# In the Shadow of Giants*

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(Preliminary and incomplete)

**Abstract**

Intellectual giants provide broad shoulders for subsequent inventors. When they tumble, however, it also casts shadow on the prospect of future research. This paper incorporates this shadow effect into a two-stage innovation process and shows that patenting the first-stage result (the basic invention) may enhance the second stage performance. Only under weak shadow effect can it be optimal to enable the doctrine of patentable subject matter (the DPSM) and reject patent protection to the basic invention. In this case, the DPSM serves to preserve the pioneering inventor's incentive to continue research activities.

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# 1 Introduction

*"By standing on the shoulders of Giants,"* Sir Isaac Newton and generations of scholars saw further and more than their predecessors. This cumulative feature of the knowledge-generation process has been recognized and formed the foundation of modern economic analysis of innovation (Green and Scotchmer, 1995, Scotchmer, 1996, O'Donoghue, 1998, Denicolò, 2000, Bessen and Maskin, 2009). The unfinished pursuits of intellectual giants, however, leave a daunting task to follow. When evaluating the possibility to construct a necessary view to interpret probability,[1] Savage (1972, p. 61) cited the limited progress made by its two most prominent enthusiasts, J. M. Kenyes and R. Carnap, and suggested that:

> That these men express any doubt at all about the possibility of narrowing a personalistic view to the point where it becomes a necessary one, after such extensive and careful labor directed toward proving this possibility, speaks loudly for their integrity; at the same time it indicates that the task they have set themselves, if possible at all, is not a light one.

In another context, Farber (2010, p. 7) also expressed the same sort of doubts based on past records:

> The search for a foundational First Amendment "brick" has been unavailing so far. If so many thoughtful legal commentators have failed to identify *the* foundational value that supports a unified First Amendment theory, the prospects for future efforts may be dim.

In other words, when intellectual giants tumble or remain silent, their legacy may cast a shadow on future explorations.

This paper addresses the impacts of this "shadow effect" on innovation and patent policy. Consider a pioneering inventor (she) and a follower (he) sequentially conducting research to achieve an invention, whose *ex ante* successful probability is less than one. The follower observes the pioneer's result and uses it as a signal to adjust his own assessment of the successful probability. Since the two players pursue the same invention, the pioneer's failure to come up with the invention sends a bad news to the follower. The follower will update the successful probability to a lower level, and thus

[1] According to Savage (1972), *"Necessary views hold that probability measures the extent to which one set of propositions, out of logical necessity and apart from human opinion, confirms the truth of another. They are generally regarded by their holders as extensions of logic, which tells when one set of propositions necessitates the truth of another."*

reduce his innovation effort. In a simple way, this captures the idea that the follower conducts his own research activity under the shadow of the pioneer. The more efforts the pioneer devotes to research, the darker is the shadow and the lower the follower's updated belief when facing the pioneer's silence.

I incorporate this shadow effect into a two-stage innovation process to discuss issues related to the patent policy. Following the literature *à la* Green and Scotchmer (1995), I assume that the completion of the first stage is a pre-requisite to start the second stage. For the purpose of policy discussion, I refer to the first stage invention as the abstract idea and the second stage invention as the application. The pioneer is the sole player at the first stage. At the second stage, the pioneer and follower sequentially engage in innovation activity as described above. (Hence the source of shadow effect is the pioneer's innovation effort at the second stage.) Assume that the second stage invention (the application) is always patentable, and will always infringe on the first stage invention (the abstract idea) should the latter become patentable. I consider how the patent protection to the abstract idea affects the innovation performance, in particular the second stage innovation rate and structure, and whether it is optimal to grant the basic patent, i.e., to protect the abstract idea with patent rights.

For the first issue, consistent with the literature, a basic patent boosts the pioneer's incentive to engage in the first stage innovation. It also reduces the pioneer's effort at the second stage, because she can use the patent to get a share of the follower's invention surplus. For the follower, sharing fruit with the pioneer causes a direct negative impact on his innovation incentive. This negative sharing effect, however, is offset by the shadow effect: The lower (second stage) effort by the pioneer after obtaining a basic patent will raise the follower's assessment of the successful probability. When the shadow effect outweighs the sharing effect, the follower has *stronger* incentive to conduct research after the abstract idea becomes patentable. It may also happen that the basic patent benefits both stages of innovation, and encourages decentralization of the innovation market (measured by the extent to which different inventions are created by different inventors.) Shadow effect thus generates different predictions than previous literature and provides a less gloomy role of basic patents.

The second issue deals with the debate of patentable subject matter in patent law. Reflecting its sequential nature, the economic literature of cumulative innovation emphasizes the needs to properly protect early stage inventions, and focuses on how to adjust patent rights to latter inventions in order to balance R&D incentives at different stages of the innovation process.[2] Patent law does not always enthusiastically embrace the strong support of basic inventions, however. The Supreme Court of the United States has long held that "*[h]e who discovers a hitherto unknown phenomenon of nature has no claim to a monopoly of it which the law recognizes. If there is to be invention from such a discovery, it must come from the application of the law of nature to a new and useful end.*"[3] Established in case law, the doctrine of patentable subject matter (henceforth, the DPSM) precludes the following from the realm of patent protection:[4]

> principles, laws of nature, mental processes, intellectual concepts, ideas, natural phenomena, mathematical formulae, methods of calculation, fundamental truths, original causes, motives, [and] the Pythagorean theorem. . . .

*Applications* of abstract ideas and principles, instead, may be patented, provided that they also satisfy other requirements such as novelty, non-obviousness, and usefulness.

To reconcile this discrepancy between economic theory and patent law (Eisenberg, 2000), I use the two-stage model to consider when it is optimal to enable the DPSM and deny patent protection to the abstract idea in order to "promote the Progress of Science and useful Arts."[5] That is, to maximize the probability of finishing the two stages and inventing the application.[6] Previous analysis immediately provides a necessary condition: The DPSM is optimal only when all levels of protection to the abstract idea retards the second the second stage innovation, which in turn requires a small shadow effect.

Suppose that this necessary condition holds. The DPSM is more likely to be optimal when, at the *second* stage, the pioneer has better innovation capacity, while the

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[2]See Scotchmer (2004) for a literature review. Bessen and Maskin (2009) argues that the patent system should be abolished in the cumulative innovation environment.

[3]Funk Bros. Seed Co. *vs.* Kalo Inoculand Co., 333 U.S. 127 (1948).

[4]*In re Bergy*, 596 F.2d 952, 201 U.S.P.Q. (BNA) 352 (C.C.P.A. 1979). See also Merges (1997). The European Patent Convention excludes the following from patentable inventions: (a) discoveries, scientific theories and mathematical methods; (b) aesthetic creations; (c) schemes, rules and methods for performing mental acts, playing games or doing business, and programs for computers; and (d) presentations of information (http://www.epo.org/patents/law/legal-texts/html/epc/1973/e/ar52.html).

[5]U.S. Constitution, Art I, sect. 8, cl. 8.

[6]For sure, one may find other justifications for the DPSM, such as the difficulty to enforce patent rights based on abstract ideas or mental process, or the somewhat ambiguous difference between "discovery" and "invention." In Gottschalk *vs.* Benson, 409 U.S. 63 (1972), the Supreme Court states that: "*It is conceded that one may not patent an idea.. . . The mathematical formula involved here has no substantial practical application except in connection with a digital computer, which means that if the judgment below is affirmed, the patent would wholly preempt the mathematical formula and in practical effect would be a patent on the algorithm itself*." This argument could be analyzed as one with patent scope, i.e., whether to allow a patent with a very broad scope such that it covers all inventions using the algorithm.

follower is less likely to make the discovery. In this model, the patent policy has to balance not only incentives of different generations of inventors, but also the *same* inventor's incentives at different innovation stages. When the follower has a rather small probability to find the application (even without the threat of basic patent), there is not much surplus to transfer from the follower to the pioneer. Patenting the abstract idea has limited benefit on the first stage innovation, and the second stage discovery probability is dominated by the pioneer's performance. When the pioneer can find the application with a significant probability, *provided that* she is willing to do so, the negative effect of such an "early reward" on her search decision can be non-negligible. The DPSM then is justified as a way to preserve the pioneer's continuing efforts in research.

This finding implies that abstract ideas or basic inventions should not be patentable if great first-mover advantage can be derived from engaging in fundamental research, while a new comer, lacking the experience at the earlier stage, faces a substantial obstacle to join the rank. But as the innovation process becomes more "democratic," i.e., as knowledge and research capacity disseminate and are no longer concentrated on a few "early stars," then it would be optimal to start patenting abstract ideas or early inventions. Alternatively, capacities possessed by the pioneer and follower may be different in kind. The pioneer may be good at perfecting the basic invention or better understanding its fundamental properties, and follower may have advantage in identifying particular use of the basic invention and adapting it to specific contexts. The relative importance of these two capacities then depends on the phase of technological progress. To the extent that further understanding the basic scientific principles has priority in primitive technology fields, basic inventions or abstract ideas should become patentable only in mature fields.

My results also provide another interpretation of the shrinking of the DPSM since the 1980s. Through a series of court decisions, particularly in computer software and biotechnology, the scope of patentable subject matters has drastically increased in the U.S. (Kuhn, 2007). Despite rapid expansions, some commentators have warned that rewarding patents to abstract ideas would do more harm than good to the long-term development in these fields. And it is an often raised hypothesis that these industries could have done better had these basic patents been denied. The shadow effect introduced in this paper nevertheless provides a theoretical argument to mitigate this concern.[7] Furthermore, if the optimal patent policy takes into account the concerns listed here, then there may be a reverse causality: abstract ideas should become patentable precisely when there is a better follower joining the development process.

There is a long and well established literature of the doctrine of patentable subject matter in the legal profession.[8] In economics, however, most studies either assume that early inventions always receive patent protection (Green and Scotchmer, 1995, Scotchmer, 1996, Denicolò, 2000), or give equal treatments to innovations at different stages (O'Donoghue, 1998). Matutes *et al.* (1996) and Kultti and Mittunen (2008) allow various levels of protection to the basic invention, including no protection, but conclude that some protection is always better. To the best of my knowledge, Harhoff *et al.* (2001) and Aoki and Nagaoka (2004) are the two exceptions that obtain no patent protection to the basic invention as the optimal policy. Assuming that firms have fixed research capacity, Harhoff *et al.* (2001) cautions that patenting basic inventions (gene in their model) may induce socially wasteful stockpile of basic inventions and delay applications. Aoki and Nagaoka (2004) allow firms to vary R&D efforts and is the most relevant paper to my analysis.[9]

Aoki and Nagaoka (2004) considers the same issue as here, namely, whether to grant patent protection to an intermediate invention that serves only as an input for future research, and obtains a pretty intuitive result that patent protection is desirable when conducting basic research is very costly. Aoki and Nagaoka (2004) adopts a two-stage paten race model as in Denicolò (2000), and assume that players have the same Poisson-type innovation technology. In this paper, I stress the asymmetry between inventors of different generations. I will also show that, when the first stage innovation cost has uniform distribution, the optimality of the DPSM does not depend on

[7]See, e.g., Merges (2007) for a discussion of these "unfulfilling" critics in the software industry.

[8]See Merges (1997) for a general discussion. A partial list of recent articles includes Gruner (2007), Kuhn (2007) and Risch (2008).

[9]Aoki and Nagaoka (2004) considers the same issue as here but in the name of utility requirement. Arguably there is some over-lapping between the utility requirement and the DPSM: an abstract idea is not patentable because it lacks "specific and substantial utility," i.e., it is not "useful for any particular practical purpose." (See USPTO, *Utility Examination Guidelines*, http://www.uspto.gov/web/menu/utility.pdf.) Indeed, in Brenner *vs.* Manson, 383 U.S. 519 (1966), the Supreme Court ruled that the Manson patent is at a too preliminary stage to be protected by a patent, and stated that *"a patent is not a hunting license. It is not a reward for the search, but compensation for its successful conclusion."* The Court's reasoning, however, contains some flavor of patent scope: *"Unless and until a process is refined and developed to this point–where specific benefit exists in current available form–there is insufficient justification for permitting an applicant to engross what may prove to be a broad field."* Risch (2008) suggests to abolish the DPSM but reinvigorate the utility requirement to assess the patentability of each invention. In practice, the utility requirement is not strictly applied. Few patent applications are rejected under this requirement.

the cost parameter (the support of the distribution) at this stage. In this regard, my analysis is complementary to the insight derived in Aoki and Nagaoka (2004).

To proceed, section 2 introduces the basic setting. Section 3 considers the influence of the shadow effect. Section 4 applies these results to determine when it is optimal to enable the DPSM. Section 5 (to be completed) considers some variations of the basic model, including research grants and academic reputations. Section 6 concludes the paper. Proofs are collected in Appendix A.

# 2 Model

A pioneer and a follower participate in a two-stage innovation process. The goal of the first stage is to create a basic invention (abstract idea or scientific knowledge) whose application is to be discovered at the second stage. As in Matutes *et al.* (1996), I assume that only the pioneer participates in the first stage, but both players may search for the application at the second-stage. To compare my results with the cumulative innovation literature, I also assume that, at the second stage, players are looking for the same application, or applications with high substitutability in terms of payoffs.

An inventor decides whether to spend an exogenous (but *ex ante* random) innovation cost. After incurring the cost, the invention arrives with some probability. At the first stage, I assume that the basic invention will be created for sure when the pioneer spends the cost $c_0$, which is distributed over $[0, \infty)$ with CDF $F_0(\cdot)$ and pdf $f_0(\cdot)$. The basic invention, or abstract idea, has no stand-alone value, and the game ends when the pioneer decides not to spend $c_0$.

After the pioneer incurs $c_0$, the game proceeds to the stage of application search. The application has a private value $\pi > 0$ and exists with a probability $\alpha \in (0, 1]$. The expected value is $v \equiv \alpha\pi$. These parameters are common knowledge between two inventors. Given existence, the pioneer (the follower) finds the application after incurring a cost $c_1 \in [0, \infty)$ ($c_2 \in [0, \infty)$, respectively). Denote the CDF and pdf of cost $c_i$ as $F_i(\cdot)$ and $f_i(\cdot)$, respectively, $i \in \{1, 2\}$. Suppose that pioneer searches first, and the true cost $c_i$ is the player's private information. An inventor cannot commit to her/his own nor observe the other's search strategy.

The distribution of search cost captures an inventor's innovation capacity. I assume that $F_i$ as well as $f_i$ are continuous and differentiable as necessary, $i \in \{0, 1, 2\}$. In addition, for all $i \in \{0, 1, 2\}$, $f_i(c) > 0$ for $0 \leq c < C_i$, with $C_i > v$. This guarantees that $0 < F_i(v) < 1$, and so even if an inventor can grab the whole expected surplus, from the *ex ante* point of view there is some probability that the inventor is not willing to engage in innovation.

As in the literature of cumulative innovation, the patent policy affects the division of surplus $\pi$ between inventors. To focus on the doctrine of patentable subject matter, I assume that the application is patentable but always infringes on the basic invention when the latter is protected by the patent rights. The only policy instrument is the level of patent rights rewarded to the basic invention.

If the pioneer discovers the application, then she obtains a patent on the application (and maybe also one on the basic invention); she enjoys the whole surplus $\pi$. If the follower makes the discovery, then patent policy determines that the pioneer receives $\theta\pi$ and the follower receives $(1-\theta)\pi$, where $\theta \in [0, \overline{\theta}]$ and $\overline{\theta} < 1$. A higher $\theta$ then implies stronger patent rights endowed to the basic invention, and the doctrine of the patentable subject matter corresponds to the case of $\theta = 0$. The upper bound $\overline{\theta}$ is assumed to be strictly less than one because generally, in case of mutual blocking patents, each patent-holder would receive a share of surplus. In other words, I exclude the extreme case where the pioneering inventor has the full bargaining power.

Note that, if the pioneer exhausts her search opportunity but does not come up with the application, i.e, if she decides not to spend $c_1$ or if $c_1$ is incurred but she doesn't find the application, then the pioneer is (weakly) better off to disclose the basic invention, for all values of patent policy $\theta$. When the DPSM is enabled and the basic invention is not patentable ($\theta = 0$), then whether the pioneer discloses the basic invention has no impact on her payoff. She won't get a share of $\pi$ whatever happens after.[10] When the basic invention is patentable with $\theta > 0$, by disclosing the basic invention and so allow the follower to engage in application search, the pioneer may receive a surplus $\theta\pi$ with some probability. Since there is no harm of disclosing the basic invention, I assume that the pioneer will always publish the basic invention.

The optimal policy $\theta$ is derived to induce technological progress, as measured by the overall probability to complete the two-stage innovation process.[11] This objective can also be justified from the concern of the social surplus. When the *application*

[10]But after disclosure of the basic invention the pioneer may receive, say, a Nobel Prize or other reputation-based reward from the scientific community for the recognition as the inventor of important scientific knowledge or breakthrough. I incorporate these "kudos" (Gans *et al.*, 2010) in Section 5 (to be finished).

[11]In Section 4, I show that the DPSM cannot maximize the joint surplus of the two inventors.

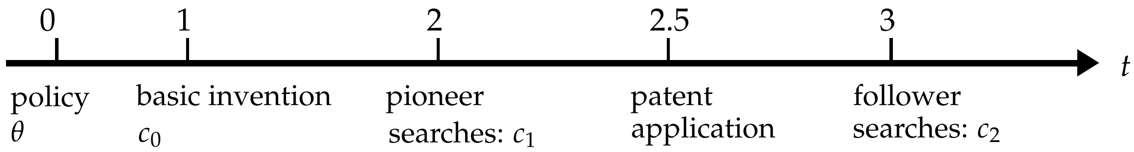


Figure 1: Timing

has significant positive externality, private parties always under-invest. It is socially desirable to raise private innovation efforts in order to achieve the application.

Figure 1 illustrates the timing of the game.

- At time 0, the patent policy $\theta$ is announced;
- at time 1, the pioneer learns the patent policy and the cost $c_0$ of conducting the first stage innovation. The game continues only if the inventor spends $c_0$ and creates the basic invention;
- at time 2, the pioneer inventor learns $c_1$ and decides whether to search the application;
- at time 2.5, the pioneer applies patents for the basic invention (if allowed), and for the application (if she finds it); and
- at time 3, if the pioneer doesn't find the application, then the follower learns his search cost $c_2$ and decides whether to search.

When the pioneer holds a basic patent and doesn't find the application, the follower may want to negotiate a license before searching for the application, between time 2.5 and 3. I postpone the discussion of licensing to Section 5.

# 3 The Shadow Effect

This section illustrates the shadow effect and how it alters the impact of patent policy $\theta$ on the innovation process. Solving the game in the backward manner, suppose that the pioneer has created the basic invention, and consider the subgame of application search.

If the pioneer incurs $c_1$ and finds the application, then she can patent the application (and maybe the basic invention) and gets the whole surplus $\pi$; the game ends. Suppose that the pioneering inventor does not come up with the application. By assumption, the follower does not know whether it is because the pioneer doesn't spend

$c_1$ to search, or because she incurs $c_1$ but the application does not exist.[12] After learning his search cost $c_2$, the follower decides whether to search with some updated belief $\hat{\alpha}$ that the application exists. Below I will show that both player' optimal search strategy takes a cut-off form. That is, an inventor will incur the search cost if and only if it is lower than a threshold value. Given this rule, when the follower believes that the pioneer's cut-off is $\tilde{c}_1$, his updated belief is

$$\hat{\alpha}(\tilde{c}_1) = \frac{\alpha[1 - F_1(\tilde{c}_1)]}{1 - \alpha + \alpha[1 - F_1(\tilde{c}_1)]} = \frac{\alpha[1 - F_1(\tilde{c}_1)]}{1 - \alpha F_1(\tilde{c}_1)}. \tag{1}$$

With probability $\alpha$, the application exists, and the pioneer finds it only if incurring the cost $c_1$; and with probability $1 - \alpha$, the application does not exist and the pioneering inventor won't be able to find it whether spending $c_1$ or not. The follower then updates his belief according to Bayes' rule as expressed in condition (1).

When the application exists for sure, $\alpha = 1$, then the follower's updated assessment of successful probability will remain at the *ex ante* level, $\hat{\alpha} = \alpha = 1$, given that $F_1(\tilde{c}_1) < 1$, as will be satisfied later. The pioneer's innovation activity has no impact on the follower's belief. But for all $\alpha \in (0, 1)$ and $\tilde{c}_1 > 0$, the follower's updated belief is lower than the *ex ante* level, $\hat{\alpha} < \alpha$. When the pioneer engages in innovation activity at this stage ($\tilde{c}_1 > 0$), her failure conveys a bad news to the follower. The shadow effect arises. In addition, for all $\alpha \in (0, 1)$,

$$\frac{\partial \hat{\alpha}}{\partial \tilde{c}_1} = -\frac{\alpha(1 - \alpha) f_1(\tilde{c}_1)}{[1 - \alpha F_1(\tilde{c}_1)]^2} < 0. \tag{2}$$

The shadow gets darker when the pioneer exerts a higher effort (a higher $\tilde{c}_1$).

Given the patent policy $\theta \in [0, \overline{\theta}]$, the follower receives a payoff $(1 - \theta)\pi$ for his discovery. He incurs $c_2$ and searches if and only if

$$\hat{\alpha}\pi(1 - \theta) - c_2 \geq 0 \;\; \Rightarrow \;\; c_2 \leq \hat{c}_2 \equiv \hat{\alpha}\pi(1 - \theta). \tag{3}$$

The follower adopts a cut-off rule, with an expected payoff (given that the pioneer doesn't find the application)

$$\hat{U}_2 = \int_0^{\hat{c}_2} [\hat{\alpha}\pi(1 - \theta) - c_2] dF_2. \tag{4}$$

[12] If the pioneer has incurred search cost but failed to find the application, by assumption it is a clear indication that the application does not exist. This "negative information" is valuable to the follower as well as the society for it prevents further wasteful search effort. See 5 for a discussion of "licensing" this information.

For the pioneer, if she spends the cost $c_1$, then with probability $\alpha$ she will find the application and enjoy the whole surplus $\pi$; and with probability $1-\alpha$ the application does not exist and the follower will not find it either. If the pioneer does not spend $c_1$, then she will receive a surplus $\theta\pi$ when the application exists and the follower searches. Suppose that the pioneer believes that the follower adopts a cut-off $\tilde{c}_2$ and so will search with probability $F_2(\tilde{c}_2)$. The pioneer searches if and only if

$$\alpha\pi - c_1 \geq F_2(\tilde{c}_2)\alpha\pi\theta \quad \Rightarrow \quad c_1 \leq \hat{c}_1 \equiv \alpha\pi[1 - F_2(\tilde{c}_2)\theta] = v[1 - F_2(\tilde{c}_2)\theta]. \tag{5}$$

The pioneer also adopts a cut-off rule, and her expected payoff at the second-stage is

$$\hat{U}_1 = \int_0^{\hat{c}_1} (\alpha\pi - c_1) dF_1 + [1 - F_1(\hat{c}_1)] F_2(\tilde{c}_2)\alpha\pi\theta. \tag{6}$$

Since players cannot commit to their search strategies (the cut-off values) and the true search cost and the decision to incur it are not observable to the other party, the proper equilibrium concept at the search subgame is rational expectation equilibrium. Slightly abusing the notation, a rational expectation equilibrium is a pair of cut-offs $(\hat{c}_1, \hat{c}_2)$ such that they are determined according to conditions (3) and (5), with the belief $\hat{\alpha}$ in condition (3) evaluated at $\tilde{c}_1 = \hat{c}_1$ according to the expression (1), and the belief $\tilde{c}_2 = \hat{c}_2$ in condition (5). Given a search equilibrium $(\hat{c}_1, \hat{c}_2)$, denote the corresponding probabilities $\hat{F}_i \equiv F_i(\hat{c}_i)$, $i \in \{1, 2\}$. For the interest of this paper, I denote $(c_1^*, c_2^*)$ as the search equilibrium under the DPSM, i.e., under the policy $\theta = 0$, with corresponding $F_i^* = F_i(c_i^*)$, $i = 1, 2$.

The DPSM guarantees a unique search equilibrium. Setting $\theta = 0$ in condition (5), the pioneer's search decision is independent of the follower's search strategy. The optimal cut-off is uniquely determined by

$$c_1^* \equiv \alpha\pi \equiv v. \tag{7}$$

This unique cut-off then pins down the follower's updated belief at search, $\hat{\alpha}(c_1^*) \equiv \alpha^*$, and the follower's optimal cut-off $c_2^*$:

$$c_2^* \equiv \alpha^*\pi. \tag{8}$$

When $\theta > 0$, the two inventors' search decisions become strategic substitutes. In equilibrium, a higher cut-off $\hat{c}_1$ will reduce $\hat{c}_2$, and *vice versa*. The pioneer, with $\theta > 0$, benefits from the follower's search. More intensive search by the follower, i.e., a higher cut-off $\hat{c}_2$ and so a larger probability $\hat{F}_2$, lowers the pioneer's search incentive.

The pioneer's cut-off $\hat{c}_1$ is decreasing in $\hat{c}_2$ for $\theta > 0$. The negative impact of $\hat{c}_1$ on $\hat{c}_2$ works through the shadow effect, namely, expression (2). This negative effect on belief depresses the follower's search incentives: $\hat{c}_2$ is decreasing in $\hat{c}_1$. As long as $\theta < 1$, the patent policy only changes the magnitude of this effect, but does not affect its presence.

The mutual dependence of search decisions may lead to multiple search equilibria. Consider an increase in $\hat{c}_1$. Along the equilibrium path, a more intensive search from the pioneer lowers the follower's belief, and so the follower's equilibrium cut-off $\hat{c}_2$. A lower search intensity from the follower in turn justifies the initial increase in $\hat{c}_1$. By the same token, expecting an increase of the follower's cut-off, the pioneer will search over a smaller range of search cost. The follower, along the equilibrium path, will a have a higher updated belief, and so is willing to raise the cut-off.

Despite the possibility of multiple equilibria, granting patent rights to the basic invention always reduces the pioneer's search incentive, $c_1^* > \hat{c}_1$ for all $\theta \in (0, \bar{\theta}]$. By $\overline{\theta} < 1$, in any search equilibrium $\hat{c}_2 > 0$ and so $\hat{F}_2 > 0$. It follows that $c_1^* = v > \hat{c}_1 = v(1 - \hat{F}_2\theta)$, for all $\theta \in (0, \overline{\theta}]$. For the follower, a lower cut-off adopted by the pioneer boosts his belief at search: $\hat{\alpha}(\hat{c}_1) > \alpha^*$, for all $\hat{c}_1 < c_1^*$. Whether $\hat{c}_2 \gtreqless c_2^*$ then depends on whether $\hat{\alpha}(1 - \theta) \gtreqless \alpha^*$. The shadow effect alleviates the negative effect of transferring surplus $\theta\pi$ from the follower to the pioneer on the former's search decision. Patent protection to the basic invention does not necessarily weaken the follower's innovation incentive.

Given the parameter $\alpha$ and search equilibrium $(\hat{c}_1, \hat{c}_2)$, the probability to discover the application is $\alpha[\hat{F}_1 + (1 - \hat{F}_1)\hat{F}_2]$. Define $\hat{E} \equiv \hat{F}_1 + (1 - \hat{F}_1)\hat{F}_2$, which measures the overall search effort, or the innovation performance at the second stage. Define the corresponding measure under the DPSM as $E^* \equiv F_1^* + (1 - F_1^*)F_2^*$. Since $c_1^* > \hat{c}_1$, the comparison between $E^*$ and $\hat{E}$ depends on the relative size of $c_2^*$ and $\hat{c}_2$. If $c_2^* \geq \hat{c}_2$, then the DPSM surely boosts the second-stage innovation performance, $E^* > \hat{E}$. If $\hat{c}_2 >> c_2^*$, however, we may have the opposite outcome, $\hat{E} > E^*$. Note that this is possible only in the presence of the shadow effect. If $\alpha = 1$, then $\hat{\alpha} = \alpha = 1$. The shadow effect disappears, and we must have $\hat{c}_i < c_i^*$ for both $i = 1$ and 2.

The shadow effect also generates different predictions of how the patent policy $\theta$ affects the "market structure" of the innovation market than those obtained by Aoki and Nagaoka (2004). Aoki and Nagaoka (2004) uses a two-stage patent race model from Denicolò (2000) and also allows the first inventor to engage in the second stage innovation.[13] Due to the assumptions of a homogeneous Poisson race and identical research capability, if the basic invention is patentable, the patent-holder has no incentive to let other inventors pursue the second stage innovation. The only meaningful policy space is a binary set, namely, whether the basic invention is patentable or not. The pioneer does not benefit from other inventor's innovation capacity. Patenting the basic invention generates a monopoly at the second stage, and increases the concentration of the innovation activity, i.e., the extent to which different inventions are created by different inventors.

By contrast, I have a "hybrid" structure where the pioneer enjoys head-start advantage at the second stage and at the same time could extract some surplus from the follower when the basic invention is patentable and her search fails. There is a richer policy space $\theta \in [0, \overline{\theta}]$, and asymmetric innovation capacities by different inventors can be captured by different distributions $F_1$ and $F_2$. In addition, the basic patent may reduce the concentration of the innovation market. Given the completion of the second stage, the probability that it is finished by the follower is $[(1-F_1^*)F_2^*]/E^*$ when the basic invention is not patentable, and $[(1-\hat{F}_1)\hat{F}_2]/\hat{E}$ when it is patentable with $\theta > 0$. Compare the two levels,

$$\frac{(1-\hat{F}_1)\hat{F}_2}{\hat{E}} > \frac{(1-F_1^*)F_2^*}{E^*} \quad \Leftrightarrow \quad F_1^*(1-\hat{F}_1)\hat{F}_2 > \hat{F}_1(1-F_1^*)F_2^*. \tag{9}$$

Since $F_1^* > \hat{F}_1$, as long as $\hat{F}_2$ is not too small relative to $F_2^*$, patenting the basic invention helps the decentralization of innovation activities.

The following proposition summarizes the results up to this point. An example of two-point search technology follows as an illustration.

*Proposition* 1. *(The Shadow effect) When $\alpha = 1$ or under the DPSM, the search equilibrium at the second innovation stage is unique. But when $\theta \in (0, \overline{\theta}]$ and $\alpha \in (0,1)$, the shadow effect may lead to multiple search equilibria.*

*The pioneer has a higher search incentive under the DPSM than under other policy $\theta \in (0, \overline{\theta}]$, $c_1^* > \hat{c}_1$. Due to the shadow effect, the impact of the patent policy on the follower's search incentive is ambiguous, $c_2^* \gtrless \hat{c}_2$. Consider stable search equilibria. When evaluated at $\theta = 0$, $d\hat{c}_1/d\theta < 0$. For $\theta > 0$, $d\hat{E}/d\theta \gtrless 0$, but not both $d\hat{c}_1/d\theta$ and $d\hat{c}_2/d\theta > 0$.*

*Example* 1. (Two-point search technology). Suppose that both the pioneer's and follower's search cost have two-point distributions, $c_i \in \{C_i, K\}$, with $K > v \geq C_i \geq 0$,

[13] They assume free entry at the first stage. The "pioneer," therefore, refers to the first inventor to finish the race and create the basic invention (or the intermediate technology as they called it).

and the probability of low search cost is $\Pr(c_i = C_i) = p_i \in (0, 1)$, $i = 1, 2$. An inventor will not incur the high search cost $K > v$. In any search equilibrium, the pioneer's (follower's) search probability is at most $\hat{F}_1 = p_1$ ($\hat{F}_2 = p_2$, respectively).

Fixing $\theta > 0$, I first show that both $(\hat{F}_1, \hat{F}_2) = (0, p_2)$ and $(p_1, 0)$ can be search equilibria. To have $(0, p_2)$ as the equilibrium, the pioneer must find it too costly to incur $C_1$, given that the following inventor will incur $C_2$. We need $C_1 > v(1 - p_2\theta)$. And for the follower to be willing to incur $C_2$, given that the pioneering inventor does not search at all, we need $C_2 \leq v(1 - \theta)$. In this search equilibrium, the follower's belief maintains at the *ex ante* level. For $(p_1, 0)$ to be the search equilibrium, the pioneer incurs $C_1$ but the follower will not search. We need $C_1 \leq v$ and $C_2 > \hat{\alpha}\pi(1 - \theta)$, where $\hat{\alpha} = \alpha(1 - p_1)/(1 - \alpha p_1) < \alpha$. We have multiple equilibria when

$$v(1 - p_2\theta) < C_1 \leq v \text{ and } \hat{\alpha}\pi(1 - \theta) < C_2 \leq v(1 - \theta). \tag{10}$$

An implication of multiple equilibria is mis-allocation of search activity. Even though the overall search performance is the same, different equilibria may entail different levels of total search cost. To see this, suppose $p_1 = p_2 = p \in (0, 1)$ and condition (10) holds. Both search equilibria have the same probability to find the application (given existence), $\hat{E} = p$, but different search costs depending on which inventor searches. When $C_1 > C_2$, then the equilibrium where only the follower searches is more cost-efficient. In fact, if $p_2 > p_1$, then this equilibrium also has a higher probability to find the application.

Lastly, suppose that $p_2 > p_1$. Under the DPSM ($\theta = 0$), the search equilibrium is unique, $(p_1, 0)$, with $E^* = p_1$. But if we let the basic invention be patentable with $\theta > 0$ such that condition (10) holds, then in the search equilibrium $(0, p_2)$, we have $\hat{E} = p_2 > E^*$. Patenting the basic invention boosts the second stage innovation performance when the "good" search equilibrium prevails. It also causes a more disintegrated innovation market. Under the DPSM, $(1 - F_1^*)F_2^*/E^* = 0$, but for $\theta > 0$ such that condition (10), $(1 - \hat{F}_1)\hat{F}_2/\hat{E} = 1$. ‖

*Remark* 1. (Impact of $\alpha$) Fixing the expected value $v$, the level of the parameter $\alpha$ captures how "abstract" the basic invention is, or how far it is from commercial applications. A lower $\alpha$ means that it is more difficult to find or develop the application, although the expected value is not affected. In the proof of Proposition 1, I show that a higher $\alpha$ does not necessarily raise the overall second stage performance. Given the pioneer's search strategy, a higher $\alpha$ will raise the follower's belief $\hat{\alpha}$ and increase his incentive to search, $d\hat{c}_2/d\alpha > 0$. When $\theta > 0$, this boost in the follower's search intensity provides a negative feedback to the pioneer's search decision, $d\hat{c}_1/d\alpha < 0$, for she can free ride on the follower's search result. The overall impact on the second stage performance $\hat{E}$ is ambiguous, and may be negative when the pioneer has better search capacity than the follower. For instance, when $c_1$ and $c_2$ have uniform distributions over $[0, \gamma_1 v]$ and $[0, \gamma_2 v]$, respectively, with $\gamma_1$ and $\gamma_2 > 1$, then $d\hat{E}/d\alpha < 0$ for $\gamma_1 < 1 + \theta$ and $\gamma_2$ large enough. ‖

# 4 When to Impose the DPSM?

Now I turn to the question of when the DPSM is an optimal policy to encourage innovation. To do so, let us consider the impact of patent policy $\theta$ on the first stage innovation incentive. Expecting a payoff $U_1$ from the search subgame, the pioneer will incur a cost $c_0$ to create the basic invention as long as $c_0 \leq U_1$. The basic invention will be produced with a probability $F_0(U_1)$, and a higher $U_1$ raises the pioneer's incentive to engage in basic research. The pioneer's expected payoff at the first stage is

$$\hat{U}_0 = \int_0^{U_1} (U_1 - c_0) dF_0. \tag{11}$$

Denote $\hat{U}_1$ and $U_1^*$ as the pioneer's payoffs in the search equilibrium when $\theta \in (0, \overline{\theta}]$ and $\theta = 0$, respectively. By previous discussion, $c_1^* > \hat{c}_1$ and $\hat{c}_2 > 0$. Together with the definition of $\hat{c}_1$,

$$\begin{aligned} U_1^* = & \int_0^{c_1^*} (v - c_1) dF_1 = \int_0^{\hat{c}_1} (v - c_1) dF_1 + \int_{\hat{c}_1}^{c_1^*} (v - c_1) dF_1 \\ < & \int_0^{\hat{c}_1} (v - c_1) dF_1 + \int_{\hat{c}_1}^{c_1^*} \hat{F}_2 v \theta dF_1 < \hat{U}_1 = \int_0^{\hat{c}_1} (v - c_1) dF_1 + (1 - \hat{F}_1) \hat{F}_2 v \theta. \end{aligned} \tag{12}$$

As in the standard cumulative innovation literature, denying patent protection to the basic invention reduces the pioneer's first stage incentive. The DPSM imposes a cost of harming the basic innovation.

*Proposition* 2. *(First stage innovation incentives) Granting patent protection to the basic invention increases the pioneer's incentive to engage in basic research.*

Considering the impact on both innovation stages, when is it optimal to impose the DPSM? Using the overall technology progress rate, $F_0(\hat{U}_1)\alpha\hat{E}$, as the policy criterion, I am interested in situations where $\theta = 0$ is the solution to the program

$\max_\theta \alpha F_0(\hat{U}_1)\hat{E}$. Fixing $\alpha$, it is equivalent to finding conditions such that $F_0(U_1^*)E^* \geq F_0(\hat{U}_1)\hat{E}$ for all $\theta \in (0, \overline{\theta}]$.

According to Proposition 2, the DPSM is detrimental to the first stage innovation incentive. If, at the second stage, $E^* < \hat{E}$ for some $\theta \in (0, \overline{\theta}]$, then the DPSM is dominated at both stages of the innovation process. A necessary condition to reject patent protection to the basic invention therefore is $\hat{E} < E^*$ for all $\theta \in (0, \overline{\theta}]$. This requires that the shadow effect cannot be strong, relative to the sharing effect.

Suppose that this necessary condition holds. Consider the overall impact of the patent policy on the technological progress:

$$
\begin{aligned}
&\frac{dF_0(\hat{U}_1)\hat{E}}{d\theta} = \hat{E} f_0(\hat{U}_1)\frac{d\hat{U}_1}{d\theta} + F_0(\hat{U}_1)\frac{d\hat{E}}{d\theta} \\
=&\hat{E} f_0(1-\hat{F}_1)v\left(\hat{F}_2 + \theta \hat{f}_2 \frac{d\hat{c}_2}{d\theta}\right) + F_0(1-\hat{F}_1)(1-\hat{F}_2)\left(\frac{\hat{f}_1}{1-\hat{F}_1}\frac{d\hat{c}_1}{d\theta} + \frac{\hat{f}_2}{1-\hat{F}_2}\frac{d\hat{c}_2}{d\theta}\right) \qquad (13) \\
=&(1-\hat{F}_1)\left[\hat{E} f_0 v\left(\hat{F}_2 + \theta \hat{f}_2 \frac{d\hat{c}_2}{d\theta}\right) + F_0(1-\hat{F}_2)\left(\frac{\hat{f}_1}{1-\hat{F}_1}\frac{d\hat{c}_1}{d\theta} + \frac{\hat{f}_2}{1-\hat{F}_2}\frac{d\hat{c}_2}{d\theta}\right)\right].
\end{aligned}
$$

The patent policy affects three decisions: besides the pioneer's innovation decision at the first stage and the follower's decision at the second stage, it also affects the pioneer's incentive at the second stage. The cumulative innovation literature, such as Green and Scotchmer (1995), emphasizes the trade-off between different generations of inventors at different innovation stages, but overlooks the *same* inventor's innovation incentives across stages. As shown in Section 3, a patent on the basic invention encourages the first stage innovation ($F_0(\hat{U}_1) > F_0(U_1^*)$) and changes the follower's innovation performance ($d\hat{c}_2/d\theta \gtreqless 0$). This early reward also *discourages* the pioneer from continuing her research activity ($\hat{F}_1 < F_1^*$). When this effect is strong enough, we may find a reason to reject patenting the basic invention.

To better illustrate the key result, consider special cost distributions. Suppose that $c_0$ follows the uniform distribution over the support $[0, \gamma_0 v]$, with $\gamma_0 > 1$, and $c_2$ follows the two-point distribution, $c_2 \in \{0, K\}$, with $K > v$ and $\Pr(c_2 = 0) = p_2 \in (0,1)$. The first simplification brings about an interesting case where the optimal policy $\theta$ is independent of the cost parameter at the first stage, namely, $\gamma_0$. Different from Aoki and Nagaoka (2004), within the class of uniform distributions, I can derive the optimality of the DPSM without referring to the difficulty of obtaining the basic invention. The two-point search technology, $c_2 \in \{0, K\}$, implies that the follower has a fixed probability $\hat{F}_2 = F_2^* = p_2$ to find the application. The second simplification is introduced to point out how the follower's capacity $p_2$ affects the trade-off between the pioneer's innovation incentives at different stages.[14] (For sure, the shadow effect is eliminated by this specification. The necessary condition of $E^* > \hat{E}$ for all $\theta > 0$, however, already imposes a constraint of weak shadow effect.)

Under these specifications, $d\hat{c}_2/d\theta = 0$ and $d\hat{c}_1/d\theta = -vp_2$. Fixing the follower's search capacity, stronger patent protection to the basic invention $\theta$ always raises the pioneer's incentive to engage in basic invention. By integration by parts and the optimal cut-off $\hat{c}_1 = v(1 - \hat{F}_2\theta) = v(1 - p_2\theta)$,

$$\begin{aligned} \hat{U}_1 &= \int_0^{\hat{c}_1} (v - c_1) dF_1 + (1 - \hat{F}_1) p_2 v\theta = (v - c_1) F(c_1)\Big|_0^{\hat{c}_1} + \int_0^{\hat{c}_1} F_1 dc_1 + (1 - \hat{F}_1) p_2 v\theta \\ &= \int_0^{\hat{c}_1} F_1 dc_1 + p_2 v\theta, \end{aligned} \tag{16}$$

and so

$$\frac{d\hat{U}_1}{d\theta} = \hat{F}_1 \frac{d\hat{c}_1}{d\theta} + p_2 v = (1 - \hat{F}_1) p_2 v > 0, \text{ when } \hat{F}_1 < 1. \tag{17}$$

This positive effect, however, comes at a cost of lower second stage innovation performance,

$$\frac{d\hat{E}}{d\theta} = (1 - p_2)\hat{f}_1 \frac{d\hat{c}_1}{d\theta} = -(1 - p_2)\hat{f}_1 p_2 v < 0. \tag{18}$$

The term $p_2 v$ appear in both $d\hat{U}_1/d\theta$ and $d\hat{E}/d\theta$. Raising patent protection to the basic invention directly affects the surplus transfer from the follower to the pioneer. Its impact is proportional to the follower's expected return from search, which is $p_2 v$ in this case. Beyond this common factor, the positive effect on the first stage innovation is also proportional to $1 - \hat{F}_1$, the probability that the pioneer does not search for

[14] Suppose that, instead, the pioneer has a fixed search capacity, $c_1 \in \{0, K\}$ with $\Pr(c_1 = 0) = p_1 \in [0, 1)$. The optimal $\theta$ then is determined according to the classical trade-off between different inventor's incentives at different stages. (When $p_1 = 0$, it corresponds to the standard model where different generations of innovations are conducted by different players.) By $d\hat{c}_1/d\theta = 0$ and so $d\hat{c}_2/d\theta = -v\phi$, where $\phi = (1 - \hat{F}_1)/(1 - \alpha\hat{F}_1)$,

$$sign\left(\frac{dF_0(\hat{U}_1)\hat{E}}{d\theta}\right) = sign\left(\frac{f_0}{F_0}[p_1 + (1 - p_1)\hat{F}_2](\hat{F}_2 - \theta\phi\hat{f}_2) - \hat{f}_2\phi\right). \tag{14}$$

If both $f_0$ and $f_2$ take uniform distributions, the sign of $dF_0(\hat{U}_1)\hat{E}/d\theta$, when evaluated at $\theta = 0$, is the same as

$$\frac{1}{p_1 v}[p_1 + (1 - p_1)F_2^*] - \frac{\phi}{\phi v} = \frac{1}{v}\left(1 + \frac{1 - p_1}{p_1}F_2^* - 1\right) > 0. \tag{15}$$

The DPSM is never optimal.

application. For the pioneer will use the basic patent to get a share of the follower's search surplus only when she does not search. On the other hand, the negative impact on the second stage innovation is also proportional to $1 - p_2$, namely, a lower search effort from the pioneer becomes a more serious problem when the follower is less likely to make the discovery. Combining the two factors, it may be optimal to maintain the DPSM when the pioneer has significant search capacity (and so $\hat{F}_1$ is high for all $\theta \in [0, \overline{\theta}]$), but not the follower (and so $p_2$ is small).

Under the specified distributions of $c_0$ and $c_2$, the sign of the first-order condition, $dF_0(\hat{U}_1)\hat{E}/d\theta$, is the same as

$$\hat{E}(1 - \hat{F}_1) - \hat{U}_1(1 - p_2)\hat{f}_1. \tag{19}$$

The first term, $\hat{E}(1 - \hat{F}_1)$, captures the incentive effect of an increase in $\theta$ on the pioneer's willingness to engage in the first stage research, $\hat{U}_1$. The second term, $\hat{U}_1(1 - p_2)\hat{f}_1$, is associated with the effect of $\theta$ on the pioneer's second stage decision $\hat{c}_1$. If the expression (19) is negative for all $\theta \in [0, \overline{\theta}]$, then the DPSM is the optimal policy.

To illustrate how the optimality of the DPSM is determined by the pioneer's search capacity, let's assume that $c_1$ also has uniform distribution over the support $[0, \gamma_1 v]$, with $\gamma_1 > 1$. When $\gamma_1$ is smaller, the support of $c_1$ shrinks, the pioneer is more likely to have smaller search cost. Fixing $\hat{c}_1$, the pioneer is more likely to make the discovery, $\hat{F}_1$ is decreasing in $\gamma_1$. This parameter also affects the density function $f_1(c_1) = 1/\gamma_1 v$. A reduction in $\gamma_1$ increases $f_1$, provided that $c_1 \in [0, \gamma_1 v]$, which in turn magnifies the negative impact of $\theta$ on $\hat{F}_1$ and thus $\hat{E}$. A lower $\gamma_1$, then, implies a better search capacity by the pioneer, and so the more likely to find the DPSM optimal.

After some calculation,

$$\hat{U}_1 = \frac{v}{2\gamma_1}\left[1 + 2(\gamma_1 - 1)\theta p_2 + (\theta p_2)^2\right]. \tag{20}$$

The sign of the first-order condition is the same as

$$\begin{aligned} 2(\gamma_1 - 1)[(\gamma_1 - 1)p_2 + 1] - (1 - p_2) - 3(1 - p_2)(p_2\theta)^2 \\ +2p_2\theta[(\gamma_1 - 1)p_2 + 1 - 2(\gamma_1 - 1)(1 - p_2)]. \end{aligned} \tag{21}$$

When $\gamma_1 \to 1^+$ and $p_2 \to 0$, the first-order condition is strictly negative for all $\theta \in [0, \overline{\theta}]$. The DPSM is the optimal policy. The following proposition slightly generalizes the result to the case where $c_2$ also has a uniform distribution, $f_2 = 1/(\gamma_2 v)$, with $\gamma_2 >$

1. In this case, $p_2$ is no more fixed, but the DPSM is optimal when $\gamma_2$ is large enough and $\gamma_1$ small enough. As the pioneer's search capacity expands and the follower's capacity shrinks, denying patent protection to the basic invention is more likely to be optimal.

*Proposition* 3. *To promote the technology progress, a necessary condition for the DPSM to be the optimal policy is that it encourages the overall efforts to search the application, $E^* > \hat{E}$, for all $\theta > 0$.*

*Suppose that $c_i$ following uniform distribution, $f_i = 1/(\gamma_i v)$, with $\gamma_i > 1$, $i \in \{0, 1, 2\}$. The DPSM is an optimal policy when $\gamma_1$ is small enough and $\gamma_2$ is large enough.*

In light of this result, the DPSM should be applied, and the basic invention should not be patentable when the pioneer has superior technology at the subsequent research stage, but not the follower. But where does this persistence of innovation dominance come from? One source of this advantage would be some knowledge the pioneer acquired during the first stage. The follower cannot benefit from this knowledge either because of its tacit nature and so the intrinsic difficulty to transfer among different inventors, or the pioneer's unwillingness to disclose and help the follower to understand this knowledge. The former in turn may relate to the landscape of the research environment, for instance, how easily it would be for a late-comer to digest the knowledge required to effectively participation in the innovation process. To the extent that, at its nascent phase, the background information of a field may not be widely distributed, but rather concentrated on very few key players, there may not be many capable followers who can readily pursue the pioneer's research line. The insight of Proposition 3 suggests that patents shouldn't be granted to basic inventions in order to maintain the pioneer's continuation effort. The latter, on the other hand, may depend on the disclosure requirement of the patent law. That is, when weak disclosure or enablement requirements significantly hampers other parties' ability to exploit the patented technology, the patent should not be granted. Although it is a common argument that patent system should be designed to diffuse technology, the reasoning here is based on a somewhat reason, namely, the pioneer's incentive to continue doing research.

Another factor that would affect the pioneer's and follower's chance to develop the application are their commercialization capacity. Although the model is developed as a two-stage innovation process, the second stage can be equivalently interpreted as one that involves not research, but commercialization activity. A party is more likely to successfully commercialize the basic invention if, say, she controls more key physical assets used to develop useful and marketable application. The assumption that the second stage result is patentable then corresponds to the protection to (tangible) property rights. And the condition identified in Proposition 3 implies that the patentability of basic invention hinges on the degree of vertical integration. It should not be patentable when the upstream pioneer extends her dominance to the downstream stage of commercialization.

*Remark* 2. (Alternative objective). This remark considers another objective function, namely, the joint surplus between the two inventors. It turns out that setting $\theta = 0$ will not maximize the joint surplus. In the search of the optimality of the DPSM, this justifies the use of technology progress as the policy objective.

Given the policy $\theta$ and the payoffs from the search subgame, $\hat{U}_1$ and $\hat{U}_2$, the joint surplus is

$$S = \int_0^{\hat{U}_1} (\hat{U}_1 - c_0) dF_0 + F_0(\hat{U}_1)(1 - \alpha \hat{F}_1)\hat{U}_2. \tag{22}$$

Since the basic invention has no stand-alone value, when the pioneer is willing to incur $c_0$, she expects a payoff $\hat{U}_1$ from the subsequent subgame. And the follower gets a payoff $\hat{U}_2$ only when the basic invention is created and the pioneer does not come up with the application. In the proof of Proposition 4, I show that, when evaluating at $\theta = 0$, a marginal increase in $\theta$ always raises the joint surplus $S$. This result does not need further restrictions on the distributions of innovation costs. Intuitively, raising $\theta$ beyond zero only exerts a negative impact on the follower's payoff $\hat{U}_2$. This negative effect, however, is canceled by a positive impact on the pioneering inventor's payoff $\hat{U}_1$. Therefore, the DPSM cannot be justified with the inventor's joint surplus as the policy objective.

*Proposition* 4. *Imposing the DPSM, i.e., setting $\theta = 0$, does not maximize the joint surplus of the two inventors.* ||

# 5 Extensions and Discussion (to be completed)

☐ **Licensing:** When the pioneer holds a patent on the basic invention, the two parties may negotiate a license between time 2.5 and 3.[15] Licensing bargaining takes place

[15]To the extent that $\pi$ reflects the maximal revenue from holding a patent on the application, there is no benefit to license this patent.

around two issues.

First, by limited liability, a license only contains a revenue-sharing rule (the royalty term) between the two parties, namely, the portion of $\pi$ transferred from the follower to the pioneer. The patent protection $\theta$ may be too strong. For instance, in the extreme case of $\theta = 1$, the follower has no incentive to search; it may be mutually beneficial to a lower royalty rate. This concern justifies the upper bound of patent protection, $\overline{\theta} < 1$, as the range of protection that would matter along the equilibrium path, after taking into account licensing.

Second, the follower is interested in the pioneer's private information, i.e., when she has incurred the search cost at the second stage (time 2). By learning this information, the follower can save on the search cost $c_2$ if the pioneer has searched yet failed, and in the case where the pioneer didn't search, the follower can raise his belief $\hat{\alpha}$ to the *ex ante* level $\alpha$.

Consider the pioneer's incentive to disclose her information.[16] Suppose that the pioneer does not incur the search cost at time 2. As long as she can get a stake from the follower's search result, e.g., when $\theta > 0$, the pioneer has a strong incentive to transmit this information to the follower in order to raise his belief and the search effort.[17] For the pioneer who has spent the search cost and learned that the application does not exist, she knows that the follower's search is doomed to fail and so loses the interests in the stake from the follower's innovation activity. The pioneer is indifferent to making (or accepting) an offer or not. By breaking this indifference in different ways, the follower may or may not learn the pioneer's private information.

The indifference, however, is not robust to some modifications of the model, e.g., if the pioneer may make mistakes in search.[18] Suppose that with some probability

[16]When the true level of $c_1$ is the pioneer's private information and whether she has spent this cost is non-verifiable, it is unclear which patent policy tool could be used to encourage the pioneer to disclosure this information. Since patents are public records, whenever the identity of following inventors are unknown *ex ante*, it may be difficult to enforce patent rights that are granted to knowledge that is used to *prevent* some activities from happening. A monetary reward might be useful, though. That is, the pioneering inventor brings the hard evidence of spending $c_1$ and receives a prize related to the follower's expected saving. But when the follower cannot be traced down to finance the monetary reward, we go beyond the scope of the patent system and public funds become necessary. I do not consider how the patent system should be designed to directly tackle this issue.

[17]When $\theta = 0$, whether the pioneer is willing to reveal this information depends on the contracting environment. For instance, if the pioneer makes the offer, then after learning the pioneer's information via license offering, the follower can simply turn down the offer and run away with the pioneer's private information. The Arrow problem applies here. But this strategy does not work when the follower makes an enforceable offer to the pioneer.

[18]An alternative way is to relax the limited liability constraint and so the follower can purchase informa-

$\varepsilon > 0$ the pioneer fails to find the application even when it exists and the search cost is spent. In this case, the pioneer's failure is still a bad news, but not as desperate as before. The qualitative feature of shadow effect is unaffected, and the pioneer will retain some interests in the follower's search activity. The updated belief about the existence of the application, after the pioneer's search failure, is

$$\alpha^{\varepsilon} = \frac{\varepsilon\alpha}{1 - \alpha + \varepsilon\alpha}. \tag{23}$$

For any $\varepsilon \in (0, 1)$, $0 < \alpha^{\varepsilon} < \alpha$. Denote the pioneer's belief, or "type" as $\alpha^P \in \{\alpha, \alpha^{\varepsilon}\}$. When the pioneer gets a share $l$ from the follower's successful search, her expected payoff is

$$\alpha^P F_2(\tilde{\alpha}^l (1 - l)\pi) l\pi, \tag{24}$$

where $\alpha^P \in \{\alpha, \alpha^{\varepsilon}\}$, and the follower forms his belief at search, $\tilde{\alpha}^l$, according to the contract term $l$ (offered by the pioneer in the signaling model, or accepted by the pioneer in the screening model). Note that, apart from the first term, the pioneer's own belief $\alpha^P$ affects her expected payoff only through its impact on the follower's belief $\tilde{\alpha}^l$ via the contract term $l$. There, at the bargaining stage, the pioneer acts to maximize $F_2(\tilde{\alpha}^l(1 - l)\pi)l\pi$, regardless of her type. The pioneer's behavior is not affected by her private information. It is then natural to select an equilibrium where both types of pioneer take the same action and so the follower learns no new information, i.e., a pooling equilibrium when the pioneer makes the offer, or no separation (bunching) when the follower makes the offer.[19] The shadow effect, and thus previous analysis, is retained when we restrict our attention to these equilibria.

☐ **Research grants and academic kudos:** Basic research is often funded by research grants, and reputation or recognition from the scientific community ("kudos," Gans *et al.* (2010)) may provide strong incentive for academic researchers.[20] The main

tion with cash, or if the follower's saving on the search cost $c_2$ is transferrable to the pioneer. The pioneer may be able to "sell" her negative information in exchange for some rent from the follower. The question, then, is whether this broader contracting space could help information transmission.

[19]When the pioneer makes the offer, by carefully structuring the follower's off-path beliefs we may have separating equilibria. However, in any such equilibrium both types of pioneer must be indifferent to the two equilibrium offers.

[20]Scientists may prefer to tackle more challenging tasks (Sauermann and Cohen, 2007, Owan and Nagaoka, 2008). This higher satisfaction from solving more difficult puzzles can be introduced by an intrinsic (psychological) reward accrued to the follower that is decreasing in the updated belief $\hat{\alpha}$. This intrinsic motivation will then offset the shadow effect. But as long as its size is not too large, previous analysis is not affected.

advantage of these alternative rewards, it is often argued, is to avoid monopolization of fundamental knowledge. By the shadow effect, however, monopoly rights over basic innovation do not necessarily hinder subsequent innovation. It follows that these non-patent mechanisms may not substitute the patent system, even if we ignore their drawbacks (e.g., the shadow cost of raising public funds to finance research grants).

Formally, let the pioneer receive a reward $B > 0$ after completing the first stage innovation. This reward does not affect the second stage decisions. First, suppose that $B$ is part of the policy instruments at the disposal of the patent authority (such as the Congress). When the innovation policy consists of a bundle $(B, \theta)$, the policy maker can use different instrument to address different innovation stages. The patent policy $\theta$ will be chosen to induce the second stage innovation, and the non-patent policy $B$ chosen to boost first stage innovation. It is still optimal to grant basic patent when the shadow effect is sufficiently strong such that $\hat{E} > E^*$ for some $\theta > 0$. The presence of the other instrument $B$ does not necessarily vindicate the DPSM.

Suppose that $B$ is not controlled by the patent authority. This may be the case of scientific kudos, or when research grants are provided by another independent agency (or even private organization as in the case of scientific prizes). The policy maker then chooses the patent policy $\theta$ taking as given the "extra" boost $B > 0$ at the first stage. The pioneer will engage in the search of basic invention (abstract idea) when $B + \hat{U}_1 \geq c_0$, where $\hat{U}_1$ is the same as before. Again, if the basic patent is beneficial to the second stage innovation (i.e., $\hat{E} > E^*$ for some $\theta > 0$), then rewarding the pioneer with the basic patent is the optimal policy.

Suppose that the basic patent is detrimental to the second stage (i.e., $\hat{E} < E^*$ for all $\theta > 0$). It may be interesting to consider how the optimal policy $\theta$ is affected by the introduction of $B$. When $B = 0$, it is optimal to reject basic patent instead of rewarding the pioneer with some $\theta > 0$ if

$$\frac{F_0(U_1^*)}{F_0(\hat{U}_1)} > \frac{\hat{E}}{E^*}, \tag{25}$$

where $\hat{U}_1$ and $\hat{E}$ are computed at the corresponding $\theta > 0$. When $B > 0$, the criterion involves comparing $F_0(B + U_1^*)/F_0(B + \hat{U}_1)$ with $\hat{E}/E^*$. How $B$ affects the optimal patent policy then depends on the shape of the distribution $F_0$. For $x$, $y$, and $\triangle > 0$ (over the relevant range) such that $x < y$, if $F_0(x + \triangle)/F_0(y + \triangle)$ is increasing (decreasing) in $\triangle$, then the introduction of non-patent reward $B > 0$ will move the optimal policy toward the DPSM (toward $\theta > 0$, respectively). For uniform distributions,

$F_0(x+\triangle)/F_0(y+\triangle) = (x+\triangle)/(y+\triangle)$, increasing in $\triangle$. Therefore, the non-patent reward crowds out basic patents.

*Proposition* 5. *(Non-patent reward to basic invention) When the basic patent enhances the performance of the second stage innovation, the introduction of non-patent rewards B does not render the DPSM an optimal policy.*

□ **Empirical findings and a short remark on multiple applications:** Here I briefly discuss two recent empirical studies of the impact of intellectual property rights on innovation. Considering human genome sequencing as the basic invention, Williams (2010) finds that those genes sequenced by a private company, Celera, were covered for some period under the company's (copyrights-based) intellectual property, and followed by fewer subsequent research than those sequenced by the public initiative, the Human Genome Project. In this particular case, the negative impact of IPRs on subsequent innovation might be attributed to the quick disclosure requirement imposed by the Human Genome Project. The Human Genome Project was mostly executed by a small number of research centers. The so-called "Bermuda rules" required gene sequence information processed under this project be submitted to the public online databas GenBank within 24 hours of sequencing.[21] Such a short time might prevent those research centers to have a "first-mover" advantage in subsequent research; the shadow effect might be less relevant in this context.

Murray *et al.* (2009) shows that more research paths were explored after the relaxation of (patent-backed) restrictions on the use and distribution of genetically engineered mice (as research tools). Here I incorporate multiple-application into previous analysis. Let the pioneer's research opportunity at the second stage be the same as before, and can only search for one application ($a$). The follower, however, can decide to search for either application $a$ or application $b$ (but not both) at cost $c_2$.[22] The *ex ante* belief that application $b$ exists is $\beta$, and the value is $\pi$, the same as application $a$. Let's consider the follower's choice between application $a$ and $b$ when the pioneer hasn't discovered at the second stage. If there is no patent protection, then the pioneer will adopt the same search cost threshold as before, $c_1^*$, and the follower's updated belief for application $a$ is $\alpha^*$. The follower will choose application $b$ when $\beta \geq \alpha^*$.

[21] For more details, see Williams (2010).

[22] As emphasized by Murray *et al.* (2009), different researchers may have different ideas about how the basic invention could be used.

Suppose that the pioneer receives a patent on the basic invention, with protection level $\theta \in (0,1)$ on both applications. If the follower decides to search for application $b$, then the pioneer's search threshold is still $c_1^*$; and it is indeed optimal for the follower to search for $b$ when $\beta(1-\theta) \geq \alpha^*(1-\theta)$, or when $\beta \geq \alpha^*$. But if the follower decides to search for application $a$, then by previous analysis the pioneer will search less often. Her search threshold reduces to $\hat{c}_1 < c_1^*$, and the follower's updated belief about application restores to $\hat{\alpha} > \alpha^*$. For $\hat{\alpha} > \beta$, the follower will optimally choose application $a$ (and adopt search threshold $\hat{c}_2$ such that $\hat{c}_1$ and $\hat{c}_2$ are compatible). Therefore, when $\alpha^* \leq \beta < \hat{\alpha}$, there are multiple equilibria after the basic invention becomes patentable. In one equilibrium, we will observe fewer applications being explored at the second stage.

*Proposition* 6. *Patent protection to the basic invention may reduce the exploration of alternative research directions at the second stage.*

□ **Endogenous order to search:** In the basic model I let the pioneer search first. An implicit assumption is that the pioneer can protect the basic invention under secrecy until her search fails, or until she decides not to search. This assumption captures some first-mover advantage and, more importantly, avoids the extreme situation where the pioneer is "forced" to disclose the basic invention even if it is *not* patentable. Here I consider whether the pioneer will exploit this advantage, or instead will want to wait until after the follower's search.

I keep the assumption that a player cannot observe the other's true search cost nor the decision to incur the cost, and that the pioneer still learns the trust cost at time 2, but add an additional stage, time 4, where the pioneering inventor can spend her cost $c_1$ to search, if she hasn't done that at time 2. For a policy $\theta \in [0, \overline{\theta}]$, denote $\hat{c}_1$ and $\hat{c}_2$ as the equilibrium cut-offs without time 4. I derive conditions under which this additional timing is irrelevant.

When endowed with this additional timing to search, the pioneer knows that if she delays search, she will need to incur $c_1$ only if the follower doesn't come up with an application. Similar to the reasoning in section 3, the pioneer can update her belief about $\alpha$ at this event. The shadow effect still applies, but with a switch of the role between two players. The pioneer's updated belief at time 4 is $\alpha \cdot \hat{\phi}$, where

$$\hat{\phi} = \frac{1 - F_2(\hat{c}_2)}{1 - \alpha F_2(\hat{c}_2)}. \tag{26}$$

Since the follower holds no claim again the pioneer, the latter will incur search at time 4 as long as $c_1 \leq \alpha\hat{\phi}\pi = \hat{\phi}v$. For $c_1 > \hat{\phi}v$, this additional timing to search is irrelevant.

Suppose that the pioneer has search cost $c_1 \leq \hat{\phi}v$. If she searches at time 2, the expected payoff is $v - c_1$. If she delays to time 4, the expected payoff is

$$F_2(\hat{c}_2)\theta v + [1 - \alpha F_2(\hat{c}_2)](\hat{\phi}v - c_1) = F_2(\hat{c}_2)\theta v + [1 - F_2(\hat{c}_2)]v - [1 - \alpha F_2(\hat{c}_2)]c_1. \quad (27)$$

When $F_2(\hat{c}_2) > 0$,

$$v - c_1 \geq F_2(\hat{c}_2)\theta v + (1 - F_2(\hat{c}_2))v - [1 - \alpha F_2(\hat{c}_2)]c_1 \Leftrightarrow c_1 \leq (1-\theta)\pi. \quad (28)$$

For $c_1$ smaller than $(1-\theta)\pi$, the pioneer will search at time 2 rather than wait.

Compare the pioneer's different thresholds. If

$$1 - \theta \geq \alpha\hat{\phi} = \alpha\frac{1 - F_2(\hat{c}_2)}{1 - \alpha F_2(\hat{c}_2)} \Rightarrow \theta \leq \frac{1-\alpha}{1 - \alpha F_2(\hat{c}_2)}, \quad (29)$$

then time 4 is irrelevant. The pioneer has any incentive to search at time 4 only for $c_1 \leq \alpha\hat{\phi}\pi$. But by $1 - \theta \geq \alpha\hat{\phi}$, and so $c_1 \leq (1-\theta)\pi$, for this range of search cost the pioneer prefers searching at time 2 than time 4.[23] When $\alpha$ and $\theta$ are not too large, such that condition (29) holds, previous results are robust to the pioneer's endogenous search timing.

If condition (29) fails, then the search equilibrium is not robust to the pioneer's additional search opportunity. The pioneer will want to delay search for $c_1 \in ((1-\theta)\pi, \hat{\phi}v]$, and only spend $c_1 \leq (1-\theta)\pi$ at time 2. The search equilibrium is characterized by three cut-offs: $c_1' = (1-\theta)\pi$, $c_2' = \hat{\alpha}(c_1')\pi(1-\theta)$, and $c_1'' = \hat{\phi}(c_2')v$, where

$$\hat{\alpha}(c_1') = \frac{\alpha[1 - F_1(c_1')]}{1 - \alpha F_1(c_1')} \text{ and } \hat{\phi}(c_2') = \frac{1 - F_2(c_2')}{1 - \alpha F_2(c_2')}. \quad (30)$$

That is, the pioneer adopts the cut-off $c_1'$ at time 2, and cut-off $c_1''$ at time 4, and the follower adopts cut-off $c_2'$. The search equilibrium is unique, but the patent policy has similar impact as before. An increase in $\theta$ will decrease $c_1'$, and has a direct negative impact on $c_2'$. But a lower $c_1'$ exerts a positive shadow effect on $c_2'$, via the follower's belief $\hat{\alpha}$. The net change in $c_2'$, then, has another shadow effect on the pioneer's second cut-off $c_1''$, via $\hat{\phi}$. The overall impact on the search performance, again, is ambiguous.

[23]The same condition also guarantees $(1-\theta)\pi \geq \hat{c}_1 = \alpha\pi[1 - \theta F_2(\hat{c}_2)]$, the cut-off obtained in section 3. That is, the additional time 4 expands the range of search cost the pioneer is willing to spend at time 2. Time 4, again, is irrelevant.

# 6 Conclusion

In this paper, I re-examine the impact of basic patents on the cumulative innovation process in the presence of the shadow effect, namely, the dimer prospect of future research when hard-working inventors leave unsolved questions. Due to the shadow effect, the basic patent more friendly to subsequent innovation and may encourage disintegration of the innovation market. To justify the DPSM, i.e., to reject patent protection to the basic invention, therefore, requires a weak shadow effect. When this is true, then the DPSM may be the optimal policy to induce pioneering inventors' continuing efforts.

For future research, it would be interesting to check the empirical validation of the predictions derived under the shadow effect, in particular that concerning the concentration level of the innovation market. The empirical support of the shadow effect, when it can be found, would prompt us to rethink the more subtle role (basic) patents play in innovation, as demonstrated in this paper.

Concerning patent policy, the sufficient condition of the optimality of the DPSM is derived under specific cost distributions. It would be important to test its robustness in more general settings. In addition, research capacity (cost distribution here) may be too abstract or too difficult to estimate and thus may not be suitable as the basis of policy recommendation. Future work should also develop other fundamental elements that are easier for policy-makers to apply and reduce the theory to practice.

Beside the implementation issue, a few other avenues for future research come to mind: multiple pioneers at the first stage innovation as in Denicolò (2000) and Aoki and Nagaoka (2004); secrecy protection to the basic invention; and the combination of the DPSM with other policy instruments, such as patent length and protection to second stage inventions, to name a few. A better understanding of the doctrine of the patentable subject matter would advance our knowledge on the optimal design of the patent system. This paper constitutes an early step.

# Appendix: Proofs

# A Proofs

☐ **Proposition 1**

*Proof.* For the comparative static results, keep $v \equiv \alpha\pi$ constant and denote $\phi \equiv (1 - \hat{F}_1)/(1 - \alpha\hat{F}_1)$. Differentiate conditions (3) and (5):

$$d\hat{c}_1 + \theta v \hat{f}_2 d\hat{c}_2 = -v\hat{F}_2 d\theta \tag{31}$$

$$-(1-\theta)v\frac{\partial\phi}{\partial\hat{c}_1}d\hat{c}_1 + d\hat{c}_2 = -v\phi d\theta + (1-\theta)v\frac{\partial\phi}{\partial\alpha}d\alpha, \tag{32}$$

where

$$\frac{\partial\phi}{\partial\hat{c}_1} = -\frac{\hat{f}_1(1-\alpha)}{(1-\alpha\hat{F}_1)^2} \leq 0 \text{ and } \frac{\partial\phi}{\partial\alpha} = \frac{\hat{F}_1(1-\hat{F}_1)}{(1-\alpha\hat{F}_1)^2} > 0, \tag{33}$$

with $\hat{f}_i \equiv f(\hat{c}_i)$, $i \in \{1, 2\}$.

On the $c_1 - c_2$ plane, a stable equilibrium $(\hat{c}_1, \hat{c}_2)$ requires that the pioneer's reaction curve have a larger slope (in absolute value) than the follower's reaction curve. That is,

$$\left|\frac{\partial\hat{c}_2}{\partial\hat{c}_1}\right|_{\hat{c}_1} > \left|\frac{\partial\hat{c}_2}{\partial\hat{c}_1}\right|_{\hat{c}_2} \Leftrightarrow \triangle \equiv 1 + \theta(1-\theta)v^2\hat{f}_2\frac{\partial\phi}{\partial\hat{c}_1} = 1 - \theta(1-\theta)v^2\frac{(1-\alpha)\hat{f}_1\hat{f}_2}{(1-\alpha\hat{F}_1)^2} > 0. \tag{34}$$

Suppose that this is true.

By Cramer's rule, the impact of an exogenous change in $\theta$ are

$$\frac{d\hat{c}_1}{d\theta} = \frac{v}{\triangle}(v\theta\phi\hat{f}_2 - \hat{F}_2) \gtrless 0 \text{ and } \frac{d\hat{c}_2}{d\theta} = \frac{v}{\triangle}[-(1-\theta)v\hat{F}_2\frac{\partial\phi}{\partial\hat{c}_1} - \phi] \gtrless 0. \tag{35}$$

When $\theta = 0$, $d\hat{c}_1/d\theta < 0$. When $\theta > 0$, if both terms are strictly positive,[24] then

$$v\theta\phi\hat{f}_2 > \hat{F}_2 > \frac{\phi}{-v(1-\theta)(\partial\phi/\partial\hat{c}_1)}, \tag{36}$$

which contradicts the requirement of $\triangle > 0$.

The overall effect of $\theta$ on $\hat{E}$ is

$$\frac{d\hat{E}}{d\theta} = (1-\hat{F}_2)\hat{f}_1\frac{d\hat{c}_1}{d\theta} + (1-\hat{F}_1)\hat{f}_2\frac{d\hat{c}_2}{d\theta} = (1-\hat{F}_1)(1-\hat{F}_2)\left(\frac{\hat{f}_1}{1-\hat{F}_1}\frac{d\hat{c}_1}{d\theta} + \frac{\hat{f}_2}{1-\hat{F}_2}\frac{d\hat{c}_2}{d\theta}\right). \tag{37}$$

The comparative static results with respect to $\alpha$ are

$$\frac{d\hat{c}_1}{d\alpha} = \frac{-(1-\theta)v}{\triangle}v\theta\hat{f}_2\frac{\partial\phi}{\partial\alpha} < 0 \text{ and } \frac{d\hat{c}_2}{d\alpha} = \frac{(1-\theta)v}{\triangle}\frac{\partial\phi}{\partial\alpha} > 0. \tag{38}$$

The impact on the overall search performance is

$$\begin{aligned}\frac{d\hat{E}}{d\alpha} &= (1-\hat{F}_1)(1-\hat{F}_2)\left(\frac{\hat{f}_1}{1-\hat{F}_1}\frac{d\hat{c}_1}{d\alpha} + \frac{\hat{f}_2}{1-\hat{F}_2}\frac{d\hat{c}_2}{d\alpha}\right) \\ &= (1-\hat{F}_1)(1-\hat{F}_2)\frac{(1-\theta)v}{\triangle}\frac{\partial\phi}{\partial\alpha}\left(\frac{\hat{f}_2}{1-\hat{F}_2} - \frac{\hat{f}_1}{1-\hat{F}_1}\hat{f}_2\theta v\right).\end{aligned} \tag{39}$$

[24]This excludes the case where $\alpha = 1$ and so $\partial\phi/\partial\hat{c}_1 = 0$.

Under the DPSM, $\theta = 0$, $dE^*/d\alpha > 0$ for $\partial\phi/\partial\alpha > 0$. When $\theta > 0$, the sign of $d\hat{E}/d\alpha$ depends on

$$\frac{\hat{f}_2}{1-\hat{F}_2} - \frac{\hat{f}_1}{1-\hat{F}_1}\hat{f}_2\theta v. \tag{40}$$

Suppose that $c_1$ and $c_2$ are distributed uniformly over $[0, \gamma_1 v]$ and $[0, \gamma_2 v]$, respectively. In this case, given that $\hat{c}_1 = v(1-\hat{F}_2\theta)$ and $\hat{c}_2 = v\phi(1-\theta)$,

$$\frac{\hat{f}_2}{1-\hat{F}_2} = \frac{1/(\gamma_2 v)}{1-[\hat{c}_2/(\gamma_2 v)]} = \frac{1}{v[\gamma_2 - \phi(1-\theta)]}, \quad \frac{\hat{f}_1}{1-\hat{F}_1} = \frac{1}{v[\gamma_1 - 1 + \hat{F}_2\theta],} \tag{41}$$

and so

$$\frac{\hat{f}_2}{1-\hat{F}_2} - \frac{\hat{f}_1}{1-\hat{F}_1}\hat{f}_2\theta v = \frac{1}{v}\left[\frac{1}{\gamma_2 - (1-\theta)\phi} - \frac{\theta}{\gamma_2(\gamma_1 - 1 + \hat{F}_2\theta)}\right]. \tag{42}$$

The sign of $d\hat{E}/d\alpha$ at $\theta > 0$ is determined by

$$\begin{aligned}\gamma_2(\gamma_1 - 1 + \hat{F}_2\theta) - \theta[\gamma_2 - (1-\theta)\phi] &= \gamma_2[\gamma_1 - (1+\theta)] + \gamma_2\theta\frac{\phi(1-\theta)v}{\gamma_2 v} + \phi\theta(1-\theta) \\ = \gamma_2[\gamma_1 - (1+\theta)] + 2\phi\theta(1-\theta) &\le \gamma_2[\gamma_1 - (1+\theta)] + \frac{1}{2},\end{aligned} \tag{43}$$

for $\phi \le 1$ and $\theta(1-\theta) \le 1/4$. Therefore, given any $\theta > 0$, $d\hat{E}/d\alpha < 0$ for $\gamma_1 < 1 + \theta$ and $\gamma_2$ large enough. *Q.E.D.*

## □ Proposition 3

*Proof.* When all three cost components have uniform distributions, but different supports, the objective function is $F_0(\hat{U}_1)\alpha\hat{E} = [\alpha/(\gamma_0 v)]\hat{U}_1\hat{E}$. Finding the analytical solutions of $\hat{U}_1$ and $\hat{E}$, the relevant part of the objective function is

$$\hat{U}_1\hat{E} = \frac{v}{2\gamma_1^2}\left[1 + 2\theta\hat{F}_2(\gamma_1 - 1) + \theta^2\hat{F}_2^2\right]\left[1 + (\gamma_1 - 1)\hat{F}_2 - \theta\hat{F}_2(1-\hat{F}_2)\right], \tag{44}$$

where $\hat{F}_2 = \hat{c}_2/(\gamma_2 v) = \phi(1-\theta)/\gamma_2$. Ignoring $v/(2\gamma_1^2)$, the objective function is proportional to

$$\begin{aligned}1 + (\gamma_1 - 1)\hat{F}_2 + \theta\hat{F}_2\Big\{&[1 + (\gamma_1 - 1)\hat{F}_2][2(\gamma_1 - 1) + \theta\hat{F}_2] - \\ &(1-\hat{F}_2)[1 + 2(\gamma_1 - 1)\theta\hat{F}_2] - \theta^2\hat{F}_2^2(1-\hat{F}_2)\Big\}.\end{aligned} \tag{45}$$

The DPSM is optimal if the whole term is decreasing in $\theta$, for all $\theta > 0$. According to the comparative static results in Proposition 1, under uniform distribution,

$$\frac{d\hat{c}_2}{d\theta} = -\frac{v}{\triangle}\left[\phi + (1-\theta)v\hat{F}_2\frac{\partial\phi}{\partial\hat{c}_1}\right] = -\phi\frac{v}{\triangle}\left[1 + \frac{v}{\gamma_2}(1-\theta)^2\frac{\partial\phi}{\partial\hat{c}_1}\right], \tag{46}$$

which is negative when $\gamma_2$ large enough, for $|\partial\phi/\partial\hat{c}_1| \leq \hat{f}_1/(1-\alpha) < \infty$ as long as $\alpha < 1$. (If $\alpha = 1$, then $\phi = 1$, a constant.) By $\hat{f}_1 = 1/(\gamma_1 v)$,

$$1 + \frac{v}{\gamma_2}(1-\theta)^2\frac{\partial\phi}{\partial\theta} \geq 1 - \frac{v}{\gamma_2}\frac{(1-\theta)^2}{(1-\alpha)\gamma_1 v}. \tag{47}$$

When $\gamma_2$ is large enough such that $\gamma_2 > 1/[(1-\alpha)\gamma_1] \geq (1-\theta)^2/[(1-\alpha)\gamma_1]$, an increase in $\theta$ will reduce $\hat{c}_2$ and so $(\gamma_1 - 1)\hat{F}_2$.

Consider the whole term associated with $\theta\hat{F}_2$. It is negative for all $\theta$ as long as both $\gamma_1 - 1$ and $\hat{F}_2$ are small enough. For instance, if $\gamma_1 - 1$ is close to zero, it becomes $\theta\hat{F}_2 - (1-\hat{F}_2) - \theta^2\hat{F}_2^2(1-\hat{F}_2) < 2\hat{F}_2 - 1 - \theta^2\hat{F}_2^2(1-\hat{F}_2)$, by $\theta \leq \overline{\theta} < 1$. When $\gamma_2$ is large enough such that $\hat{F}_2 \leq v/(\gamma_2 v) \leq 1/2$, it is strictly negative. Or, if $\gamma_1 - 1 = 1/4$, then, since $\theta \leq \overline{\theta} < 1$,

$$\begin{aligned} &[1 + (\gamma_1 - 1)\hat{F}_2][2(\gamma_1 - 1) + \theta\hat{F}_2] - (1-\hat{F}_2)[1 + 2(\gamma_1 - 1)\theta\hat{F}_2] \\ &= -\frac{1}{2} + \frac{3\theta}{4}\hat{F}_2^2 + (\frac{9}{8} + \frac{\theta}{2})\hat{F}_2 < -\frac{1}{2} + \frac{3}{4}\hat{F}_2^2 + \frac{13}{8}\hat{F}_2, \end{aligned} \tag{48}$$

which is strictly negative if $\gamma_2$ is large enough such that $\hat{F}_2$ is smaller than, say, 1/8.

*Q.E.D.*

## □ Proposition 4

*Proof.* The impact of the policy $\theta$ on the total surplus is

$$\frac{dS}{d\theta} = F_0(\hat{U}_1)\frac{d\hat{U}_1}{d\theta} + (1-\alpha\hat{F}_1)f_0(\hat{U}_1)\hat{U}_2\frac{d\hat{U}_1}{d\theta} - F_0(\hat{U}_1)\left[\alpha\hat{f}_1\hat{U}_2\frac{d\hat{c}_1}{d\theta} - (1-\alpha\hat{F}_1)\frac{d\hat{U}_2}{d\theta}\right]. \tag{49}$$

By the envelope theorem, the direct effect of $\theta$ on an inventor's choice variable can be ignored:

$$\frac{d\hat{U}_1}{d\theta} = \frac{\partial\hat{U}_1}{\partial\hat{c}_2}\frac{d\hat{c}_2}{d\theta} + \frac{\partial\hat{U}_1}{\partial\theta} = (1-\hat{F}_1)v\left[\hat{F}_2 + \theta\hat{f}_2\frac{d\hat{c}_2}{d\theta}\right], \tag{50}$$

$$\frac{d\hat{U}_2}{d\theta} = \frac{\partial\hat{U}_2}{\partial\hat{c}_1}\frac{d\hat{c}_1}{d\theta} + \frac{\partial\hat{U}_2}{\partial\theta} = \hat{F}_2 v\left[(1-\theta)\frac{\partial\phi}{\partial\hat{c}_1}\frac{\partial\hat{c}_1}{\partial\theta} - \phi\right]. \tag{51}$$

At $\theta = 0$, $d\hat{U}_1/d\theta = (1 - \hat{F}_1)\hat{F}_2 v > 0$. By the comparative static results in the proof of Proposition 1, $d\hat{c}_1/d\theta = -\hat{F}_2 v < 0$. When $\theta = 0$, the only negative term in $dS/d\theta$ is the one associated with $\phi$ in $d\hat{U}_2/d\theta$, i.e.,

$$F_0(\hat{U}_1)(1 - \alpha\hat{F}_1)(-\hat{F}_2 v\phi) = -F_0(\hat{U}_1)(1 - \hat{F}_1)\hat{F}_2 v, \tag{52}$$

which is exactly canceled by the first term in $dS/d\theta$, for

$$F_0(\hat{U}_1)\frac{d\hat{U}_1}{d\theta}|_{\theta=0} = F_0(\hat{U}_1)(1 - \hat{F}_1)\hat{F}_2 v. \tag{53}$$

Therefore, $dS/d\theta > 0$ at $\theta = 0$. *Q.E.D.*

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