Entrepreneurial Optimism, Self-Financing, and Capital-Market Efficiency*

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Abstract

We consider a capital market signaling model in which some entrepreneurs are optimistic–they overestimate the quality of their projects–and some pessimistic. Investors know the fractions of high-quality projects, optimistic and pessimistic entrepreneurs. We find that entrepreneurs' biases raise the equity price of non self-financed projects, lower the equity price of partially self-financed projects, and lower the fraction of partial self-finance. We provide conditions under which entrepreneurs' biases raise aggregate self-finance. Finally, we show that entrepreneurial optimism improves capital market efficiency when risk aversion is sufficiently high and the ratio of optimistic to pessimistic entrepreneurs is not too high.

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1 Introduction

Entrepreneurs undertake and carry out risky projects. To do that they look for investors willing to finance part of their projects and thereby share the projects' risk. Capital market efficiency crucially depends on informational asymmetries between market participants. When investors are risk-neutral, entrepreneurs are risk-averse, and the value of entrepreneurs' projects is known by both sides of the market, entrepreneurs obtain 100% outside finance and the capital market is efficient (first-best solution).

In reality, information is asymmetric and entrepreneurs know the value of their own projects better than investors do. If entrepreneurs with good-quality projects have no ability to signal to investors the value of their projects, then in general the capital market equilibrium is inefficient. Akerlof (1970) shows that when informational asymmetries are substantial the market may even fail to exist (adverse selection). Spence (1973) shows that signaling can reduce the welfare loss associated with asymmetric information in labor markets.

Leland and Pyle (1977) show that when entrepreneurs have private information about the project, the amount of their own funds invested in the project will be interpreted as a signal of its quality. In equilibrium, the higher the quality of the project, the greater the amount of equity that will be retained by the entrepreneur, and the higher will be the market valuation of the firm. However, signaling is costly because entrepreneurs are risk-averse and those with high quality projects do not obtain full-insurance. Thus, signalling reduces the inefficiencies caused by asymmetric information in capital markets but at a cost (second-best solution).

Entrepreneurs' accurate beliefs about the quality of projects are the cornerstone of the signaling mechanism. Only if entrepreneurs know the true value of a project can they truthfully signal it to investors. Yet, scholarly work shows that entrepreneurs are overconfident about their skills and typically overestimate the chances that their projects will be successful (e.g. Wu and Knott, 2006).

Entrepreneurs are considered to be overly optimistic because they are not deterred by the evidence of unfavorable returns to entrepreneuship. Hamilton (2000) finds that the expected financial returns to self-employment is 35% below that that of paid employment. Moskovitz and Vissing-Jorgensen (2002) find that the returns from entrepreneurship are, on average, not different from the return on a diversified publicly traded portfolio (private equity puzzle).

Cooper et al. (1988) find that 68% of American entrepreneurs thought that the odds of their business succeeding were better than for others in the same sector, 27% thought that the odds were similar and only 5% thought they were worse. Pinfold (2001) finds similar results in a survey on new business founders in New Zealand. Landier and Thesmar (2009) compare French entrepreneurs' expectations of future sales to data on sales growth derived from tax files. They find that, for businesses started in 1997, the realizations of 27%, 53% and 20% of entrepreneurs exceeded, matched and were below their expectations, respectively.[1]

Entrepreneurial optimism is often invoked as a potential explanation for errors in entrepreneurs' decisions. Roll (1986) argues that optimism can lead to value destroying mergers and acquisitions. Malmendier and Tate (2008) show empirically that overconfident CEOs overestimate their ability to generate returns and, as a result, overpay for target companies and undertake value-destroying mergers. Heaton (2002) shows that optimistic managers overinvest when they have abundant internal funds whereas they cut investment when they need external financing since they view it as extremely costly. Malmendier and Tate (2005 and 2011) find empirical support for Heaton's (2002) predictions.

This paper provides a formal argument for entrepreneurs' excessive self-financing of their own enterprise and shows that entrepreneurial optimism can improve capital market efficiency. To do that we consider a particular case of Leland and Pyle's

[1]Entrepreneurs are more optimistic than other individuals. Busenitz and Barney (1997) and Lowe and Ziedonis (2006) find that entrepreneurs are more optimistic than managers. Arabsheibani et al. (2000) find that self-employed are more optimistic than employees. Gentry and Hubbard (2000), Hurst and Lusardi (2004), Puri and Robinson (2007), Friedman (2007), and others show that optimistic individuals are more likely to become entrepreneurs.

(1977) capital market signaling model in the spirit of Rochet and Freixas (2008). We assume that some entrepreneurs are correct but others are mistaken about the quality of their projects. Optimistic (pessimistic) entrepreneurs believe to have a good (bad) quality project, when, in fact, they have a bad (good) quality project. Investors observe entrepreneurs' self-financing decisions and know the fractions of good-quality projects, optimistic and pessimistic entrepreneurs. Entrepreneurs are assumed to be risk averse and investors to be risk neutral.

We start by showing that the entrepreneurs' biases lower the equity price gap, i.e., the difference between the equity price of partially self-financed projects and non self-financed projects. In a separating equilibrium, optimistic entrepreneurs and realistic entrepreneurs with high-quality projects partially self-finance their projects whereas pessimistic entrepreneurs and realistic entrepreneurs with low-quality projects do not self-finance their projects. The optimal response of investors to the fact that optimism raises the proportion of low-quality projects in the group who partially self-finance their projects whereas pessimism raises the proportion of high-quality projects in the group who does not self-finance their projects is to lower the equity price offered to the first group and raise the equity price offered to the second group.

Next, we show that entrepreneurs' biases lower the fraction of partial self-finance. The lower equity price gap makes partial self-financing less attractive to entrepreneurs who perceive to have a low-quality project. As a consequence, entrepreneurs who perceive to have a high-quality project need to self-finance a lower fraction of the project to separate themselves from entrepreneurs who perceive to have a low-quality project in an incentive compatible manner than they would have to if all entrepreneurs were rational.

We proceed by characterizing the impact of entrepreneurial optimism on aggregate self-finance. We show that entrepreneurs' biases raise aggregate self-finance as long as the ratio of optimistic to pessimistic entrepreneurs is not too low. This finding provides a rationale for entrepreneurs' excessive self-finance.

The main result of the paper is that if entrepreneurs' risk aversion is sufficiently high and the ratio of optimistic to pessimistic entrepreneurs is not too high, then capital market efficiency is higher with biased entrepreneurs than with rational ones. This result stands in contrast with de Meza and Southey (1996) and Manove and Padilla (1999) who show that entrepreneurial optimism reduces market efficiency. The intuition behind our result is as follows.

The existence of biased entrepreneurs implies that realistic entrepreneurs with high-quality projects need a lower fraction of self-finance to signal their type to investors. Hence, realistic entrepreneurs with high-quality projects bear a lower risk than in the rational case. Since entrepreneurs are risk averse this leads to a drop in the cost of partial self-financing. However, the existence of biased entrepreneurs lowers the equity price offered by investors to realistic entrepreneurs with high-quality projects. When entrepreneurs' risk aversion is sufficiently high and the ratio of optimistic to pessimistic entrepreneurs is not too high, realistic entrepreneurs with high-quality projects are better off because there is large drop in the cost of partial self-finance and a small drop in the equity price.

This result is consistent with the theory of the second best. According to this theory, introducing a new distortion–entrepreneurs' biases–in an environment where another distortion is already present–private information about project's value–, may increase welfare. In section 4 we show that this finding is compatible with empirical levels of absolute risk aversion and the fractions of optimistic and pessimistic entrepreneurs in Landier and Thesmar (2009).

Our paper contributes to the literature that explores the impact of optimism on entrepreneurs' decisions and market outcomes. This literature has mostly focused on the adverse individual consequences of entrepreneurial optimism. Less attention has been given to the impact that optimism has on the interaction between entrepreneurs and investors and on market efficiency. Our paper also contributes to the literature on capital market signaling. We show that entrepreneurs' biases weaken the main qualitative implication of the Leland and Pyle (1977) model: the quantity of self-finance held by the entrepreneur may not be increasing in the value of the project.

2 The Model

Consider an economy consisting of a large number of entrepreneurs and investors. Each entrepreneur has a risky project, requiring a fixed investment of 1 and yielding random gross returns $\tilde{R} = 1 + \tilde{r}(\theta)$. Net returns $\tilde{r}(\theta)$ follow a normal distribution of mean θ and variance σ^2. The variance σ^2 is the same for all projects, whereas θ can take two values: a low value θ_1 if the project's quality is low and a high value θ_2 if the project's quality is high, where $\theta_2 > \theta_1 > 0$. The project's net mean returns θ are known by the entrepreneur.

Entrepreneurs are risk averse and investors are risk neutral. Entrepreneurs have a constant absolute risk aversion (CARA) utility function $u(W) = -e^{-\rho W}$, where $\rho > 0$ is the entrepreneurs' coefficient of risk-aversion and W is entrepreneurs' final wealth. Entrepreneurs have enough initial wealth $W_0 > 1$ to self-finance their project. However, self-financing a risky project is costly to entrepreneurs because they are risk averse. Furthermore, self-finance is more costly for entrepreneurs with low quality projects because the net mean returns from self-finance are lower than those of entrepreneurs with high quality projects.

There are four types of entrepreneurs. Realistic entrepreneurs with low-quality projects have low value projects and know it. Realistic entrepreneurs with high-quality projects have high value projects and know it. Optimistic entrepreneurs believe they have high value projects when in fact they have low value projects. Pessimistic entrepreneurs believe they have low value projects when in fact they have high value projects. Let $\pi = \Pr(\theta = \theta_2) \in (0, 1)$ be the fraction of high quality projects, $\nu \in [0, \pi]$ the fraction of pessimistic entrepreneurs, and $\kappa \in [0, 1 - \pi]$ the fraction of optimistic entrepreneurs. Investors cannot observe a project's net mean returns θ nor entrepreneurs' beliefs. Entrepreneurs and investors know ν, κ, and the distribution of θ.

Let γ be the fraction of a project's self-finance. Investors observe γ and consequently infer the project's quality. Investors then price each project according to the inferred quality. Entrepreneurs seek to maximize their perceived expected utility according to their perceptions.

In a separating equilibrium, self-finance choices are determined by entrepreneurs' beliefs about the net mean returns of their project. Entrepreneurs who believe to have a high quality project self-finance a fraction $\gamma > 0$ of their project. Since self-financing has no value other than signaling, entrepreneurs who believe to have a low quality project do not self-finance it ($\gamma = 0$).[2]

Among all entrepreneurs who choose a fraction of self-finance $\gamma > 0$, investors know that a fraction $\beta = \frac{\kappa}{\pi+\kappa-\nu}$ has low quality projects and a fraction $1-\beta = \frac{\pi-\nu}{\pi+\kappa-\nu}$ has high quality projects. Among all entrepreneurs who do not self-finance the project, investors know that a fraction $\alpha = \frac{\nu}{1-\pi-\kappa+\nu}$ has high quality projects and a fraction $1-\alpha = \frac{1-\pi-\kappa}{1-\pi-\kappa+\nu}$ has low quality projects. Hence, investors' posterior belief that a project's quality is high after having observed γ is

$$\mu(\theta_2|\gamma) = \begin{cases} \alpha, & \text{if } \gamma = 0 \\ 1-\beta, & \text{if } \gamma > 0 \end{cases} .$$

Competition in the capital market implies that investors break even. Therefore, the equity price offered to each group of entrepreneurs (those who choose $\gamma = 0$ and those who choose $\gamma > 0$) is a weighted average of low and high quality projects' net mean returns. Hence, investors' strategy is to offer the following equity price

$$P(\gamma) = E[\tilde{r}(\gamma)] = \begin{cases} \theta_1 + \alpha\Delta\theta, & \text{if } \gamma = 0 \\ \theta_2 - \beta\Delta\theta, & \text{if } \gamma > 0 \end{cases} ,$$

with $\Delta\theta \equiv \theta_2 - \theta_1$.

Define the equity price gap, ΔP, as the difference between the equity price of self-financed projects and the equity price of not self-financed projects, that is,

$$\Delta P = (\theta_2 - \beta\Delta\theta) - (\theta_1 + \alpha\Delta\theta) = (1-\alpha-\beta)\Delta\theta. \tag{1}$$

We see from (1) that the equity price gap with biased entrepreneurs–when $\alpha+\beta > 0$–is lower than the equity price gap with rational entrepreneurs–$\alpha + \beta = 0$. This

[2] We assume that there is no pooling equilibrium where all entrepreneurs obtain 100 percent outside financing at equity price $P = (1-\pi)\theta_1 + \pi\theta_2$. This is the case if $1-\pi > \frac{\rho\sigma^2}{2\Delta\theta}$.

happens because the presence of optimistic entrepreneurs lowers the equity price of self-financed projects and the presence of pessimistic entrepreneurs raises the equity price of projects that are not self-financed.

In a separating equilibrium the price of equity for self-financed projects must be higher than the price of equity for projects that are not self-financed. This condition is satisfied if $\alpha + \beta < 1$, or, using the definitions of α and β

$$(1 - \pi)\nu + \pi\kappa < (1 - \pi)\pi. \tag{2}$$

Condition (2) says that if the fractions of optimistic and pessimistic entrepreneurs are sufficiently small, self-finance can serve as a signal of project's quality. When the fraction of biased entrepreneurs is too high, condition (2) is violated and separating equilibria may no longer exist. We assume from now on that condition (2) is satisfied.[3]

In a separating equilibrium, pessimistic entrepreneurs and realistic entrepreneurs with a low quality project do not envy optimistic entrepreneurs and entrepreneurs with a high quality project, that is

$$u(W_0 + \theta_1 + \alpha\Delta\theta) \geq Eu[W_0 + (1 - \gamma)(\theta_2 - \beta\Delta\theta) + \gamma\tilde{r}(\theta_1)] \tag{3}$$

The left-hand side of (3) is the utility that pessimistic entrepreneurs and realistic entrepreneurs with a low quality project obtain from selling the entire project to investors. In this case entrepreneurs' final wealth is the sum of initial wealth W_0 and the equity price $\theta_1 + \alpha\Delta\theta$ paid by investors. The right-hand side of (3) is the utility that pessimistic entrepreneurs and realistic entrepreneurs with a low quality project expect to obtain if they partially self-finance their project. Such entrepreneurs expect to obtain as final wealth the sum of initial wealth W_0, the revenue obtained from selling fraction $1 - \gamma$ of the project to investors at equity price $\theta_2 - \beta\Delta\theta$, and the

[3]In Cooper et al. (1988) we have $\kappa = 0.68$ and $\nu = 0.05$ and condition (2) is not satisfied for any $\pi \in (0, 1)$. In Landier and Thesmar (2009) we have $\kappa = 0.27$ and $\nu = 0.20$ and condition (2) is satisfied for any $\pi \in (0.338, 0.592)$.

revenue obtained from keeping fraction γ of the project with net random returns $\tilde{r}(\theta_1)$.

Furthermore, in a separating equilibrium optimistic entrepreneurs and realistic entrepreneurs with a high-quality project do not envy pessimistic entrepreneurs and realistic entrepreneurs with a low-quality project, that is

$$Eu[W_0 + (1-\gamma)(\theta_2 - \beta\Delta\theta) + \gamma\tilde{r}(\theta_2)] \geq u(W_0 + \theta_1 + \alpha\Delta\theta) \tag{4}$$

The left-hand side of (4) is the utility that optimistic entrepreneurs and realistic entrepreneurs with a high-quality project expect to obtain from partially self-financing their project. Such entrepreneurs expect to get as final wealth the sum of initial wealth W_0, the revenue obtained from selling fraction $1-\gamma$ of the project to investors at equity price $\theta_2 - \beta\Delta\theta$, and the revenue obtained from keeping fraction γ of the project with net random returns $\tilde{r}(\theta_2)$. The right-hand side of (4) is the utility that optimistic entrepreneurs and realistic entrepreneurs with a high-quality project obtain by selling the entire project to investors. In this case entrepreneurs' final wealth is the sum of initial wealth W_0 and the equity price $\theta_1 + \alpha\Delta\theta$ paid by investors.

We can rewrite the expected utilities in (3) and (4) as utilities noting that if $u(\tilde{x}) = -e^{-a\tilde{x}}$ with $\tilde{x} \sim N(\mu, \sigma^2)$, then $E[u(\tilde{x})] = -e^{-a\mu + \frac{a^2}{2}\sigma^2} = u(a\mu - \frac{a^2}{2}\sigma^2)$. Hence, in a separating equilibrium

$$u(W_0 + \theta_1 + \alpha\Delta\theta) \geq u\left[W_0 + (1-\gamma)(\theta_2 - \beta\Delta\theta) + \gamma\theta_1 - \gamma^2\frac{\rho\sigma^2}{2}\right] \tag{5}$$

and

$$u\left[W_0 + (1-\gamma)(\theta_2 - \beta\Delta\theta) + \gamma\theta_2 - \gamma^2\frac{\rho\sigma^2}{2}\right] \geq u(W_0 + \theta_1 + \alpha\Delta\theta) \tag{6}$$

must be satisfied.

There exist a continuum of separating equilibria parametrized by a fraction of self-finance γ fulfilling (5) and (6). These equilibria can be Pareto-ranked. We focus our analysis on the least cost separating equilibrium–the one with the lowest fraction of self-finance–because it Pareto-dominates the other separating equilibria.

3 Self-Finance

We now analyze the impact of entrepreneurs' biases on self-finance. We start by showing that the fraction of partial self-finance with biased entrepreneurs is lower than if all entrepreneurs were rational. We then provide conditions under which entrepreneurs' misperceptions raise aggregate self-finance.

In the least cost separating equilibrium, (5) holds with equality while (6) is slack.[4] Therefore, the fraction of partial self-finance by optimistic entrepreneurs and realistic entrepreneurs with high-quality projects, γ_B, is obtained by solving

$$W_0 + \theta_1 + \alpha\Delta\theta = W_0 + (1-\gamma)(\theta_2 - \beta\Delta\theta) + \gamma\theta_1 - \gamma^2 \frac{\rho\sigma^2}{2} \tag{7}$$

with respect to γ. Doing that we obtain

$$\gamma_B = \frac{\Delta\theta}{\rho\sigma^2}\left[-(1-\beta) + \sqrt{(1-\beta)^2 + 2(1-\alpha-\beta)\frac{\rho\sigma^2}{\Delta\theta}}\right].$$

When all entrepreneurs are rational, the fraction of partial self-finance by entrepreneurs with high-quality projects, γ_R, is given by

$$\gamma_R = \frac{\Delta\theta}{\rho\sigma^2}\left(-1 + \sqrt{1 + 2\frac{\rho\sigma^2}{\Delta\theta}}\right).$$

Our first result compares γ_B to γ_R.

Proposition 1: *The fraction of partial self-finance with biased entrepreneurs is lower than that with rational entrepreneurs, i.e.,* $\gamma_B < \gamma_R$.

In the least cost separating equilibrium with rational entrepreneurs, those with low-quality projects are indifferent between self-financing fraction γ_R of their projects and not self-financing their projects. As we have seen, the presence of biased

[4] Condition (6) is satisfied for all γ less than or equal to $\bar{\gamma} = \frac{\Delta\theta}{\rho\sigma^2}\left[\beta + \sqrt{\beta^2 + 2(1-\alpha-\beta)\frac{\rho\sigma^2}{\Delta\theta}}\right]$. In the least cost separating equilibrium condition (6) is slack since $\gamma_B < \bar{\gamma}$.

entrepreneurs lowers the equity price of self-financed projects and raises the equity price of non self-financed projects. This makes self-financing less attractive to entrepreneurs who perceive to have a low-quality project. As a consequence, entrepreneurs who perceive to have a high-quality project need to self-finance a lower fraction of the project to separate themselves from entrepreneurs who perceive to have a low-quality project in an incentive compatible manner than they would have to if all entrepreneurs were rational.

We now study the impact of entrepreneurs' biases on aggregate self-finance. Aggregate self-finance with biased entrepreneurs is equal to

$$S_B = (\pi - \nu + \kappa)\gamma_B, \tag{8}$$

that is, the sum of self-finance by realistic entrepreneurs with high-quality projects and optimistic entrepreneurs. Aggregate self-finance with rational entrepreneurs is equal to

$$S_R = \pi\gamma_R. \tag{9}$$

The change in aggregate self-finance is obtained by subtracting (9) from (8):

$$\begin{aligned} S_B - S_R &= (\pi - \nu + \kappa)\gamma_B - \pi\gamma_R \\ &= -(\pi - \nu)(\gamma_R - \gamma_B) - \nu\gamma_R + \kappa\gamma_B. \end{aligned} \tag{10}$$

We see from (10) that entrepreneurs' biases have three effects on aggregate self-finance. First, realistic entrepreneurs with good-quality projects self-finance a lower fraction of their projects than they would if all entrepreneurs were rational. Second, pessimistic entrepreneurs do not self-finance their projects whereas they would partially self-finance them if all entrepreneurs were rational. Third, optimistic entrepreneurs partially self-finance their projects whereas they would not self-finance them if all entrepreneurs were rational. The first two effects lower aggregate self-finance and the third effect raises it.

Proposition 2 summarizes the impact of entrepreneurs' biases on aggregate self-finance.

Proposition 2. *Aggregate self-finance with biased entrepreneurs is higher than with rational entrepreneurs when either (i) some entrepreneurs are optimistic and no entrepreneur is pessimistic or (ii) some entrepreneurs are optimistic, some are pessimistic, and the ratio of optimistic to pessimistic entrepreneurs satisfies*

$$\frac{\kappa}{\nu} > \frac{\Delta\theta}{\rho\sigma^2}\left(1 + 2\frac{\rho\sigma^2}{\Delta\theta} - \sqrt{1 + 2\frac{\rho\sigma^2}{\Delta\theta}} + \frac{\rho\sigma^2}{\Delta\theta}\frac{\pi}{1-\pi}\right). \tag{11}$$

If some entrepreneurs are optimistic and no entrepreneur is pessimistic, then the increase in self-finance by optimistic entrepreneurs dominates the reduction in self-finance by realistic entrepreneurs with high-quality projects. If some entrepreneurs are optimistic, some are pessimistic, and the ratio of optimistic to pessimistic entrepreneurs is greater than the lower bound defined in condition (11), then the increase in self-finance by optimistic entrepreneurs dominates the decrease in self-finance by realistic entrepreneurs with high-quality projects and by pessimistic entrepreneurs.[5]

Figure 1 illustrates part (ii) of Proposition 2. In the horizontal axis we have the coefficient of absolute risk aversion and in the vertical axis the ratio of optimistic to pessimistic entrepreneurs. We set $\pi = 0.5$, $\Delta\theta = 40$, and $\sigma^2 = 80$.

[Figure 1]

Any point above the curve satisfies condition (11). Hence, aggregate self-finance with biased entrepreneurs is higher (lower) than with rational entrepreneurs in the area above (below) the curve, i.e., the area with the positive (negative) sign.

4 Capital Market Efficiency

In this section we characterize the impact of entrepreneurs' biases on capital market efficiency. To do that we compare welfare with biased entrepreneurs to that with

[5] If some entrepreneurs are optimistic, some are pessimistic, and the inequality of Proposition 2 goes in the opposite direction, then aggregate self-finance with biased entrepreneurs is lower than with rational entrepreneurs.

rational entrepreneurs. Welfare is the weighted average of the expected utilities of each group of entrepreneurs since investors break even.

To evaluate the expected utility of a biased entrepreneur, we take the perspective of an outside observer who knows the actual projects' value. We denote $E[u(\theta_1|\theta_1)]$ the expected utility of a realistic entrepreneur with a low-quality project, $E[u(\theta_2|\theta_1)]$ the expected utility of an optimistic entrepreneur, $E[u(\theta_1|\theta_2)]$ the expected utility of a pessimistic entrepreneur, and $E[u(\theta_2|\theta_2)]$ the expected utility of a realistic entrepreneur with a high-quality project. Hence, welfare with biased entrepreneurs is

$$W_B = (1-\pi-\kappa)\,E[u(\theta_1|\theta_1)] + \kappa E[u(\theta_2|\theta_1)] + \nu E[u(\theta_1|\theta_2)] + (\pi-\nu)\,E[u(\theta_2|\theta_2)]. \tag{12}$$

The expected utilities of entrepreneurs with low- and high-quality projects when all entrepreneurs are rational are denoted by $E[u(\theta_1)]$ and $E[u(\theta_2)]$, respectively. Hence, welfare with rational entrepreneurs is

$$W_R = (1-\pi)\,E[u(\theta_1)] + \pi E[u(\theta_2)].$$

Our first welfare result compares the utilities of the four different types of entrepreneurs in the biased model to the utilities of entrepreneurs with low- and high-quality projects in the rational model.

Proposition 3: *In the least cost separating equilibrium of the model with biased entrepreneurs:*
(i) If $\alpha \geqq 0$, *then* $E[u(\theta_1|\theta_1)] \geqq E[u(\theta_1)]$;
(ii) If $\alpha \gtreqless \gamma_R$ *then* $E[u(\theta_1|\theta_2)] \gtreqless E[u(\theta_2)]$;
(iii) If $\alpha \geqq 0$, *then* $E[u(\theta_2|\theta_1)] \geqq E[u(\theta_1)]$;
(iv) If $\alpha \gtreqless \gamma_R - \gamma_B$, *then* $E[u(\theta_2|\theta_2)] \gtreqless E[u(\theta_2)]$.

Proposition 3 part (i) tells us that a realistic entrepreneur with a low-quality project attains at least the same utility as an entrepreneur with a low-quality project in the rational model. When there exist pessimistic entrepreneurs, realistic entrepreneurs with low-quality projects benefit from a higher price of equity.

Part (ii) tells us that a pessimistic entrepreneur attains a higher (lower) utility than an entrepreneur with a high-quality project in the rational model if the fraction of pessimistic entrepreneurs is sufficiently high (low). Pessimistic entrepreneurs face a lower price of equity and a lower risk since they do not self-finance their projects. When the fraction of pessimistic entrepreneurs satisfies $\alpha > \gamma_R$ ($\alpha < \gamma_R$), the risk reduction effect dominates (is dominated by) the lower equity price effect.

Part (iii) tells us that an optimistic entrepreneur attains at least the same utility as an entrepreneur with a low-quality project in the rational model. Optimistic entrepreneurs face a higher price of equity and a higher risk since they partially self-finance their projects. When there exist pessimistic entrepreneurs, the higher equity price effect dominates the risk increase effect.

Finally, part (iv) tells us that a realistic entrepreneur with a high-quality project attains a higher (lower) utility than an entrepreneur with a high-quality project in the rational model when $\alpha > \gamma_R - \gamma_B$ ($\alpha < \gamma_R - \gamma_B$). Realistic entrepreneurs with high-quality projects face a lower price of equity but they also face lower risk since they need a lower fraction of partial self-finance to signal their type. When the fraction of pessimistic entrepreneurs satisfies $\alpha > \gamma_R - \gamma_B$ ($\alpha < \gamma_R - \gamma_B$), the risk reduction effect dominates (is dominated by) the lower equity price effect.

Proposition 4: *If some entrepreneurs are optimistic and no entrepreneur is pessimistic, then capital market efficiency is lower than when all entrepreneurs are rational.*

When some entrepreneurs are optimistic and no entrepreneur is pessimistic, the utility of realistic entrepreneurs with low-quality projects and the utility of optimistic entrepreneurs are equal to the utility of entrepreneurs with low-quality projects in the rational model. However, the utility of realistic entrepreneurs with high-quality projects is lower than that of entrepreneurs with high-quality projects in the rational model. Hence, capital market efficiency is reduced.

Proposition 5. *If some entrepreneurs are optimistic, some are pessimistic, the coefficient of absolute risk aversion satisfies*

$$\rho > \frac{1}{\Delta\theta}, \tag{13}$$

and the ratio of optimistic to pessimistic entrepreneurs satisfies

$$\frac{\kappa}{\nu} < \frac{1}{\rho^2\sigma^2}\left[\left(1+\sqrt{1+\frac{2\rho\sigma^2}{\Delta\theta}}\right)\left(\rho^2\sigma^2\frac{\pi}{1-\pi}+\frac{\rho\sigma^2}{\Delta\theta}+1\right)+\frac{\rho\sigma^2}{\Delta\theta}\right], \tag{14}$$

then capital market efficiency is higher than when all entrepreneurs are rational.

Proposition 5 tells us that if entrepreneurs' coefficient of absolute risk aversion is sufficiently high, i.e., ρ is greater than the lower bound defined in condition (13), and the ratio of optimistic to pessimistic entrepreneurs is not too high, i.e., κ/ν is less than the upper bound defined in condition (14), then entrepreneurs' biases raise capital market efficiency. The intuition behind this result is as follows.

The existence of biased entrepreneurs implies that realistic entrepreneurs with high-quality projects need lower partial self-finance to signal their type to investors. Hence, realistic entrepreneurs with high-quality projects bear a lower risk than in the rational case. However, if the ratio of optimistic to pessimistic entrepreneurs is too high, then there is a sharp fall in the price of equity offered by investors to realistic entrepreneurs with high-quality projects which has an unfavorable impact on their utility. Thus, if risk aversion is sufficiently high and the ratio of optimistic to pessimistic entrepreneurs is not too high, then the favorable impact of misperceptions on the need for partial self-financing dominates the unfavorable impact on the price of equity.

If the two inequalities of Proposition 5 go in the opposite direction, then entrepreneurs' biases lower capital market efficiency.

Figure 2 illustrates Proposition 5 when $\pi = 0.5$, $\Delta\theta = 40$, and $\sigma^2 = 80$. The vertical line represents condition (13) and the curve represents condition (14).

[Figure 2]

Any point to the right of the vertical line satisfies condition (13). Any point below the curve satisfies condition (14). Hence, welfare with biased entrepreneurs is higher in the area with the positive sign. Conversely, welfare with biased entrepreneurs is lower in the area with the negative sign. The two question marks denote the areas in which the impact of entrepreneurs' biases on welfare is ambiguous.

Empirical evidence shows that the coefficient of absolute risk aversion is usually below 0.1.[6] In Landier and Thesmar (2009) the ratio of optimistic to pessimistic entrepreneurs is $\kappa/\nu = 27/20 = 1.35$. Hence, this parameterization of the model shows that Proposition 5 is compatible with empirical evidence on absolute risk aversion and on the fractions of optimistic to pessimistic entrepreneurs.

5 Extensions

In this section we discuss four possible extensions of the model.

5.1 Low-Quality Projects Yield Negative Mean Returns

If entrepreneurs are rational and low-quality projects yield negative net mean returns, that is, $\theta_1 < 0 < \theta_2$, there exist separating equilibria where low-quality projects are not self-financed and high-quality projects are partially self-financed. Investors know that projects that are not self-financed have negative net mean returns and so they are not willing to offer a positive equity price for those projects. Therefore, entrepreneurs do not undertake low-quality projects.

If some entrepreneurs are biased, there exist separating equilibria where realistic entrepreneurs with low-quality projects and pessimistic entrepreneurs do not self-finance their projects and where realistic entrepreneurs with high-quality projects and optimistic entrepreneurs partially self-finance their projects.

[6] Higher coefficients of absolute risk aversion imply unreasonably high levels of risk aversion and are rarely observed. See Cohen and Einav (2007) and Sydnor (2010).

If the net mean return of a low quality project is slightly negative, i.e., $-\alpha\theta_2/(1-\alpha) < \theta_1 < 0$, then the net mean return of non self-financed projects is strictly positive and investors are willing to offer a positive equity price for these projects. Thus, biased beliefs raise the equity price for non self-financed projects and lower the equity price for partially self-financed projects and the qualitative nature of the results remains unchanged.

In contrast, if the net mean return of a low quality project is substantially negative, i.e., $\theta_1 \leq -\alpha\theta_2/(1-\alpha)$, then the mean return of non self-financed projects is strictly negative and investors are not willing to offer a positive equity price for these projects. Therefore, realistic entrepreneurs with low-quality projects and pessimistic entrepreneurs do not undertake their projects. Proposition 6 shows that in this case, entrepreneurs' biases always lower capital market efficiency.

Proposition 6. *If the net mean returns of low-quality projects satisfy $\theta_1 \leq -\frac{\alpha\theta_2}{1-\alpha}$, then welfare is lower with biased entrepreneurs than with rational entrepreneurs.*

The intuition behind this result is straightforward. The existence of pessimistic entrepreneurs implies that some high-quality projects are not undertaken and this represents a welfare loss. Additionally, the existence of optimistic entrepreneurs implies that realistic entrepreneurs with high-quality projects attain a lower utility than entrepreneurs with high-quality projects in the rational model. This happens because the unfavorable lower price of equity effect dominates the favorable lower fraction of self-finance effect.

5.2 Endogenous Effort

In our model entrepreneurs are endowed with a low or a high-quality project and choose the level of self-finance. In reality, entrepreneurial effort is also an important factor for project returns. We consider an extension of the model where a project's net returns depend on its quality, the effort put in by the entrepreneur, and where quality and effort are complements. Entrepreneurs choose effort to maximize net project returns minus cost of effort.

The assumption that quality and effort are complements implies that entrepreneurs who perceive to have high-quality projects put in higher effort than those who perceive to have low-quality projects. This increases the return of projects taken by optimistic entrepreneurs and reduces the return of projects taken by pessimistic entrepreneurs by comparison with the model where effort is not a choice variable. As a consequence, the equity price gap in the model with endogenous effort and biased entrepreneurs is greater than that in the model with exogenous effort and biased entrepreneurs. So, the main qualitative findings of this paper extend to the case with endogenous effort.[7]

5.3 Credit Constraints

When some entrepreneurs are credit constrained, that is, they do not have enough initial wealth to self-finance their projects (i.e. $W_0 < 1$), the fraction of entrepreneurs self-financing part of their projects can change but the qualitative nature of our findings remains the same.

5.4 High-Quality Projects are Riskier

It is also possible to generalize the model by assuming that high-quality projects are riskier than low-quality ones. Indeed, projects' net returns are usually proportional to the projects' risk. If that is the case we have $\tilde{r}(\theta_2) \sim N(\theta_2, \sigma_2^2)$ and $\tilde{r}(\theta_1) \sim N(\theta_1, \sigma_1^2)$, where $\theta_2 > \theta_1 > 0$ and $\sigma_2^2 > \sigma_1^2 > 0$. We find that the results change quantitatively but not qualitatively.

[7] One difference concerns the ex-post utility of optimistic entrepreneurs. When effort is exogenous, optimistic entrepreneurs are not worse off than entrepreneurs with low-qualitiy projects in the rational model. In contrast, when effort is endogenous, optimistic entrepreneurs exert an excessive effort and might end up worse off than entrepreneurs with low-quality projects in the rational model.

6 Discussion

In this section we explain how our paper contributes to the literature on signaling, capital market efficiency, and corporate decisions. We also discuss policy implications of our findings.

6.1 Signaling

We consider a particular version of Leland and Pyle' (1977) capital market signaling model in the spirit of Rochet and Freixas (2008). In Rochet and Freixas (2008) project quality, θ, can take only two values, θ_1 or θ_2, whereas in Leland and Pyle (1977) θ has a continuous distribution on a closed interval. In both models there is a unique stage in the capital raising process, only the project's net mean returns is entrepreneurs' private information, and entrepreneurs are perfectly informed about the project's quality.

There have been several extensions to the Leland and Pyle (1977) model, e.g. by considering several stages in the capital raising process, by allowing for the possibility of non-linear contracts, by seeking "robust" contracts, and by focusing on particular aspects of the IPO process. For an excellent survey of this signaling literature see Daniel and Titman (1995).

In our model a project's quality can take two values as in Rochet and Freixas (2008) but we relax the assumption that all entrepreneurs are perfectly informed about the quality of their project. Our model shows that entrepreneurial optimism can weaken the main qualitative implication of the Leland and Pyle (1977) model: the quantity of self-finance held by the entrepreneur may not be increasing in the value of the project. This happens because optimistic entrepreneurs partially self-finance low-quality projects and pessimistic entrepreneurs do not self-finance high-quality projects.

6.2 Market Efficiency

In de Meza and Southey (1996) risk neutral entrepreneurs must choose the right mix of self-finance and debt-finance from risk neutral banks to develop their projects. Banks and realistic entrepreneurs know a project's true probability of success but optimistic entrepreneurs overestimate it. When all entrepreneurs are realists information is symmetric and the market is efficient. Hence, optimism is a distortion in an environment otherwise free of distortions and so it lowers efficiency.

Manove and Padilla (1999) study the impact of entrepreneurial optimism on market efficiency using a screening model. Risk neutral banks use collateral requirements and interest rates to screen risk neutral entrepreneurs with good projects from those with bad ones. In their model optimistic entrepreneurs are willing to fully collateralize their loans and so collateral cannot be used to separate them from the realists. Collateral serves to protect the banks against the errors of optimistic entrepreneurs, but competition between banks reduces interest rates, which further encourages optimists. As a consequence banks lend too much, and thus entrepreneurial optimism reduces market efficiency.

We study the impact of entrepreneurial optimism on market efficiency using a signaling model. We find that entrepreneurial optimism reduces the efficacy of signaling by entrepreneurs. Hence, entrepreneurial optimism is a distortion in our model as well as in de Meza and Southey (1996) and in Manove and Padilla (1999). We also find that entrepreneurial optimism reduces market efficiency when entrepreneurs' risk aversion is sufficiently low and the ratio of optimistic to pessimistic entrepreneurs is too high.

In contrast to de Meza and Southey (1996) and Manove and Padilla (1999), we show that entrepreneurial optimism might improve market efficiency. The reason behind this different result is that the presence of biased entrepreneurs can create a positive externality for realistic entrepreneurs with high-quality projects. When risk aversion is sufficiently high and the ratio of optimistic to pessimistic entrepreneurs is not too high, realistic entrepreneurs with high-quality projects are better off because there is large drop in the cost of partial self-finance and a small drop in the equity price.

6.3 Corporate Decisions

In our framework optimistic entrepreneurs self-finance a larger fraction of their project than they would if they were realistic because optimistic entrepreneurs overestimate the net mean return of their project. A first implication of this finding is that optimistic entrepreneurs undertake low-quality projects because they believe such projects to be high-quality. The literature on mergers and acquisitions confirms this result.

Our finding on projects' self-finance also suggests that optimistic entrepreneurs prefer self-finance to external funds or investment in public equity because they overestimate the returns of their enterprise. The literature on corporate investments provides insights consistent with this result.

6.4 Policy Implications

Our results have the following policy implications. A policy intervention aimed at eliminating entrepreneurs' biases is necessary whenever these lower welfare.[8] This is the case if there are some optimistic entrepreneurs but no pessimistic entrepreneurs in the economy (Proposition 4), if entrepreneurs' risk aversion is low and the ratio of optimistic to pessimistic entrepreneurs is high (Proposition 5), and when low-quality projects yield sufficiently negative net mean returns (Proposition 6). However, when entrepreneurs' biases raise welfare no policy intervention is needed, i.e. the optimal policy is *laissez-faire*. This is the case when entrepreneurs are sufficiently risk averse

[8] Cooper et al. (1988) and Kahneman and Lovallo (1993) argue that optimism is best alleviated by introducing an "outside" view, one capable of realizing all the reasons the "inside" view might be wrong. Outside experts can make entrepreneurs aware of the risks they take by self-financing their projects. External evaluation of projects by financial intermediaries (e.g. banks) may also reduce optimism.

and the ratio of optimistic to pessimistic entrepreneurs is not too high (Proposition 5).

7 Conclusion

We consider a capital market signaling model in which some entrepreneurs are optimistic–they overestimate the quality of their projects–and some pessimistic. Investors know the fractions of high-quality projects in the economy as well as the fractions of optimistic and pessimistic entrepreneurs.

The model shows that entrepreneurial optimism can weaken the main qualitative implication of the Leland and Pyle (1977) model: the quantity of self-finance held by the entrepreneur may not be increasing in the value of the project.

We find that entrepreneurs' biases raise the equity price of non self-financed projects, lower the equity price of partially self-financed projects, and lower the fraction of partial self-finance. We also show that entrepreneurs' biases raise aggregate self-finance when the ratio of optimistic to pessimistic entrepreneurs is not too low.

Finally, we find that entrepreneurial optimism can raise capital market efficiency if entrepreneurs' risk aversion is high and the ratio of optimistic to pessimistic entrepreneurs is not too high. Of course, capital market efficiency does not always rise when entrepreneurs have biased beliefs. Entrepreneurial optimism lowers capital market efficiency when either (i) there are no pessimistic entrepreneurs, or (ii) risk aversion is low and the ratio of optimistic to pessimistic entrepreneurs is high, or (iii) low-quality projects yield sufficiently negative net mean returns.

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9 Appendix

Proof of Proposition 1:

When some entrepreneurs are biased $\alpha+\beta \in (0,1)$ and, from (7), the optimal fraction of self-finance is defined implicitly as:

$$\frac{\gamma_B^2}{1-\gamma_B}\frac{\rho\sigma^2}{2\Delta\theta}+\frac{\alpha}{1-\gamma_B}=1-\beta, \tag{15}$$

with $\gamma_B \in (0,1)$. When all entrepreneurs are rational $\alpha=\beta=0$ and the optimal fraction of self-finance is defined implicitly as

$$\frac{\gamma_R^2}{1-\gamma_R}\frac{\rho\sigma^2}{2\Delta\theta}=1, \tag{16}$$

with $\gamma_R \in (0,1)$. We consider two cases: (i) $\beta > 0$ and $\alpha \geq 0$ and (ii) $\beta = 0$ and $\alpha > 0$.

(i) If $\beta > 0$, then the RHS of (15) is less than the RHS of (16). This implies that the LHS of (15) is less than the LHS of (16). That is,

$$\frac{\gamma_B^2}{1-\gamma_B}\frac{\rho\sigma^2}{2\Delta\theta}+\frac{\alpha}{1-\gamma_B}<\frac{\gamma_R^2}{1-\gamma_R}\frac{\rho\sigma^2}{2\Delta\theta}. \tag{17}$$

If $\alpha \geq 0$, then the second term in the LHS of (17) is non-negative. Hence, (17) implies

$$\frac{\gamma_B^2}{1-\gamma_B}<\frac{\gamma_R^2}{1-\gamma_R}. \tag{18}$$

Since $\frac{x^2}{1-x}$ is strictly increasing in x for $x \in (0,1)$, then (18) implies $\gamma_B < \gamma_R$.

(ii) If $\beta = 0$ then (17) holds as equality. If (17) holds as equality and $\alpha > 0$, then (18) is satisfied and $\gamma_B < \gamma_R$.

Q.E.D.

Proof of Proposition 2:

(i) We need to show that if $\alpha = 0$ and $\beta \in (0,1)$, then $S_B > S_R$. If $\alpha = \nu = 0$, then $\kappa = \beta\pi/(1-\beta)$ and $S_B = (\pi+\kappa)\gamma_B = \pi\gamma_B/(1-\beta)$, where

$$\gamma_B = \frac{1}{\rho\lambda}\left[-(1-\beta)+\sqrt{(1-\beta)^2+2(1-\beta)\rho\lambda}\right],$$

with $\lambda \equiv \sigma^2/\Delta\theta$. We have that $S_R = \pi\gamma_R$ where $\gamma_R = \frac{1}{\rho\lambda}\left(-1+\sqrt{1+2\rho\lambda}\right)$. Hence, $S_B > S_R$ is equivalent to

$$\frac{\pi}{1-\beta}\gamma_B > \pi\gamma_R$$

or

$$-(1-\beta)+\sqrt{(1-\beta)^2+2(1-\beta)\rho\lambda} > -(1-\beta)+(1-\beta)\sqrt{1+2\rho\lambda}$$

or

$$(1-\beta)^2+2(1-\beta)\rho\lambda > (1-\beta)^2+2(1-\beta)^2\rho\lambda$$

or

$$(1-\beta) > (1-\beta)^2,$$

which is true since $\beta \in (0,1)$.

(ii) The amount of self-finance as a function of κ and ν is given by:

$$S_B(\kappa,\nu) = (\pi-\nu+\kappa)\gamma_B(\kappa,\nu). \tag{19}$$

A first-order Taylor series expansion of $S_B(\kappa,\nu)$ around $(0,0)$ is given by:

$$S_B(\kappa,\nu) \approx S_B(0,0) + \left.\frac{\partial S_B}{\partial\kappa}\right|_{(0,0)}\kappa + \left.\frac{\partial S_B}{\partial\nu}\right|_{(0,0)}\nu,$$

where $S_B(0,0) = S_R$. We need to find out the two partial derivatives. From (19) we have

$$\begin{aligned}
\left.\frac{\partial S_B}{\partial\kappa}\right|_{(0,0)} &= \gamma_B(\kappa,\nu)|_{(0,0)} + \left.\frac{\partial S_B}{\partial\gamma_B}\left(\frac{\partial\gamma_B}{\partial\alpha}\frac{\partial\alpha}{\partial\kappa}+\frac{\partial\gamma_B}{\partial\beta}\frac{\partial\beta}{\partial\kappa}\right)\right|_{(0,0)} \\
&= \gamma_R + (\pi-\nu+\kappa)\left[-\frac{1}{\rho\lambda}\left(\frac{1+\rho\lambda}{\sqrt{1+2\rho\lambda}}-1\right)\right]\left.\frac{1}{\pi}\right|_{(0,0)} \\
&= \gamma_R - \frac{1}{\rho\lambda}\left(\frac{1+\rho\lambda}{\sqrt{1+2\rho\lambda}}-1\right).
\end{aligned}$$

From (19) we have

$$\left.\frac{\partial S_B}{\partial \nu}\right|_{(0,0)} = -\gamma_B(\kappa,\nu)|_{(0,0)} + \frac{\partial S_B}{\partial \gamma_B}\left(\frac{\partial \gamma_B}{\partial \alpha}\frac{\partial \alpha}{\partial \nu} + \frac{\partial \gamma_B}{\partial \beta}\frac{\partial \beta}{\partial \nu}\right)\Bigg|_{(0,0)}$$

$$= -\gamma_R + (\pi - \nu + \kappa)\left[-\frac{1}{\sqrt{1+2\rho\lambda}}\right]\frac{1}{1-\pi}\Bigg|_{(0,0)}$$

$$= -\gamma_R - \frac{1}{\sqrt{1+2\rho\lambda}}\frac{\pi}{1-\pi}.$$

Hence, we have

$$S_B(\kappa,\nu) - S_R \approx \left[\gamma_R - \frac{1}{\rho\lambda}\left(\frac{1+\rho\lambda}{\sqrt{1+2\rho\lambda}} - 1\right)\right]\kappa - \left(\gamma_R + \frac{1}{\sqrt{1+2\rho\lambda}}\frac{\pi}{1-\pi}\right)\nu.$$

The term inside square brackets is positive since $\rho\lambda > 0$ implies $\gamma_R > \frac{1}{\rho\lambda}\left(\frac{1+\rho\lambda}{\sqrt{1+2\rho\lambda}} - 1\right)$. Note: From the definition of γ_R we have that $\rho\lambda > 0$ implies $\frac{1}{\rho\lambda}\left(-1+\sqrt{1+2\rho\lambda}\right) > \frac{1}{\rho\lambda}\left(\frac{1+\rho\lambda}{\sqrt{1+2\rho\lambda}} - 1\right)$. Thus, $S_B(\kappa,\nu) > S_R$ as long as

$$\left[\gamma_R - \frac{1}{\rho\lambda}\left(\frac{1+\rho\lambda}{\sqrt{1+2\rho\lambda}} - 1\right)\right]\kappa > \left(\gamma_R + \frac{1}{\sqrt{1+2\rho\lambda}}\frac{\pi}{1-\pi}\right)\nu$$

or

$$\frac{\kappa}{\nu} > \frac{\gamma_R + \frac{1}{\sqrt{1+2\rho\lambda}}\frac{\pi}{1-\pi}}{\gamma_R - \frac{1}{\rho\lambda}\left(\frac{1+\rho\lambda}{\sqrt{1+2\rho\lambda}} - 1\right)}. \tag{20}$$

Substituting $\gamma_R = \frac{1}{\rho\lambda}\left(-1+\sqrt{1+2\rho\lambda}\right)$ in (20), multiplying both sides of (20) by $\frac{\rho\lambda\sqrt{1+2\rho\lambda}}{\rho\lambda\sqrt{1+2\rho\lambda}}$, and simplifying terms we obtain

$$\frac{\kappa}{\nu} > \frac{1}{\rho\lambda}\left(1 + 2\rho\lambda - \sqrt{1+2\rho\lambda} + \rho\lambda\frac{\pi}{1-\pi}\right).$$

or, substituting $\lambda = \frac{\sigma^2}{\Delta\theta}$

$$\frac{\kappa}{\nu} > \frac{\Delta\theta}{\rho\sigma^2}\left(1 + 2\frac{\rho\sigma^2}{\Delta\theta} - \sqrt{1 + 2\frac{\rho\sigma^2}{\Delta\theta}} + \frac{\rho\sigma^2}{\Delta\theta}\frac{\pi}{1-\pi}\right).$$

Q.E.D.

Proof of Proposition 3:

Realistic entrepreneurs with low-quality projects and pessimistic entrepreneurs sell their projects at equity price $\theta_1 + \alpha\Delta\theta$, therefore their utilities are given by

$$E[u(\theta_1|\theta_1)] = E[u(\theta_1|\theta_2)] = -e^{-\rho(W_0+\theta_1+\alpha\Delta\theta)}. \tag{21}$$

The expected utility of an optimistic entrepreneur is given by

$$E[u(\theta_2|\theta_1)] = -e^{-\rho\left[W_0+(1-\gamma_B)(\theta_2-\beta\Delta\theta)+\gamma_B\theta_1-\gamma_B^2\frac{\rho\sigma^2}{2}\right]},$$

which, using (7), can be simplified to

$$E[u(\theta_2|\theta_1)] = -e^{-\rho(W_0+\theta_1+\alpha\Delta\theta)}. \tag{22}$$

Finally, the expected utility of a realistic entrepreneur with a high-quality project is

$$E[u(\theta_2|\theta_2)] = -e^{-\rho\left[W_0+(1-\gamma_B)(\theta_2-\beta\Delta\theta)+\gamma_B\theta_2-\gamma_B^2\frac{\rho\sigma^2}{2}\right]},$$

which, using the fact that $\gamma_B^2 = \frac{2\Delta\theta}{\rho\sigma^2}\left[(1-\beta)(1-\gamma_B)-\alpha\right]$, can be simplified to

$$E[u(\theta_2|\theta_2)] = -e^{-\rho(W_0+\theta_1+\alpha\Delta\theta+\gamma_B\Delta\theta)}. \tag{23}$$

The expected utility of an entrepreneur with a low-quality project when all entrepreneurs are rational is

$$E[u(\theta_1)] = -e^{-\rho(W_0+\theta_1)}, \tag{24}$$

and the expected utility of an entrepreneur with a high-quality project when all entrepreneurs are rational is

$$E[u(\theta_2)] = -e^{-\rho\left[W_0+(1-\gamma_R)\theta_2+\gamma_R\theta_2-\gamma_R^2\frac{\rho\sigma^2}{2}\right]},$$

which, using the fact that $\gamma_R^2 = \frac{2\Delta\theta}{\rho\sigma^2}(1-\gamma_R)$, can be simplified to

$$E[u(\theta_2)] = -e^{-\rho(W_0+\theta_1+\gamma_R\Delta\theta)}. \tag{25}$$

(i) From (21) and (24) we have

$$E[u(\theta_1|\theta_1)] - E[u(\theta_1)] = e^{-\rho(W_0+\theta_1)}\left(1 - e^{-\rho\alpha\Delta\theta}\right).$$

Therefore, if $\alpha \geqq 0$, then $E[u(\theta_1|\theta_1)] \geqq E[u(\theta_1)]$.

(ii) From (25) and (21) it follows that

$$E[u(\theta_1|\theta_2)] - E[u(\theta_2)] = e^{-\rho(W_0+\theta_1)}\left(e^{-\rho\gamma_R\Delta\theta} - e^{-\rho\alpha\Delta\theta}\right).$$

Therefore, if $\alpha \gtreqqless \gamma_R$ then $E[u(\theta_1|\theta_2)] \gtreqqless E[u(\theta_2)]$.

(iii) From (24) and (22) it follows that

$$E[u(\theta_2|\theta_1)] - E[u(\theta_1)] = e^{-\rho(W_0+\theta_1)}\left(1 - e^{-\rho\alpha\Delta\theta}\right).$$

Therefore, if $\alpha \geqq 0$, then $E[u(\theta_2|\theta_1)] \geqq E[u(\theta_1)]$.

(iv) From (25) and (23) we have

$$E[u(\theta_2|\theta_2)] - E[u(\theta_2)] = e^{-\rho(W_0+\theta_1)}\left(e^{-\rho\gamma_R\Delta\theta} - e^{-\rho(\gamma_B\Delta\theta+\alpha\Delta\theta)}\right).$$

Therefore, if $\alpha \gtreqqless \gamma_R - \gamma_B$, then $E[u(\theta_2|\theta_2)] \gtreqqless E[u(\theta_2)]$.

Q.E.D.

Proof of Proposition 4:

Welfare when some entrepreneurs are optimistic, $\kappa > 0$, and no entrepreneur is pessimistic, $\nu = 0$, is given by:

$$W_B = (1 - \pi - \kappa)\, E[u(\theta_1|\theta_1)] + \kappa E[u(\theta_2|\theta_1)] + \pi E[u(\theta_2|\theta_2)].$$

We know from parts (i) and (iii) of Proposition 4 that if $\alpha = 0$, then $E[u(\theta_1|\theta_1)] = E[u(\theta_2|\theta_1)] = E[u(\theta_1)]$. We also know from part (iv) of Proposition 3 that if $\alpha = 0$ and $\gamma_R > \gamma_B$, then $E[u(\theta_2|\theta_2)] < E[u(\theta_2)]$. Hence, $W_B < W_R$.

Q.E.D.

Proof of Proposition 5:

Making use of (12), (21), (22) and (23), we have

$$W_B = -e^{-\rho(W_0+\theta_1+\alpha\Delta\theta)}\left[(1-\pi+\nu)+(\pi-\nu)\,e^{-\rho\gamma_B\Delta\theta}\right], \tag{26}$$

where

$$\gamma_B = \frac{1}{\rho\lambda}\left[-(1-\beta)+\sqrt{(1-\beta)^2+2(1-\alpha-\beta)\rho\lambda}\right], \tag{27}$$

and

$$\alpha = \frac{\nu}{1-\pi-\kappa+\nu}, \tag{28}$$

and

$$\beta = \frac{\kappa}{\pi+\kappa-\nu}. \tag{29}$$

Taking a first-order Taylor series expansion of $W_B(\kappa,\nu)$ around $(0,0)$ we obtain:

$$W_B(\kappa,\nu) \approx W_B(0,0) + \left.\frac{\partial W_B}{\partial \kappa}\right|_{(0,0)}\kappa + \left.\frac{\partial W_B}{\partial \nu}\right|_{(0,0)}\nu, \tag{30}$$

where $W_B(0,0) = W_R$. We need to find out the two partial derivatives. From (26) we have

$$\left.\frac{\partial W_B}{\partial \kappa}\right|_{(0,0)} = \frac{\partial W_B}{\partial \gamma_B}\left.\left(\frac{\partial \gamma_B}{\partial \alpha}\frac{\partial \alpha}{\partial \kappa}+\frac{\partial \gamma_B}{\partial \beta}\frac{\partial \beta}{\partial \kappa}\right)\right|_{(0,0)} + \left.\frac{\partial W_B}{\partial \alpha}\frac{\partial \alpha}{\partial \kappa}\right|_{(0,0)}.$$

From (26) we have

$$\left.\frac{\partial W_B}{\partial \gamma_B}\right|_{(0,0)} = \rho\Delta\theta\pi e^{-\rho(W_0+\theta_1)}e^{-\rho\gamma_R\Delta\theta}$$

From (28) we obtain

$$\left.\frac{\partial \alpha}{\partial \kappa}\right|_{(0,0)} = \left.\frac{\nu}{(1-\pi-\kappa+\nu)^2}\right|_{(0,0)} = 0.$$

From (29) we have

$$\left.\frac{\partial \beta}{\partial \kappa}\right|_{(0,0)} = \left.\frac{\pi-\nu}{(\pi+\kappa-\nu)^2}\right|_{(0,0)} = \frac{\pi}{\pi^2} = \frac{1}{\pi}.$$

From (27) we obtain

$$\begin{aligned}\left.\frac{\partial \gamma_B}{\partial \beta}\right|_{(0,0)} &= \left.\frac{1}{\rho\lambda}\left[1+\frac{1}{2}\frac{-2(1-\beta)-2\rho\lambda}{\sqrt{(1-\beta)^2+2(1-\alpha-\beta)\rho\lambda}}\right]\right|_{(0,0)} \\ &= \left.\frac{1}{\rho\lambda}\left[1-\frac{(1-\beta)+\rho\lambda}{\sqrt{(1-\beta)^2+2(1-\alpha-\beta)\rho\lambda}}\right]\right|_{(0,0)} \\ &= -\frac{1}{\rho\lambda}\left(\frac{1+\rho\lambda}{\sqrt{1+2\rho\lambda}}-1\right).\end{aligned}$$

Hence

$$\left.\frac{\partial W_B}{\partial \kappa}\right|_{(0,0)} = -e^{-\rho(W_0+\theta_1)}\frac{\Delta\theta}{\lambda}\left(\frac{1+\rho\lambda}{\sqrt{1+2\rho\lambda}}-1\right)e^{-\rho\gamma_R\Delta\theta}. \tag{31}$$

From (26) we have

$$\begin{aligned}\left.\frac{\partial W_B}{\partial \nu}\right|_{(0,0)} &= -e^{-\rho(W_0+\theta_1)}\left(1-e^{-\rho\gamma_R\Delta\theta}\right) \\ &\quad + \left.\frac{\partial W_B}{\partial \gamma_B}\left(\frac{\partial \gamma_B}{\partial \alpha}\frac{\partial \alpha}{\partial \nu}+\frac{\partial \gamma_B}{\partial \beta}\frac{\partial \beta}{\partial \nu}\right)\right|_{(0,0)} + \left.\frac{\partial W_B}{\partial \alpha}\frac{\partial \alpha}{\partial \nu}\right|_{(0,0)}.\end{aligned}$$

From (28) we obtain

$$\left.\frac{\partial \alpha}{\partial \nu}\right|_{(0,0)} = \left.\frac{1-\pi-\kappa}{(1-\pi-\kappa+\nu)^2}\right|_{(0,0)} = \frac{1-\pi}{(1-\pi)^2} = \frac{1}{1-\pi}.$$

From (29) we have

$$\left.\frac{\partial \beta}{\partial \nu}\right|_{(0,0)} = \left.\frac{\kappa}{(\pi+\kappa-\nu)^2}\right|_{(0,0)} = 0.$$

From (27) we obtain

$$\left.\frac{\partial \gamma_B}{\partial \alpha}\right|_{(0,0)} = \left.\frac{1}{\rho\lambda}\left[\frac{1}{2}\frac{-2\rho\lambda}{\sqrt{(1-\beta)^2+2(1-\alpha-\beta)\rho\lambda}}\right]\right|_{(0,0)} = -\frac{1}{\sqrt{1+2\rho\lambda}}.$$

From (26) we have

$$\left.\frac{\partial W_B}{\partial \alpha}\right|_{(0,0)} = \rho\Delta\theta e^{-\rho(W_0+\theta_1)}\left(1-\pi+\pi e^{-\rho\gamma_R\Delta\theta}\right)$$

Therefore

$$\left.\frac{\partial W_B}{\partial \alpha}\frac{\partial \alpha}{\partial \nu}\right|_{(0,0)} = \rho\Delta\theta e^{-\rho(W_0+\theta_1)}\left(1+\frac{\pi}{1-\pi}e^{-\rho\gamma_R\Delta\theta}\right).$$

$$\begin{aligned}\left.\frac{\partial W_B}{\partial \nu}\right|_{(0,0)} &= -e^{-\rho(W_0+\theta_1)}\left(1-e^{-\rho\gamma_R\Delta\theta}\right)\\ &\quad -e^{-\rho(W_0+\theta_1)}\rho\Delta\theta e^{-\rho\gamma_R\Delta\theta}\left(\frac{1}{\sqrt{1+2\rho\lambda}}\frac{\pi}{1-\pi}\right)\\ &\quad +e^{-\rho(W_0+\theta_1)}\rho\Delta\theta\left(1+\frac{\pi}{1-\pi}e^{-\rho\gamma_R\Delta\theta}\right).\end{aligned}$$

Hence

$$\begin{aligned}\left.\frac{\partial W_B}{\partial \nu}\right|_{(0,0)} = -e^{-\rho(W_0+\theta_1)}&\Big[1-e^{-\rho\gamma_R\Delta\theta}-\rho\Delta\theta\\ &-\rho\Delta\theta\left(1-\frac{1}{\sqrt{1+2\rho\lambda}}\right)\frac{\pi}{1-\pi}e^{-\rho\gamma_R\Delta\theta}\Big]\end{aligned}\tag{32}$$

Substituting (31) and (32) into (30) we obtain

$$\begin{aligned}W_B(\kappa,\nu)-W_R \approx -e^{-\rho(W_0+\theta_1)}&\Bigg\{\frac{\Delta\theta}{\lambda}\left(\frac{1+\rho\lambda}{\sqrt{1+2\rho\lambda}}-1\right)e^{-\rho\gamma_R\Delta\theta}\kappa\\ &+\left[1-\rho\Delta\theta-e^{-\rho\gamma_R\Delta\theta}-\rho\Delta\theta\left(1-\frac{1}{\sqrt{1+2\rho\lambda}}\right)\frac{\pi}{1-\pi}e^{-\rho\gamma_R\Delta\theta}\right]\nu\Bigg\}.\end{aligned}$$

or,

$$\begin{aligned}W_B(\kappa,\nu)-W_R \approx -e^{-\rho(W_0+\theta_1)}&\Bigg\{(1-\rho\Delta\theta)\nu+\Bigg\{\frac{\Delta\theta}{\lambda}\left(\frac{1+\rho\lambda}{\sqrt{1+2\rho\lambda}}-1\right)\kappa\\ &-\left[1+\rho\Delta\theta\left(1-\frac{1}{\sqrt{1+2\rho\lambda}}\right)\frac{\pi}{1-\pi}\right]\nu\Bigg\}e^{-\rho\gamma_R\Delta\theta}\Bigg\}.\end{aligned}\tag{33}$$

From (33) we obtain two sufficient conditions for $W_B(\kappa,\nu) > W_R$:

$$\rho > \frac{1}{\Delta\theta},$$

and

$$\frac{\kappa}{\nu} < \frac{\lambda}{\Delta\theta}\frac{\sqrt{1+2\rho\lambda}+\left(\sqrt{1+2\rho\lambda}-1\right)\frac{\pi}{1-\pi}\rho\Delta\theta}{1+\rho\lambda-\sqrt{1+2\rho\lambda}}.\tag{34}$$

Rewriting the RHS of (34)

$$RHS = \frac{\lambda}{\Delta\theta} \frac{\sqrt{1+2\rho\lambda} + \left(\sqrt{1+2\rho\lambda} - 1\right) \frac{\pi}{1-\pi}\rho\Delta\theta}{1 + \rho\lambda - \sqrt{1+2\rho\lambda}}.$$

Multiplying both sides by $\frac{1+\rho\lambda+\sqrt{1+2\rho\lambda}}{1+\rho\lambda+\sqrt{1+2\rho\lambda}}$ and simplifying the denominator we have

$$RHS = \frac{\lambda}{\Delta\theta} \frac{\left(1 + \rho\lambda + \sqrt{1+2\rho\lambda}\right) \left[(1 + \frac{\pi}{1-\pi}\rho\Delta\theta)\sqrt{1+2\rho\lambda} - \frac{\pi}{1-\pi}\rho\Delta\theta\right]}{\rho^2\lambda^2},$$

or,

$$RHS = \frac{\left(1 + \rho\lambda + \sqrt{1+2\rho\lambda}\right) \left[(1 + \frac{\pi}{1-\pi}\rho\Delta\theta)\sqrt{1+2\rho\lambda} - \frac{\pi}{1-\pi}\rho\Delta\theta\right]}{\rho^2\Delta\theta\lambda}.$$

Multiplying terms in the numerator we obtain

$$\begin{aligned} RHS = \Bigg[&(1+\rho\lambda)\,(1 + \frac{\pi}{1-\pi}\rho\Delta\theta)\sqrt{1+2\rho\lambda} - \frac{\pi}{1-\pi}\rho\Delta\theta\,(1+\rho\lambda) \\ &+ (1 + \frac{\pi}{1-\pi}\rho\Delta\theta)(1+2\rho\lambda) - \frac{\pi}{1-\pi}\rho\Delta\theta\sqrt{1+2\rho\lambda}\Bigg] \div (\rho^2\Delta\theta\lambda). \end{aligned}$$

Multiplying $(1+\rho\lambda)\left(1 + \frac{\pi}{1-\pi}\rho\Delta\theta\right)\sqrt{1+2\rho\lambda}$ in the numerator we have

$$\begin{aligned} RHS = \Bigg[&\left(1 + \rho\lambda + \frac{\pi}{1-\pi}\rho\Delta\theta + \frac{\pi}{1-\pi}\rho^2\lambda\Delta\theta\right)\sqrt{1+2\rho\lambda} - \frac{\pi}{1-\pi}\rho\Delta\theta\,(1+\rho\lambda) \\ &+ (1 + \frac{\pi}{1-\pi}\rho\Delta\theta)(1+2\rho\lambda) - \frac{\pi}{1-\pi}\rho\Delta\theta\sqrt{1+2\rho\lambda}\Bigg] \div \left(\rho^2\Delta\theta\lambda\right). \end{aligned}$$

Cancelling out $+\frac{\pi}{1-\pi}\rho\Delta\theta\sqrt{1+2\rho\lambda} - \frac{\pi}{1-\pi}\rho\Delta\theta\sqrt{1+2\rho\lambda}$ in the nominator we get

$$\begin{aligned} RHS = \Bigg[&\left(1 + \rho\lambda + \frac{\pi}{1-\pi}\rho^2\lambda\Delta\theta\right)\sqrt{1+2\rho\lambda} - \frac{\pi}{1-\pi}\rho\Delta\theta\,(1+\rho\lambda) \\ &+ (1 + \frac{\pi}{1-\pi}\rho\Delta\theta)(1+2\rho\lambda)\Bigg] \div \left(\rho^2\Delta\theta\lambda\right). \end{aligned}$$

Multiplying terms in the nominator we obtain

$$\begin{aligned} RHS = \Bigg[&\left(1 + \rho\lambda + \frac{\pi}{1-\pi}\rho^2\lambda\Delta\theta\right)\sqrt{1+2\rho\lambda} - \frac{\pi}{1-\pi}\rho\Delta\theta - \frac{\pi}{1-\pi}\rho^2\lambda\Delta\theta \\ &+ 1 + 2\rho\lambda + \frac{\pi}{1-\pi}\rho\Delta\theta + 2\frac{\pi}{1-\pi}\rho^2\lambda\Delta\theta\Bigg] \div \left(\rho^2\Delta\theta\lambda\right). \end{aligned}$$

Simplifying terms in the nominator we have

$$RHS = \frac{\left(1 + \rho\lambda + \frac{\pi}{1-\pi}\rho^2\lambda\Delta\theta\right)\sqrt{1+2\rho\lambda} + 1 + 2\rho\lambda + \frac{\pi}{1-\pi}\rho^2\lambda\Delta\theta}{\rho^2\Delta\theta\lambda}.$$

Noting that $1 + 2\rho\lambda + \frac{\pi}{1-\pi}\rho^2\lambda\Delta\theta = (1 + \rho\lambda + \frac{\pi}{1-\pi}\rho^2\lambda\Delta\theta) + \rho\lambda$ we obtain

$$RHS = \frac{\left(1 + \rho\lambda + \frac{\pi}{1-\pi}\rho^2\lambda\Delta\theta\right)\sqrt{1+2\rho\lambda} + (1 + \rho\lambda + \frac{\pi}{1-\pi}\rho^2\lambda\Delta\theta) + \rho\lambda}{\rho^2\Delta\theta\lambda}.$$

Evidencing out $(1 + \rho\lambda + \frac{\pi}{1-\pi}\rho^2\lambda\Delta\theta)$ we get

$$RHS = \frac{(1 + \rho\lambda + \frac{\pi}{1-\pi}\rho^2\lambda\Delta\theta)(1 + \sqrt{1+2\rho\lambda}) + \rho\lambda}{\rho^2\Delta\theta\lambda}.$$

Finally, replacing λ by $\frac{\sigma^2}{\Delta\theta}$ we have

$$RHS = \frac{1}{\rho^2\sigma^2}\left[\left(1 + \sqrt{1 + \frac{2\rho\sigma^2}{\Delta\theta}}\right)\left(\rho^2\sigma^2\frac{\pi}{1-\pi} + \frac{\rho\sigma^2}{\Delta\theta} + 1\right) + \frac{\rho\sigma^2}{\Delta\theta}\right]$$

Q.E.D.

Proof Proposition 6: Let the net mean returns of low-quality projects θ_1 be negative, that is $\theta_1 < 0 < \theta_2$. When all entrepreneurs are rational the equity price is

$$P(\gamma) = E[\tilde{r}(\gamma)] = \begin{cases} 0, & \text{if } \gamma = 0 \\ \theta_2, & \text{if } \gamma > 0 \end{cases}.$$

Investors do not finance low-quality projects because they yield negative net mean returns. Therefore, entrepreneurs with low-quality projects do not undertake their projects. When some entrepreneurs are biased the equity price is

$$P(\gamma) = E[\tilde{r}(\gamma)] = \begin{cases} 0, & \text{if } \gamma = 0 \text{ and } \theta_1 + \alpha\Delta\theta \leq 0 \\ \theta_1 + \alpha\Delta\theta, & \text{if } \gamma = 0 \text{ and } \theta_1 + \alpha\Delta\theta > 0 \\ \theta_2 - \beta\Delta\theta, & \text{if } \gamma > 0 \end{cases},$$

with $\Delta\theta \equiv \theta_2 - \theta_1$. If $\theta_1 \leq -\frac{\alpha\theta_2}{1-\alpha}$, then investors are not willing to offer a positive equity price for non self-financed projects. So, realistic entrepreneurs with low-quality projects and pessimistic entrepreneurs do not undertake their projects. In this case, the condition for a separating equilibrium is

$$\theta_2 - \beta\Delta\theta > 0. \tag{35}$$

We assume from now on that (35) is satisfied. The incentive compatibility condition for realistic entrepreneurs with low-quality projects and pessimistic entrepreneurs is

$$u(W_0) \geq Eu[W_0 + (1-\gamma_B)(\theta_2 - \beta\Delta\theta) + \gamma_B \tilde{r}(\theta_1)].$$

In the least cost separating equilibrium this inequality is binding. Hence, using the property of normal returns and the fact that utilities are strictly increasing in final wealth we have

$$W_0 = W_0 + (1-\gamma_B)(\theta_2 - \beta\Delta\theta) + \gamma_B\theta_1 - \gamma_B^2\frac{\rho\sigma^2}{2}, . \tag{36}$$

Rearranging terms in (36) we obtain that γ_B is given by

$$\gamma_B^2\frac{\rho\sigma^2}{2} + \Delta\theta\gamma_B = \theta_2 - \beta\Delta\theta(1-\gamma_B). \tag{37}$$

Hence, when entrepreneurs are rational, $\alpha = \beta = 0$, γ_R is given by

$$\gamma_R^2\frac{\rho\sigma^2}{2} + \Delta\theta\gamma_R = \theta_2. \tag{38}$$

It follows from (37) and (38) that $\beta \geqq 0$ implies $\gamma_R \geqq \gamma_B$. The expected utilities of a realistic entrepreneur with a low-quality project and of a pessimistic entrepreneur are the same and given by

$$E[u(\theta_1|\theta_1)] = E[u(\theta_1|\theta_2)] = -e^{-\rho W_0}, \tag{39}$$

The expected utility of an optimistic entrepreneur is given by

$$E[u(\theta_2|\theta_1)] = -e^{-\rho\left[W_0 + (1-\gamma_B)(\theta_2 - \beta\Delta\theta) + \gamma_B\theta_1 - \gamma_B^2\frac{\rho\sigma^2}{2}\right]},$$

which, using (36), can be simplified to

$$E[u(\theta_2|\theta_1)] = -e^{-\rho W_0}. \tag{40}$$

Finally, the expected utility of a realistic entrepreneur with a high-quality project is

$$E[u(\theta_2|\theta_2)] = -e^{-\rho\left[W_0+(1-\gamma_B)(\theta_2-\beta\Delta\theta)+\gamma_B\theta_2-\gamma_B^2\frac{\rho\sigma^2}{2}\right]},$$

which, using the fact that $\gamma_B^2 = \frac{2}{\rho\sigma^2}[\theta_2 - \Delta\theta(1-\beta)\gamma_B - \beta\Delta\theta]$, can be simplified to

$$E[u(\theta_2|\theta_2)] = -e^{-\rho(W_0+\Delta\theta\gamma_B)}. \tag{41}$$

The expected utility of an entrepreneur with a low-quality project when all entrepreneurs are rational is

$$E[u(\theta_1)] = -e^{-\rho W_0}, \tag{42}$$

and the expected utility of an entrepreneur with a high-quality project when all entrepreneurs are rational is

$$E[u(\theta_2)] = -e^{-\rho\left[W_0+(1-\gamma_R)\theta_2+\gamma_R\theta_2-\gamma_R^2\frac{\rho\sigma^2}{2}\right]},$$

which, using the fact that from (38) $\gamma_R^2 = \frac{2}{\rho\sigma^2}(\theta_2 - \Delta\theta\gamma_R)$, can be simplified to

$$E[u(\theta_2)] = -e^{-\rho(W_0+\Delta\theta\gamma_R)}. \tag{43}$$

Welfare with biased entrepreneurs is given by

$$W_B = (1-\pi-\kappa)\,E[u(\theta_1|\theta_1)] + \nu E[u(\theta_1|\theta_2)] + \kappa E[u(\theta_2|\theta_1)] + (\pi-\nu)\,E[u(\theta_2|\theta_2)].$$

Making use of (39), (40) and (41), we have

$$W_B = -e^{-\rho W_0}\left[(1-\pi+\nu) + (\pi-\nu)e^{-\rho\Delta\theta\gamma_B}\right]$$

Welfare with rational entrepreneurs is given by

$$W_R = \pi E[u(\theta_2)] + (1-\pi)E[u(\theta_1)]$$

Making use of (42) and (43) we have

$$W_R = -e^{-\rho W_0}\left[(1-\pi) + \pi e^{-\rho\Delta\theta\gamma_R}\right]$$

Hence,

$$W_B - W_R = e^{-\rho W_0} \left[\pi \left(e^{-\rho\Delta\theta\gamma_R} - e^{-\rho\Delta\theta\gamma_B}\right) + \nu \left(e^{-\rho\Delta\theta\gamma_B} - 1\right)\right] \tag{44}$$

The term multiplying π in (44) is negative or zero because $\Delta\theta\gamma_R \geq \Delta\theta\gamma_B \Leftrightarrow \gamma_R \geq \gamma_B$ which is true. The term multiplying ν in (44) is negative because $\Delta\theta\gamma_B > 0 \Leftrightarrow \gamma_B > 0$ which is true. Hence, $W_B < W_R$ as long as $\nu > 0$ or $\kappa > 0$.

Q.E.D.

10 Figures

Figure 1: Impact of Entrepreneurs' Biases on Aggregate Self-Finance when $\pi = 0.5$, $\Delta\theta = 40$, and $\sigma^2 = 80$.

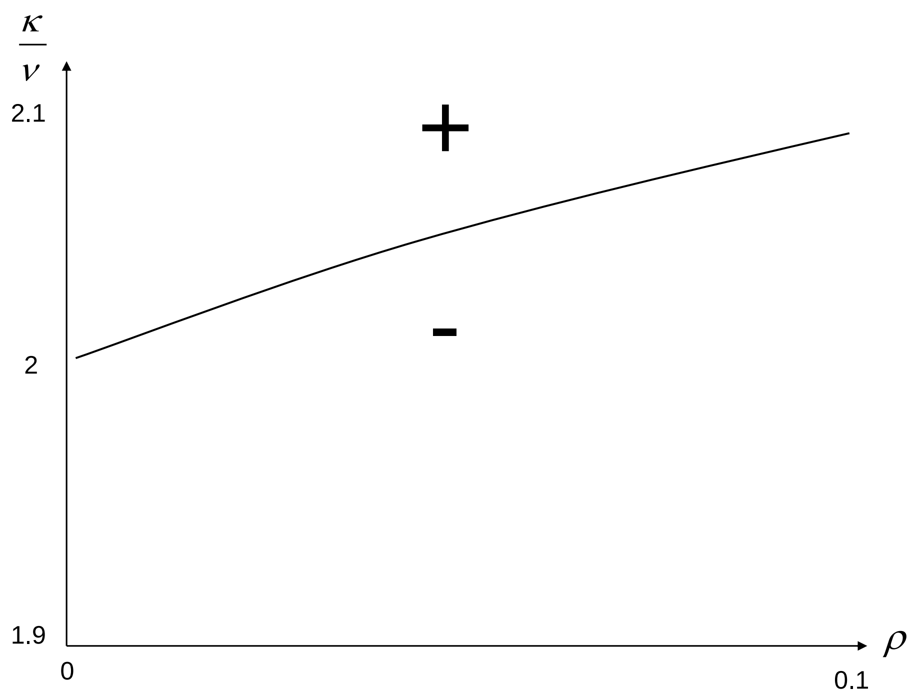

Figure 2: Impact of Entrepreneurs' Biases on Capital Market Efficiency when $\pi = 0.5$, $\Delta\theta = 40$, and $\sigma^2 = 80$.