Dynamic Interaction between Markets for Leasing and Selling Automobiles

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Abstract. This paper studies dynamic interactions between price variations in leasing and selling markets for automobiles. We develop our model by solving a differential game between multiple Bertrand-type competing firms offering differentiated products to forward-looking agents. This setting nests a parsimonious VAR(1) specification of the dynamics between the two markets. In the empirical application we use monthly US data from 2002 to 2011 on consumer price indices for leased and new cars and trucks, respectively. The results show that variations in monthly selling market prices significantly lead rapidly dissipating changes in the opposite direction in leasing market prices. We discuss the practical implications of our model and estimates by augmenting a standard lease valuation formula with terms representing the leased asset value changes that can be expected on the basis of present variations in automobile selling market prices.

Keywords : Interacting markets; Automobiles; Differential Games; Leasing; Valuation.

JEL Codes : D43, L62, E32, C73

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1. Introduction

For households in developed countries the automobile is usually the second largest asset purchased after a house and the most commonly held non-financial asset (Aizcorbe, Kennickell, and Moore, 2003). It is estimated that in the US one third of all cars sold is financed via leasing (e.g., see Hendel and Lizzeri, 2002; Johnson and Waldman, 2003) while a comparable proportion of sales involves cash transactions (Mannering, Winston, and Starkey, 2002; Dasgupta, Siddarth, and Silva-Risso, 2007). Leasing markets and cash markets (also known as selling markets) for automobiles are closely related and important in size, but their exact association is not yet fully understood. Although some theoretical models exist (see Bulow, 1982, 1986; Bucovetsky and Chilton, 1986; Purohit and Staelin, 1994; Purohit, 1997; Desai and Purohit, 1998, 1999; Saggi and Vettas, 2000; Huang, Yang and Anderson, 2001), they are mostly static in nature and assume perfect substitutability. Moreover, to the best of our knowledge, no attempt has been made to investigate empirically the relationship between leasing and selling markets for automobiles. The objective of the present paper is to shed light on this relationship at both a theoretical and empirical level under more generic assumptions which permit for dynamic interactions and imperfect substitutability. Our results motivate us to develop a new dynamic leasing asset pricing approach for automobiles whereby shocks in selling market prices are allowed to have a dissipative effect on leasing market prices and residual values.

In the next section we review the relevant literature. Section 3 lays out our model for describing the interaction between price changes for automobiles in leasing and selling markets. Section 4, estimates empirically a simple version of this model using monthly US CPI data and discusses the implications of the results for leasing valuation. The final section concludes the paper.

2. Literature Review: The Relationship Between Leasing and Selling Markets

The earliest attempts in understanding the association between leasing and selling markets originate in the investigation of decisions made by agents in the markets for durable goods under the so-called durable goods monopoly problem (see Coase, 1972; Stokey, 1981; Bulow, 1982, 1986; Gul, Sonneschein, and Wilson, 1986; Bucovetsky and Chilton, 1986; Purohit and Staelin, 1994; Purohit, 1997; Desai and Purohit, 1998, 1999; Saggi and Vettas, 2000). Most of these papers assume that leasing and selling are perfect substitutes with market participants that are indifferent between these two alternatives. Moreover, the focus of these studies is to investigate the conditions under which leasing is the optimal strategy in the context of different market structures. A related strand of literature examines the relationship between the markets for new and used automobiles. From a static perspective, Bresnahan (1981), Berry, Levinsohn, and Pakes (1995), Goldberg (1995) and Petrin (2002) gauge the market power of introducing new products in the automobile industry. However, as argued by Blanchard and Melino (1986) it is important to employ a dynamic approach for at least two reasons discussed below. First, dynamics may arise in durable goods models of two interacting markets where used cars constitute stock variables which are imperfect substitutes to new cars. For example, Berkovec (1985) uses the econometric estimates of a short-run model to forecast sales and other automobile industry variables. Rust (1985, 1986) concentrates on dynamic consumer demand in durable goods with new, used and scrappage markets for automobiles. Transaction costs in a dynamic setting are considered by Konishi and Sandfort (2002), Stolyarov (2002) and Schiraldi (2011). Esteban and Shum (2007) model the production decision of a firm in a discrete dynamic oligopoly setting in which automobile prices are endogenously determined. Adda and Cooper (2000a) build a dynamic stochastic discrete choice model of car ownership at the individual level in order to study the output and public finance effects of subsidies on automobile demand. Eberly (1994) and Attanasio (2000) study (*S, s*) models of household automobile demand with transaction costs and liquidity constraints. Second, forward-looking dynamics may arise also in the demand side of the durable goods market on the basis of consumer expectations of future prices for new cars. In this case consumers are not myopic for the future since they consider their expected utility while making their primary decisions on if and when to buy. Chen, Esteban, and Shum (2008, 2010) construct a calibrated equilibrium time consistent dynamic oligopoly model of a durable goods market, which comprises both sources of dynamics mentioned previously. In particular, Chen, Esteban, and Shum (2008) ignore the dynamics by evaluating the bias in estimating the structural parameters of a static model. Chen, Esteban, and Shum (2010) incorporate transaction costs in the used market which make purchases important on the demand side.

A prominent issue in the durable goods markets is the possibility of oscillatory behavior. Sobel (1991) and Conlisk, Gerstner, and Sobel, (1984) consider a new group of consumers, with a heterogeneity of tastes, which enters the market sequentially and leads the monopolist to fluctuate the equilibrium price periodically (Sobel, 1984, studies the same problem in an oligopoly setting). Board (2008) considers the pricing behavior of a durable goods monopolist for a new good where agents can strategically time their purchases and where the demand fluctuates exogenously over time. Janssen and Karamychev (2002) allow for information asymmetry in a dynamic competitive model of identical generations entering the market over time. Caplin and Leahy (2006) develop an (S, s) model of oscillations in demand which reflects fluctuations in the number of consumers who purchase the durable goods as well as of variations in the demand of a single consumer. They use this model to analyze the equilibrium dynamics of prices, the number of purchases and the size of purchases of the durable goods. Empirical evidence by Bils and Klenow (1998) confirms that durable goods prices have a tendency to move procyclically relative to prices of nondurable goods. Blanchard and Melino (1986) construct a competitive equilibrium model with representative consumers and firms. Their intention is to understand the common cyclical behavior of prices and quantities in a certain market for automobiles. Finally, Adda and Cooper (2000b) concentrate on the demand side and estimate a VAR(1) model of aggregate income, relative prices of cars and consumer preference shocks. They report that the impulse response function exhibits dampened oscillations in response to an income shock. This is explained on the basis of two reasons. First, due to non-convex adjustment costs with heterogeneous consumers, the endogenous growth of the stock of cars can generate replacement cycles and subsequent oscillations in sales. Second, the oscillations can arise from the serial correlation in income and prices.

3. Model Formulation

In this section we build a framework for modeling in a dynamic manner the interaction between leasing and selling market prices for automobiles. Such a dynamic setting has been studied previously only at a theoretical level by Huang, Yang, and Anderson, (2001). They construct a dynamic monopoly model of leasing, selling and used goods markets, respectively, with finite duration under an infinite time horizon and nontrivial transaction costs. Although our approach does not consider transaction costs and used goods, we assume a more realistic oligopoly setting. Our approach is closely related to that of Esteban and Shum (2007) although they concentrate on modeling the interaction between new and used car markets. We also differ from Esteban and Shum (2007) in the focus of our models. Specifically, they assume a discrete time approach and analyze the stage in which firms determine the new car designs. Our continuous time model deals with the stage in which producers set prices conditional on product types. This distinction makes the model of Esteban and Shum (2007) backward looking, since the production choices of the firm today depend on cars produced in the past. In our setting firms are forward looking since their price choices depend on the future.

Another important characteristic of our approach is that unlike most of the previous literature it does not treat selling and leasing of automobiles as perfect substitutes. As pointed out by Dasgupta, Siddarth, and Silva-Risso (2007), the assumption of substitutability is unrealistic for at least three reasons. First, automobile leasing contracts differ significantly from selling contracts in that the former typically comprise of several terms and conditions, such as the price, the interest rate, the installment and the maturity of the contract. Second, differences in the discount factor used by consumers can also lead to differences in the evaluation of leasing versus selling decisions. Third, in the case of leasing the decision also involves non-financial clauses related to, for example, operating and maintenance costs.

For simplicity, we assume a market with a fixed number of n lessors and m sellers of automobiles which offer homogeneous services in each market, respectively. However, leasing services are assumed to be differentiated from selling services. So, the market lease (sell) rate in any period is unique and is the lowest rate offered by the leasing (selling) firms:

$$R_L^{market}(t) = \min\{R_L^1(t), R_L^2(t), \cdots R_L^n(t)\}$$

$$R_k^{market}(t) = \min\{R_k^1(t), R_k^2(t), \cdots R_k^m(t)\} \quad (1)$$

where $R^i\ (t)$ is the rate of firm *i* at time *t* and the subscripts *L* and *k* denote leasing and selling, respectively. If more than one firms set the market rate for either leasing or selling then the revenues are split equally among them. For notational convenience we denote the market rates $R_L^{market}(t) \equiv R_L(t)$ and $R_k^{market}(t) \equiv R_k(t)$. As argued by Dudine, Hendel, and Lizzeri, (2006), the dynamic demand is driven by both the durability of the product and by the anticipation of consumers for the future prices. However, in the present setting, as Chen, Esteban, and Shum (2008), we assume for simplicity that the consumer decisions whether to buy an automobile depend on their expectations about future market prices which create forward-looking dynamics in the demand function, and, subsequently in the decisions of the firms. In this manner, the demand depends on the current rate level as well as on its time derivative.

As Goldberg (1995), we focus on the second stage of a two stage game. Specifically, since the market is an oligopoly with differentiated products, the supply decisions and the market equilibria involve two stages. First, a long-run stage, in which firms determine the product-mix and the quality of their products, and second, a short-run stage in which producers set prices given their product types. Since automobiles are durable goods, the representative lessor, who charges the lowest rate in period *t*, faces the following intertemporal problem:

$$\max_{R_L(t)} \int_0^\infty e^{-\rho t}(R_L(t) - c_L)q_L(t)dt$$

$$\text{s. t } q_L(R_L(t), R_k(t), \dot{R}_L(t), \dot{R}_k(t)) \quad (2)$$

where dot denotes the time-derivative of the variable and c_L is the opportunity cost of capital (WACC) of the leasing firm. In order to maximize his profits the representative lessor chooses the instantaneous lease rate $R_L(t) = \frac{dP_L}{P_L} = \frac{L(t)}{P_L(t)}$,or, in discrete time $R_L(t) = \frac{\Delta P_L}{P_{L,t-1}} = \frac{P_{L,t} - P_{L,t-1}}{P_{L,t-1}}$. $L(t)$ is the default free lease payment paid at the beginning of the period; $P_L(t)$ and $P_k(t)$ represent the market values of the leased and the purchased asset, respectively, at the beginning of period. The representative seller faces the following problem:

$$\max_{R_k(t)} \int_0^\infty e^{-\rho t}(R_k(t) - c_k)q_k(t)dt$$

$$\text{s.t. } q_k(R_k(t), R_L(t), \dot{R}_k(t), \dot{R}_L(t)). \quad (3)$$

As with the lessor, the representative seller chooses the instantaneous sell rate, $\mathrm{R_k(t)} = \frac{\mathrm{dP_k}}{\mathrm{P_k}}$, or, $\mathrm{R_k(t)} = \frac{\Delta \mathrm{P_k}}{\mathrm{P_{k,t-1}}} = \frac{\mathrm{P_{k,t}} - \mathrm{P_{k,t-1}}}{\mathrm{P_{k,t-1}}}$ in order to maximize his profits.[1] In line with Goldberg (1995), firms are assumed to be free of quantity constraints while attempting to maximize the present value of profits between consecutive market periods. They use the same discount factor $e^{-\rho t} \epsilon$ (0, 1), where ρ denotes the common rate of discounting and corresponds to their Weighted Average Cost of Capital (WACC), i.e., $\rho = c_L = \mathrm{c}_k$. Finally, the demand functions are linear and set equal to:[2]

$$\begin{aligned} q_L(t) &= \theta_o + \theta_1 R_L(t) + \theta_2 R_k(t) + \theta_3 \dot{R}_L(t) + \theta_4 \dot{R}_k(t) \\ q_k(t) &= \lambda_o + \lambda_1 R_k(t) + \lambda_2 R_L(t) + \lambda_3 \dot{R}_k(t) + \lambda_4 \dot{R}_L(t) \end{aligned} \tag{4}$$

The parameters θ_o, λ_o are always positive; θ_1 and λ_1 are always negative since the demand for good *i* is downward sloping in its own rate. The demand structure at hand allows a range of different degrees of substitutability between the goods. In general, we consider that selling and leasing are substitutes when $\theta_2, \lambda_2 > 0$, while they are complements when $\theta_2, \lambda_2 < 0$. However, as recently discussed by De Jaegher (2009) the above definitions refer specifically to weak symmetric gross substitutes and weak symmetric gross complements, respectively. The definition in general of substitutability and complementarity requires that not only the signs but also the absolute values of the coefficients θ_2, λ_2 to be the same. De Jaegher considers the case when θ_2, λ_2 have opposite signs in which two goods are strong asymmetric substitutes. In our setting, selling would be a substitute for leasing and leasing would be a complement to selling, or, vice versa. If, for example, θ_2<0 and λ_2>0 then the demand for leasing is a decreasing function of the purchase rate so leasing is a gross substitute of purchasing. At the same time, the demand for purchasing is an increasing function of leasing rate so that purchasing is a gross complement of leasing.

In our model consumers form expectations in a perfect foresight manner according to $\frac{dR^e}{dt} = \frac{dR}{dt}$. This constitutes the deterministic equivalent of the rational expectation hypothesis and allows us to avoid complications related to adverse selection (see Akerlof, 1970; Hendel and Lizzeri, 1999). If consumers expect rates to continue rising, their desire to hold money is reduced so θ_3 and λ_3 will be positive. However, if they expect that rates will continue falling then they restrain from buying and θ_3 and λ_3 will be negative. The interpretation of the signs for θ_4 and λ_4 will depend on the substitutability of selling and leasing. For example, when they are substitutes, if consumers expect that the sell rate will continue rising then they increase their demand now. Assuming constant supply, this leads to a rise in sell rates which in turn increases the demand for

[1] Since the seller has already chosen $P_{\mathrm{k},t-1}$ in the previous period, the assumption of choosing $R_{\mathrm{k,t}}$ is equivalent to the assumption of choosing $P_{\mathrm{k},t}$.

[2] The linear demand structure arises from a quadratic and strictly concave utility function (see Dixit, 1979; Singh and Vives, 1984).

leasing services. In this case, θ_4 is positive. Conversely, if consumers expect sell rates to fall then θ_4 will be negative. A similar line of arguments can be made in interpreting the sign of λ_4.

The first order conditions of the optimization problems underhand which correspond to the best response functions are the following:

$$\dot{R}_L(t) + a_1\dot{R}_k(t) + a_2 R_L(t) + a_3 R_k(t) = a$$

$$\dot{R}_k(t) + b_1\dot{R}_L(t) + b_2 R_k(t) + b_3 R_L(t) = b \qquad (5)$$

where:

$$\alpha_1 \equiv \frac{\theta_4}{\theta_3},\ \alpha_2 \equiv \frac{2\theta_1+\rho\theta_3}{\theta_3},\ \alpha_3 \equiv \frac{\theta_2}{\theta_3},\text{ and } \alpha \equiv \frac{(\theta_1+\rho\theta_3)\rho-\theta_o}{\theta_3},$$

$$b_1 \equiv \frac{\lambda_4}{\lambda_3},\ b_2 \equiv \frac{2\lambda_1+\rho\lambda_3}{\lambda_3},\ b_3 \equiv \frac{\lambda_2}{\lambda_3},\text{ and } b \equiv \frac{(\lambda_1+\rho\lambda_3)\rho-\lambda_o}{\lambda_3} \qquad (6)$$

The first order conditions are obtained by substituting the demand functions into the objective functions and then solving the maximization problems using the calculus of variations technique. The obtained system of first order differential equations is not in normal form and requires further transformation. The corresponding homogeneous system is:

$$\dot{R}_L(t) + a_1\dot{R}_k(t) + a_2 R_L(t) + a_3 R_k(t) = 0$$

$$\dot{R}_k(t) + b_1\dot{R}_L(t) + b_2 R_k(t) + b_3 R_L(t) = 0 \qquad (7)$$

or, in matrix form:

$$\begin{bmatrix} 1 & a_1 \\ b_1 & 1 \end{bmatrix}\begin{bmatrix} \dot{R}_L(t) \\ \dot{R}_k(t) \end{bmatrix} + \begin{bmatrix} a_2 & a_3 \\ b_3 & b_2 \end{bmatrix}\begin{bmatrix} R_L(t) \\ R_k(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

This can be reduced to an equivalent first order system in normal form as following:

$$\begin{bmatrix} \dot{R}_L(t) \\ \dot{R}_k(t) \end{bmatrix} = -\begin{bmatrix} 1 & a_1 \\ b_1 & 1 \end{bmatrix}^{-1}\begin{bmatrix} a_2 & a_3 \\ b_3 & b_2 \end{bmatrix}\begin{bmatrix} R_L(t) \\ R_k(t) \end{bmatrix}$$

The reduction requires that the matrix $\begin{bmatrix} 1 & a_1 \\ b_1 & 1 \end{bmatrix}$ is not singular, i.e., that $1 - a_1 b_1 = 1 - \frac{\theta_4}{\theta_3}\frac{\lambda_4}{\lambda_3} \neq 0$. So, we end up with the following homogeneous system of differential equations in normal form:

$$\dot{R}_L(t) = \varphi_{11} R_L(t) + \varphi_{12} R_k(t)$$

$$\dot{R}_k(t) = \varphi_{21} R_L(t) + \varphi_{22} R_k(t) \qquad (8)$$

where:

$$\varphi_{11} \equiv \frac{a_1 b_3 - a_2}{1 - a_1 b_1}, \qquad \varphi_{12} \equiv \frac{a_1 b_2 - a_3}{1 - a_1 b_1}$$

(9)

$$\varphi_{21} \equiv \frac{b_1 a_2 - b_3}{1 - a_1 b_1}, \qquad \varphi_{22} \equiv \frac{b_1 a_3 - b_2}{1 - a_1 b_1}$$

In the above system the two rates of return interact with each other linearly since their first time derivatives are proportional to a linear combination of their levels. The values of the coefficients φ_{ij} determine the contribution that the levels of the variables make to their growth. Specifically, φ_{12} and φ_{21} relate the growth of the return of one variable to the level of return of the other variable. So, a negative value of φ_{12} indicates the negative contribution of the level of the sell rate to the growth of the leasing rate, in the sense that the presence of selling reduces the growth of the lease rate. In other words, if consumers cannot buy the good then the rate of growth for the return of leasing will be higher. In this way, a negative value of φ_{12} reduces the power of the leasing firms in the market. Coefficients φ_{11} and φ_{22} indicate the effect of the level of return on its own rate of growth. If we consider the particular solution of system (5) in matrix form we have:

$$\begin{bmatrix} 1 & a_1 \\ b_1 & 1 \end{bmatrix} \begin{bmatrix} \dot{R}_L(t) \\ \dot{R}_k(t) \end{bmatrix} + \begin{bmatrix} a_2 & a_3 \\ b_3 & b_2 \end{bmatrix} \begin{bmatrix} R_L(t) \\ R_k(t) \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$$

This can be reduced to an equivalent first order system in normal form as following:

$$\begin{bmatrix} \dot{R}_L(t) \\ \dot{R}_k(t) \end{bmatrix} = -\begin{bmatrix} 1 & a_1 \\ b_1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} a_2 & a_3 \\ b_3 & b_2 \end{bmatrix} \begin{bmatrix} R_L(t) \\ R_k(t) \end{bmatrix} + \begin{bmatrix} 1 & a_1 \\ b_1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} a \\ b \end{bmatrix}$$

This assumes that the matrix $\begin{bmatrix} 1 & a_1 \\ b_1 & 1 \end{bmatrix}$ is not singular, i.e., that: $1 - a_1 b_1 = 1 - \frac{\theta_4}{\theta_3}\frac{\lambda_4}{\lambda_3} \neq 0$. Finally, we obtain the following system of differential equations in normal form:

$$\dot{R}_L(\mathrm{t}) = \mathrm{K}_1 + \varphi_{11}\mathrm{R}_\mathrm{L}(\mathrm{t}) + \varphi_{12}\mathrm{R}_\mathrm{k}(\mathrm{t})$$

(10)

$$\dot{R}_k(\mathrm{t}) = \mathrm{K}_2 + \varphi_{21}\mathrm{R}_\mathrm{L}(\mathrm{t}) + \varphi_{22}\mathrm{R}_\mathrm{k}(\mathrm{t})$$

where $K_1 \equiv \frac{a - a_1 b}{1 - a_1 b_1}$, $K_2 \equiv \frac{b - a b_1}{1 - a_1 b_1}$ and φ_{ij} are defined as previously. The characteristic polynomial of the system's matrix $\Phi \equiv \begin{bmatrix} \varphi_{11} & \varphi_{12} \\ \varphi_{21} & \varphi_{22} \end{bmatrix}$ can be written as $\varphi^2 - (\varphi_{11} + \varphi_{22})\varphi + (\varphi_{11}\varphi_{22} - \varphi_{12}\varphi_{21}) = 0$. The signs of the coefficients φ_{ij} allow us to classify the dynamics of the two interacting markets in four interesting cases:

Case 1. Stable Node

Arises when $\Delta = tr(\Phi)^2 - 4det(\Phi) > 0$ where $tr(\Phi) = \varphi_1 + \varphi_2 < 0$ and $det(\Phi) = \varphi_1\varphi_2 > 0$. There are two ways in which the trace can be negative. First, $\varphi_{11}, \varphi_{22}$ are both negative. This means that we have competition only internally within each of the two markets and not between them. For instance, an increase in the lease rate $R_L(t)$ has an inverse impact on the growth of both $R_L(t)$ and $R_k(t)$ since the derivatives $\dot{R}_L(t), \dot{R}_k(t)$ are negative. Second, either one of $\varphi_{11}, \varphi_{22}$ is negative while the other is positive with the negative being higher in absolute value. Although there is competition within only one market, the level of competition is high enough to compensate for any growth trend in the returns of the other market. In other words, the competition is present only in one market but it is large enough to lead both markets towards their steady-state rates of returns. As an example, suppose that there is internal competition in the lease market, i.e., $\varphi_{11} < 0$, while in the selling market returns are increasing, i.e., $\varphi_{22} > 0$. For the determinant to be positive, the term $-\varphi_{12}\varphi_{21}$ must be positive since $\varphi_{11}\varphi_{22}$ is negative. So, $\varphi_{12}, \varphi_{21}$ must have opposite signs. When real roots are positive, $\varphi_{11}\varphi_{22} > 0$, an unstable node arises. The returns in both markets arise indefinitely and system deviates from its steady state. So in order to exclude the possibility of a bubble in the markets we require that the real roots are negative.

Case 2. Saddle Point

Arises only when $\Delta = tr(\Phi)^2 - 4det(\Phi) > 0$ and $det(\Phi) = \varphi_1\varphi_2 < 0$. This means that the interaction effect $\varphi_{12}\varphi_{21}$ is positive and greater than the product $\varphi_{11}\varphi_{22}$ and can be realized in two different ways. First, if there is a high level of co-operation between the two markets, $\varphi_{12} > 0, \varphi_{21} > 0$, which dominates the system dynamics. In this manner, the interaction effect overcomes the positive combined effect, $\varphi_{11}\varphi_{22}$. Second, if the combined effect $\varphi_{11}\varphi_{22}$ is negative, i.e., there is internal competition within one market and growth in the other making the interaction effect nonnegative.

Case 3. Focus

Arises when $\Delta = tr(\Phi)^2 - 4det(\Phi) = (\varphi_{11} - \varphi_{22})^2 + 4\varphi_{12}\varphi_{21} < 0$ and $tr(\Phi) = \varphi_1 + \varphi_2 \neq 0$. The negative discriminant means that $(\varphi_{11} - \varphi_{22})^2$ does not exceed in absolute value $4\varphi_{12}\varphi_{21}$. Consequently, $\varphi_{12}, \varphi_{21}$ must have opposite signs. Such a situation may arise if, for example, the lease market benefits from its interaction with the selling market, i.e., $\varphi_{12} > 0$, while the selling market is damaged by its interaction with the lease market, i.e., $\varphi_{21} < 0$, and in the lease market there is internal competition ($\varphi_{11} < 0$) while the selling market is growing ($\varphi_{22} > 0$). If the discriminant is equal to zero the interpretation is similar but the interaction of the markets follows a node rather than focus equilibrium. In order to exclude the possibility of instability and bubbles we require that the real parts of the roots are negative and the trace is negative in the case of the focus and node, respectively.

Case 4. Centre

Arises when $\Delta = tr(\Phi)^2 - 4det(\Phi) < 0$ and $tr(\Phi) = \varphi_1 + \varphi_2 = 0$. The negative discriminant is explained as in the previous case and the trace condition can occur in two ways. First, $\varphi_{11} = \varphi_{22} = 0$, i.e., the rates of returns in both markets are unchanged. Second, $\varphi_{11} = -\varphi_{22}$, i.e., the intensity of the internal competition in the one market is equal and opposite to that of the growth in the other market.

A more general specification of our setting can be obtained by adding interaction terms in the demand functions:

$$q_L(t) = \theta_o + \theta_1 R_L(t) + \theta_2 R_k(t) + \theta_3 \dot{R}_L(t) + \theta_4 \dot{R}_k(t) + \theta_5 R_L(t) R_k(t)$$

(11)

$$q_k(t) = \lambda_o + \lambda_1 R_k(t) + \lambda_2 R_L(t) + \lambda_3 \dot{R}_k(t) + \lambda_4 \dot{R}_L(t) + \lambda_5 R_L(t) R_k(t)$$

In this way the partial effect of the demand for the lease services with respect to the lease rate depends on the magnitude of the sell rate:

$$\frac{\partial q_L(t)}{\partial R_L(t)} = \theta_1 + \theta_5 R_k(t)$$

If $\theta_5 > 0$, given that $\theta_1 < 0$, then an increase in $R_L(t)$ yields a lower decrease in demand for the lease services for higher sell rates. So, θ_1 reflects the partial effect of lease rates on demand when the sell rate is zero. The partial effect of sell rates on demand is:

$$\frac{\partial q_L(t)}{\partial R_k(t)} = \theta_2 + \theta_5 R_L(t)$$

If $\theta_5 > 0$, assuming that selling and leasing are substitutes, i.e., $\theta_2 > 0$, then an increase in $R_k(t)$ yields a higher increase in demand for the lease services for higher lease rates. Our best response functions are now as following:

$$\dot{R}_L(t) + a_1 \dot{R}_k(t) + a_2 R_L(t) + \gamma_3 R_k(t) + \alpha_5 R_L(t) R_k(t) = a$$

(12)

$$\dot{R}_k(t) + b_1 \dot{R}_L(t) + b_2 R_k(t) + \gamma_4 R_L(t) + b_5 R_L(t) R_k(t) = b$$

where:

$$\gamma_3 \equiv \frac{(\theta_2 - \theta_5 \rho)}{\theta_3}, \alpha_5 = \frac{2\theta_5}{\theta_3}$$

(13)

$$\gamma_4 \equiv \frac{(\lambda_2 - \lambda_5 \rho)}{\lambda_3}, b_5 = \frac{2\lambda_5}{\lambda_3}$$

and $a_1, a_2, a, b_1, b_2, b,$defined as previously. With the corresponding homogeneous system becoming:

$$\dot{R}_L(t) + a_1\dot{R}_k(t) + a_2R_L(t) + \gamma_3R_k(t) + \alpha_5R_L(t)R_k(t) = 0$$

$$\dot{R}_k(t) + b_1\dot{R}_L(t) + b_2R_k(t) + \gamma_4R_L(t) + b_5R_L(t)R_k(t) = 0 \tag{14}$$

This system can be reduced to an equivalent first order system in normal form as follows

$$\begin{bmatrix}\dot{R}_L(t)\\ \dot{R}_k(t)\end{bmatrix} = -\begin{bmatrix}1 & a_1\\ b_1 & 1\end{bmatrix}^{-1}\begin{bmatrix}a_2 & \gamma_3\\ \gamma_4 & b_2\end{bmatrix}\begin{bmatrix}R_L(t)\\ R_k(t)\end{bmatrix} - \begin{bmatrix}1 & a_1\\ b_1 & 1\end{bmatrix}^{-1}\begin{bmatrix}a_5/2 & a_5/2\\ b_5/2 & b_5/2\end{bmatrix}\begin{bmatrix}R_L(t)R_k(t)\\ R_L(t)R_k(t)\end{bmatrix}$$

Assuming again that the matrix $\begin{bmatrix}1 & a_1\\ b_1 & 1\end{bmatrix}$ is not singular. In this manner, we end up with the following homogeneous system of differential equations in normal form:

$$\dot{R}_L(t) = \chi_{11}R_L(t) + \chi_{12}R_k(t) + \chi_{13}R_L(t)R_k(t)$$

$$\dot{R}_k(t) = \chi_{21}R_L(t) + \chi_{22}R_k(t) + \chi_{23}R_L(t)R_k(t) \tag{15}$$

where:

$$\chi_{11} \equiv \frac{a_1\gamma_4 - a_2}{1 - a_1b_1}, \chi_{12} \equiv \frac{a_1b_2 - \gamma_3}{1 - a_1b_1}, \chi_{13} \equiv \frac{a_1b_5 - a_5}{1 - a_1b_1}$$

$$\chi_{21} \equiv \frac{b_1a_2 - \gamma_4}{1 - a_1b_1}, \chi_{22} \equiv \frac{b_1\gamma_3 - b_2}{1 - a_1b_1}, \chi_{23} \equiv \frac{a_5b_1 - b_5}{1 - a_1b_1} \tag{16}$$

For $\chi_{12} = \chi_{21} = 0$, we have:

$$\dot{R}_L(t) = R_L(t)(\chi_{11} + \chi_{13}R_k(t))$$

$$\dot{R}_k(t) = R_k(t)(\chi_{22} + \chi_{23}R_L(t)) \tag{17}$$

When χ_{13}, χ_{23} have opposite signs we obtain the famous Lotka-Volterra specification (Goodwin, 1967, 1990). In line with the biological origin of the Lotka-Volterra model and its standard interpretation in economics, in our case this would mean that leasing and selling markets have a prey-predator relationship. Although here we study car prices, such relationships in terms of market shares have been investigated in the literature. For example, Levinthal and Purohit (1989) develop a theoretical model which allows new sales of durable goods to predate the sales of used one's. If we consider the particular solution of system (12) in normal form we have:

$$\begin{bmatrix}\dot{R}_L(t)\\ \dot{R}_k(t)\end{bmatrix} = -\begin{bmatrix}1 & a_1\\ b_1 & 1\end{bmatrix}^{-1}\begin{bmatrix}a_2 & \gamma_3\\ \gamma_4 & b_2\end{bmatrix}\begin{bmatrix}R_L(t)\\ R_k(t)\end{bmatrix} - \begin{bmatrix}1 & a_1\\ b_1 & 1\end{bmatrix}^{-1}\begin{bmatrix}a_5/2 & a_5/2\\ b_5/2 & b_5/2\end{bmatrix}\begin{bmatrix}R_L(t)R_k(t)\\ R_L(t)R_k(t)\end{bmatrix} + \begin{bmatrix}1 & a_1\\ b_1 & 1\end{bmatrix}^{-1}\begin{bmatrix}a\\ b\end{bmatrix}$$

or,

$$\dot{R}_L(t) = K_1 + \chi_{11}R_L(t) + \chi_{12}R_k(t) + \chi_{13}R_L(t)R_k(t)$$

$$\dot{R}_k(t) = K_2 + \chi_{21}R_L(t) + \chi_{22}R_k(t) + \chi_{23}R_L(t)R_k(t) \tag{18}$$

where χ_{ij}, K_1, K_2 are defined as previously.

4. Empirical Application

4.1 A model of Dynamic Interaction between Leasing and Selling Markets

Our sample is drawn from the Bureau of Labor Statistics (BLS) databases and corresponds to the US which is the largest automobile market internationally. The period covered is January 2002 to May 2011, a total of 113 monthly observations expressed in constant prices of December 2001.[3] Two city-average Consumer Price Indices (CPI) are used which correspond to seasonally adjusted price levels of New Cars and Trucks (*NEW*) and Leased Cars and Trucks (*LEAS*). The later is a component of the new and used motor vehicles expenditure class, which is part of the CPI's private transportation component in the transportation major group and it covers leases on all classes of new consumer vehicles. The CPI data collector describes each selected vehicle lease in detail including seven aspects of the lease contract: the vehicle make, nameplate, model, engine, transmission, options and lease terms. The lease terms include characteristics such as the number of months of the lease term, the down payment, the residual value, the depreciation amount and the total rent charge. The sample is updated by one model year each September through November in order to maintain the same age vehicles over time. If a production model is discontinued, it is replaced by a comparable model. A complete resampling is scheduled every 5 years. Finance charges are not included in the CPI as well as any incentives associated with low-interest financing, are excluded from the discount or rebate amount.[4] The value that the CPI uses in *LEAS* is an estimated transaction price that reflects the vehicle base price, destination charge, options, dealer preparation charges, applicable taxes, depreciation, and lease rent charge (the finance fee portion of a monthly lease payment, similar to interest on a loan).

[3] The discussion closely follows Labor/Bureau of Labor Statistics, www.bls.gov

[4] The formation of the Leased cars and trucks index is based on the calculation of total monthly lease payment. The formula, which uses the U.S. Department of Labor/Bureau of Labor Statistics, for the calculation of total monthly lease payment is the following: Total Monthly Lease Payment = (Base Price of Leased Vehicle) + (Transportation to Dealer) + (Total Price of Packages & Options) + (Dealer Preparation and Miscellaneous Charges) + (Additional Dealer Markup) – (Dealer Concession or Discount), which is equal with: (Capitalized Cost) (similar to the purchase price of a vehicle) – (Down payment) – (Rebate) – (Other Capitalized Cost Reductions) + (Tax) + (Other Additions to Capitalized Cost), which in turn is equal with: (Adjusted Capitalized Cost, amount used to calculate base monthly payment) – (Residual Value, value of the vehicle at the end of the lease) and this is equal with: (Depreciation Amount, the total amount charged for the decline in value) + (Total Lease Rent Charge, the finance fee, similar to interest), which finally equals with: (Total of Base Monthly Payments/ Lease Term, the number of months in the lease), or (Base Monthly Payment) + (Monthly Sales/Use Tax).

The estimated transaction price also includes the respondent's estimate for the price markup, dealer concession or discount, and consumer rebate.

A casual inspection of the CPI levels suggests some kind of inverse co-evolution between the two series under study along with a smooth variation which is consistent with nonstationarity. As shown in Table 1, panel stationarity tests of CPI levels assuming a common or separate unit root processes confirm that both *NEW* and *LEAS* are integrated. Cointegration analysis suggests that no long term equilibrium relationship exists between the levels of the two CPI series (results are available upon request by the authors). So, CPI levels are used to calculate monthly rates of returns for selling (*RNEW*) and leasing (*RLEAS*), respectively, as simple percentage changes and these are then used in the subsequent analysis. Stationarity tests show that *RNEW* and *RLEAS* are *I*(0) indicating that the original series is *I*(1).

Figure 1. Automobile selling prices (*NEW*) and leasing prices (*LEAS*)

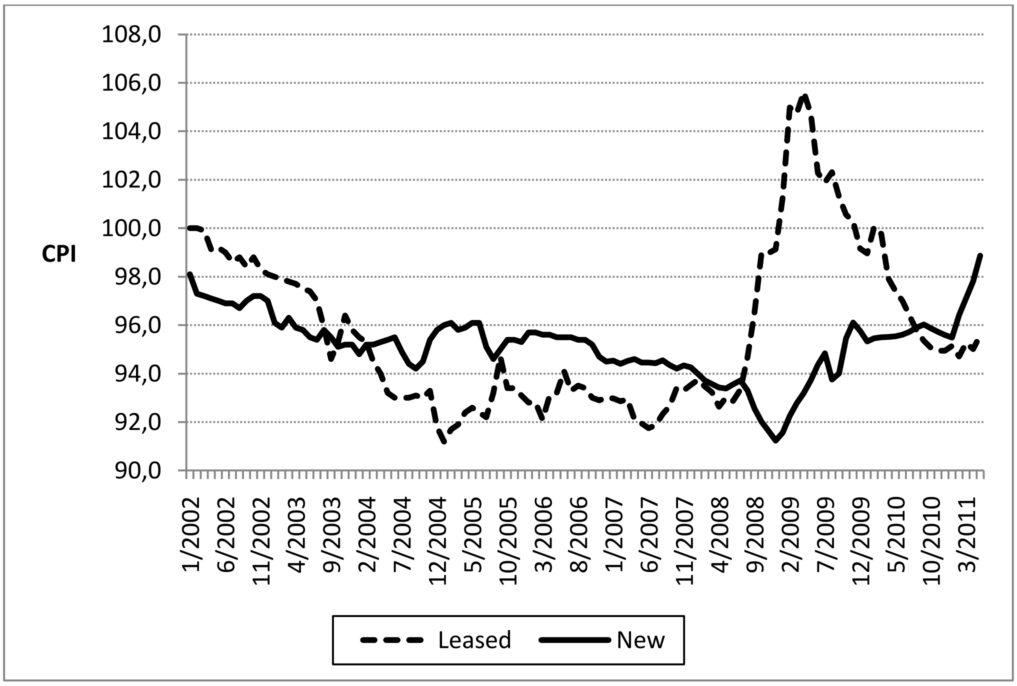

Table 1. Stationarity analysis of automobile selling prices (*NEW*) and leasing prices (*LEAS*)

Method	**Test statistic**	***p*-value**
H_0: Common unit root process		
Levin, Lin and Chu (2002) *t**	0.4818	0.6850
H_0: Individual unit root process		
Im, Pesaran and Shin (2003) W-stat	-0.0392	0.4844
Maddala and Wu (1999), ADF Fisher Chi-square	3.0436	0.5506
Choi (2001) PP Fisher Chi-square	3.7628	0.4391

Probabilities for Fisher tests are computed using an asymptotic Chi-square distribution. All other tests assume asymptotic normality.

Descriptive statistics of the returns appear in Table 2. The results suggest that both series are positively skewed and leptokurtic. The maximum positive (negative) change was 1.53% or 3.64 standard deviations (-1.15% or 2.74 s.d.) for *RNEW* and occurred during the recent crisis period on October 2009 (March 2008). Similarly, for RLEAS the maximum (minimum) was 3.67% or 4.5 s.d. (-2.32% or -2.86 s.d.) on February 2009 (June 2009). The Pearson correlation coefficient between the two return series is -10.84% which is statistically insignificant at the 10% level and suggests no contemporaneous relationship. However, the null hypothesis that *DNEW* does not Granger-cause *DLEAS* is rejected with a test *F*-statistic of 6.0358 for 1 lag which is significant at the 1.56% level. The hypothesis in the opposite direction cannot be rejected at conventional levels of significance.

Table 2. Descriptive Statistics of monthly changes in automobile selling prices (*RNEW*) and leasing prices (*RLEAS*)

	RNEW	***RLEAS***
Mean	0.0001	-0.0004
Median	0.0000	-0.0011
Maximum	0.0153	0.0367
Minimum	-0.0115	-0.0232
Std.Dev.	0.0042	0.0081
Skewness	0.3726	1.1606
Kurtosis	4.5428	7.3385
Jarque-Bera	13.6990	112.9836
p-value	0.0011	0.0000

The derived system of differential equations (8) of our model can be written in discrete time as follows:

$$R_{L,t+1} = \beta_{11} R_{L,t} + \varphi_{12} R_{k,t} + u_{L,t+1}$$

(19)

$$R_{k,t+1} = \varphi_{21} R_{L,t} + \beta_{22} R_{k,t} + u_{k,t+1}$$

where $\beta_{11} \equiv (1 + \varphi_{11})$ and $\beta_{22} \equiv (1 + \varphi_{22})$ .This is a VAR model, of lag order one as our theory dictates, in reduced form. The corresponding structural VAR is:

$$\begin{bmatrix} 1 & a_1 \\ b_1 & 1 \end{bmatrix} \begin{bmatrix} R_{L,t+1} \\ R_{k,t+1} \end{bmatrix} + \begin{bmatrix} a_2 - 1 & a_3 - a_1 \\ b_3 - b_1 & b_2 - 1 \end{bmatrix} \begin{bmatrix} R_{L,t} \\ R_{k,t} \end{bmatrix} = \begin{bmatrix} u_{L,t+1} \\ u_{k,t+1} \end{bmatrix}$$

Estimation of this VAR model via OLS and subsequent elimination of the insignificant coefficients led to the following results (standard errors appear in brackets below estimates):

.

$$\underset{}{\mathrm{R}_{L,t+1}} = \underset{(0.0901)}{0.2643}\ \mathrm{R}_{L,t} - \underset{(0.1790)}{0.4408}\ \mathrm{R}_{\mathrm{k,t}} + u_{L,t+1},\ R^2_{adj} = 0.124$$

$$\mathrm{R}_{k,t+1} = \underset{(0.0888)}{0.3978}\ \mathrm{R}_{k,t} + u_{k,t+1},\ R^2_{adj} = 0.153$$

The estimated coefficients allow us to draw several interesting conclusions. It appears that leasing market price changes are inversely related to prices changes in the selling market from the previous month ($\varphi_{12} < 0$). From a biological perspective, this is characterized as a "predatory" relationship of selling market over the leasing market. In line with the Granger causality results obtained previously, selling market price changes do not seem do depend on past leasing market price changes ($\varphi_{21} = 0$). Both leasing and selling market price changes are moderately persistent with the autoregressive coefficients being positive. The modulus of both roots is less than unity so we have a stable equilibrium point (stable node; see Case 1 in Section 3). Since both roots are real and distinct, shocks will dissipate in a monotone rather than fluctuating manner. Estimation of a nonlinear version of the interaction between selling and leasing did not produce significant results.

4.2 Implications for Leasing Contract Valuation

The standard framework of lease valuation (Myers, Dill and Bautista, 1976) adopts discounted cash flow analysis to derive the equilibrium rental rate:[5]

[5] An alternative is the user cost theory approach of Miller and Upton (1976). A number of other valuation models have been proposed in order to account for credit risk in lease contracts (see, for example, Grenadier, 1996; Ambrose and Yildirim 2008; Agarwal *et al.*, 2011*)* or for various optionalities in leasing contracts (see,

$$\mathrm{P}_{\mathrm{L},\mathrm{t}} = \mathrm{P}_{\mathrm{L},0} - \sum_{\mathrm{t}=1}^{\mathrm{n}} \frac{\mathrm{L}_{\mathrm{t}}}{(1+\rho)^{\mathrm{n}}} - \frac{\mathrm{RV}_{\mathrm{n}}}{(1+\rho)^{\mathrm{n}}} \quad (20)$$

Where, RV_n is the expected residual value of the asset in period *n*. By employing a uniform lease payment we obtain the Myers, Dill and Bautista (MDB) formula:

$$L(t) = \frac{\rho}{1-(1+\rho)^n}[P_{L,0} - P_{L,t} - \frac{RV_n}{(1+\rho)^n}]$$

or, equivalently, the lease rate:

$$\frac{L(t)}{P_L(t)} = \frac{\rho}{(1+\rho)^n - 1}[1 - \frac{P_{L,0}}{P_{L,t}} + \frac{RV_n}{P_{L,t}(1+\rho)^n}] \quad (21)$$

Given our model and empirical results, an obvious shortcoming of this valuation approach is that it treats the leasing market autonomously and ignores any interactions with the selling market. The remainder of this section will incorporate our findings concerning the interaction between the leasing and selling markets in the MBD valuation approach.

From (19) we can derive the motion for the system of lease and sell rates from the following complementary function:

$$\mathrm{R}_{\mathrm{L},\mathrm{t}} = A_1\beta_1^t + A_2\beta_2^t$$

$$\mathrm{R}_{\mathrm{k},\mathrm{t}} = B_1\beta_1^t + B_2\beta_2^t \quad (22)$$

where:

$$B_1 \equiv A_1\frac{\beta_1 - \beta_{11}}{\varphi_{12}},$$

$$B_2 \equiv A_2\frac{\beta_2 - \beta_{11}}{\varphi_{12}}, \quad (23)$$

Moreover, since $\beta_1 - \beta_{11} = 0$, we obtain:

$$\mathrm{R}_{\mathrm{L},\mathrm{t}} = A_1\beta_1^t + A_2\beta_2^t$$

$$\mathrm{R}_{\mathrm{k},\mathrm{t}} = B_2\beta_2^t$$

The arbitrary constants A_i are determined by the initial conditions of the system as follows:

$$A_2 = \frac{R_{k,0}\varphi_{12}}{\beta_2 - \beta_{11}}$$

$$R_{L,0} = A_1 + A_2 \quad (24)$$

for example, McConnell and Schallheim, 1983; Schallheim and McConnell, 1985; Grenadier, 1995;Trigeorgis, 1996). For empirical applications see Schallheim *et al.* (1987) and Giaccotto, Goldberg, and Hegde, (2007).

where $R_{L,0}$ and $R_{k,0}$ have already been defined as $\mathrm{R}_{L,0} = \frac{\Delta \mathrm{P_L}}{\mathrm{P_{L,-1}}} = \frac{\mathrm{P_{L,0}} - \mathrm{P_{L,-1}}}{\mathrm{P_{L,-1}}}$ and $\mathrm{R}_{k,0} = \frac{\Delta \mathrm{P_k}}{\mathrm{P_{k,-1}}} = \frac{\mathrm{P_{k,0}} - \mathrm{P_{k,-1}}}{\mathrm{P_{k,-1}}}$. So, having estimated $R_{L,t}$ and $R_{k,t}$, we can obtain $P_{L,t}$ and $P_{k,t}$ as:

$$\hat{P}_{L,t} = P_{L,t-1}\left(1 + \hat{R}_{L,t}\right) = P_{L,t-1}(1 + A_1\beta_1^t + A_2\beta_2^t)$$
$$\hat{P}_{k,t} = P_{k,t-1}\left(1 + \hat{R}_{k,t}\right) = P_{k,t-1}(1 + B_2\beta_2^t) \tag{25}$$

Now, the following quantity:

$$G_t = P_{L,t} - \hat{P}_{L,t} = P_{L,t} - P_{L,t-1}\left(1 + \hat{R}_{L,t}\right) = P_{L,t} - P_{L,t-1}(1 + A_1\beta_1^t + A_2\beta_2^t) \tag{26}$$

represents a capital gain or loss which results from the interaction between the leasing and selling markets and could be used to augment the MBD leasing valuation formula. In other words, this term reflects an opportunity cost in the sense that the price of the leased asset changes and this is something that should be accounted for. Another reasonable adjustment that should be made concerns the residual value since this is an expectation of the stochastic value which the asset will have in the termination of the contract (e.g., Trigeorgis, 1996, assumes that the residual value follows an Ornstein–Uhlenbeck process). The residual value is corrected here on the basis of the interaction with the selling market by using the cumulative changes in the leasing market prices $\prod_{t=1}^{n}\left(1 + \hat{R}_{L,t}\right)$. Finally, the overall effect of the interaction with the selling market can be captured by the following augmented lease valuation formula:

$$P_{L,t} = P_{L,0} - \sum_{t=1}^{n} \frac{L_t{'}}{(1+\rho)^n} - \frac{RV_n \prod_{t=1}^{n}\left(1+\hat{R}_{L,t}\right)}{(1+\rho)^n} - \sum_{t=1}^{n} \frac{G_t}{(1+\rho)^n} \tag{27}$$

We shall use a hypothetical example in order to illustrate the application and practical importance for valuation of the interaction between leasing and selling markets. Assume that we are considering the valuation of a contract for a car with a base price $P_{L,0}$ = €30,000 which will be leased over a 6 month period with a terminal residual value RV_n equal to €25,000 (83.3% of the base price). Lease payments are due at the end of each month and the lease is financed at a monthly rate of 1%. Without taking into account the interaction between the two markets, the traditional MDB formula gives a monthly lease payment equal to €1112.74. Assume now that we are at October 2009 when the selling market price level increased by 1.535% compared to the previous month (or 20.06% in annual terms). We can use this information to recursively predict lease rates over the next 6 months on the basis of the estimated model from the previous subsection. The predicted lease rates are -0.86971%, -0.49903%,-0.23897%, -0.10575%, -0.04489% and -0.01861%, respectively, or a total compound (average) expected drop of 1.77% (0.3%).These predictions are close to the actual rates of -0.27942%,-1.12581%,-0.19465%,1.10043%,-0.28486% and -1.84133% which correspond to a total compound (average) change of -2.61% (-0.44%).If we use these values in the augmented MDB formula we obtain a monthly lease payment of €1505.64 which is higher by €382.9 (or 35.1%) than the previous one. If the standard MDB formula is used and the predictions of lease rates from our estimated model are realized then the lessor will underprice the lease payment. This translates into a negative monthly internal rate of return of -1.21% (instead of a positive 1%) which corresponds to an annual loss of -13.61% (instead of a 12.68% profit). Using the actual rather than predicted lease rates gives an ex post fair monthly payment of €1578.70 which is close to the estimate from the augmented MDB model. These calculations suggest that our results have significant practical implication for pricing leasing contracts.

5. Conclusions

This paper describes a novel theoretical framework which leads to an interactive relationship between leasing and selling markets for automobiles. This framework extends previous approaches by allowing forward-looking firms set in an oligopoly while leasing and selling are not assumed to be perfect substitutes. The simplest specification justified is a VAR (1) model of lease and sell rates which is estimated using monthly US data. Results confirm a one-way interacting relationship whereby sell rates Granger-cause lease rates. We also show how this interaction can be incorporated within standard lease pricing formulas. A numerical example demonstrates that our findings have non-trivial practical implications for lease pricing.

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