3}{2}b^2R\bar{R}_e^2-b(1-\beta)\lambda G\bar{R}_e}$$

Since $b = \beta + (1-\beta)k^2$, the above equation can be simplified as

$$\begin{aligned}
\frac{de_1}{d\beta} &= \frac{1}{2}(1-k^2)\bar{R}_e^2 + \frac{\left[-b\bar{R}_e^4 + bR\bar{R}_e^3 + \frac{1}{2}(1-\beta)\lambda G\bar{R}_e^2 - \frac{\frac{1}{2}b\lambda G\bar{R}_e^2}{1-k^2}\right](1-k^2)}{2b\bar{R}_e^2 - \frac{3}{2}bR\bar{R}_e - (1-\beta)\lambda G} \\
&= \frac{\frac{1}{2}(1-k^2)\bar{R}_e^2\left[2b\bar{R}_e^2 - \frac{3}{2}bR\bar{R}_e - (1-\beta)\lambda G\right] + \left[-b\bar{R}_e^2 + bR\bar{R}_e + \frac{1}{2}(1-\beta)\lambda G - \frac{\frac{1}{2}b\lambda G}{1-k^2}\right]\bar{R}_e^2(1-k^2)}{2b\bar{R}_e^2 - \frac{3}{2}bR\bar{R}_e - (1-\beta)\lambda G} \\
&= \frac{(1-k^2)\bar{R}_e^2\left[b\bar{R}_e^2 - \frac{3}{4}bR\bar{R}_e - \frac{(1-\beta)}{2}\lambda G\right] + \left[-b\bar{R}_e^2 + bR\bar{R}_e + \frac{(1-\beta)}{2}\lambda G - \frac{\frac{1}{2}b\lambda G}{1-k^2}\right]\bar{R}_e^2(1-k^2)}{b\bar{R}_e(2\bar{R}_e - \frac{3}{2}R) - (1-\beta)\lambda G} \\
&= \frac{(1-k^2)\bar{R}_e^2\left[b\bar{R}_e^2 - \frac{3}{4}bR\bar{R}_e - \frac{(1-\beta)}{2}\lambda G - b\bar{R}_e^2 + bR\bar{R}_e + \frac{(1-\beta)}{2}\lambda G - \frac{\frac{1}{2}b\lambda G}{1-k^2}\right]}{b\bar{R}_e(2\bar{R}_e - \frac{3}{2}R) - (1-\beta)\lambda G} \\
&= \frac{(1-k^2)\bar{R}_e^2\left[\frac{1}{4}bR\bar{R}_e - \frac{\frac{1}{2}b\lambda G}{1-k^2}\right]}{b\bar{R}_e(2\bar{R}_e - \frac{3}{2}R) - (1-\beta)\lambda G} = \frac{\frac{1}{4}b\bar{R}_e^2\left[R\bar{R}_e(1-k^2) - 2\lambda G\right]}{b\bar{R}_e(2\bar{R}_e - \frac{3}{2}R) - (1-\beta)\lambda G}
\end{aligned}$$

If $\lambda G < \frac{R\bar{R}_e(1-k^2)}{2}$ & $\lambda G < \frac{b\bar{R}_e(2\bar{R}_e - \frac{3}{2}R)}{1-\beta}$, $\frac{de_1}{d\beta} > 0$; if $\frac{R\bar{R}_e(1-k^2)}{2} < \lambda G < \frac{b\bar{R}_e(2\bar{R}_e - \frac{3}{2}R)}{1-\beta}$, $\frac{de_1}{d\beta} < 0$; if $\lambda G > \frac{R\bar{R}_e(1-k^2)}{2}$ & $\lambda G > \frac{b\bar{R}_e(2\bar{R}_e - \frac{3}{2}R)}{1-\beta}$, $\frac{de_1}{d\beta} > 0$.

8.8 Proof of proposition 7

Take a total derivative of equation $\bar{B}^* = k(z-k)h(\bar{B}^*)(R - \bar{h}(\bar{B}^*))$ of variable $\bar{B}^*$ and z. We have

$$d\bar{B}^* = k\bar{h}(R-\bar{h})dz + k(z-k)(R-2\bar{h})\bar{h}'dB$$

rearrange the above equation, we get

$$\frac{d\bar{B}^*}{dz} = \frac{k\bar{h}(R-\bar{h})}{1 - k(z-k)(R-2\bar{h})\bar{h}'}$$

we know that $\bar{h}$ is increasing in λG,i.e., $\bar{h}' > 0$, and since $\bar{h} > \frac{1}{2}R, z > k$, therefore the denominator of the RHS of above equation is positive, $1 - k(z-k)(R-2\bar{h})\bar{h}' > 0$. Also we have got that $\bar{h} \leqslant R$,therefore $\frac{d\bar{B}^*}{dz} \geqslant 0$. And we know that $\frac{\partial z}{\partial \alpha} < 0$, therefore, then $\frac{d\bar{B}^*}{d\alpha} \leqslant 0$, which implies that as α increases, the cutoff value of λG to satisfy the incentive participation constraint decreases.