Learning, Fiscal Policy and the Yield Curve

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Abstract

This paper analyzes the effects of changes in government debt on the term structure of interest rates. A structural vector-autoregression is used to estimate the effects of government debt on the yield curve: a 1% rise in real debt to GDP is found to increase the three-month and ten-year rates by 30 and 21 basis points respectively. These effects are difficult to obtain in rational expectations models. They can, however, be partly derived in a general equilibrium model in which the government issues riskless debt and the optimizing agents are adaptive learners. Long-term exponentially maturing debt in the model is calibrated to match the average maturity of U.S. Treasury debt since the 1980s. To test the empirical consistency of the model, the implied term structure of yields is tested for the Expectations Hypothesis; rejections of the Hypothesis, consistent with the U.S. experience, are obtained. Positive effects of government debt on asset yields are generated since the agents do not learn the principle of Ricardian equivalence exactly, and perceive increases in holdings of government bonds as a rise in their net wealth.

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1 Introduction

Does government debt affect the yield curve of interest rates? Anecdotal evidence suggests that there are significant effects of changes in government debt on the term structure. Blinder and Yellen (2001) highlight two U.S. government budget-related episodes that affected the yield curve: in February 1993, Clinton introduced a budget reduction package, and for the U.S. nominal yield curve, the ten year yield fell by 1.3% between October 1992 and 1993. A second Clinton budget agreement in November 1999 declared that the Social Security surplus would be "off-budget" raised the fiscal bar, and the yield curve slope fell by approximately 1%.

An empirical investigation of the relation between yields on bonds and government debt, using vector autoregression analysis (described below), also confirms this. Innovations in the level of public debt issued by the government are found to have significant, positive effects on yields.[2]

When fiscal policy is Ricardian, the intertemporal budget constraint of the government is satisfied, irrespective of the evolution of government purchases and other endogenous variables such as goods and asset prices.[3] That is, fiscal shocks have no effects on the intertemporal budget constraints of optimizing households. In addition, when only lump-sum taxes are imposed, the hypothesis of Ricardian equivalence holds. In such a model, it is difficult to generate any effects of fiscal shocks on asset prices or yields.

In this paper, I explore whether quantitatively significant effects of government debt on yields, of the kind documented below, can be generated in a model with optimizing agents, Ricardian fiscal policy and lump-sum taxes. The central assumption made here is that agents are boundedly rational - they understand their individual decision problems, but need to make forecasts of the variables exogenous to their decisions such as future tax obligations. When these agents have rational expectations, their forecasts of the relevant variables are the same as those predicted by the model.

Under the bounded rationality assumption, this is no longer true: the conditional forecasts of agents are incorrect with respect to the model's predictions. In particular, while it is still true that the present value of the agents' future tax obligations is equal to the value of the government debt, the agents no longer understand this. In this case, increases in government debt are perceived as an increase in net wealth of the agent. As they choose to consume more, yields on government bonds, which are the only means of saving, increase.

The main contributions of the analysis are the following: I first construct a structural vector auto-regression to estimate the effects of government debt on the yield curve of interest rates, using data from the United States for the period $1984 - 2009$. Changes in total public holdings of government debt are found to have statistically significant effects on the yield curve: one quarter after

[2] Prior literature has also attempted to discern the effects of government debt and other fiscal variables on yields. Given the variety of specifications used, the results have been inconclusive.

[3] See Woodford (1998, 2000) for an extensive discussion.

a 1% increase in the real government debt-to-GDP ratio, the three-month interest rate increases by 12 basis points, and the ten-year rate by 7 basis points. These rates rise to 30 and 21 basis points respectively in the third quarter after the shock.

Next, I construct a dynamic stochastic general equilibrium model with boundedly rational agents and long-term, exponentially maturing government debt and examine whether it can replicate the findings of the empirical analysis. As discussed above, although the optimizing agents understand their individual decision problem, they are required to make forecasts of variables exogenous to their decisions by running a regression on the history of observed aggregate variables. In doing so, they make systematic forecasting errors. For the benchmark calibration of the model, it is found that the three-month rate rises by a maximum of 13.1 basis points (in the first quarter) after a 1% increase in the real debt-to-GDP ratio, and the ten-year rate rises by 5.5 basis points.

Given that the learning model partly accounts for the empirical evidence, I illustrate how Ricardian equivalence fails in the model: changes in holdings of government debt are perceived as increases in net wealth.[4] The conditions under which Ricardian equivalence holds are also explored. With very high levels of indebtedness and long-term debt approaching the maturity of consols, standard monetary and fiscal policy configurations - a Taylor rule and Ricardian fiscal policy - may still not guarantee that Ricardian equivalence will be learnt by the agents.

Finally, I analyze the degree to which bond yields generated by the model are consistent with the Expectations Hypothesis. This is motivated by the following: rejections of the Hypothesis have been a robust feature of yield data; since this analysis is interested in generating effects of fiscal shocks on yields, it must also be able to model yields in an empirically consistent way. The model implied term structure, in its benchmark calibration, is able to reconcile the rejection of the Expectations Hypothesis.

Related Literature The implications of fiscal policy in models of boundedly rational agents has been explored elsewhere in the literature. The analysis here is closely related to Eusepi and Preston (2008, 2010), who consider the constraints imposed on monetary and fiscal policies when agents make forecasts of aggregate variables using a recursive least squares learning algorithm. Evans, Honkapohja and Mitra (2008) consider the effects of anticipated fiscal policy in a framework with adaptive learning agents. They find that interest rate dynamics under the learning model differ from those under the rational expectations analog. In a related work, the authors (Evans, Honkapohja and Mitra, 2010) explore conditions under which Ricardian equivalence holds.

The effect of fiscal variables on the yield has also recently been examined in the context of factor models of the term structure. Dai and Phillippon (2006) introduce deficit shocks as an observable factor in addition to other observable factors (corresponding to the output gap, inflation and the federal funds rate), and a latent factor. They find that in response to a positive deficit shock, the

[4] If expectations were rational, Ricardian equivalence would hold.

increase in the long interest rate (ten-year) can be attributed to both a change in the risk premia and as well as the expected short rates.

Bikbov and Chernov (2006) find that the long (ten-year) yield dropped by 1.5% between late 1992 and 1993 (the budget related episodes described in the introduction), and the short yield remained constant.[5] For the second period, they show that the slope of the curve dropped by 2%. Using a factor model, the authors generate this effect on the yield curve using a latent factor that is found to be significantly positively correlated to the annual growth in public debt.

The paper is organized as follows: A description of the data, empirical strategy and results is presented in section two. The model is set up in section three, and section four discusses the quantitative results. Section five presents robustness analyses, and section six concludes.

2 Empirical Methodology

2.1 Data

The following variables are constructed for the U.S. data for the period $1984 - 2009$: detrended log of the per capita real GDP, inflation, the three-month treasury bill rate, and the ten-year nominal yield. The GDP and inflation measures are taken from Bureau of Economic Analysis. A measure of the total debt held by the public is constructed from the Monthly Statement of the Public Debt of the United States, available from the Treasury.[6] I also construct measures of detrended log of the per capita real government expenditures and tax receipts which are used in the robustness analysis in section five. A detailed description of the data and sources used is provided in Appendix A.1. Figures 1 and 2 show the evolution of the different series used in the analysis over the sample period $1984 - 2009$.

2.2 Effect of Government Debt on Yields

The dynamics of the vector $X_t = [debt_t, gdp_t, \pi_t, i_{3mon,t}, i_{10,t}]$ are modelled using a structural vector auto-regression (VAR) with three lags:

$$T\tilde{X}_t = C + A\tilde{X}_{t-1} + \varepsilon_t, \tag{1}$$

[5] The authors use an unsmoothed Fama-Bliss approximation of the zero coupon bond prices of maturities three months to ten years.

[6] The public debt outstanding used here consists of "all federal debt held by individuals, corporations, state or local governments, foreign governments, Government Account Series Deposit Funds, and other entities outside the United States Government less Federal Financing Bank securities. Types of securities held by the public include, but are not limited to, Treasury Bills, Notes, Bonds, TIPS, United States Savings Bonds, State and Local Government Series securities, and Government Account Series Securities held by Deposit Funds." (Monthly Public Statement of Debt, Treasury Direct, February 2010).

where $\tilde{X}_t \equiv [X_t, X_{t-1}, X_{t-2}]$, T has non-zero elements in the first five rows and an identity matrix in the lower triangular of dimension 10×10. A consists of the estimated coefficients from the VAR in the first five rows, and an identity matrix in the lower rows and C is vector of constants. The $debt_t$ variable is real debt outstanding as a fraction of the GDP. In order to isolate effects of changes in current debt on interest rates, the total debt outstanding is divided by last period's price and real GDP.[7] Other variables used in the VAR: gdp_t is the detrended log of the real GDP, π_t is the inflation, and $i_{3mon,t}$ and $i_{10,t}$ are the three-month and ten-year yields respectively. In order to identify the debt shock, the reduced form residual corresponding to real government debt, u_t^d is written as:

$$u_t^d = \mu_{d,gdp} u_t^{gdp} + \mu_{d,\pi} u_t^{\pi} + \mu_{d,i_{3mon}} u_t^{i_{3,mon}} + \mu_{d,i_{10}} u_t^{i_{10}} + \varepsilon_t^d. \quad (2)$$

The μ coefficients are used to denote automatic responses of real debt to innovations in output, prices and interest rates as well as the discretionary responses of policy to the same innovations. Structural debt shocks corresponding to government are denoted by ε_t^d. The elasticities μ are derived from the data. The identification procedure follows Perotti (2004).

Figures 3(a) and 3(b) respectively show the response of the three-month interest rate and the ten-year nominal rate to a 1% rise in the real debt, along with the bootstrap 95% confidence intervals. By construction of the debt measure, the responses isolate the effects of an increase in the debt, and not a response to output or prices. Both the short and long interest rates rise, and the effects are statistically significant, upto four quarters. The most significant increase in both the rates occurs in the third quarter after the positive debt shock occurs. The variance decomposition for the interest rates at different forecast horizons is shown in table 1.

These findings are in line with the literature, although direct comparisons cannot be made due to different specifications. Perotti (2004) finds for a structural VAR, which includes government spending, taxes, output, inflation and the ten-year rate, that the ten-year rate rises in response to the spending shock. Dai and Phillippon (2006) construct a deficit shock and find that a 1% increase in the deficit measure constructed[8] leads to a 35 basis point increase in the ten-year rate after three years. They also find that the ten-year yield rises in response to a pure spending shock. The authors use a factor model (with a latent factor) of the yield curve, unlike the structural VAR used here. Canzoneri, Cumby and Diba (2002) include the ten-year and the federal funds rate, and find that the ten-year yield immediately rises by 45 basis points in response to a spending shock equal to 1% of GDP, and by 40 basis points in the long run. The robustness of the results when government spending and tax shocks are included are shown in section six.

[7] That is, I consider $(B_t/P_{t-1})/GDP_{t-1}$, where B_t is the level of debt outstanding, and GDP_{t-1} is the real GDP.

[8] The authors use Blanchard and Perotti (2002) and Perotti (2004) identification for fiscal shocks, which are used to construct the deficit measure.

Related Literature The above analysis belongs to a large literature which has attempted to determine the effects of fiscal variables on interest rates. I present a brief overview here. Engen and Hubbard (2004) present a survey of several analyses. They make a distinction between regressing measures of changes in the debt or the deficit, on the relevant interest rate measure, and the level of debt. The regression specification with debt is considered to be consistent with their theoretical model. In the regression analysis, the authors find that a rise in the five-year projection of the level of federal government debt increases the forward real ten-year treasury yield in a statistically significant way, as does the five-year projection of the deficit. In other regressions, the authors consider the effect of increases in the current debt and deficit on the current real treasury yield, and do not find any statistically significant effects. Laubach (2003) estimates the relation between long-horizon forecasts of the relevant interest rates and fiscal variables, using projections reported by the Congressional Budget Office and the Office of Management and Budget as proxies for expectations of future fiscal policy. Elmendorf (1993) and Cohen and Garnier (1991) also use projections of fiscal policy, based on the assumption that these act as proxies of private expectations about future fiscal policy. Five- and ten-year projections of future nominal interest rates are computed from the zero-coupon yield curve. Regressions of the future interest rates on the deficit-to-GDP ratio yield positive, statistically significant results. The results are robust to other specifications of fiscal variables and different forecast horizons. Gale and Orzag (2003) contend that fiscal deficits affect interest rates by having an impact on national savings.

Perotti (2004) uses a structural VAR to estimate the effects of fiscal policy on GDP, inflation, interest rates in a set of OECD countries while making identifying assumptions about the effects of government spending and taxes on output, inflation and interest rates. He finds that in response to a positive government spending shock, the ten-year interest rate in the U.S. between $1980:1-2001:4$, rises and then falls.

Evans (1987) and Plosser (1982, 1987) also construct VARs and find no statistically significant effects of deficits on interest rates. Elmendorf (1993) raises the concern that since these VARs focus only on a limited number of variables, and are only backward looking, they may not capture complete effects of fiscal variables on interest rates. Canzoneri, Cumby and Diba (2002) include the ten-year and the federal funds rate, and find positive, statistically significant effects of fiscal shocks on yields.

Romer and Romer (2009) contend that the tax variables identified in the literature are not exogenous, and the tax changes may be simply responding to changes in output. Therefore, the authors use the narrative approach to construct a series of the legislated tax changes which are exogenous. In a three-variable VAR, the authors find that the three-month interest rate responds to tax shocks.

3 Model

The framework consists of households, firms, the central bank and the government, and is an extension of the model used in Eusepi and Preston (2008) and chapter one. This generalizes the former analysis by modelling dynamics of long-term bond prices.

3.1 Optimizing Agents

Households There is a continuum of households $i \in [0,1]$, which maximize discounted sum of future expected utility:

$$\max_{\{C_t^i, B_{1,t}^i, B_{2,t}^i, B_t^i\}} \tilde{E}_t \sum\nolimits_{j=0}^{\infty} \beta^j \left(U(C_{t+j}^i; \xi_{t+j}) - \int_0^1 v(h_{t+j}^i(k); \xi_{t+j}) dk \right). \tag{3}$$

The consumption index, C_t^i, is defined over the consumption of i over the k goods:

$$C_t^i = \left(\int_0^1 c_t^i(k)^{\frac{\theta-1}{\theta}} dk \right)^{\frac{\theta}{\theta-1}}, \tag{4}$$

where θ is the elasticity of substitution, and $c_t^i(k)$ denotes household i's consumption of good k. The aggregate preference shocks are denoted with ξ_t. The household supplies $h_t^i(k)$ hours of work to firm k, and obtains disutility $v(h)$ for doing so. The utility function U is concave and the disutility function v is convex.

Here $\tilde{E}_t$ is used to denote subjective expectations and rational expectations will be denoted using E_t. In this analysis, the deviation from rational expectations is that probability distribution used by optimizing households to form conditional forecasts may differ from the distribution under the true model. That is, while $\tilde{E}_t$ constitutes a complete model that can be used to specify joint probabilities, and satisfies all the probability axioms, it will imply different joint probabilities than the rational expectations operator E_t. The specification of $\tilde{E}_t$ will be described in section 3.4 below. In the present framework, all households and firms (described below) are assumed to have the same subjective expectations $\tilde{E}_t$.[9]

Asset markets are incomplete and the households have access to three kinds of riskless bonds issued by the government. These are of maturities one- and two- periods, and a long-term nominal bond, with an exponentially declining maturity (Woodford, 1998; 2000). This last bond may be considered as a portfolio which consists of assets with an exponentially declining maturity that pay off at later dates. The amount of bonds that are issued in period t, and mature at period $t+j$, is given by $\rho \geq 0$. Then the weight of the bond in the portfolio that pays one unit in the first period after the purchase is one, the weight of the two-period bond is ρ, of the three-period bond is ρ^2 and

[9] The framework can be extended to allow for different subjective expectations among households, that is $\tilde{E}_t^i$.

analogously for the rest. The one-period bond is a special case of this with $\rho = 0$. I will consider an economy that has zero inflation in steady state, and in this case, the duration of the long bond is $1/(1-\beta\rho)$.

Each household i optimally chooses its holdings of the one- and two-period bonds, $B^i_{1,t}$ and $B^i_{2,t}$, and its holdings of the long bond, B^i_t. As households do not own capital, wealth, denoted by $\tilde{W}^i_t$, can only be held in the form of these riskless bonds. Using $P^B_{1,t}$ and $P^B_{2,t}$ to denote the prices of the one- and two-period bonds, P^B_t as the price of the bond portfolio at time t, and T^i_t to represent the lump sum tax obligations of household i, the flow budget constraint of household i is:

$$P_t C^i_t + P^B_t B^i_t + P^B_{1,t} B^i_{1,t} + P^B_{2,t} B^i_{2,t} \leq \tilde{W}^i_t + P_t Y^i_t - T^i_t, \tag{5a}$$

$$\tilde{W}^i_{t+1} = \left[1 + \rho P^B_{t+1}\right] B^i_t + B^i_{1,t} + P^B_{1,t+1} B^i_{2,t} \tag{5b}$$

Here P_t is the composite price index and Y^i_t is the nominal income of the household i:

$$P_t = \left(\int_0^1 p_t(k)^{1-\theta} dk\right)^{\frac{1}{1-\theta}}; \; P_t Y^i_t = W_t h^i_t + \int_0^1 \Pi_t(k) dk. \tag{6}$$

Here W_t is the competitive wage, and $\Pi_t(k)$ denotes the profits from k accruing to the household.

The No-Ponzi condition holds:

$$\lim_{j \longrightarrow \infty} \tilde{E}_t P^B_{1,t,t+j} \tilde{W}^i_{t+j+1} \geq 0. \tag{7}$$

Here $P^B_{1,t,t+j} = \prod_{k=0}^{j} P^B_{1,t+k}$. For the subjective expectations $\tilde{E}_t$, it is assumed that the household cannot make arbitrage profits from its holdings of the government bonds over the infinite horizon.

The timing of the household's decision problem is as follows: given the subjective expectations operator, $\tilde{E}_t$, households make their forecasts of the relevant aggregate variables at time t. Specifically, the households forecast their future incomes, tax obligations, inflation and bond prices. They use these forecasts to make their optimal decisions at time t, which in turn determines market data in period t, such as the market clearing prices of bonds. This data is included in the updated information set of households at time $t+1$, and is used in the construction of conditional expectations of aggregate variables in this period. The subsequent actions follow.

Finally, the optimization problem of household i is to choose $\{c^i_t(k), h^i_t(k), B^i_{1,t}, B^i_{2,t}, B^i_t\}$ for all k to maximize the present discounted sum of utilities subject to the constraints in (5) and (7), taking as given $\{p_{t+j}(k), w_{t+j}(k), \Pi_{t+j}(k), R_{t,t+j}, \xi_{t+j}\}$ for all j, for the subjective expectations operator $\tilde{E}_t$.

Firms There are k differentiated goods, and monopolistic competition among the firms which supply them. The production technology for output $y_t(k)$ of firm k is specified as:

$$y_t(k) = A_t f(h_t(k)), \tag{8}$$

where $A_t > 0$ is the time-dependent, exogenous technology shock, and f is an increasing, concave function. Labor $h_t(.)$ is the variable factor of production. Under monopolistic competition, the demand function faced by firm k is:

$$y_t(k) = Y_t \left(\frac{p_t(k)}{P_t}\right)^{-\theta}, \tag{9}$$

where Y_t is an index of aggregate demand, and the individual firm's price $p_t(k)$ is determined taking Y_t and P_t as given. Following Calvo (1983), $0 < \alpha < 1$ of the good prices remain unchanged every period and the probability of new prices being set $(1-\alpha)$ every period is assumed to be time-independent. The aggregate price index, for optimal price p_t^* is:

$$P_t = \left[(1-\alpha)p_t^{*1-\theta} + \alpha P_{t-1}^{1-\theta}\right]^{\frac{1}{1-\theta}}. \tag{10}$$

The presence of incomplete markets implies the existence of a non-unique discount factor. The firms are assumed to discount future profits using the ratio of the marginal utility of aggregate consumption in the future to the marginal utility of aggregate consumption today:

$$R_{t,t+j} = \beta^j \frac{P_t}{P_{t+j}} \frac{U_C(C_{t+j};\xi_{t+j})}{U_C(C_t;\xi_t)}. \tag{11}$$

All households are assumed to get an equal share of the profit and wage income. Then, the discount factor defined in terms of the marginal valuation of an additional unit of aggregate income by the households implies that in the approximation of the model, the average discount rate will be identical.

Finally, the optimization problem of the firm i is:

$$\max_{p_t(k)} \tilde{E}_t \sum\nolimits_{j=0}^{\infty} \alpha^j R_{t,t+j}(\Pi^i_{t+j}(p_t(i))), \tag{12}$$

where the profit function is $\Pi^i_t(p) = p(Y_t P_t^\theta / p^\theta) - w_t(i) f^{-1}[p(Y_t P_t^\theta / p^\theta)/A_t]$.

In this framework, a firm's problem then is to choose $p_t(i)$ to maximize the discounted sum of profits, taking as given $\{Y_{t+j}, P_{t+j}, w_{t+j}(k), A_{t+j}, R_{t,t+j}\}$ for all j. The firm understands its demand function in (9), and only needs to forecast aggregate demand.

Monetary and Fiscal Authorities The central bank is assumed to perfectly observe current inflation and output, and specifies a Taylor rule for the evolution of the one-period interest rate in the economy. A wider set of monetary policy rules can easily be accommodated in this framework. Here, I restrict my attention to the Taylor rule (Taylor, 1999).

Government expenditures are financed by issuances of riskless bonds and lump-sum taxes. The liabilities of the government at time t are given by:

$$\tilde{W}^s_{t+1} = \left[1 + \rho P^B_{t+1}\right] B^s_t + B^s_{1,t} + P^B_{1,t+1} B^s_{2,t}, \tag{13}$$

and the flow budget constraint of the government is:

$$P^B_t B^s_t + P^B_{1,t} B^s_{1,t} + P^B_{2,t} B^s_{2,t} \leq \tilde{W}^s_t - T_t. \tag{14}$$

Here B^s_t, $B^s_{1,t}$ and $B^s_{2,t}$ are the supplies of the long bond, one-period bond and two-period riskless bonds, and T_t is the total tax collection of the government. The government is assumed to impose lump-sum taxes only.

The full description of the model requires that the amount of liabilities which need to be financed every period by the government, and the composition of debt issued are specified. Taxes are a proportion of the net liabilities outstanding at time t, and only the long-term asset is assumed to be issued in non-zero net supply by the government:

$$\frac{T_t}{P_t} = \bar{T} \left(\frac{(\tilde{W}^s_t / P_{t-1})}{\bar{W}^s} \right)^{\phi_\tau} ; \phi_\tau \geq 0, \tag{15}$$

where $\bar{T}$ is the steady state level of surpluses (as government spending is set to zero). This formulation allows for both passive and active formulations of fiscal policy. The absence of government spending implies that tax receipts are the primary surpluses in the economy. Finally, in the total debt issued, only the long bond is assumed to be in non-zero net supply.

3.2 Equilibrium

The first order conditions from the household $i's$ optimization problem yield:

$$P^B_{1,t} = \tilde{E}_t \left[\beta \frac{U_C(C^i_{t+1}; \xi_{t+1})}{U_C(C^i_t; \xi_t)} \frac{P_t}{P_{t+1}} \right], \tag{16a}$$

$$P^B_{2,t} = \tilde{E}_t \left[\beta \frac{U_C(C^i_{t+1}; \xi_{t+1})}{U_C(C^i_t; \xi_t)} \frac{P_t}{P_{t+1}} P^B_{1,t+1} \right], \tag{16b}$$

$$P^B_t = \tilde{E}_t \left[\beta \frac{U_C(C^i_{t+1}; \xi_{t+1})}{U_C(C^i_t; \xi_t)} \frac{P_t}{P_{t+1}} \left(1 + \rho P^B_{t+1}\right) \right]. \tag{16c}$$

The Euler equations in (16a) and (16b) are derived using the optimal holdings of the one- and two-period bonds for household i. The final Euler equation (16c) relates the price of long bond to the one-period bond price, and future price of the long bond. Assuming that under the subjective expectations operator, the conditional expectation of future prices of the long bond satisfies the condition: $\lim_{j\longrightarrow\infty}\rho^j\tilde{E}_t R_{t,t+j}P^B_{t+j}=0$, the price of the long bond can be expressed as:

$$P_t^B=\tilde{E}_t\left[\sum\nolimits_{j=0}^{\infty}R_{t,t+j}\rho^{j-1}\right], \tag{17}$$

and equivalently written in terms of the $j-$ period bond prices:

$$P_t^B=\tilde{E}_t\left[\sum\nolimits_{j=0}^{\infty}P^B_{j,t}\rho^{j-1}\right]. \tag{18}$$

More generally, following Lucas (1978), the price of any $n-$period riskless bond in zero net supply is:

$$P^B_{n,t}=\tilde{E}_t\left[\beta\frac{U_C(C^i_{t+1};\xi_{t+1})}{U_C(C^i_t;\xi_t)}\frac{P_t}{P_{t+1}}P^B_{n-1,t+1}\right]. \tag{19}$$

Since the expectation of future marginal utility of consumption is different from the rational expectations analog, the perceived path of the one-period interest rate will be different from the corresponding yield under rational expectations.

The household will also choose the labor hours supplied:

$$\frac{v_h(h_t(k);\xi_t)}{U_C(C^i_t;\xi_t)}=\frac{w_t(k)}{P_t}. \tag{20}$$

The intertemporal budget constraint of household i is satisfied, and the total expenditures are optimally allocated by i as:

$$c^i_t(k)=C^i_t\left(\frac{p_t(k)}{P_t}\right)^{-\theta}. \tag{21}$$

For the firm, the optimality conditions entails choosing the optimal price p_t^* :

$$\tilde{E}_t\sum\nolimits_{j=0}^{\infty}\alpha^j R_{t,t+j}Y_{t+j}P^{\theta}_{t+j}\left(p^*_t(i)-\frac{\theta}{\theta-1}P_{t+j}s_{t,t+j}(i)\right)=0, \tag{22}$$

where $s=(v_h(f^{-1}(y/A;\bar{\xi})/U_C(Y;\bar{\xi})A)(1/f'(f^{-1}(y)))$ and $\bar{\xi}_t\equiv(\xi_t,A_t)'$.

Finally, the goods and asset market clearing conditions must hold:

$$\begin{aligned}\int_0^1 C^i_t di &= Y_t; \\ \int_0^1 B^i_t di &= B^s_t,\end{aligned} \tag{23}$$

for every bond issued by the fiscal authority.

For the model described above, an equilibrium is the path of the endogenous processes such that the optimality conditions hold and the market clearing conditions are satisfied, for a given expectations operator $\tilde{E}_t$.

3.3 Approximation

This analysis is concerned with examining the effects of alternative expectations formation on the relation between fiscal policy and the yield curve. To isolate this effect, I abstract from the risk-premia hypothesis by considering a first order approximation of the model around its deterministic steady state. I consider a first order log-linear approximation around the steady state output level $\bar{Y}$, and the price of the portfolio, $\beta/(1-\beta\rho)$. The fiscal authority is assumed to issue the long-term, exponentially maturing riskless bond in non-zero net supply; the one- and two-period assets are issued in zero net supply. The steady state ratio of the net supply of total wealth to output is denoted as $s_W = \bar{W}/\bar{Y}$.

The optimal consumption decision rule for household i, derived in Appendix A.2., is:

$$\begin{aligned}\hat{C}_t^i &= s_c^{-1} s_T \left(\hat{w}_t^i - \hat{\pi}_t\right) \\ &+ s_c^{-1} \tilde{E}_t \sum\nolimits_{j=0}^{\infty} \beta^j \left((1-\beta) \left[\hat{Y}_{t+j}^i - s_T \hat{\tau}_{t+j} \right] - \beta \left(\sigma - s_T\right) \left(\hat{\imath}_{1,t+j} - \hat{\pi}_{t+j+1}\right) \right),\end{aligned} \tag{24}$$

where $\tau_t = T_t/P_t$, $s_c = \bar{C}/\bar{Y}$ and $s_T = \bar{\tau}/\bar{Y}$. Summing consumption and wealth holdings over the the i households, imposing the market clearing conditions, and rewriting the consumption decision rule in terms of the output gap yields:

$$\begin{aligned}\hat{x}_t &= \tilde{E}_t \sum\nolimits_{j=0}^{\infty} \beta^j \left[\begin{array}{c} (1-\beta)\hat{x}_{t+j+1} - \sigma\beta(\hat{\imath}_{t+j} - \tilde{E}_t \hat{\pi}_{t+j+1}) \\ + \hat{r}_{t+j+1}^n \end{array} \right] \\ &+ s_T \left[\begin{array}{c} \frac{(\hat{w}_t - \hat{\pi}_t)}{\beta} - \frac{\hat{\tau}_t}{\beta} \\ + \tilde{E}_t \sum_{j=0}^{\infty} \left[(\hat{\imath}_{1,t+j} - \tilde{E}_t \hat{\pi}_{t+j+1}) - (1-\beta)\hat{\tau}_{t+j+1} \right] \end{array} \right].\end{aligned} \tag{25}$$

Here, since government debt is issued in non-zero net supply, it appears in the determination of the current output gap. The log deviations of government debt are denoted using $\hat{w}_t = \log((W_t/P_{t-1})/\bar{W})$.

Under rational expectations, Ricardian equivalence holds: the households recognize that intertemporal budget constraint of the government must be satisfied. That is, the present value of debt is equal to the discounted sum of future tax collections. In determining permanent income effects from the optimal consumption decision rule in (25), the intertemporal budget constraint of the government, captured by the second term, is zero. The intertemporal optimization of the households ensures the budget constraint of the government holds, and this applies regardless of the specification of fiscal policy. In this case, any increases in the net issuance of the government debt will not affect the consumption decisions of the households.

When expectations are near-rational, household i is required to form forecasts of its future tax obligations, and the evolution of the bond prices, conditional on its information set. Out of the rational expectations equilibrium, it is no longer required for each household to understand that Ricardian equivalence must hold - an individual household may not recognize that its tax obligations are the same as all other households, and that the present value of government debt must equal the discounted sum of tax obligations. As it makes small expectational errors in its conditional forecasts of the state variables, government bonds will be *perceived* as net wealth, in the context of Barro (1974).

Term Structure From the Euler equation in (16b), the price of a two-period bond is:

$$\hat{P}^B_{2,t} = \left[\hat{P}^B_{1,t} + \tilde{E}_t \hat{P}^B_{1,t+1}\right], \tag{26}$$

and this is generalized to the price of an $n-$period bond as:

$$\hat{P}^B_{n,t} = \left[\hat{P}^B_{1,t} + \tilde{E}_t \hat{P}^B_{n-1,t+1}\right]. \tag{27}$$

This can be formulated in terms of one-period bond prices as:

$$\hat{P}^B_{n,t} = \left[\hat{P}^B_{1,t} + \tilde{E}_t \hat{P}^B_{1,t+1} + ... + \tilde{E}_t \hat{P}^B_{1,t+(n-1)}\right]. \tag{28}$$

The corresponding $n-$period interest rates are:

$$\hat{\imath}_{n,t} = \frac{1}{n}\left[\hat{\imath}_{1,t} + \tilde{E}_t \hat{\imath}_{1,t+1} + ... + \tilde{E}_t \hat{\imath}_{1,t+(n-1)}\right]. \tag{29}$$

as $\hat{\imath}_{n,t} = -\hat{P}^B_{n,t}/n$. This is log pure version of the Expectations Hypothesis, with the subjective expectations operator $\tilde{E}_t$.

Firms The full linearization of the optimization problem of the firm is derived in appendix A.2. The optimal price p^*_t can be written as:

$$\hat{p}^*_t = \tilde{E}_t \sum\nolimits_{j=0}^{\infty} (\alpha\beta)^j \left[\frac{1-\alpha\beta}{1+\omega\theta}(\omega + \sigma^{-1})\hat{x}_{t+j} + \hat{\pi}_{t+j}\right]. \tag{30}$$

This is rewritten using the price index as:

$$\hat{\pi}_t = \kappa \hat{x}_t + \tilde{E}_t \sum\nolimits_{j=0}^{\infty} (\alpha\beta)^j \left[\kappa\alpha\beta\hat{x}_{t+j+1} + (1-\alpha)\beta\hat{\pi}_{t+j+1}\right], \tag{31}$$

where $\kappa = ((1-\alpha)/\alpha)((1-\alpha\beta)/(1+\omega\theta))(\omega+\sigma^{-1}) > 0$, and ω is the elasticity of the marginal cost of production to the output (also defined in A.2.).

Monetary Authority The one-period interest rate evolves according to the rule:

$$\hat{\imath}_{1,t} = \bar{\imath}_t + \phi_x \hat{x}_t + \phi_\pi \hat{\pi}_t, \tag{32}$$

where $\bar{\imath}_t$ is stochastic intercept term, and is denoted as the monetary policy shock.

Fiscal Authority The government flow budget constraint is approximated as:

$$\hat{w}_t = \frac{1}{(1-\frac{\bar{\tau}}{\bar{w}})} \left(\hat{w}_{t-1} - \hat{\pi}_{t-1} - \frac{\bar{\tau}}{\bar{w}} \hat{\tau}_{t-1} \right) + \left(\rho \hat{P}_t^B \bar{P}^B - \hat{P}_{t-1}^B \right). \tag{33}$$

This can also be expressed in terms of the one-period interest:

$$\begin{aligned} \hat{w}_t \quad &= \quad \frac{1}{(1-\frac{\bar{\tau}}{\bar{w}})} \left(\hat{w}_{t-1} - \hat{\pi}_{t-1} - \frac{\bar{\tau}}{\bar{w}} \hat{\tau}_{t-1} \right) \\ &\quad + \left[\frac{(1-\beta\rho)}{1-\rho} \hat{\imath}_{t-1} - \left(\frac{\beta^2\rho^2}{1-\rho} \right) \sum\nolimits_{j=0}^{\infty} (\beta\rho)^j \, \hat{\imath}_{1,t+j} \right]. \end{aligned} \tag{34}$$

In case of only one-period debt, the wealth of the government is pre-determined at time t, in contrast to the case with long-term debt issuances. Finally, the approximation of tax policy yields:

$$\hat{\tau}_t = \phi_\tau \hat{w}_t. \tag{35}$$

3.4 Adaptive Learning

The complete description of the model requires specifying a forecasting model for the optimizing agents, which can be used to construct forecasts of the variables exogenous to the optimization problems of households and firms: output gap, inflation, government debt, the interest rates on different riskless bonds, taxes and the vector of exogenous disturbances $r_t = (\hat{r}_t^n, \bar{\imath}_t)'$.

From (18), the price of the bond portfolio can be expressed as a sum of the $j-$ period bond prices. These are a function of the one-period bond price (following from (28)), and subsequently, of the one-period interest rate. Additionally, the interest rates on the bonds of different maturities are a sum of the one-period interest rate over the maturity of the bonds (from (29)). That is, under the subjective beliefs of the household, the Expectations Hypothesis of the term structure holds. Therefore, in subsequent analysis, I assume that only the one-period interest rate needs to be forecasted by the agents. Conditional expectations of the interest rates corresponding to longer bonds are constructed using (29).

Given this assumption, the set of variables that is forecasted by the optimizing agents is $z_t \equiv \{\hat{x}_t, \hat{\pi}_t, \hat{w}_{t+1}, \hat{\imath}_{1,t}, \hat{\tau}_t\}$, along with the exogenous disturbances in r_t. Agents are assumed to include the total value of outstanding government debt, $\hat{w}_{t+1}$, in z_t since it will help them in better forecasting the evolution of other variables, such as taxes. I assume that the disturbances are independently and identically distributed processes and are known by the optimizing agents.

Formation of Expectations Beliefs are formed using least squares learning dynamics: agents run a linear regression of the past observed variables to be forecasted on the corresponding history of the vector of variables that can be used as the basis for a forecast in the future. Before describing the evolution of beliefs, and the forecasting rule used by the agents, it is useful to discuss the form of the rational expectations equilibrium. Since only Ricardian fiscal policy will be considered here,[10] the following proposition presents the conditions required for the determinacy of the rational expectations equilibrium:

Proposition 1 *There exists a unique determinate rational expectations equilibrium if and only if*

$$1 < \phi_\tau < \frac{1+\beta}{1-\beta} \text{ and } \kappa(\phi_\pi - 1) + \phi_x(1-\beta) > 0. \tag{36}$$

Proof. Appendix A.3. ■

Therefore, under the policy configuration considered, the non-fiscal endogenous variables (output gap, inflation and the one-period interest rate) will only depend on the exogenous disturbances, $r_t = (r_t^n, \bar{\imath}_t)$. This is the Minimum State Variable form of the rational expectations equilibrium.

Following Evans and Honkapohja (2001) and Eusepi and Preston (2008), I assume that the perceived data generating process for the variables to be forecasted corresponds to the Minimum State Variable form:

$$z_t = a_t + b_t\hat{w}_t + c_t r_{t-1} + \eta_t, \tag{37}$$

where $a_t = [a_t^{\hat{x}}, a_t^{\hat{\pi}}, a_t^{\hat{w}}, a_t^{\hat{\imath}_1}, a_t^{\hat{\tau}}]'$ is used to denote the households uncertainty about the average of the aggregate variables. The b_t matrix indicates how these variables depend on real government debt outstanding in period t, and c_t denotes the dependence on the state variables in r_t. The η_t matrix is a vector of i.i.d. shocks, and η_{t+1} is assumed to be unforecastable in period t. For instance,

$$\hat{\imath}_{1,t} = a_t^i + b_t^{i,w}\hat{w}_t + c_t^{i,r} r_{t-1} + \eta_t^i, \tag{38}$$

is the perceived data generating process for the one-period interest rate. Under Ricardian fiscal policy, the Minimum State Variable solution will imply $a_t^i = b_t^{i,w} = 0$.

[10] An analysis of active and passive formulations of government policy with adaptive learning is presented in Eusepi and Preston (2008).

Updating of Beliefs Given the perceived data generating process in (37), after observing current data, households update their estimates of $\Omega_t = \{a_t, b_t, c_t\}$ using a recursive least squares estimator, following Marcet and Sargent (1989). The algorithm is written as:

$$\begin{aligned}\Omega_t &= \Omega_{t-1} + g^{-1}\Upsilon_{t-1}^{-1} q_{t-1}[z_t - \Omega_{t-1}' q_{t-1}]'; \\ \Upsilon_t &= \Upsilon_{t-1} + g^{-1}[q_{t-1}q_{t-1}' - \Upsilon_{t-1}],\end{aligned} \tag{39}$$

where $q_{t-1} = [1, \hat{w}_{t+1}, r_t]_{t=0}^{t-1}$, and Υ_t is the variance-covariance matrix of the coefficients in Ω_t. The only difference from the rational expectations case is the updating or gain coefficient, g, which controls the rate at which new information affects beliefs. A constant gain parameter g implies that the household puts greater weight on the more recent observations in the updating procedure.

Using the recursive estimator, subjective forecasts of z_t are formed. For instance, the one-period ahead forecast is:

$$\tilde{E}_t z_{t+1} = a_{t-1} + b_{t-1}^{i,w}\hat{w}_{t+1} + c_{t-1}^{i,r} r_t, \ \forall n \geq 1. \tag{40}$$

This corresponds to the households running a constant coefficient vector autoregression to form their beliefs: at time t, they do not take into account the fact that their belief coefficients will be updated in the future. This modelling strategy can be justified using an anticipated utility argument as in Kreps (1989), and has been widely used in the learning literature (Evans and Honkapohja, 2001 and Sargent, 1993).

True Data Generating Process Substituting forecasts of the vector z_t in (40) into the structural relations determining the aggregate dynamics of the output gap and inflation, one-period interest rate, government liabilities and the tax rule yields the actual data generating process for z_t, consistent with the process perceived by the households. The true data generating process is:

$$z_t = T^0(a_{t-1}) + T^w(b_{t-1})\hat{w}_t + T^r(c_{t-1})r_{t-1}. \tag{41}$$

For this data generating process, the one-period ahead conditional expectations are:

$$E_t z_{t+1} = T^0(a_{t-1}) + T^w(b_{t-1})\hat{w}_{t+1} + T^r(c_{t-1})r_t. \tag{42}$$

The T matrices are functions of the model parameters.

Finally, the model of the economy can be summarized: equilibrium dynamics determined by (25) and (31), the determination of asset prices in (29), the interest rate rule in (32), the fiscal budget constraint and tax rule in (33) and (35), along with the system of equations used for forecasting in (39) and (40).

Expectational Stability The fixed point of the $T-$mappings in (41) is a self-consistent equilibrium: beliefs generating the data must confirm those beliefs. This corresponds to the rational expectations equilibrium when the class of forecasting models is such that the optimal forecasting rule given subjective beliefs (such as in (40)) belong to this class.

The self consistent equilibrium is Expectationally Stable (E-stable) if it is the locally stable rest point of the dynamics defined by the ordinary differential equation:

$$\dot{\Omega} = T(\Omega) - \Omega. \tag{43}$$

This ensures that the households' beliefs about the right forecasting model evolve over time to correct the discrepancy between their current beliefs given by (a_t, b_t) and the actual data that is generated as a result of their beliefs given by $T(a_t, b_t)$. Thus, conditions can be determined so that households will asymptotically converge to the rational expectations equilibrium.

Proposition 2 *The conditions for determinacy of the rational expectations equilibrium are necessary and sufficient for Expectational Stability when $\rho = 0$ and $\alpha \longrightarrow 0$.*

Proof. Eusepi and Preston (2010, Technical Appendix). ■

The above proposition also establishes the conditions for Ricardian equivalence. When the conditions for Expectational Stability are satisfied, Ricardian equivalence will only hold in case of decreasing gain adaptive learning, once the belief coefficients have converged to their rational expectations analogs. When the agents put a small, constant positive weight on past observations, Ricardian equivalence will not hold as the intercept and slope coefficients converge to a stationary distribution, and remain dispersed around the rational expectations means. That is, agents will continue to make systematic forecasting errors, and perceive holdings of government debt as net wealth.

Since from (29) long interest rates are linear functions of the one-period yield and its expected future realizations, $\hat{\imath}_{1,t}$ is the only relevant yield to be forecasted. Therefore, the conditions for determinacy of the rational expectations equilibrium, as well as E-stability under adaptive learning, apply here also. With positive exponentially maturing debt ($\rho > 0$), however, the policy configurations of an active monetary policy along with Ricardian fiscal policy need not imply E-stability. In section (4.2) below, I discuss the regions in which this occurs.

4 Results and Analysis

4.1 Model Parameters

Three sets of model parameters must be calibrated for the numerical analysis - the parameters of the New-Keynesian model along with the properties of the exogenous shocks; the gain parameter;

the average maturity of the debt issued by the fiscal authority, and the steady state level of debt. For the first set of parameters, I use the values from the New-Keynesian literature as in chapter one: the frequency of price adjustment, α is 0.75, corresponding to a yearly price adjustment by firms; the discount factor β is 0.99, implying a quarterly real interest rate of approximately 4%. The intertemporal elasticity of substitution is set at one. The shock processes have been assumed to be identically and independently distributed, and for the preference, technology and monetary policy shocks, standard deviations of the are 0.05, 0.01 and 0.02 respectively. These are taken from the estimates obtained by Rabanal and Rubio-Ramirez (2005) for the U.S., assuming that a rational expectations DSGE model is the true representation of the U.S. economy.

There is little consensus in the literature on how the gain should be calibrated, and several attempts have been made at disciplining this parameter. Orphanides and Williams (2005) use a gain of 0.02, while Eusepi and Preston (2008*b*) estimate a value of 0.002 for a real business cycle model with constant gain adaptive learning. The gain is a central parameter in model - a very small positive gain, such as $g = 0.002$, implies that a near constant weight is put on past observations, while a higher gain of 0.02 implies that a weight of 0.55 is put on an observation thirty quarters in the past.

In order to calibrate the gain parameter for the present analysis, the systematic forecasting errors generated by the model are used. The Survey of Professional Forecasters (SPF), administered by the Federal Reserve Bank of Philadelphia, reports forecasts of aggregate economic variables such as inflation and the ten-year rate for different forecasting horizons, as well as the corresponding forecasting errors made. The gain can be chosen to minimize the deviations between different moments of survey data errors and the learning model can be used. Here, I use the gain that minimizes the deviation in the autocorrelation of the one-quarter ahead forecast errors in inflation. For SPF data, the autocorrelation in one-period ahead forecast errors for CPI inflation is 0.12 for the period $1990 : 2 - 2008$. For a gain of 0.006, the model implied autorcorrelation in one-period ahead forecast errors in inflation is 0.105. This is the benchmark gain for the following analysis.

Finally, the average maturity of government debt issued, and the steady state level of the debt-to-GDP ratio must be calibrated. For the U.S., the average maturity of Treasury debt was approximately twenty quarters (although it has recently declined to 16 quarters). Therefore, I will set the ρ parameter to imply an average duration of debt equal to twenty quarters, that is at 0.95. The benchmark annual steady state level of debt is set at 40% of the GDP.

For numerical simulations, I initialize the model at the rational expectations beliefs. To ensure that the results of adaptive learning analysis do not depend on the transitory dynamics, as the dynamics of the endogenous variables converges to the rational expectations equilibrium, the following analysis is considered only in the region where the dynamics have converged to a stationary distribution around the rational expectations equilibrium. The model is simulated for 3000 periods for 1000 draws, and its implications are considered for the last 100 periods, corresponding to 25 years of data.

4.2 E - Stability Analysis

As mentioned above, there are policy configurations for which the model may not be E-stable, even though fiscal policy is Ricardian and monetary policy follows the Taylor rule. Figure 4 shows combinations of the steady state level of debt and average maturity of government debt issued for which the E-stability occurs. For very long maturity debt, as ρ becomes closer to one (a consol will imply $\rho = 1$), and a high ratio of steady state debt to output, the economy would not converge to a stationary distribution around the rational expectations equilibrium, for small constant gains.

In the regions where the E-stability condition is satisfied, distribution of the intercept and slope coefficient converges to a stationary distribution, normally distributed around the rational expectations equilibrium. Households' beliefs about the averages of the aggregate variables are dispersed around the rational expectations mean, exhibiting the self-referential nature of the model.

4.3 Effects of Fiscal Shocks

In order to determine the effects of increases in government debt on endogenous variables in the learning model, I use dynamic response analysis. The median responses of the variables $\{\hat{x}_t, \hat{\pi}_t, , \hat{\imath}_{1,t}, \hat{\imath}_{10,t}\}$ following a 1% increase in government debt, $\hat{w}_{t+1}$, are analyzed.

Figures 5(a) and 5(b) report the dynamics of the short and long interest rates (the three-month and the ten-year rates), and the corresponding responses for output and inflation. Under rational expectations, these effects will be zero for all periods, as Ricardian equivalence holds - agents understand that an increase in holdings of government debt will not lead them to alter their consumption and savings decisions. Therefore, neither the short nor long yields change in response to the positive debt shock. In the first period of the increase in debt, the responses under the learning model and rational expectations are approximately identical as conditional forecasts of future aggregate variables are still made using coefficient estimates (of a and b) from the past period. That is, the effect of the debt shock is approximately zero.

In the period after the impact under learning, however, the increase in government debt is perceived by households as an increase in the net wealth, and leads to an increase in consumption. This is seen in the response of the output gap in figure 5(c). As the optimizing households choose to consume more, actual prices of riskless bonds must fall, and the corresponding yields rise. Although both the short and long yields rise, the increase in the latter is smaller since the longer yields are an average of the short yields over the maturity of the bond. In the first quarter after the debt shock, the short yield rises by 13.1 basis points, and the long yield rises by 5.5 basis points. Due to the forecasting rule used in (40), all the persistence in dynamics of the variables under consideration is due to the evolution of debt dynamics in the learning model.

When the level of indebtedness increases, keeping all other parameters at their original values, the qualitative implications of the model are similar although the magnitude of responses are larger than the benchmark case considered. When the annual debt-to-GDP ratio is 72% the three-month rate rises by 21 basis points and the ten-year rate rises by 9 basis points in the first quarter after the shock.

For the benchmark level of indebtedness, as the average maturity of debt shortens, the responses become smaller. In the case when only one-period debt is issued in non-zero supply ($\rho = 0$), the three-month rate rises by 9 basis points, and the long rate rises by 2 basis points.

How does Learning Work? In order to isolate the effects of learning, aggregate output gap is expressed as a function of the net real value of debt holdings, and the permanent income term that would obtain in case of rational expectations in (25).[11] I consider the case with one-period debt, to examine the impact of a positive debt shock on consumption (and subsequently on interest rates). Figure 6 shows the response of the net real value of wealth holdings in response to a 1% increase in wealth.

In response to the positive debt shock, the net real value of wealth holdings increases in the learning model. This is in contrast to the rational expectations analog of the model, in which the positive wealth shock does not change the real value of holdings. The consumption of the household therefore does not change, as it does in the learning model. Subsequently, savings decisions remain unaffected under rational expectations. In the learning model, as savings fall, the yields on riskless bonds rise.

Connections with the literature The results found here can be related to findings of factor models of the yield curve that use fiscal variables. Bikbov and Chernov (2004) explicitly distinguish between the contribution of real activity and inflation to the level, slope, and curvature of the yield curve, and of the residual factors. Of these residual factors, which are orthogonal to the macro variables, one is found to be correlated with the growth in public debt. The residuals explain 50% of the slope of the yield curve, and the effect of the latent factor related to public debt is most evident for long yields. The authors find that the fiscal shock contributes the most deviations from the Expectations Hypothesis - that is, the effect of fiscal shocks is manifested at the long end of the yield curve by affecting the risk premia.

Dai and Phillippon (2006) introduce a deficit shock explicitly as a factor in their no-arbitrage factor model consisting of the help wanted index, inflation, the federal funds rate and a latent factor. They find that the response of the ten-year rate to a positive deficit shock can be decomposed into the effect on the expectations of short yields, and the risk premia. After four years, the positive risk premia explains up to a third of the increase in long yields. Therefore, in factor models of

[11] This decomposition is used by Eusepi and Preston (2008a).

yield curve, changes in risk premia have been identified as the primary channel through which fiscal variables affect the slope of the yield curve and long yields.

Eusepi and Preston (2010*b*) analyze the effects of a shock to inflation expectations, when agents are assumed to have full knowledge of the monetary policy regime, and fiscal policy is Ricardian. The impact effects of output, inflation and interest rates are smaller and more persistent when the economy has a higher level of steady state indebtedness, relative to the low indebtedness regime.

4.4 Implications for the Yield Curve: Testing the Expectations Hypothesis

To test the empirical consistency of the model on another dimension, I test the predictions of the model for the Expectations Hypothesis. I test whether the Hypothesis is rejected here, as it is found to be in U.S. data. I also show the predictions for the Expectations Hypothesis at different levels of indebtedness and varying average durations of the long-term debt issued. This is motivated by two facts: the real debt-to-GDP ratio of the U.S. economy has varied over the post-war period (from approximately 30% to 65%) and the average duration of U.S. Treasury debt has shortened from 116 months in 1945 to 60 months in 2009. As documented in chapter one, the Campbell-Shiller coefficients in the U.S. have varied between 1972 − 2009. While it would be of interest to explore what the empirical coefficients are for the U.S. when the debt levels are varying, the sample periods are too small to generate precise results. Therefore, I only present the predictions of the model for the yield curve for different debt and maturity levels. The model predicts that the deviations from the Expectations Hypothesis increase, when for a given level of indebtedness, the average maturity of debt increases. Alternatively, holding the average maturity of debt constant, as the level of indebtedness increases, the deviations get larger.

The Expectations Hypothesis is tested using the Campbell-Shiller regression:

$$\hat{\imath}_{n-1,t+1} - \hat{\imath}_{n,t} = \bar{\alpha} + \frac{\lambda}{n-1}(\hat{\imath}_{n,t} - \hat{\imath}_{1,t}) + e_t. \tag{44}$$

If the Expectations Hypothesis holds, the slope coefficient λ is statistically not different from one. However, for U.S. nominal data, λ has been found to be less than one at short maturities, and negative at longer maturities. A widely accepted interpretation of this finding has been that the regression does not allow for a time-varying risk premia,[12] and the omission of this variable leads to a bias in the slope coefficient. Chapter one shows that these rejections can be attributed to the misspecification of the regression in (44), that arises due to the assumption of rational expectations. In an optimizing framework, when agents use adaptive learning to form their conditional expectations about long yields, they make systematic forecasting errors, leading to a violation of the orthogonality condition between the regression error and the short yield in (44). Since the regression does not allow for these systematic errors, there exists a downward bias in the slope

[12] The intercept $\bar{\alpha}$ corresponds to a constant risk-premia.

coefficient. The implications of the model presented in this paper, with respect to the Expectations Hypothesis rejections, are analogous. That is, constructing long yields from the analysis, and using these for the regression in (44) leads to a negative bias with respect to one in γ.

Using the dynamics of the one-period interest rate generated by the learning model, I construct the long yield using (29). These are used to test the Expectations Hypothesis using the Campbell-Shiller regression in (44).

Table 2 reports the slope coefficients for the Campbell-Shiller regression, along with the percentage of times the Expectations Hypothesis is rejected, for the benchmark gain.

The implications for estimates of λ when (a) the level of indebtedness varies, and (b) the average duration of debt issued changes are now analyzed. I consider low and high levels of indebtedness (the annual ratio of debt-to-GDP varies between zero and seventy percent of the GDP), and different average maturities of government debt. Setting $\rho = 0$ implies that only one-period debt is being issued by the government, while $\rho = 0.95$ matches the average maturity structure of U.S. debt. The resulting Campbell-Shiller slope coefficients and the frequency of rejections of the Expectations Hypothesis are shown in table 3.

When the steady state level of debt-to-GDP is zero, the terms in the optimal consumption decision rule in (25) corresponding to future expectations about the real tax obligations will be zero, and smaller forecasting errors are made by the agents. Subsequently, the coefficient estimates for γ are less negative than for the benchmark model. This is also indicated by the autocorrelation in forecast errors of inflation, which falls to 0.07 from 0.105, and there are smaller feedback effects than the benchmark case. As the level of indebtedness increases, however, the coefficients become more negative.

Alternatively, if only one-period debt is issued by the government in non-zero supply, wealth at time t is a pre-determined variable, for any level of indebtedness. In this case, the λ coefficients for the (44) regression are more than the corresponding estimates for the benchmark case. Intuitively, as the average maturity of debt shortens, the agents need to forecast their debt holdings over a shorter time horizon, and make smaller systematic forecasting errors relative to the case where very long debt is issued. The bias in the slope coefficient, with respect to one, is smaller. For a given level of steady state debt, as the average duration of government debt increases (encapsulated by an positive, increasing ρ), the point estimates of the slope coefficients become more negative.

5 Robustness Analysis

To compare the empirical results in section 2.2 with the literature, I analyze the responses of short and long yields when government spending or tax shocks are included in the structural VAR: the relevant vector is $X_t = [debt_t, g_t, gdp_t, \pi_t, i_{3mon,t}, i_{10,t}]$. Here g_t is detrended log of the per capita real government expenditures. In order to identify these fiscal shocks, I follow the strategy used in

Blanchard and Perotti (2002) and Perotti (2004). The reduced form residual for real government spending, u_t^g is written as:

$$u_t^g = \mu_{g,gdp} u_t^{gdp} + \mu_{g,\pi} u_t^{\pi} + \mu_{g,i_{3mon}} u_t^{i_{3,mon}} + \mu_{g,i_{10}} u_t^{i_{10}} + \varepsilon_t^g. \tag{45}$$

As above, μ coefficients are used to denote automatic responses of government spending to innovations in output, prices and interest rates as well as the discretionary responses of policy to the same innovations. Structural fiscal shocks corresponding to government are denoted by ε_t^g. The key identification step then involves specifying the elasticities μ. The within-quarter response of government spending to real GDP within the quarter is assumed to be zero, as Blanchard and Perotti (2002) argue that there is no evidence of substantial automatic response of government spending to GDP. For the U.S. data, Perotti (2004) estimates that the price elasticity $\mu_{g,\pi}$ is -0.5 and the interest elasticity $\mu_{g,i_{3mon}}$ is zero.

For a 1% increase in government debt, the three-month rate increases by 13 basis points, and the ten-year rises by 5 basis points in the first quarter after the increase. The maximum increase in the ten-year rate (it rises by 14 basis points) occurs in the second quarter after the rise in government debt (figure 7). The effects are significant only upto the third quarter after the initial increase in government debt. Table 4 shows the corresponding variance decompositions. Changing the elasticity of inflation with respect to government spending will alter the effect of government shocks on inflation and therefore the ten-year yield. However, increasing the elasticity of government spending to inflation from -0.5 to -0.25 or lowering it to -0.75 is found to change the response of yields to debt shocks in a negligible way.

When tax receipts are introduced into the VAR specification, instead of government spending, elasticities of net taxes to output and prices must be specified. To identify the tax shocks, the reduced form residuals corresponding to real net taxes u_t^{τ} are analogous to real government spending:

$$u_t^{\tau} = \mu_{\tau,gdp} u_t^{gdp} + \mu_{\tau,\pi} u_t^{\pi} + \mu_{\tau,i_{3mon}} u_t^{i_{3,mon}} + \mu_{\tau,i_{10}} u_t^{i_{10}} + \varepsilon_t^{\tau}. \tag{46}$$

For the period $1960 : 1 - 2001 : 4$, Perotti (2004) estimates $\mu_{\tau,gdp}$ to be 1.85 and $\mu_{\tau,\pi}$ as 1.25 respectively.[13] The interest elasticity of net taxes is assumed to be zero.[14] The maximum increase in the three-month interest rate, in response to a 1% increase in debt is 12 basis points in the first quarter after the increase. The ten-year rate rises by 10 basis points in the first quarter of the increase, and the maximum response is in the second quarter at 21 basis points. The responses are

[13] For two subsamples, 1960:1-1980:1 and 1980:2-2001:4, the elasticities are $(1.75, 1.09)$ and $(1.97, 1.40)$ respectively.

[14] The original Perotti (2004) exercise uses the ten-year interest rate, instead of the three-month rate used here. Constructing the structural VAR with the long interest rate does not lead to a significant difference in the dynamic response analysis. Also, the assumption about zero contemporaneous effects of taxes on the interest rate is not uncontroversial. See Blanchard and Perotti (2002) and Perotti (2004) for discussion.

significant upto four quarters. Thus, the results documented in the empirical analysis above are robust to reasonable changes in parameter values and specifications.

It may be noted here that the identification of fiscal shocks using the Blanchard and Perotti (2002) and Perotti (2004) approach is not uncontroversial. The narrative approach of Ramey and Shapiro (1997) to identify spending shocks, and more recently, of Romer and Romer (2009) to identify tax shocks may provide a cleaner analysis. Dai and Phillippon (2006) use Ramey and Shapiro dates, and find that the nominal short rate increases in response to a deficit shock, and the nominal long rate increases over time.

6 Conclusions

The preceding analysis has attempted to provide a theoretical motivation for the reported empirical effects of fiscal variables on the yield curve of interest rates. The central feature of the model is that optimizing make systematic forecasting errors while forming conditional expectations about future fiscal policy. In this case, government debt is perceived as net wealth by the agents, using these incorrect forecasts to make optimal consumption and savings decisions, leads to variations in yields in response to changes in debt. Such effects are absent in the rational expectations analog of the model considered here, as the agents understand that Ricardian equivalence holds exactly.

One unexplored feature of the model so far has been the effects on yields of expected deficits. As Laubach (2003) reports, increases in expected deficits have a positive, statistically significant effect on the yield curve. This could be accommodated in the present framework by allowing the forecasting rule to assign a non-trivial weight on a measure of expected deficits. This is left to future research.

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Appendices

Appendix A.1.

Variable	Source
Real Gross Domestic Product (Chained 2005 Dollars)	Bureau of Economic Analysis (BEA) Table 1.1.6
Government expenditures and receipts	BEA, Tables 3.2, 3.3, 3.9.5
Interest Rates	Board of Governors of the Federal System
Public Debt	Monthly Statement of Public Debt, Treasury Direct

Government expenditures consist of federal and state expenditures (lines 6 and 21 in table 3.9.5 respectively). Federal receipts are the difference between current receipts (line 1) and sum of transfers to persons (line 23), grants-in-aid to state and local governments (line 26) and interest payments (line 28) in table 3.2. State receipts are computed as the difference between current receipts (line 1) and sum of government social benefits to persons (line 23) and interest payments (line 24) in table 3.3.

Appendix A.2.

The first order approximation of the optimality conditions of the households and firms is described below. The linearization is constructed around the following steady state: $\xi = 0$, $Y_t = \bar{Y}$ (defined below) and $\bar{P}_1^B = \beta$ (or $\bar{\imath}_1 = (1-\beta)/\beta$) with $\bar{\pi} = 1$. The hat variables denote the log deviations of the respective variable from its steady state value. For the one-period interest rate, the log deviation is defined as $\hat{\imath}_{1,t} = \log[(1 + i_{1,t})/(1 + \bar{\imath}_1)]$.

The first order approximation of the household's Euler equation for the one-period asset price in (14) yields:

$$\hat{C}_t^i = \tilde{E}_t \hat{C}_{t+1}^i - \sigma(\hat{\imath}_{1,t} - \hat{\pi}_{t+1}) + (g_t - \tilde{E}_t g_{t+1}). \tag{47}$$

The flow budget constraint in (6) is iterated forwards, and its approximation is:

$$\tilde{E}_t \sum\nolimits_{j=0}^{\infty} \beta^j \hat{C}_{t+j}^i = \hat{W}_t^i + \tilde{E}_t \sum\nolimits_{j=0}^{\infty} \beta^j \hat{Y}_{t+j}^i. \tag{48}$$

Substituting (47) recursively into (48) yields (22):

$$\hat{C}_t^i = (1-\beta)\hat{W}_t^i + (1-\beta)\tilde{E}_t \sum\nolimits_{j=0}^{\infty} \beta^j \left[\hat{Y}_{t+j}^i - \sigma\beta(\hat{\imath}_{1,t+j} - \hat{\pi}_{t+j+1}) + \beta(g_{t+j} - g_{t+j+1})\right].$$

To obtain (23), apply the market clearing conditions in (21):

$$\hat{Y}_t = (1-\beta)\tilde{E}_t \sum\nolimits_{j=0}^{\infty} \beta^j \left[\hat{Y}_{t+j} - \sigma\beta(\hat{\imath}_{1,t+j} - \hat{\pi}_{t+j+1}) + \beta(g_{t+j} - g_{t+j+1})\right] \tag{49}$$

The output gap is defined as $\hat{x}_t = \log(Y_t/Y_t^n)$, and is used to rewrite (49) as:

$$\hat{x}_t = \tilde{E}_t \sum_{j=0}^{\infty} \beta^j \left[(1-\beta)\hat{x}_{t+j+1} - \sigma\beta(\hat{\imath}_{1,t+j} - \tilde{E}_t\hat{\pi}_{t+j+1}) + \hat{r}_{t+j+1}^n\right], \tag{50}$$

which is the expression in (23), and the natural rate of interest $\hat{r}_t^n = (\hat{Y}_{t+1}^n - g_{t+1}) - (\hat{Y}_t^n - g_t)$.

Before deriving the approximation to the firm's optimization problem, the real marginal cost function is defined using $s_{t,t+j}$ as firm $k's$l marginal cost in period $t+j$:

$$s(y, Y, \bar{\xi}) = \frac{v_h(f^{-1}(y/A;\xi)}{u_c(Y,\xi)A} \frac{1}{f'(f^{-1}(y))}, \tag{51}$$

where $\bar{\xi} \equiv (\xi, A)$ is a vector of preference and technology shocks.

When prices are fully flexible, the price of firm k is a markup over its real marginal cost:

$$\frac{p_t(k)}{P_t} = \mu s(y_t(k), Y_t, \bar{\xi}_t), \tag{52}$$

where $\mu = \theta/(\theta-1)$.Then, in equilibrium, the firms will face the symmetric problem, so that the price set by each firm k is P_t and its output is Y_t. This implies that $s(Y_t^n, Y_t^n, \bar{\xi}_t) = \mu^{-1}$. The natural rate of output Y_t^n is thus defined. This relation is also used to define the steady state level of output $\bar{Y}$ such that $s(\bar{Y}, \bar{Y}, 0) = \mu^{-1}$.

The linearization of (51) gives:

$$\hat{s}_{t,t+j}(k) = \omega\hat{y}_{t+j}(k) + \sigma^{-1}\hat{Y}_{t+j} - (\omega + \sigma^{-1})\hat{Y}_{t+j}^n, \tag{53}$$

where $\omega > 0$ is the elasticity of the real marginal cost function $s(.)$ with respect to $y_t(k)$. Aggregating the above relation yields:

$$\hat{s}_{t+j} = (\omega + \sigma^{-1})(\hat{Y}_{t+j} - \hat{Y}_{t+j}^n). \tag{54}$$

This implies the following relation between the real marginal cost of producing $y_t(k)$ and the aggregate output Y_t :

$$\hat{s}_{t,t+j}(k) = \hat{s}_{t+j} - \omega\theta\left[\hat{p}_t(k) - \sum_{m=t+1}^{t+j} \hat{\pi}_m\right]. \tag{55}$$

Finally, to derive (28), differentiate (13) with respect to $p_t(k)$:

$$\tilde{E}_t \sum_{j=0}^{\infty} \alpha^j Q_{t,t+j} Y_{t+j} P_{t+j}^{\theta} \left[p_t^*(k) - \mu P_{t+j} s_{t,t+j}(k)\right] = 0. \tag{56}$$

The discount factor $Q_{t,t+j}$ is defined in (15) and using the relation in (55) gives:

$$\hat{p}_t^* = \tilde{E}_t \sum_{j=0}^{\infty} (\alpha\beta)^j \left[\frac{1-\alpha\beta}{1+\omega\theta}(\omega+\sigma^{-1})\hat{x}_{t+j} + \hat{\pi}_{t+j}\right], \tag{57}$$

which is the relation in (28). The can be rewritten in terms of (29) using the approximation to the aggregate price index in (11): $\hat{\pi}_t = \hat{p}_t^*(1-\alpha)/\alpha$.

Appendix A.3.

The relevant system of equations under rational expectations is given by:

$$\hat{x}_t = E_t\hat{x}_{t+1} - \sigma(\hat{\imath}_{1,t} - E_t\hat{\pi}_{t+1} - r_t^n) \tag{58a}$$

$$\hat{\pi}_t = \kappa\hat{x}_t + \beta E_t\hat{\pi}_{t+1} \tag{58b}$$

$$\hat{\imath}_{1,t} = \bar{\imath}_t + \phi_x\hat{x}_t + \phi_\pi\hat{\pi}_t \tag{58c}$$

$$\hat{\tau}_t = \phi_\tau\hat{w}_t \tag{58d}$$

$$\begin{aligned}\beta\rho\hat{w}_{t+1} = {} & \hat{w}_t\left[1+\rho(1-\phi_\tau(1-\beta))\right] \\ & -\frac{1}{\beta}(1-\phi_\tau(1-\beta))\hat{w}_{t-1} + \frac{1}{\beta}\hat{\pi}_{t-1} \\ & -\frac{(1-\beta\rho)}{1-\rho}\hat{\imath}_{t-1} + \left(\frac{\beta^2\rho^2}{1-\rho}\right)\hat{\imath}_t - \rho\hat{\pi}_t + \beta\rho\frac{(1-\beta\rho)}{1-\rho}\hat{\imath}_t\end{aligned} \tag{58e}$$

The interest rate and tax rule are into the relations describing the output gap, inflation and wealth. Re-writing in the form $E_tX_{t+1} = AX_t + ae_t$, the conditions for the determinacy of the rational expectations equilibrium are:

$$1 < \phi_\tau < \frac{1+\beta}{1-\beta} \text{ and } \kappa(\phi_\pi - 1) + \phi_x(1-\beta) > 0. \tag{59}$$

Figures and Tables

Figure 1: Fiscal Variables

1a: Government Debt ($debt_t$) | 1b: Government Expenditures (g_t)

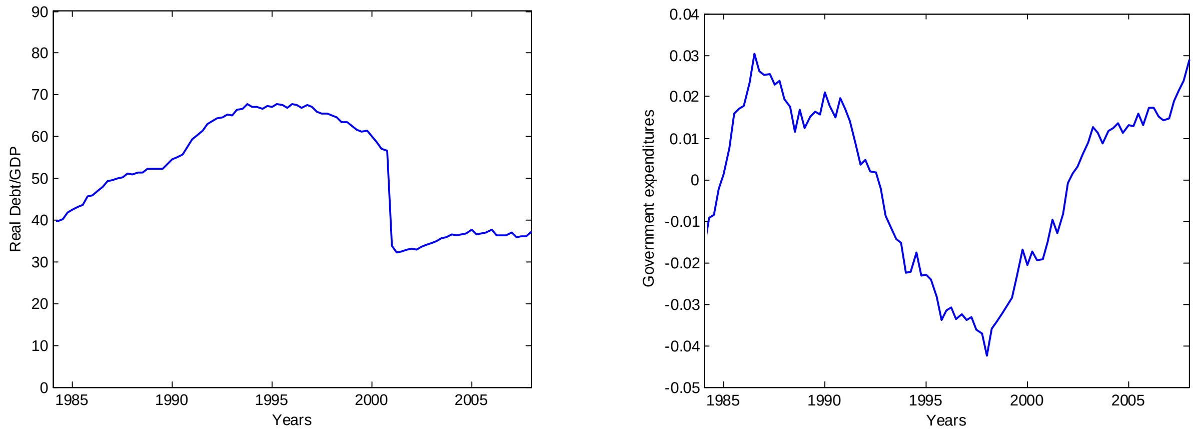


Figure 2: Interest Rates

2a: 3-month Rate | 2b: 10-year rate

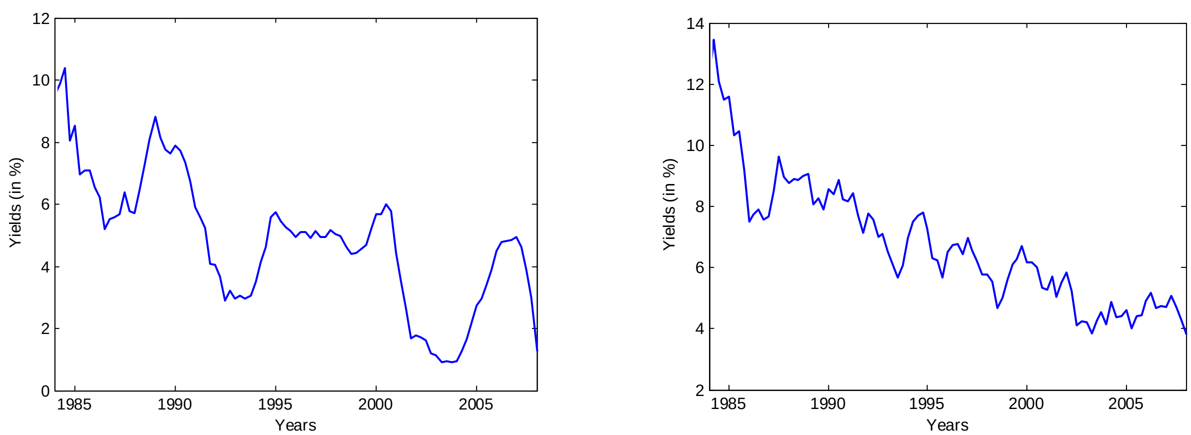
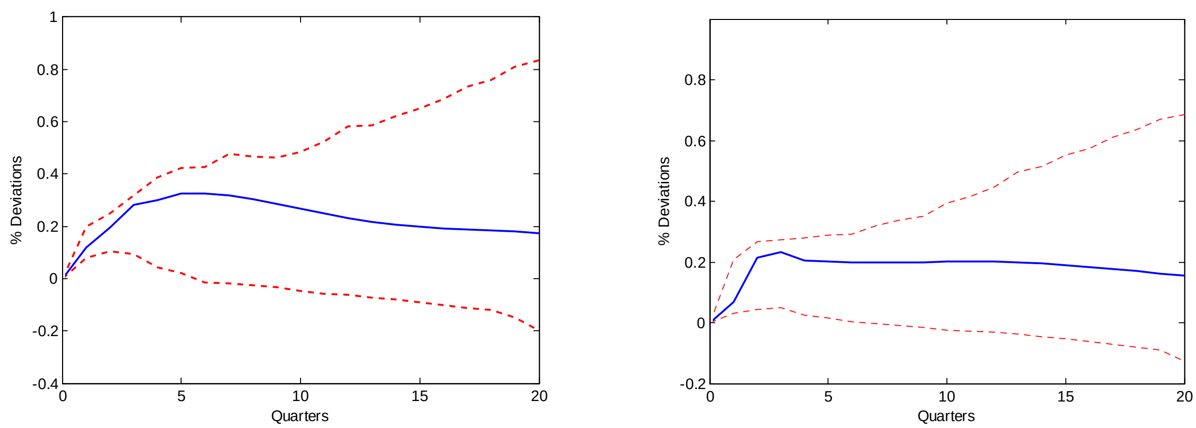


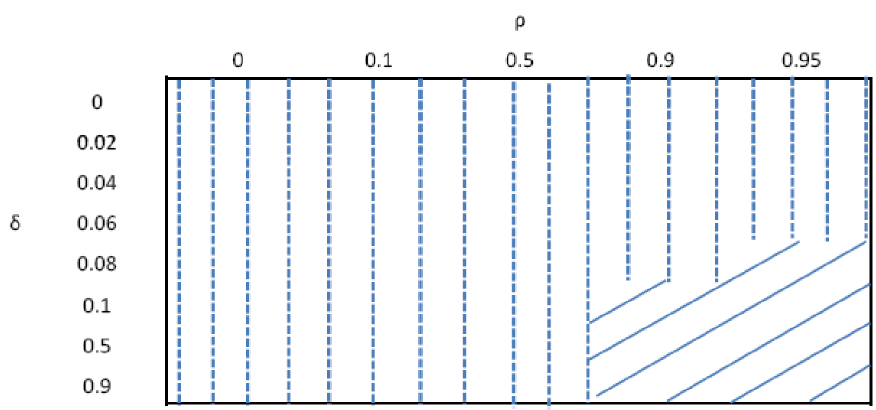
Figure 3: Impulse Response to a 1% Increase in Debt

3a: 3-month Rate | 3b: 10-year rate



Note: The blue lines denote the point estimates and the red lines denote the 95% confidence interval bands.

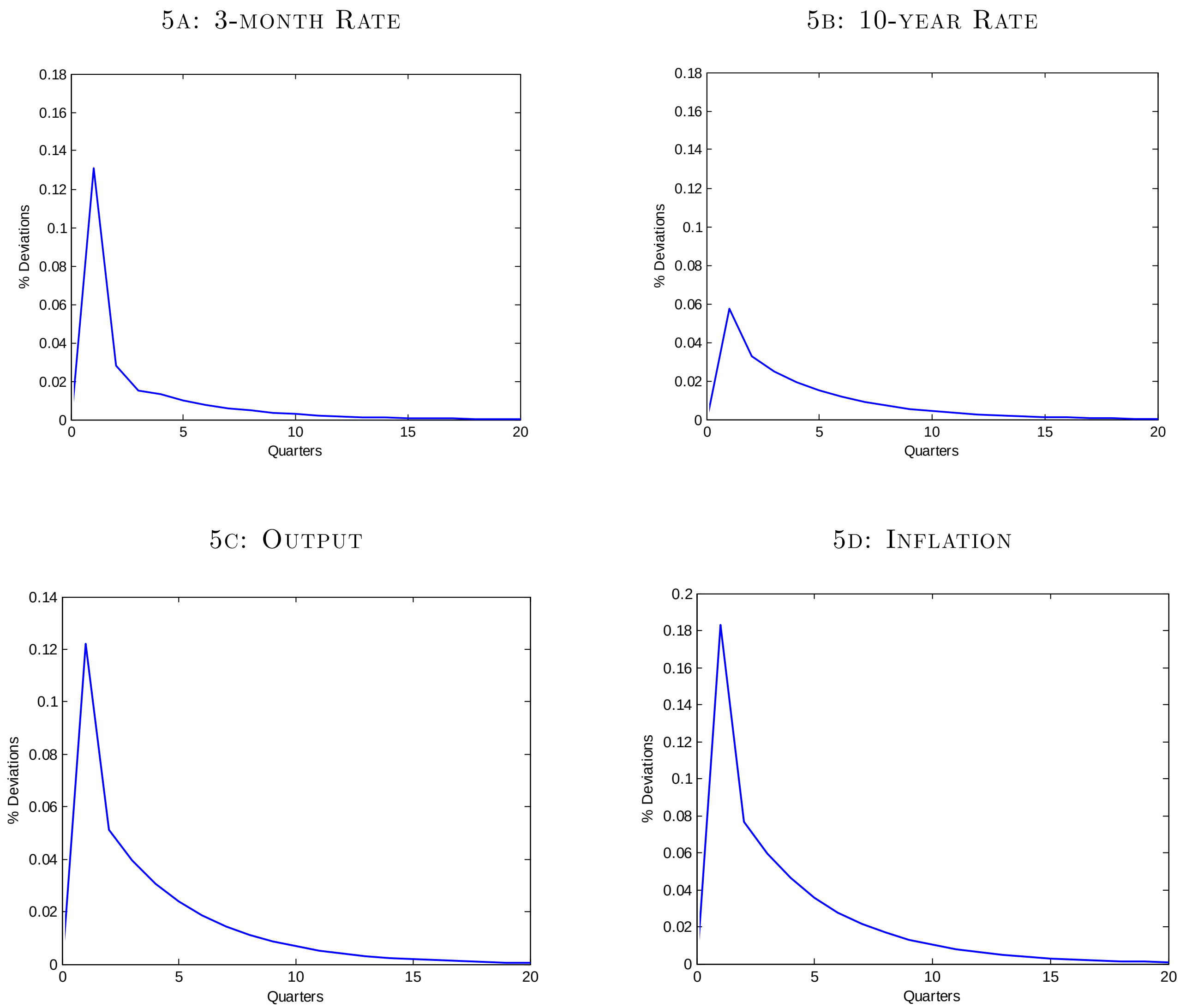
Figure 4: Regions of E-Stability



Other parameter values: $\beta = 0.99$, $\alpha = 0.75$, $\sigma = 1$, $\phi_\pi = 1.5$, $\phi_x = 0$, $\phi_\tau = 1.5$

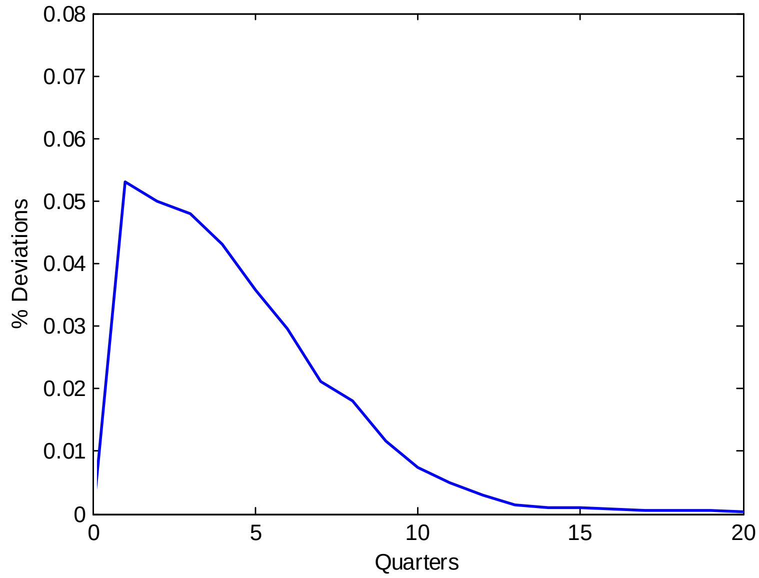
Note: The dashed lines denote the E-Stable regions. δ is the steady state ratio of surpluses to output, ρ is the average maturity of long-term debt.

Figure 5: Impulse Responses to 1% increase in Debt



Note: These respones are for the case with $\rho = 0.95$ and gain of 0.006.

Figure 6: Impulse Response of Net Wealth to 1% increase in One Period Debt



Note: These respones are for the case with $\rho = 0$ and gain of 0.006.

Figure 7: Impulse Response to a 1% Increase in Debt

7a: 3-month Rate

7b: 10-year rate

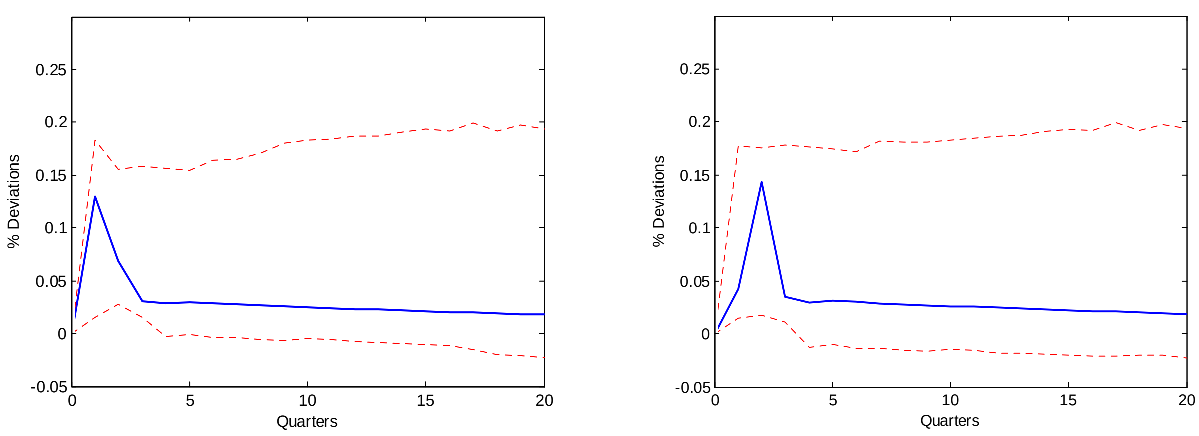


Table 1a: Variance Decomposition of 3−month Interest Rate (corresponding to Figure 3(a))

Variance Pd. (*in qtrs*)	Debt $debt_t$	Output $\hat{y}_t$	Inflation $\hat{\pi}_t$	3-mon rate $i_{3,mon}$	10-year rate i_{10}
1	6.98	10.40	15.38	67.23	0.00
5	15.11	36.35	13.48	31.51	3.55
10	12.55	53.24	11.23	15.09	7.88

Ordering: $[debt_t, gdp_t, \pi_t, i_{3mon,t}, i_{10,t}]$

Table 1b: Variance Decomposition of 10−year Interest Rate (corresponding to Figure 3(b))

Variance Pd. (*in qtrs*)	Debt $debt_t$	Output $\hat{y}_t$	Inflation $\hat{\pi}_t$	3-mon rate $i_{3,mon}$	10-year rate i_{10}
1	0.01	16.08	12.19	4.12	67.59
5	2.03	21.11	18.39	1.34	57.13
10	5.97	24.23	23.80	1.18	44.82

Ordering: $[debt_t, gdp_t, \pi_t, i_{3mon,t}, i_{10,t}]$

Table 2: Campbell-Shiller Slope Coefficients for Nominal Yields

$n(Years)$	$U.S.Data$	$g_2 = 0.006$	
	γ	γ	*Rej.*
2	-0.01	0.89	11%
3	-0.25	0.81	15%
4	-0.45	0.65	18%
5	-0.63	0.40	21%
6	-0.79	0.09	25%
7	-0.91	-2.36	57%
8	-1.01	-3.07	59%
9	-1.09	-3.35	56%
10	-1.09	-4.19	57%

Table 3: Campbell-Shiller Slope Coefficients for Nominal Yields for g_{BM}

	$\rho = 0$		$\rho = 0.95$	
Annual Debt/GDP	γ_2	γ_{10}	γ_2	γ_{10}
0%	0.94	-0.23	0.94	-0.23
	(12%)	(12%)	(12%)	(12%)
40%	0.91	-0.65	0.89	-4.19
	(12%)	(34%)	(11%)	(57%)
72%	0.89	-1.24	0.86	-5.07
	(16%)	(41%)	(15%)	(69%)

Table 4a: Variance Decomposition of 3−month Interest Rate (corresponding to Figure 7(a))

Variance Pd. (*in qtrs*)	Debt $debt_t$	Govt g_t	Output $\hat{y}_t$	Inflation $\hat{\pi}_t$	3-mon rate $i_{3,mon}$	10-year rate i_{10}
1	6.62	0.75	15.93	13.37	63.31	0.00
5	15.52	0.55	40.49	10.21	30.11	3.09
10	15.92	0.872	50.57	8.57	16.10	7.95

Ordering: $[debt_t, g_t, gdp_t, \pi_t, i_{3mon,t}, i_{10,t}]$

Table 4b: Variance Decomposition of 10−year Interest Rate (corresponding to Figure 7(b))

Variance Pd. (*in qtrs*)	Debt $debt_t$	Govt g_t	Output $\hat{y}_t$	Inflation $\hat{\pi}_t$	3-mon rate $i_{3,mon}$	10-year rate i_{10}
1	0.21	4.10	13.97	10.83	5.38	65.47
5	1.51	1.86	21.45	13.00	2.07	60.09
10	6.19	1.65	24.78	14.74	2.46	50.15

Ordering: $[debt_t, g_t, gdp_t, \pi_t, i_{3mon,t}, i_{10,t}]$