Maximum-Revenue versus Optimum-Welfare Tariffs: A Delegation Game with Many Countries

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Abstract

A number of large countries are importers of a product, produced by a perfectly competitive industry, and use import tariffs to improve their terms of trade. In the trade policy game between the importers of the product, import tariffs are strategic complements rather than strategic substitutes as in the usual trade policy game between importers and exporters. As in Clarke and Collie (2008), delegating to a policy maker who sets the tariff to maximize tariff revenue can yield higher welfare than setting the tariff to maximize welfare, and all countries delegating to a policy maker who maximizes tariff revenue can be a Nash equilibrium of the delegation game. We show that in the Nash equilibrium either all or none of the countries will delegate to a policy maker who maximizes tariff revenue. Hence, we show that if the number of countries is greater than four then all countries delegating to a policy maker who maximizes tariff revenue is never a Nash equilibrium. In fact, with more than four countries, the only Nash equilibrium is where all countries set their import tariffs to maximize welfare.

1 Introduction

Conventional wisdom under perfect competition is that a country that maximizes tariff revenue will set a higher tariff than if it was maximizing welfare, and that it will have lower welfare when maximizing revenue than if it was maximizing welfare; see Johnson (1951-52). However, in a trade policy games where there are strategic interactions between the trade policies of countries, there is the possibility that delegating to a revenue-maximizing policy-maker will change the outcome of the game in a favorable way. This possibility will be explored in a number of trade policy games in this paper.

In conventional two-country trade policy games, there is generally one importer and one exporter of a product, and the trade taxes of the two countries are generally strategic substitutes. As pointed out by Panagariya and Schiff (1994, 1995), this ignores the strategic interdependence between exporters of the same product and, with two exporters of a product, they showed that export taxes are generally strategic complements. Recently, Zissimos (2009) showed that import tariffs are strategic complements when there are many importers of the same product. An implication of export taxes being strategic complements considered by Panagariya and Schiff (1994, 1995) is that revenue-maximizing export taxes may yield higher welfare for exporting countries than welfare-maximizing export taxes. By choosing to maximize revenue rather than welfare, a country commits to a higher export tax and as a result the other exporter increases its export tax. Yilmaz (1999) applied this analysis to the cocoa market and found that revenue-maximizing export taxes did yield higher welfare than welfare-maximizing export taxes. Clarke and Collie (2008) extended the trade policy game considered by Panagariya and Schiff (1994, 1995) by adding an initial stage where the two countries decide whether to delegate the setting of export taxes to a policy maker who maximizes revenue or one who maximizes welfare. This explains how a country can commit to maximizing revenue rather than welfare. They showed that while both countries maximizing revenue would often yield higher welfare for both countries, this was not always a Nash equilibrium of the initial stage of the game.

The analysis in section two of this paper extends the game of Clarke and Collie (2008) to consider import tariffs in a perfectly competitive model rather than export taxes in an oligopolistic model, and, more significantly, it considers the case of more than two importers. As with export taxes, it is shown that all importers maximizing revenue may yield higher welfare than all importers maximizing welfare. Also, it is shown that in the Nash equilibria of the initial stage either all the countries maximize welfare or all the countries maximize revenue. Hence, it is shown that if there are more than four countries then the only Nash equilibrium of the initial stage of the game is where all countries maximize welfare. All countries maximizing revenue is only a unique Nash equilibrium of the initial stage of the game when there are two countries.

When a country uses an optimum tariff to improve its terms of trade then there is always the possibility of retaliation. In a conventional trade policy game with two countries setting tariffs, Johnson (1953-54) showed that, while both countries would be worse off in the Nash equilibrium in tariffs than under free trade, one country could gain if it had more market power than the other country. Kennan and Riezman (1988) and Syropoulos (2002) show that a country can be better off in the Nash equilibrium in tariffs than under free trade if it is sufficiently large relative to the other country.

The analysis in section three of this paper considers a trade policy game between two importers setting import tariffs and two exporters setting export taxes where each country can choose between delegating to a revenue-maximizing policy-maker or a welfare maximizing policy-maker. For an importing (exporting) country, delegating to a revenue-maximizing agent commits the country to setting a higher tariff which has two effects on the outcome of the game. First, it leads the other importer (exporter) to set a higher import tariff (export tax) since import tariffs (export taxes) of the two importers (exporters) are strategic complements, which has a positive effect on the welfare of the country. Second, it leads the exporters (importers) to lower their export taxes (import tariffs) since the import tariffs and export taxes are strategic substitutes, which also has a positive effect on welfare of the country. When all countries maximize welfare, the importers (exporters) will be better off in the Nash equilibrium in trade policies than under free trade when they have sufficient market power. When the countries can delegate to a revenue maximizing policy-maker, the importers (exporters) will delegate to a revenue-maximizing agent when they have sufficient market power and as a result will be even better off than if they were maximizing welfare so the gains for the winners are increased. However, world welfare when either exporters or importers delegate to a revenue-maximizing policy-maker will be lower than when all countries maximize welfare.

The analysis in section four of this paper considers a version of the game from section two with incomplete information. It is shown that incomplete information makes it less likely that countries will delegate to revenue-maximizing policy-makers.

2 The Model with Many Importers

There are $n \geq 2$ identical countries that import a homogenous product. Let the set of countries be denoted by $N = \{1, ..., n\}$. Demand of country $i \in N$, q_i, is linear in domestic price, p_i:

$$q_i = A - Bp_i \tag{1}$$

where A and B are positive constants. A policy maker of country i sets a specific import tariff, t_i, on the imported product. Hence, the domestic price of country i is equal to the world price of the product, P, plus the import tariff, t_i:

$$p_i = P + t_i. \tag{2}$$

World supply of the product, Q, is also linear in the world price:

$$Q = a + bP \tag{3}$$

where $b > 0$.

Given the import tariffs, $(t_i)_{i=1}^n$, the equilibrium world price equates world supply with demand:

$$Q = \sum_{i=1}^{n} q_i. \tag{4}$$

Substituting (1) to (3) into (4) gives the expression for the equilibrium world price:

$$P = \frac{An - a - B\sum_{i=1}^{n} t_i}{b + Bn}.$$

It follows that an increase in import tariffs improves the terms of trade of the importing countries by lowering the world price on their imports. Using the above expression for the equilibrium world price, demand of country i can be written as a function of import tariffs $(t_i)_{i=1}^n$:

$$q_i = \frac{Ab + Ba + B^2\sum_{j\neq i} t_j - B\left(B\left(n-1\right)+b\right)t_i}{b + Bn}.$$

To ensure that each country imports a strictly positive quantity of the product, we assume that $Ab + Ba > 0$. The demand function resembles the one that is usually employed when modelling a differentiated product oligopoly: the demand is decreasing in own price (here, tariff), but increasing in competitors' prices (tariffs).

We study a two-stage game. In the first stage, the government of each importing country $i \in N$ independently and simultaneously decides whether to delegate the policy maker of that country to maximize the import tariff revenue or the welfare of that country. The former for country i is given by

$$R_i = q_i t_i,$$

while the latter is the sum of import tariff revenue and the consumer surplus:

$$W_i = R_i + \frac{q_i^2}{2B}.$$

In the second stage, the policy maker of each importing country $i \in N$ independently and simultaneously decides on the import tariff, t_i. It is assumed that, prior to making his decision on tariffs, each policy maker observes not just the first stage decision of his own government but of all the governments.

2.1 The Tariff Setting Stage

We solve the two-stage game backwards by solving for the Nash equilibrium of the tariff setting stage first. The policy maker of country i maximizes his objective by choosing t_i, while treating all t_j, $j \neq i$, as given.

Suppose the policy maker of country i maximizes the revenue, R_i. We can calculate the first order condition and rearrange it to obtain a best reply function for such a revenue maximizing policy maker:

$$t_i = r_R\left(\sum\nolimits_{j \neq i} t_j\right) = \frac{B^2 \sum_{j \neq i} t_j + Ab + Ba}{2B\left(b + B\left(n-1\right)\right)}. \tag{5}$$

Suppose the policy maker of country i maximizes the welfare, W_i. Again, from the first order condition, we can express the best reply function corresponding to such a welfare maximizing policy maker:

$$t_i = r_W\left(\sum\nolimits_{j \neq i} t_j\right) = \frac{B^2 \sum_{j \neq i} t_j + Ab + Ba}{\left(b + B\left(n+1\right)\right)\left(b + B\left(n-1\right)\right)}. \tag{6}$$

The best reply function of either type of policy maker is increasing in the import tariffs set by the other countries, implying that tariffs are strategic complements. Further, the best response only depends on the sum of import tariffs set by all other countries. The comparison of (5) and (6) reveals that the best reply function under revenue maximization is always steeper than the one under welfare maximization.

One can also verify that the second order derivative with respect to t_i is strictly negative for either objective function, implying that import tariffs found in equations (5) and (6) indeed maximize the corresponding objective functions.

Suppose that the policy makers of m out of n countries maximize revenue and the rest maximize welfare.

Proposition 1 *The tariff-setting game has a unique Nash equilibrium, in which each revenue maximizing policy maker sets the import tariff equal to*

$$t_R = \frac{(b+Bn)^2 (Ab+Ba)}{B\left((b+Bn)^2 (2(b+B(n-1)) - B(m-1)) + B^2 (m-n)(2b+2Bn-B)\right)}, \tag{7}$$

while each welfare maximizing policy maker sets the import tariff equal to

$$t_W = \frac{(2b+2Bn-B)(Ab+Ba)}{(b+Bn)^2 (2(b+B(n-1)) - B(m-1)) + B^2 (m-n)(2b+2Bn-B)}. \tag{8}$$

Proof. First we argue that any equilibrium must be symmetric, in the sense that all revenue maximizing policy makers will set the same import tariff, and all welfare maximizing policy makers will also set a common import tariff. Suppose the policy makers of countries i and j are delegated to maximize revenue. Then it is true that

$$r_R\left(\sum\nolimits_{k\in N\backslash i} t_k\right) + \sum\nolimits_{k\in N\backslash i} t_k = r_R\left(\sum\nolimits_{k\in N\backslash j} t_k\right) + \sum\nolimits_{k\in N\backslash j} t_k.$$

Substituting in the expression of the best response function from (5) and simplifying, it follows that $t_i = t_j$ in the equilibrium. Analogous argument establishes that $t_i = t_j$ holds if the policy makers of countries i and j are delegated to maximize welfare.

Hence, the Nash equilibrium is given by the following system of linear equations

$$t_R = \frac{B^2\left((m-1)\,t_R + (n-m)\,t_W\right) + Ab + Ba}{2B\left(b + B\,(n-1)\right)},$$

$$t_W = \frac{B^2\left(mt_R + (n-m-1)\,t_W\right) + Ab + Ba}{\left(b + B\,(n+1)\right)\left(b + B\,(n-1)\right)},$$

which has a unique solution given by (7) and (8).

Also, since either objective function is strictly concave in own import tariff, the best response is always single valued and no Nash equilibrium in mixed strategies exists. ■

Comparison of (7) and (8) reveals that $t_R > t_W$ holds in the Nash equilibrium. Further, both t_R and t_W are increasing in the number of countries that maximize revenue. When m countries maximize revenue, the Nash equilibrium welfare of revenue and welfare maximizing countries, respectively, are

$$\begin{aligned} W_R\,(m) &= \frac{(b+Bn)^2\,(Ab+Ba)^2\,(-B+b+Bn)\,(-B+3b+3Bn)}{2B\left((b+Bn)^2\,(2b-B\,(m-1)+2B\,(n-1)) + B^2\,(m-n)\,(2b-B+2Bn)\right)^2} \\ &= \frac{B\,(b+B\,(n-1))\,(3b+B\,(3n-1))}{2\,(b+Bn)^2}\,(t_R\,(m))^2\,, \qquad (9) \end{aligned}$$

$$\begin{aligned} W_W\,(m) &= \frac{(Ab+Ba)^2\,(2b-B+2Bn)^2\,(B+b+Bn)\,(-B+b+Bn)}{2B\left((b+Bn)^2\,(2b-B\,(m-1)+2B\,(n-1)) + B^2\,(m-n)\,(2b-B+2Bn)\right)^2} \\ &= \frac{(b+B\,(n-1))\,(b+B\,(n+1))}{2B}\,(t_W\,(m))^2\,. \qquad (10) \end{aligned}$$

It immediately follows that $W_W\,(m)$ and $W_R\,(m)$ are increasing in m.[1] One can also verify that $W_W\,(m) > W_R\,(m)$ holds for all m. This indicates an element of free rider problem: for a given m, each country prefers to belong to the group of countries that maximize their welfare. Of course, it does not imply that no country will want to delegate its policy maker to maximize revenue. The set of Nash equilibria of the delegation stage are analyzed in the next section.

[1] One can also show that in the equilibrium, $q_i \propto t_i$ and $R_i \propto (t_i)^2$ for all $i \in N$, where the coefficients of proportionality do not depend on m. Hence, q_i, R_i, as well as consumer surplus for all $i \in N$ are also increasing in m.

2.2 The Delegation Stage

We now turn to the first stage when the governments simultaneously and independently decide whether to delegate the policy makers to maximize revenue or welfare. We restrict attention to pure strategy Nash equilibria in the delegation game. There exists a pure strategy Nash equilibrium in which exactly m^* countries choose to maximize revenue if the following inequalities hold

$$\begin{aligned} W_R(m^*) &\geq W_W(m^*-1), \\ W_W(m^*) &\geq W_R(m^*+1). \end{aligned}$$

Define

$$\Omega(m) \equiv \frac{W_R(m+1)}{W_W(m)}$$

for $m = 0, ..., n-1$. Then m^* for $0 < m^* < n$ is a Nash equilibrium if the following condition holds

$$\Omega(m^*-1) \geq 1 \geq \Omega(m^*), \tag{11}$$

while $m^* = 0$ is a Nash equilibrium if $\Omega(0) \leq 1$ holds, and $m^* = n$ is a Nash equilibrium if $\Omega(n-1) \geq 1$ holds.

Let $\theta \equiv B/b$.Then $\Omega(m)$ can be written as

$$\begin{aligned} \Omega(m) &= \frac{(b+Bn)^2(-B+3b+3Bn)}{(2b-B+2Bn)^2(B+b+Bn)} \\ &\quad \times \left[\frac{(b+Bn)^2(2b-B(m-1)+2B(n-1))+B^2(m-n)(2b-B+2Bn)}{(b+Bn)^2(2b-Bm+2B(n-1))+B^2(m+1-n)(2b-B+2Bn)}\right]^2 \\ &= \frac{(3-\theta+3n\theta)(1+n\theta)^2}{(1+\theta+n\theta)(2-\theta+2n\theta)^2} \\ &\quad \times \left[\frac{(1+n\theta)^2(2-\theta m+2\theta(n-1))+\theta^2(m-n)(2-\theta+2n\theta)+\theta(1+n\theta)^2}{(1+n\theta)^2(2-m\theta+2\theta(n-1))+\theta^2(m-n)(2-\theta+2n\theta)+\theta^2(2-\theta+2n\theta)}\right]^2. \end{aligned} \tag{12}$$

Lemma 2 *$\Omega(m)$ is strictly increasing in m.*

Proof. The term in square brackets in (12) can be written as

$$f(m) = \frac{g(m)+c}{g(m)+d}$$

where

$$\begin{aligned} g(m) &= (1+n\theta)^2 (2-\theta m+2\theta(n-1)) + \theta^2 (m-n)(2-\theta+2n\theta), \\ c &= \theta(1+n\theta)^2, \\ d &= \theta^2 (2-\theta+2n\theta). \end{aligned}$$

Then

$$\frac{d\Omega}{dm} = \frac{(3-\theta+3n\theta)(1+n\theta)^2}{(1+\theta+n\theta)(2-\theta+2n\theta)^2} \times 2f(m) f'(m)$$

where

$$f'(m) = \frac{g'(m)(d-c)}{(g(m)+d)^2}.$$

The inequality $f'(m) > 0$ holds because $g'(m) = d - c = (2n\theta - \theta + 2)\theta^2 - \theta(n\theta+1)^2 = -\theta(-\theta+n\theta+1)^2 < 0$. Further, $f(m) > 1$ because $g(m) > (n-m)\left\{\theta(1+n\theta)^2 - \theta^2(2-\theta+2n\theta)\right\} \geq 0$ and $c > d$. Thus, it follows that $\Omega(m)$ is a strictly increasing function of m. ■

Lemma 2 immediately allows us to establish certain properties about the Nash equilibria of the game in the delegation stage. Since $\Omega(m)$ is increasing in m, the condition in (11) cannot hold and, consequently, there does not exist an asymmetric Nash equilibrium in the game, that is, there is no Nash equilibrium for $0 < m^* < n$. The monotonicity of $\Omega(m)$ also implies that either $\Omega(0) \leq 1$ or $\Omega(n-1) \geq 1$ must hold. Hence, the game in the delegation stage necessarily possesses a symmetric Nash equilibrium, that is, either $m^* = 0$, or $m^* = n$, or both hold. Further, it follows that if $\Omega(n-1) < 1$ ($\Omega(0) > 1$) then $\Omega(m) < 1$ ($\Omega(m) > 1$) for all m and maximizing welfare (revenue) is a strictly dominant strategy. We summarize these results in the following proposition.

Proposition 3 *The game in the delegation stage only has a symmetric Nash equilibrium in pure strategies. If the pure strategy Nash equilibrium is unique, then the game is dominance solvable and this equilibrium is also the unique mixed strategy Nash equilibrium and the unique correlated equilibrium.*

2.3 Comparative Statics

$\Omega(m)$ in (12) depends on two parameters: n and θ. To emphasize it, now we will write $\Omega(m; n, \theta)$ instead of $\Omega(m)$. Next, we analyze how the set of Nash equilibria depends on these parameters.

Lemma 4 *For all $n \geq 2$ and for all $\theta > 0$ the following is true:*

$$\frac{d\Omega\left(0;n,\theta\right)}{dn} < 0, \quad \frac{d\Omega\left(0;n,\theta\right)}{d\theta} > 0,$$
$$\frac{d\Omega\left(n-1;n,\theta\right)}{dn} < 0, \quad \frac{d\Omega\left(n-1;n,\theta\right)}{d\theta} > 0.$$

The above lemma allows to establish the following results.

Proposition 5 *There exists a Nash equilibrium to the delegation stage game, in which all countries delegate their policy makers to maximize welfare, for all $\theta > 0$ when $n \geq 3$. This equilibrium is unique for all $\theta > 0$ when $n \geq 5$.*

Proof. To prove the first part of the proposition, we need to show that $\Omega\left(0;n,\theta\right) \leq 1$ for all $\theta > 0$ when $n \geq 3$. Since $\Omega\left(0;n,\theta\right)$ is increasing in θ, it is enough to show that $\lim_{\theta\to\infty}\Omega\left(0;n,\theta\right) \leq 1$ when $n \geq 3$:

$$\lim_{\theta\to\infty}\Omega\left(0;n,\theta\right) = \frac{n^4\left(3n-1\right)\left(2n-1\right)^2}{\left(n+1\right)\left(2n-1\right)^2\left(2n^2-2n+1\right)^2}.$$

When $n = 3$, $\lim_{\theta\to\infty}\Omega\left(0;n,\theta\right) = \frac{162}{169}$ and, since $\Omega\left(0;n,\theta\right)$ is decreasing in n, the claim holds. (One can verify that $\lim_{\theta\to\infty}\Omega\left(0;n,\theta\right)$ is still a decreasing function in n.)

To prove the second part of the proposition, we need to show that $\Omega\left(n-1;n,\theta\right) < 1$ for all $\theta > 0$ when $n \geq 5$. Since $\Omega\left(n-1;n,\theta\right)$ is increasing in θ, it is again enough to show that $\lim_{\theta\to\infty}\Omega\left(n-1;n,\theta\right) < 1$ when $n \geq 5$:

$$\lim_{\theta\to\infty}\Omega\left(n-1;n,\theta\right) = \frac{\left(n+n^2-1\right)^2\left(3n-1\right)}{n^2\left(2n-1\right)^2\left(n+1\right)}.$$

When $n = 5$, $\lim_{\theta\to\infty}\Omega\left(n-1;n,\theta\right) = \frac{5887}{6075}$ and, since $\Omega\left(n-1;n,\theta\right)$ is decreasing in n, the claim holds. (One can verify that $\lim_{\theta\to\infty}\Omega\left(n-1;n,\theta\right)$ is also a decreasing function in n.) ■

A Nash equilibrium, in which all countries delegate their policy makers to maximize revenue, can only occur for $n = 2, 3, 4$. Further, this equilibrium can only be unique for $n = 2$. To find a range of values of θ, for which the revenue maximization equilibrium exists, we find the values of θ when $\Omega\left(n-1;n,\theta\right) = 1$ for $n = 2, 3, 4$. They are approximately 1.7759, 2.6357, and 11.5413, respectively. To find a range of values of θ, for which the revenue

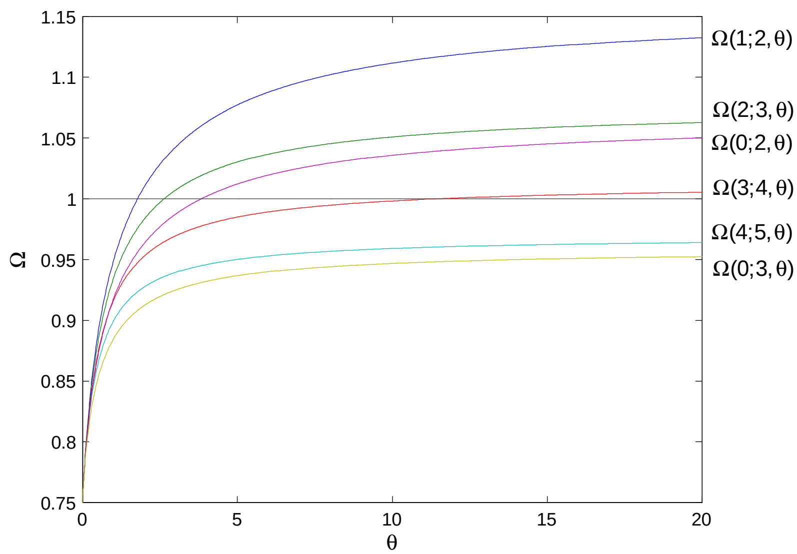

Figure 1: Nash equilibria of the delegation game

maximization equilibrium is unique, we find the value of θ when $\Omega\,(0;2,\theta) = 1$. It happens when $\theta \approx 3.8376$. The results, which are summarized in following proposition, then follow from the monotonicity of $\Omega\,(0;n,\theta)$ and $\Omega\,(n-1;n,\theta)$ in θ. The results of the proposition can also be inferred from Figure 1.

Proposition 6 *The set of Nash equilibria to the game in the delegation stage depending on the values of n and θ are as follows:*

	The set of Nash equilibria		
	$m^* = 0$	$m^* = 0$ *and* $m^* = n$	$m^* = n$
$n = 2$	$(0, 1.7759)$	$[1.7759, 3.8376]$	$(3.8376, \infty)$
$n = 3$	$(0, 2.6357)$	$[2.6357, \infty)$	-
$n = 4$	$(0, 11.5413)$	$[11.5413, \infty)$	-
$n \geq 5$	$(0, \infty)$	-	-

2.4 Welfare

In the previous section we have established that for a wide range of parameter values there is a unique equilibrium, in which all countries maximize their welfare. We now analyze if the inability by countries to coordinate on revenue maximization results in a lower welfare. Therefore, we compare welfare from the outcomes when all countries delegate their policy makers to maximize revenue and welfare, respectively. Define

$$\Phi(n,\theta) \equiv \frac{W_R(n)}{W_W(0)} = \frac{(3-\theta+3\theta n)\left(n^2\theta^2+2n\theta-n\theta^2+1\right)^2}{(\theta+1+\theta n)\left(2+\theta(n-1)\right)^2(1+\theta n)^2}.$$

Lemma 7 *For all $n \geq 2$ and for all $\theta > 0$ the following is true:*

$$\frac{d\Phi(n,\theta)}{dn} > 0 \text{ and } \frac{d\Phi(n,\theta)}{d\theta} > 0.$$

Proposition 8 *For each $\theta > 0$, there exists $n(\theta)$ such that for all $n > n(\theta)$, $\Phi(n,\theta) > 1$. In particular, when $\theta > 0.318\,09$, then $\Phi(n,\theta) > 1$ for all $n \geq 2$.*

Proof. As the number of countries tends to infinity, $\Phi(n,\theta)$ converges to 3 for all $\theta > 0$: $\lim_{n\to\infty}\Phi(n,\theta) = 3$. This establishes the first part of proposition. When $n \geq 2$, $\Phi(n,\theta) = 1$ for $\theta = 0.318\,09$. The second part of proposition then follows from the monotonicity of $\Phi(n,\theta)$ in n and θ. ■

Clearly, $n(\theta)$ mentioned in the above proposition is implicitly defined by $\Phi(n,\theta) = 1$. Figure 2 represents $\Phi(n,\theta) = 1$ for $n = 2, ..., 20$. Since $\Phi(n,\theta)$ is strictly increasing in both arguments, $n(\theta)$ (or $\theta(n)$ in the figure) is strictly declining. Note, however, θ does not converge to 0 for any finite n, since $\lim_{\theta\to 0}\Phi(n,\theta) = \frac{3}{4}$.

The results of Propositions 6 and 8 together imply that for a wide range of values, the countries are facing a Prisoners' dilemma situation: even so the welfare is higher when all countries delegate their policy makers to maximize import tariff revenue, the countries end up delegating their policy makers to maximize welfare in the unique Nash equilibrium. Therefore, we can conclude that the delegation game often fails to achieve its objective, which is to introduce more cooperation in setting tariffs among countries. On the other hand, the results of Propositions 6 and 8 also imply that when both symmetric equilibria exist, the revenue maximizing equilibrium Pareto dominates the welfare maximizing equilibrium. This can help the countries to coordinate on their choice of equilibrium.

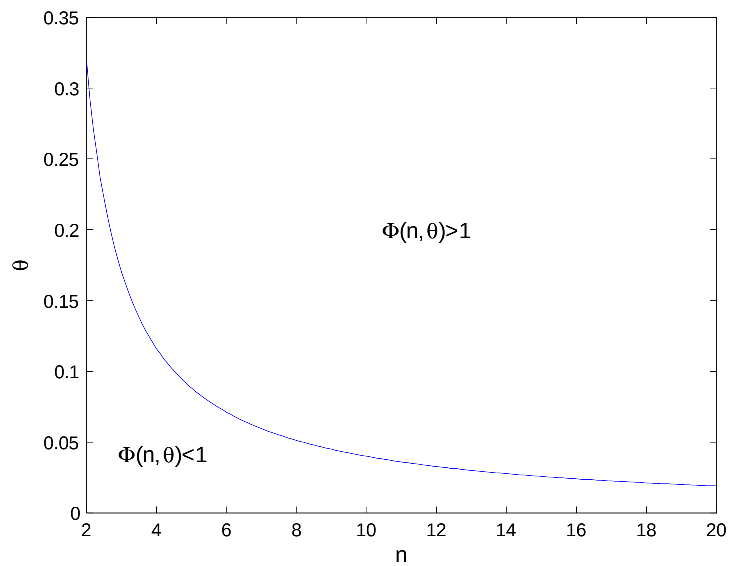

Figure 2: Graph of $\Phi(n, \theta) = 1$

3 The Model with Many Importers and Exporters

We now extend the benchmark model, in which besides n importing countries, we additionally introduce l exporting countries. Let $L = \{1, ..., l\}$ denote the set of exporting countries. Each exporting country $j \in L$ sets its specific export tax, e_j. The supply of country j is given by

$$Q_j = a + b(P - e_j).$$

In the equilibrium, the world supply equals the world demand

$$\sum_{j=1}^{l} Q_j = \sum_{i=1}^{n} q_i.$$

This equilibrium condition implies the following expression for the equilibrium world price:

$$P = \frac{b \sum_{j=1}^{l} e_j - B \sum_{i=1}^{n} t_i + An - al}{Bn + bl}.$$

The demand of importing country $i \in N$ can now be written as

$$q_i = \frac{(Ab + Ba)\, l - BbE + B^2 \sum_{j \in N \setminus \{i\}} t_j - Bt_i \,(B\,(n-1) + bl)}{Bn + bl}$$

where $E \equiv \sum_{j=1}^{l} e_j$. The best response functions of revenue and welfare maximizing importing countries, respectively, are

$$\begin{aligned} t_i &= r_R^M \left(\sum_{j \in N \setminus \{i\}} t_j, E \right) = \frac{B^2 \sum_{j \in N \setminus \{i\}} t_j + (Ab + Ba)\, l - BbE}{2B\,(B\,(n-1) + bl)}, \\ t_i &= r_W^M \left(\sum_{j \in N \setminus \{i\}} t_j, E \right) = \frac{B^2 \sum_{j \in N \setminus \{i\}} t_j + (Ab + Ba)\, l - BbE}{(B\,(n+1) + bl)\,(B\,(n-1) + bl)}. \end{aligned}$$

Note that if we set $E = 0$ and $l = 1$ we obtain the corresponding expressions from the benchmark model.

Let us treat the strategies of policy makers of exporting countries as given, that is, keep E fixed, and solve for the equilibrium values of import tariffs of revenue and welfare maximizing countries, respectively. As before, suppose that the policy makers of m out of n importing countries maximize revenue and the rest maximize welfare. The equilibrium import tariffs are

$$\begin{aligned} t_R &= \frac{(Bn + bl)^2\,((Ab + Ba)\, l - BbE)}{B\left((Bn + bl)^2\,(2\,(bl + B\,(n-1)) - B\,(m-1)) + B^2\,(2Bn - B + 2bl)\,(m-n)\right)}, \\ t_W &= \frac{(2Bn - B + 2bl)\,((Ab + Ba)\, l - BbE)}{(Bn + bl)^2\,(2\,(bl + B\,(n-1)) - B\,(m-1)) + B^2\,(2Bn - B + 2bl)\,(m-n)}. \end{aligned}$$

The equilibrium imports of revenue and welfare maximizing countries as functions of import tariffs are given, respectively, by

$$\begin{aligned} q_R^M &= \frac{B\,(Bn - B + bl)}{Bn + bl} t_R, \\ q_W^M &= (Bn - B + bl)\, t_W, \end{aligned}$$

while the corresponding equilibrium welfare is

$$\begin{aligned} W_R^M &= \frac{B\,(bl + B\,(n-1))\,(3bl + B\,(3n-1))}{2\,(bl + Bn)^2} (t_R)^2, \\ W_W^M &= \frac{(bl + B\,(n-1))\,(bl + B\,(n+1))}{2B} (t_W)^2. \end{aligned}$$

Let us now consider exporting countries. The supply of exporting country $i \in L$, given the equilibrium world price, can be written as

$$q_i = \frac{(Ab + Ba)\, n - BbT + b^2 \sum_{j \in L \setminus \{i\}} e_j - be_j \left(Bn + b\,(l-1)\right)}{Bn + bl}$$

where $T \equiv \sum_{i=1}^{n} t_i$. One can check that if we replace e_j by t_j, T by E, n by l, and exchange B with b except in the term $Ab + Ba$, we obtain the demand function of importing country. That is, the problem for exporting countries is a 'mirror image' of the importers' problem. Therefore, the best response functions for revenue and welfare maximizing exporters can be written as

$$\begin{aligned} e_i &= r_R^X \left(T, \sum_{j \in L \setminus \{i\}} e_j \right) = \frac{b^2 \sum_{j \in L \setminus \{i\}} e_j + (Ab + Ba)\, n - BbT}{2b \left(Bn + b\,(l-1)\right)}, \\ e_i &= r_W^X \left(T, \sum_{j \in L \setminus \{i\}} e_j \right) = \frac{b^2 \sum_{j \in L \setminus \{i\}} e_j + (Ab + Ba)\, n - BbT}{\left(Bn + b\,(l-1)\right)\left(Bn + b\,(l+1)\right)}. \end{aligned}$$

Suppose that the policy makers of k out of l exporting countries maximize revenue and the rest maximize welfare. The equilibrium export tariffs, for a fixed T, are

$$\begin{aligned} e_R &= \frac{(Bn + bl)^2 \left((Ab + Ba)\, n - BbT\right)}{b \left((Bn + bl)^2 \left(2\,(Bn + b\,(l-1)) - b\,(k-1)\right) + b^2 \left(2Bn - b + 2bl\right)(k - l) \right)}, \\ e_W &= \frac{(2Bn - b + 2bl) \left((Ab + Ba)\, n - BbT\right)}{(Bn + bl)^2 \left(2\,(Bn + b\,(l-1)) - b\,(k-1)\right) + b^2 \left(2Bn - b + 2bl\right)(k - l)}. \end{aligned}$$

while the corresponding equilibrium welfare is

$$\begin{aligned} W_R^X &= \frac{b \left(Bn + b\,(l-1)\right)\left(3Bn + b\,(3l-1)\right)}{2\,(bl + Bn)^2} (e_R)^2, \\ W_W^X &= \frac{\left(Bn + b\,(l-1)\right)\left(Bn + b\,(l+1)\right)}{2b} (e_W)^2. \end{aligned}$$

Given that $T = mt_R + (n-m)\, t_W$ and $E = ke_R + (l-k)\, e_W$, we have that

$$T = \frac{\left((Ab + Ba)\, l - BbE\right) \left\{ m\,(Bn + bl)^2 + (n-m)\, B\,(2Bn - B + 2bl) \right\}}{B \left((Bn + bl)^2 \left(2\,(bl + B\,(n-1)) - B\,(m-1)\right) + B^2 \left(2Bn - B + 2bl\right)(m - n) \right)} \quad (13)$$

$$E = \frac{\left((Ab + Ba)\, n - BbT\right) \left\{ k\,(Bn + bl)^2 + (l-k)\, b\,(2Bn - b + 2bl) \right\}}{b \left((Bn + bl)^2 \left(2\,(Bn + b\,(l-1)) - b\,(k-1)\right) + b^2 \left(2Bn - b + 2bl\right)(k - l) \right)}. \quad (14)$$

Using equations (13) and (14), we can solve for the values of T and E, and then we can recover the values of other variables.

To simplify the analysis, we now fix $n = l = 2$. After some tedious calculations, the welfare of revenue and welfare maximizing importers, depending on the values of m and k, is given by the following expressions (where we have suppressed the superscript M):

$$
\begin{aligned}
W_R(2,2) &= \frac{B(B+2b)(2B+b)^2(5B+6b)(Ab+Ba)^2}{2(B+b)^2(4B^3+13B^2b+4Bb^2)^2}, \\
W_R(1,2) &= \frac{8B(B+2b)(2B+b)^2(5B+6b)(Ab+Ba)^2(B+b)^2}{(20B^5+103B^4b+164B^3b^2+96B^2b^3+16Bb^4)^2}, \\
W_R(2,1) &= \frac{B(B+2b)(5B+6b)(Ab+Ba)^2(12B^3+28B^2b+21Bb^2+5b^3)^2}{2(B+b)^2(16B^5+96B^4b+164B^3b^2+103B^2b^3+20Bb^4)^2}, \\
W_R(1,1) &= \frac{B(B+2b)(5B+6b)(Ab+Ba)^2(12B^3+28B^2b+21Bb^2+5b^3)^2}{2(B+b)^2(20B^5+115B^4b+192B^3b^2+115B^2b^3+20Bb^4)^2}, \\
W_R(2,0) &= \frac{B(B+2b)(5B+6b)(Ab+Ba)^2(2B^2+3Bb+b^2)^2}{2(B+b)^2(2B^4+12B^3b+15B^2b^2+4Bb^3)^2}, \\
W_R(1,0) &= \frac{2B(B+2b)(5B+6b)(Ab+Ba)^2(2B^2+3Bb+b^2)^2}{(B+b)^2(5B^4+28B^3b+34B^2b^2+8Bb^3)^2}, \\
W_W(1,2) &= \frac{2(B+2b)(3B+2b)(Ab+Ba)^2(6B^2+11Bb+4b^2)^2}{B(20B^4+103B^3b+164B^2b^2+96Bb^3+16b^4)^2}, \\
W_W(0,2) &= \frac{(B+2b)(2B+b)^2(3B+2b)(Ab+Ba)^2}{2B(4B^3+15B^2b+12Bb^2+2b^3)^2}, \\
W_W(1,1) &= \frac{(B+2b)(3B+2b)(Ab+Ba)^2(36B^3+96B^2b+79Bb^2+20b^3)^2}{8B(20B^5+135B^4b+307B^3b^2+307B^2b^3+135Bb^4+20b^5)^2}, \\
W_W(0,1) &= \frac{(B+2b)(3B+2b)(Ab+Ba)^2(12B^2+16Bb+5b^2)^2}{8B(8B^4+42B^3b+62B^2b^2+33Bb^3+5b^4)^2}, \\
W_W(1,0) &= \frac{(B+2b)(3B+2b)(Ab+Ba)^2(6B^2+11Bb+4b^2)^2}{2B(5B^4+33B^3b+62B^2b^2+42Bb^3+8b^4)^2}, \\
W_W(0,0) &= \frac{(B+2b)(2B+b)^2(3B+2b)(Ab+Ba)^2}{8B(B^3+5B^2b+5Bb^2+b^3)^2}.
\end{aligned}
$$

Figure 3 illustrates $\frac{W_R(m,k;\theta)}{W_W(m-1,k;\theta)}$ for $m = 1, 2$ and $k = 0, 1, 2$, where $\theta = B/b$

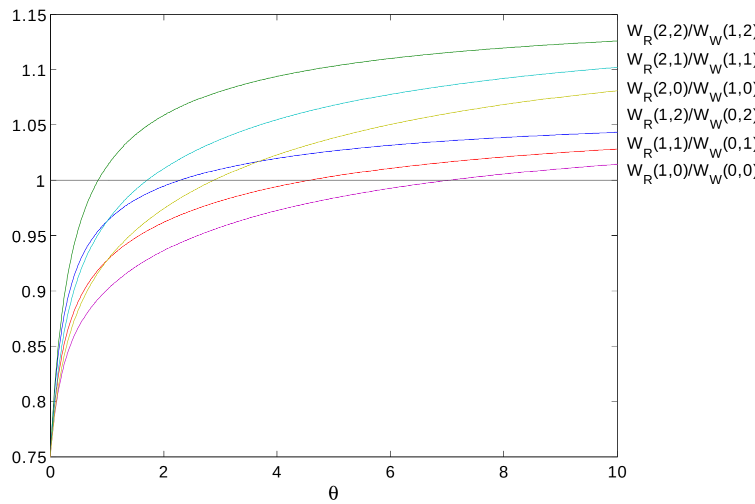

Figure 3: Function $\frac{W_R(m,k;\theta)}{W_W(m-1,k;\theta)}$

as before. It shows that the delegation game still does not have asymmetric equilibria such that one importer maximizes revenue while the other importer maximizes welfare. Therefore, we can focus on the cases when both importers either maximize revenue or welfare. The relevant threshold values of θ are as follows. If both exporters maximize revenue, then the government of importing country is indifferent between delegating revenue and welfare maximization for $\theta = 0.841\,59$ if the government of the other importing country delegates to maximize revenue, that is, θ solves $W_R\,(2,2;\theta) = W_W\,(1,2;\theta)$, while it is indifferent between delegating revenue and welfare maximization for $\theta = 2.\,291\,6$ if the government of the other importing country delegates to maximize welfare, that is, θ solves $W_R\,(1,2;\theta) = W_W\,(0,2;\theta)$. The relevant threshold values of θ are $2.\,884$ and $7.\,045\,4$, respectively, when both exporters maximize welfare. It follows that if the exporters maximize welfare instead of revenue, then the governments of importing countries will also find it optimal to maximize welfare for a larger range of values of θ.

Due to the symmetry of the problem, the threshold values for exporters are the inverse values of those for importers.

Given the threshold values, we can now identify the equilibria of the delegation game. The equilibrium strategies are summarized in the following table for different values of θ:

θ	(Importers, Exporters)	$1/\theta$
$(0, 0.141\,94)$	(Welfare, Revenue)	$(7.045\,4, \infty)$
$[0.141\,94, 0.346\,74]$	(Welfare, Revenue), (Welfare, Welfare)	$[2.884, 7.045\,4]$
$(0.346\,74, 0.436\,38)$	(Welfare, Welfare)	$(2.291\,6, 2.884)$
$[0.436\,38, 0.841\,59)$	(Welfare, Welfare)	$[1.188\,2, 2.291\,6)$
$[0.841\,59, 1.188\,2]$	(Welfare, Welfare), (Revenue, Revenue)	$[0.841\,59, 1.188\,2]$
$(1.188\,2, 2.291\,6]$	(Welfare, Welfare)	$(0.436\,38, 0.841\,59]$
$(2.291\,6, 2.884)$	(Welfare, Welfare)	$(0.346\,74, 0.436\,38)$
$[2.884, 7.045\,4]$	(Welfare, Welfare), (Revenue, Welfare)	$[0.141\,94, 0.346\,74]$
$(7.045\,4, \infty)$	(Revenue, Welfare)	$(0, 0.141\,94)$

The first thing to note is that the equilibria are not anymore monotonic in θ unlike the benchmark model with non-strategic exporters. Second, when two equilibria exist, one of the equilibria is welfare superior for either the importers, or the exporters or both. For example, $W_R(2,0) > W_W(0,0)$ for $\theta \in [2.884, 7.045\,4]$. Therefore, the importers prefer the equilibrium (Revenue, Welfare) over (Welfare, Welfare). (While it is opposite for the exporters since $W_W(0,0) > W_W(0,2)$ for all θ.) Similarly, the exporters prefer (Welfare, Revenue) over (Welfare, Welfare) when $\theta \in [0.141\,94, 0.346\,74]$. Also, since $W_W(0,0) > W_R(2,2)$ for all θ, (Welfare, Welfare) Pareto dominates (Revenue, Revenue) when $\theta \in [0.841\,59, 1.188\,2]$.

Assuming that the importers can coordinate on the equilibrium (Revenue, Welfare) when $\theta \in [2.884, 7.045\,4]$, or that the exporters can coordinate on the equilibrium (Welfare, Revenue) when $\theta \in [0.141\,94, 0.346\,74]$, or all countries can coordinate on the equilibrium (Welfare, Welfare) when $\theta \in [0.841\,59, 1.188\,2]$, we can predict the following equilibrium welfare for an importer:

$$W_E^M = \begin{cases} W_W(0,2) & \text{if} \quad 0 < \theta \leq 0.346\,74 \\ W_W(0,0) & \text{if} \quad 0.346\,74 < \theta < 2.884 \\ W_R(2,0) & \text{if} \quad 2.884 \leq \theta \end{cases}$$

Under free trade, the demand of importer $i \in N$ is

$$q_i = \frac{(Ab + Ba)\, l}{Bn + bl}$$

and the welfare is

$$W_F^M = \frac{1}{2B}\left(\frac{(Ab + Ba)\, l}{Bn + bl}\right)^2.$$

When $n = l = 2$, the expression for welfare is given by

$$W_F^M = \frac{(Ab + Ba)^2}{2B(B+b)^2}.$$

Figure 4 plots

$$\frac{W_E^M}{W_W^M(0,0)} = \begin{cases} \frac{4(\theta+1)^2\left(4\theta+\theta^2+1\right)^2}{\left(12\theta+15\theta^2+4\theta^3+2\right)^2} & \text{if } \ 0 < \theta \leq 0.346\,74 \\ 1 & \text{if } \ 0.346\,74 < \theta < 2.\,884 \\ \frac{4(\theta+1)^2(5\theta+6)\left(4\theta+\theta^2+1\right)^2}{(3\theta+2)\left(15\theta+12\theta^2+2\theta^3+4\right)^2} & \text{if } \ 2.\,884 \leq \theta \end{cases}$$

and

$$\frac{W_E^M}{W_F^M} = \begin{cases} \frac{(3\theta+2)(\theta+1)^2(\theta+2)(2\theta+1)^2}{\left(12\theta+15\theta^2+4\theta^3+2\right)^2} & \text{if } \ 0 < \theta \leq 0.346\,74 \\ \frac{(3\theta+2)(\theta+2)(2\theta+1)^2}{4\left(4\theta+\theta^2+1\right)^2} & \text{if } \ 0.346\,74 < \theta < 2.\,884 \\ \frac{(5\theta+6)(\theta+1)^2(\theta+2)(2\theta+1)^2}{\left(15\theta+12\theta^2+2\theta^3+4\right)^2} & \text{if } \ 2.\,884 \leq \theta \end{cases},$$

and the corresponding expressions for exporting country.

Note that $W_E^M = W_F^M$ when $\theta = 1.\,247\,3$. Thus, the importers are better off from imposing tariffs compared to the free trade regime when $\theta > 1.\,247\,3$, while the exporters are worse off. The conclusions are reversed for $\theta < 1/1.\,247\,3 = 0.801\,73$. Only for the values of $\theta \in [0.801\,73, 1.\,247\,3]$ are both importers and exporters better off from free trade. Note, however, Figure 4 might be misleading in the following sense. When for example θ approaches 0, W_E^M (and $W_W^M(0,0)$) converge to W_F^M. On the other hand, $W_E^X > W_F^X$ (and $W_W^X(0,0) > W_F^X$) when θ converges to 0. Thus, it appears that under protectionism, the importers are no worse off than under free trade, while the exporters definitely gain compared to the free trade when θ is close to 0. However, when B and hence θ approach 0, the welfare of importer tends to infinity and it completely dwarfs the welfare of exporting country. Consequently, the joint welfare of importers and exporters is always higher under free trade as shown by the dashed line in Figure 5. It plots the following functions:

$$\frac{W_E^M + W_E^X}{W_W^M(0,0) + W_W^X(0,0)} = \begin{cases} \frac{4(\theta+1)\left(17\theta+23\theta^2+6\theta^3+2\right)\left(4\theta+\theta^2+1\right)^2}{\left(11\theta+2\theta^2+2\right)\left(12\theta+15\theta^2+4\theta^3+2\right)^2} & \text{if } \ 0 < \theta \leq 0.346\,74 \\ 1 & \text{if } \ 0.346\,74 < \theta \leq 2.\,884 \\ \frac{4(\theta+1)\left(23\theta+17\theta^2+2\theta^3+6\right)\left(4\theta+\theta^2+1\right)^2}{\left(11\theta+2\theta^2+2\right)\left(15\theta+12\theta^2+2\theta^3+4\right)^2} & \text{if } \ 2.\,884 < \theta \end{cases}$$

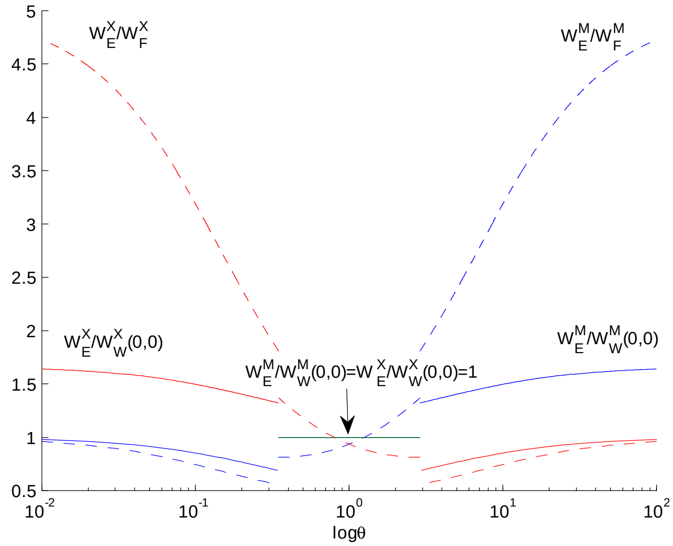

Figure 4: Functions $\frac{W_E^i}{W_W^i(0,0)}$ and $\frac{W_E^i}{W_F^i}$ for $i = M, X$

and

$$\frac{W_E^M + W_E^X}{W_F^M + W_F^X} = \begin{cases} \frac{(2\theta+1)(\theta+2)(\theta+1)\left(17\theta+23\theta^2+6\theta^3+2\right)}{\left(12\theta+15\theta^2+4\theta^3+2\right)^2} & \text{if} \quad 0 < \theta \leq 0.346\,74 \\ \frac{(2\theta+1)(\theta+2)\left(11\theta+2\theta^2+2\right)}{4\left(4\theta+\theta^2+1\right)^2} & \text{if} \quad 0.346\,74 < \theta \leq 2.884 \\ \frac{(2\theta+1)(\theta+2)(\theta+1)\left(23\theta+17\theta^2+2\theta^3+6\right)}{\left(15\theta+12\theta^2+2\theta^3+4\right)^2} & \text{if} \quad 2.884 < \theta \end{cases}.$$

Figure 5 also shows that jointly importers and exporters are worse off when they can delegate tariff/tax setting to policy makers, which is not surprising as revenue maximizing policy makers will set higher tariffs or taxes, which in turn will result in higher deadweight loss.

4 The Delegation Game with Incomplete Information

We now consider an incomplete information version of the delegation game of section two. To simplify analysis, we set $n = 2$. We assume now that the

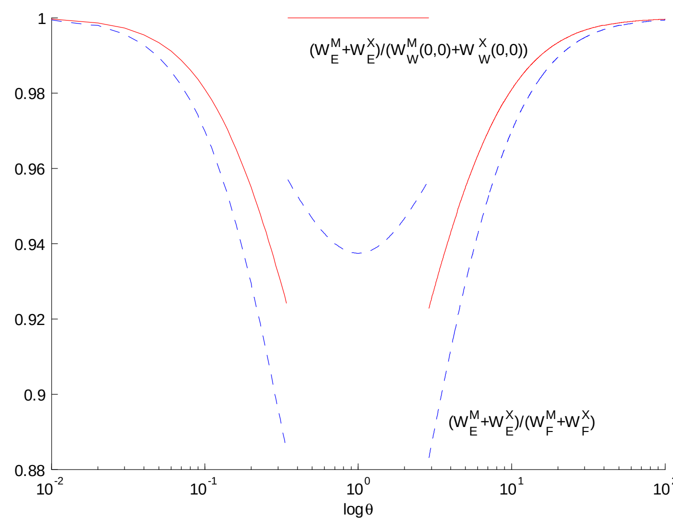

Figure 5: Functions $\frac{W_E^M+W_E^X}{W_W^M(0,0)+W_W^X(0,0)}$ (solid line) and $\frac{W_E^M+W_E^X}{W_F^M+W_F^X}$ (dashed line)

demand of country i is given by

$$q_i = A_i - Bp_i,$$

where A_1 and A_2 are drawn independently according to the same distribution function. We assume that only the policy maker of country i observes the realization of A_i.

Since, in general, $A_1 \neq A_2$, the demand function of country i now becomes

$$q_i\left(t_i, t_j; A_i, A_j\right) = \frac{Ba + (B+b)\,A_i - BA_j + B^2 t_j - B\,(B+b)\,t_i}{2B+b},$$

where we have emphasized that the demand depends on tariffs and random realizations of demand parameters. We assume that the support of distribution function is such that in equilibrium, both countries import a positive amount of product for all realizations of A_i, $i = 1, 2$.

We start by solving the game of tariff setting stage. Now the strategy of country i's policy maker is a function $t_i\left(A_i\right)$ that assigns for each realized value of A_i a corresponding import tariff. Suppose the policy maker of country i is delegated to maximize revenue. The objective function that the policy maker is maximizing is

$$E_{A_j}\left[\frac{Ba+(B+b)A_i-BA_j+B^2t_j(A_j)-B(B+b)t_i}{2B+b}t_i\right].$$

From the first order condition we obtain the expression for the optimal t_i, given A_i and the expected value of tariff set by the other country:

$$t_i=\frac{B\left(a-\bar{A}\right)+(B+b)A_i+B^2E\left[t_j(A_j)\right]}{2B(B+b)}, \tag{15}$$

where $\bar{A}\equiv E[A_j]$. Suppose the policy maker of country i is delegated to maximize welfare. The objective function that the policy maker is maximizing can be written as

$$E_{A_j}\left[\frac{Ba+(B+b)A_i-BA_j+B^2t_j(A_j)-B(B+b)t_i}{2B+b}\right.$$
$$\left.\times\frac{Ba+(B+b)A_i-BA_j+B^2t_j(A_j)+B(3B+b)t_i}{2B(2B+b)}\right].$$

From the first order condition we again obtain the expression for the optimal t_i, given A_i and the expected value of tariff set by the other country:

$$t_i=\frac{B\left(a-\bar{A}\right)+(B+b)A_i+B^2E\left[t_j(A_j)\right]}{(B+b)(3B+b)}. \tag{16}$$

Note that the optimal tariff both in (15) and (16) is linear in A_i. Thus, the equilibrium strategy is a linear function of the form $t_i(A_i)=\alpha_i+\beta_iA_i$ for either objective. Further, it immediately follows that $\beta_i=\frac{1}{2B}$ if the policy maker is delegated to maximize revenue, and $\beta_i=\frac{1}{3B+b}$ if delegated to maximize welfare.

Suppose the policy makers of both countries have been delegated to maximize revenue. We exploit the symmetry of the problem and look for a value of α, common to both policy makers, and which we denote by α_{RR}. It is given by the following equation:

$$\alpha_{RR}=\frac{B\left(a-\bar{A}\right)+B^2\left(\alpha_{RR}+\frac{\bar{A}}{2B}\right)}{2B(B+b)},$$

for which the solution is

$$\alpha_{RR} = \frac{2a - \bar{A}}{2B + 4b}.$$

Hence, the symmetric equilibrium strategy when both policy makers are delegated to maximize revenue is

$$t_{RR}(A_i) = \frac{2a - \bar{A}}{2B + 4b} + \frac{A_i}{2B}.$$

Suppose both policy makers have been delegated to maximize welfare. We again look for a symmetric equilibrium, that is, for a common value of α, which now is denote by α_{WW}. It is given by the following equation:

$$\alpha_{WW} = \frac{B\left(a - \bar{A}\right) + B^2\left(\alpha_{WW} + \frac{\bar{A}}{3B+b}\right)}{(B+b)(3B+b)},$$

for which the solution is

$$\alpha_{WW} = B\frac{(3B+b)\,a - (2B+b)\,\bar{A}}{(3B+b)\left(4Bb + 2B^2 + b^2\right)}.$$

Hence, the symmetric equilibrium strategy when both policy makers are delegated to maximize welfare is

$$t_{WW}(A_i) = B\frac{(3B+b)\,a - (2B+b)\,\bar{A}}{(3B+b)\left(4Bb + 2B^2 + b^2\right)} + \frac{A_i}{3B+b}.$$

Suppose one of the policy makers has been delegated to maximize revenue, while the other has been delegated to maximize welfare. Let the equilibrium values of αs of revenue and welfare maximizing policy maker, respectively, be denoted by α_{RW} and α_{WR}. (The first letter in a subscript indicates the objective of the country that we are considering at that moment, while the second letter indicates the objective of the other country. We will maintain this convention throughout.) They are given by the following system of equations:

$$\begin{aligned}
\alpha_{RW} &= \frac{B\left(a - \bar{A}\right) + B^2\left(\alpha_{WR} + \frac{\bar{A}}{3B+b}\right)}{2B(B+b)}, \\
\alpha_{WR} &= \frac{B\left(a - \bar{A}\right) + B^2\left(\alpha_{RW} + \frac{\bar{A}}{2B}\right)}{(B+b)(3B+b)}.
\end{aligned}$$

The solution is

$$\alpha_{RW} = \frac{2(2B+b)^2 a - (6Bb + 5B^2 + 2b^2)\bar{A}}{20Bb^2 + 28B^2 b + 10B^3 + 4b^3},$$
$$\alpha_{WR} = B\frac{(3B+2b)(3B+b)a - (5Bb + 5B^2 + b^2)\bar{A}}{(10Bb^2 + 14B^2 b + 5B^3 + 2b^3)(3B+b)}.$$

Hence, the equilibrium strategies of revenue and welfare maximizing policy maker, respectively, are

$$t_{RW}(A_i) = \frac{2(2B+b)^2 a - (6Bb + 5B^2 + 2b^2)\bar{A}}{20Bb^2 + 28B^2 b + 10B^3 + 4b^3} + \frac{A_i}{2B},$$
$$t_{WR}(A_i) = B\frac{(3B+2b)(3B+b)a - (5Bb + 5B^2 + b^2)\bar{A}}{(10Bb^2 + 14B^2 b + 5B^3 + 2b^3)(3B+b)} + \frac{A_i}{3B+b}.$$

Let us define $\epsilon_i = A_i - \bar{A}$. Then, the equilibrium strategy of country i's policy maker can be expressed as a function of ϵ_i: $\tilde{t}_i(\epsilon_i) = t_i\left(\bar{A}\right) + \beta_i \epsilon_i$. (Obviously, $\tilde{t}_i(\epsilon_i)$, as well as $\tilde{q}_i(\epsilon_i, \epsilon_j)$ and $\tilde{W}_i(\epsilon_i, \epsilon_j)$ below also depend on the first stage decisions of both governments.) Similarly, the demand of country i in the equilibrium as a function of ϵ_i and ϵ_j is

$$\tilde{q}_i(\epsilon_i, \epsilon_j) = q_i\left(t_i\left(\bar{A}\right), t_j\left(\bar{A}\right); \bar{A}, \bar{A}\right) + \frac{B\left(B\beta_j - 1\right)\epsilon_j - (B+b)\left(B\beta_i - 1\right)\epsilon_i}{2B+b}.$$

Then, expected payoff of country i is given by

$$\begin{aligned}
&E\left[\tilde{W}_i(\epsilon_i, \epsilon_j)\right] \\
=\ & E\left[\tilde{q}_i(\epsilon_i, \epsilon_j)\,\tilde{t}_i(\epsilon_i) + \frac{1}{2B}\left(\tilde{q}_i(\epsilon_i, \epsilon_j)\right)^2\right] \\
=\ & q_i\left(t_i\left(\bar{A}\right), t_j\left(\bar{A}\right); \bar{A}, \bar{A}\right) t_i\left(\bar{A}\right) + \frac{1}{2B}\left(q_i\left(t_i\left(\bar{A}\right), t_j\left(\bar{A}\right)\right)\right)^2 \\
& + E\left[-\frac{(B+b)\left(B\beta_i - 1\right)\beta_i \epsilon_i^2}{2B+b} + \frac{1}{2B}\frac{B^2\left(B\beta_j - 1\right)^2 \epsilon_j^2 + (B+b)^2\left(B\beta_i - 1\right)^2 \epsilon_i^2}{(2B+b)^2}\right],
\end{aligned}$$

where we have used the fact that ϵ_i and ϵ_j are independent and have zero mean. Let $\sigma^2 \equiv E\left[\epsilon_i^2\right] = E\left[\epsilon_j^2\right]$. Then,

$$\begin{aligned}
E\left[\tilde{W}_i(\epsilon_i, \epsilon_j)\right] =\ & q_i\left(t_i\left(\bar{A}\right), t_j\left(\bar{A}\right); \bar{A}, \bar{A}\right) t_i\left(\bar{A}\right) + \frac{1}{2B}\left(q_i\left(t_i\left(\bar{A}\right), t_j\left(\bar{A}\right)\right)\right)^2 \\
& + \frac{B^2\left(B\beta_j - 1\right)^2 - (B+b)\left(B\beta_i - 1\right)\left(B + b + B\beta_i(3B+b)\right)}{2B(2B+b)^2}\sigma^2.
\end{aligned}$$

One can make the following useful observation. If one evaluates the equilibrium strategies of the second stage at $A_i = \bar{A}$, one obtains the equilibrium tariff values of the complete information game found in (7) and (8) with A replaced by $\bar{A}$. For example, $t_{RR}\left(\bar{A}\right)$ is equal to the expression in (7) when $n = m = 2$. In which case, $q_i\left(t_i\left(\bar{A}\right), t_j\left(\bar{A}\right); \bar{A}, \bar{A}\right) t_i\left(\bar{A}\right) + \frac{1}{2B}\left(q_i\left(t_i\left(\bar{A}\right), t_j\left(\bar{A}\right)\right)\right)^2$ is the welfare of country i in the complete information game when $A = \bar{A}$. This allows us conveniently to summarize the expressions for expected welfare in the incomplete information version of the game:

$$\begin{aligned}
\bar{W}_{RR} &= W_R(2) + \frac{8Bb + 6B^2 + 3b^2}{8B(2B+b)^2}\sigma^2, \\
\bar{W}_{RW} &= W_R(1) + \frac{26Bb^3 + 118B^3b + 84B^2b^2 + 61B^4 + 3b^4}{8B(2B+b)^2(3B+b)^2}\sigma^2, \\
\bar{W}_{WR} &= W_W(1) + \frac{20Bb^2 + 33B^2b + 19B^3 + 4b^3}{8B(2B+b)^2(3B+b)}\sigma^2, \\
\bar{W}_{WW} &= W_W(0) + \frac{(2B+b)^2}{2B(3B+b)^2}\sigma^2,
\end{aligned}$$

where $\bar{W}_{RR}, ..., \bar{W}_{WW}$ represent $E\left[\tilde{W}_i(\epsilon_i, \epsilon_j)\right]$ depending on the first stage decisions of the governments.

We can conclude that expected welfare is increasing in the variance of A_i whatever are the first stage decisions of governments. Thus, for example, if a distribution F second order stochastically dominates a distribution G, then the welfare will be higher if A_i for $i = 1, 2$ were drawn according to the distribution G instead of F. To get some intuition behind this result, note that the equilibrium welfare of country is a convex function of the equilibrium domestic tariff in the complete information version of the game in (9) and (10). Therefore, the randomness of tariff due to randomness in A_i results in a higher expected utility compared to complete information model. Note, though, σ^2 cannot be arbitrary large as we have assumed that for all realizations of A_1 and A_2, the countries will have strictly positive imports. Therefore, restrictions on the support of A_i also imply certain restrictions on the variance of A_i.

Now we consider the first stage of the game. Let

$$K \equiv \frac{(B+b)^3}{8B(2B+b)^2(3B+b)}.$$

Then, it is true that

$$\begin{aligned}\bar{W}_{RR} - \bar{W}_{WR} &= W_R(2) - W_W(1) - K\sigma^2, \\ \bar{W}_{RW} - \bar{W}_{WW} &= W_R(1) - W_W(0) - K\sigma^2.\end{aligned}$$

Based on the above equations, we can make several conclusions. First, the delegation stage still does not have asymmetric (pure strategy) Nash equilibria. For the asymmetric equilibrium to exist, the following inequalities must hold:

$$\begin{aligned}W_R(2) &\leq W_W(1) + K\sigma^2, \\ W_R(1) &\geq W_W(0) + K\sigma^2,\end{aligned}$$

which in turn imply the following

$$\Omega(1) = \frac{W_R(2)}{W_W(1)} \leq 1 + \frac{K}{W_W(1)} \leq 1 + \frac{K}{W_W(0)} \leq \frac{W_R(1)}{W_W(0)} = \Omega(0).$$

The middle inequality follows from the fact that $W_W(m)$ is increasing in m. However, we have obtained a contradiction since $\Omega(m)$ is strictly increasing in m. Second, from the previous argument either $W_R(2) \geq W_W(1) + K\sigma^2$, or $W_R(1) \leq W_W(0) + K\sigma^2$, or both hold. Therefore, the symmetric Nash equilibrium exists and if it is unique, it is a dominant strategy equilibrium. Third, it also follows that, compared with the complete information version of the game, the Nash equilibrium, in which both countries maximize revenue, is 'less likely', while the Nash equilibrium, in which both countries maximize welfare, is 'more likely' under incomplete information. It is formalized in the following proposition.

Proposition 9 *Let S_{RR} and S_{WW} denote the set of values (B, b), for which the Nash equilibria $m^* = 2$ and $m^* = 0$, respectively, hold in the complete information version of the game when $A = \bar{A}$. Let $\bar{S}_{RR}$ and $\bar{S}_{WW}$ denote the corresponding sets in the incomplete information version of the game. Then, $\bar{S}_{RR} \subset S_{RR}$ and $S_{WW} \subset \bar{S}_{WW}$.*

On the other hand, the governments are now less likely to be in a Prisoners' dilemma situation. From

$$\bar{W}_{RR} - \bar{W}_{WW} = W_R(2) - W_W(0) - \frac{(10B^4 + 20B^3b + 15B^2b^2 + 6Bb^3 + b^4)}{8B(6B^2 + 5Bb + b^2)^2}\sigma^2$$

it follows that even if $W_R(2) > W_W(0)$ holds, it still can be that $\bar{W}_{RR} \leq \bar{W}_{WW}$.

We also briefly consider another version of the model. Suppose that A_1 and A_2 are drawn between the first and second stages, and both policy makers observe the realizations of both demand parameters before deciding on tariffs. The governments still do not know the values of A_1 and A_2 when making their decisions in the first stage. Note that this is a game of complete information. One can verify that the solutions to this model are the same as those in the original model with $Ab + Ba$ replaced by $B(a + A_i - A_j) + bA_i$ when $n = 2$. In particular, all expressions for expected welfare would contain the same term $E\left[(B(a + A_i - A_j) + bA_i)^2\right]$, which would cancel out when calculating $\Omega(m)$. But this implies that the uncertainty about A_1 and A_2 does not affect the Nash equilibria of the delegation stage of the game. Thus, we conclude that the results of the incomplete information game are driven not merely by the governments' uncertainty about the demand parameters, but rather they depend on the presence of private information.

We conjecture that all these conclusions also extend to $n > 2$.

5 Conclusions

In a trade policy game with many importers in a perfectly competitive model, it has been shown all importing countries delegating to revenue-maximizing policy makers may increase the welfare of all importing countries. However, it was shown that all importing countries delegating to revenue-maximizing policy makers is not a Nash equilibrium if there are more than four importing countries.

In a trade policy game with two importers setting import tariffs and two exporters setting export taxes, it was shown that the importers (exporters) would delegate to a revenue-maximizing policy-maker if they have sufficient market power. This magnifies the gains for the countries with sufficient market power to win the trade war and magnifies the losses for the countries that lose the trade war.

In a trade policy game with incomplete information, it was shown that incomplete information makes it less likely that countries will delegate to revenue-maximizing policy-makers.

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