Conflict Networks[‡]

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Conflict parties are frequently involved in more than one conflict simultaneously. In this paper the structure of local conflicts is modeled as a conflict network where rivals invest in conflict specific technology to attack their respective neighbors. We proof that there exists a unique equilibrium and examine the relation between total conflict investment (a proxy for conflict intensity) and underlying network characteristics. We also identify a class of conflict networks where peaceful conflict resolution results in reduced conflict intensity and show that in other conflict networks this result might not hold because countervailing indirect network effects can be substantial.

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1 Introduction

Violent conflicts and wars have been frequent and devastating phenomena in human history that spurred the interest of social scientists for centuries. Theoretical approaches based on game-theoretic methods and stylised models of conflicts between rational hostile actors provided valuable insights into the strategic aspects of warfare and conflict. The predominant set up in this type of strategic models considered usually two or more conflict parties that were involved in single and isolated conflicts between them. The nature of the respective conflict was then analysed depending on specific idiosyncratic characteristics of the involved conflict parties, see Garfinkel and Skaperdas (2006), Konrad (2009), as well as Anderton and Carter (2009) for surveys. In human history, however, conflict parties were frequently involved in more than one conflict at a given time that implied interdependencies between distinctive conflicts and conflict parties. These interdependencies had a substantive impact on the nature and the intensity of the respective conflicts. The following list provides some historically relevant examples of conflict structures where the conflict parties had to deal with several rivals in different conflicts simultaneously: (i) conflicts between a center and its periphery (e.g., the Roman empire), (ii) conflicts among rivals of similar size and power; for instance, the so called peer-polity interactions, where different polities interact among each other without any polity being or becoming dominant (e.g., the Myceneaen states), (iii) conflicts among groups or ideologies, where members of different groups perceive each other as enemies (e.g., World War II). Therefore, the nature of the conflict in these cases was shaped to some extent by the underlying structure of interdependent conflictive relations that we are going to interpret as respectively, star-shaped, regular, and bipartite conflict networks in our analysis.

The focus of our study is the analysis of this interrelated structure of conflicts and its consequences for conflict intensity. Hence, we extend the existing literature on strategic conflict analysis by assuming that a conflict party may be involved in two or more different conflicts involving different opponents at the same time. As our conflict model is based on interrelated bilateral conflicts the overall conflict structure can be represented as a network, where conflict parties are linked if they are in a conflictive relation among each other.[1] For each bilateral conflict a conflict party can affect the probability of winning against a particular direct rival by investing in conflict specific technology (e.g., military equipment, or mercenaries). Therefore, the structure of interrelated conflicts is a non-cooperative multi-agent conflict game, consisting of several distinctive bilateral conflicts

[1]The model is formulated in general terms such that different interpretations for the underlying conflictive environment are possible: For instance, lobbying of several firms for several distinctive issues at different authorities could also be interpreted as a network of bilateral conflicts where two firms are connected if they lobby for the same issue. Also a set of competing multi-product firms that fight for market share in different product markets by investing in R&D or marketing has a similar structure in the sense that two firms are connected if they are competitors on the same market.

that are simultaneously played on a fixed and given network. The network structure of conflictive relations implies that the investment decision of a conflict party may not only affect the decision of direct rivals but also of other parties that are not directly involved. This interrelated structure results in local externalities because the strategic behavior by agents is affected by changes in strategic behavior of direct and also indirect opponents. Hence, our objective is to clarify the relation between the structure of those interrelated conflicts and equilibrium behavior which allows us to analyse the dependence of conflict intensity (measured as the total equilibrium investment in conflict technology by all opponents) on different network characteristics using comparative statics.

We establish existence and uniqueness in pure strategies for the conflict network game which facilitates the comparative static analysis. Based on the typical setup from the conflict literature we derive closed form expressions for conflict intensity for three important classes of conflict networks which are regular, star-shaped, and complete bipartite networks. Those types of conflict networks are related with the mentioned historical conflict cases (where the specific similarities are discussed in the relevant sections). For these network classes an intuitive relation holds between the underlying network characteristics and conflict intensity: Within each considered class, conflict intensity is increasing in the number of conflictive relations and the density of the network. Moreover, local externalities are important; for example, the center in a star-shaped network is worse of if the number of rivals in the periphery increases due to negative network externalities, that are (at the same time) beneficial for each agent in the periphery. Network externalities also affect the comparison of conflict intensity *across* different conflict classes. The concept of eigenvector centrality facilitates this comparison across the three different classes with respect to their induced conflict intensity. We use this result to show by example that an unambiguous ranking of those classes with respect to conflict intensity does not exist and clarify the role of network externalities in this context.

The game-theoretic set up can also be interpreted from the perspective of peaceful conflict resolution. Peaceful resolution of conflicts is here conceived as an exogenous ad-hoc deletion of specific conflictive links within the conflict network. For the three considered classes the established positive relation between conflict intensity and the number of bilateral conflicts implies that peaceful conflict resolution is beneficial because total conflict intensity is reduced. This result is valid within and across the considered classes of conflict network. Hence, we identify a class of conflict networks where peaceful conflict resolution is always beneficial.

Extending these results to more general networks is a complex issue due to the fact that in equilibrium no closed form solution exists. Nevertheless, it is possible to characterise equilibrium behavior in general networks in an indirect way which provides some insights with respect to the importance of direct (first order) and indirect (second order) effects of peaceful conflict resolution. While first order effects of resolving conflict are always beneficial with respect to conflict intensity, the consequences of second order effects are mostly countervailing with varying extent that is highly dependent on the underlying network structure. To demonstrate this dependence we provide an example where peaceful conflict resolution might in fact lead to adverse consequences on conflict intensity due to substantial and counteracting second order effects driven by network externalities. Hence, peaceful conflict resolution might actually result in an increase of conflict intensity if the embeddedness of conflict parties into a local structure of multiple conflicts is not taken into account.

Moreover, our model can be extended along other dimensions. We discuss the robustness of our results with respect to more general contest success functions and cost functions and analyse the consequences of additional sources of heterogeneity. Finally, we relax the assumption on pure link specific action in the sense that conflict specific technology can (to some extent) be applied in other bilateral conflicts. In this case conflict investment has a multi-purpose (or strategic) characteristic that can be interpreted as investments into intercontinental ballistic missiles or R&D in weapon systems in general. We show that this modified type of conflict technology is equivalent to the original model with heterogeneous cost functions.

Related Literature

Besides the literature on conflicts and wars our approach is related to the recent network literature that considers games played on fixed and given networks; for example, public good provision in Bramoullé and Kranton (2007), Bramoullé et al. (2010), R&D investment in Goyal and Moraga-González (2001), and the characterization of the central player in Ballester et al. (2006). We depart from this literature in two important aspects: First, in our set-up the individual action is link-specific (which makes the strategy space of an agent multi-dimensional) because conflict investment is specific for each bilateral conflict.[2] This is in contrast to the usual assumption of a uni-dimensional strategy space of an agent (which is therefore common for all neighbors) that is typically made in those types of models.[3] Our extension provides a richer structure that allows us to analyse explicitly how a specific agent strategically reallocates her conflict investment among its different bilateral conflicts.

[2] This assumption can be justified by referring to the different nature of each bilateral conflict; e.g., naval forces are more suited than air forces in specific conflicts. Moreover, even if the same type of force is suitable in various conflicts, the conflict party has to decide where they should be employed. Link-specificity then simply implies that forces cannot be employed at two distinct locations at the same time. This assumption is relaxed in section 5.2.2.

[3] For a recent exception with multi-dimensional individual strategy space in a network context see Goyal et al. (2008), while Arbatskaya and Mialon (2010) consider this type of extension in a two player-contest game. There also exist some similarities with Colonel Blotto games, see Roberson (2006), Hart (2008) and a recent survey in Kovenock and Roberson (2010). However, these models are usually based on a two-player set-up and do not consider different local structures of interaction.

The second difference concerns the application of a so called contest success function in a network framework. This type of function has a simple and intuitive interpretation and is frequently used in the literature on conflict analysis; for instance, in Esteban and Ray (2008) to model ethnic conflicts, in Skaperdas (1992) to model the trade-off between coercive and productive activities, and in Beviá and Corchón (2010) to analyse strategic formation of peace agreements.

According to our knowledge our approach is the first one to embed this type of conflict model in a local network structure. From a technical perspective the resulting conflict game has the non-standard property that it is neither an aggregative nor a supermodular game.[4] In combination with the fact that the strategy space is multi-dimensional this implies that common existence proofs that are based on these characteristics, e.g. Galeotti et al. (2010) or Cornes and Hartley (2005), are not applicable in this context. The alternative existence and uniqueness result presented here is instead based on the notion of diagonal strict concavity introduced by Rosen (1965). Especially in network games, where the strategic relation is bilateral and additively separable, our approach might therefore be a valuable and convenient alternative for establishing equilibrium existence and uniqueness results.

The rest of the paper is structured as follows. In the next section we set up a general model of conflict networks and proof, in section 3, that a unique equilibrium exists for our framework. We then analyse extensively the three mentioned classes of conflict networks in section 4, and compare conflict intensity across those classes. We also reinterpret and discuss our results from the perspective of peaceful conflict resolution. In section 5 we discuss whether the results can be extended to more general conflict networks. Additionally, we confirm the robustness of our results with respect to more general functional forms and analyse the consequences of additional sources of heterogeneity and different types of conflict technologies. Finally, section 6 concludes by pointing out some limitations and further research possibilities. All proofs are gathered in an appendix.

2 The Model

There is a set $N = \{1, \ldots, n\}$ of conflicting parties (from now on called agents) that are embedded in a fixed structure of bilateral conflicts; that is, each agent $i \in N$ is engaged into bilateral conflicts with some other agents called rivals. The set of rivals of agent i is denoted by $N_i \subseteq N \setminus \{i\}$ which implies that agent i is involved in $n_i = |N_i|$ conflicts. The underlying structure of bilateral conflicts can be interpreted as a fixed network which is represented by a graph $\mathbf{g}$ consisting of nodes (agents) and links (conflicts). Hence, if agent i is in conflict with rival j then $g_{ij} = 1$. If there is no conflictive relation between them then $g_{ij} = 0$. It is assumed that a bilateral conflict is symmetric in the sense that

[4] The reaction functions are non-monotonic and cannot be expressed in terms of the aggregated strategies of all the other players.

$g_{ij} = g_{ji}$ for all $i \neq j$. The set N_i of rivals of agent i can then be formally defined as $N_i = \{j \in N \setminus \{i\} : g_{ij} = 1\}$.

The outcome of each bilateral conflict is probabilistic and depends on the investment in conflict specific technology by the respective rivals. The investment of agent i in the conflict against rival $j \in N_i$ is denoted by $x_{ij} \in \Re_+$ and the n_i-dimensional vector of conflict spendings of agent i against all her rivals (her strategy) by $\mathbf{x_i} = (x_{ij})_{j \in N_i}$. The vector of conflict spendings that is directed against agent i by all of her respective rivals is denoted by $\mathbf{x_{-i}} = (x_{ji})_{j \in N_i}$.

Our study is focused on the analysis of the effects of the network structure on equilibrium outcome. As the network structure by itself induces endogenously heterogeneity on the agents (depending on their location), we exclude all other sources of heterogeneity in our model to be able to concentrate exclusively on this channel.[5] Hence, it is assumed that potential gains and losses in each bilateral conflict are symmetric in the following sense: If agent i wins the conflict against a direct rival $j \in N_i$ she obtains an amount $V \in \Re_{++}$ of resources of the respective rival j, otherwise an amount V of her own resources are transferred to the winning agent j. In other words, each bilateral conflict is modeled as a transfer contest where contested resources are transfered from the winner to the loser, see Appelbaum and Katz (1986), Hillman and Riley (1989), and Leininger (2003). This assumption reflects the frequently observed fact that underlying motivations for conflict are contested natural resources, or territory, and that looting is and was a frequently observed behavior of the winning conflict party.[6]

The outcome of each bilateral conflict is governed by a probability function that maps the conflict specific investments of two opposing rivals into a probability to win the respective conflict; that is, agent i wins the bilateral conflict against rival $j \in N_i$ with probability $p_{ij} = p(x_{ij}, x_{ji}) \in [0, 1]$, which is twice differentiable, increasing, and strictly concave in own spendings x_{ij} for each level of spending x_{ij} by the respective opponent j. As $p_{ij} = 1 - p_{ji}$ the probability function is also decreasing and strictly convex in the spending x_{ji} of the respective rival.[7] Moreover, this probability is symmetric in the sense that if two direct rivals i and j spend the same amount, $x = x_{ij} = x_{ji}$, then they will win the conflict with the same probability: $p_{ij} = p_{ji} = p(x, x) = \frac{1}{2}$.

Investing in conflict specific technology against rivals is also related with a cost $c(\mathbf{x_i})$ that is a continuous, increasing, and convex function with $c(0, \ldots, 0) = 0$. The expected payoff function of agent i is additively separable in costs and expected wins and losses of

[5]In section 5.2 we discuss the consequences of additional sources of heterogeneity that affect the cost functions of the agents.

[6]In Collier and Hoeffler (2004), for instance, it is shown that economic factors ('greed'), like primary commodities and opportunity costs for conflict activity, have more predictive power for the outbreak of civil war than political factors ('grievance'), like inequality or ethnic polarization.

[7]Note, that there is no assumption on the cross derivative of the probability function such that the set up is sufficiently general to include specifications where conflict spendings are strategic complements, or substitutes, or mixtures of both.

all bilateral conflicts in which she is involved:[8]

$$\pi_i(\mathbf{x_i}, \mathbf{x_{-i}}; \mathbf{g}) = \sum_{j \in N_i} p_{ij} V - \sum_{j \in N_i} p_{ji} V - c(\mathbf{x_i}).$$

For notational simplicity we reformulate this expression as follows:

$$\pi_i(\mathbf{x_i}, \mathbf{x_{-i}}; \mathbf{g}) = W(\mathbf{x_i}, \mathbf{x_{-i}}; \mathbf{g}) - c(\mathbf{x_i}), \tag{1}$$

where $W(\mathbf{x_i}, \mathbf{x_{-i}}; \mathbf{g}) = 2V \sum_{j \in N_i} p_{ij} - n_i V$ denotes the expected 'revenue' of conflict for agent i; that is, the aggregate expected amount of transfered resources that agent i wins or looses in all her bilateral conflicts. Note that, due to the fact that each conflict is modeled as a transfer contest where losers have to compensate the winner, the total expected revenue of the overall conflict game (or, in other words, the aggregate value of contested and transfered resources) is zero independently of the network structure:

$$\sum_{i \in N} W(\mathbf{x_i}, \mathbf{x_{-i}}; \mathbf{g}) = 0. \tag{2}$$

This implies that equilibrium behavior does purely depend on the strategic response to the network structure and is not confounded by the fact that different network structures induce different values of aggregated resources.

Our objective is the analysis of overall conflict intensity, denoted by $X^*(\mathbf{g})$, and formally defined as the aggregate level of conflict investment in equilibrium by all agents in all bilateral conflicts:

$$X^*(\mathbf{g}) = \sum_{i \in N} X_i^*(\mathbf{g}) = \sum_{i \in N} \sum_{j \in N_i} x_{ij}^*(\mathbf{g}),$$

where $X_i(\mathbf{g}) = \sum_{j \in N_i} x_{ij}(\mathbf{g})$ is the aggregate conflict investment of agent i against all her rivals $j \in N_i$. The variation of conflict intensity for different networks $\mathbf{g}$ can then be determined by analyzing how $X^*(\mathbf{g})$ depends on the variables that characterise the respective network structure.

3 Equilibrium Analysis

The equilibrium analysis of the conflict network game relies on the notion of 'diagonal strict concavity' as established in Goodman (1980). Using his definition we can establish existence and uniqueness of equilibrium using Rosen (1965), where it is proved that diagonal strict

[8] Hence, we follow the interpretation of Bueno de Mesquita (1981) in the sense that the conflict investment of a conflict party is determined as if it would have been the decision of a rational, expected-utility maximizing foreign policy decision maker, see the mentioned publication for a theoretical and empirical justification of this assumption based on extensive international conflict data.

concavity is a necessary and sufficient condition for the existence of a unique equilibrium in pure strategies. The definition of diagonal strict concavity is based on the joint (and weighted) payoff function:

$$\sigma(\mathbf{x}, \mathbf{r}) = \sum_{i \in N} r_i \, \pi_i(\mathbf{x_i}, \mathbf{x_{-i}}), \tag{3}$$

where $\mathbf{x} = (\mathbf{x_1}, \ldots, \mathbf{x_n})$, $\mathbf{r} = (r_1, \ldots, r_n)$, and $r_i \geq 0$. Theorem 1 and 2 in Rosen (1965) imply that a unique equilibrium exists in a concave n-person game with orthogonal constraint set[9] if and only if the function $\sigma(\mathbf{x}, \mathbf{r})$ is diagonally strictly concave. Intuitively, this technical condition guarantees that an agent has more control over her payoff than the other players. In Goodman (1980) it is shown that the following conditions on the payoff functions are equivalent for diagonally strict concavity of function $\sigma(\mathbf{x}, \mathbf{r})$:

(i) $\pi_i(\mathbf{x_i}, \mathbf{x_{-i}})$ is strictly concave in $\mathbf{x_i}$ for all $\mathbf{x_{-i}}$,

(ii) $\pi_i(\mathbf{x_i}, \mathbf{x_{-i}})$ is convex in $\mathbf{x_{-i}}$ for all $\mathbf{x_i}$,

(iii) $\sigma(\mathbf{x}, \mathbf{r})$ is concave in $\mathbf{x}$ for some $\mathbf{r}$ with $r_i > 0$ for all $i \in N$.

The conflict network game satisfies the requirement of a concave n-person game by the assumptions on the payoff function. Hence, the following proposition establishes existence and uniqueness by proving that the conflict network game is also diagonally strictly concave.

Proposition 1 *There exists a unique equilibrium in the conflict network game.*

The existence and uniqueness result is an important precondition to carry out the comparative statics analysis. Using comparative statics also requires the characterization of conflict intensity $X^*(\mathbf{g})$ in closed form for different network types and therefore more structure on the payoff function. Hence, we consider the following specification which combines a contest rule stemming from the literature on conflicts (with a link-specific action space whose dimensionality is determined by the underlying conflict network) and a quadratic cost function.[10] The cost function for agent $i \in N$ is of the following form:

$$c(\mathbf{x_i}) = c(X_i) = (X_i)^2 = \left(\sum_{j \in N_i} x_{ij} \right)^2. \tag{4}$$

This cost function captures the externalities of the network structure because the marginal cost of conflict technology for a specific bilateral conflict also depends on the investment in

[9] A constraint set is orthogonal if it is uncoupled. This is the case in the conflict game because the strategy space of each individual does not depend on the strategies of her rivals. Note also that in Rosen (1965) the strategy space for each $i \in N$ is convex and compact, which is, in principle, not the case in the conflict game defined above (here it is the non-negative orthant). However, we can construct a (sufficiently high) upper limit $\bar{x}$ such that all strategies $x_{ij} > \bar{x}$ are strictly dominated (for instance by choosing $x_{ij} = 0$) due to the fact that $W(\mathbf{x_i}, \mathbf{x_{-i}}) \in [-n_i V, n_i V]$ is bounded while $c(\mathbf{x_i})$ is unbounded. Hence, without loss of generality we can restrict attention to the strategy space $[0, \bar{x}]^{n_i}$ of non-dominated strategies for each individual $i \in N$ which is convex and compact.

[10] In section 5.2 we show that most of the results are robust with respect to more general contest rules or cost functions.

all other bilateral conflicts in which agent i is involved. Intuitively, the assumption on the cost function could be attributed to a production technology where agent i has access to a centralised (and convex) production process where conflict technology for all her different bilateral conflicts has to be produced. Agent i then allocates the centrally produced conflict technology output to the different bilateral conflicts in which she is involved.

The outcome of a bilateral conflict is realised according to a so called contest success function in the style of Tullock (1980), which is frequently applied in models of conflict and contests; see the mentioned surveys in the introduction and Skaperdas (1996) for an axiomatization.[11] Under this contest success function the winning probability of agent i in the bilateral conflict against rival j is simply determined as her relative investment in conflict specific technology for the respective conflict:[12]

$$p_{ij} = \begin{cases} \frac{x_{ij}}{x_{ij}+x_{ji}} & \text{if } x_{ij}+x_{ji} > 0, \\ 1/2 & \text{if } x_{ij}+x_{ji} = 0. \end{cases} \tag{5}$$

The payoff function of agent $i \in N$ is therefore specified as follows:

$$\pi_i(\mathbf{x_i}, \mathbf{x_{-i}}; \mathbf{g}) = 2V \sum_{j \in N_i} p_{ij} - n_i V - \left(\sum_{j \in N_i} x_{ij} \right)^2 . \tag{6}$$

For the specified conflict network game the following interiority result holds due to the convexity of the cost function:

Lemma 1 *The equilibrium in the conflict network game as specified in Eq. (6) is interior.*

Interiority of equilibrium implies that each agent will invest strictly positive amounts in conflict specific technology in all of her conflictive relations. This already indicates that equilibrium investment turns out to be socially inefficient: Contested resources are merely reallocated while all agents invest positive amounts and therefore face real costs. Hence, the aggregate expected equilibrium payoff will be negative. This captures the idea that conflicts are generally destructive and socially undesirable. In principle, a conflict party has also the option not to invest anything in a bilateral conflict which would reduce socially wasteful investment. However, this option can be exploited by their rival and therefore will never occur in equilibrium which implies that investing in conflict specific technology resembles a prisoner's-dilemma structure.

[11] This probability function leads to non-monotonic best reply functions; i.e., conflict investments are neither strategic substitutes nor complements. Hence, the conflict game as specified here is not supermodular.

[12] Note that this functional form satisfies properties (i), (ii) and (iii) of the existence proof in Proposition 1. However, due to the discontinuity at point $(0, 0)$ the payoff function is not everywhere continuous such that the conflict game does not fit exactly the setup as introduced before. Nonetheless, the same argument as in Myerson and Wärneryd (2006) can be applied to show that this lack of continuity is not problematic: Basically, the contest success function in Eq. (5) can be obtained as the limit of the function $\tilde{p}_{ij} = \frac{\tilde{x}_{ij}+a}{\tilde{x}_{ij}+\tilde{x}_{ji}+2a}$ for $a \to 0$ with $\alpha > 0$. This alternative function $\tilde{p}_{ij}$ is everywhere continuous and satisfies also all the conditions of Proposition 1 such that the existence result applies.

Interiority also implies that equilibrium investment is the solution to the following system of first order conditions:

$$\frac{x_{ki}^*(\mathbf{g})}{(x_{ik}^*(\mathbf{g}) + x_{ki}^*(\mathbf{g}))^2} V = X_i^*(\mathbf{g}), \qquad \text{for all } k \in N_i, \text{ and all } i \in N. \tag{7}$$

This is a non-linear system with $\sum_{i \in N} n_i$ equations that does not yield closed form solutions for general conflict structures. Therefore we will concentrate our analysis at first on three important classes of more structured conflict networks, which are regular, star-shaped, and complete bipartite networks, that allows us to derive closed form solutions of the above system. The extension to more general networks is then discussed in section 5.1.

4 Characteristic Classes of Conflict Networks

Regular, star-shaped, and complete bipartite conflict networks are distinct with respect to their grade of symmetry. In our framework asymmetry is induced through the underlying network structure in the sense that agents with a high number of conflictive relations can potentially gain and also loose more resources than agents with less conflicts. In the following three subsections we consider those three types of network classes separately: In the first subsection our focus is on highly symmetric conflict structures like regular conflict networks, where each agent has the same number of conflicts. In the second subsection we consider highly asymmetric conflict networks like star-shaped networks where a center is in conflict with agents on the periphery. An intermediate class, analysed in the third subsection, are complete bipartite conflict networks that consist of two hostile coalitions where members of one coalition are in conflict with each member of the opposed coalition. Finally, in the last two subsections we combine the previous analysis by deriving the relation between conflict intensity and peaceful conflict resolution across those three classes of conflict networks.

4.1 Regular Conflict Networks

Regular conflict networks are characterised by their high degree of symmetry among rivals. This symmetry property among social entities in a local environment is also the crucial element in the concept of 'peer polity interaction'. This concept was introduced in Renfrew and Cherry (1986) to describe the historical fact that complex societies often developed through interaction of autonomous and homogeneous social units that were not related to each other in forms of dominance and subordination. Peer polity interaction also included warfare and conflict. Historical examples that could be subsumed under this concept are:

> The Mycenaean states, the later small city-states of the Aegean and the Cyclades, or the centers of the Maya Lowlands, that interact on an approximately

Figure 1: Regular Conflict Structures: Ring (left) and Complete Network (right)

> equal level. [...] The evolution of such clusters of peer polities is conditioned not by some dominant neighbor, but more usually by their own mutual interaction, which may include both exchange and conflict. Tainter (1988, p. 201)

Our focus is on hostile interaction among peer polities and we associate the symmetric nature of conflictive peer polity interaction with a regular conflict network. Formally, a graph $\mathbf{g}^R$ is called *regular of degree* d if each agent $i \in N$ has the same number d of opponents: $n_i = d$ for all $i \in N$. Hence, a regular graph $\mathbf{g}^R$ can be characterised by its degree d and the total number n of agents. The corresponding class R of regular networks incorporates cases such as the fully connected complete network, where $d = n - 1$, and a ring structure, where $d = 2$, compare Figure 1.

The following proposition describes the relation between those characteristics and conflict intensity for the class R of regular networks.

Proposition 2 *In conflict networks of class R:*

(i) Conflict intensity $X^*(\mathbf{g}^R)$ is increasing in its degree d and in the total number n of agents in the network.

(ii) Conflict intensity $X^*(\mathbf{g}^R)$ in a regular network $\mathbf{g}^{R1}$ is higher than in $\mathbf{g}^{R2}$ if and only if

$$n_1\sqrt{d_1} > n_2\sqrt{d_2}.$$

(iii) Individual conflict investment $x_i^*(\mathbf{g}^R)$ and expected equilibrium payoff $\pi_i(\mathbf{x_i^*}, \mathbf{x_{-i}^*}; \mathbf{g}^R)$ is decreasing in d and does not depend on n. Moreover, expected equilibrium payoff is negative for all agents.

While the interpretation of the first result is straight forward in the sense that a higher number of conflicts induces higher conflict intensity, the second result provides more insights into the relation of the two sources of differences among regular networks; that is, degree and total number of players. The comparison between two distinctive regular conflict networks suggests that the number of players contributes qualitatively more to conflict intensity than the number of direct rivals. However, individual behavior is independent of the overall size of the networks.

In equilibrium all agents choose the same level of conflict investment which implies that they win each bilateral conflict with the same probability. Hence, individual equilibrium payoff is negative for each agent because an agent also faces the cost of conflict spending. This situation is socially inefficient because universal peace (i.e., refraining from conflict investment) would result in zero expected payoff. The fact that such a conflict structure induces socially inefficient results is also acknowledged in the historical analysis of the above mentioned examples:

> Successful competition by any Mycenaean polity would yield little real return. The result was probably constant investment in defense, military administration, and petty warfare, with any single polity rarely experiencing a significant return on that investment. (ibid., p. 204).

Figure 2: A Star-Shaped (left) and a Bipartite Conflict Network (right)

4.2 Star-Shaped Conflict Networks

We now focus our attention on asymmetric conflict structures that are star-shaped; that is, one agent is in conflict with all other remaining rivals while none of the rivals is in conflict with each other, see the left part of figure 2. This class of conflict networks has a center-periphery structure which is reminiscent of historical empires, for instance the Roman Empire, that was frequently in conflict with rivals at their periphery.

Formally, a star-shaped conflict network consists of a center agent c who is in conflict with all other agents such that $g_{ci} = 1$ for all $i \in N_c$ and $N_c = N \setminus \{c\}$. All agents of set N_c at the periphery are only in conflict with the center but not with each other, $g_{ij} = 0$ for all $i, j \neq c$ and thus $n_i = 1$ for all $i \in N_c$. This implies that there are in total $n - 1$ bilateral conflicts in the star network. Hence, the class of star networks, from now on denoted by S, is completely characterised by $n_c = n - 1$, the number of agents in the periphery.

The payoff of the center agent c can be written as

$$\pi_c(\mathbf{x_c}, \mathbf{x_i}; \mathbf{g}^S) = \sum_{i \in N_c} \frac{x_{ci}}{x_{ci} + x_{ic}} 2V - (X_c)^2 - (n-1)V, \tag{8}$$

and the corresponding payoff by an agent $p \in N_c$ in the periphery is

$$\pi_p(x_{pc}, x_{cp}; \mathbf{g}^S) = \frac{x_{pc}}{x_{cp} + x_{pc}} 2V - (x_{pc})^2 - V. \tag{9}$$

The following proposition summarises equilibrium behavior and outcomes in this class of star-shaped conflict networks. As star-shaped conflict networks are subsets of complete bipartite conflict networks, Proposition 3 is a special case of Proposition 4. Hence, the proof of Proposition 4 applies here as well.

Proposition 3 *In conflict networks of class S:*

(i) Conflict intensity $X^*(\mathbf{g}^S)$ is increasing in the number n_c of agents in the periphery.

(ii) For the center agent individual conflict investment $x^*_{cp}(\mathbf{g}^S)$ is decreasing in n_c, while aggregate conflict investment of the center $X^*_c(\mathbf{g}^S)$ is increasing in n_c. Equilibrium probability $p^*_c(\mathbf{g}^S)$ and expected payoff $\pi_c(\mathbf{x}^*_\mathbf{c}, \mathbf{x}^*_\mathbf{p}; \mathbf{g}^S)$ of the center is decreasing in n_c. For the periphery agent the same relation holds with respect to individual conflict investment, while the relation is reversed for equilibrium probability and payoff.

The results stated in proposition 3 imply that the center agent is worse off if she faces more bilateral conflicts with the periphery. This is intuitive because additional rivals of the center agent will also invest in the bilateral conflict which forces the center agent to invest more in total conflict spendings (i.e., $X^*_c(\mathbf{g}^S)$ is increasing in the number of agents in the periphery) and to redistribute her conflict investment among more conflictive relations. The result is an increase in marginal costs which explains why $x^*_{cp}(\mathbf{g}^S)$ and therefore also $p^*_{cp}(\mathbf{g}^S)$ is decreasing in n_c. Hence, additional conflicts for the center imply that expected equilibrium payoff $\pi^*_c(\mathbf{g}^S)$ is strictly decreasing in the number of rivals. This negative externality for the center is due to the specific network structure and is also reflected by the historically observed tendency of expanding empires to collapse at some point in time because expansion requires more total investment for an increasing number of conflicts. These types of investment frequently induce marginal increasing costs for the center agent as is argued in Tainter (1981) based on a detailed analysis of several historical cases:

> The economics of territorial expansion dictate, as a simple matter of mathematical probability, that an expanding power will ultimately encounter a frontier beyond which conquest and garrisoning are unprofitable. [...] The combined factors of increased costliness of conquest, and increased difficulty of administration with distance from the capital, effectively require that at some point a policy of expansion must end. (ibid, p. 148 f.)

4.3 Complete Bipartite Conflict Networks

An intermediate case with respect to the symmetry of the underlying conflict structure are networks where the members of two hostile coalitions are in conflict among each other. Hence, each agent of a coalition is in conflict with all the members of the hostile coalition, as represented in the right part of figure 2. This type of conflict structure resembles an ideological bipolar conflict because members of the two hostile coalitions perceive each other as enemies.[13] Moreover, the common ideology among coalition members implies that there are no conflictive relations among agents of the same ideology. Historical examples of conflicts that fit to this description are the massive ideological conflicts in the 20th century, for instance World War II, where each member of the Axis Powers was (at least at some point in time) in conflict with nearly each member of the Allies.

A complete bipartite network, denoted by B, consists of two sets (coalitions) of agents, Y and Z, that each have $y = |Y|$ and $z = |Z|$ members. All members of set Y are in conflict with each member of set Z and vice versa, such that $g_{ij} = 1$ for all $i \in Y$ and all $j \in Z$. Agents of the same coalition are not in conflict among each other: $g_{ij} = 0$ for all $i, j \in Y$ or $i, j \in Z$ which also implies that $N_i = Z$ for all $i \in Y$ and vice versa.

The payoff function of an agent $i \in Y$ in a complete bipartite network B can be stated as follows:

$$\pi_i(\mathbf{x_i}, \mathbf{x_{-i}}; \mathbf{g}^B) = \sum_{j \in Z} \frac{x_{ij}}{x_{ij} + x_{ji}} 2V - (X_i)^2 - zV, \tag{10}$$

and vice versa for an agent that is a member of coalition Z. Note also, that the star-shaped network is a special case of a bipartite network (i.e., $S \subseteq B$) where one coalition only consists of one (center) agent c; e.g., $Y = c$ and $Z = N_c$. Therefore, the following proposition generalises Proposition 3.

Proposition 4 *In conflict networks of class B:*

(i) Conflict intensity $X^*(\mathbf{g}^B)$ is increasing in the number y and z of each coalition.

(ii) Conflict intensity $X^*(\mathbf{g}^B)$ in a complete bipartite network $\mathbf{g}^{B1}$ is higher than in $\mathbf{g}^{B2}$ if and only if the total number of bilateral conflicts in $\mathbf{g}^{B1}$ is higher than in $\mathbf{g}^{B2}$:

$$y_1\, z_1 > y_2\, z_2.$$

(iii) A larger coalition is beneficial for its members; that is, each member i of the more numerous coalition invests less in aggregate conflict investment $X_i^*(\mathbf{g}^B)$, wins each bilateral conflict with higher probability $p_i^*(\mathbf{g}^B)$, and has higher equilibrium payoff $\pi_i(\mathbf{x}_\mathbf{i}^*, \mathbf{x}_\mathbf{-i}^*; \mathbf{g}^B)$.

[13] As mentioned earlier our basic assumption is that the underlying conflict network is exogenously given. For a theoretical model that shows how a bipolar coalition structure can be the stable equilibrium outcome in a coalition formation game embedded in a conflict framework, see Jackson and Morelli (2007).

In a complete bipartite conflict network a coalition becomes more powerful if it has more members. The intuition for this result coincides with the interpretation provided for the star-shaped network. Assuming without loss of generality that $y > z$, Proposition 4 implies that a member of coalition Y will exert relatively less in aggregated conflict investments but more against each of her respective rivals: $x^*_y(\mathbf{g}^B) > x^*_z(\mathbf{g}^B)$, but at the same time $X^*_y(\mathbf{g}^B) < X^*_z(\mathbf{g}^B)$ due to the fact that coalition Y is more numerous which implies that members of coalition Z face relatively more rivals. Hence, the marginal cost of members of the less numerous coalition are higher, however, they loose their conflicts with a higher expected probability. This relation implies that equilibrium payoff is increasing in the number of members of the respective own coalition. Moreover, if the lead in coalition membership is sufficiently large (out of the perspective of coalition Y, if $y > z(3+\sqrt{5})/2 \approx 2.62\,z$) then it is possible that the conflict game results in positive equilibrium payoff for the members of the more numerous coalition.

4.4 Conflict Intensity and Centrality

In the previous sections equilibrium outcomes were analysed separately for each of the considered conflict classes. In this section we present a result that allows us to compare conflict intensity across the three classes R, B, and S. This cross-comparison is facilitated by establishing a relation between conflict intensity and network centrality, here eigenvector centrality, that holds for the union of the three considered classes, denoted by $C \equiv R \cup S \cup B$.[14] For a discussion of eigenvector centrality and its properties, see Bonacich (1987, 2007).

The following additional notation is used: The symmetric adjacency matrix G represents graph $\mathbf{g}$ and has elements g_{ij} where $g_{ii} = 0$ for all $i \in N$ (because no agent is in a conflictive relation with herself).[15] The largest eigenvalue of G, denoted by $\mu(G)$, is real-valued and positive because G is symmetric. By the Perron-Frobenius theorem the components $(\mu_1(G), \ldots, \mu_n(G))$ of the eigenvector that corresponds to the largest eigenvalue $\mu(G)$ are all positive and frequently interpreted as a centrality measure of the respective nodes of graph $\mathbf{g}$. Solving the characteristic equation for the considered classes of conflict networks implies that:

- for regular networks: $\mu(\mathbf{g}^R) = d$ and $\mu_i(\mathbf{g}^R) = 1$ for all $i \in N$.
- for star-shaped networks: $\mu(\mathbf{g}^S) = \sqrt{n-1}$, and

$$\begin{aligned} \mu_i(\mathbf{g}^S) &= 1 \text{ for all } i \in N_c, \\ \mu_c(\mathbf{g}^S) &= \sqrt{n-1}. \end{aligned} \tag{11}$$

[14] As already mentioned: A star-shaped network is a special case of a complete bipartite network ($S \subseteq B$). We include it in the definition for completeness.

[15] To save on notation we identify a network class with its adjacency matrix.

- for complete bipartite networks: $\mu(\mathbf{g}^B) = \sqrt{y\,z}$, and, assuming without loss of generality that $y > z$:

$$\mu_i(\mathbf{g}^B) = 1 \qquad \text{for all } i \in Y,$$
$$\mu_j(\mathbf{g}^B) = \sqrt{\frac{y}{z}} \qquad \text{for all } j \in Z.$$

Based on this notation the results in Proposition 2-4 can be reformulated based on the introduced notation. This leads to a single expression for conflict intensity and individual conflict investment that now holds for the joint class C of conflict networks:

$$X^*(\mathbf{g^C}) = \sum_{i \neq j} \frac{g_{ij}}{2} \sqrt{\frac{V}{\mu(G^C)}}, \qquad \text{and} \tag{12}$$

$$x^*_{ij}(\mathbf{g^C}) = \frac{\mu_j(G^C)}{\mu_i(G^C) + \mu_j(G^C)} \sqrt{\frac{V}{\mu(G^C)}}. \tag{13}$$

The following results are directly derived from inspection of these expressions:

Corollary 1 *For conflict networks of class C conflict intensity $X^*(\mathbf{g^C})$ is increasing in the number of conflicts in the respective network and decreasing in the largest eigenvalue of its adjacency matrix. Individual conflict spending $x^*_{ij}(\mathbf{g^C})$ is decreasing in individual eigenvector centrality and the largest eigenvalue.*

The corollary facilitates the comparison of equilibrium outcomes across the considered classes R, S, or B. This type of comparison is used in the following example to clarify the consequences of network externalities on conflict intensity across different network classes. Basically, the example shows that increasing the number of agents in ring- and star-shaped conflict networks does not induce proportional changes in conflict intensity. In other words, it is not possible to rank different classes of conflict networks with respect to their induced conflict intensity. The reason for this difference lies in the underlying network externalities that have a different impact depending on the respective network structure.

Figure 3: Ring-shaped network $\mathbf{g^1}$ (left) and star-shaped network $\mathbf{g^2}$ (right)

Example 1

Two types of network structures are considered: $\mathbf{g}^1$ is a ring-shaped network (i.e., a regular network of class R with degree $d = 2$), $\mathbf{g}^2$ is a star-shaped network of class S, see Figure 3. Both conflict networks are assumed to have the same number of bilateral conflicts:[16]

$$\sum_{i \neq j} \frac{\mathbf{g}^1_{ij}}{2} = \sum_{i \neq j} \frac{\mathbf{g}^2_{ij}}{2}. \tag{14}$$

The corollary in combination with Eq. (12) implies that the difference in conflict intensity between $\mathbf{g}^1$ and $\mathbf{g}^2$ is solely determined by the relation of their largest eigenvalues: $X^*(\mathbf{g}^1) > X^*(\mathbf{g}^2)$ iff $\mu(G^1) < \mu(G^2)$. The largest eigenvalue of the ring-shaped network $\mathbf{g}^{\mathbf{1}}$ is equal to its degree, $\mu(G^1) = 2$, and therefore independent of the number of involved agents. However, the largest eigenvalue of the star-shaped network $\mathbf{g}^{\mathbf{2}}$ is equal to the centrality of its center, $\mu(G^2) = \mu_c(G^2) = \sqrt{n}$, which is increasing in the number of involved agents. Hence, for $n < 4$ the star-shaped network induces higher conflict intensity which can be related to the fact that there is one additional agent involved in $\mathbf{g}^1$ in comparison to $\mathbf{g}^2$. However, there is a countervailing effect due to network externalities that affect the center in the star-shaped conflict network: Being more central implies more rivals, higher aggregate conflict investment and therefore also larger marginal costs for the center. Hence, individual conflict investment of the center declines which tends to reduce conflict investments by agents on the periphery. The increase in conflict intensity in $\mathbf{g}^2$ is dampened by this second order effect. In fact, for $n > 4$ the relation of conflict intensity between $\mathbf{g}^1$ and $\mathbf{g}^2$ is reversed: $X^*(\mathbf{g}^1) > X^*(\mathbf{g}^2)$. Hence, for a sufficiently high number of bilateral conflicts the network externality that is induced through the center agent on each agent of the periphery becomes so dominant that conflict intensity is lower in the star-shaped network in comparison to the ring-shaped network. The observations derived from this example show that it is not possible to derive an unambiguous ranking of the considered classes of conflict networks with respect to relative conflict intensity.

4.5 Peaceful Conflict Resolution

The results derived so far imply that conflict networks with a higher number of conflicts tend to induce higher conflict intensity. In this subsection we are going to switch the perspective by analyzing the consequences of decreasing the number of conflictive relations for a given conflict network. We interpret the ad-hoc deletion of conflictive links between

[16] The condition of having the same number of conflicts implies that a ring-shaped network with n agents is compared with a star-shaped network that involves $n+1$ agents; i.e., there is always one agent more involved in $\mathbf{g}^{\mathbf{2}}$ than in $\mathbf{g}^{\mathbf{1}}$.

rivals in a given conflict network as peaceful conflict resolution.[17] Formally, the graph $\mathbf{g}$ that remains after peaceful conflict resolution is a subset of the original graph $\mathbf{g}'$. The crucial question is how conflict intensity is affected by changing exogenously the network structure, or, in other words, whether peaceful conflict resolution is beneficial in the sense that conflict intensity is reduced. The following proposition answers this question under the condition that the original and the resulting conflict network belong to class C of regular, star-shaped, or bipartite conflict networks.

Proposition 5 *For conflict networks of class C resolving conflictive links implies a reduction in conflict intensity:*

$$\textit{If } \mathbf{g} \subset \mathbf{g}', \textit{ where } \mathbf{g} \in C \textit{ and } \mathbf{g}' \in C, \textit{ then } X^*(\mathbf{g}) < X^*(\mathbf{g}'). \tag{15}$$

Proposition 5 implies that peaceful conflict resolution for the considered class C of conflict networks is in fact beneficial because conflict intensity is reduced. This result is not trivial because besides the beneficial effects of resolving a conflict (and thereby removing an opportunity to invest in conflict specific technology) there might exist substantial second order effects of indirectly affected agents. Those indirect effect come into existence because directly affected agents face less rivals and will therefore spend less in aggregate conflict investment. This implies decreased marginal costs for the respective agents that will therefore invest comparatively more in each of their remaining conflicts. In other words, they will reallocate conflict resources to the remaining conflicts which induces their rivals to invest more in conflict specific technology as well. The net effect on conflict intensity (i.e., decreased aggregate conflict investment by directly affected agents and increased conflict investment by indirectly affected agents) depends crucially on the underlying network structure. Hence, the main significance of Proposition 5 is to identity a class of conflict networks (that is the considered class C) where second order effects will never dominate the beneficial first order effects from peacefully resolving conflictive relations. The discussion in the next section will reveal that outside the considered class C of conflict networks second order effects can be substantial.

[17]Note that, although two direct rivals would profit from voluntary refraining of investing in conflict specific technology, this strategy is not credible due to the prisoner's dilemma like situation (unilaterally there is no enforceable commitment device). Hence, an exogenously third party intervention is sometimes needed; for instance, peace enforcing operations by international organizations to resolve specific conflictive relations.

5 Extensions

In this section we discuss some extensions of our framework. In the first subsection we analyse equilibrium behavior in more general conflict networks which allows us to address the issue of peaceful conflict resolution in conflict networks outside of class C. In the next subsection, we discuss the robustness of our results with respect to more general payoff functions and analyse the consequences of additional dimensions of heterogeneity. Finally, we will show how more general conflict technologies, incorporating multi-purpose conflict technology that can be used in different bilateral conflicts at the same time, can be integrated into our model.

5.1 General Conflict Networks

The analysis of general conflict networks is complex because the non-linear system of first order conditions in Eq. (7) cannot be solved in closed form outside of class C of conflict networks. Hence, we rely on indirect results to characterise equilibrium behavior in networks outside of this class.

Combining the two first order conditions for two direct rivals i and j implies that in equilibrium:[18]

$$\frac{x_{ij}^*}{x_{ji}^*} = \frac{X_j^*}{X_i^*}, \text{ and} \tag{16}$$

$$x_{ij}^* + x_{ji}^* = \frac{V}{X_i^* + X_j^*}. \tag{17}$$

These expressions lead to the following two observations that characterise conflict spending in equilibrium:

Observation 1 *There is an inverse relation with respect to individual versus aggregate conflict investment between two direct rivals, see Eq. (16).*

Observation 2 *There is an inverse relation with respect to conflict investment in a specific conflict versus total aggregate conflict investment of the respective rivals: Investments in a specific conflict are higher for those conflicts where involved agents have low aggregated investments, see Eq. (17).*

A similar relation holds with respect to the probability to win a specific conflict. This probability can be expressed in terms of aggregate conflict spending of the two rivals:

$$p(x_{ij}^*, x_{ji}^*) = \frac{X_j^*}{X_i^* + X_j^*} \tag{18}$$

[18]The dependence of equilibrium conflict investment on graph $\mathbf{g}$ is suppressed in this section for notational convenience.

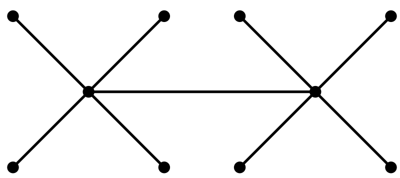
Hence, the agent that spends more in the aggregate in comparison to her direct rival is less likely to win the respective conflict. Although this result seems to be counter-intuitive at first sight it corresponds to the results already derived in the previous sections, for instance, for the center agent in a star-shaped network: Having high aggregated conflict investments (due to the high number of conflicts with the periphery) in comparison to an agent at the periphery implies also that marginal costs for the center are relatively high such that her conflict investments in each bilateral conflict are comparatively low. The consequence for the center is a low probability to win the respective conflict in equilibrium.

The characterization of equilibrium behavior in Eq. (17) can also be used to derive additional insights into the determinants of conflict intensity. Summing up Eq. (17) over all bilateral conflicts in the respective conflict network yields an alternative (indirect) expression for conflict intensity:

$$X^* = \sum_{i \in N} X_i^* = \sum_{i \neq j} \frac{g_{ij}}{2} \frac{V}{X_i^* + X_j^*}. \tag{19}$$

This expression can be used to clarify the importance of second order effects for conflict intensity in general conflict networks. From the perspective of peaceful conflict resolution the ad-hoc deletion of conflictive links affects X^* in two ways: First, there is a direct and beneficial first order effect because the network structure changed which is formally reflected by the fact that at least two factors g_{ij} and g_{ji} take on a value of zero which reduces conflict intensity (i.e., at least two terms in the sum of Eq. (19) are eliminated). Second, there are also indirect (countervailing) second order effects because all agents in the network will react to the change in aggregate conflict investment by the directly affected agents. This implies that all remaining terms in the sum might be altered. For general networks it is not clear which effect dominates because the indirect effects depend in a complex way on the network structure $\mathbf{g}$ which is demonstrated in the following example.

Figure 4: Conflict Network $\mathbf{g}^{\mathbf{S_1}}$ before peaceful conflict resolution

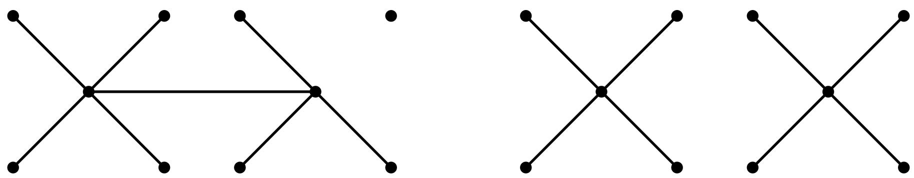


Example 2

In this example conflict network $\mathbf{g}^{\mathbf{S_1}}$ consists of two star-shaped networks where the two centers are also in a conflictive relation with each other, see Figure 4. Both center agents have an identical numbers of $n_c = n-1$ agents in the periphery such that there are in total $2n$ agents in network $\mathbf{g^{S_1}}$.

We are interested in the consequences of resolving one bilateral conflict in $\mathbf{g^{S_1}}$ with respect to conflict intensity. Basically, there are two options in this case: In option A a conflict between one of the centers and a periphery agent is resolved. The resulting network is denoted by $\mathbf{g^{S_A}}$ and is displayed on the left of Figure 5. In option B the central conflict between the two center agents is resolved which results in network $\mathbf{g^{S_B}}$ with two isolated stars, see right part of Figure 5.[19]

Figure 5: Conflict Network $\mathbf{g^{S_A}}$ (left) and $\mathbf{g^{S_B}}$ (right) after conflict resolution



Observation 2 implies that option A resulting in $\mathbf{g^{S_A}}$ leads to a higher first order effect.[20] For $n = 5$ this implies, for example, a first order effect of withhold conflict spending of 0.663 for option A compared to 0.477 for option B.[21] However, directly affected agents will reallocate resources to the remaining conflicts and their rivals will react to this change in behavior. The resulting second order effects amount to an increase in total conflict investments of 0.138 for option A in comparison to 0.354 for option B which demonstrates that second order effects can be substantial. Aggregating first and second order effects implies that the net effect of peaceful conflict resolution is higher for option A because not only the first order effect is larger but also the countervailing second order effect is less dominant.

The following table shows that option A generates a larger decline in conflict intensity for different numbers of periphery agents and is therefore the more effective alternative with respect to peaceful conflict resolution. As already argued, this can be attributed to large first order effects of withhold conflict investments in combination with relative low second order effects due to reallocations of conflict resources. While the large first order effect can

[19] Some conference participants suggested that this constellation resembles the situation at the end of the cold war when some political scientists expected a substantial decrease in worldwide conflict investment; e.g. Fukuyama (1992).

[20] The aggregate conflict investment of a center agent is higher than for a periphery agent, therefore conflict investment in the periphery conflict is higher than conflict investment in the central conflict. Resolving the link with the periphery therefore results in higher withhold conflict investments; i.e., the beneficial first order effect is larger.

[21] The numerical results in this section are based on the assumption that $V = 1$. For different values of V numerical results for conflict intensity are scaled up by a constant factor without affecting the relative comparison.

be explained by Observation 2, the large difference of second order effects between option A and B must be attributed to the difference in network structure.

n	$\mathbf{X}^*(\mathbf{g}^{\mathbf{S_1}})$	$\mathbf{X}^*(\mathbf{g}^{\mathbf{S_A}})$	$\mathbf{X}^*(\mathbf{g}^{\mathbf{S_B}})$
5	5.780	5.256	5.657
8	8.643	8.185	8.607
9	9.530	9.087	9.514
10	10.392	9.961	10.392
11	11.232	10.812	11.247
12	12.053	11.642	12.080
50	36.862	36.543	37.041
100	62.469	62.232	62.771

Table 1: Conflict intensity for $\mathbf{g}^{\mathbf{S_1}}$, $\mathbf{g}^{\mathbf{S_A}}$, and $\mathbf{g}^{\mathbf{S_B}}$

More importantly, Table 1 reveals that neglecting these second order effects can have adverse consequences for peaceful conflict resolution: For $n > 10$ conflict intensity in $\mathbf{g}^{\mathbf{S_B}}$ is actually higher than in the original network $\mathbf{g}^{\mathbf{S_1}}$; that is, peaceful conflict resolution according to option B does actually imply an increase in conflict intensity! This can be attributed to the fact that under option B both centers are directly affected by conflict resolution; therefore, both centers are a source for countervailing second order effects.[22] For option A these second order effects are less substantial, as only one (the right) center agent is affected directly. Her periphery agents react similar as under option B by increasing conflict investment; however, the decrease in aggregate conflict investment of the indirectly affected (left) center is weaker in comparison to option B. Hence, the reactions of the respective periphery agents are also relatively weak such that the overall change due to countervailing second order effects is less drastic under option A.

Observation 1 additionally provides some advice on how to localise the bilateral conflict that would induce the largest (beneficial) first order effect with respect to conflict intensity. In the presented example resolving this link also resulted in comparatively low second order effects. Whether this insight can be generalised to more complex network structures would go beyond the scope of this paper and is therefore left for future research.

5.2 General Conflict Technology

The conflict network game as presented in section 4 is based on specific functional forms for contest success functions and cost functions. These functional forms guarantee analytical tractability and closed form solutions for the considered classes of conflict networks.

[22] As aggregate conflict investment by the directly affected center agents has declined, resources are shifted to the remaining conflicts, and periphery agents will react by increasing conflict investments as well.

Nevertheless, our main results are also robust with respect to more general specifications. For instance, convex cost functions of the form $c(\mathbf{x_i}) = X_i^r$ with $r > 1$, as well as more general contest success functions of the form $p_{ij} = \frac{x_{ij}^s}{x_{ij}^s + x_{ji}^s}$ with $s \in (0, 1]$ do not affect our theoretical results from Proposition 2 to 5.[23]

The assumption that agents are homogeneous with respect to payoff-functions is more important for the derived results. In our set up heterogeneity among agents is implicitly induced by the relevant position in the conflict network. Hence, adding other sources of heterogeneity (for instance, differences in cost functions) might influence our results. If this second source of heterogeneity is sufficiently important then it will dominate the effects on conflict investment stemming from different locations in the network. To analyse this relation we introduce an agent specific idiosyncratic cost parameter $k_i \in \Re_{++}$ in the following way:

$$c_i(\mathbf{x_i}) = k_i \cdot \left(\sum_{j \in N_i} x_{ij} \right)^2 \quad \text{for all } i \in N.^{24} \tag{20}$$

The idiosyncratic cost parameter k_i allows us to model different types of individual conflict technologies (e.g., economies of scope in conflict technologies and multi-conflict technology) and to analyse their impact on network externalities, equilibrium conflict investment, and conflict intensity.[25] The propositions in the following subsections demonstrate that for specific marginal cost parameters the differences in individual conflict investment due to different locations in the network can be exactly neutralised.

5.2.1 Economies of scope in conflict technology

Economies of scope in conflict technology imply that the marginal cost of conflict investment is decreasing in the number of conflictive relations of the respective agent. This type of conflict technology can be modeled with the help of the idiosyncratic cost parameter by setting $k_i = 1/n_i$ for all $i \in N$ where k_i is in fact decreasing in the number of direct rivals. Based on this modification the results for class C of conflict networks can be recalculated which leads to the following result.

[23] It is straightforward but cumbersome to generalise Proposition 2 to 5 based on this general framework. However, Corollary 1 cannot be extended easily because the relation to eigenvector centrality is sensitive with respect to quadratic cost functions, compare Ballester et al. (2006) for a brief discussion on this issue. It should also be mentioned that for $r \to 1$ the externality that is induced through the network structure vanishes because for $r = 1$ the marginal cost of investing in a specific bilateral conflict is independent of the other conflict investments of an agent.

[24] The conflict game based on this modified cost function is strategically equivalent to a contest game with heterogeneous valuations, where an agent i perceives the value of contested resources differently than its respective rivals (The transformation $\tilde{\pi}_i(\mathbf{x_i}, \mathbf{x_{-i}}; \mathbf{g}) = \frac{1}{k_i}\left[W(\mathbf{x_i}, \mathbf{x_{-i}}; \mathbf{g}) - c_i(\mathbf{x_i})\right]$ provides the respective payoff function).

[25] Proposition 1 and Lemma 1 are still applicable because diagonal strict concavity is not affected by this modification.

Proposition 6 *For conflict networks of class C economies of scope in conflict technology (with $k_i = 1/n_i$ for all $i \in N$) imply that network externalities disappear. Individual conflict investment is then independent of the network structure:*

$$x^*_{ij}(\mathbf{g^C}) = \frac{1}{2}\sqrt{V} \text{ for all } j \in N_i \text{ and all } i \in N.$$

Moreover, conflict intensity only depends on the number of conflictive relations:

$$X^*(\mathbf{g^C}) = \sum_{i \neq j} \frac{g_{ij}}{2}\sqrt{V}.$$

Proposition 6 demonstrates that there exist degrees of agents' heterogeneity such that equilibrium investment is identical for each agent, irrespectively of the type of network R, S, B and the type of agent (e.g., center, periphery, or member of a specific coalition). Identical individual conflict investment also implies that conflict intensity is only driven by the number of conflictive relations in the network. Hence, the dependence on the network structure as derived in Corollary 1 is irrelevant under this specification because network externalities are exactly neutralised: The heterogeneity from the cost function due to economies of scope exactly balances out the heterogeneity induced by different locations in the network structure.

5.2.2 Multi-conflict technology

In the conflict network game the conflict technology is assumed to be specific for each bilateral conflict. This assumption is relaxed in this subsection based on the observation that conflict technologies might also have multi-purpose characteristics; that is, they can be applied (or are effective) simultaneously in different bilateral conflicts in which a conflict party might be involved. Examples for this type of multi-conflict technology are R&D in weapon systems or the deterrence potential of nuclear weapons of mass destruction. This agent specific public good aspect of conflict technology can be captured by modifying the input of the contest success function for a bilateral conflict. It now depends on conflict input $a_{ij}(\mathbf{x_i})$, which is a convex combination of conflict specific technology, x_{ij}, and total conflict investment of the respective agent, $X_i = \sum_{j \in N_i} x_{ij}$:

$$a_{ij}(\mathbf{x_i}) = \gamma_i x_{ij} + (1 - \gamma_i) X_i \text{ for all } j \in N_i, \text{ and all } i \in N. \tag{21}$$

Under this modification the bilateral conflict outcome does not only depend on conflict specific investment but additionally (and to some extent) on total conflict investment, where $\gamma_i \in [0, 1]$ denotes the degree of multi-conflict applicability of the respective conflict technology.[26] We denote the modified payoff function of agent i in line with eq. (6) by:

$$\pi_i(\mathbf{x_i}, \mathbf{x_{-i}}; \mathbf{g}) = 2V \sum_{j \in N_i} \frac{a_{ij}(\mathbf{x_i})}{a_{ij}(\mathbf{x_i}) + a_{ji}(\mathbf{x_j})} - n_i V - (X_i)^2 . \tag{22}$$

The following proposition reveals that the modified conflict game is equivalent to the conflict game with heterogeneous cost functions as it is introduced in Eq. (20). Moreover, it is shown that a higher degree of multi-conflict applicability can be interpreted as a decrease in the individual cost parameter of the respective conflict party. Hence, if the conflict technology of an agent has some degree of this multi-conflict characteristic then this is equivalent to a reduction in marginal costs for the respective agent. Moreover, a specific degree of multi-conflict applicability is equivalent to the economies of scope case considered in Section 5.2.1 which implies (in line with Proposition 6) that network externalities would then disappear.

Proposition 7 *The conflict game with multi-conflict technology is equivalent to a conflict game with specific heterogeneous cost parameters $k_i = k(n_i, \gamma_i)$ for all $i \in N$. Moreover:*

(i) A higher degree of multi-purpose utilizability (a lower γ_i) corresponds to a lower individual cost parameter: $\frac{\partial k_i}{\partial \gamma_i} > 0$ for all $i \in N$.

(ii) For class C of conflict networks there exists a degree of multi-conflict applicability γ^* such that network externalities disappear.

6 Concluding Remarks

While strategic models of conflicts between two or more conflict agents are intensively studied, the embeddedness of conflict parties into a local structure of conflictive relations has not been gained much attention so far. In our approach we analyse the latter situation by joining a simple model from the conflict literature with the recent network literature. Although simple, the conflict framework has some properties which make the resulting conflict network game non-standard; for instance, it is neither an aggregative nor a supermodular game. We address these issues by relying on an alternative existence proof based on the concept of diagonal concavity which is convenient to apply in our context. Additionally, we analyse equilibrium behavior in the prominent network classes of regular, star-shaped, and complete bipartite conflict networks that reflect in a stylised way some important types of historically observed conflicts.

Our analysis confirms the intuitive statement that more conflictive relations imply higher conflict intensity. However, there also exist important network externalities which come

[26] The case $\gamma_i = 1$ coincides with the original model, where conflict technology is purely conflict specific, while the polar case $\gamma_i = 0$ reflects a conflict technology that is either highly versatile and operationally flexible or strategic in nature such that it can be applied in each potential conflict (for instance, a highly flexible and mobile air force).

into play if conflict intensity is compared across different types of conflict networks. This dependence between conflict intensity and network characteristics is especially important for peaceful conflict resolution, interpreted here as exogenously resolving conflictive links. While we can identify a class of conflict networks where peaceful conflict resolution is always beneficial with respect to conflict intensity we also emphasise the role of indirect second order effects resulting from network externalities. We show by example that disregarding these indirect network effects can result in detrimental outcomes with respect to conflict intensity. Hence, a careful analysis of the underlying indirect dependencies of conflict rivals should precede the attempts of peacefully resolving specific conflictive relations.

The robustness of the theoretical results is intensively discussed with respect to different extensions that include more general conflict networks, and more general conflict technologies (e.g., multi-conflict technology and economies of scope). However, in our framework we also assume that the underlying conflict network is given ex-ante without specifying how it came into existence. Hence, the formation of the conflict network could be an interesting extension; for instance, following the network formation game suggested in Bala and Goyal (2000) or Galeotti and Goyal (2010). Some recent attempts along these lines, where network formation games are combined with (more stylised) models of conflictive relations, are presented in Hiller (2011) and Goyal and Vigier (2011) that could be interpreted as complementary approaches to our model.

Appendix

Proof of Proposition 1. The assumptions on the payoff functions guarantee that the conflict network game is a concave n-person game. To proof that it is also diagonally strictly concave it suffices to show that conditions (i), (ii), and (iii) are satisfied. In this proof the expression $\pi_i(\mathbf{x_i}, \mathbf{x_{-i}}) = W(\mathbf{x_i}, \mathbf{x_{-i}}) - c(\mathbf{x_i})$ is used.[27]

(i) $W(\mathbf{x_i}, \mathbf{x_{-i}})$ is strictly concave in $\mathbf{x_i}$ for all $\mathbf{x_{-i}}$ and for all $i \in N$ because its Hessian is negative definite; i.e., it is a (diagonal) matrix with entries $\frac{\partial^2 W(\mathbf{x_i},\mathbf{x_{-i}})}{\partial x_{ij}^2} = \frac{\partial^2 p_{ij}}{\partial x_{ij}^2} 2V < 0$ on the diagonal and $\frac{\partial^2 W(\mathbf{x_i},\mathbf{x_{-i}})}{\partial x_{ij} \partial x_{ik}} = \frac{\partial^2 p_{ij}}{\partial x_{ij} \partial x_{ik}} = 0$ elsewhere for all $j, k \in N_i$ with $j \neq k$. As $c(\mathbf{x_i})$ is convex for all $i \in N$, the payoff function $\pi_i(\mathbf{x_i}, \mathbf{x_{-i}})$ is the sum of a constant, a strictly concave and a concave function in $\mathbf{x_i}$. Hence, it is strictly concave in $\mathbf{x_i}$ for all $\mathbf{x_{-i}}$ and for all $i \in N$.

(ii) Note that $W(\mathbf{x_i}, \mathbf{x_{-i}}) = 2V \sum_{j \in N_i} p_{ij} - n_i V$. Each factor p_{ij} of this sum is strictly convex in x_{ji} because $\frac{\partial^2 p_{ij}}{\partial x_{ji}^2} > 0$ by assumption. Moreover, it is convex in $\mathbf{x_{-i}}$ for all $\mathbf{x_i}$ because its Hessian is positive semi-definite; i.e., it is a (diagonal) matrix with positive entries, $\frac{\partial^2 p_{ij}}{\partial x_{ji}^2} > 0$, or zero entries, $\frac{\partial^2 p_{ij}}{\partial x_{ki}^2} = 0$, on the diagonal and zero entries, $\frac{\partial^2 p_{ij}}{\partial x_{ji} \partial x_{ki}} = 0$,

[27] For reasons of notational simplicity the dependence of the payoff functions on graph $\mathbf{g}$ is suppressed in the following paragraphs.

elsewhere for all $j, k \in N_i$ with $j \neq k$. This implies that $W(\mathbf{x_i}, \mathbf{x_{-i}})$ is a sum of functions that are all convex in $\mathbf{x_{-i}}$ for all $\mathbf{x_i}$. Hence, $W(\mathbf{x_i}, \mathbf{x_{-i}})$ is also convex in $\mathbf{x_{-i}}$ for all $\mathbf{x_i}$. As the cost function does not depend on conflict spending of the rivals, the function $\pi_i(\mathbf{x_i}, \mathbf{x_{-i}})$ is therefore convex in $\mathbf{x_{-i}}$ for all $\mathbf{x_i}$.

(iii) Assume that $r_i = r > 0$ for all $i \in N$. Then Eq. (3) simplifies substantially due to the fact that the aggregate value of contested resources is zero, compare Eq. (2):

$$\sigma(\mathbf{x}, \mathbf{r}) = -r \sum_{i \in N} c(\mathbf{x_i})$$

By assumption, the cost function is convex in own conflict spending. Hence, the function $\sigma(\mathbf{x}, \mathbf{r})$ is a sum of concave functions which is also concave. ■

Proof of Lemma 1. The proof consists of the following two claims:

1. *Claim: Two direct rivals cannot exert zero conflict investment in equilibrium in their respective bilateral conflict.*

   Consider an arbitrary strategy profile $(\mathbf{x_i}, \mathbf{x_{-i}})$, where $x_{ij} = x_{ji} = 0$ and $j \in N_i$. Consider now the following strategy $\mathbf{x'_i} = (x_{i1}, \ldots, x'_{ij}, \ldots, x_{in_i})$ where $x'_{ij} = \epsilon$ for ϵ sufficiently small. This is a profitable deviation because $\pi_i(\mathbf{x'_i}, \mathbf{x_{-i}}; \mathbf{g}) > \pi_i(\mathbf{x_i}, \mathbf{x_{-i}}; \mathbf{g})$ as $p(x'_{ij}, x_{ji}) = 1 > p(x_{ij}, x_{ji}) = 1/2$ while $lim_{\epsilon \to 0} c(\mathbf{x'_i}) = c(\mathbf{x_i})$.

2. *Claim: An agent cannot exert zero conflict investment in equilibrium against a rival with positive conflict investment.*

   Assume by contradiction that there exists an equilibrium strategy profile $\mathbf{x}^* = (\mathbf{x_1^*}, \ldots, \mathbf{x_i^*}, \ldots, \mathbf{x_j^*}, \ldots, \mathbf{x_n^*})$ with $j \in N_i$ where agent i invests $x^*_{ij} = 0$ and its rival j invests $x^*_{ji} > 0$ in the respective bilateral conflict. The following strategy is a profitable deviation for agent j: $\mathbf{x'_j} = (x'_{ji}, \mathbf{x^*_{j-i}})$ where $x'_{ji} \in (0, x^*_{ji})$ and $\mathbf{x^*_{j-i}} = \{x^*_{jk}\}_{k \in N_j/i}$, i.e., agent j only reduces conflict spending against rival i without altering conflict investment in all other conflicts. Note that $p(x'_{ji}, x^*_{ij}) = p(x^*_{ji}, x^*_{ij}) = 1$ while $c(\mathbf{x'_j}) < c(\mathbf{x^*_j})$ and therefore $\pi_j(\mathbf{x'_j}, \mathbf{x^*_{-j}}; \mathbf{g}) > \pi_j(\mathbf{x^*_j}, \mathbf{x^*_{-j}}; \mathbf{g})$. Hence, $\mathbf{x}^*$ cannot be an equilibrium strategy profile.

As in equilibrium neither one nor two direct rivals will exert zero conflict investment, the equilibrium investment must be strictly positive by both rivals which holds for all agents in the respective network. ■

Proof of Proposition 2. By Lemma 1 the equilibrium of the conflict game is interior. The following expression for conflict investment $x^* \equiv x^*_{ij}$ for all $i \neq j$ solves the system of first order conditions:

$$x^*(\mathbf{g}^R) = \frac{1}{2}\sqrt{\frac{V}{d}}, \qquad \text{for all } i \in N.$$

Total conflict intensity is defined as the aggregate conflict spending in equilibrium:

$$X^*(\mathbf{g}^R) = \sum_{i \in N} \sum_{j \in N_i} x^*(\mathbf{g}^R) = n\, d\, x^*(\mathbf{g}^R) = \frac{n}{2}\sqrt{d\,V}.$$

The last expression is increasing in the total number of agents and also in its degree d. Simplifying the inequality $X^*(\mathbf{g}^{R1}) > X^*(\mathbf{g}^{R2})$ yields the condition presented in (ii).

As the equilibrium is symmetric, the probability to win (or loose) each bilateral conflict is identical for all agents; i.e., $p^*_{ij} = \frac{1}{2}$ for all $i \neq j$. Hence, expected equilibrium payoff is:

$$\pi(\mathbf{x}^*_\mathbf{i}, \mathbf{x}^*_{-\mathbf{i}}; \mathbf{g}^R) = -d\frac{V}{4}, \qquad \text{for all } i \in N.$$

Clearly, $x^*(\mathbf{g}^R)$ and $\pi(\mathbf{x}^*_\mathbf{i}, \mathbf{x}^*_{-\mathbf{i}}; \mathbf{g}^R)$ are decreasing in d and independent of n which establishes part (iii) of the proposition. ■

Proof of Proposition 4. By Lemma 1 the equilibrium is interior. Inspection of the first order conditions reveals that each member of the same coalition invests the same amount in each of her conflicts; e.g., for $i \in Y$: $x^*_y(\mathbf{g}^B) = x^*_{ij}(\mathbf{g}^B)$ for all $j \in Z$ and all $i \in Y$. A symmetric observation holds for all agents $j \in Z$. Hence, there exist only two levels of individual equilibrium conflict investment in a bipartite conflict network:

$$x^*_i(\mathbf{g}^B) = p^*_i(\mathbf{g}^B)\sqrt{\frac{V}{\sqrt{y\, z}}} \qquad \text{for } i \in \{Y, Z\}\ , \tag{23}$$

where $p^*_y(\mathbf{g}^B) = \frac{\sqrt{y}}{\sqrt{y}+\sqrt{z}}$ for members of coalition Y and $p^*_z(\mathbf{g}^B) = 1 - p^*_y(\mathbf{g}^B)$ for members of coalition Z. Note also, that $\frac{\partial p^*_y(\mathbf{g}^B)}{\partial y} > 0$, $\frac{\partial p^*_y(\mathbf{g}^B)}{\partial z} < 0$, and that $p^*_y(\mathbf{g}^B) > p^*_z(\mathbf{g}^B)$ if and only if $y > z$. Total conflict investment of an agent $i \in Y$ is:

$$X^*_i(\mathbf{g}^B) = \frac{\sqrt{V\sqrt{y\, z}}}{1 + \sqrt{\frac{y}{z}}}.$$

This expressions is strictly increasing in y as long as $y < z$ and becomes strictly decreasing for $y > z$. Using the derived solutions to calculate expected equilibrium payoff for an agent $i \in Y$ as specified in Eq. (10) yields:

$$\pi_i(\mathbf{x}^*_\mathbf{i}, \mathbf{x}^*_{-\mathbf{i}}; \mathbf{g}^B) = \frac{z(y - z - \sqrt{y\, z})}{(\sqrt{y} + \sqrt{z})^2}V,$$

which is strictly increasing in y. Note that this relation also implies that, for $i \in Y$ and $j \in Z$, we have that $\pi_i(\mathbf{x}^*_\mathbf{i}, \mathbf{x}^*_{-\mathbf{i}}; \mathbf{g}^B) > \pi_j(\mathbf{x}^*_\mathbf{j}, \mathbf{x}^*_{-\mathbf{j}}; \mathbf{g}^B)$ if and only if $y > z$.

Conflict intensity in a complete bipartite network can be expressed as:

$$X^*(\mathbf{g}^B) = \sqrt{(y\,z)^{\frac{3}{2}} V}. \tag{24}$$

This expression is obviously increasing in y, as well as in z, the number of agents in each coalition, which proves (i). Solving the inequality $X^*(\mathbf{g}^{B1}) > X^*(\mathbf{g}^{B2})$ yields the condition stated in (ii). ■

Proof of Proposition 5. By proposition 2, 3, and 4 peaceful conflict resolution implies reduced conflict intensity if the resulting conflict network (after peaceful conflict resolution) remains within the same class. It remains to check whether this result also holds across the considered classes. For peaceful conflict resolution of star-shaped conflict networks the proof is trivial because the resulting conflict network after peaceful conflict resolution is always star-shaped. Also, peaceful conflict resolution of bipartite networks is clearly beneficial if the resulting network is star-shaped because star-shaped networks are subclasses of bipartite networks (Proposition 4 can be applied directly). The remaining two cases are therefore:

1. Case: $\mathbf{g} \subset \mathbf{g}'$, with $\mathbf{g} \in R$ and $\mathbf{g}' \in B$

   Peaceful conflict resolution is beneficial if $X^*(\mathbf{g}) < X^*(\mathbf{g}')$. We calculate the largest possible conflict intensity $\bar{X}^*(\mathbf{g}) = \max_{\mathbf{g} \subset \mathbf{g}'} X^*(\mathbf{g})$ for a regular network $\mathbf{g}$ (with maximal degree $\bar{d}$ and maximal number $\bar{n}$ of agents) that results from a bipartite network with y (z) members of coalition Y (Z) through deleting links. Assume without loss of generality that $y < z$. Then $\bar{d} = \min\{y, z\} = y$, and $\bar{n} = 2\min\{y, z\} = 2y$. Hence, maximal conflict intensity for a regular network that results from a bipartite network is $\bar{X}^*(\mathbf{g}) = \sqrt{y^3 V}$. This is clearly less than $X^*(\mathbf{g}') = \sqrt{(yz)^{\frac{3}{2}} V}$, which proves the statement.

2. Case: $\mathbf{g} \subset \mathbf{g}'$, with $\mathbf{g} \in B$ and $\mathbf{g}' \in R$

   We derive the largest possible conflict intensity $\bar{X}^*(\mathbf{g}) = \max_{\mathbf{g} \subset \mathbf{g}'} X^*(\mathbf{g})$ of the resulting bipartite network with y (z) members of coalition Y (Z) that stems from a regular network $\mathbf{g}$ of degree d with n agents. We then show that $\bar{X}^*(\mathbf{g}) < X^*(\mathbf{g}')$ which proves the statement.

   The bipartite network must satisfy the following inequalities: $y + z \leq n$, $y \leq d$, and $z \leq d$, where one of this equation must be strict.[28] Conflict intensity for a bipartite network, $X^*(\mathbf{g}) = \sqrt{(yz)^{\frac{3}{2}} V}$ will be maximal if $y = z$ because $(yz)^{\frac{3}{2}}$ is a concave and symmetric function. Hence, there are two cases to check for the inequality $\bar{X}^*(\mathbf{g}) < X^*(\mathbf{g}')$ to be satisfied:

[28] If all expressions are satisfied with equality then the regular network with $n = 2d$ is identical to a complete bipartite network with $y = z = d$ which would violate the assumption that $\mathbf{g} \subset \mathbf{g}'$.

- $y = z < d$ and $y + z = n$ which implies that $y = z = \frac{n}{2}$ and $n < 2d$: Based on this information the inequality $\bar{X}^*(\mathbf{g}) < X^*(\mathbf{g'})$ can be reduced to: $(\frac{n}{2})^3 < \frac{n^2 d}{4}$, which is satisfied because $n < 2d$.
- $y = z = d$ and $y + z < n$ which implies that $n > 2d$: Based on this information the inequality $\bar{X}^*(\mathbf{g}) < X^*(\mathbf{g'})$ can be reduced to: $d^3 < \frac{n^2 d}{4}$, which is satisfied because $n > 2d$.

As the inequality $\bar{X}^*(\mathbf{g}) < X^*(\mathbf{g'})$ is satisfied for both cases, this relation also holds for $X^*(\mathbf{g}) < X^*(\mathbf{g'})$, which proves the statement.

■

Proof of Proposition 6. The proof of Proposition 2 and 4 can be adopted to the modified setup with the extended cost function. Individual conflict investment in equilibrium is then:

$$x^*_{ij}(\mathbf{g}^R) = x^*_{ij}(\mathbf{g}^B) = \frac{1}{2}\sqrt{V} \text{ for all } j \in N_i, \text{ and all } i \in N,$$

while conflict intensity can be expressed as:

$$\begin{aligned} X^*(\mathbf{g}^R) &= n\, d\, x^*_{ij}(\mathbf{g}^R) = \frac{n\, d}{2}\sqrt{V} = \sum_{i \neq j} g_{ij}\sqrt{V}, \\ X^*(\mathbf{g}^B) &= y\ z\ x^*_Y(\mathbf{g}^B) + z\ y\ x^*_Z(\mathbf{g}^B) = y\ z\ \sqrt{V} = \sum_{i \neq j} g_{ij}\sqrt{V}. \end{aligned}$$

Hence, individual conflict investment but also overall conflict intensity is identical in all networks of class C if the number of conflictive relations is the same. ■

Proof of Proposition 7. Note that Eq. (22) can be expressed as:

$$a_{ij}(\mathbf{x_i}) = x_{ij} + (1 - \gamma_i)(X_i - x_{ij}).$$

Summation over all direct rivals $j \in N_i$ of agent i yields:

$$A_i(\mathbf{x_i}) = \sum_{j \in N_i} a_{ij}(\mathbf{x_i}) = X_i + (1 - \gamma_i)(n_i - 1)X_i = (n_i - \gamma_i(n_i - 1))X_i.$$

Solving for X_i allows us to express the payoff function in Eq. (22) solely in terms of the input variables $(\mathbf{a_i}, \mathbf{a_{-i}})$:

$$\pi_i(\mathbf{a_i}, \mathbf{a_{-i}}; \mathbf{g}) = 2V \sum_{j \in N_i} \frac{a_{ij}}{a_{ij} + a_{ji}} - n_i V - k_i \cdot (A_i)^2\,,$$

where the cost parameter k_i is defined as: $k_i = k(n_i, \gamma_i) = \left(\frac{1}{n_i - \gamma_i(n_i - 1)}\right)^2$. Hence, the conflict game with multi-conflict technology is strategically equivalent to a conflict game with individually heterogeneous cost parameters as specified in Eq. (20). Note also, that conflict technology which is purely conflict specific (i.e., where $\gamma_i = 1$) implies that $k(n_i, 1) = 1$ which coincides with the original model.

Based on the expression for k_i it is also obvious that $\frac{\partial k_i}{\partial \gamma_i} = \frac{2(n_i-1)}{(n_i-\gamma_i(n_i-1))^3} > 0$. Therefore, a higher degree of multi-purpose applicability can be interpreted as a lower individual cost parameter, which proves part (i) of the proposition. Part (ii) is established by observing that $\gamma_i^* = \frac{\sqrt{n_i}}{\sqrt{n_i}+1}$ implies $k_i = \frac{1}{n_i}$. This coincides with the economies of scope in conflict technology-case considered in Proposition 6 where it is shown that differences in equilibrium behavior due to network locations are neutralised by exactly this specification of the cost parameter. ■

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