Economics Smoking Bans

V. Bhaskar
University College London

Ted To
U.S. Bureau of Labor Statistics

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Abstract

Comprehensive smoking bans in public places have become increasingly popular in the developed world. However, there is little systematic economic analysis of the pros and cons of such policies. We consider a model where consumers have heterogeneous preferences regarding smoking, and where firms must choose whether to allow smoking or prohibit it. Under plausible parameter values, we find that the welfare optimal policy, at least from a non-paternalistic point of view, is to have heterogeneity in policies, with some firms allowing smoking while others prohibit. Nevertheless, the non-cooperative equilibrium may well have firms choosing the same policy, e.g., to permit smoking.

1 Introduction

Comprehensive smoking bans in public places have become increasingly popular in the developed world. Doctors and health professionals have been at the center of the debates and campaigns for smoking bans. However, there is little systematic economic analysis of the pros and cons of such policies, a fact that is somewhat surprising given the centrality of questions regarding externalities and public goods in this regard.

At a conceptual level, it is useful to make a distinction between two types of public spaces and facilities. The first are facilities such as train stations or airports, that are effective monopolies, in the sense that the consumer has little choice but to use these facilities – let us call these monopoly public spaces. These may be contrasted with privately owned public spaces that operate in a competitive environment, such as bars and restaurants. The case for banning smoking in monopoly public spaces may be relatively straightforward. Non-smokers (as well as smokers) have little choice but to use airports and train stations, and therefore there is a case for ensuring that they are not exposed to passive smoking. Smokers may be adversely affected by such a ban, but it is in the nature of the monopoly that the demands of only one group can be satisfied, and there a persuasive case that the dangers and nuisance of passive smoking outweighs the inconvenience to smokers.

The case of privately owned public spaces in a competitive environment is different. Typically, there is large variety of bars and restaurants that one may patronize, and these are owned and operated by private profit maximizing individuals. Since customers are free to patronize or not any such bar, and since bars are free to compete for customers via their choice of smoking policies, an absolutist case for a ban is less compelling. As economists, one must therefore ask, what sort of market failure arises, which may justify comprehensive smoking bans, or perhaps some other alternative policy response.

Two facts need to be noted at the outset. First, while private bars and restaurants have always been free to impose their own restrictions on smoking, this option has been rarely exercised, if at all. As far as we are aware, almost no bar or pub, and very few restaurants chose to restrict smoking.[1] Second, the smoking ban appears to be widely popular in the developed countries where it has been imposed. This is not so surprising given that well below 50% of the population smoke in these countries.

The main contribution of this paper is to provide an explanation for this puzzling phenomenon. We consider a model where consumers have heterogeneous preferences regarding smoking, and where firms must choose whether to allow smoking or prohibit it. Under plausible parameter values, we find that the welfare optimal policy, at least from a non-paternalistic point of view, is to have heterogeneity in policies, with some firms allowing smoking while others prohibit. Nevertheless, the non-cooperative equilibrium may well have firms choosing the same policy, e.g., to permit smoking. While this model is directly motivated by the issue of smoking, it also sheds light on the incentives of firms to provide socially appropriate quality levels when consumers are heterogeneous in the degree to which they value quality. Specifically, it suggests that there may be

[1] There may be some instances of private firms instituting bans, in somewhat different contexts. Airlines are a case in point?

excessive uniformity relative to the quality configurations that would be chosen by a social planner.

2 Background

We now turn to possible arguments that may be given for banning smoking in private bars and restaurants.

1. Passive smoking or the externality/public good argument: allowing smoking in a pub is unpleasant and unhealthy for non-smokers, since air quality is a public good. However, by the same token preventing smoking imposes a negative externality on smokers. Importantly, in choosing smoking policy, restaurants and bars are more accurately thought of as providing club goods since customers are free to choose which restaurant to patronize. Thus a proper examination of this argument requires us to model the amenity choice decisions of bars and restaurants.

2. Banning smoking is good for smokers: This is an argument that is often made by health professionals, who say that by discouraging smoking, one is in fact helping smokers who have self control problems. The extreme version of this argument is either squarely paternalistic, in the sense that the actual preferences of smokers do not matter – if they don't know that smoking is bad for them, too bad. A more sophisticated version is based on the premise that smokers have self control problems – the ex ante self would like to commit by frequenting a place where smoking is not permitted, but if this is not available, ex post, the person would succumb to temptation. Thus the need for regulation to ensure that bars are available where smoking is not permitted. However, the sophisticated version of this argument needs to confront the same problem as argument (1) above – as long as some bars prohibited smoking, smokers with self control problems would have a commitment device.

3 The model

We now set out a simple model of the provision of amenities that have a public good element by pubs in a competitive environment. Our purpose in setting out this model is two-fold. First, we would like to examine how the unregulated market behaves, and whether it ensures the appropriate provision of smoking versus non-smoking pubs, as compared to the social optimum. This also enables us to examine what forms of regulation or intervention may be appropriate, and specifically, whether a ban on smoking in pubs is welfare improving. Our second purpose is to derive empirical predictions from the model, in terms of the effects of the smoking ban upon sales, profits and prices.

Our model is intended to capture the following features. First, cigarette smoke has a large public good element, in the sense that if a pub permits customers to smoke, this has an adverse effect on non smokers (or smokers who have quit, who may be tempted to smoke again). Conversely, if a pub prohibits smoking, this has an adverse effect upon smokers, who maybe forced to go outdoors in the rain or cold to smoke. While pubs may be able to choose a mix of smoking and non-smoking rooms, the heart of the problem appears to be the fact that facilities cannot be tailored so as to perfectly satisfy both types of consumer, so that the public good element remains.[2] We shall therefore simplify and adopt a binary specification, where each pub must choose either to permit smoking or to prohibit it. Second, consumers are heterogeneous in the valuation of this amenity. Non-smokers dislike cigarette smoke, and may also differ in the intensity of their preferences in this dimension. Smokers prefer a smoking pub, and here again, one can allow the intensity of smoking preferences to vary. Finally, we shall also allow for an element of horizontal differentiation, so that consumers prefer to frequent a pub that is located "close" to them, where closeness may have a geographical element but may also refer to other characteristics of the pub. This enables a pub to have an element of market power over and above that arising from possible differences in amenity choice.

More specifically, we set out a model of localized monopolistic competition, that is a generalization of the Hotelling-Salop class of models. Consumers and firms are modeled using an undirected graph, where the nodes or vertices represent firms and consumers are located on the edges. The graph is regular and connected, with each pub is connected to $k+1$ other pubs, by an edge of length ℓ. Consumers are uniformly distributed on each such edge, and effectively have a choice between the two pubs that are located at the vertices of the edge. In order to avoid boundary problems, we shall assume that the graph is infinite. Some examples may clarify. When $k+1=2$, we have a version of the Salop model, where each firm is competes with its two neighbors on a circle of infinite length. When $k+1=4$, we have competition on the plane – the city is such that streets run North-South and East-West, with pubs located at each intersection. Each pub competes with its four neighbors, on adjacent intersections. We may also allow for higher values of k, although these may not have an immediate spatial interpretation. The case of $k+1=1$ corresponds to the Hotelling model – the graph consists of disconnected line segments, where each pub has only one competitor.

Turning to consumers, we assume that each consumer gets the same "base utility" v from frequenting a pub, and some reservation utility $\bar{v}$ from the outside option, which may be interpreted as staying at home or having a drink in some private premises. We assume that v is high relative to $\bar{v}$. We assume that in case the consumer exercises the outside option, he or she can smoke or ensure that the environment is smoke free, in line with his/her preferences. As we shall see, this implies that the effective reservation utility of a consumer is type dependent, since it depends upon her preferences regarding smoking. Consumers differ in three distinct dimensions, as set out below:

First, they differ in location, being uniformly distributed on the interval $[0,\ell]$, where 0 and ℓ index the locations of the two nodes or pubs. Consumers incur a "transportation cost" $\tau(d)$, that is a strictly increasing function of the distance traveled. One example is of linear transportation costs, i.e., $\tau(d)=\hat{t}d$, where $\hat{t}>0$ is the cost per unit distance. A second popular example is quadratic transportation costs, i.e., $\tau(d)=\hat{t}d^2$. Our analysis applies to either of these versions of transportation cost, and indeed, to any generalized quadratic transportation cost function, $\tau(d)=\nu\hat{t}d+(1-\nu)\hat{t}d^2$. Define $t=\hat{t}\ell$, so that edges are normalized to length 1, with the associated transportation cost parameter t.

[2] Indeed, U.S. Department of Health and Human Services (2006) concluded that "current HVAC systems cannot fully control exposures to secondhand smoke unless a complete smoking ban is enforced."

The second dimension of consumer heterogeneity is in smoking preferences: some get a positive benefit from being in a pub that allows smoking, while others incur a disutility from being exposed to tobacco smoke. To model this, let the set of types of smoking preferences belong to the set $\{1, 2, \ldots, m\} \cup \{\mathbf{S}, \mathbf{N}\}$ and let u_i denote the additional benefit that a consumer of type i gets from the pub being a smoking one – u_i may be positive or negative. For $i \in \{1, 2, \ldots, m\}$, $|u_i| \leq \max\{\tau(1), v - \bar{v}\}$. A "die-hard" smoker has index **S**, and is such that $u_{\mathbf{S}}$ is large – in particular, $u_{\mathbf{S}} \gg \max\{\tau(1), v - \bar{v}\}$. This implies that he strictly prefers, by a margin, to patronize a smoking pub, even if he has to travel the maximum distance in order to do so. Furthermore, if he cannot go to a smoking pub, he prefers his outside option, since by staying at home he gets $\bar{v} + u_{\mathbf{S}} > v$. Similarly, a "die hard" non-smoker has index **N**, and $-u_{\mathbf{N}} \gg \max\{\tau(1), v - \bar{v}\}$, i.e., she strictly prefers to patronize a non-smoking pub, even if she has to travel the maximum distance in order to do so, and also prefers to stay at home if a pub allows smoking. Types with index between 1 and m are *responsive*, i.e., given a choice they may choose a smoking or a non-smoking pub depending upon their location and upon prices. Let α_i be the measure of type i, and normalize to unity the total number of responsive consumers, i.e., $\sum_{i=1}^{m} \alpha_i = 1$.

One may also allow for a third type, potential smokers, with a self control problem. This may describe, for example, someone who has quit smoking. Ex post, such a person will smoke with positive probability if they are permitted to do so. However, the ex ante self of such a person would prefer not to smoke. If the ex ante self is *naïve* (in the terminology of Rabin), she assumes that she will succumb to temptation. In this case, the ex ante self has a value of u_i that is liable to be negative, simply because she does not want to be in a smoking environment. Alternatively, the ex ante self could be *sophisticated*, and foresee that she is likely to smoke if permitted to do so. In this case, the value of u_i for the ex ante self is likely to be substantially more negative, since it incorporates the anticipated cost of losing self control. In either case, our positive analysis applies to such types — the interpretation being that the choice would be made on the value of u_i for the ex ante self rather than the ex post self. Similarly, the welfare results would also apply without modification, as long as we use as our individual welfare criterion the utility of the ex ante self rather than the ex post self, as is usual in the literature on self control problems.

Our first aim is to examine the market provision of smoking versus non-smoking pubs, and to examine its efficiency properties. We model competition between pubs via the following extensive form game. First, pubs simultaneously choose whether to be smoking or non-smoking. They then observe the choices made by all pubs in the market, and choose a price.[3] On every edge, consumers choose which, if any, of the two local pubs to patronize.

3.1 Uniformity versus Maximal Heterogeneity

Our analysis will focus primarily on three types of configurations. First, we may have a configuration where every pub chooses the same policy regarding smoking. If every pub permits smoking, we call this configuration *universal smoking*,

[3] This is a somewhat strong assumption – a plausible alternative assumption is that a pub only observes the choices made by its immediate neighbors. Our qualitative results also hold under this informational assumption – indeed, our essential argument is strengthened, as we shall see.

and label this **S**. Similarly, if no pub permits smoking, we have *universal non-smoking*, **N**. The third configuration that we focus on is *maximal heterogeneity*, where on every edge there is one smoking and one non-smoking pub, which we label **H**.

Pricing in any uniform equilibrium is straightforward. Let us measure prices net of marginal costs, which are assumed to be constant. Every pub will charge a price equal to t, on every edge, the two competing pubs will serve half the consumers. If we have universal smoking, this will be the responsive consumers and the die-hard non-smokers. Profits in each pub will therefore be $\frac{t(1+\alpha_{\mathbf{S}})}{2}$ on every edge. On the other hand, suppose that we have *universal non-smoking,* equilibrium prices will be t and in this case the die-hard smokers will stay out, so that profits per edge equal $\frac{t(1+\alpha_{\mathbf{N}})}{2}$. Clearly, demand is greater and firms are better off under universal smoking as compared to universal non-smoking if and only if $\alpha_{\mathbf{S}} > \alpha_{\mathbf{N}}$.

Now let us consider maximal heterogeneity, where on every edge, each consumer has a choice between a smoking pub located at one end and a non-smoking one located at the other end. Thus every smoking firm competes directly only with non-smoking firms, and vice versa. We now turn to demand for a non-smoking firm, indexed by **N**, from the set of responsive consumers.

Consider a responsive consumer of type i, who gets a benefit u_i from smoking. If the prices of the two firms are not too different, the marginal consumer of type i who is indifferent between the two firms belongs to the interior of the unit interval, and is given by[4]

$$x_i = \frac{1}{2} + \frac{p_{\mathbf{S}} - p_{\mathbf{N}} - u_i}{2t}.$$

Aggregating across all the types of consumers, total demand for firm **N** from consumers on this edge,[5] as a function of $p_{\mathbf{S}}$ and $p_{\mathbf{N}}$, is given by

$$\begin{aligned} D_{\mathbf{N}} &= \alpha_{\mathbf{N}} + \sum_{i=1}^{n} \alpha_i \left(\frac{1}{2} + \frac{p_{\mathbf{S}} - p_{\mathbf{N}} - u_i}{2t} \right) \\ &= \alpha_{\mathbf{N}} + \frac{1}{2} + \frac{p_{\mathbf{S}} - p_{\mathbf{N}} - \theta t}{2t}, \end{aligned} \tag{1}$$

where θ denotes the "average" valuation of smoking in the responsive population, normalized relative to the transport cost, t.

$$\theta = \frac{1}{t} \sum_{i=1}^{n} \alpha_i u_i.$$

Similarly, demand for a smoking firm on this edge is given by

$$D_{\mathbf{S}} = \alpha_{\mathbf{S}} + \frac{1}{2} + \frac{p_{\mathbf{N}} - p_{\mathbf{S}} + \theta t}{2t}. \tag{2}$$

[4] More precisely, $x_i^* = 0$ if the solution to this equation is less than zero, and $x_i^* = 1$ if the solution to this equation is greater than 1.

[5] Since there are $k+1$ edges, total demand must be multiplied by this number. However, since each of firm **N**'s competitors is a smoking firm that chooses the same price, firm **N**'s maximization problem is equivalent to maximizing profit per edge.

From the first order condition for profit maximization, we derive the best responses of the two firms:

$$p_{\mathbf{N}} = \frac{(1+2\alpha_{\mathbf{N}}-\theta)t}{2} + \frac{p_{\mathbf{S}}}{2}. \tag{3}$$

$$p_{\mathbf{S}} = \frac{(1+2\alpha_{\mathbf{S}}+\theta)t}{2} + \frac{p_{\mathbf{N}}}{2}. \tag{4}$$

Equilibrium prices are given by

$$p_{\mathbf{N}}^{*} = \frac{(3+4\alpha_{\mathbf{N}}+2\alpha_{\mathbf{S}}-\theta)t}{3}. \tag{5}$$

$$p_{\mathbf{S}}^{*} = \frac{(3+4\alpha_{\mathbf{S}}+2\alpha_{\mathbf{N}}+\theta)t}{3}. \tag{6}$$

The average price in the market equals

$$t\left(1+\alpha_{\mathbf{S}}+\alpha_{\mathbf{N}}\right) > t.$$

Thus we see that average prices in the market are larger in a heterogeneous amenity equilibrium, as long as $\alpha_{\mathbf{S}}+\alpha_{\mathbf{N}}$ is positive. Note that the profits of the two types of firm will generally be different. For example, if there are more smokers than non-smokers, firms that permit smoking will charge higher prices and have higher profits.

We now turn to welfare under the three configurations. Our welfare criterion is the sum of consumer surplus and producer surplus per edge. Under universal smoking, **S**, welfare is given by

$$W_{\mathbf{S}} = \left(v+\theta t-\frac{t}{4}\right) + \alpha_{\mathbf{S}}\left(v+u_{\mathbf{S}}-\frac{t}{4}\right) + \alpha_{\mathbf{N}}\bar{v}.$$

The first term is the corresponds to the utility (plus profits) from the responsive consumers, the sum of the consumption benefit, plus the average utility from smoking, minus the transport cost. The second term is the same expression for die-hard smokers, while the third term is that for die-hard non-smokers, who get their outside option. Similarly, under universal non-smoking, **N**, welfare equals

$$W_{\mathbf{N}} = \left(v-\frac{t}{4}\right) + \alpha_{\mathbf{N}}\left(v-\frac{t}{4}\right) + \alpha_{\mathbf{S}}\left(\bar{v}+u_{\mathbf{S}}\right).$$

Thus the welfare difference between universal smoking and universal non-smoking equals

$$\theta t + \left(\alpha_{\mathbf{S}}-\alpha_{\mathbf{N}}\right)\left(v-\frac{t}{4}-\bar{v}\right).$$

Under maximal heterogeneity **H**, prices are different across firms. In consequence, consumers will not be allocated optimally to firms. We therefore compute two different expressions for welfare. First best welfare, $W_{\mathbf{H}}^{FB}$, assumes that consumers are allocated optimally. This is given by

$$W_{\mathbf{H}}^{FB} = \left(v+\frac{\theta t}{2}-\frac{t}{4}\left(1-\sigma^2-\theta^2\right)\right) + \alpha_{\mathbf{S}}\left(v+u_{\mathbf{S}}-\frac{t}{2}\right) + \alpha_{\mathbf{N}}\left(v-\frac{t}{2}\right).$$

Thus the difference in welfare between maximal heterogeneity and universal smoking equals

$$W_{\mathbf{H}}^{FB} - W_{\mathbf{S}} = \left(-\frac{\theta t}{2} + \frac{t}{4}\left(\sigma^2 + \theta^2\right)\right) - \alpha_{\mathbf{S}}\left(\frac{t}{4}\right) + \alpha_{\mathbf{N}}\left(v - \frac{t}{2} - \bar{v}\right).$$

The effect of heterogeneity is to reduce the welfare of the die-hard smokers, since they now have only one pub to choose, and thus face increased transport costs. It raises the welfare of the die-hard non-smokers by a larger magnitude, since they no longer have to stay away from the market. Thus the net effect on the die-hard consumers is positive unless the measure of smokers in the die-hard category is substantially larger than the measure on non-smokers. More interesting is the welfare effect on the responsive consumers. If the variance of the normalized utility from smoking ($\frac{u_i}{t}$) is large relative to the mean (i.e., if σ^2 is large relative to θ), then heterogeneity increases welfare among them. Thus, if preferences are sufficiently heterogeneous, either among the die-hard consumers or amongst the responsive consumers, then heterogeneity is preferable to homogeneity.

Second best welfare is the welfare that arises under actual market behavior of consumers, and is given by

$$\begin{aligned} W_{\mathbf{H}} &= \left(v + \frac{\theta t}{2} - \frac{t}{4}\left(1 - \sigma^2 - \theta^2\right) - \frac{t}{9}(\alpha_{\mathbf{S}} - \alpha_{\mathbf{N}} + \theta)^2\right) \\ &\quad + \alpha_{\mathbf{S}}\left(v + u_{\mathbf{S}} - \frac{t}{2}\right) + \alpha_{\mathbf{N}}\left(v - \frac{t}{2}\right). \end{aligned}$$

Thus second best welfare is lower than first best welfare by the term $\frac{t}{9}(\alpha_{\mathbf{S}} - \alpha_{\mathbf{N}} + \theta)^2$; this is related to the difference in prices between smoking and non-smoking firms being $\frac{2}{3}t(\alpha_{\mathbf{S}} - \alpha_{\mathbf{N}} + \theta)$.

To summarize, we have investigated prices and welfare when firms follow homogeneous policies, and when they are maximally heterogeneous. If consumers are sufficiently heterogeneous, so that the variance of their utilities from smoking is large, then welfare optimality requires maximal heterogeneity, so that every consumer has a choice between smoking and non-smoking pubs.

The configurations we have investigated are the most interesting, from a welfare and policy point of view. To see this, consider an arbitrary configuration, where some firms permit smoking, while others do not. On any edge, one has three possibilities. If both firms permit smoking, first best welfare on this edge corresponds to that in configuration **S**. If both firms do not permit smoking, first best welfare on this edge corresponds to **N**. Finally, if there is one smoking and one non-smoking pub, first best welfare corresponds to **H**. Thus, abstracting from any price differences between firms on the edge, welfare in any arbitrary configuration corresponds to convex combination of the the welfare levels from these three types of configurations. Thus, if configuration **H** is welfare optimal among the three configurations that we have focused on, it is also welfare optimal globally among the set of all configurations. A more formal statement and proof this claim is in the appendix.

Equilibrium analysis of other configurations is more complex, and requires imposing some regularities.

4 Equilibrium configurations

We now turn to the question, what will the policies chosen by the firms in equilibrium? We analyze the following extensive form game. First, firms choose simultaneously whether or not to allow smoking, i.e., a policy belonging to $\{\mathbf{S}, \mathbf{N}\}$. Second, each firm observes the choices made by every other firm in the market, and then firms choose prices. We have already solved for prices in the subgame following universal smoking, universal non-smoking and maximal heterogeneity. We now consider what happens when one firm deviates from these configurations.

4.1 Is universal smoking an equilibrium?

In order to investigate whether universal smoking is an equilibrium consider the incentives of an individual pub to deviate from this configuration and prohibit smoking. Let us index this pub by 0, and let 1 denote any of its immediate competitors. The deviation by firm 0, whereby it now prohibits smoking, will change the composition of demand, making demand less elastic, both for itself and for each of its $k+1$ competitors. Let us index these competitors by 1 (we use the same index since they are all symmetrically placed). However, each firm with index 1 also competes with k firms other than firm 0 – let us index each of these firms by the number 2. Thus the best response of firm 0 depends upon p_1, the price chosen by each of the firms indexed by 1; the best response of firm 1 depends upon p_0 and upon p_2, and so on. Let n index the distance of a firm from firm 0, and let p_n denote the optimal price of this firm. Equilibrium prices are given by a sequence $(p_n)_{n=0}^{\infty}$, where for every $n > 0$, p_n is a best response to p_{n-1} and p_{n+1}, and p_0 is a best response to p_1. In other words, equilibrium prices are given by the solution to a second-order difference equation, that we now proceed to derive.

Consider first the interval $[0, 1]$, that lies between the deviating firm 0 and and its neighbor 1. Consider a responsive consumer of type i, who gets a benefit u_i from smoking. If the prices of the two firms are not too different, the marginal consumer of type i who is indifferent between the two firms belongs to the interior of the unit interval, and is given by[6]

$$x_i = \frac{1}{2} + \frac{p_1 - p_0 - u_i}{2t}.$$

Aggregating across all the types of consumers, total demand for firm 0 from consumers on this edge,[7] as a function of p_1 and p_0, is given by

$$D_0 = \alpha_{\mathbf{N}} + \frac{1}{2} + \frac{p_1 - p_0 - \theta t}{2t}. \tag{7}$$

Profits per edge are given by $\Pi_0 = p_0 D_0$. From the first order condition for profit maximization, pub 0's best response, as a function of pub 1's price is given by

$$p_0 = a_0 + \frac{p_1}{2}.$$

[6]More precisely, $x_i^* = 0$ if the solution to this equation is less than zero, and $x_i^* = 1$ if the solution to this equation is greater than 1.

[7]Since there are $k+1$ edges, total demand must be multiplied by this number. However, since every firm located at 1 chooses the same price, firm 0's maximization problem is equivalent to maximizing profit per edge.

where

$$a_0 = \frac{(2\alpha_{\mathbf{N}} + 1 - \theta)t}{2}.$$

Consider now firm 1. On one segment, it competes with firm 0, and has a monopoly over the die-hard smokers on this segment. On k other segments, it competes with firm 2, which has the same policy and allows smoking. Its best response is given by

$$p_1 = a_1 + \frac{\beta}{2}p_0 + \frac{(1-\beta)}{2}p_2, \tag{8}$$

$$a_1 = \frac{[(k+1) + (k+2)\alpha_{\mathbf{S}} + \theta]\,t}{2[(k+1) + k\alpha_{\mathbf{S}}]}.$$

$$\beta = \frac{1}{1 + k(1 + \alpha_{\mathbf{S}})}.$$

Note that $\beta < \frac{1}{k+1}$, i.e., firm 0 has less weight than each of its other competitors, since firm 1's shared demand with firm 0 is less than its shared demand with the other competitors, indexed by 2.

Consider now a pub of index $n > 1$. Such a pub allows smoking, and also competes only with firms that allow smoking. Thus its demand function is symmetric, between the price p_{n-1} and each of the k firms that charge p_{n+1}. Thus its best response gives equal weight to each of its competitors, and is given by

$$p_n = \frac{t}{2} + \frac{\lambda}{2}p_{n-1} + \frac{(1-\lambda)}{2}p_{n+1}, n > 1, \tag{9}$$

$$\lambda = \frac{1}{1+k}.$$

To solve for equilibrium prices following a deviation by firm 0 requires solving for the infinite sequence of prices that satisfy the above difference equation system. This is a second order difference equation, with the solution:

$$\hat{p}_n = t + K_{\mathbf{S}}\xi^n$$

where

$$\xi = \frac{1 - \sqrt{1 - \lambda(1-\lambda)}}{1-\lambda}$$

is the smaller root of the characteristic equation[8] and

$$K_{\mathbf{S}} = \frac{2a_0\beta + 4a_1 - (2+\beta)t}{\xi[4 - \beta - 2(1-\beta)\xi]}.$$

Thus the deviation price for firm 1 is:

$$\hat{p}_1 = t + \frac{2a_0\beta + 4a_1 - (2+\beta)t}{(4-\beta) - 2(1-\beta)\xi}.$$

Recall that $\hat{p}_0 = a_0 + \hat{p}_1/2$ and demand per edge is D_0 so that profits per edge are $\hat{\pi} = \hat{p}_0 D_0(\hat{p}_0, \hat{p}_1)$. In the uniform smoking equilibrium, profits per edge are $t(1+\alpha_{\mathbf{S}})/2$ and therefore uniform smoking is an equilibrium whenever $t(1+$

[8]The larger root, $\frac{1+\sqrt{1-\lambda(1-\lambda)}}{1-\lambda}$, can be ruled out since it would imply that prices increase without bound with distance from the deviating firm.

$\alpha_{\mathbf{S}})/2 \geq \hat{\pi}$. We are now in a position to discuss the incentives of the deviating firm to unilaterally disallow smoking, given that all firms in the market permit smoking. Given the complexity of the expression for a firm's deviation profits, necessary and sufficient conditions for the existence of the uniform smoking equilibrium are difficult to attain. Nevertheless, it is possible to derive some general, necessary conditions for existence.

Consider competition on any edge between firm 0 and firm 1. Since the competing firms are choosing different smoking policies, this raises average prices – since each firm now has a captive market (the die-hard smokers or non-smokers), this reduces the elasticity of demand and softens price competition. Total demand also rises by $\alpha_{\mathbf{N}}$, the mass of die-hard non-smokers. However, the effects on individual demands can be quite different across firms. At equal prices, the firm instituting a non-smoking policy loses a measure $\alpha_{\mathbf{S}}/2$ of die-hard smokers and gains $\alpha_{\mathbf{N}}$ die-hard non-smokers – this can represent a fall in demand if the measure of die-hard smokers amongst pub customers is significantly larger than the measure of die-hard non-smokers. Furthermore, demand from responsive consumers decreases by $\theta/2$. Thus if the *mean utility* from smoking of a responsive consumer, θ, is positive, the non-smoking firm experiences a direct fall in demand that is proportional to θ.

Let us begin with some special cases. Suppose that $\alpha_{\mathbf{N}} = \alpha_{\mathbf{S}} = 0$ and $\theta > 0$, so that there are no die-hard consumers and the mean utility of a responsive consumer from smoking is positive. In this case, the deviating firm always has a lower profit. The direct demand effect on the firm, at initial prices is negative, since the number of non-smokers it attracts is less than the number of smokers it loses. This can be verified from the expression for D_0, equation (7). The consequent rise in demand for firm 1 causes an increase in equilibrium prices for firm 1, and a reduction in price for firm 0. Firm 1's price increase is muted, since firm 1 also competes with firm 2, and so on. However, the overall effect on firm 0's profits is negative.

Since the price effects are continuous in the parameters, this implies that the effect on profits for the deviating firm is negative as long as $\theta > 0$ provided that $\alpha_{\mathbf{N}}$ and $\alpha_{\mathbf{S}}$ are sufficiently small. Thus the unique equilibrium in this case for all firms to permit smoking.

Let us assume that the direct demand effects on the deviating firm are negative. That is, we assume that $\theta > 0$ and that $\alpha_{\mathbf{S}} > 2\alpha_{\mathbf{N}}$. That is, the mass of die hard smokers is greater than twice the mass of die hard non-smokers, and the mean utility of a responsive consumer is such that he gets a benefit from smoking. This implies that a non-smoking pub suffers a direct loss of demand, since the number of non-smokers it attracts is less than the number of smokers it loses. However, there is a benefit in the form of higher prices – the presence of die hard consumers, both smokers and non-smokers, implies that both firms charge higher prices, so that price competition is less intense. The net effect depends upon the relative numbers of smokers and non-smokers, as well as the relative numbers of die-hard versus responsive consumers. If the number of die hard consumers is large, then the effect of higher prices more than offsets the loss in market share; conversely, if the number of responsive consumers is relatively large, then the price response is small, and fails to offset the negative effect on profits of the loss of market share.

More interesting is the effect of market competition upon the price effect. We say that firm competition is more intense if k is larger. That is, a larger value of k implies that each firm competes directly with more local firms. As k becomes larger, p_1 declines, and so does p_0, so that the price effects of the deviation become smaller. Thus as $k \to \infty$, the price effect tends to zero and the deviating firm suffers a loss as long as the direct demand effect is negative.

Despite the fact that we cannot derive necessary and sufficient conditions for the existence of the uniform smoking equilibrium, normalizing $t = 1$ and for specific parameterizations of θ and k it is possible examine necessary and sufficient conditions for existence graphically. For example, in Figure 1, with $k = 1$ and $\theta = -0.25, 0, 0.25$, and Figure 2, with $k = 2, 3, 4$ and $\theta = 0$, the gray shaded regions represent combinations of $\alpha_{\mathbf{S}} > \alpha_{\mathbf{N}}$ where the uniform smoking equilibrium exists.[9] The effect of an increase in θ is to shift the intercept upwards. Fixing $\theta = 0$, the effect of increasing k is to pivot the region upward around the origin.

4.2 Is Maximal Heterogeneity an equilibrium?

To investigate if maximal heterogeneity is an equilibrium, let us consider a firm who deviates from a non-smoking to a smoking policy. Let p_0 be the price of this firm. Since this firm interacts only with firms that allow smoking, its optimal price is given by

$$p_0 = a_0 + \frac{1}{2}p_1, \tag{10}$$

where,

$$a_0 = \frac{t}{2}$$

and p_1 denotes the price charged by each of its neighbors. Note that it competes for the die hard smokers with each of these firms, and thus the expression for a_0.

Now consider any of the deviant's immediate neighbors. This interacts with the deviant smoking firm and with k non-smoking firms, indexed by 2. Its optimal price is given by

$$p_1 = a_1 + \frac{\gamma}{2}p_0 + \frac{(1-\gamma)}{2}p_2,$$

where

$$\gamma = \frac{1 + \alpha_{\mathbf{S}}}{k + 1 + \alpha_{\mathbf{S}}},$$

and

$$a_1 = \frac{t\,[k\theta + (k+1) + (2k+1)\alpha_{\mathbf{S}}]}{2(k+1) + \alpha_{\mathbf{S}}}.$$

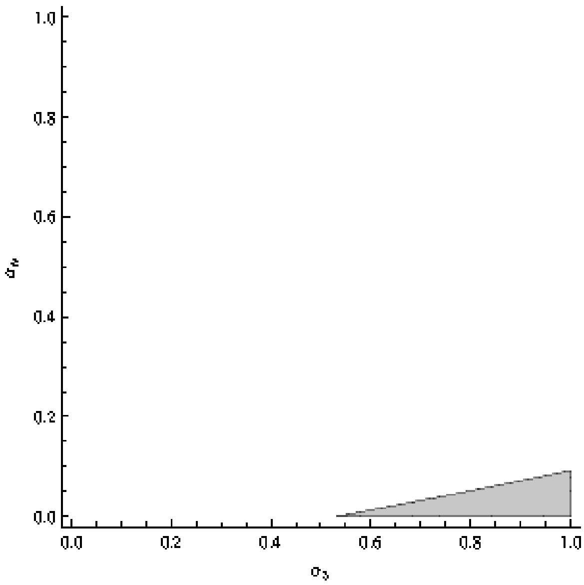
Now consider a firm at distance n from the deviant firm, $n > 1$. Such a firm interacts only with firms of a different type, i.e., if it permits smoking, it interacts only with non-smoking firms, and if it does not it only interacts with smoking firms. Its optimal price is given by

$$p_n = a_n + \frac{\lambda}{2}p_{n-1} + \frac{(1-\lambda)}{2}p_{n+1}, n > 1, \tag{11}$$

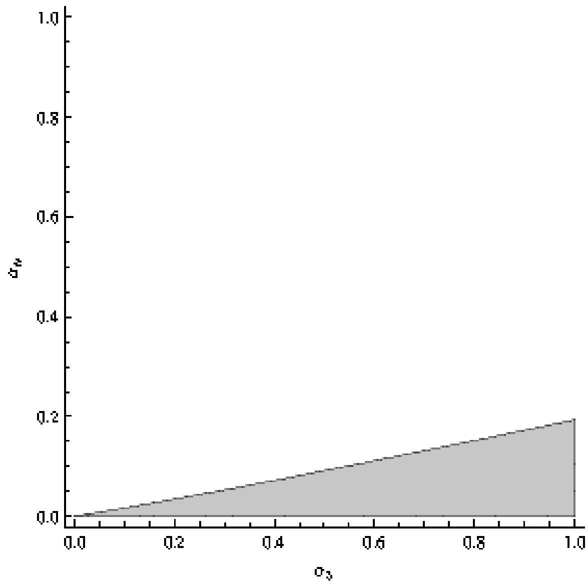
[9]The case where $\alpha_{\mathbf{S}} < \alpha_{\mathbf{N}}$ is analogous and the regions of existence for the uniform non-smoking equilibrium can be seen by switching the axes.

Figure 1: Regions of existence: $k = 1$; $\theta = -0.25, 0, 0.25$

(a) $\theta = -0.25$



(b) $\theta = 0.0$



(c) $\theta = 0.25$

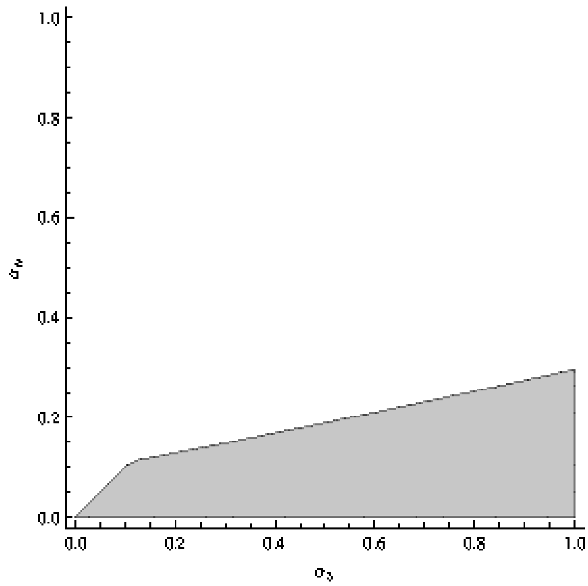
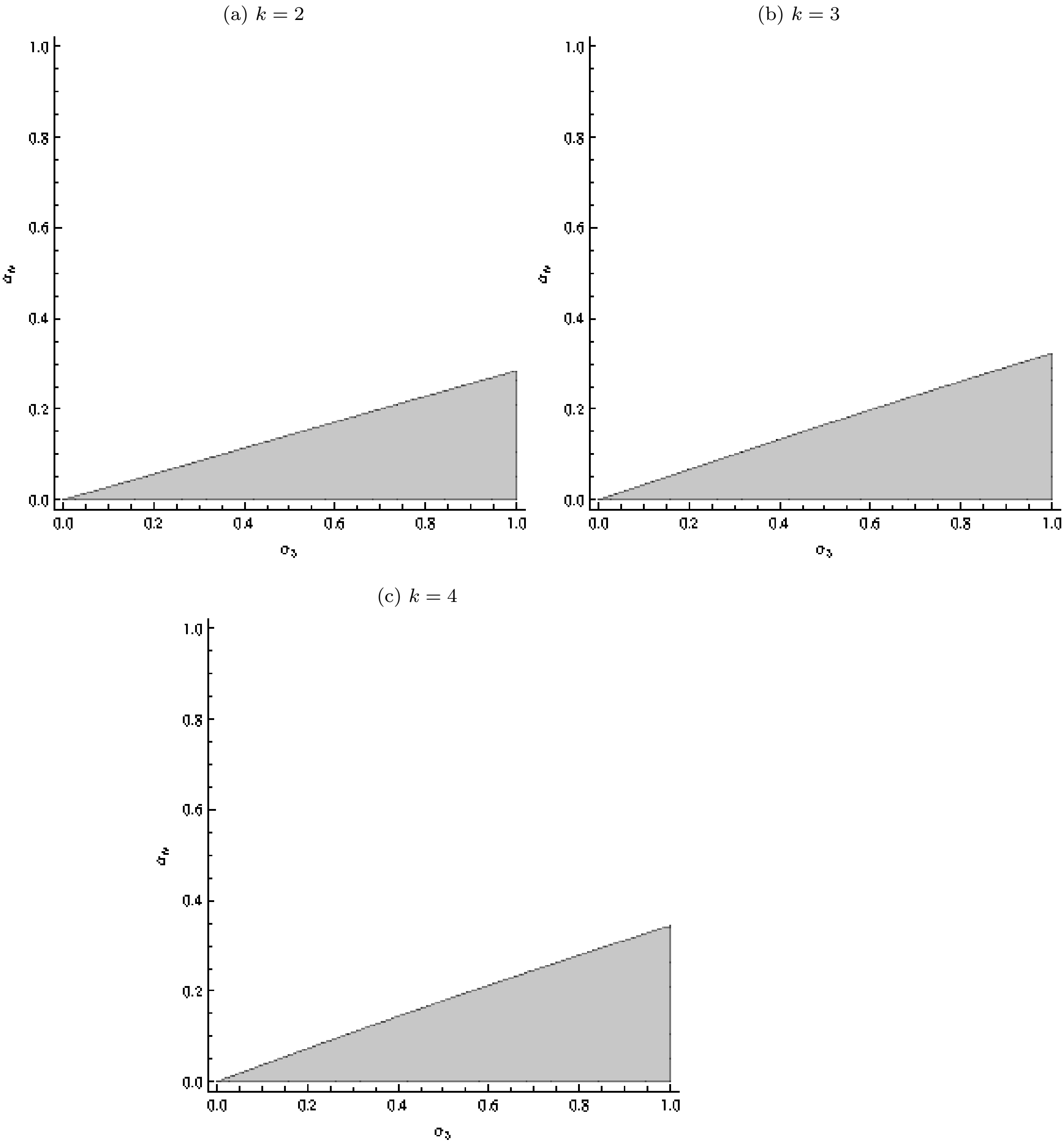


Figure 2: Regions of existence: $k = 2, 3, 4$; $\theta = 0$



where

$$\lambda = \frac{1}{k+1}. \tag{12}$$

and

$$a_n = \begin{cases} a_{\mathbf{N}} = \frac{(1+2\alpha_{\mathbf{N}}-\theta)t}{2} \text{ if } n \text{ is even} \\ a_{\mathbf{S}} = \frac{(1+2\alpha_{\mathbf{S}}+\theta)t}{2} \text{ if } n \text{ is odd} \end{cases}$$

The general solution to the second order difference equation is given by

$$\hat{p}_n = \begin{cases} b_{\mathbf{N}} + K_{\mathbf{H}}\xi^n \text{ if } n \text{ is even} \\ b_{\mathbf{S}} + K_{\mathbf{H}}\xi^n \text{ if } n \text{ is odd} \end{cases}$$

From the characteristic equation,

$$\xi = \frac{1 - \sqrt{1 - \lambda(1-\lambda)}}{(1-\lambda)}.$$

$$b_{\mathbf{N}} = \frac{4a_{\mathbf{N}} + 2a_{\mathbf{S}}}{3}.$$

$$b_{\mathbf{S}} = \frac{4a_{\mathbf{S}} + 2a_N}{\mathbf{3}}.$$

$$b_{\mathbf{S}} + K_{\mathbf{H}}\xi = \frac{4a_1 + 2\beta a_0}{4-\beta} + \frac{2(1-\beta)}{4-\beta}(b_{\mathbf{N}} + K_{\mathbf{H}}\xi^2).$$

This allows us to solve for $K_{\mathbf{H}}$:

$$K_{\mathbf{H}} = \frac{4a_1 + 2\beta a_0 + 2(1-\beta)b_{\mathbf{N}} - (4-\beta)b_{\mathbf{S}}}{\xi[4-\beta-2(1-\beta)\xi]}.$$

$$\hat{p}_1 = b_{\mathbf{S}} + \frac{4a_1 + 2\beta a_0 + 2(1-\beta)b_{\mathbf{N}} - (4-\beta)b_{\mathbf{S}}}{(4-\beta) - 2(1-\beta)\xi}.$$

Thus we have a solution for $\hat{p}_1$, and from (10), $\hat{p}_0$ in terms of the parameters $a_0, a_1, b_{\mathbf{S}}, b_{\mathbf{N}}, \beta$ and ξ.

We are now in a position to discuss the incentives of the deviating firm to unilaterally allow smoking, given that its immediate competitors permit smoking, and given the heterogeneity of amenity provision. As with uniform smoking, conditions that are both necessary and sufficient for the existence of maximal heterogeneity are difficult to attain but we can still derive some necessary conditions.

Consider competition on any edge between firm 0 and firm 1. Let us assume that $\alpha_{\mathbf{S}}$ is large relative to $\alpha_{\mathbf{N}}$ and that $\theta > 0$. The direct of the deviation is to increase market share for the deviating firm, while the indirect effect is to reduce its competitor's price, since competition is now more intense. However, since firm 1 also competes with other non-smoking firms (indexed by 2), that have not changed their policies, this mitigates the price reducing effect of the deviation. Furthermore, since each of the firms indexed by 2 compete with $k+1$ other firms, only one of index 2 and therefore affected by the deviation in the first instance, this further mitigates the price reducing effect. More generally, p_n is linear function of p_{n-1} and p_{n+1}, where the weight on p_{n-1} is $\frac{1}{k+1}$, implying that the price reducing effects are small. The fact that there is heterogeneity in the market, keeping prices high, implies that the price reducing effects of the deviation will be mitigated, and the deviating firm can free ride on these high prices.

Consider first the special case where $\alpha_{\mathbf{N}} = \alpha_{\mathbf{S}} = 0$ and $\theta > 0$, so that there are no die-hard consumers and the mean utility of a responsive consumer from smoking is positive. In this case, the deviating firm always has higher profits. The direct demand effect on the firm, at initial prices is positive, and the price effects do not offset this. Thus the unique equilibrium in this case is one where every firm permits smoking. Since the price effects are continuous in the parameters, this implies that the effect on profits for the deviating firm is positive as long as $\theta > 0$ provided that $\alpha_{\mathbf{N}}$ and $\alpha_{\mathbf{S}}$ are sufficiently small. Thus if the number of die-hard consumers is small, and if $\theta > 0$, the unique equilibrium is one where all firms to permit smoking.

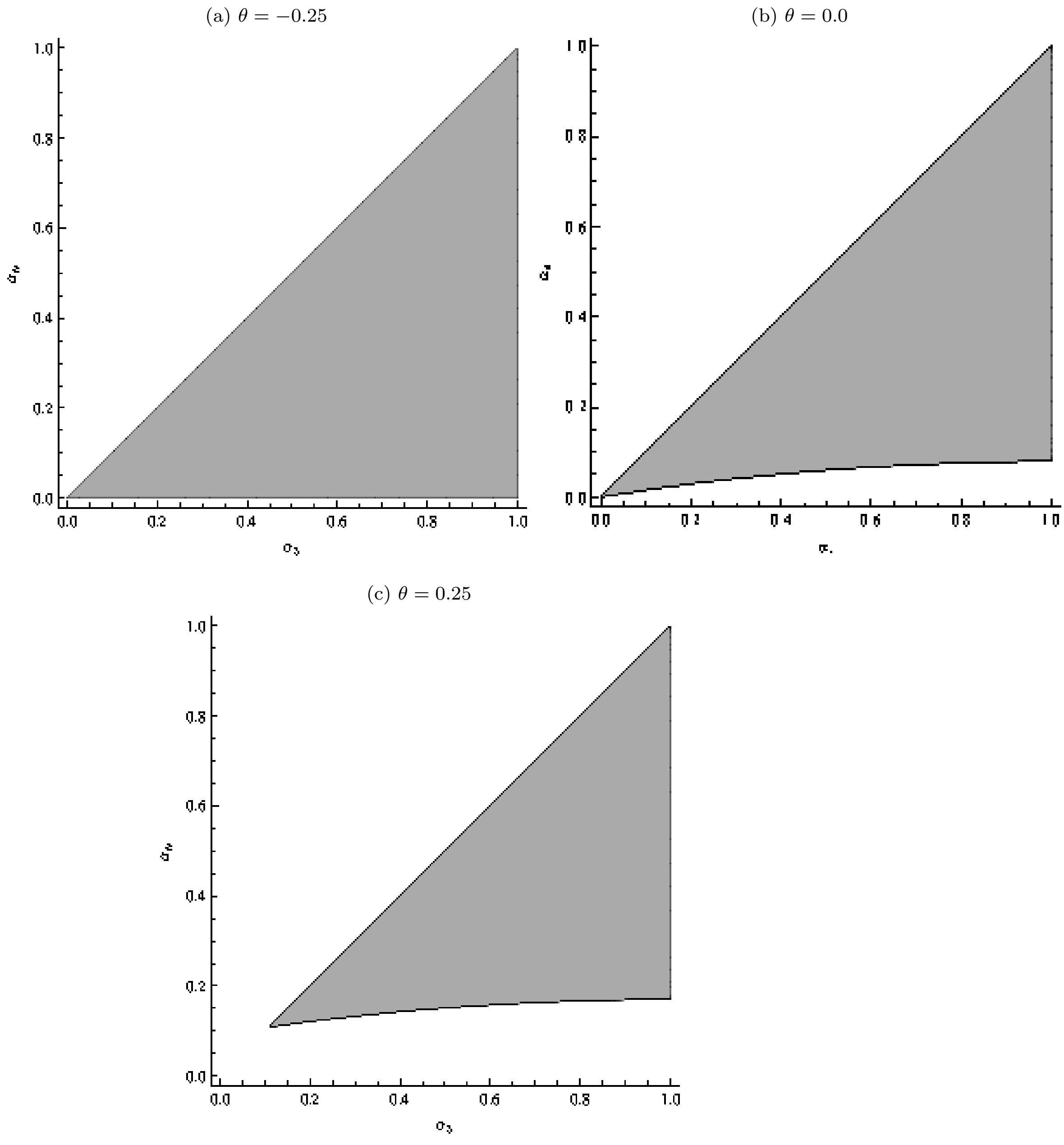
Let us assume that the direct demand effects on the deviating firm are negative. That is, we assume that $\theta > 0$ and that $\alpha_{\mathbf{S}} > 2\alpha_{\mathbf{N}}$. That is, the mass of die hard smokers is greater than twice the mass of die hard non-smokers, and the mean utility of a responsive consumer is such that he gets a benefit from smoking. This implies that a non-smoking pub suffers a direct loss of demand, since the number of non-smokers it attracts is less than the number of smokers it loses. However, there is a benefit in the form of higher prices – the presence of die hard consumers, both smokers and non-smokers, implies that both firms charge higher prices, so that price competition is less intense. The net effect depends upon the relative numbers of smokers and non-smokers, as well as the relative numbers of die-hard versus responsive consumers. If the number of die hard consumers is large, then the effect of higher prices more than offsets the loss in market share; conversely, if the number of responsive consumers is relatively large, then the price response is small, and fails to offset the negative effect on profits of the loss of market share.

More interesting is the effect of market competition upon the price effect. We say that firm competition is more intense if k is larger. That is, a larger value of k implies that each firm competes directly with more local firms. As k becomes larger, p_1 declines, and so does p_0, so that the price effects of the deviation become smaller. Thus as $k \to \infty$, the price effect tends to zero and the deviating firm suffers a loss as long as the direct demand effect is negative.

Proposition 1 *If $\theta > 0$, the unique equilibrium has universal smoking provided that $\alpha_{\mathbf{S}}$ and $\alpha_{\mathbf{N}}$ are sufficiently small. If $\alpha_{\mathbf{S}} + \frac{\theta}{t} > 2\alpha_{\mathbf{N}}$, the unique equilibrium has universal smoking if market competition is sufficiently intense, i.e., k is sufficiently large.*

As before, we cannot derive necessary and sufficient conditions for the existence of the uniform smoking equilibrium but normalizing $t = 1$ and for specific parameterizations of θ and k we can examine these conditions for existence graphically. For example, in Figure 3, with $k = 1$ and $\theta = -0.25, 0, 0.25$, and Figure 4, with $k = 2, 3, 4$ and $\theta = 0$, the gray shaded regions represent combinations of $\alpha_{\mathbf{S}} > \alpha_{\mathbf{N}}$ where the maximal heterogeneity equilibrium exists. The effect of an increase in θ is to shift the intercept upwards. Fixing $\theta = 0$, the effect of increasing k is to pivot the region upward around the origin. Moreover, examining Figure 1 and 3, one might surmise that both the maximal heterogeneity and the uniform smoking equilibria can coexist. Indeed, examining Figures 5,

Figure 3: Regions of existence: $k = 1$; $\theta = -0.25, 0, 0.25$



(a) $\theta = -0.25$

(b) $\theta = 0.0$

(c) $\theta = 0.25$

Figure 4: Regions of existence: $k = 2, 3, 4$; $\theta = 0$

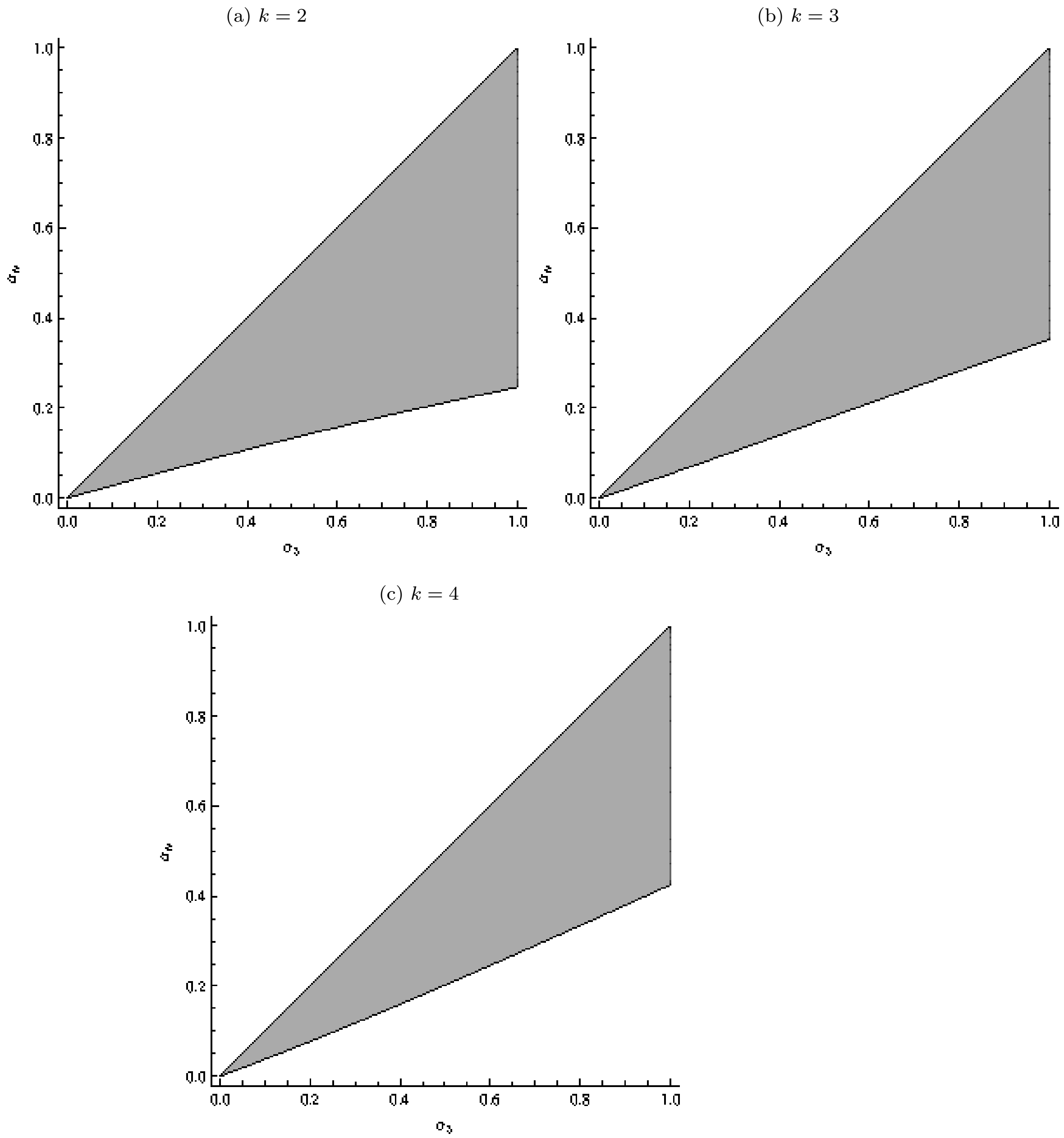


Table 1: Numeric computations: Equilibria for $k = 1$

$\alpha_{\mathbf{S}}$	$\alpha_{\mathbf{N}}$	θ	Max. het.	Unif. sm.	Unif. non-sm.	Other eq.
0.05	0.0	−0.3	N	N	Y	N
0.05	0.0	−0.2	N	N	Y	N
0.05	0.0	−0.1	Y	N	N	Y
0.25	0.05	0.2	N	Y	N	N
0.25	0.05	0.3	N	Y	N	N
0.25	0.1	0.2	N	Y	N	N
0.25	0.1	0.3	N	Y	N	N
0.25	0.15	−0.3	Y	N	N	Y
0.25	0.15	−0.2	Y	N	N	Y
0.25	0.15	0.0	Y	N	N	Y
0.25	0.15	0.2	Y	N	N	Y
0.25	0.15	0.3	Y	Y	N	N
0.25	0.25	−0.3	Y	N	N	Y
0.25	0.25	−0.2	Y	N	N	Y
0.25	0.25	0.0	Y	N	N	Y
0.25	0.25	0.2	Y	N	N	Y
0.25	0.25	0.3	Y	N	N	Y
0.5	0.0	−0.1	N	Y	N	N
0.5	0.0	−0.2	Y	Y	N	Y
0.5	0.0	−0.3	Y	N	N	Y
0.5	0.0	0.1	N	Y	N	N
0.5	0.0	0.2	N	Y	N	N
0.5	0.0	0.3	N	Y	N	N

the light gray region represents uniform smoking, the slightly darker gray region represents maximal heterogeneity and the dark gray region represents the coexistence of maximal heterogeneity and uniform smoking. Indeed, numeric approximations of the set of all equilibria[10] are presented in Table 1 and suggests that not only can uniform smoking and maximal heterogeneity coexist, but that other, more complicated equilibria can exist as well.

Finally, while the numeric approximations suggest that equilibria always exist for the case $k = 1$,[11] it is not obvious that equilibria exist for $k > 1$. Examining Figure 2 reveals that for $k = 3, 4$ and $\theta = 0$, there is a region where neither the maximal heterogeneity nor the uniform smoking equilibria exist.

[10]The methodology used to approximate the set of equilibria is presented in Appendix A.

[11]Table 1 presents only a small subset of all parameter combinations for which the set of equilibria were computed. The complete set includes all $\alpha_{\mathbf{S}} = 0.0, 0.05, 0.1, \ldots 1.0$, $\alpha_{\mathbf{N}} = 0.0, 0.05, 0.1, \ldots \alpha_{\mathbf{S}}$ and $\theta = -0.3, -0.2, \ldots, 0.3$.

Figure 5: Regions of existence: $k = 1$; $\theta = -0.25, 0, 0.25$

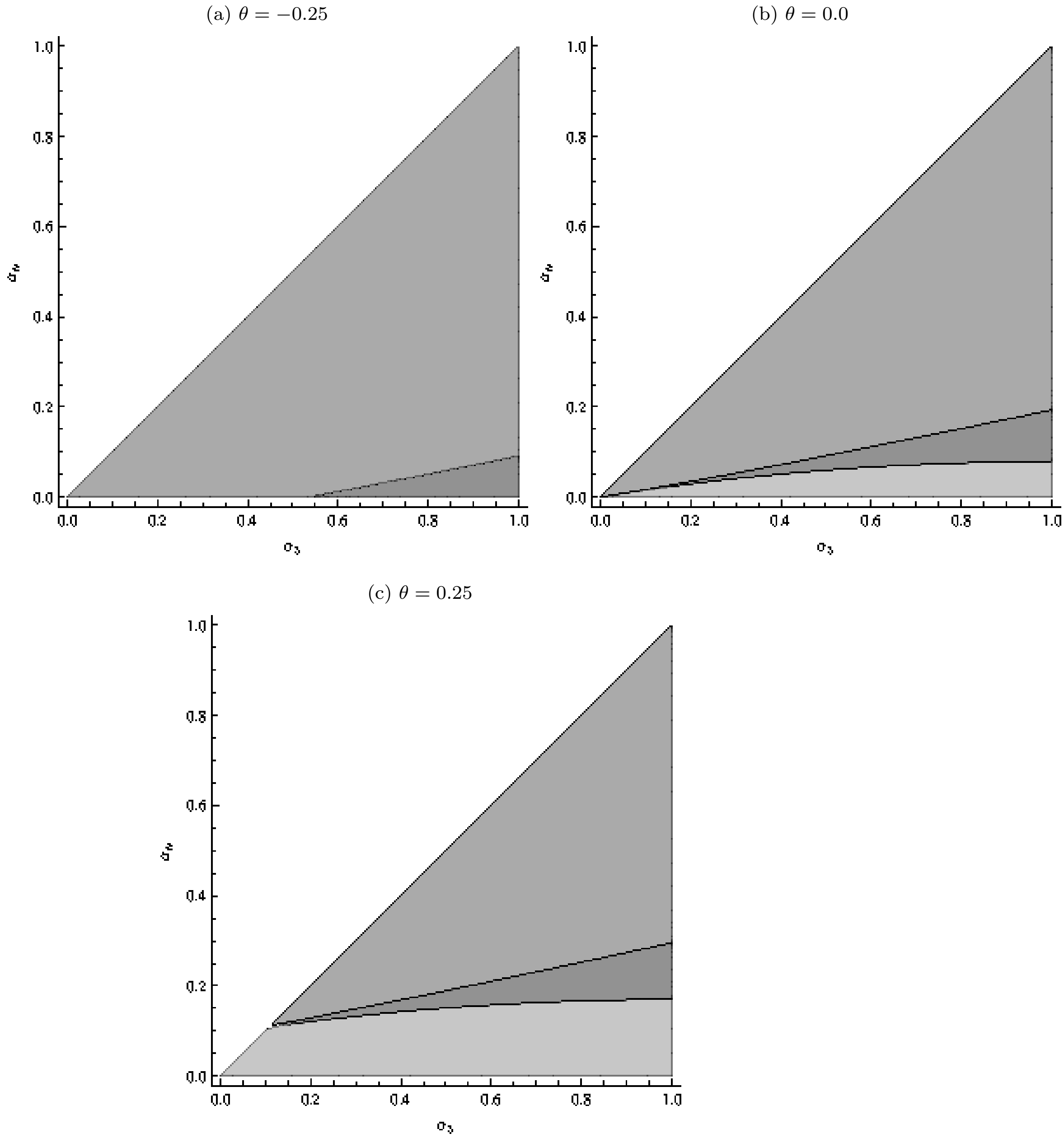
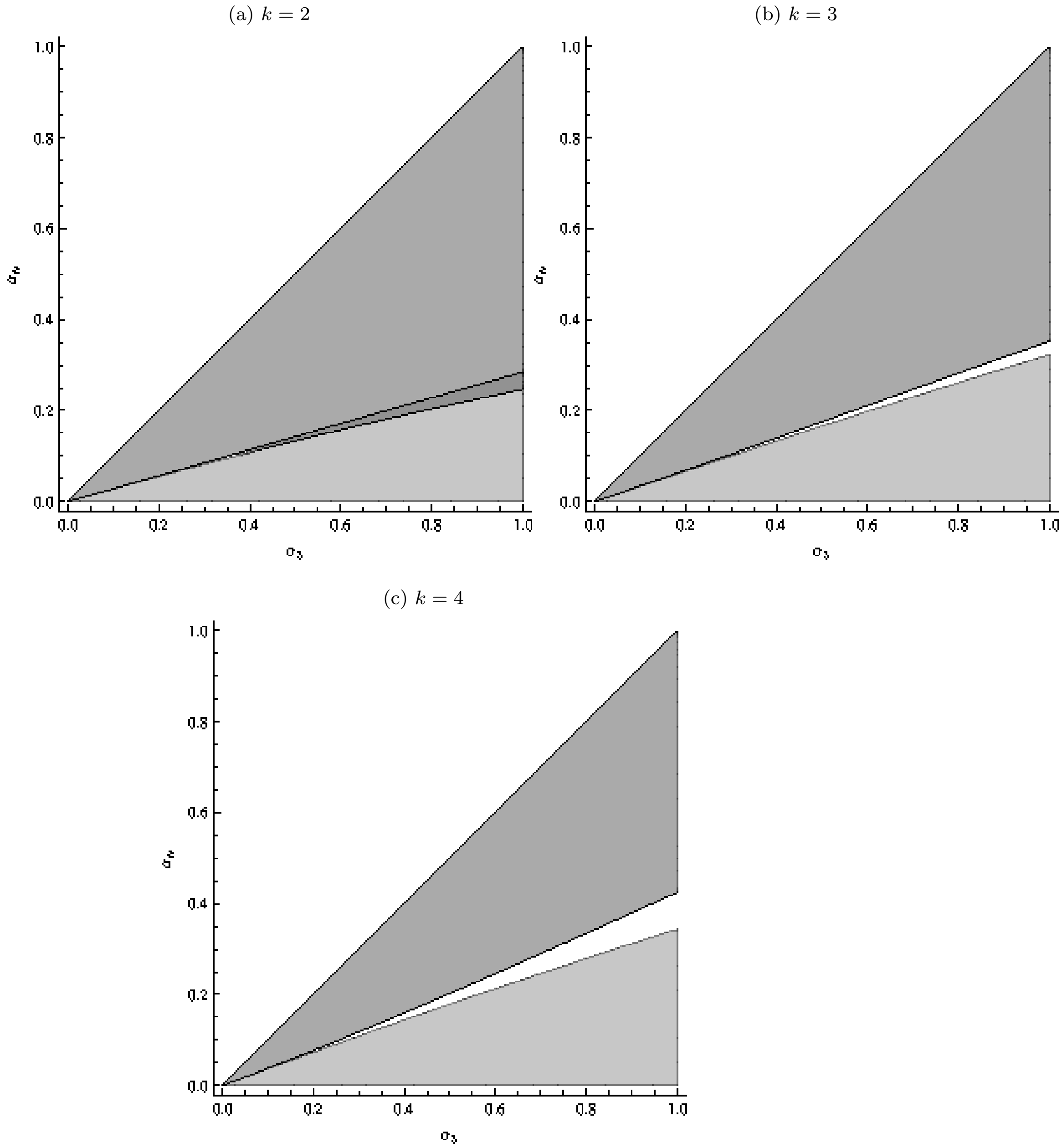


Figure 6: Regions of existence: $k = 2, 3, 4$; $\theta = 0$



Appendix

A Numerical analysis

We can numerically approximate the set of equilibria of this model for the case $k = 1$. Let $\eta = (\eta_0, \eta_1, \ldots, \eta_{n-1})$ be an N-vector of configuration of smoking and non-smoking firms where $\eta_n = \mathbf{S}$ if the firm allows smoking and $\eta_n = \mathbf{N}$ if not. Assume that the configuration of firms is such that firms 0 through $N-1$ have types $\eta_0, \eta_1, \ldots, \eta_{N-1}$ and that firms N through $2N-1$ have types $\eta_0, \eta_1, \ldots, \eta_{N-1}$, etc.

Although the set of potential equilibrium configurations is infinite, it is straightforward to numerically compute price equilibria for the case $k = 1$ (an infinite line). In such models, as competitors become more distant, their impact on equilibrium prices falls exponentially. That is, if a single firm changes its smoking policy, its impact on rival firms falls exponentially with distance. Thus we can compute subgame pricing equilibria for arbitrarily complicated repeated configurations. For example, there are 8 permutations of patterns of three, "**SSS**", "**SSN**", "**SNS**", "**SNN**", "**NSS**", "**NSN**", "**NNS**", "**NNN**" and repeating "**SNN**" gives us the configuration "...**SNNSNNSNN**..." For repeated patterns of length N where N is sufficiently large and for specific parameterizations, these equilibria are easily computed—the subgame price equilibrium is identical to that for an N firm Salop model. Given equilibrium profits for the pricing subgames, equilibrium configurations can be evaluated by comparing profits from a particular pattern to deviation profits for each firm within the pattern. As long as the pattern is sufficiently long, ripple effects will be small, with negligible effects on firms at the edge of the pattern. Since the pattern is repeated, every firm's optimal smoking policy can be evaluated from the center of the pattern. Moreover, equilibrium patterns can be overlapped to generate yet longer equilibrium configurations.

Note that any given firm with index n (modulo N) has two direct competitors and the solution to its profit maximizing problem yields a reaction function with the following form:

$$p_n = a_n + \frac{\beta_n}{2} p_{(n-1)\,\%\,N} + \frac{1-\beta_n}{2} p_{(n+1)\,\%\,N},$$

where "$x \,\%\, y$" is "x modulo y", η_n^3 is the three firm configuration $(\eta_{n-1}, \eta_n, \eta_{n+1})$ and

$$a_n = \begin{cases} \frac{t}{2} & \text{if } \eta_n^3 \in \{(\mathbf{N},\mathbf{N},\mathbf{N}), (\mathbf{S},\mathbf{S},\mathbf{S})\} \\ \frac{(1+2\alpha_{\mathbf{S}}+\theta)t}{2} & \text{if } \eta_n^3 = (\mathbf{N},\mathbf{S},\mathbf{N}) \\ \frac{(1+2\alpha_{\mathbf{N}}-\theta)t}{2} & \text{if } \eta_n^3 = (\mathbf{S},\mathbf{N},\mathbf{S}) \\ \frac{(2+3\alpha_{\mathbf{S}}+\theta)t}{2(1+\alpha_{\mathbf{S}})} & \text{if } \eta_n^3 \in \{(\mathbf{S},\mathbf{S},\mathbf{N}), (\mathbf{N},\mathbf{S},\mathbf{S})\} \\ \frac{(2+3\alpha_{\mathbf{N}}+\theta)t}{2(1+\alpha_{\mathbf{N}})} & \text{if } \eta_n^3 \in \{(\mathbf{S},\mathbf{N},\mathbf{N}), (\mathbf{N},\mathbf{N},\mathbf{S})\} \end{cases}$$

and

$$\beta_n = \begin{cases} \frac{1}{2} & \text{if } \eta_n^3 \in \{(\mathbf{N},\mathbf{N},\mathbf{N}), (\mathbf{S},\mathbf{S},\mathbf{S}), (\mathbf{N},\mathbf{S},\mathbf{N}), (\mathbf{S},\mathbf{N},\mathbf{S})\} \\ \frac{1+\alpha_{\mathbf{N}}}{2+\alpha_{\mathbf{N}}} & \text{if } \eta_n^3 = (\mathbf{N},\mathbf{N},\mathbf{S}) \\ \frac{1+\alpha_{\mathbf{S}}}{2+\alpha_{\mathbf{S}}} & \text{if } \eta_n^3 = (\mathbf{S},\mathbf{S},\mathbf{N}) \\ \frac{1}{2+\alpha_{\mathbf{S}}} & \text{if } \eta_n^3 = (\mathbf{N},\mathbf{S},\mathbf{S}) \\ \frac{1}{2+\alpha_{\mathbf{N}}} & \text{if } \eta_n^3 = (\mathbf{S},\mathbf{N},\mathbf{N}) \end{cases}$$

For firm 0,

$$p_0 = a_0 + \frac{\beta_0}{2} p_N + \frac{1-\beta_0}{2} p_1,$$

and for firm $n > 0$,

$$p_n = a_n + \frac{\beta_n}{2} p_{n-1} + \frac{1-\beta_n}{2} p_{n+1}, \quad \text{(A1)}$$

where the indices n are modulo N. This is a system of N equations and N unknowns, the solution to which is straightforward.

To see that distant competitors have little impact on one another, notice that when a deviating firm n changes its smoking policy and price, firm $n+1$ faces both a different strategic situation (a_{n+1} and β_{n+1} change) and different prices at location n and $n+2$. Suppose that firm $n+1$ changes its price by $\triangle p_{n+1}$ in response (the argument for $n-1$ is analogous). Firm $n+2$ only faces different prices (a_{n+2} and β_{n+2} remain the same) and its best response changes by approximately $\triangle p_{n+2} \approx \triangle p_{n+1}\beta_{n+2}/2$.[12] Similarly firm $n+3$ changes its price by approximately $\triangle p_{n+3} \approx \triangle p_{n+2}\beta_{n+3}/2 \approx \triangle p_{n+1}\beta_{n+2}\beta_{n+3}/4$ and firm $n+4$ by about $\triangle p_{n+4} \approx \triangle p_{n+3}\beta_{n+4}/2 \approx \triangle p_{n+1}\beta_{n+2}\beta_{n+3}\beta_{n+4}/8$, etc. For $\alpha_S, \alpha_N < 1$, $1/3 < \beta_{n+i} < 2/3$ and for small α_S, α_N $\beta_{n+i} \approx 1/2$. It follows that the price response to a deviation rapidly converges to the non-deviation price as we move away from the deviating firm. In general, if $\alpha_S, \alpha_N < 1$ then $|\triangle p_{n+1}|/6^{j-1} < |\triangle p_{n+j}| < |\triangle p_{n+1}|/3^{j-1}$ and if $\beta_{n+i} \approx 1/2$ then $\triangle p_{n+j} \approx \triangle p_{n+1}/4^{j-1}$ for $j > 1$. Thus if N is sufficiently large, we can compute the equilibria for arbitrary repeated patterns of length N.

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[12]This is a very rough argument in the sense that firm $n+2$ chooses its price as a best response to the prices of both firms $n+1$ and $n+3$, not just firm $n+1$.