Skills, Mismatch and Inequality: Labour Market Frictions and Costly Technology.

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Abstract

Mismatch can be considered as misallocation, and may have effects on both productivity and inequality. This paper provides a quantitative assessment of a model of mismatch for a frictional labour market in which workers differ by skill level, jobs differ by skill requirement and creating a vacancy with a given skill-requirement entails a fixed cost (of technology adoption). We calibrate the model to US data. In contrast to the existing literature on mismatch, we calibrate to observable targets, such as skill-specific unemployment rates and wage premia, rather than unobservable or poorly identified parameters such as the value of leisure, vacancy posting costs and relative productivity. Using this calibration strategy we show that the equilibrium with mismatch cannot arise for any feasible combination of parameter values; whereas regrading and segmentation are plausible outcomes. Fixed costs of technology adoption, which are omitted from previous studies, play a critical role. Without fixed costs the equilibria proposed in the literature do not exist, because a firm is unable to commit to hold open a job with a given skill requirement in the event that it meets a worker of a different skill level. We show that a much richer set of equilibria arises if a firm must pay a fixed cost to open a vacancy with a particular skill-requirement. As well as equilibria with mismatch and with segmentation, new equilibria may arise in which a firm regrades its vacancy so that its skill-requirements match the skill level of the worker it has met. Finally, we demonstrate that the existence of multiple equilibria in previous work is simply an artefact of the calibration framework adopted therein.

1 Introduction.

How do frictional labour markets behave when workers have heterogeneous skills and jobs have differing skill-requirements? What can account for skill-specific differences in labour market outcomes such as wages and unemployment? What difference does the nature of interaction between workers of different skill levels (and vacancies with different skill-requirements) make? One approach is to treat the heterogenous labour market as segmented, so that jobs are skill-specific. Then one skill group impacts on another primarily through demand.[1] Another possibility arises when at least one job may be undertaken by more than one skill-level of worker and at least one side of the labour market does not segregate prior to search. In this case the interaction between workers with different skills (and between jobs with different skill requirements) in the search and matching process may be a key mechanism in generating misallocation of resources (mismatch) and influencing aggregate productivity. Inequality, as described in Table 1, may also be influenced by this process. For example, if high-skilled workers may undertake low skill-requirement jobs, this may act to crowd out low-skilled workers (increasing the congestion effects for low-skilled workers engaged in search). At the same time, the pattern of spillovers is potentially intricate; the fact that high-skilled workers are prepared to undertake low-skilled tasks may increase the profitability of opening low skill-requirement vacancies at the expense of high skill requirement vacancies and alter the proportion of such jobs in equilibrium.

Albrecht and Vroman (2002, henceforth AV) is a leading example of a model which can generate mismatch.[2] They argue that equilibria

[1] For example, interaction effects may arise from complementarity, in the final good production function, between intermediate inputs produced by each skill group.

[2] Their simple, intuitively appealing framework melds the dynamic concerns of the equilibrium unemployment literature with the static concerns of the literature on assortative matching. Two other distinct frameworks have been used to examine the effect of matching frictions on labour market inequalities. In one, typified by Mortensen and Pissarides (1999), workers with different skill levels operate in separate labour markets. Mismatch is ruled out explicitly, and interaction between skill types is driven instead by the extent to which they are complements in final goods production. This is the same mechanism at work in models with frictionless labour markets. An alternative, exemplified by Marimon and Zillibotti (1999), considers

exhibiting either mismatch or segmentation by skills may arise in such a setting;[3] multiple equilibria may arise also. Their model has been applied to a range of issues.[4] However, empirical evaluation has been limited.

We contribute to the study of skill-specific mismatch in understanding labour market inequalities in two ways. Firstly, on the theoretical side, we show that, neither of the candidate equilibria which AV identify (with or without mismatch) exist under the assumptions adopted by AV (and the subsequent literature). This is because in the AV framework firms are unable to commit to hold open a job with a specific skill-requirement, and encounter workers of either skill level. Instead they find it optimal to regrade vacancies to suit the workers they encounter.[5] We then modify the basic AV model so that the firm commits to hold open a job of a particular skill-requirememt through the payment of a sunk cost at the time that the unfilled vacancy is first opened.[6] Then

mismatch in a continuum of skill types on both sides of the market. AV's approach is closer to the theoretical framework underpinning the empirical literature on skill-biased technical change and is the subject of more extensive related literature.

[3]Their mismatch based equilibrium arises if high-skilled workers find it optimal to match with a vacancy with low skill-requirements, while the segmentation equilibrium arises if high-skilled workers would turn down this opportunity to form a match with a low skill-requirement vacancy in favour of continued search. This differs from the ex ante segmentation described in the opening paragraph, in which search by firms and workers occurs in skill-specific pools.

[4]Other authors have explored the efficiency properties of the resulting equilibrium, Blasquez and Jansen (2008), and applied their framework to study on-the-job search, Dolado et al. (2008), trade, specialisation and offshoring, Davidson et al. (2008) and economic fluctuations, Khalifa (2008).

[5]To see this, consider a firm which has opened a low-skill requirement vacancy. In common with much of the literature on equilibrium unemployment, AV assume that firms make a *flow* payment, at each point in time, to hold open a vacancy. But then, if such a firm encounters a high-skilled worker, it faces no cost associated with upgrading the skill-requirement of the vacancy to suit the high-skilled worker, while the surplus from upgrading exceeds either that from maintaining the original low skill-requirement job and engaging in mismatch, or from rejecting the match and continuing to search for a suitable worker -as would arise in the equilibrium with segmentation. In addition, it is possible that high-skill requirement vacancies may be downgraded if a firm meets a low skilled worker.

[6]This sunk cost can be thought of as the cost of undertaking the investment required to introduce a skill-specific production technology. This is consistent with observations that investment activity is lumpy and irreversible and that labour adjustment decisions are correlated with investment activity.

mismatch equilibria and segmentation equilibria similar to those studied by AV and others re-emerge as part of a much richer set of feasible equilibria, which includes several equilibria exhibiting regrading.[7] We characterise these equilibria and explain how, for given values of parameters, the nature of the equilibrium exhibited by the model depends directly on the magnitude of the sunk cost of opening a vacancy.

We also contribute to the empirical analysis of models of mismatch. Existing empirical analysis takes the existence of an empirically plausible equilibrium with mismatch as given; it cannot be construed as a test of the model framework. Here, by contrast, the focus is directed towards testing the model, to investigate which, if any of the equilibria generated by the model is consistent with the data.[8] We identify two related problems with previous empirical work. First, authors tend to target unobserved, poorly identified or poorly measured parameters, such as vacancy costs, the value of leisure and skill-specific productivity differences but are then unable to match key obervables such as skill-specific unemployment rates unless further free parameters are introduced. Such an approach cannot be used for model evaluation, as it presupposes the validity of the model. Secondly, the equilibrium of particular interest here, mismatch, arises in a frictional model of equilibrium unemployment with skill-specific heterogeneity. As such, at the bare minimum the model should be capable of matching observable unemployment and skill premium data. Therefore, we calibrate the model with fixed costs

[7] As well as equilibria exhibiting mismatch or segmentation (each without regrading), akin to those studied by AV, the set of feasible equilibria includes a further mismatch equilibrium, featuring downgrading of high-skill requirement jobs upon encountering low skill workers, additional segmentation equilibria can arise either with upgrading of low skill-requirement jobs, or with downgrading of low skill-requirement jobs or with regrading of both high and low skill-requirement jobs.

[8] One response to the state of the empirical analysis of the AV framework is that it is primarily designed to give insight rather than being an empirical construct, after all the two-skill framework is clearly restrictive, skill is unlikely to be binary, or even unidimensional.. However, the two skill set up is consistent with the approach adopted in the frictionless literature, see Acemoglu and Autor (2010). This argument that the AV model was not intended to be taken to the data is rather undermined in the literature by the work of Albrecht and Vroman (2002) and Dolado et al. (2008), Khalifa (2008) and others who choose 'plausible' values for the model parameters for their numerical work.

of vacancy creation to match observable features of US data, such as skill specific unemployment rates and wage premia, and use the equilibrium conditions (specific to each of the candidate equilibria) to pin down unobserved, poorly identified or poorly measured parameters. We then proceed to ask: if we insist on matching these observable features of the data, are there any combinations of unobservable parameters for which any of the candidate equilibria exist? And, if so, does the data support the existence of mismatch in the context of the AV model? That is, unlike earlier work, this approach permits us to examine the validity of any candidate equilibria in the AV class of models as a means of accounting for the data, and provides a direct test of whether the AV model provides an empirically plausible model of mismatch.[9] We find that, for US data, the answers to these two questions are yes, and no, respectively. The equilibria which are supported by the data turn out to be segmentation without regrading (for low worker bargaining power) and segmentation with upgrading (for high worker bargaining power). This result is robust to a number of plausible variations in model specification. We also investigate why the mismatch equilibrium, although computationally feasible, fails to satisfy the necessary conditions for existence. At the more empirically plausible low values for worker bargaining power, the critical issue is whether high skilled workers prefer mismatch to segmentation. Using our calibration strategy, the critical result is that the calibrated value of leisure in the mismatch equilibrium is so high that it is never worthwhile for a high-skilled worker to undertake mismatch; the high-skilled worker always prefers to continue to search for a high skill-requirement position. To the extent that one views skill-based mismatch to be an important phenomenon, an alternative to the AV model is required.

Finally, our strategy deals with the difficulties posed by multiple equilibria, which AV (and others) claimed were an outcome of their model. The existence of multiple equilibria would make empirical analysis more difficult to undertake. We show that multiple equilibria are an artefact

[9] We do not question the existence or significance of mismatch as an empirical phenomenon. Rather, this is a model specific question: Is the equilibrium of an AV model exhibiting mismatch without regrading, capable of accounting for obserbale patterns in US labour market data?

of the calibration framework adopted by both AV, and subsequent work. For example, when considering the existence of the equilibrium with segmentation one must check the following necessary condition: that in the *candidate equilibrium* a firm with a low skill requirement vacancy would prefer to continue to search (for a low-skilled individual) against the *alternative equilibrium* that they engage in mismatch with a high-skilled worker). This calculation uses the value of unemployment for a high-skilled worker. I argue that AV and others claim the existence of multiple equilibria because they use an inappropriate value of unemployment for the high skill worker. In particular they evaluate the alternative equilibrium (mismatch) using the value of unemployment that arises for the candidate equilibrium. It is this that generates ranges of parameters for which both mismatch and segmentation are feasible. Instead, I argue that the alternative equilibrium (against which one evaluates a candidate equilibrium) should be evaluated using the values of variables that would result *if that alternative equilibrium occurred*. As such multiple equilibria are infeasible.

In the next section, I outline the theoretical model, demonstrate the result that the equilibria studied to date in the literature do not exist, and characterise the equilibria which do exist once costs of skill-specific technology adoption are introduced. Section 3 illustrates the impact of fixed costs of technology adoption on the existence of the various equilibria that can arise. Section 4 outlines the calibration strategy in detail, while Section 5 describes the key results. Section 6 concludes and offers suggestions for further work

2 A Model of Technology Adoption and Mismatch.

Consider a model in which workers exhibit skill-differences, jobs exhibit different skill requirements, and firms face fixed costs of vacancy creation. This generalises AV (2002) by allowing for a one-off sunk cost associated with creating a vacancy with a specific skill-requirement. This may prevent a firm changing the technology (or skill-requirement) associated with its job to suit the skill level of the first worker with whom it makes contact. I show that this model with sunk vacancy creation costs has a large variety of equilibria which may involve mismatch, segmentation and regrading.[10] I show that equilibria (without regrading) featuring mismatch or segmentation, which have been the focus of the current literature, are infeasible under the maintained assumption of that literature, namely that the costs of adopting a skill specific technology are zero.[11] I show that the existence of a particular type of equilibrium depends on the magnitude of the costs of adopting the technology associated with a particular skill type.

2.1 Basic Setup

Consider a continuous-time model with infinitely-lived, risk-neutral agents, who discount the future at the interest rate r. The measure of workers is normalised to one, a fraction μ of workers are low-skilled, (denoted by the subscript $_L$), the remainder, $1-\mu$, are high-skilled, (denoted by $_H$). The final good, Y is produced by combining intermediate skill-specific outputs according to the production function[12]

$$Y = A_H Y_H + A_L Y_L,$$

where A_j, $j \in \{L, H\}$ represents the efficiency with which intermediate good produced using a technology with minimum skill-requirement j

[10] The most interesting among these equilibria involve i) mismatch without regrading, ii) segmentation without regrading, iii) segmentation with upgrading, in which low-skill requirement jobs can be upgraded to high-skill requirement jobs, and iv) segmentation with downgrading in which high-skill requirement jobs may be downgraded to low-skill requirement jobs.

[11] To see this, consider a firm which has opened a low-skill requirement vacancy. In common with much of the literature on equilibrium unemployment, AV assume that firms make a flow payment, at each point in time, to hold open a vacancy. But then, if such a firm encounters a high-skilled worker, it faces no costs associated with upgrading the skill-requirement of the vacancy to suit the high-skilled worker, while the surplus from upgrading exceeds both that from maintaining the original low skill-requirement job and engaging in mismatch, and that from rejecting the match and continuing to search for a suitable worker (as would occur in the equilibrium with segmentation). In addition, it is possible that high-skill requirement vacancies may be downgraded if a firm meets a low skilled worker.

[12] This production function assume that the output of high-skill requirement and low skill requirement positions are perfect substitutes in the production of final goods. This eliminates interaction effects between skilled and unskilled workers associated with the structure of the final goods production function and allow us to focus on interaction arising through labour market frictions.

can be used in final good production . This captures the productivity of employment in an activity with a particular skill-requirement. I assume that the efficiency of high-skilled workers exceeds that of low skilled workers $A_H > A_L$. If final good producers are price takers in their product market and in the markets for intermediate goods, then the maximisation problem for final goods producers can be written as

$$\max_{Y_L, Y_H} PY - p_L Y_L - p_H Y_H,$$

where p_j, $j \in \{L, H\}$ represents the relative price of intermediate good j. Then, provided both high and low skill intermediate outputs are produced, $p_H/p_L = A_H/A_L > 1$.[13]

A job is either vacant or filled. A job is characterised by its skill requirement, that is, by the minimum skill required of a worker who is to undertake the job. When a vacancy is filled, 1 unit of skill–requirement specific intermediate good, $Y_{i,j}$, is produced whenever the skill level, i, of the worker is greater than or equal to the minimum skill-requirement, j, of the job, otherwise no (intermediate) output can be produced.[14] This intermediate good production technology is represented as:

$$Y_{i,j} \equiv Y_j = \begin{cases} 0 & \text{if} \quad i = L, j = H \\ 1 & \text{otherwise} \end{cases}. \tag{1}$$

Both low-skilled and high-skilled unemployed workers search in a common pool, of measure, u, to find a vacancy (regardless of its skill requirement). Firms with vacancies also search in a common pool, of

[13]As an alternative one could assume that a match producing type j output produces A_j units of output. The structure given in the text is adopted because of the ease with which it can be generalised to the case where skill-specific intermediate goods are imperfect substitutes in production.

[14]AV assume that the skill-requirement of a job is a non-negative number which is chosen by the firm. Yet, given the available production technology and the distribution of skills across workers, the only skill-requirements that any firm creating a vacancy would choose are 1) the skill requirement of a low skill job will equal the skill level of low skilled worker, and 2) the skill requirement of a high-skilled job = the skill level of high-skilled workers. The framework described in the text generates these outcomes and also provides a tractable approach in the face of imperfect substitution between skill-requirement specific intermediate outputs in final goods production.

measure v, for unemployed workers. The random matching technology is described by the matching function $m(u,v)$, which is linearly homogeneous, so that $m(u,v) = vm\left(\frac{1}{\theta},1\right) = vq(\theta)$, where $\theta = \frac{v}{u}$, with $\frac{\partial q}{\partial \theta} < 0$. Vacancies meet workers at the rate $q(\theta)$. Vacancies for high skill-requirement positions sometimes encounter low-skilled workers who are not qualified for those jobs. Suppose a fraction γ of all unemployed workers are low-skilled, then the effective arrival rate for high-skilled vacancies is $[1-\gamma]\,q(\theta)$. The rate at which unemployed workers of all types meet vacancies is $\theta q(\theta)$. Although low-skilled workers encounter vacancies at the same rate as do high-skilled workers, they are not qualified for employment in jobs with high-skill requirements. The effective arrival rate of vacancies is $\phi\theta q(\theta)$, where ϕ the fraction of vacancies which are for low skill-requirement jobs.

When a vacant job is created, the firm chooses the skill requirement to maximise the value of that vacancy, so that search by firms is directed. The choice of a specific skill requirement entails a one off cost, which can be thought of as the cost of adopting a skill-specific production technology. This cost is sunk. Initially, assume that this cost of vacancy creation, K, is independent of the level of skill-requirement that a firm chooses. The number of vacancies at each skill-requirement is determined by a free entry condition, so that new vacancies with a particular skill-requirement continue to be created until the value of that vacancy, V_j, $j \in \{L, H\}$ equals, K, the fixed cost of adopting the associated technology.[15] The presence of a fixed cost of opening a vacancy of particular type acts as a commitment device. If adopting a skill-specific technology entails a sunk cost, then a firm which encounters a worker whose skill-level differs from the skill-requirement of the position on offer, will only regrade the job and adopt the technology specific to that worker if the value of doing so outweighs the sunk cost incurred by so doing. By contrast, if flow costs of posting vacancies are incurred per unit time, as in the Diamond-Mortensen-Pissarides framework, then, with heteroge-

[15] The lump sum structure could be easily justified given the lack of detailed knowledge over the structure of adjustment costs generally. Fujita and Ramey (2007) show lump-sum costs of vacancy creation generate vacancy persistence in a Diamond-Mortensen-Pissarides matching model of equilibrium unemployment.

neous workers, a firm will always regrade and adopt the technology most appropriate to the first worker that it meets, since regrading is costless, and provides a higher payoff than the alternatives.

To facilitate comparison with the existing literature, we assume that, besides the sunk cost of technology adoption, K, firms pay a flow fixed cost, c, per unit time, to hold open a vacancy; again we do not allow c to be skill-specific. We also follow Albrecht and Vroman and assume both that production entails a flow cost, κ. Then Albrecht and Vroman's model is a special case (in which $K = 0$, and $\kappa = c$) of our more general model.

2.2 Match Formation and Wage Determination

Broadly speaking we need to characterise equilibria with mismatch and equilibria with segmentation; this is complicated by the fact that either of these situations may involve regrading of low-skill requirement and/or high skill-requirement jobs. We focus on the steady state.

2.2.1 The Surplus and the Conditions for Match Formation

Once we allow the possibility of regrading, a meeting between an unemployed worker with skill level i and a firm which has created a vacancy with skill requirement j results in the formation of an employment relationship under two alternative sets of conditions.[16] First, if no regrading occurs, then the joint surplus, $S_{i,j}$, generated by this match (absent regrading) must be non-negative *and* must exceed the surplus, $S_{i,i}^k$, that can be obtained if the skill-requirement of the vacancy is regraded to match the skill-level of the unemployed worker whom the firm has met.[17] These conditions require $S_{i,j} \geq \max\left\{S_{i,i}^k, 0\right\}$. Secondly, if a match arises as a result of regrading, then the surplus, $S_{i,i}^k$, must be non-negative and must exceed the surplus $S_{i,j}$, that arises without regrading, $S_{i,i}^k \geq \max\{S_{i,j}, 0\}$.

To proceed we evaluate these match formation conditions in terms of the capital value of the transition involved in match formation for each

[16] By contrast AV, who ignore the possibility of regrading, argue that match formation requires only one, less restrictive, condition $S_{i,j} \geq 0$.

[17] Here $k \in \{Up, Down\}$, allows for both upgrades and downgrades.

party to the match. We use the following notation for the value functions pertaining to workers and firms associated with particular states. The value of unemployment for a worker of type i is U_i. The value of employment for a worker of skill-level i in a job with skill-requirement j is $W_{i,j}$, in the absence of regrading and $W^k_{i,i}$ if the skill-requirement j position is regraded once a type i worker is encountered. This notation differentiates between values with and without regrading. For example $W_{L,L}$ and $W^{Down}_{L,L}$ both refer to the value of an low skilled worker employed in a low skill-requirement job, and both arise in the same equilibrium (e.g. segmentation with downgrading), but $W_{L,L}$ is the value to a low skilled worker of employment in a position that was originally created as a low skilled position, whereas $W^{Down}_{L,L}$ refers to the value of a worker in the case in which the vacancy was originally a high skill-requirement position, but was downgraded to match the skills of the low-skilled worker.[18] The value of a vacancy for a job with skill-requirement j is V_j and the value to a firm of a job with skill requirement j filled by a worker of skill level i is $J_{i,j}$, in the absence of regrading and $J^k_{i,i}$ when regrading occurs.[19]

Using this notation and the definition of the surplus, we write a necessary condition for match formation without regrading as (2) and a necessary condition for match formation with regrading as (3), reflecting the sunk cost of regrading the job.

$$S_{i,j} \equiv W_{i,j} + J_{i,j} - U_i - V_j \geq \max\left\{S^k_{ii}, 0\right\} \tag{2}$$

$$S^k_{i,i} \equiv W^k_{i,i} + J^k_{i,i} - K - U_i - V_j \geq \max\left\{S_{ij}, 0\right\}. \tag{3}$$

2.2.2 Bargaining and Wages

Suppose, as is standard, that the wage in a viable match is determined by Nash Bargaining. Then, without regrading, the wage $w_{i,j}$ for a worker

[18] Of course *all* these value functions, U_i , $W_{i,j}$, $J_{i,j}$ etc. will vary across equilibria, depending in part on whether regrading occurs. Rather than introduce that extra notational complexity, the notation here distinguishes simply between the value of different states which may arise within the same equilibrium.

[19] The free entry condition drives the value of all vacancies to $V_L = V_H = K$, irrespective of regrading.

with skill level i in a job with skill requirement j firm, is determined by the condition (4), which apportions to the worker a share β of the surplus $S_{i,j}$. If regrading occurs, the wage, w_{ii}^k, is determined by condition, (5).

$$W_{i,j} - U_i = \beta \left[W_{i,j} + J_{i,j} - U_i - V_j\right] \tag{4}$$

$$W_{i,i}^k - U_i = \beta \left[W_{i,i}^k + J_{i,i}^k - K - U_i - V_j\right] \tag{5}$$

Next, define the asset value equations of workers and firms (which generalise those of AV). We focus on the steady state. A low skill requirement vacancy has value

$$rV_L = -c + q(\theta) \left[\begin{array}{c} \gamma \left[J_{L,L} - V_L\right] + \\ \left[1-\gamma\right] \max\left\{0, J_{H,L} - V_L, J_{H,H}^{Up} - K - V_L\right\} \end{array} \right]. \tag{6}$$

Equation (6) says that the flow return to a vacancy for a low-skill requirement job is the sum of the flow cost per unit time of posting a such a vacancy, $-c$ and the expected capital gain from forming a match. This second term is the weighted value of filling the vacancy, where the weights reflect the arrival rates of low-skilled workers, $\gamma q(\theta)$, and a high-skilled workers, $[1-\gamma]\, q(\theta)$, respectively. An encounter with a low-skilled worker results in a match which produces a capital gain $J_{LL} - V_L$. If the firm encounters a high-skilled worker, it chooses the most valuable of the several options available. In particular, a capital gain of 0 results if the high-skilled worker does not find it worthwhile to undertake employment in a low skill requirement job *and* the firm does not find it optimal to upgrade the skill requirements of the job to that of the work; the capital gain $J_{H,L} - V_L$ represents the payoff when the high-skilled worker finds it worthwhile to undertake employment in the low skill-requirement position while firms find it too costly to upgrade the skill-requirements of the job to match the skill level of the worker. The term $J_{H,H}^{Up} - K - V_L$ represents the capital gain when firms upgrade the skill requirements of the job to match the skill level of the worker.[20] [21]

For a high skill-requirement vacancy, the asset value satisfies:

$$rV_H = -c + q(\theta)\begin{bmatrix} \gamma \max\left\{J_{L,L}^{Down} - K - V_H, 0\right\} \\ +[1-\gamma][J_{H.H} - V_H] \end{bmatrix}. \tag{7}$$

That is, the return on a high skill-requirement vacancy is the sum of the flow cost of posting the vacancy, $-c$, and the expected instantaneous capital gain from forming a match, which is the weighted value of filling the vacancy. These weights $\gamma q(\theta)$ and $[1-\gamma]q(\theta)$, reflect the arrival rates of low-skilled workers and high-skilled workers, respectively. An encounter with a low-skilled worker results in a match which produces a capital gain $J_{LL} - V_L$. In the event of meeting a high-skilled worker the firm chooses the most profitable of the several options available. In particular, a capital gain of $J_{L,L}^{Down} - K - V_H$ results if the firm downgrades the skill requirement of the job to match the skills of the low-skilled worker; a capital gain of 0 results when downgrades are too costly, since the production technology precludes a low skilled worker from producing output in a high skill requirement task. An encounter with a high-skilled worker results in a match which produces a capital gain $J_{H,H} - V_H$.[22]

The flow value, $rJ_{i,j}$, of a job with skill-requirement j filled by a worker of skill-level i, without regrading, reflects the sum of the operating profit flow, $p_j - c - w_{i,j}$ (revenue, p_j, less flow fixed costs of operation, κ, and wages, $w_{i,j}$) and the expected capital loss arising from exogenous separation, $\delta[V_j - J_{i,j}]$. This is summarised in equation (8). The value to a firm of a job regraded to skill requirement i in order to suit a worker of skill level i, is given by equation (9). In general $J_{i,i} \neq J_{i,i}^k$, because if the cost of regrading, K, exceeds zero then surplus with regrading differs from that without, and the wage with regrading $w_{i,i}^k$ will differ from that

[20] An equilibrium may exhibit both upgrading (of low skill requirement jobs) and downgrading (of high skill requirement jobs), but then the decision on whether to downgrade is made by a firm with a high-skill requirement vacancy. This is considered in equation (7).

[21] AV, who do not consider the possibility of regrading, use the simpler condition $rV_L = -c + q(\theta)[\gamma[J_{L,L} - V_L] + [1-\gamma]\max\{J_{H,L} - V_L, 0\}]$.

[22] AV use the simpler condition $rV_H = -c + q(\theta)[1-\gamma][J_{H.H} - V_H]$.

without regrading $w_{i,i}$.

$$rJ_{i,j} = p_j - c - w_{i,j} + \delta \left[V_j - J_{i,j} \right] \tag{8}$$

$$rJ_{i,i}^k = p_i - c - w_{i,i}^k + \delta \left[V_i - J_{i,i}^k \right] \tag{9}$$

For a level i-skilled worker, the value of employment, without regrading, in a job requiring skill j (provided $\{i, j\} \neq \{L, H\}$) is given in equation (10). While the value of employment to a level i-skilled worker in a regraded job requiring skill level i is summarised in (11). Again, if $K > 0$, $W_{i,i}^k \neq W_{i,i}$; bargaining ensures that $w_{i,i}^k \neq w_{i,i}$.

$$rW_{i,j} = w_{i,j} + \delta \left[U_i - W_{i,j} \right]. \tag{10}$$

$$rW_{i,i}^k = w_{i,i}^k + \delta \left[U_i - W_{i,i}^k \right] \tag{11}$$

Finally, consider the value of unemployment. For a high-skilled worker the value of unemployment is the sum of the flow value of leisure, b, and the expected capital gain from the possibility of forming an employment relationship. A high-skilled worker encounters a job with high skill-requirement at a rate $[1 - \phi] \theta q(\theta)$, in that case the capital gain is $W_{H,H} - U_H$. A high-skilled worker encounters vacant low skill requirement jobs at a rate $\phi \theta q(\theta)$. Then the firm decides whether or not to upgrade the job to meet the skill level of the worker. So the expected capital gains in that case are

$$I \max \{W_{H,L} - U_H, 0\} + [1 - I] \max \left\{ W_{H,H}^{Up} - U_H, 0 \right\},$$

where I is an indicator function which denotes when the firm finds it optimal to regrade the vacancy, such that $I = 1$ if $J_{H,L} - V_L \geq J_{H,H}^{Up} - K - V_L$ and $I = 0$ otherwise. Here $W_{H,L} - U_H$ represents the capital gain if the firm decides not to upgrade the low skill-requirement job and the high-skilled worker accepts it (and the gain if the high-skilled worker rejects the upgraded job is 0), while $W_{H,H}^{Up} - U_H$ represents the capital gain when the high-skilled worker decides to accept the regraded job in the case that the firm does decide to upgrade the skill-requirements of the job (and the capital gain if the worker rejects that upgraded job is 0). Clearly, a firm will only upgrade the skill-requirement of the vacancy if the worker is going to accept the upgraded position. Thus, the criterion that the firm uses to determine whether to upgrade a low skill-requirement vacancy to a high skill-requirement vacancy, namely $\max\left\{J_{H,L}-V_L, J_{H,H}^{Up}-K-V_L, 0\right\}$ can be written as $\max\left\{W_{H,L}-U_H, W_{H,H}^{Up}-U_H, 0\right\}$. So we write the asset value equation of unemployment for high-skilled workers as (12), where (in relation to AV) the possibility of regrading introduces $W_{H,H}^{Up}-U_H$ into the max operation. By a similar line of argument the value of an unemployed low skilled worker satisfies condition (13).

$$rU_H = b+\theta q(\theta)\begin{bmatrix}\phi\max\left\{W_{H,L}-U_H, W_{H,H}^{Up}-U_H, 0\right\}\\ +[1-\phi]\left[W_{H,H}-U_H\right]\end{bmatrix}, \tag{12}$$

$$rU_L = b+\theta q(\theta)\begin{bmatrix}\phi\left[W_{L,L}-U_L\right]+\\ [1-\phi]\max\left\{W_{L,L}^{Down}-U_L, 0\right\}\end{bmatrix}. \tag{13}$$

2.2.3 Match Formation, Wages and Capital Gains

We can substitute asset value conditions (8) and (10) into equation (2) to re-express the match formation conditions in flow terms. Thus a necessary condition for a match to be formed without regrading is (14). Using (9) and (11) into (3), a necessary condition for match formation on regrading from a skill requirement j vacancy to a skill requirement i vacancy is (15)

$$p_j - c \geq rU_i + rV_j, \tag{14}$$

$$p_i - c \geq rU_i + rV_i + [r+\delta]\left[V_j - K - V_i\right]. \tag{15}$$

Using conditions (8) and (10) in (4) the wage of a worker of type i employed in a job with skill requirement j (in the absence of regrading) is given by (16), while, combining (9) and (11) into (3) if regrading from a skill requirement j to a skill-requirement i job occurs then the wage of the type i worker is given by (17). Notice that regrading provides a new channel through which within-skill-group wage inequality may arise.

$$w_{i,j} = \beta [p_j - c - rV_j] + [1 - \beta] rU_i, \tag{16}$$

$$w^k_{i,i} = \beta [p_i - c - rV_i + [r + \delta] [V_i - K - V_j]] + [1 - \beta] rU_i. \tag{17}$$

We also note the following capital gains for firms and workers, as a result of the formation of a match, with regrading and without.

$$J_{i,j} - V_j = \frac{[1 - \beta] [p_j - c - rV_j - rU_i]}{r + \delta} \tag{18}$$

$$W_{i,j} - U_i = \frac{\beta [p_j - c - rV_j - rU_i]}{r + \delta} \tag{19}$$

$$J^k_{i,i} - K - V_j = \frac{[1 - \beta] [p_i - c - rV_i - rU_i + [r + \delta] [V_i - K - V_j]]}{r + \delta} \tag{20}$$

$$W^k_{i,i} - U_i = \frac{\beta [p_i - c - rV_i - rU_i + [r + \delta] [V_i - K - V_j]]}{r + \delta} \tag{21}$$

2.3 Equilibrium

We wish to characterise the equilibrium values of four endogenous variables, the aggregate unemployment rate, u, the fraction of the unemployed who are low-skilled, γ, labour market tightness, θ, and the fraction of vacancies which are for low skill requirement jobs, ϕ. The model generates a number of equilibria of interest. We characterise these equilibria here. Under the assumption that K is a known parameter, the nature of the equilibrium that arises depends on the magnitude of K, the sunk cost of creating a vacancy for a job with a particular skill requirement. Since the equilibrium with mismatch without regrading has attracted the greatest attention, we begin by considering the impact of fixed costs on this equilibrium in order to tease out the similarities with and differences to the earlier literature. AV refer to the former case as a *cross-skill-matching* equilibrium, and to the case where no mismatch occurs as an equilibrium with *ex post segmentation*; we follow this notation. We also characterise four other interesting equilibria, which feature i) segmentation with upgrading of low skill-requirement jobs, ii) segmentation with downgrading of high skill-requirement jobs (only), iii) segmentation with both upgrading of low skill-requirement jobs and downgrading of high skill-requirement jobs, and iv) mismatch with downgrading of high skill-requirement jobs.[23]

2.3.1 Mismatch Without Regrading

To rule out regrading we require that (i) $J_{H,L} - V_L > J_{H,H}^{Up} - K - V_L$ to rule out upgrading and (ii) $0 > J_{L,L}^{Down} - K - V_H$ to rule out downgrading. In addition to AV's condition for the existence of an equilibrium with mismatch holds: $J_{H,L} - V_L > 0$.

Equilibrium Conditions The equilibrium is pinned down by two mass balance conditions, one for high skilled workers and one for low skilled workers, and two free entry conditions, one for jobs with high skill requirements and one for jobs with low skill requirements.[24]

For low-skilled workers the flow of workers from unemployment into employment through match formation must equal the flow from employment to unemployment through separations. This is summarised in equation (22). Total separations for low-skilled workers (per unit time) are the product of the separation rate, δ, and the mass of low-skilled workers in employment, $\mu - \gamma u$.[25] The outflow of low-skilled workers to employment is the product of the rate at which low-skilled workers encounter suitable employment opportunities (jobs with low skill-requirements), $\phi\theta q(\theta)$ and the mass of low skilled unemployed workers, γu. For high-skilled workers the equivalent equation, (23), is obtained

[23] In addition when the costs of technology adoption are sufficiently high, there are two further equilibria: i) those with no low-skill requirement jobs, in which even low-skilled individuals do not find it worthwhile to undertake low skill-requirement jobs and ii) those with no feasible jobs whatever (in which both skilled and unskilled workers prefer unemployment). These equilibria are somewhat less plausible and less interesting, so I do not discuss them. However, they do generate further necessary conditions that a candidate equilibrium must satisfy; these necessary conditions are exploited in the empirical analysis.

[24] While the labour market flow conditions are identical to those of AV, the free entry conditions differ in that they allow for the costs of skill-specific technology adoption, K.

[25] This is the measure of low-skilled workers in the workforce less the number of low-skilled workers who are unemployed, γu. This, in turn, is the product of unemployment and the fraction of the unemployed who are low-skilled of the workforce who are low skilled.

by a similar argument.

$$\phi\theta q(\theta)\gamma u = \delta[\mu - \gamma u]. \tag{22}$$

$$\theta q(\theta)[1-\gamma]u = \delta[1-\mu-[1-\gamma]u] \tag{23}$$

Next, consider the free entry conditions. Using (18) the asset value equations for low skill requirement and high-skill requirement vacancies (6) and (7) can be written as

$$[r+\delta][rV_L + c] = [1-\beta]q(\theta)[p_L - \kappa - rV_L - \gamma rU_L - [1-\gamma]rU_H] \tag{24}$$

$$[r+\delta][rV_H + c] = [1-\beta]q(\theta)[1-\gamma][p_H - \kappa - rV_H - rU_H] \tag{25}$$

To proceed we need to eliminate rU_i from (24) and (25). The value of unemployment for low-skilled individuals is obtained using (13) and (19) to give (26). Similarly for high-skilled individuals, equations (12) and (19) give (27)

$$rU_L = \frac{[r+\delta]b + \beta\phi\theta q(\theta)[p_L - \kappa - rV_L]}{r+\delta+\beta\phi\theta q(\theta)} \tag{26}$$

$$rU_H = \frac{[r+\delta]b + \beta\theta q(\theta)\begin{bmatrix}\phi[p_L-\kappa-rV_L]+\\ [1-\phi][p_H-\kappa-rV_H]\end{bmatrix}}{r+\delta+\beta\theta q(\theta)} \tag{27}$$

The two free entry conditions require the value of a vacancy (of either skill requirement) equal the sunk cost of technology adoption. That is

$$V_L = K, \quad V_H = K. \tag{28}$$

We follow AV (2002) in exploiting the equal value feature of the free entry conditions (i.e. $V_H = V_L$) by combining (24) and (25) to give

$$[1-\gamma][p_H - p_L] = \gamma[p_L - \kappa - rK - rU_L]$$

Using (26), $p_L - \kappa - rK - rU_L = \frac{[r+\delta][p_L-\kappa-rK-b]}{r+\delta+\beta\phi\theta q(\theta)}$. So the equal value condition can be re-expressed as (29)

$$[1-\gamma]\begin{bmatrix}[p_H-p_L]\,\beta\phi\theta q(\theta)+\\ [r+\delta]\,[p_H-\kappa-rK-b]\end{bmatrix}=[r+\delta]\,[p_L-\kappa-rK-b] \quad (29)$$

For high skill-requirement jobs, substituting the free entry condition $V_H = K$ in (25) yields

$$[r+\delta]\,[rK+c]=[1-\beta]\,q(\theta)\,[1-\gamma]\,[p_H-\kappa-rK-rU_H]$$

Using (27) we see that $p_H-\kappa-rK-rU_H=\frac{[r+\delta][p_L-\kappa-rK-b]}{[1-\gamma][r+\delta+\beta\theta q(\theta)]}$, so the free entry condition to high skill-requirement jobs can be written

$$[rK+c]=[1-\beta]\,q(\theta)\,\frac{[p_L-\kappa-rK-b]}{[r+\delta+\beta\theta q(\theta)]} \quad (30)$$

Conditions (**??**), (23), (29) and (30) governing the equilibrium with mismatch take the same general form as those in AV (2002). In the limit as $K\to 0$, the equilibrium conditions become identical to AV.

Necessary Conditions Next, we need to establish necessary conditions for the existence of an equilibrium featuring mismatch without regrading for arbitrary $K\geq 0$. For this equilibrium to exist, we must rule out equilibria in which segmentation occurs and also rule out those with regrading. To rule out regrading, we must rule out both the upgrading of low skill-requirement jobs and the downgrading of high skill-requirement jobs. To rule out segmentation we require that the surplus from a match between a high skilled worker and a low-skill requirement job be positive: $J_{H,L}-V_L\geq 0$ or equivalently;[26]

$$p_L-\kappa-rK-rU_H\geq 0. \quad (31)$$

To rule out upgrading, a firm with a low skill-requirement vacancy must, on meeting a high-skilled worker, make a greater capital gain through

[26] I do not substitute for rU_i, $i\in\{H,L\}$ in this section. It turns out that whether rU_i is evaluated with the parameter values from the candidate equilibrium, or with parameter values for the alternative equilibrium determines whether multiple equilibria arise in the model, see Section 3 below.

mismatch at the existing skill requirement than by paying K to upgrade the vacancy by adopting the technology suitable for high skill requirement workers. This amounts to the condition $J_{H,L}-V_L > J^{Up}_{H,H}-K-V_L$. Using (18), (20) and the free entry conditions gives

$$p_L - \kappa - rK - rU_H > p_H - \kappa - [2r+\delta]\,K - rU_H. \tag{32}$$

To rule out downgrading, a firm with a high skill-requirement vacancy, on encountering a low-skilled worker, must not make a greater capital gain by incurring the cost K of adopting the skill requirement suitable to the low-skilled worker and forming the match, than by retaining the high-skill requirement vacancy and continuing to search for a high-skilled worker.[27] This is summarised by the condition $J^{Down}_{L,L} - K - V_H < 0$. Using (20) and the free entry conditions gives

$$p_L - \kappa - [2r+\delta]\,K - rU_L < 0 \tag{33}$$

Notice that in the limit case, as $K \to 0$, condition (32) ruling out upgrades is violated, and condition (33) ruling out downgrades is only satisfied if low skill requirement matches are sufficiently unprofitable that they are never formed (even without regrades). In other words, for the limit case, $K=0$, considered by AV, the equilibrium with mismatch without regrading can not exist. Regrading occurs instead.

Summary Once the possibility of regrading is admitted, an equilibrium with mismatch is characterised by equations (22), (23), (29) and (30) subject to conditions (31), (32) and (33). The wage premium (of high-skilled workers over low-skilled workers) is then given by (34), in which the individual $w_{i,j}$ are determined using (16). Here within-skill-group wage inequality arises because of mismatch.

$$wp = \frac{\phi w_{H,L} + [1-\phi]\,w_{H,H}}{w_{L,L}} \tag{34}$$

[27] If this latter condition fails then an alternative equilibrium arises featuring mismatch with downgrading - see below. However, it appears likely that such an outcome can be ruled out on empirical grounds.

Finally, to ensure that both low-skill workers and high-skilled workers are willing to undertake employment (at all) we require

$$p_L - \kappa - rK - rU_L \geq 0 \tag{35}$$

$$p_H - \kappa - rK - rU_H \geq 0 \tag{36}$$

2.3.2 Segmentation Without Regrading

Necessary Conditions An equilibrium featuring segmentation without regrading differs from that studied in the previous section in one key way: a high-skilled worker rejects any low skill-requirement position that he/she encounters. This necessary condition can be written as $J_{H,L} - V_L < 0$. Using (18) and the free entry condition (28) this can be written as (37). Although (37) was considered in isolation by AV, the need to rule out regrading was not. An absence of regrading requires (i) that upgrading a low skill-requirement vacancy on encountering a high skill requirement vacancy generates a loss, and (ii) that downgrading a high skill-requirement vacancy on encountering a low-skilled worker generates a loss. That is (i) $J^{Up}_{H,H} - K - V_L < 0$, or equation (38), and (ii) $J^{Down}_{L,L} - K - V_H < 0$, or equation (39).

$$0 > p_L - \kappa - rK - rU_H \tag{37}$$

$$0 > p_H - \kappa - [2r + \delta] K - rU_H \tag{38}$$

$$0 > p_L - \kappa - [2r + \delta] K - rU_L \tag{39}$$

Once again the AV equilibrium is infeasible in the limit case, $K \to 0$. Letting $K \to 0$, (37) collapses to AV's necessary condition, but (38), which rules out upgrades fails (except in the uninteresting and implausible case in which high-skilled workers will not undertake employment in high-skill requirement jobs), while (39), which rules out downgrades also fails (except when low skilled workers refuse to undertake low skill-requirement jobs. The equilibrium is again characterised by four equations: two mass balance equations (one for each type of worker) and one free entry condition for vacancies of each skill-requirement, reflecting the cost of skill-specific technology adoption.[28]

Equilibrium When conditions (37)-(39) are satisfied, workers of each type never mismatch and no regrading occurs. It follows that the mass balance equation, (40), for low-skilled workers is the same as the case of mismatch without regrading considered in Section (2.3.1). Compared with the equilibrium exhibiting mismatch without regrading, outflows of high-skilled workers from unemployment occur at the lower rate $[1-\phi]\,\theta q(\theta)$, since high-skilled workers do not take up employment when they encounter low skill-requirement positions. So the mass balance equation for high-skilled workers under ex post segmentation without regrading is (41).

$$\phi\theta q(\theta)\,\gamma u = \delta\left[\mu - \gamma u\right] \tag{40}$$

$$[1-\phi]\,\theta q(\theta)\,[1-\gamma]\,u = \delta\left[1-\mu-[1-\gamma]\,u\right] \tag{41}$$

The asset value equations for vacancies and unemployed workers are

$$rV_L = -c + \frac{[1-\beta]\,q(\theta)\,\gamma\left[p_L - c - rV_L - rU_L\right]}{r+\delta}, \tag{42}$$

$$rV_H = -c + \frac{[1-\beta]\,q(\theta)\,[1-\gamma]\left[p_H - c - rV_H - rU_H\right]}{r+\delta} \tag{43}$$

$$rU_L = \frac{[r+\delta]\,b + \beta\phi\theta q(\theta)\left[p_L - c - rV_L\right]}{r+\delta+\beta\phi\theta q(\theta)} \tag{44}$$

$$rU_H = \frac{[r+\delta]\,b + [1-\phi]\,\beta\theta q(\theta)\left[p_H - c - rV_H\right]}{r+\delta+[1-\phi]\,\beta\theta q(\theta)} \tag{45}$$

The two free entry conditions require the value of a vacancy (for either skill-requirement) equal the sunk cost of vacancy creation, $V_L = V_H = K$. Combining this information with (42), (43), (44) and (45) the free entry condition for vacancies for low skill requirement positions becomes:

$$rK + c = [1-\beta]\,q(\theta)\,\gamma\frac{\left[p_L - c - rK - b\right]}{r+\delta+\beta\phi\theta q(\theta)}; \tag{46}$$

[28] These pin down unemployment, u, the fraction of the unemployed who are low-skilled, γ, overall labour market tightness, θ, and the fraction of vacancies which correspond to low skill-requirement positions, ϕ.

while the free entry condition in vacancies for high skill requirement positions becomes

$$rK + c = [1-\beta]\, q(\theta)\, [1-\gamma] \frac{[p_H - c - rK - b]}{r + \delta + \beta [1-\phi]\, \theta q(\theta)}. \tag{47}$$

Summary So segmentation without regrading is characterised by equations (40), (41), (46) and (47), subject to the constraints, (37), (38) and (39). The wage premium in this case is given by (48), where the match specific wages are given by (16)

$$wp = \frac{w_{H,H}}{w_{L,L}} \tag{48}$$

2.3.3 Segmentation With Upgrading

Necessary Conditions An equilibrium featuring segmentation with upgrading arises when i) a firm with a low skill requirement jobs which encounters a high-skilled worker decides to pay the fixed cost, K, of technology adoption to raise the skill-requirement of the job to suit that worker, and ii) a high skill-requirement vacancy is not downgraded if a low-skilled worker is encountered. That is, we require not only $\max\left\{J_{H,L} - V_L, J_{H,H}^{Up} - K - V_L, 0\right\} = J_{H,H}^{Up} - K - V_L$, but also $\max\left\{J_{L,L}^{Down} - K - V_H, 0\right\} = 0$. Notice that when an equilibrium with segmentation and upgrading arises, the alternative equilibrium without regrading may exhibit mismatch or segmentation depending on whether $J_{H,L} - V \gtrless 0$. Therefore, the necessary conditions to ensure upgrading rather than mismatch or segmentation (when a firm with a low skill requirement job encounters a high-skilled worker) are $J_{H,H}^{Up} - K - V_L \geq J_{H,L} - V_L$ *and* $J_{H,H}^{Up} - K - V_L \geq 0$. To rule out downgrading we require $J_{L,L}^{Down} - K - V_H < 0$. Using the free entry conditions $V_L = K$ and $V_H = K$, in conjunction with equations (18) and (20), these necessary conditions become, respectively:

$$p_H - \kappa - [2r + \delta] K - rU_H \geq p_L - \kappa - rK - rU_H \tag{49}$$

$$p_H - \kappa - [2r + \delta] K - rU_H \geq 0 \tag{50}$$

$$0 > p_L - \kappa - [2r + \delta] K - rU_L \tag{51}$$

Equilibrium Equilibrium is characterised by two mass balance equations (one for each type of worker) and two free entry conditions for vacancies, one for each skill-requirement.

Outflows of high-skilled workers from unemployment occur at the rate $\theta q(\theta)$, since, with job upgrading, such workers take up employment in high skill-requirement jobs whether or not the vacancies were initially created as high skill-requirement positions. So the mass balance equation for flows of high-skilled workers is (52). The fact that low-skilled workers only match with vacancies which were originally created as low skill-requirement positions, means that the mass balance equation, (53), for low skilled workers is the same as in the equilibria without regrading discussed in preceding sections:

$$\theta q(\theta)[1-\gamma]u = \delta[1-\mu-[1-\gamma]u] \tag{52}$$

$$\phi\theta q(\theta)\gamma u = \delta[\mu-\gamma u]. \tag{53}$$

In an equilibrium with upgrading the asset value equation for a low skill requirement vacancy differs from the previous cases

$$rV_L = -c + q(\theta)\left[\gamma[J_{LL}-V_L] + [1-\gamma]\left[J_{H,H}^{Up} - K - V_L\right]\right].$$

The flow value of such a vacancy still reflects the product of the probability that an encounter with a worker arises and the expected capital gain associated with a given match. However, now, on meeting a high-skilled worker, whose arrival rate is $[1-\gamma]\theta q(\theta)$, a firm makes capital gain $J_{H,H}^{Up} - K - V_L$, by upgrading to a high skill-requirement technology and forming the match. Using (18) and (20) the asset value of a low skill-requirement vacancy can be written as (54). The flow value of a high skill-requirement vacancy, (55), takes the same form as in the cases without regrading considered in previous sections, since upgrading of low skill-requirement vacancies does not imply downgrading of high skill-requirement vacancies:

$$[rV_L - c] = \frac{[1-\beta]}{[r+\delta]} q(\theta) \begin{bmatrix} \gamma [p_L - c - rV_L - rU_L] + \\ [1-\gamma] \begin{bmatrix} p_H - c - rV_H - rU_H + \\ [r+\delta][V_H - K - V_L] \end{bmatrix} \end{bmatrix} \tag{54}$$

$$[rV_H + c] = \frac{[1-\beta]}{[r+\delta]} q(\theta) [1-\gamma][p_H - c - rV_H - rU_H] \tag{55}$$

For a high-skilled worker, the value of unemployment reflects the idea that all meetings generate employment in high skill requirement positions. However, a fraction ϕ of such meetings lead the firm offering the vacancy to incur the cost, K, of upgrading its production technology to suit the high skilled worker. In that case the capital gain available to a worker is $W_{H,H}^{Up} - U_H$. This is differs from (and is likely lower than) the capital gain obtained from employment in a high skill requirement vacancy for which no upgrading cost was incurred, $W_{H,H} - U_H$. So

$$rU_H = b + \theta q(\theta) \left[\phi \left[W_{H,H}^{Up} - U_H\right] + [1-\phi][W_{H,H} - U_H]\right].$$

Substuting for $W_{H,H}^{Up} - U_H$ and $W_{H,H} - U_H$, via (21) and (19) gives (56).

$$rU_H = \frac{[r+\delta] b + \beta\theta q(\theta) \begin{bmatrix} p_H - c - rV_H + \\ \phi [r+\delta][V_H - K - V_L] \end{bmatrix}}{[r + \delta + \beta\theta q(\theta)]}. \tag{56}$$

For a low-skilled worker the form of the flow value of unemployment is unaffected by the presence of upgrading, so it takes the same form as in the previous two sections:

$$rU_L = \frac{[r+\delta] b + \beta\phi\theta q(\theta)[p_L - c - rV_L]}{r + \delta + \beta\phi\theta q(\theta)}. \tag{57}$$

Now we impose the two free entry conditions, $V_L = K$ and $V_H = K$. We follow AV's approach to the equilibrium featuring mismatch without regrading and use the equal value condition $V_L = V_H$ in conjunction with a free entry condition for high-skill requirement vacancies $V_H = K$.

Combining these conditions with the asset value equations (54) and (55) with the flow value of unemployment for each skill level, equations (56) and (57), and then simplifying, gives the equal value condition (58) and the free entry condition, (59), for high skill-requirement vacancies

$$\gamma\left[p_L - c - rK - b\right] = \left[1-\gamma\right]\left[r + \delta + \beta\phi\theta q\left(\theta\right)\right]K. \tag{58}$$

$$\left[rK + c\right] = \left[1-\beta\right]q\left(\theta\right)\left[1-\gamma\right]\frac{p_H - c - rK - b + \phi\beta\theta q\left(\theta\right)K}{\left[r + \delta + \beta\theta q\left(\theta\right)\right]}. \tag{59}$$

Summary So segmentation with upgrading is the solution to (52), (53), (58) and (59), subject to the conditions, (49), (50) and (51). In that case the skill premium is given by equation (60), where the terms $w_{i,j}$ are determined by (16) and that in $w_{i,j}^k$ is given by (17). Wage inequality across high-skill workers exists even though all end up in high skill-requirement occupations. The cost of regrading a job is the source of wage inequality within this environment.

$$wp = \frac{\phi w_{H,H}^{Up} + \left[1-\phi\right]w_{H,H}}{w_{L,L}} \tag{60}$$

2.3.4 Segmentation with Downgrading

Necessary Conditions Downgrading occurs when a firm with a vacancy for a high skill requirement position finds it optimal, on meeting a low-skilled worker, to pay the fixed cost K to adopt the technology of a low skill-requirement job. This is equivalent to the condition $J_{L,L}^{Down} - K - V_H \geq 0$. If, in addition, meetings between high-skilled workers and low skill-requirement jobs do not lead to employment relationships then both mismatch and regrading must be ruled out when such meetings occur. So the following conditions must both be satisfied i) segmentation-not-mismatch requires $0 > J_{H,L} - V_L$; ii) segmentation-not-upgrading requires $0 > J_{H,H}^{Up} - K - V_L$. Using (18) (20) and the free entry conditions $V_L = V_H = K$ these necessary conditions can be written more fully as (61), (62) and (63) respectively.

$$p_L - \kappa - [2r + \delta] K - rU_L \geq 0 \tag{61}$$

$$0 > p_L - \kappa - rK - rU_H \tag{62}$$

$$0 > p_H - \kappa - [2r + \delta] K - rU_H \tag{63}$$

Equilibrium To characterise the equilibrium we need one mass balance equation for each skill level of worker and one free entry condition for each vacancy skill requirement level.

For low-skilled workers, contact with low skill-requirement vacancies and high skill-requirement vacancies, which arrive at rates $\phi\theta q(\theta)$ and $[1 - \phi] \theta q(\theta)$ respectively, both result in match formation. So the mass balance equation is (64)

$$\theta q(\theta) \gamma u = \delta [\mu - \gamma u]. \tag{64}$$

An equilibrium featuring segmentation with (only) downgrading is one in which a high skilled worker only enters a match if he/she encounters a high skill-requirement vacancy. So (65), the steady state mass balance equation for high-skilled workers, is identical to that in the equilibrium featuring segmentation without regrading described above:

$$[1 - \phi] \theta q(\theta) [1 - \gamma] u = \delta [1 - \mu - [1 - \gamma] u]. \tag{65}$$

To proceed we determine the flow values of vacancies and unemployment, which can be combined with free entry conditions. Since neither upgrading nor mismatch occurs if a firm with a low skill-requirement vacancy encounters a high-skilled worker, the asset value equation for a low skill-requirement vacancy takes the same form as in the equilibrium featuring segmentation without regrading described earlier: $rV_L = -c + q(\theta) \gamma [J_{L,L} - V_L]$. Using (18) this can be written as

$$rV_L + c = \frac{[1 - \beta]}{[r + \delta]} q(\theta) \gamma [p_L - \kappa - rV_L - rU_L]. \tag{66}$$

The flow value of a high skill-requirement vacancy directly incorporates the effect of downgrading. If a firm with a high skill-requirement vacancy encounters a low-skilled worker then the capital gain from forming a match is $J_{L.L}^{Down} - K - V_H$, but if it encounters a high-skilled worker the capital gain is $J_{H.H} - V_H$. So:

$$rV_H = -c + q(\theta)\left[\gamma\left[J_{L.L}^{Down} - K - V_H\right] + [1-\gamma][J_{H.H} - V_H]\right].$$

which can be expressed as

$$rV_H + c = \frac{[1-\beta]}{[r+\delta]} q(\theta) \begin{bmatrix} \gamma \begin{bmatrix} p_L - \kappa - rV_L - rU_L + \\ [r+\delta][V_L - K - V_H] \end{bmatrix} \\ + [1-\gamma][p_H - \kappa - rV_H - rU_H] \end{bmatrix}. \tag{67}$$

For low-skilled workers all encounters with firms result in match formation, regardless of the (initial) skill-requirement of the vacancy. The flow value of unemployment to a low-skilled worker reflects the flow income from unemployment, b, the rate, $\phi\theta q(\theta)$, at which meetings occur with firms offering low skill-requirement vacancies multiplied by the associated capital gain, $W_{L,L} - U_L$, to a worker on forming that match, and the product of the rate, $[1-\phi]\theta q(\theta)$, at which low-skilled workers encounter high skill-requirement vacancies which are downgraded to low skill-requirement positions and the resulting capital gain $W_{L,L}^{Down} - U_L$: $rU_L = b + \theta q(\theta)\left[\phi[W_{L,L} - U_L] + [1-\phi]\left[W_{L,L}^{Down} - U_L\right]\right]$. Using (19) and (21) this can be simplified to

$$rU_L = \frac{[r+\delta]b + \beta\theta q(\theta)\begin{bmatrix} p_L - \kappa - rV_L + \\ [1-\phi][r+\delta][V_L - K - V_H] \end{bmatrix}}{r + \delta + \beta\theta q(\theta)}. \tag{68}$$

For high-skilled workers, the value of unemployment reflects the fact that only meetings with high skill-requirement vacancies generate employment. Such vacancies arise at the rate $[1-\phi]\theta q(\theta)$, and, upon match formation, yield the worker a capital gain of $W_{H,H} - U_H$. So the asset value equation is $rU_H = b + \theta q(\theta)[1-\phi][W_{H,H} - U_H]$.Using expression (19) this can be written as

$$rU_H = \frac{[r+\delta]\,b + [1-\phi]\,\beta\theta q(\theta)\,[p_H - \kappa - rV_H]}{r+\delta+[1-\phi]\,\beta\theta q(\theta)}. \tag{69}$$

Now, using (68), substitute for rU_L in the asset value equation (66) and impose the free entry conditions $V_L = V_H = K$. Then the free entry condition for low skill-requirement vacancies is equation (70)

$$rK + c = \frac{[1-\beta]\,q(\theta)\,\gamma\,[[p_L - c - rK - b] - [1-\phi]\,\beta\theta q(\theta)\,K]}{r+\delta+\beta\theta q(\theta)} \tag{70}$$

Since $V_L = V_H = K$, the expressions (66) and (67) can be combined to give

$$\gamma\,[r+\delta]\,K = [1-\gamma]\,[p_H - c - rK - rU_H]$$

Using (69) this simplifies to the condition

$$K = \frac{[1-\gamma]\,[p_H - c - b]}{r+\gamma\,[\delta+[1-\phi]\,\beta\theta q(\theta)]} \tag{71}$$

Summary So, in (64), (65), (70) and (71) we have four independent equations in the four unknowns u,γ, θ and ϕ. For this equilibrium to exist the necessary conditions (61), (62) and (63) must be satisfied. Then the wage premium will given by (72). In this equilibrium there is within-group inequality between low-skilled workers, even though they can only be employed in low-skill requirement jobs. These wage differences arise because some low skilled workers form employment relationships with regraded vacancies: vacancies which have been downgraded, having originally been involved high-skill requirements.

$$wp = \frac{w_{H,H}}{\phi w_{L,L} + [1-\phi]\,w_{L,L}^{Down}}. \tag{72}$$

2.3.5 Segmentation with Upgrading and Downgrading

As just discussed, when a firm with a high skill-requirement position encounters a low-skilled worker, downgrading requires $J_{L,L}^{Down} - K - V_H > 0$. If, in addition, a meeting between a high-skilled worker and a low skill-requirement job leads the firm to pay the fixed cost K to upgrade the skill requirement of the job to suit that of the worker then the value of upgrading must exceed that of the alternatives mismatch and segmentation without upgrading. This requirement is summarised in the conditions i) upgrading not-mismatch: $J_{H,H}^{Up} - K - V_L > J_{H,L} - V_L$ and ii) upgrading-not-segmentation: $J_{H,H}^{Up} - K - V_L > 0$. Using (18) (20) and the free entry conditions $V_L = V_H = K$ these necessary conditions can be written more fully as (73), (74) and (75) respectively.

$$p_L - \kappa - [2r + \delta] K - rU_L > 0, \tag{73}$$

$$p_H - \kappa - [2r + \delta] K - rU_H > p_L - \kappa - rK - rU_H \tag{74}$$

$$p_H - \kappa - [2r + \delta] K - rU_H > 0 \tag{75}$$

Once again the equilibrium is characterised using one mass balance equation for each skill-level of worker and one free entry condition for each skill-requirement level. For low-skilled workers, contact with either low skill-requirement vacancies or high skill-requirement vacancies, which arrive at rates $\phi\theta q(\theta)$ and $[1-\phi]\theta q(\theta)$, results in match formation. So, in this case, the mass balance equation becomes (76). For high-skilled workers, contact with either low skill-requirement or high skill-requirement vacancies will lead to match formation, so the steady state mass balance equation, (77), is identical to that in the equilibrium featuring mismatch without regrading described above (even though in the present case no mismatch arises):

$$\theta q(\theta) \gamma u = \delta [\mu - \gamma u] \tag{76}$$

$$\theta q(\theta) [1-\gamma] u = \delta [1 - \mu - [1-\gamma] u] \tag{77}$$

Since workers enter employment regardless of the skill requirement of the vacancy they meet, the flows equations (76) and (77) do not involve ϕ, the fraction of vacancies that are for low-skill requirement positions when originally created. The flow equation for low skilled workers can be rearranged as $\theta q(\theta) = \delta [\mu - \gamma u] / \gamma u$. Substituting this into the flows equation for high skilled workers gives $\gamma = \mu$, and also

$\theta q(\theta) = \delta[1-u]/u$.

The two free entry conditions require the value of a vacancy (for either skill requirement) to equal the sunk cost of technology adoption, $V_L = V_H = K$. As upgrading maxmises a firm's payoff, the firm with a low skill-requirement vacancy forms a match regardless of whether the worker whom it encounters is high-skilled or low-skilled. So the asset value equation for a low skill requirement vacancy is

$$rV_L = -c + q(\theta)\left[\gamma[J_{L,L} - V_L] + [1-\gamma]\left[J^{Up}_{H,H} - K - V_L\right]\right]$$

Substituting for $J_{L,L} - V_L$ and $J^{Up}_{H,H} - K - V_L$ using expressions (18) and (20), gives

$$rV_L + c = \frac{[1-\beta]}{r+\delta} q(\theta) \left[\begin{array}{c} \gamma[p_L - \kappa - rV_L - rU_L] + \\ [1-\gamma]\left[\begin{array}{c} p_H - \kappa - rV_H - rU_H \\ +[r+\delta][V_H - K - V_L] \end{array}\right] \end{array}\right]. \quad (78)$$

The flow value of a high skill-requirement vacancy reflects the presence of downgrading. If a firm with a high skill-requirement vacancy encounters a low-skilled worker, the capital gain from forming a match is $J^{Down}_{L.L} - K - V_H$, and if it encounters a high-skilled worker, the capital gain is $J_{H.H} - V_H$, as before. So:

$$rV_H = -c + q(\theta)\left[\gamma\left[J^{Down,Up}_{L.L} - K - V_H\right] + [1-\gamma][J_{H.H} - V_H]\right].$$

which, using (18) and (20), can be expressed as

$$rV_H + c = \frac{[1-\beta]}{r+\delta} q(\theta) \left[\begin{array}{c} \gamma\left[\begin{array}{c} p_L - \kappa - rV_L - rU_L + \\ [r+\delta][V_L - K - V_H] \end{array}\right] \\ +[1-\gamma][p_H - \kappa - rV_H - rU_H] \end{array}\right]. \quad (79)$$

We could proceed in a similar manner to the equilibria considered in previous sections: determine the flow value of unemployment for high-skilled individuals and for low-skilled individuals, and combine this with the asset values fior high skill-requirement and low-skill requirement vacancies, (78) and (79) and the free entry conditions. However, exploiting the equal value condition approach of AV, $V_L = V_H = K$, (78) and (79) can be reduced to the condition $\gamma = [1 - \gamma]$. This is consistent with the earlier result that $\mu = \gamma$ only if $\mu = 0.5$. Moreover, since the system of equations provides no information on ϕ thereis insufficient information to determine unique equilibrium values for u, ϕ and θ .[29]

2.3.6 Mismatch With Downgrading

Finally consider the equilibrium which exhibits mismatch with downgrading. If mismatch arises when a firm with a low skill-requirement vacancy meets a high-skilled worker, then the resulting capital gain, $J_{H,L} - V_L$ must exceed the gain from either segmentation or upgrading. This requires two inequalities to be satisfied: $J_{H,L} - V_L \geq 0$, $J_{H,L} - V_L \geq J_{H,H}^{Up} - K - V_L$. Using (18) and (20), we summarise this as conditions (80) and (81). If downgrading arises when a firm with a high-skill-requirement encounters a low-skilled worker, then the weak inequality $J_{L,L}^{Down} - K - V_L \geq 0$ holds. Using (20) this can be expressed as (82)

$$p_L - \kappa - rK - rU_H \geq 0 \tag{80}$$

$$p_L - \kappa - rK - rU_H \geq p_H - \kappa - [2r + \delta] K - rU_H \tag{81}$$

$$p_L - \kappa - [2r + \delta] K - rU_L \geq 0 \tag{82}$$

For high-skilled workers, contact with both low skill-requirement and high skill-requirement vacancies lead to match formation, so the steady state mass balance equation (83) is identical to that in the cross-skill matching equilibrium described above. For low-skilled workers, contact with low skill-requirement vacancies and high skill-requirement vacancies, which arrive at rates $\phi\theta q(\theta)$ and $[1 - \phi] \theta q(\theta)$ respectively, both result in match formation, so the mass balance equation is (84)

$$\theta q(\theta) [1 - \gamma] u = \delta [1 - \mu - [1 - \gamma] u] . \tag{83}$$

$$\theta q(\theta) \gamma u = \delta [\mu - \gamma u] \tag{84}$$

[29] This situation may be remedied if other parameters are skill-specific, such as the separation rate, the costs of vacancy creation and bargaining power, see Section 4.

As in the case of an equilibrium featuring segmentation with upgrading and downgrading, these equations imply $\gamma = \mu$, and also $\theta q(\theta) = \delta [1-u]/u$.

Now the asset value equation for vacancies with low skill-requirements takes the same form as in the equilibrium featuring mismatch without regrading. Using (18) and (20) we find

$$rV_L + c = \frac{[1-\beta]}{r+\delta} q(\theta) \begin{bmatrix} \gamma [p_L - \kappa - rV_L - rU_L] \\ +[1-\gamma][p_L - \kappa - rV_L - rU_H] \end{bmatrix}. \tag{85}$$

The flow value of a high skill requirement vacancy takes the same form as that in the previous section due to the presence of downgrading. When a firm with a high-skill requirement vacancy encounters a low skilled worker, the capital gain is $J_{L.L}^{Down} - K - V_H$, but when it encounters a high skilled worker the capital gain is $J_{H.H} - V_H$, as before. So:

$$rV_H = -c + q(\theta) \left[\gamma \left[J_{L.L}^{Down} - K - V_H \right] + [1-\gamma][J_{H.H} - V_H] \right].$$

which, using (18) and (20), can be expressed as

$$rV_H + c = \frac{[1-\beta]}{r+\delta} q(\theta) \begin{bmatrix} \gamma \begin{bmatrix} p_L - c - rV_L - rU_L + \\ [r+\delta][V_L - K - V_H] \end{bmatrix} \\ +[1-\gamma][p_H - c - rV_H - rU_H] \end{bmatrix}. \tag{86}$$

The two free entry conditions require the value of a vacancy (for either skill requirement) equal the sunk cost of technology adoption: $V_L = V_H = K$. Notice that, as in the previous section, equations (85) amd (86) can be combined in the equal value condition. In this case mismatch with downgrading gives equation (87). Since one of the two free entry conditions provides no additional information on u, ϕ and θ compared with that available from the mass balance equations, (83) and (84), there is insufficient information to determine the equilibrium of the system in this case.

$$K = \frac{[1-\gamma][p_H - p_L]}{\gamma [r+\delta]} \tag{87}$$

3 Vacancy Creation Costs: A Sensitivity Analysis

What is the impact of fixed costs of vacancy creation? Here I illustrate the role of fixed costs of vacancy creation by developing a numerical, using the parameter values adopted by AV and their calibration strategy. The aim is to put into perspective the results of the existing literature on mismatch due to differences in skill-requirements. To that end I conduct a sensitivity analysis to determine which (if any) candidate equilibria arise as vacancy creation costs vary, holding other parameters constant. In particular, when these costs are suppressed, i.e. $K = 0$, the relevance of mismatch without regrading and of segmentation without regrading in the original AV environment is revealed.

Table 1 Here

The parameters set in this approach are $\{r, \delta, \alpha, \beta, b, c, \kappa, K, p_H, p_L\}$. The assigned values, taken from the analysis AV (2002) are summarised in Table 1. In the light of recent work on the empirical properties of the canonical DMP model, some of these parameters appear unusual. The choice of a real interest rate of 5%, suggests that AV's numerical work should be viewed as applied to data at annual frequencies. This would be broadly consistent with a separation rate of 20% (whereas at quarterly frequencies, a separation rate of 10% is standard in recent empirical analysis of the DMP framework, see Shimer (2005)). The match elasticity of $\alpha = 0.5$ is standard and relatively uncontroversial. Worker bargaining strength, β, is set to 0.5, this too is standard, but is the source of controversy in the DMP literature. The value of leisure, b is set to 0.1; the flow cost of vacancy posting, c and the flow fixed cost of production, κ, are both set to 0.3 (most search models suppress the latter). The productivity of low-skilled workers is normalised to unity. Relative productivity for high-skilled workers, p_H/p_L, is set at 1.3. AV (ignoring the role of sunk costs of vacancy creation) argue that for this value of relative productivity both the equilibrium with mismatch and the equilibrium with segmentation (i.e. multiple equilibria) exist. Finally, AV include a multiplicative free parameter in the matching function, $A = 2$, representing the efficiency of the matching technology.[30]

Figure 1 Here

The pattern of the resulting equilibria is summarised in Figure 1. We use an indicator function to illustrate whether, for given K, the candidate equilibrium satisfies the necessary conditions; this takes the value 1 if, for given K, the necessary conditions are not violated, but takes the value 0 if any of the necessary conditions for the candidate equilibrium are violated. Consistent with the arguments presented in Section 2, and contrary to the existing literature, neither the equilibrium with mismatch (without regrading) nor the equilibrium with segmentation (without regrading) exists when $K = 0$. Instead, for low but strictly positive K there are multiple equilibria, one with downgrading of high-skill requirement vacancies, and another with upgrading of low skill-requirement vacancies.[31] As the fixed cost of creating vacancies rises equilibria without regrading emerge to replace the equilibria exhibiting (i) mismatch or (ii) segmentation). There is a small region in which mismatch exists as a unique equilibrium. Then at higher costs of vacancy creation, multiple equilibria occur again as an equilibrium with ex post segmentation arises. This suggests that the existence of mutiple equilibria might well confound attempts to determine which equilibrium, if any, is consistent with the data.

Figure 2 Here

Figure 2 illustrates the aggregate and skill-specific unemployment rates and between skill wage premia which arise in the model as K is varied. The figure also indicates the average values (across time) of these variables in US data. The jumps in the values of aggregate unemployment, u, low-skilled unemployment, u_L, and the wage premium, wp,

[30] Dolado et al. (2008) offer an alternative parameterisation that is closer to the "standard" values used to calibrate the DMP model to quarterly US data, see Shimer (2005). In particular, $r = 0.01$, $\delta = 0.1$ and $\kappa = 0$ and $A = 1$. Qualitatively this delivers similar results to those obtained from here.

[31] Notice that the equilibrium with both upgrading and downgrading does not exist here. This is because it is not possible to compute a solution (not because it violates one of the necessary conditions).

arise when new equilibria arise as K is increased. From the perspective of the empirical plausibility of the model, the key feature is that none of the equilibria, for any value of K, is capable of matching US data in relation to these variables.

4 Calibration and Experimental Design

Here I critically appraise the empirical strategy typically used in the literature on mismatch and propose an alternative data-consistent approach to evaluate the feasibility of the model and its various equilibria.

4.1 Critique of Existing Approach

Previous work on the AV model and related studies broadly follows the pattern of the previous Section. Authors assign values to the following parameters: $\{r, \delta, \alpha, \beta, b, c, \kappa, K, p_H, p_L\}$ to generate an equilibrium exhibiting mismatch and use the relevant equilibrium conditions to pin down equilibrium values of unemployment, u, the fraction of the unemployed who are low-skilled, γ, labour market tightness, θ, and the fraction of vacancies which are for low-skill requirement positions, ϕ. Finally, necessary conditions for a particular candidate equilibrium, such as (31), are checked - although given the manner in which parameters are typically chosen this last step is something of a formality.

There are a number of problems with this approach. (i) While some of these parameters, such as $\{r, \alpha, \delta\}$, can be pinned down reliably, others can not. Yet these unobservable or poorly measured parameters are assigned independently of each other and without reference to obervables, in order to generate an equilibrium of interest (i.e. the one with mismatch).[32] Then observable targets, such as skill-specific unemployment rates, are not attained, but this is considered neither as a puzzle to be addressed nor as a sufficiently significant flaw to warrant rejection of the model. In short, this approach does not provide a test of the model. (ii) While empirical work on the Diamond-Mortensen-Pissarides

[32] To achive this, authors do one or more of the following: i) adopt values from the literature on equilibrium unemployment with homogeneous workers; ii) assign parameters freely; iii) introduce additional free parameters.

(DMP) model provides a natural reference point, the associated values for unobserved parameters are specific to that model, and indeed to the particular calibration strategy used, and need not carry across to an environment with heterogeneous workers and (potentially) mismatch. (iii) Even in the context of the DMP model, there is considerable disagreement over the appropriate values of parameters such as the value of leisure, b, and worker bargaining power, β, as illustrated by the controversy over the cyclical properties of the DMP model, see Shimer (2005) and Hagedorn and Manowskii (2008). (iv) The approach adopted in AV and subsequent work can generate multiple equilibria, and it is then unclear how one should procede to evaluate the empirical content of equilibria exhibiting mismatch.

4.2 A Strategy for Calibration and Evaluation

Here we propose a simple 3-step approach. The aim is to allow the adequacy of the model as an account of the data to be investigated. (i) We calibrate the model to observable targets; a candidate equilibrium which cannot match these targets is rejected as computationally infeasible. (ii) The targets for observables are used in conjunction with equilibrium conditions and accounting identities to pin down values of the unobserved parameters for each of the computationally feasible equilibria (in turn).[33] (iii) To evaluate which of the computable equilibria arises, I examine the necessary conditions in conjunction with the targetted and computed parameters. A candidate equilibrium only arises if it satisfies all the necessary conditions and the associated parametyer values are feasible (e.g. non-negative).[34] To illiustrate this strategy, we focus on the equilibrium exhibiting mismatch without regrading.[35]

[33] So the computed values of the unobservable parameters are specific not just to the model, but to the particular candidate equilibrium that is being considered.

[34] A computable equilibrium which satisfies all the necessary conditions for existence is said to feasible, whilst a computable equilibrium which does not satisfy all the necessary conditions is said to be infeasible.

[35] This involves the equilibrium conditions (22), (23), (29) and (30), as well as the necessary conditions (31), (32), and (33). In addition we require that low skilled and high-skilled workers both find the environment sufficiently attractive to enter into employment relationships (35) and (36). We also use an accounting identity for the skill decomposition of unemployment and an expression for the wage premium (34)

4.2.1 Calibration

We calibrate the model to match US data, with the length of a period equal to one quarter. The experiment invariant parameters, with targets, are summarised in Table 2. Parameters whose calibrated values vary across experiments are summarised in Table (3). One of the advantages of this approach is that we adopt an agnostic position on the magnitudes of these controversial parameters, and are prepared to allow them to be determined by the data and the model. However, at the minimum some sign restrictions should be satisfied, all parameters should be non-negative and certain parameters, e.g. γ, ϕ, can only lie in the unit interval.

Tables 2 & 3 here

Several parameters $\{r, \alpha, \delta\}$ are relatively uncontroversial. We set the real interest rate $r = 0.01$, consistent with an annual interest rate of 4%. The matching function is assumed to take a Cobb-Douglas form, to be linearly homogeneous and to have an elasticity parameter $\alpha = 0.5$, consistent with evidence surveyed by Petrongolo and Pissarides (2001). To enhance comparability with the existing literature, the separation rate is intially assumed homogeneous across workers. We set $\delta = 0.1$, 10% of jobs are destroyed each quarter, following Shimer (2005) and others.[36]

The critical issue is how to calibrate the remaining poorly observed and/or controversial parameters, in particular b,c, K, κ, p_H and p_L and pin down the remaining unknowns μ, u_L, wp, γ, θ and ϕ. To reduce the scale of the problem let us normalise some parameters. Firstly, set the absolute productivity of low skilled workers, $p_L = 1$, since a knowledge of the *relative* productivity of skilled and unskilled is sufficient in accounting for the wage premium. Secondly, we follow the standard practice of the equilibrium unemployment literature where the flow fixed cost of production, $\kappa = 0$. Thirdly, for the model presented in Section 2

for a given candidate equilibrium

[36] In the light of concerns over the relevance of parameter restrictions obtained in the homogeneous worker set up of the DMP model, we later relax this assumption and allow separation rates to exhibit skill-specific differences.

there are two different sources of costs associated with vacancies, K, the one-off fixed cost of vacancy creation, and the flow fixed cost of posting vacancies, c. In the DMP framework an absence of heterogeneity between workers (and hence jobs) makes the distinction between K and c irrelevant - at least when considering the steady state.[37] Yet, as demonstrated in Section 2, once workers have heterogeneous skills and firms can create skill-specific vacancies, the distinction becomes critical - even when dynamics are not of interest. However, our focus on the steady state makes it difficult to identify c and K separately, and in any case evidence on c is thin and model-specific, which makes it difficult to rely on information from other studies. So for simplicity we set $c = 0$, and focus on K.[38] Of the remaining unknowns, μ, the fraction of unskilled workers, u the aggregate unemployment rate, u_L the unemployment rate among the low-skilled, wp the wage premium of skilled over unskilled, can be pinned down directly in the data. Then γ, θ, ϕ, b, K and p_H can then be computed in a recursive fashion using a combination of 6 accounting identities and equilibrium conditions from the equilibrium featuring mismatch without regrading, see below.

We begin with the accounting identity relating aggregate unemployment to unemployment among specific skill groups

$$u \equiv \mu u_L + [1 - \mu] u_H$$

where u_i is the unemployment rate among individuals with skill-level i. The number of low-skilled unemployed, captured above by μu_L, can also be written as γu, the product of the unemployment rate and the fraction of unemployed who are low skilled. In US data $u = 0.06$, $u_L = 0.08$ and $\mu = 0.66$, so

$$\gamma = \mu \frac{u_L}{u} = 0.88. \tag{88}$$

[37] However, even in the DMP environment, one-off fixed costs of vacancy creation can matter out of steady state. Fujita and Ramey (2007), in a contribution to the controversy over the cyclical properties of the DMP model, calibrate fixed costs of vacancy creation to match the persistence properties of vacancies in a DMP environment, while retaining the traditional flow cost of vacancy creation.

[38] Whereas other studies introduce free parameters we normalise these (to unity in the case of multiplicative parameters or zero in the case of additive parameters).

Notice that by insisting that we match the aggregate and skill-specific unemployment rates, the share of low-skilled amongst the unemployed, γ, must take this value *regardless of which equilibrium holds*.

The mass balance equations for high skilled and low-skilled workers pin down labour market tightness, θ and the fraction of jobs which are high-skilled, ϕ.[39] The values for θ and ϕ are specific to the candidate equilibrium.[40] For the equilibrium exhibiting mismatch, equation (23) gives

$$\theta q(\theta) = \delta \frac{[1 - \mu - [1 - \gamma] u]}{[1 - \gamma] u} = 4.62$$

so to match these targets, given a matching function with elasticity $\alpha = 0.5$, labour market tightness, θ, is 21.365. Therefore, ϕ, the fraction of vacancies with high skill-requirement, must satisfy the mass balance equation, (22), for low-skilled individuals:

$$\phi = \frac{\delta [\mu - \gamma u]}{\theta q(\theta) \gamma u} = 0.249$$

Next we use the equal value condition (29) and the free entry for high skill-requirement vacancies (30) and an expression for the wage premium for high-skilled workers (34) to pin down equilibrium specific values for the value of leisure, b, the size of fixed costs of vacancy creation, K, and the relative productivity of high-skilled workers, p_H. These three equations must be solved simultaneously.

There is one remaining parameter, β, which captures worker bargaining power. This has been a source of recent controversy in the analysis of labour market fluctuations using the DMP matching model, see Shimer (2005), Hagedorn and Manowskii (2008). If our main interest were in the cyclical properties of the model then it would be straightforward, in principle, to β assign to target some dynamic property of the model, as in the DMP literature. However, our concern here is with whether the AV

[39] The structure in the case of mismatch without regrading is recursive, but for other candidate equilibria, these two equations may need to be solved simultaneously, depending on the form of the mass balance equations.

[40] Some equilibria require all workers who encounter jobs will form matches, regardless of the initial skill-requirement of the vacancy that they encounter. For these it is not possible to determine ϕ in this way. As a result the system is under-determined.

model, and particularly the equilibrium exhbiting mismatch is even capable of explaining the steady state. This is, in a sense, logically prior to any question about the cyclical properties of the model, which typically concerns itself with deviations around such a steady state. Therefore instead of pinning down "the" value of β we sidestep the controvery in the literature by examining whether there is any feasible range of values of β for which the AV model is capable of matching the data. For the purposes of assessing the model in this way, β is a suitable choice of free parameter, since, unlike say K, it must take a value on a finite interval, in particular $\beta \in [0,1]$. So, for a given value of $\beta \in [0,1]$, given the parameters from Table (2), and the equilibrium-specific values of θ and ϕ, we use (29), (30) and (34) to pin down expeiment-specific values for b, K, and p_H.

4.2.2 Necessary Conditions and Model Evaluation

We use the necessary conditions, such as (31), (32) and (33), as a means to test which, if any, of the equilibria which may arise in the model are consistent with US data. To do this we must evaluate all the necessary conditions for each candidate equilibrium at each set of calibrated parameters (i.e. for each $\beta \in [0,1]$ and associated γ, θ, ϕ, b, K and p_H). For any candidate equilibrium, the necessary conditions indicate whether that equilibrium arises by comparing its consequences against the consequences from an alternative equilibrium. We implement the tests of existence in the same way. The full implications of this approach, which rules out multiple equilibria, are most easily understood by means of an example.

Let us consider both the equilibrium exhibiting mismatch without regrading and the equilibrium exhibiting segmentation without regrading. One of the conditions for the existence of an equilibrium featuring mismatch (without regrading) involves ruling out the equilibrium with segmentation (without regrading) using (31). One of the conditions for the existence of an equilibrium featuring segmentation (without regrading) involves ruling out the equilibrium featuring mismatch (without regrading) using (**??**). Notice that the first condition for an equilibrium featuring mismatch without regrading is the opposite of the first condition for an equilibrium featuring segmentation. So if one holds, the other should not.

What this means in practice is that when evaluating the feasibility of a candidate equilibrium against an alternative equilibrium, terms referring to the situation in which the candidate equilibrium holds should be evaluated at the parameter values which occur when candidate equilibrium arises, but terms arising under the alternative should be evaluated at the values that would be taken *if the alternative equilibrium were to arise*. So in the case of mismatch versus segmentation, the necessary condition should be

$$p_L^{Mismatch} - \kappa^{Mismatch} - rK^{Mismatch} - rU_H^{Mismatch} \geq 0 \tag{89}$$

while in the case of segmentation versus mismatch the necessary condition should be

$$p_L^{Mismatch} - \kappa^{Mismatch} - rK^{Mismatch} - rU_H^{Mismatch} < 0. \tag{90}$$

By contrast, when determining the existence of a candidate equilibrium AV evaluate both the terms arising in the candidate equilibrium and thos arising under the alternative as if they arise unser the candidate equilibrium. This does not affect the outcome for the condition ruling out segmentation in a mismatch equilibrium, so (89) is implemented, but for the segmentation equilibrium, AV take all parameters (and variables) to be those which hold in the equilibrium with segmentation. That is, AV evaluate whether the following condition is violated[41]

$$p_L^{Segmentation} - \kappa^{Segmentation} - rK^{Segmentation} - rU_H^{Segmentation} < 0. \tag{91}$$

[41] To be precise, AV use the stategy outlined in Section (3), so the mismatch without regrading equilibrium and the equilibrium featuring segmentation without regrading would each be evaluated at (common) parameters $\{\mu, \alpha, \delta, r, c, K, \kappa, b, \beta, p_H, p_L\}$. So, while u, γ, ϕ, θ and wp would vary across equilibria exhibiting mismatch and segmentation, this would show up only in the term rU_i in the necessary conditions. Nonetheless, our critique of the multiple equilibrium interpretation of their model would still apply.

But in this inequality, the left hand side computes the value that arises in the event that the *mismatch* equilibrium arises using the parameter values that apply in the *segmentation* equilibrium. Of course, it is then quite possible for both (89) and (91) to be satisfied, so that multiple equilibria arise. However, this is an artefact of the implementation strategy for the model, rather than some deep property of the model itself. It is instantly clear that, as a by-product, our approach (using (89) and (90)) rules out multiple equilibria, and makes the task of evaluating which (if any) of the model's equilibria is supported by the data considerably easier.

In implementing this strategy to evaluate the necessary conditions for a particular equilibrium using the parameter values appropriate to the specific alternative considered by each condition, there is an obvious difficulty to be dealt with when one of those alternative equilibria cannot be computed. For example, none of the candidate equilibria featuring downgrading exist: because there isn't sufficient information to compute the outcome for the cases featuring segmentation with both upgrading and downgrading or mismatch with downgrading.[42] To proceed we assume that if a necessary condition involves an alternative for which no solution to the equilibrium can be computed, then the necessary condition is deemed to be satisfied.[43] If we assume otherwise, then it is never possible to evaluate all the necessary conditions associated with any of the equilibria.

[42] The equilibria that are computationally infeasible arise when workers of a given skill-level match with the employer that they meet regardless of the skill-requirement of the vacancy that was initially created. This occurs in the following candidate equilibria: Segmentation with Upgrading and Downgrading, Mismatch with Downgrading. In both these cases, ϕ does not appear in the mass balance equation for either skilled or unskilled workers. Then there are insufficient conditions to determine $\{\phi, K, b\}$. Also, the flows identities reduce to the conditions $\gamma = \mu$, and $\theta q(\theta) = \delta [1-u]/u$. Notice that this requires that the fraction of low-skilled among the unemployed equal the fraction of low-skilled in the labour force as a hold. This is clearly in conflict with the data (which require $\gamma = \mu \frac{u_L}{u}$). Only an equilibrium in which one or other group of workers (and/or firms) do not form employment matches in some circumstances can reconcile $\gamma = \mu$ and $\gamma = \mu \frac{u_L}{u}$.

[43] In principle this can give rise to multiple equilibria (in which segmentation with upgrading and segmentation with downgrading are both feasible - so segmentation with both upgrading and downgrading might arise if only it were computationally feasible).

Finally, we also require that any candidate equilibrium satisfy the following participation constraints for unskilled and skilled workers: $p_L - \kappa - rK - rU_L \geq 0$, and $p_H - \kappa - rK - rU_H \geq 0$. These constraints never bind.

4.3 Summary

To summarise, I use a target driven calibration strategy to assess the empirical relevance of the equilibria of the model described in Section (2). For a given $\beta \in [0, 1]$, using the approach outline in this Section, I determine, where computationally feasible, the parameters associated with each candidate equilibriium of the model. Provided the computed parameter values satisfy standard sign restrictions, I examine the necessary conditions. A computationally feasible equilibrium, for which the parameter values satisfy standard sign restrictions, is said to exist if it also satisfies the necessary conditions associated with that equilibrium. Moreover, using the necessary conditions to partition parameter space appropriately, at most a single equilibrium will exist for any $\beta \in [0, 1]$ at least to the extent that all equilibria can be computed.

5 Results.

There are 4 key results. (i) If the costs of vacancy creation are sunk, the model *is* capable of matching the skill-specific unemployment rate and wage premium data for the US, as some candidate equilibrium exists for each feasible value of $\beta \in [0, 1]$. (ii) Provided $\beta > 0$, i.e. if workers have some bargaining power, the equilibrium with mismatch cannot account for the data, because the candidate solution (with mismatch) fails to satisfy the appropriate conditions ruling out other equilibria. (iii) For values of β between 0 and 0.5 an equilibrium featuring segmentation without regrading is consistent with the data. For higher values of β, the equilibrium featuring segmentation with upgrading emerges. (iv) Equilibria featuring downgrading are never capable of accounting for observed unemployment and wage premia. We investigate these issues in more detail below.

5.1 Feasibility of Calibrated Parameter Values

Figure (3) illustrates the values of calibrated parameters, b, p_H, K, ϕ, θ across the candidate equilibria.[44] This diagram serves to illustrate (for $\beta \in [0,1]$) situations in which a candidate equilibrium can only be calibrated to the data by requiring parameters to take infeasible values, and allows us to compare the implied values to those obtained in other environments, such as the Diamond-Mortensen-Pissarides framework. For the values of the calibrated parameters to be feasible, we require $p_H > 1$, so that the productivity of skilled workers(in high skill-requirement positions exceeds that of low-skilled workers in low skill-requirement positions ($p_L = 1$). We also require the share of vacancies that are for low-skill requirement positions, ϕ, to lie on the unit interval, $[0,1]$. Similarly for any equilibrium in which low-skilled workers form matches we require that the value of leisure, b, is less than the productivity of low-skilled worker in employment, so $b \in [0,1)$. Finally feasible values for labour market tightness, θ, and K, the cost of vacancy creation / technology adoption, are positive.

Using this information, the equilibrium featuring segmentation with downgrading is infeasible at all $\beta \in [0,1]$ because it requires the fraction of vacancies which are for low skill requirement positions to be negative((= -3.0193) to avoid cluttering and distorting the figures information on this equilibrium is not incorporated in Figure (3).[45] Equally, the equilibrium featuring segmentation with upgrading is infeasible for $\beta < 0.6$ as is requires negative values for the sunk cost of technology adoption.

[44] Note that no information is provided for the equilibria featuring segmentation with upgrading and downgrading, or the equilibrium featuring mismatch with downgrading, because in these cases there is insufficient information to pin down unique combinations of parameter values that enables these equilibria to capture observed unemployment rates and wage premia.

[45] From the results in Figure (3) and the discussion in Section (2) we can rule out *any* equilibrium involving downgrading due to (i) infeasible parameters or (ii) underidentification. There is insufficient information to determine the equilibria featuring segmentation with both upgrading and downgrading and that exhibiting mismatch with downgrading. This is because in both cases each type of worker forms a match regardless of which type of vacancy it encounters. Therefore the flows equations do not pin down the fraction of jobs that are for low skill requirement positions and the system is undertermined.

By contrast in the equilibrium featuring mismatch without regrading is feasible (except for $\beta \in (0.45, 0.55)$), while the equilibrium featuring segmentation without regrading always generates values for the calibrated parameters that are feasible.

The values of the calibrated parameters for the equilibrium featuring segmentation without regrading are largely unaffected by changes in worker bargaining power. Calibrating the parameters to observable features using the equilibrium conditions arising from the equilibrium featuring segmentation without regrading, high-skilled workers are required to be around 50% more productive than their low-skilled counterparts. Interestingly, in the light of the controversy in the recent literature on the DMP model, the value of leisure is close to the productivity of low-skilled workers, but this holds for all $\beta \in [0, 1]$, not just for low worker bargaining power. The calibrated value of the share of low skill-requirement jobs in total vacancies is around 1/5, while overall labour market tightness is around 33. These two values are obtained directly from the labour market flows conditions (40) and (41). While their computation does not involve the free entry conditions or the wage premium condition, they are clearly consistent wit hthe idea that there are relatively high returns to opening vacancies for high skill-requirement positions. On the other hand the notion that 4/5 of the vacancies can only be performed by 1/3 of the labour force seems empirically implausible.

For the equilibrium featuring mismatch without regrading the labour market flows equations differ from the equilibrium featuring segmentation without regrading in that high-skilled workers form matches regardless of the skill-requirement of the vacancy that they encounter. So the value of labour market tightness required to match labour market facts is lower, $\theta = 21$, while the fraction of jobs that are for low skill requirement positions is higher, $\phi = 0.25$, than in the equilibrium with segmentation. Also, unlike the equilibrium with segmentation the calibrated values of b, p_H and K are sensitive to β. The most interesting ranges of β are $\beta \in (0, 0.45) \cup (0.55, 1.0)$, for which the values of b, p_H and K decline as β rises.

For the equilibrium featuring segmentation with upgrading, the flows equations are identical to the situation when there is mismatch without regrading. For the range of values of β ($\in (0.6, 1]$) for which the equilibrium featuring segmentation with upgrading is feasible, the value of leisure is broadly similar to the situation for segmentation without regrading, and independent of β however, the productivity of high-skilled workers increases from around 1 to 1.5 as β rises from 0.6 to1. At the same time (and consistent with the idea that the equilibrium with upgrading can arise, the calibrated value of K is extremely low.

5.2 Assessing the Necessary Conditions

Next we need to check whether the necessary conditions are satisfied (for any given value of $\beta \in [0, 1]$) for each of the 3 candidate equilibria which can generate feasible calibrated parameter values. Figure (4) illustrates the feasibility of the remaining equilibria across the full range of values of β. For each candidate equilibrium, for a given value of $\beta \in [0, 1]$, we evaluate the necessary conditions required to rule out other equilibria.[46] As noted in Section (4.2.2) the necessary conditions that we can evaluate for the 3 feasible candidate equilibira appear in pairs the condition ruling out segmentation in favour of mismatch is the reverse of the condition ruling out mismatch in favour of segmentation. So we list each condition only once. The upper panel of Figure (4) is the condition for examining mismatch versus segmentation (framed with mismatch as the candidate and segmentation as the alternative). The middle panel considers the condition ruling out segmentation (the alternative) in favour of upgrading (the candidate), the lower panel illustrates the necessary condition ruling out upgrading (the alternative) in favour of mismatch (candidate). Only when $\beta = 0$, i.e. the rather uninteresting limit case in which workers have no bargaining power, and for $\beta \in (0.45, 0.55)$ is the condition ruling out segmentation in favour of mismatch satisfied . Otherwise, for $\beta \in (0, 0.45) \cup (0.55, 1.0)$, the equilibrium featuring mismatch without regrading violates the necessary condition ruling out segmentation (31): high-skilled workers prefer to reject low-skill requirement

[46] We omit the requirements that high-skilled and low-skilled workers participate, as these are never binding when an equilibrium is otherwise feasible.

positions, in favour of waiting for a high-skill-requirement position. At $\beta \in (0.45, 0.55)$, while the condition ruling out segmentation in favour of mismatch is not violated, it turns out that the parameter values are infeasible. The condition ruling out upgrading in favour of mismatch, (32) is violated for similar values of β, but for $\beta < 0.55$, the parameter values are infeasible in the equilibrium featuring segmentation with upgrading. So when workers have high bargaining power, the sunk costs of upgrading are sufficiently low and the relative productivity of high-skilled workers is sufficiently high that it is worthwhile for a firm with a low skill-requirement vacancy to upgrade it rather than try to engage in mismatch. The middle and lower panels offer further insight on these other candidate equilibria. The equilibrium with segmentation without regrading, although generating feasible parameters for all β, violates the condition ruling out upgrading for $\beta \in (0.6, 1.0)$, since for those values of β a low value of adoption costs K support the existence of an equilibrium with upgrading that matches observable features of the data.[47]

Why is mismatch infeasible? To understand this we examine the necessary conditions (for mismatch) to see which conditions are violated (and why). Here we limit discussion to values of $\beta \in [0, 0.45]$ which appear to be of greatest empirical relevance. This range of values of β mismatch is rejected in favour of segmentation without regrading (while upgrading is infeasible). The relevant necessary condition is

$$p_L - rK - rU_H = \frac{[r+\delta]\,[p_L - rK - b] - [1-\phi]\,\beta\theta q(\theta)\,[p_H - p_L]}{r+\delta+\beta\theta q(\theta)} \geq 0$$

The denominator is positive, and so the validity of the inequality (and the existence of the equilibrium featuring mismatch without regrading) depends on the sign of the numerator. How does this vary as β varies? Although β enters this condition directly only once, the calibrated values of b, p_H and K depend on β. The value of b varies non-monotonically with β over the range $b \in [0.65, 0.81]$, K declines monotonically from

[47] Meanwhile the equilibrium featuring segmentation with upgrading which arises at high β, does not occur at low values of bargaining power because in matching the observed skill-specific unemployment rates and the wage premia, it generates infeasible (negative) parameter values, for the costs of creating vacancies.

$K = 9.17$ to 0.91 as β rises ffrom 0.05 to 0.55, while p_H declines monotonically from 1.7 to 1.2. For the equilibrium with mismatch $r + \delta = 0.11$, $\theta q(\theta)$ is approximately 5, ϕ is approximately 0.25 and p_L is normalised to one, so the numerator in the above expression is

$$0.11 \cdot [1 - 0.01 \cdot K(\beta) - b(\beta)] - 0.75 \cdot \beta \cdot 5 \cdot [p_H(\beta) - 1]$$

Using this information, we see that the first term in the numerator $0.11 \cdot [1 - 0.01 \cdot K(\beta) - b(\beta)]$ takes a value between 0.01 and 0.03.[48] As a result the sign of the numerator depends on the magnitude of the second term, $[1 - \phi]\beta\theta q(\theta)[p_H - p_L] = 0.75 \cdot \beta \cdot 5 \cdot [p_H(\beta) - 1]$. First notice that this term is positive (since $p_H > p_L$, $0 < \phi < 1$, $\beta \in [0, 1]$ and $\theta > 0$). Then since $\theta q(\theta) \simeq 5$ and $\phi \simeq 0.25$, it follows that for mismatch to exist, we require $[0.75]\beta \cdot 5 \cdot [p_H - p_L] < [r + \delta][p_L - rK - b] \simeq 0.03$. Given the relative changes in $p_H - p_L$ and β, this appears most likely to be satisfied for very low values of β. In particular, as $\beta \to 0$, $[r + \delta][p_L - rK - b] \simeq 0.01$, so for mismatch to be valid, assuming that $p_H - p_L$ (=0.75) we would require have $\beta < 0.01/[(0.75) \cdot 5 \cdot (0.75)] \simeq 0.003$. Interestingly this suggests that mismatch might be plausible at the low values of worker bargaining power suggested by Hagedorn and Manowskii (2008), but not at the conventional value of $\beta = 0.5$.[49]

5.3 Summary

In summary for a candidate equilibrium to exist (at a given value of $\beta \in [0, 1]$) we require that all the calibrated parameter values are feasible and also that all necessary conditions for that candiadte equilibrium hold. We summarise this information in Figure (5) which uses an indicator function to illustrate whether the candidate equilibrium generates infeasible parameter values at that value of β; this takes the value 1 if,

[48] As β rises rK delines monotonically from 0.09, to 0.01. For $\beta = 0.05$, $b \simeq 0.8$, and $rK \simeq 0.09$, so $[r + \delta][p_L - b - rK] \simeq 0.01$, for $\beta = 0.5$, $b \simeq 0.8$ and $rK \simeq 0.01$, so $[r + \delta][p_L - b - rK] \simeq 0.02$. b is minimised ($b \simeq 0.65$) at around $\beta \simeq 0.3$, then $rK \simeq 0.3$, so $[r + \delta][p_L - b - rK] \simeq 0.03$.

[49] Of course in the limit as $\beta \to 0$, b may rise, and so may p_H. This would tend to reduce still further any threshold value of β for which an equilibrium featuring mismatch without regrading exists.

for given β, all the parameter values for a candidate equilibrium are feasible and the necessary conditions are not violated, but takes the value 0 if any parameter is infeasible or if any of the necessary conditions for the candidate equilibrium are violated. For the low values of $\beta < 0.5$ that appear to be of greatest relevance equilibria with mismatch cannot account for the stylised facts, whereas equilibria. For this range of values (of β) the equilibrium featuring segmentation without regrading involves relatively high costs of vacancy creation, while the equilibrium featuring segmentation with upgrading is infeasible (as the calibrated parameter values are infeasible). At high relatively high values of worker bargaining power, in fact rather higher than used in the DMP literature, the observed equilibrium features equilibrium with upgrading. When this equilibrium arises it features a value of leisure roughly equal to the low-skilled worker productivity, high skill productivity that is roughly equal to low skill productivity (for $\beta \simeq 0.55$) rising to around 1.5 times that of low skilled workers as $\beta \to 1$, and (unsurprisingly) very low costs of vacancy creation (and regrading), K, (100 times smaller than in the DMP literature).[50]

6 Summary and Discussion

In this paper we analyse a model of mismatch for a frictional labour market in which workers differ by skill level, jobs differ by skill requirement and creating a vacancy with a given skill-requirement entails a fixed cost (of technology adoption). We assume that there are two skill levels, that low-skilled workers are unable to perform high skill-requirement activities, but that all other matches may, in principle, be formed. Fixed costs of technology adoption, which are omitted from previous studies, play a critical role. Without fixed costs the equilibria proposed in the literature do not exist, because a firm is unable to commit to hold open a job with a given skill requirement in the event that it meets a worker of a different skill level. We show that a much richer set of equilibria

[50]Form a qualitative viewpoint, these results are robust to extensions which allow for the separation rate to be skill-specific, and for the costs of technology adoption to be skill-requirement specific.

arises if a firm must pay a fixed cost to open a vacancy with a particular skill-requirement. As well as equilibria with mismatch and with segmentation, new equilibria may arise in which a firm regrades its vacancy so that its skill-requirements match the skill level of the worker it has met.

We calibrate the model to US data. In contrast to the existing literature on mismatch, we calibrate to observable targets, such as skill-specific unemployment rates and wage premia, rather than unobservable or poorly identified parameters such as the value of leisure, vacancy posting costs and relative productivity. We demonstrate that the existence of multiple equilibria in previous work is simply an artefact of the calibration framework adopted therein. Using this calibration strategy we show that the equilibrium with mismatch cannot arise for any feasible combination of parameter values; whereas regrading and segmentation are plausible outcomes.

The proximate reason for the absence of mismatch is the magnitude of the value of leisure (close to the productivity of low-skilled workers and that of mismatched high-skilled workers) and the extent of the wage premium, which reflects the extent to which the productivity of the high-skilled exceeds that of the low skilled. Given US data mismatch (without regrading) is ruled out because the observed wage premium is sufficiently high, and the value of leisure so close to the high-skilled worker's productivity when mismatched, that any high-skilled worker would reject a low-paying match with a low skill requirement vacancy in favour of continued unemployment while searching for a high-skill requirement job. Instead the labour market segments along lines of skill at low values of worker bargaining power (when upgrading generates infeasible parameter values).[51]

One might be tempted to interpret the presence of segmentation as justification for the simple segmented labour markets approach adopted in much of the literature on the labour market inequality, see, for example, the basic model in Acemoglu and Autor (2010). However, the

[51] This preference for segmentation is robust to a number of variants of the model which permit skill-specific job-separation rates and skill-requirement specific vacancy creation costs.

ex post segmentation that is present in the model in this paper arises endogenously - through high-skilled orkers choosing not to form a match after having met a low skill-requirement vacancy, whereas the traditional literature on (technological change and) labour market inequality imposes segmentation exogenously. A natural question then is whether it is preferable to endogenise the segmentation decision or simply to ignore the possibility of mismatch and proceed to impose segmentation ex ante. However, microeconometric evidence supports the notion of mismatch (even using the simple two-skill classification adopted here), see inter alia Khalifa (2008). And at the same time mismatch appears to be an interesting and important issue when thinking about productivity, misallocation and inequality. One possibility is that a richer set up, for example, allowing for a finer treatment of skill-specific hetereogeneity, or for alternative forms of heterogeneity, might help to reconcile the theoretical framework with the empirical evidence on mismatch.

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	Meaning	Value
r	real interest rate	0.05
α	Elasticity of matching function	0.5
δ	Separation rate	0.2
β	Worker bargaining power	0.5
b	Value of leisure	0.1
c	Flow cost of vacancy posting	0.3
κ	Flow fixed cost of production	0.3
p_H	High-skilled worker productivity in high skill-requirement job	1.3
p_L	Low-skilled worker productivity in low skill-requirement job	1
A	Scaling factor in matching function	2

Table 1: Albrecht Vroman (2002) Parameter Values

	Meaning	Value	Target
r	real interest rate	0.01	Annual real rate = 4%
α	Elasticity of matching function	0.5	Petrongolo & Pissarides (2001)
δ	Separation rate	0.1	Shimer (2005)
μ	Labour force share of low-skilled	0.67	Data
γ	Unemployment share of low-skilled	0.88	Data reconcile u and u_L
p_L	Low-skilled worker productivity in low skill-requirement job	1	Normalisation
c	Flow cost of vacancy posting	0	Normalisation
κ	Flow fixed cost of production	0	Normalisation
A	Scaling factor matching function	1	Normalisation

Table 2: Experiment-Invariant Parameter Values

	Meaning
ϕ	Vacancy share with low skill-requirem't
θ	Overall labour market tightness
b	Value of Leisure
p_L	Low-skilled worker productivity in low skill-requirement job
K	Fixed cost of vacancy creation

Table 3: Parameters which are not Expeiment Invariant

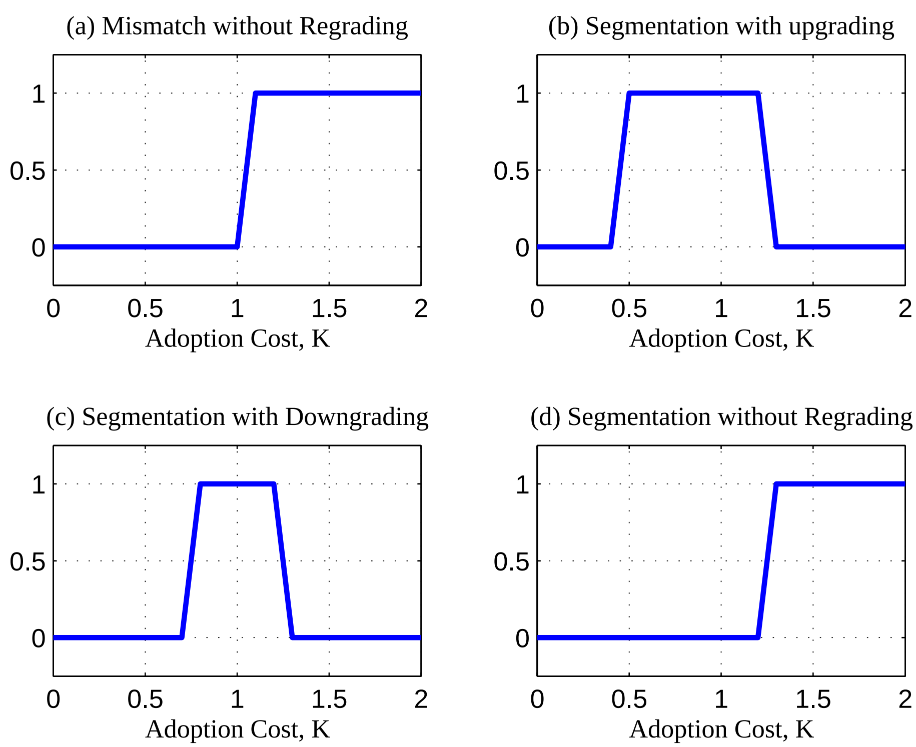

Figure 1: Effect of Vacancy Creation Costs on Existence of Equilibria (AV parameters)

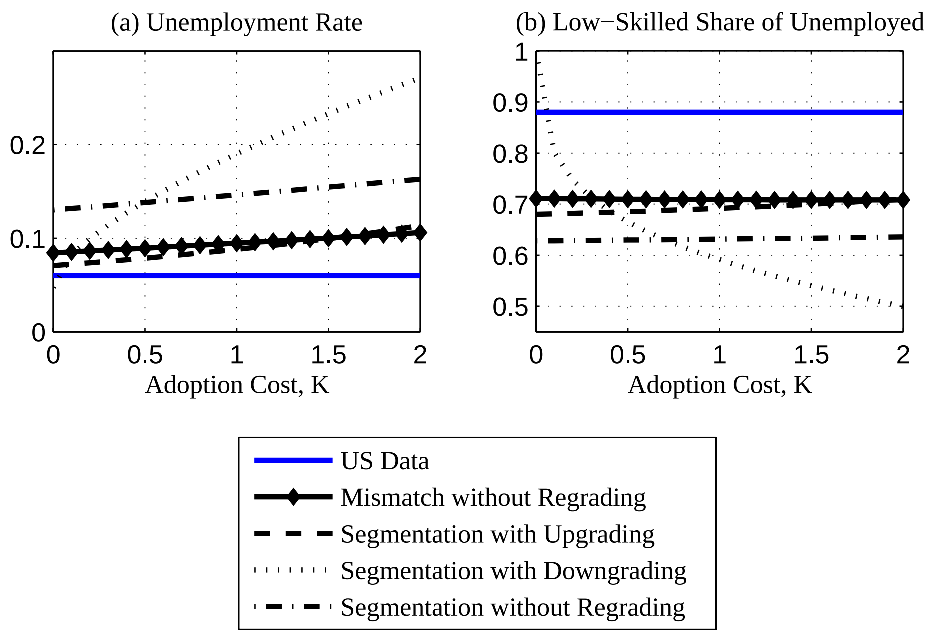

Figure 2: Labour Market Effects of Vacancy Creation Costs

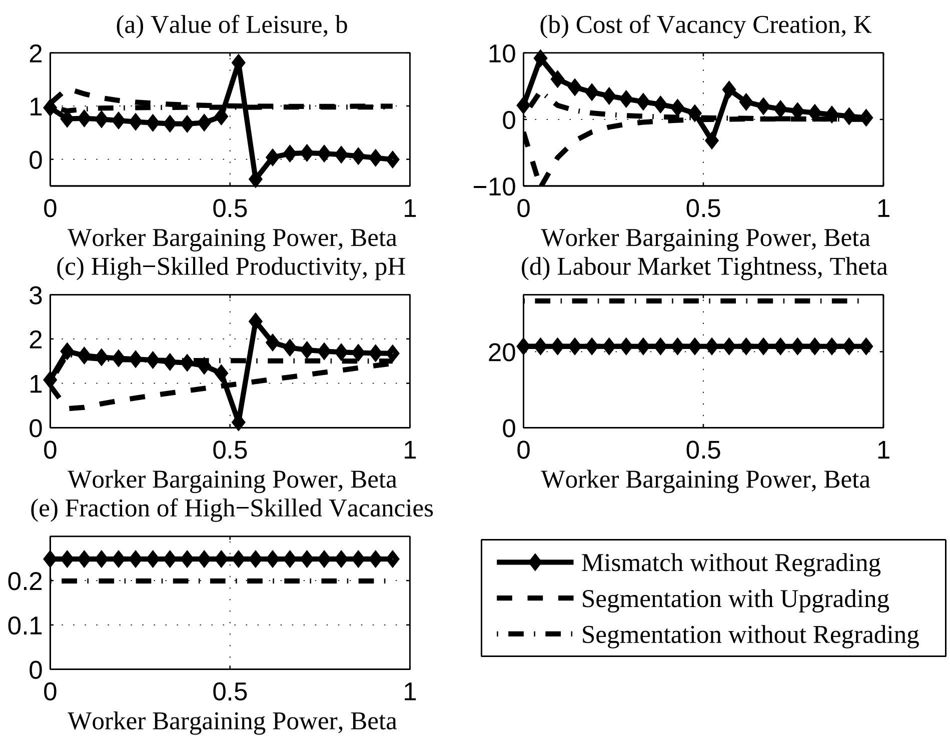

Figure 3: Experiment-Specific Parameter Values

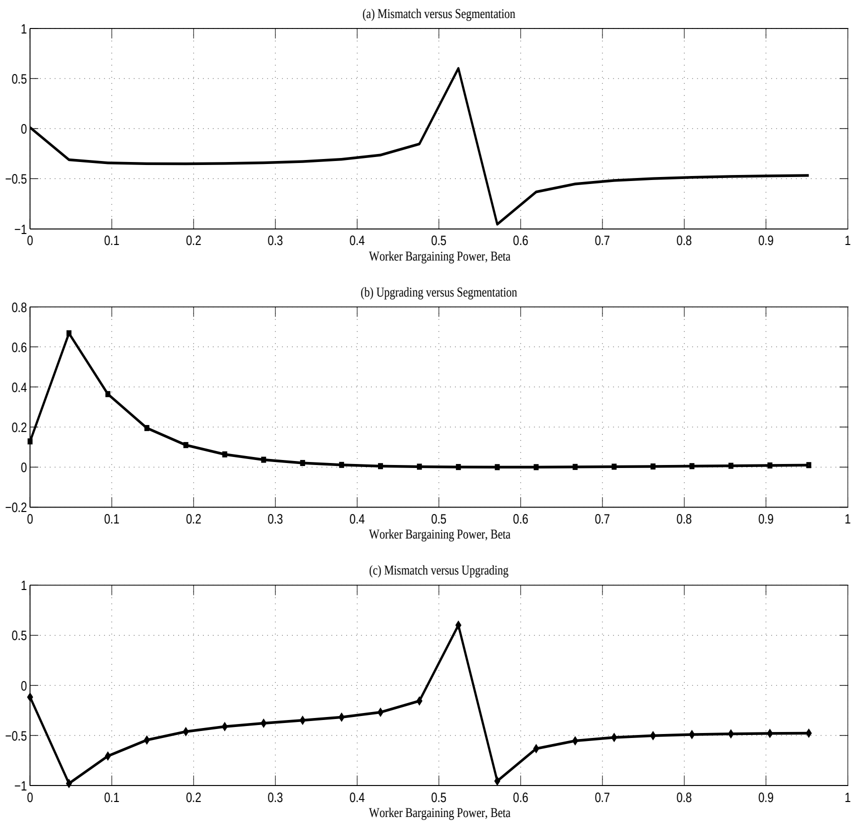

Figure 4: Necessary Conditions

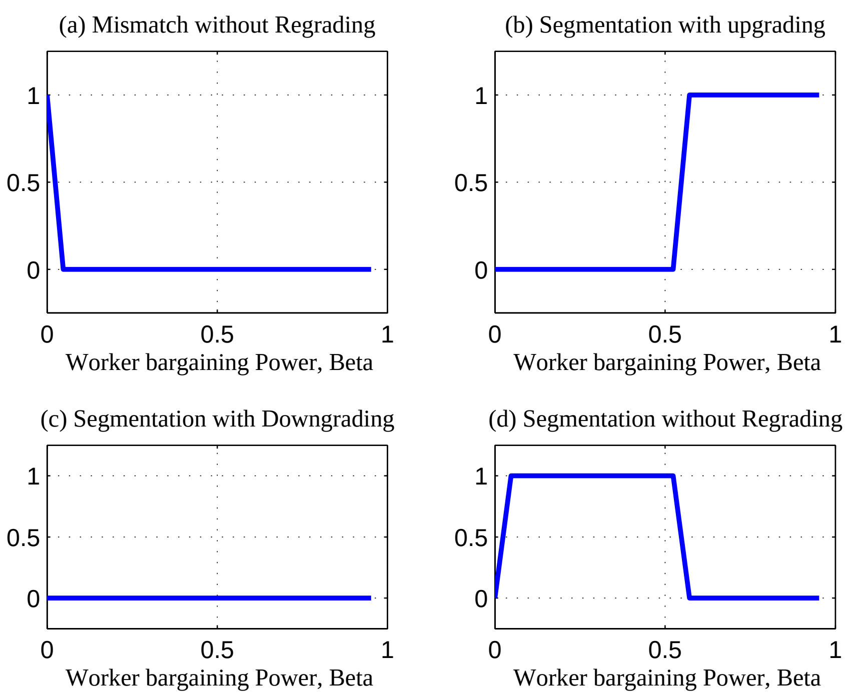

Figure 5: Existence of Candidate Equilibria