Mortgage Amortization and Amplification

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Abstract

Mortgages characterized by negative or low early amortization schedules amplify the macroeconomic effects of a housing risk shock. We analyze the role of amortization in a two-sector DSGE model with housing risk and endogenous default. Mortgage loan contracts extend to two periods and have adjustable rates. The fraction of principal to be repaid in the first period can vary. As the fraction of principal to be paid in the first period falls, steady-state mortgages and leverage increase and the impact of a housing risk shock on consumption and output is amplified. Borrowers prefer negative amortization. If free to choose the amortization schedule for their mortgages, borrowers would repay most of the principal in the last period of the contract. Low early repayments of principal give borrowers the flexibility to default in the second period having incurred small sunk costs.

Keywords: Housing; Mortgage default; Mortgage risk

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1 Introduction

The recent financial crisis and the ensuing Great Recession have their roots in the bursting of the housing bubble in the United States. Academic and policy discussions have pointed to a number of possible contributions to the housing bubble, among which changes in methods of housing finance. Bernanke (2010) points specifically to several changes in the mortgage market. First, a significant increase in the percentage of new mortgage applications for adjustable-rate mortgage (ARM) products. Second, the appearance of exotic mortgage products such as interest-only ARMs, long-amortization ARMs, negative amortization ARMs, and pay-option ARMs. These nonstandard mortgage products share a feature: the reduction in the initial monthly payment relative to standard fixed-rate mortgage contracts. Bernanke (2010) shows that initial monthly payments for these alternative mortgage instruments could be as low as 14% of a comparable fixed-rate mortgage payment for a negative amortization ARM and even lower for a pay-option ARM. Moreover, the percentage of variable-rate mortgages originated with various exotic features and extended to non-prime borrowers increased rapidly from 2000 to 2006. Exotic mortgage products including interest-only mortgages, pay-option ARMs and 40-year balloon mortgages increased from 7% in 2005Q1 to 32% of total originations in 2006Q2.[1] Using micro-level data Miam and Sufi (2009) document that subprime areas experienced rapid growth in mortgage credit from 2002 to 2005 despite a decline in relative or even absolute income growth. At the same time, the expansion in mortgage credit to subprime areas is closely correlated with the increase in securitization of subprime mortgages. Overall these findings suggest that, by allowing low initial monthly payments, nonstandard mortgage contracts expanded the supply of mortgages and possibly played an important role in the building of the housing bubble.

If nontraditional mortgage contracts are a likely key explanation of the housing bubble, they may also have contributed to the depth of the recession that followed the bursting of the bubble. The goal of our paper is to analyze whether and how these nonstandard mortgage contract features affect the transmission of a housing risk shock. We consider ARMs and focus on the amortization schedule. In our model loan contracts extend to two periods and they specify the fraction of principal that must be repaid in the first period of the contract. By varying

[1] Source: Alternative Mortgage Originations from Inside Mortgage Finance Publications.

this fraction from zero to one we encompass many amortization schedules. When the fraction of principal to be repaid in the first period is close to one, we have a high-early amortization schedules according to which most of the mortgage must be repaid early in the contract. As the fraction of principal to be repaid in the first period falls to zero, we have low early or even negative amortization – the latter occurring when the first-period payment does not even cover interest costs on the entire mortgage.

To analyze the role of mortgage amortization for the transmission of shocks, we build a two-sector DSGE model with housing. There are two households that differ in terms of their discount factor. Savers have a higher discount factor and lend to Borrowers, who have a lower discount factor. Household preferences are defined over non-durable consumption, housing services and hours worked. Borrowers pledge their homes as collateral for mortgages. We assume that loan contracts are nonrecourse in our model, as it is the case in a number of U.S. states. This means that lender's recovery in case of default is strictly limited to the collateral. Every period Borrowers experience an idiosyncratic housing shock that is private information. Borrowers that experience low realizations of the idiosyncratic default on their debts; non-defaulting Borrowers pay an adjustable rate on their loans. Lenders pay a monitoring cost and seize the houses of defaulting Borrowers. The spread between the adjustable mortgage rate and the risk-free rate is the external finance premium paid by Borrowers.

Our mortgage contract extends to two periods and it specifies the fraction of the principal that must be repaid in the first period of loans. If default occurs in the repayment of the first installment, the entire mortgage contract is defaulted on. As a result, there is no second period repayment and the Borrower loses his house. The first part of our analysis takes the amortization schedule as exogenously given. As the fraction of principal to be repaid in the first period falls, mortgages and leverage by Borrowers increase. Intuitively, the possibility to default in the second period while making a small first-period repayment gives Borrower the incentive to become more leveraged.

We assume that housing risk is time-variant and we analyze the dynamic response to a housing risk shock, namely an unanticipated increase in the standard deviation of the idiosyncratic housing shock. This increase in the dispersion raises the default rates, the external finance premia and generates a credit crunch. Borrowers experience a significant worsening of their financial situation, which forces them to de-leverage and cut both non-durable consumption and housing investment. Low early and negative amortization intensify the negative effects a housing risk shock on aggregate consumption and output. Both steady-state and dynamic effects contribute to the amplification. Low early repayments come hand-in-hand with high leverage ratios. When Borrowers need to de-leverage, they cut consumption and housing investment more aggressively, thereby depressing aggregate demand more. At the same time, Borrower's capacity to borrow is reduced as the LTV ratio is strongly pro-cyclical with low early repayments.

The last part of our paper analyzes the case where Borrowers can choose the fraction of principal to be repaid in the first period. In our framework, impatient agents are borrowing constrained so they have well-defined preferences over the repayment schedule. If free to choose, Borrowers would repay 98% of the principal in the second period. In our model this corresponds to a negative amortization schedule. Borrowers prefer to postpone the bulk of principal repayment to the end of the contract so as to retain the possibility to default without having already made large payments and therefore without incurring large sunk costs.

2 Related Literature

A growing literature embeds durable goods in an otherwise standard New Keynesian model. Barsky, House and Kimball (2007) show that price stickiness of durable goods plays a key role in the transmission mechanism of monetary policy. More precisely, if prices of durable goods are sticky, the model behaves as if most prices are sticky. On the other hand, if prices of durable goods are flexible, the model behaves as if most prices are flexible. When durable prices are flexible, the durable goods sector contracts in response to a monetary expansion, thereby offsetting the expansion in the non-durable goods sector and leaving GDP unchanged. Erceg and Levin (2006) use VAR evidence to document positive sectoral co-movement as well as higher sensitivity of the durable good sector (relative to the non-durable one) to the nominal interest rate in response to a monetary shock. To match this evidence with the model impulse responses, Erceg and Levin (2006) assume wage stickiness and the same degree of price stickiness in the durable and non-durable sector. Carlstrom and Fuerst (2006) underline the existence of a co-movement puzzle following a monetary policy shock since negative sectoral co-movement and price stickiness in the durable sector are both counterfactual. They suggest to add adjustment costs à la Topel and Rosen (1988) in the durable sector. Monacelli (2009) and Sterk (2010) analyze whether introducing credit market frictions can help to solve the co-movement puzzle. In Monacelli (2009) durable goods are used as collateral and sectoral outputs co-move in response to a monetary shock provided the durable sector displays some degree of price stickiness. Sterk (2010), on the other hand, finds that credit market frictions as in Monacelli (2009) makes it more difficult rather than easier to generate sectoral co-movements following a monetary policy tightening. Even though we do not focus on monetary policy shocks, our two-sector model with housing and non-durable goods incorporates the necessary features suggested by this literature to generate co-movement.

Another strand of literature incorporates financial frictions à la Kiyotaki and Moore (1997) into a model with housing, sticky prices, and two households with different discount factors. To ensure the existence of an equilibrium, Iacoviello (2005) assumes an exogenous borrowing constraint according to which impatient agents can borrow a fraction of the expected discounted future value of their houses. Iacoviello and Neri (2010) build and estimate a DSGE model with housing. In Iacoviello (2005) and Iacoviello and Neri (2010) loans are always fully repaid and there is no default on mortgages. Forlati and Lambertini (2011) follow Iacoviello (2005) and Iacoviello and Neri (2010) and build a model with two household groups and housing as a durable good used as collateral but allow for idiosyncratic risk in housing investment. Their household problem is akin to that of entrepreneurs in Carlstrom and Fuerst (1997) and Bernanke, Gertler and Gilchrist (1999) and endogenous default on mortgages arises in equilibrium. Forlati and Lambertini (2011) show that an unanticipated increase in idiosyncratic housing risk generates a recession. Aoki, Proudman and Vlieghe (2004) also introduce a financial accelerator á la Bernanke and Gertler in a model with housing. In their model, unlike Forlati and Lambertini (2011), risk-neutral home owners buy houses and rent them to consumers and their focus on the transmission of a monetary policy shock. Our model draws from Forlati and Lambertini (2011) in featuring idiosyncratic housing investment risk and endogenous default on mortgages and extends to allow for sticky wages and adjustment costs in the housing sector, as suggested by Carlstrom and Fuerst (2006), to capture sectorial co-movement consistent with the empirical evidence.

The novel feature of our framework is that we allow for different amortization schedules and analyze the role of amortization in the transmission of shocks and amplification. More precisely, we extend the mortgage contract in Forlati and Lambertini (2011) to two periods and encompass different amortization schedules. A related paper is Calza, Monacelli and Stracca (2011), which studies how the transmission mechanism of monetary policy is affected by the structure of housing finance. They present VAR evidence that monetary shocks are amplified in countries characterized by more flexible or developed mortgage markets. To rationalize these findings they build a DSGE model with durable and non-durable goods and an exogenous borrowing constraint along the lines of Iacoviello (2005). They allow for one- and two-period contracts, where the latter features a fixed interest rate and equal repayments in the first and second period, and show that consumption falls more in the model with one-period contracts following a monetary policy shock. Our framework differs from Calza et al. (2011) in a number of ways, as it features endogenous default and adjustable mortgage rates. More importantly, we allow for and focus on the role of different amortization schedules.

Some recent papers examine the effects of risky shocks. For example, Christiano, Motto and Rostagno (2009) augment a standard monetary DSGE model to include financial markets and a financial accelerator and fit the model to European and U.S. data. They analyze an increase in the standard deviation of idiosyncratic risk in loans to entrepreneurs. In our setting, idiosyncratic risk is in mortgage loans. Iacoviello (2010) introduces the banking sector in a model with housing and studies an exogenous shock to how much borrowers repay. This repayment shock is exogenous and different from default because borrowers do not lose their houses following a negative repayment shock.

3 The Model

Our starting point is a model with patient and impatient households that consume non-durable goods and housing services and work, whose features have been analyzed in Iacoviello (2005), Iacoviello and Neri (2010) and Monacelli (2009). Unlike these works, we allow for idiosyncratic risk and endogenous default on mortgages, which generates an endogenous participation constraint. Hence, our model is close to the one in Forlati and Lambertini (2011). Here we allow for two-period Adjustable Rate Mortgages (ARMs) with different amortization schedules. More precisely, the mortgage contract specifies the loan amount and the fraction x (plus interests) to be repaid at the end of the first period. The remaining fraction $1-x$ of the loan (plus interests) is repaid at the end of the second period. By varying the parameter x we can encompass different amortization scenarios. $x = 0$ represents the case where no repayment is done in the first period and the entire loan plus interests is repaid at the end of the second period. This schedule, as well as all schedules with x below the interest rate, implies negative amortization. $x = 0.5$ implies a linear amortization schedule; $x = 1$ is the extreme case of declining balance amortization where the entire repayment is done at the end of the first period. The first part of our analysis will take x as given; the last part of the analysis allows Borrowers to choose x optimally.

3.1 Households

The economy is populated by a continuum of households distributed over the $[0, 1]$ interval. A fraction ψ of identical households has discount factor β while the remaining fraction $1 - \psi$ has discount factor $\gamma > \beta$. We are going to refer to the households with the lower discount factor as Borrowers, as these households value current consumption relatively more than the other agents and therefore want to borrow. We are going to refer to households with the higher discount factor as Savers.

Borrowers

Borrowers have a lifetime utility function given by

$$\sum_{t=0}^{\infty} \beta^t E_0 \left\{ U \left(X_t, N_{C,t}, N_{H,t} \right) \right\}, \quad 0 < \beta < 1, \tag{1}$$

where $N_{C,t}$ is hours worked in the non-durable sector, $N_{H,t}$ is hours worked in the housing sector, and X_t is an index of non-durable and durable consumption services defined as

$$X_t \equiv \left[(1-\alpha)^{\frac{1}{\eta}} C_t^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} H_{t+1}^{T\,\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \tag{2}$$

where C_t denotes consumption of non-durable goods, H_{t+1}^T denotes consumption of housing services, α is the share of housing in the consumption index and $\eta \geq 0$ is the elasticity of substitution between housing and non-durable services. The Borrower household housing stock at the end of period t is the sum of the housing stocks (net of depreciation) purchased in the last two periods and connected to the outstanding mortgage contracts:

$$H_{t+1}^{T} \equiv \frac{1}{2}\left\{H_{t+1} + H_t(1-\delta)[1 - G_t(\bar{\omega}_{1,t})]\right\}, \tag{3}$$

where T stands for total, H_{t+1} is the housing stock purchased in period t, H_t is the housing stock purchased in period $t-1$ minus the fraction lost because of default in period t, $G_t(\bar{\omega}_{1,t})$, and net of depreciation δ. We will derive explicitly the term $G_t(\bar{\omega}_{1,t})$ later. We assume that housing services are equal to the housing stock. Assuming that services are a fraction of the stock does not affect the results qualitatively.

We assume the following period utility function:

$$U(X_t, N_t) \equiv \ln X_t - \frac{\nu}{1+\varphi} N_t^{1+\varphi} = \ln X_t - \frac{\nu}{1+\varphi}\left[N_{C,t}^{1+\xi} + N_{H,t}^{1+\xi}\right]^{\frac{1+\varphi}{1+\xi}}, \qquad \varphi, \xi \geq 0. \tag{4}$$

Our specification for the disutility of labor follows Iacoviello and Neri (2010) in allowing that hours in the non-durable and housing sector are imperfect substitutes, as consistent with the evidence found by Horvath (2000). For $\xi = 0$ hours in the non-durable and housing sector are perfect substitutes. On the other hand, positive values of ξ result in wages not being equalized in the two sectors and the substitution of hours across sectors in response to wage differentials being reduced. The parameter φ is the inverse of the Frisch labor supply elasticity.

The Borrower household consists of many members that are divided into two ex-ante identical groups. These two groups alternate in being assigned the resources to purchase new houses and finalize current mortgage contracts. More precisely, the household decides total housing investment H_{t+1} and the state-contingent mortgage rates to be paid on the mortgages. The household then assigns equal resources to each i-th member of group t to purchase the housing stock H_{t+1}^{i}, where $\int_i H_{t+1}^{i} di = H_{t+1}$. The $i-$th member finalizes the mortgage contract connected to the housing stock H_{t+1}^{i} following the instructions of the household and manages his housing stock. The mortgage contract lasts two periods during which the member cannot purchase new houses or finalize new or additional contracts. This contract specifies the total loan amount L_{t+1} and the fraction to be repaid at the end of the first period x. This mortgage therefore generates two loans:

$$L_{1,t+1} \equiv xL_{t+1}, \quad L_{2,t+1} \equiv (1-x)L_{t+1}. \tag{5}$$

$L_{1,t+1}$ is repaid at the beginning of period $t+1$ while $L_{2,t+1}$ is repaid at the beginning of period $t+2$. Group t will re-enter the mortgage and housing market in period $t+2$, once the current mortgage contract has run its course, and we will refer to them as generation $t+2$. Group $t+1$ purchases houses and finalizes mortgage contracts in period $t+1$, and so on and so forth. Below we describe in detail the decisions of group t.

Housing investment is risky. After the mortgage contract is finalized, each member experiences each period an idiosyncratic shock that affects his housing value. In period $t+1$ the group-t, i-th household member experiences an idiosyncratic shock $\omega^i_{1,t+1}$ such that the housing value in period $t+1$ is $\omega^i_{1,t+1}P_{H,t+1}(1-\delta)H^i_{t+1}$, where δ is the rate of depreciation of houses; provided default has not taken place in period $t+1$, in period $t+2$ he experiences an idiosyncratic shock $\omega^i_{2,t+2}$ such that the housing value in period $t+2$ is $\omega^i_{2,t+2}P_{H,t+2}(1-\delta)^2H^i_{t+1}$. Notice that, for simplicity, we have assumed that, at the end of the period, the housing stock and mortgages left after default are equally redistributed among all members. We label the shock ω_j because it affects the housing value $j=1$ or 2 periods after the mortgage has been finalized. This idiosyncratic risk captures the fact that housing prices display geographical variation even in the absence of aggregate shocks. Alternatively, one can think of idiosyncratic effects to the housing stock such as damages, (un-modeled) home improvements, etc. The random variables $\omega^i_{j,t+1}$ are i.i.d. across members of the same group and log-normally distributed with a cumulative distribution function $F_{t+1}(\omega^i_{t+1})$, which obeys standard regularity conditions and we assume to be same for ω_1 and ω_2.[2] The mean and variance of $\ln \omega^i_{j,t+1}$ are chosen so that $E_t(\omega^i_{j,t+1}) = 1$ at all times for both $j=1,2$. This implies that while there is idiosyncratic risk at the household-member level, there is no risk at either the group or household level and $E_t(\omega^i_{j,t+1}H^i_{t+1}) = H_{t+1}$. We are going to assume that housing investment riskiness can change over time, namely that the standard deviation $\sigma_{\omega,t}$ of $\ln \omega^i_{j,t}$ is subject to an exogenous shock

[2] The c.d.f. is continuous, at least once-differentiable, and it satisfies

$$\frac{\partial \omega h(\omega)}{\partial \omega} > 0,$$

where $h(\omega)$ is the hazard rate.

and displays time variation. The random variable $\omega^i_{j,t+1}$ is observed by the $i-$th member and the household but can only be observed by lenders after paying a monitoring cost.

After idiosyncratic shocks are realized, the household member decides whether to repay the first installment of his mortgage or to default. Intuitively, loans connected to housing stocks that experienced high realizations of the idiosyncratic shock are repaid while loans connected to housing stocks with low realizations are defaulted on. It is easier to start from the repayment decision in the last period of the contract. Let $\bar{\omega}_{2,t+2}$ be the threshold value of the idiosyncratic shock for which the member of group t is willing to repay the second installment of his loan. The incentive compatibility constraint in period $t+2$ is

$$\bar{\omega}_{2,t+2}(1-\delta)^2 P_{H,t+2} H_{t+1}[1-G_{t+1}(\bar{\omega}_{1,t+1})] = (1+R_{Z2,t+2})L_{2,t+1}[1-F_{t+1}(\bar{\omega}_{1,t+1})]. \tag{6}$$

The right-hand side of (6) is the payment that must be made in period $t+2$. $R_{Z2,t+2}$ is the two-period state-contingent adjustable rate that non-defaulting Borrowers pay on the two-period loans that have survived default in period $t+1$, $L_{2,t+1}[1-F_{t+1}(\bar{\omega}_{1,t+1})]$. The left-hand side of (6) is the housing value of the marginal member, namely the member who experiences $\bar{\omega}_{2,t+2}$ and he is indifferent between repaying and defaulting. This housing value is the housing stock purchased in period t net of depreciation and net of the housing stock lost to default in period $t+1$, $G(\bar{\omega}_{1,t+1})$. We explicitly derive and explain this term later. Loans connected to $\omega^i_{2,t+2} \in [\bar{\omega}_{2,t+2}, \infty]$ are repaid. On the other hand, loans connected to $\omega^i_{2,t+2} \in [0, \bar{\omega}_{2,t+2})$ are underwater mortgages, namely mortgages for which the value of the house is lower than the loan associated to it. These members have negative equity in their houses and, as a result, they default on these loans. Lenders pay a monitoring cost to assess and seize the collateral connected to the defaulted loan. It is the presence of monitoring that induces Borrowers to truthfully reveal their idiosyncratic shock and justifies the incentive compatibility constraint (6).[3] The household members that default on their mortgages lose their housing stocks.

Consider now the first repayment decision of group t. The mortgage contract requires that the fraction x of $L_{t+1}, L_{1,t+1}$ is repayed in period $t+1$. The incentive compatibility constraint

[3]See the seminal work of Townsend (1979).

in period $t+1$ is

$$\bar{\omega}_{1,t+1}(1-\delta)P_{H,t+1}H_{t+1} = (1+R_{Z1,t+1})L_{1,t+1}+E_{t+1}\left\{Q_{t+1,t+2}[1-F_{t+2}(\bar{\omega}_{2,t+2})](1+R_{Z2,t+2})L_{2,t+1}\right\}. \tag{7}$$

On the right-hand side we find the present expected value of current and future mortgage payments for the members of group t. These include the current repayment of the one-period loan $L_{1,t+1}$ at the state-contingent, adjustable rate $R_{Z1,t+1}$ and the present value (using the Borrower's discount factor $Q_{t+1,t+2}$, which will be derived later in Appendix A) of the second repayment, taking into account the probability of defaulting in period $t+2$. The left-hand side of (7) is the value of the house for the member that experiences the idiosyncratic shock $\bar{\omega}_{1,t+1}$. This is the marginal member who is indifferent between repaying the first installment of his mortgage and defaulting. All mortgages connected to $\omega^i_{1,t+1} \in [0, \bar{\omega}_{1,t+1})$ are underwater and defaulted on. Loans connected to $\omega^i_{1,t+1} \in [\bar{\omega}_{1,t+1}, \infty)$ are repaid. Defaulting household members lose their housing stocks. If a household member defaults on $L_{1,t+1}$, the entire mortgage is terminated; the housing stock is seized by lenders and the remaining part of the mortgage is revoked.

So far we have described the repayment decision process for the Borrower household members of group t. The same decision process holds for the Borrower household members of group $t+1$, who purchase houses H_{t+2} and finalize total mortgages L_{t+2}, broken down in one-period mortgages $L_{1,t+2}$ and two-period mortgages $L_{2,t+2}$. Two types of loans come up for repayment in every period, each type belonging to a different group. More precisely, in period $t+1$ loans $L_{1,t+1}$, which belong to group t, and loans $L_{2,t}$, which belong to group $t-1$, come up for repayment and can be defaulted on.

A few comments on our assumptions are in order at this point. Mortgages are nonrecourse in our model. This means that mortgages are secured by the pledge of collateral (the house) and the lender's recovery is strictly limited to the collateral. Defaulting Borrowers are not personally liable for the difference between the loan and the collateral value. This is a natural assumption in our model because housing is the only asset held by Borrowers. In addition to this, nonrecourse debt is broadly applicable to most U.S. states, especially those that experienced soaring mortgage delinquencies, and the focus of our paper is on the United States.

In Bernanke et al. (1999) the monitoring cost is equal to a fraction of the realized gross payoff to the defaulting firm's capital. We follow Bernanke et al. (1999) and assume that the monitoring cost in our model is equal to the fraction μ of the housing value. This assumption has two important implications. The first implication is that the foreclosure cost is proportional to the value of the house under foreclosure. The second implication is that mortgage default causes a decline in the housing stock and services, which are destroyed due to monitoring. This second implication generates a (rather unrealistic) rebound in housing demand. For this reason our analysis will show housing demand gross and net of monitoring costs.

Regarding the defaulting household members, we follow the literature on matching and assume there is perfect insurance among household members so that consumption of non-durable goods and housing services are ex-post equal across all members of the Borrower household. Hence, Borrower household members are ex-post identical.

We can now put the two groups together and formulate the budget constraint at period t for the Borrower household:

$$P_{C,t}C_t + P_{H,t}H_{t+1} + [1 - F_t(\bar{\omega}_{1,t})](1 + R_{Z1,t})L_{1,t} + [1 - F_{t-1}(\bar{\omega}_{1,t-1})][1 - F_t(\bar{\omega}_{2,t})](1 + R_{Z2,t})L_{2,t-1} =$$

$$L_{1,t+1} + L_{2,t+1} + W_{C,t}N_{C,t} + W_{H,t}N_{H,t} + (1-\delta)^2 \left[1 - G_{t-1}(\bar{\omega}_{1,t-1})\right] \left[1 - G_t(\bar{\omega}_{2,t})\right] P_{H,t}H_{t-1}, \quad (8)$$

where $P_{C,t}$ is the price of non-durable goods, $P_{H,t}$ is the price of housing and H_{t+1} is the housing stock purchased at t. $L_{1,t+1}$ and $L_{2,t+1}$ are the loans finalized by generation t Borrowers to be repaid in period $t+1$ and $t+2$, respectively. On the left-hand side of the budget constraint we find the use of resources, which includes the purchase of consumption goods and housing and the repayments of loans. $1 - F_t(\bar{\omega}_{1,t})$ is the fraction of one-period loans $L_{1,t}$ taken in period $t-1$ that is repaid to lenders and $R_{Z1,t}$ is the state-contingent interest rate paid on such loans by non-defaulting Borrowers. Similarly, $[1 - F_{t-1}(\bar{\omega}_{1,t-1})][1 - F_t(\bar{\omega}_{2,t})]$ is the fraction of two-period loans $L_{2,t-1}$ taken in period $t-2$ that is repaid to lenders; these loans have survived default both in $t-1$ and t. $W_{C,t}$ is the nominal wage in the consumption good sector and $W_{H,t}$ is the nominal wage in the housing sector. Borrower's revenues include the new loans $L_{1,t+1}, L_{2,t+1}$ finalized by the group-t members as well as the final housing stock of generation $t-2$ members, who re-enter the housing and mortgage markets as generation t. This final housing stock is equal to the original purchase H_{t-1} net of depreciation and the fraction $G_{t-1}(\bar{\omega}_{1,t-1}), G_t(\bar{\omega}_{2,t})$ lost to default in period t and $t-1$, respectively. We explicitly derive these terms later. The housing stock of group-$t-1$ members does not appear in the period t budget constraint because this generation does not purchase houses in period t.

We consider one- and two-period mortgage contracts that guarantee lenders a pre-determined rate of return on their total loans. As in Bernanke et al. (1999), the idea is that Savers have access to alternative assets that pay a risk-free rate return, which pin down the return on mortgages. Savers make one-period loans $L_{1,t+1}$ and two-period loans $L_{2,t+1}$ to Borrowers and demand the gross rates of return $1 + R_{L1,t}$ and $1 + R_{L2,t}$, respectively. These rates of return are pre-determined at t and non-state contingent. Hence, the time t participation constraint of lenders for one-period loans is given by:

$$(1 + R_{L1,t})L_{1,t+1} = \int_{0}^{\bar{\omega}_{1,t+1}} \omega_{1,t+1}(1-\mu)(1-\delta)P_{H,t+1}H_{t+1}f_{t+1}(\omega_1)d\omega_1 \qquad (9)$$

$$+ \int_{\bar{\omega}_{1,t+1}}^{\infty} (1 + R_{Z1,t+1})L_{1,t+1}f_{t+1}(\omega_1)d\omega_1,$$

where $f_t(\omega_1)$ is the probability density function of ω_1, which is time variant because it is subject to an exogenous shock to its standard deviation. The return on one-period loans is equal to the housing stock net of monitoring costs and depreciation of defaulting Borrower members (the first term on the right-hand side of (9)) and the repayment by non-defaulting members (the second term on the right-hand side of (9)). After idiosyncratic and aggregate shocks have realized, the threshold value $\bar{\omega}_{1,t+1}$ and the state-contingent mortgage rate $R_{Z1,t+1}$ are determined so as to satisfy the participation constraint above. Hence, the mortgage contract is characterized by adjustable interest rates. The participation constraint holds state-by-state and not in expected terms. An aggregate state that raises $\bar{\omega}_{1,t+1}$ and thereby default generates an increase in the adjustable rate $R_{Z1,t+1}$ paid by non-defaulting members in order to satisfy the participation constraint (9) in that state. This implies that periods characterized by high mortgage default rates are also accompanied by high mortgage interest rates in our model.

The time t participation constraint of lenders for two-period loans is given by

$$(1 + R_{L2,t})L_{2,t+1} = \int_{0}^{\bar{\omega}_{2,t+2}} \omega_{2,t+2}(1-\mu)(1-\delta)^2[1 - G_{t+1}(\bar{\omega}_{1,t+1})]P_{H,t+2}H_{t+1}f_{t+2}(\omega_2)d\omega_2$$

$$+ \int_{\bar{\omega}_{2,t+2}}^{\infty} (1 + R_{Z2,t+2})[1 - F_{t+1}(\bar{\omega}_{1,t+1})]L_{2,t+1}f_{t+2}(\omega_2)d\omega_2. \qquad (10)$$

As before, the pre-determined rate of return on two-period loans comes from seizing the housing stock of defaulting members and repayment by non-defaulting ones. Both the housing stock of defaulting members and the loans repaid by non-defaulting ones are suitably adjusted for the default that occurred in period $t-1$.

Let

$$G_{t+1}(\bar{\omega}_{j,t+1}) \equiv \int_0^{\bar{\omega}_{j,t+1}} \omega_{j,t+1} f_{t+1}(\omega_j) d\omega_j, \qquad j=1,2 \tag{11}$$

be the expected value of the idiosyncratic shock conditional on the shock being less than or equal to the threshold value $\bar{\omega}_{j,t+1}$, multiplied by the probability of default, and let

$$\Gamma_{t+1}(\bar{\omega}_{j,t+1}) \equiv \bar{\omega}_{j,t+1} \int_{\bar{\omega}_{j,t+1}}^{\infty} f_{t+1}(\omega_j) d\omega_j + G_{t+1}(\bar{\omega}_{j,t+1}), \qquad j=1,2. \tag{12}$$

Using these definitions and the participation constraints (9) and (10) the Borrower budget constraint in real terms can be written as

$$C_t + p_{H,t} H_{t+1} + \frac{l_{1,t}}{\pi_{C,t}}(1+R_{L1,t-1}) + \frac{l_{2,t-1}}{\pi_{C,t}\pi_{C,t-1}}(1+R_{L2,t-2}) = l_{1,t+1} + l_{2,t+1} \tag{13}$$

$$+p_{H,t}H_t(1-\delta)(1-\mu)G_t(\bar{\omega}_{1,t}) + p_{H,t}H_{t-1}(1-\delta)^2[1-G_{t-1}(\bar{\omega}_{1,t-1})][1-\mu G_t(\bar{\omega}_{2,t})] + w_{C,t}N_{C,t} + w_{H,t}N_{H,t},$$

where $p_{H,t}$ is the relative price of houses in terms of non-durable consumption at t, $\pi_{C,t}$ is non-durable-good inflation and $w_{C,t}, w_{H,t}$ are real wages in the C and H sector, respectively, in terms of $P_{C,t}$. $l_{1,t+1} \equiv L_{1,t+1}/P_{C,t}$ are real one-period loans finalized at t, $l_{2,t+1} \equiv L_{2,t+1}/P_{C,t}$ are real two-period loans finalized at t, etc.

Making use of definitions (11), (12) and the incentive compatibility constraints (7) and (6), the participation constraint at t on the two-period loans can be written in real terms as follows

$$(1+R_{L2,t})\frac{l_{2,t+1}}{\pi_{C,t+1}\pi_{C,t+2}} = p_{H,t+2}H_{t+1}(1-\delta)^2[1-G_{t+1}(\bar{\omega}_{1,t+1})][\Gamma_{t+2}(\bar{\omega}_{2,t+2}) - \mu G_{t+2}(\bar{\omega}_{2,t+2})]. \tag{14}$$

The participation constraint on one-period loans in real terms is

$$(1+R_{L1,t})\frac{l_{1,t+1}}{\pi_{C,t+1}} = p_{H,t+1}H_{t+1}(1-\delta)[\Gamma_{t+1}(\bar{\omega}_{1,t+1}) - \mu G_{t+1}(\bar{\omega}_{1,t+1})] \tag{15}$$

$$-H_{t+1}(1-\delta)^2[1-G_{t+1}(\bar{\omega}_{1,t+1})]E_{t+1}\left\{Q_{t+1,t+2}\, p_{H,t+2}\pi_{C,t+2}[\Gamma_{t+2}(\bar{\omega}_{2,t+2}) - \mu G_{t+2}(\bar{\omega}_{2,t+2})]\right\}.$$

We define the loan-to-value (LTV henceforth) ratio as

$$\Gamma_{1,t+1}(\bar{\omega}_{1,t+1}) - \mu G_{t+1}(\bar{\omega}_{1,t+1}). \tag{16}$$

Substituting (14 into (15) one can see that the LTV ratio measures the total mortgage (principal plus interests) as a fraction of the net housing value. Models with exogenous borrowing constraints typically feature a constant LTV ratio. In our model the LTV ratio varies endogenously.

We model imperfectly competitive labor markets that generate a wage-inflation Philips curve as in Schmitt-Grohe and Uribe (2007). Labor decisions are taken by two unions in the household, one for each sector, which monopolistically supplies labor to a continuum of labor markets indexed by $j \in [0,1]$ in each sector. The union that supplies labor to sector C decides the wage to charge in each labor market j in C and it is assumed to satisfy demand, namely

$$N^j_{C,t} = \left(\frac{w^j_{C,t}}{w_{C,t}}\right)^{-\eta_w} N^d_{C,t}, \tag{17}$$

where $w^j_{C,t}$ denotes the real wage charged by the union in labor market j in sector C at time t, $w_{C,t}$ is the index of real wages prevailing in sector C, $N^d_{C,t}$ is the aggregate demand for Borrowers' labor by firms in the C sector, and $N^j_{C,t}$ is the supply of labor in market j of sector C. This demand is formally derived later in the section describing firms. The union takes the aggregate demand $N^d_{C,t}$ and the wage index $w_{C,t}$ as given when it decides the wage to charge in labor market j, $w^j_{C,t}$. In addition, the total number of hours supplied in sector C must be equal to the sum of the hours supplied in each market j:

$$N_{C,t} = \int_0^1 N^j_{C,t}\, dj = N^d_{C,t} \int_0^1 \left(\frac{w^j_{C,t}}{w_{C,t}}\right)^{-\eta_w} dj. \tag{18}$$

This constraint is taken into account by the household in its maximization problem. The Borrowers' union in sector H solves a similar problem.

We introduce wage stickiness in the model by assuming that each union can optimally set wages only in a fraction $\varrho_i \in (0,1), i = C, H$, of randomly chosen labor markets. In these labor markets, the union can freely set $w^j_{C,t}$; we assume no wage indexation so that, in the other labor markets, the wage remains equal to that of the last period. For simplicity, we assume the same degree of wage stickiness in the two sectors so that $\varrho_C = \varrho_H = \varrho$.

For the first part of the analysis we consider the case where the fraction of one-period loans out of total loans, x, is a parameter exogenously given. This can be interpreted as an institutional constraint embedded in mortgage contracts. This implies that the Borrower can choose total real loans l_{t+1} but not the composition. In other words, the Borrower cannot choose the amortization structure of his mortgage. Borrowers maximize (1) subject to the budget constraint (13), the participation constraints (14) and (15), the loan structure (5), and the labor market constraint (18) for sector C and its counterpart for sector H with respect to the variables $C_t, H_{t+1}, N_{C,t}, N_{H,t}, l_{t+1}, \bar{\omega}_{1,t+1}, \bar{\omega}_{2,t+2}, w^j_{C,t}, w^j_{H,t}$. The respective first-order conditions are spelled out in Appendix A.

Savers

We denote Savers' variables with a ˜. Savers maximize lifetime utility

$$\max \sum_{t=0}^{\infty} \gamma^t E_0 \left\{ U(\widetilde{X}_t, \widetilde{N}_{C,t}, \widetilde{N}_{H,t}) \right\}, \quad 0 < \beta < \gamma < 1, \tag{19}$$

where $\widetilde{X}_t$ is defined similarly to (2). We assume that α, the stochastic weight of housing in the consumption index, and the utility function of Savers are identical to those of Borrowers. Savers maximize lifetime utility subject to the sequence of budget constraints:

$$\begin{aligned} \widetilde{C}_t + p_{H,t}\widetilde{H}_{t+1} + p_{A_l,t}A_{l,t+1} + \widetilde{l}_{t+1} = (1-\delta)p_{H,t}\widetilde{H}_t + (p_{A_l,t} + r_{A_l,t})\, A_{l,t} + (1 + R_{L,t-1})\frac{\widetilde{l}_t}{\pi_{C,t}} \\ + \widetilde{w}_{C,t}\widetilde{N}_{C,t} + \widetilde{w}_{H,t}\widetilde{N}_{H,t} + \widetilde{\Delta}_t \end{aligned} \tag{20}$$

where $A_{l,t+1}$ is the stock of land owned by Savers, $p_{A_{l,t}}$ denotes the real land price and $r_{A_{l,t}} \equiv \frac{R_{A_l,t}}{P_{C,t}}$, where $R_{A_l,t}$ is the rental price at which land is rented to the intermediated good producers of the housing sector. Moreover $\widetilde{\Delta}_t$ denote profits in the intermediate goods sector, which are taken as given.

As for Borrowers, Savers' labor decisions are taken by two unions, one for each sector, which monopolistically supply labor to a continuum of labor markets. Each union can optimally choose wages in the fraction $\widetilde{\varrho}$ of randomly chosen labor markets; in the other markets wages remain unchanged. For simplicity, we assume that the degree of wage stickiness in the two sectors are equal $\widetilde{\varrho}_C = \widetilde{\varrho}_H = \widetilde{\varrho}$. The maximization problem faced by Savers' unions is identical to the problem faced by Borrowers' unions and we do not repeat it here.

Savers maximize (19) subject to the budget constraint (20) with respect to $\widetilde{C}_t$, $\widetilde{H}_{t+1}$, $\widetilde{N}_{C,t}$, $\widetilde{N}_{H,t}$, $\widetilde{l}_{1,t+1}$, $\widetilde{l}_{2,t+1}$, $A_{l,t+1}$, $\widetilde{w}^j_{C,t}$, $\widetilde{w}^j_{H,t}$. The first-order conditions are summarized in Appendix A.

3.2 Firms and Technology

Both the non-durable C and the housing H sector have intermediate and final good producers.

Final Good Producers

Final good producers are perfectly competitive and produce $Y_{j,t}$, $j = C, H$. The technology in the $j-$th final good sector is given by

$$Y_{j,t} = \left(\int_0^1 Y_{j,t}(i)^{\frac{\varepsilon_j - 1}{\varepsilon_j}} di \right)^{\frac{\varepsilon_j}{\varepsilon_j - 1}}, \tag{21}$$

where $\varepsilon_j > 1$ is the elasticity of substitution among intermediate goods in sector j. Standard profit maximization implies that the demand for intermediate good i is given by

$$Y_{j,t}(i) = \left(\frac{P_{j,t}(i)}{P_{j,t}} \right)^{-\varepsilon_j} Y_{j,t}, \ \forall i \tag{22}$$

where the price index is

$$P_{j,t} = \left(\int_0^1 P_{j,t}(i)^{1-\varepsilon_j} di \right)^{\frac{1}{1-\varepsilon_j}}.$$

Intermediate Good Sectors

There are two intermediate good sectors $j \in \{C, H\}$ and in each intermediate sector there is a continuum of firms, each producing a differentiated good $i \in [0, 1]$. These firms are monopolistically competitive. We assume that intermediate good firms in the non-durable sector readjust their price according to a Calvo-type mechanism. Hence, in any given period, a firm in sector

C may reset its price with probability $1-\theta_C$. Conversely the prices in the housing sector are fully flexible. We also assume firm-level adjustment costs in the housing sector.

Non-Durable Sector

Intermediate good firm i in the C sector produces according to the following production function:

$$Y_{C,t}(i) = A_{C,t}\left[\zeta^{\frac{1}{\varsigma}} N_{C,t}(i)^{\frac{\varsigma-1}{\varsigma}} + (1-\zeta)^{\frac{1}{\varsigma}} \widetilde{N}_{C,t}(i)^{\frac{\varsigma-1}{\varsigma}}\right]^{\frac{\varsigma}{\varsigma-1}}, \quad i \in \{C\}, \tag{23}$$

where $A_{C,t}$ is the stochastic level of technology in sector j and $N_{C,t}(i)$ and $\widetilde{N}_{C,t}(i)$ are the two labor types supplied respectively by Borrowers and Savers. $\zeta \in (0,1)$ is the labor share of Borrowers in the production function and $\varsigma > 0$ is the elasticity of substitution across labor inputs. When ς goes to infinity, labor inputs become perfect substitutes. For simplicity these two parameters are assumed to be equal across sectors.

In period t firm i chooses labor and, if given the possibility, it re-optimizes its nominal price $P^*_{C,t}(i)$ so as to maximize the expected discount sum of nominal profits over the period during which its price remains unchanged. The maximization problem as well as the first-order conditions relative to $N_{C,t+k|t}(i), \widetilde{N}_{C,t+k|t}(i)$ and $P^*_{C,t}(i)$ are reported in Appendix A. In our model marginal costs are a CES index of wages net of productivity; since wages are equal across firms in the sector, marginal costs are also equal across firms.

Housing Sector

Intermediate good firm i technology in the H is described by the following production function:

$$Y_{H,t}(i) = A_{H,t} A_{l,t}^{1-\kappa}(i)\left[\zeta^{\frac{1}{\varsigma}} N_{H,t}(i)^{\frac{\varsigma-1}{\varsigma}} + (1-\zeta)^{\frac{1}{\varsigma}} \widetilde{N}_{H,t}(i)^{\frac{\varsigma-1}{\varsigma}}\right]^{\frac{\varsigma\kappa}{\varsigma-1}}, \quad i \in \{H\} \tag{24}$$

where $A_{H,t}$ is the stochastic level of technology in sector H, $A_{l,t}(i)$ is the stock of land used as input in housing production and $N_{H,t}(i)$ and $\widetilde{N}_{H,t}(i)$ are the two labor types supplied respectively by Borrowers and Savers. As for the C sector, $\zeta \in (0,1)$ is the labor share of Borrowers in the production function and $\varsigma > 0$ is the elasticity of substitution across labor inputs; κ represents the labor share in the housing production.

Given the technology described by (24), firms in the H sector maximize the expected discount value of current and future profits, namely:

$$E_t \sum_{k=0}^{\infty} \Lambda_{t,t+k} \Big\{ P_{H,t}(i) Y_{H,t}(i) - P_{H,t} g\left(Y_{H,t}(i) - Y_{H,t-1}(i)\right) - W_{H,t} N_{H,t}(i) - \widetilde{W}_{H,t} \widetilde{N}_{H,t}(i) \tag{25}$$

$$-R_{A_l,t} A_{l,t}(i) + mc_{H,t}(i) P_{H,t} \left[A_{H,t} A_{l,t}^{1-\kappa}(i) \left[\zeta^{\frac{1}{\varsigma}} N_{H,t}(i)^{\frac{\varsigma-1}{\varsigma}} + (1-\zeta)^{\frac{1}{\varsigma}} \widetilde{N}_{H,t}(i)^{\frac{\varsigma-1}{\varsigma}} \right]^{\frac{\varsigma\kappa}{\varsigma-1}} - Y_{H,t}(i) \right] \Big\},$$

where $g(Y_{H,t}(i) - Y_{H,t-1}(i))$ are firm-level adjustment costs as in Topel and Rosen (1988) such that $g(0) = g'(0) = 0$ and $g''(0) = \chi$. $R_{A_l,t}$ is the nominal rental price of land; the demand and the stochastic discount factor are respectively given by

$$Y_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon_H} Y_{H,t}, \qquad \Lambda_{t,t+k} \equiv \frac{\gamma^t \widetilde{\lambda}_{BC,t}}{\gamma^k \widetilde{\lambda}_{BC,t+k}} \frac{P_{C,t+k}}{P_{C,t}}.$$

We assume that prices are perfectly flexible in the housing sector. In fact, Iacoviello and Neri (2010) estimate a DSGE housing model of the United States and find a degree of price stickiness in the housing sector equal to zero. Each period firm i chooses labor $N_{H,t}(i)$, $\widetilde{N}_{H,t}(i)$, the amount of land to rent $A_{l,t}(i)$, and the price $P^*_{H,t}(i)$. The first-order conditions are reported in Appendix A.

3.3 Monetary Policy

We assume that monetary policy follows a Taylor-type rule for the one-period nominal interest rate:

$$\frac{1+R_{L1,t}}{1+R_{L1}} = A_{M,t} \left[\pi_{C,t}^{\phi_\pi} \right]^{1-\phi_r} \left[\frac{1+R_{L1,t-1}}{1+R_L} \right]^{\phi_r}, \quad \phi_\pi > 1,\ \phi_r < 1, \tag{26}$$

where R_{L1} is the steady-state nominal interest rate, ϕ_π is the coefficient on the inflation target, ϕ_r is the coefficient on the lagged interest rate, and $A_{M,t}$ is a monetary policy shock. In our benchmark calibration monetary policy targets inflation in the non-durable goods sector and implements interest-rate smoothing.

3.4 Market Clearing

Equilibrium in the non-durable goods market requires that production of the final non-durable good equals aggregate demand:

$$Y_{C,t} = \psi C_t + (1-\psi)\widetilde{C}_t. \tag{27}$$

Similarly, equilibrium in the housing market requires

$$Y_{H,t} = \psi \left\{ H_{t+1} - (1-\delta)(1-\mu)G_t(\bar{\omega}_{1,t})H_t - (1-\delta)^2[1-\mu G_t(\bar{\omega}_{2,t})][1-G_{t-1}(\bar{\omega}_{1,t-1})]H_{t-1} \right\}$$

$$+(1-\psi)\left[\widetilde{H}_{t+1} - (1-\delta)\widetilde{H}_t\right] + g(Y_{H,t} - Y_{H,t-1}). \tag{28}$$

Output in the housing sector net of monitoring costs is equal to

$$Y_{H,t}^N = Y_{H,t} - \psi\mu\left[(1-\delta)G_t(\bar{\omega}_{1,t}) + (1-\delta)^2 G_t(\bar{\omega}_{2,t})[1-G_{t-1}(\bar{\omega}_{1,t-1})]\right] H_t. \tag{29}$$

Equilibrium in the labor market requires

$$\int_0^1 N_{j,t}(i)di = \psi N_{j,t} \quad j \in \{C,H\}, \tag{30}$$

$$\int_0^1 \widetilde{N}_{j,t}(i)di = (1-\psi)\widetilde{N}_{j,t} \quad j \in \{C,H\}, \tag{31}$$

while the equilibrium in the credit market requires

$$\psi l_t = (1-\psi)\widetilde{l}_t. \tag{32}$$

Land is in fixed supply

$$A_{l,t} = \bar{A}_l. \tag{33}$$

We define total output as

$$Y_t = Y_{C,t} + p_{H,t}Y_{H,t}. \tag{34}$$

Notice that our measurement of total output reflects variations in the relative price of housing. National account statistics, on the other hand, measure GDP at constant relative prices.

3.5 Exogenous Shocks

There are four exogenous shocks in our model. Aggregate productivity in the two sectors and the monetary policy shock evolve according to the following first-order autoregressive processes

$$\ln A_{C,t} \quad = \rho_C \ln A_{C,t-1} + \epsilon_{C,t}, \quad \rho_C \in (-1, 1), \tag{35}$$

$$\ln A_{H,t} \quad = \rho_H \ln A_{H,t-1} + \epsilon_{H,t}, \quad \rho_H \in (-1, 1), \tag{36}$$

$$\ln A_{M,t} \quad = \rho_M \ln A_{M,t-1} + \epsilon_{M,t}, \quad \rho_M \in (-1, 1), \tag{37}$$

where $\epsilon_C, \epsilon_H, \epsilon_M$ are i.i.d. innovations with mean zero and standard deviation $\sigma_C, \sigma_H, \sigma_M$, respectively, and ρ_C, ρ_H, ρ_M are persistence parameters.

As for the idiosyncratic risk in the housing sector, we follow Bernanke et al. (1999) and assume that ω_t is distributed log-normally:

$$\ln \omega_{j,t} \sim N(-\frac{\sigma^2_{\omega,t}}{2}, \sigma^2_{\omega,t}), \qquad j = 1, 2. \tag{38}$$

In words, we assume that the idiosyncratic shocks in the first and second period of the mortgage are drawn from the same distribution. As stated earlier, the mean of the distribution is chosen so that $E_t(\omega_{j,t+1}) = 1$ for $j = 1, 2$. We are going to analyze the case where the standard deviation of idiosyncratic housing investment risk exogenously increases. To do this, we assume that the standard deviation of $\ln \omega_{j,t}$ is itself an exogenous shock subject to a first-order autoregressive process

$$\ln \frac{\sigma_{\omega,t}}{\sigma_\omega} = \rho_\sigma \ln \frac{\sigma_{\omega,t-1}}{\sigma_\omega} + \epsilon_{\sigma_{\omega,t}}, \tag{39}$$

where $\epsilon_{\sigma_{\omega,t}}$ is an i.i.d. shock with mean zero and finite standard deviation σ_{σ_ω} and ρ_σ is the serial correlation coefficient. This assumption captures the fact that housing investment is risky and this risk can change exogenously over time.

Private agents know these exogenous processes and use them to form correct expectations.

4 Steady-State Analysis

4.1 Benchmark Calibration

The parameters values for our benchmark calibration are specified in Table 1. We follow Monacelli (2009) in choosing the values for the discount factors for Borrowers and Savers, the rate of depreciation for housing and the elasticity of substitution between non-durable goods and housing services. The Savers' discount factor γ is set equal to 0.99 and Borrowers' discount factor β is set equal to 0.98. We choose an annual depreciation rate for housing of 4 percentage points, implying $\delta = 0.01$. The elasticity of substitution between non-durable consumption and housing is $\eta = 1$, which implies a Cobb-Douglas specification for the composite consumption index X_t.

U.S. private fixed investment in structures, residential and nonresidential, has been on average 5 percent of GDP from 1960 to 2009, while during the period 2000 to 2007 it averaged 8 percent of GDP. We set the parameter α that measures the share of housing in the consumption bundle equal to 0.16, so that the housing sector represents 8 percent of total output at the steady state. The Saver discount factor pins down the steady-state interest rate at $R_L = 0.0101$ on a quarterly basis. This implies an annual risk-free interest rate equal of 4.1 percentage points. The inverse of the Frisch elasticity of labor supply φ is set equal to one, as in Barsky et al. (2007) and as typical in the macro literature. As for the parameter ξ that measures the degree of substitutability between hours worked in the two sectors, we set it equal to 0.871. This is the appropriate weighted average of the ξ for Borrowers and Savers estimated by Iacoviello and Neri (2010).

We assume that housing prices are fully flexible. For non-durable goods, θ_C is set equal to 0.67 to imply that firms in the non-durable sector change their prices on average every nine months. ϱ, the Calvo probability for wages in the C and H sectors is set equal to 0.73, which implies that wages are on average changed less often than prices in the non-durable and housing sectors. For monetary policy, we set $\phi_\pi = 1.5$, as standard in the literature. For the benchmark calibration we set $\phi_r = 0.9$ because interest rate inertia mimics the zero lower bound, which was reached in 2009Q1. The serial correlation of the monetary policy shock is $\rho_M = 0$. We assume that the Borrower and Saver groups have equal size so that $\psi = 0.5$.

Parameter	Value	Description
γ	0.99	Discount factor of Savers
β	0.98	Discount factor of Borrowers
ψ	0.5	Relative size of Borrower group
δ	0.01	Rate of depreciation for housing
ε_C	7.5	Elasticity of substitution for C goods
ε_H	7.5	Elasticity of substitution for H goods
ς	3	Elasticity of substitution across labor inputs
ζ	0.5	Share of Borrower labor in the production function
ξ	0.871	Elasticity of substitution across labor types
α	0.16	Share of housing in consumption bundle
ν	2.5	Disutility from work
η	1	Elasticity of substitution between C and H goods
φ	1	Inverse of elasticity of labor supply
θ_C	0.67	Calvo probability in C
ϱ	0.73	Calvo probability wages in C, H
ϕ_π	1.5	Taylor-rule coefficient on inflation
ϕ_r	0.9	Taylor-rule coefficient on past nominal interest rate
ρ_C	0.9	Serial correlation of productivity shocks in C
ρ_H	0.9	Serial correlation of productivity shocks in H
ρ_M	0	Serial correlation of monetary policy shocks
σ_ω	0.1	Standard deviation of idiosyncratic shocks
μ	0.077	Monitoring cost

Table 1: Benchmark Calibration

For technology, we follow Calza et al. (2011) and set the elasticity of substitution among intermediate goods ε_j equal to 7.5 in each sector. Labor inputs are imperfect substitutes in production and the elasticity of substitution across Borrower's and Saver's labor is $\varsigma = 3$. We also assume that the share of Borrower's labor in the production function ζ is equal to 0.5. The serial correlation of the productivity shocks in the non-durable and housing sectors are chosen to be $\rho_C = 0.9$ and $\rho_H = 0.9$, respectively.

Regarding the mortgage market, we need to specify values for the parameters x, σ_ω and μ; at the same time we want to match the pre-crisis delinquency rate and LTV ratio. The U.S. LTV ratio was equal to 77 percentage points on 2006Q4 and its average value between 1973Q1 and 2010Q4 was 76 percentage points.[4] According to the National Delinquency Survey of the Mortgage Banker Association, seriously delinquent mortgages are all mortgages more than 90 days past due or in foreclosure. U.S. seriously delinquent mortgages averaged 2.3 percent of total mortgages between 1979Q1 and 2010Q4 and they represented 2.2 percent of total mortgages in 2006Q4. In our model, the LTV ratio and the delinquency rate are non-linear functions of σ_ω and μ. Higher monitoring costs reduce loans and thereby the LTV ratio and the default rate; higher idiosyncratic volatility lowers mortgage loans and the LTV ratio and raises the default rate. We choose the standard deviation of idiosyncratic housing price shocks σ_ω to be equal to 0.1 at the steady state. Given the chosen value for σ_ω, we set monitoring costs equal to 0.077 to find a steady-state LTV ratio between 75.5 and 78.5 percentage points, depending on the value of the parameter x, which matches the pre-crisis LTV ratio of 77 percentage points. We believe that the shocks to the standard deviation of relative house prices are persistent but there is no previous work we can rely on. Christiano et al. (2009) estimate the persistence of the idiosyncratic productivity shock for the United States to be 0.85. We set $\rho_\sigma = 0.9$.

The leverage ratio for Borrowers at the steady state is calculated as

$$\text{Leverage Ratio } = \frac{l}{l + w_C N_C + w_H N_H},$$

which measures the fraction of total expenses financed by total loans, namely consumption of C and H plus loan repayment over loans. The leverage ratio captures the dependence of Borrowers

[4]Source: Terms on Conventional Single-Family Mortgages, Monthly National Averages, All Homes, Federal Housing Finance Agency.

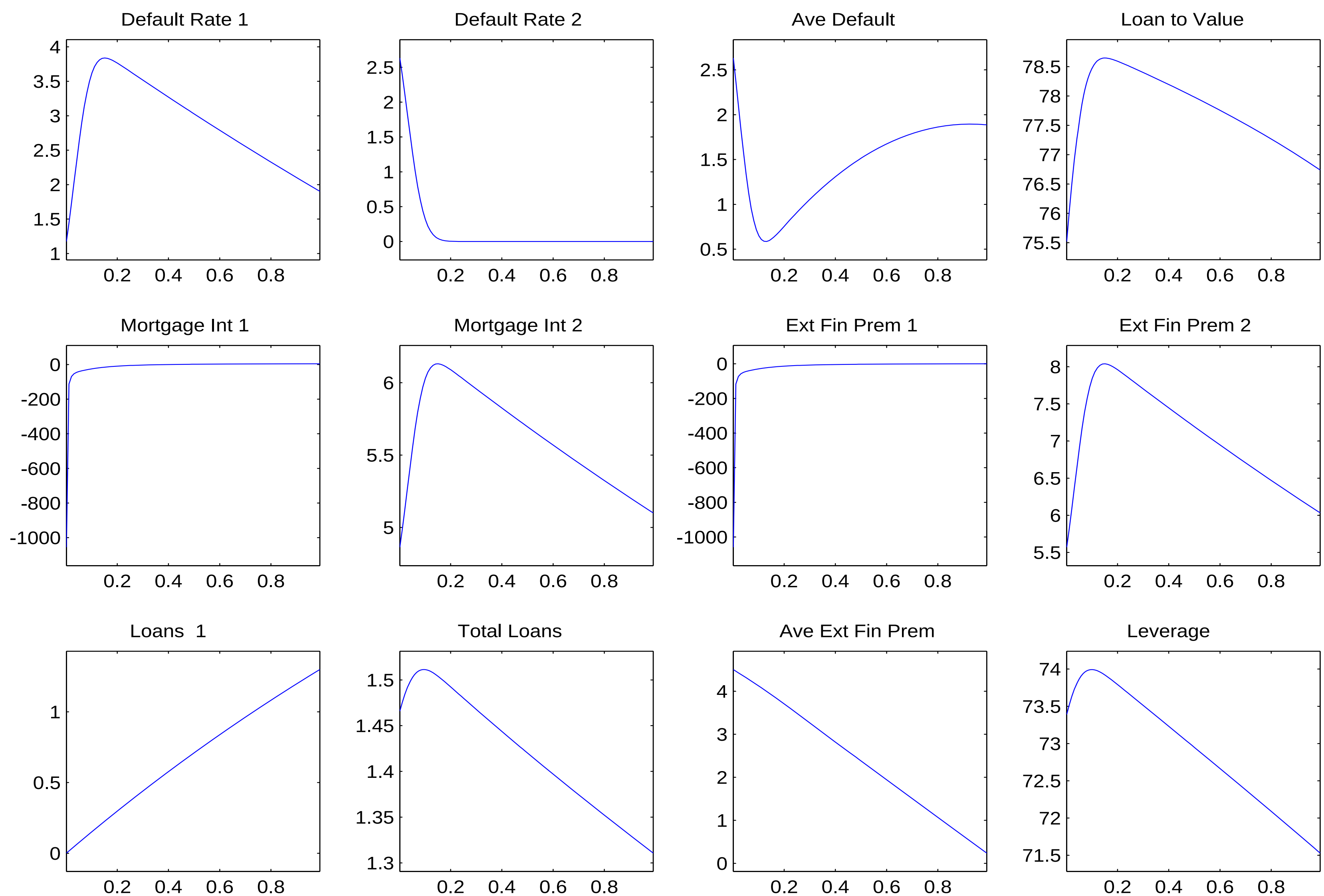
from external funding. Average default is measured is the weighted average of default rates on one- and two-period loans.

4.2 Comparative Statics

The parameter x is the fraction of the total mortgage L that must be repaid in the first period. This parameter measures how much amortization is concentrated in the first period and therefore early in the mortgage term. $x = 0$ implies that the entire mortgage, principal plus interests, is repaid at the end of the second period with no intermediate repayment or amortization. In general, we are going to refer to low values of x as the case of *low* early amortization in the sense that a small fraction of the mortgage is amortized early in the term. Notice that low early amortization may include negative amortization in our model, which occurs when the first-period repayment is less than the accrued interest on the principal. On the other hand, high values of x correspond to high early or front-loaded amortization as most of the principal is repaid early on in the term.

Figures 1 and 2 show the steady-state of our model as x varies in the interval $(0, 1)$ under our baseline calibration. Figure 1 plots the financial variables, most of which exhibit non-monotonic behavior. Borrower's incentives depend on whether default on two-period loans is zero or positive in equilibrium. Consider the case of low early amortization, namely x small. For low values of x the Borrower has an incentive not to default in the first period, hold on to the housing stock for an additional period (and get utility out of it) and default in the second period. When the first-period repayment is low, the value of postponing default while getting housing services anyway is high. This explains why the first-period default rate starts low and increases with x, while the second-period default rate starts high and falls with x. When the first-period repayment is low, the value of the housing stock seized as collateral is high enough to support low or even negative first-period adjustable mortgage rates, i.e. the non-defaulting members may even pay less than their principal. The default rate on first-period loans is *high* relative to the repayment because the decision to default depends on the present value of all liabilities, namely current and future payments due. On the other hand, the adjustable mortgage rate guarantees the pre-determined rate of return to Savers.

Consider now the case where x rises from a value close to zero. The incentive to postpone



Default rate, external finance premium, mortgage interest rate and nominal interest rate are annual and in percentage points. The loan-to-value and leverage ratios are in percentage points.

Figure 1: Steady State as x varies in the interval $(0, 1)$: Financial Variables

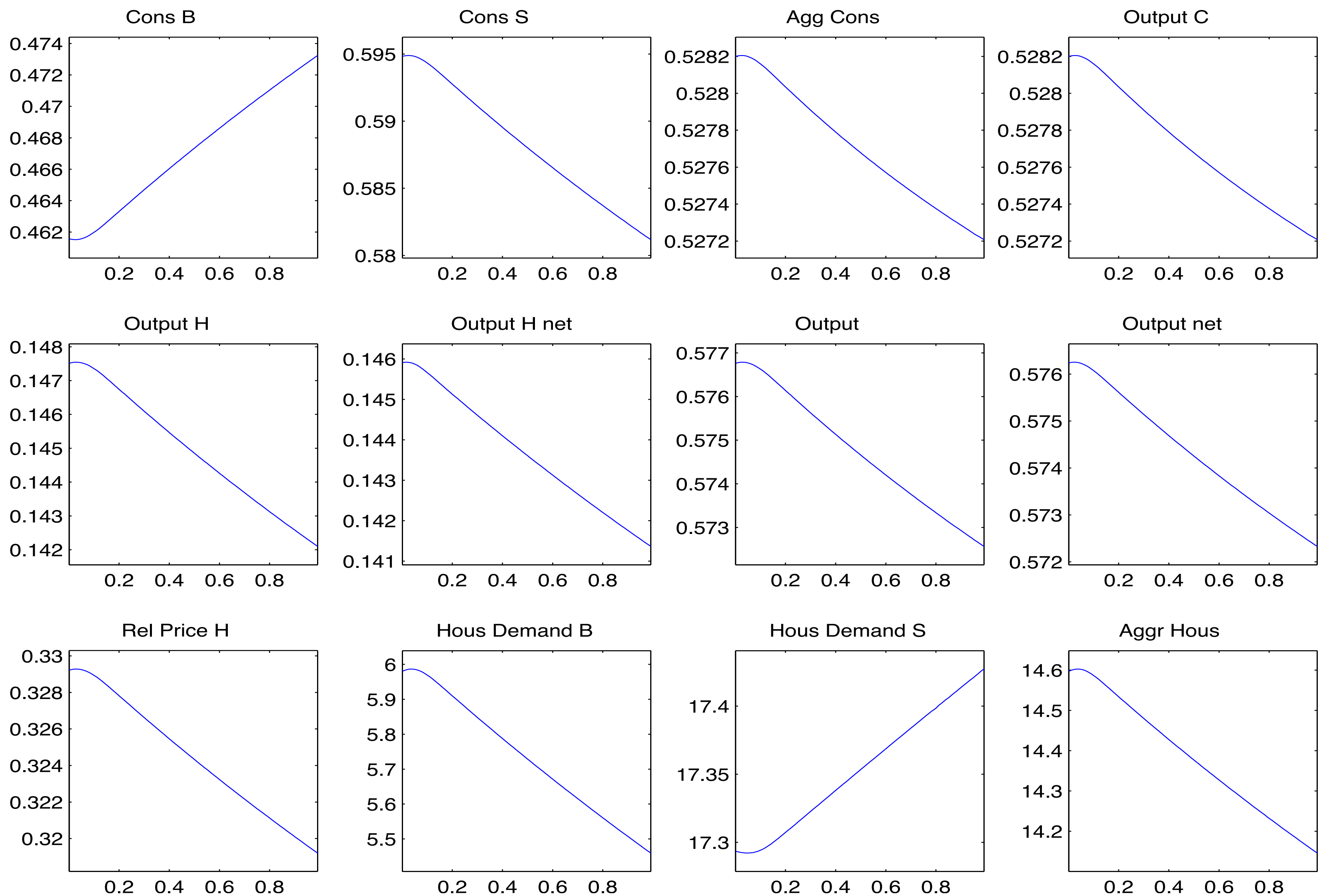


Figure 2: Steady State as x varies in the interval $(0, 1)$: Macroeconomic Variables

default to the second period gradually disappears. As the first-period repayment becomes larger, the Borrower raises default on one-period loans and reduces it on two-period loans. The adjustable mortgage rate on one-period loans increases quickly with first-period repayment while that on two-period loans increases driven by the volume of two-period loans. The external finance premia follow the behavior of the adjustable rates. Repaid one-period loans act as a sunk cost on second-period decisions. Beyond a certain value of x the Borrower finds housing investment less attractive, as he cannot hold on to the house for two periods by paying little in the first installment and defaulting on the second one. Hence, total loans and Borrower housing demand peak at a low value of x and then fall.

The incentive described above continues as long as the default rate in the second period is positive. Once the default rate on two-period loans is zero, default happens only on the first repayment. Further increases in x reduce the default rate on one-period loans as well as the total amount of lending. Larger first-period installments require higher adjustable mortgage rates R_{Z1} to be paid by non-defaulting Borrowers, which in turn reduce the default rate. At this point the Borrower can better smooth borrowing costs by reducing his leverage. The mortgage rate on two-period loans and the LTV ratio fall. This reduces his demand for housing and for loans.

Figure 2 shows the behavior of other endogenous variables as x varies in the interval $(0, 1)$ under the baseline calibration. As x increases, lower housing demand by the Borrower is compensated by higher demand for non-durable goods. The housing price decreases, thereby inducing the Saver to substitute consumption for non-durable goods with housing. Overall, output falls in both sectors.

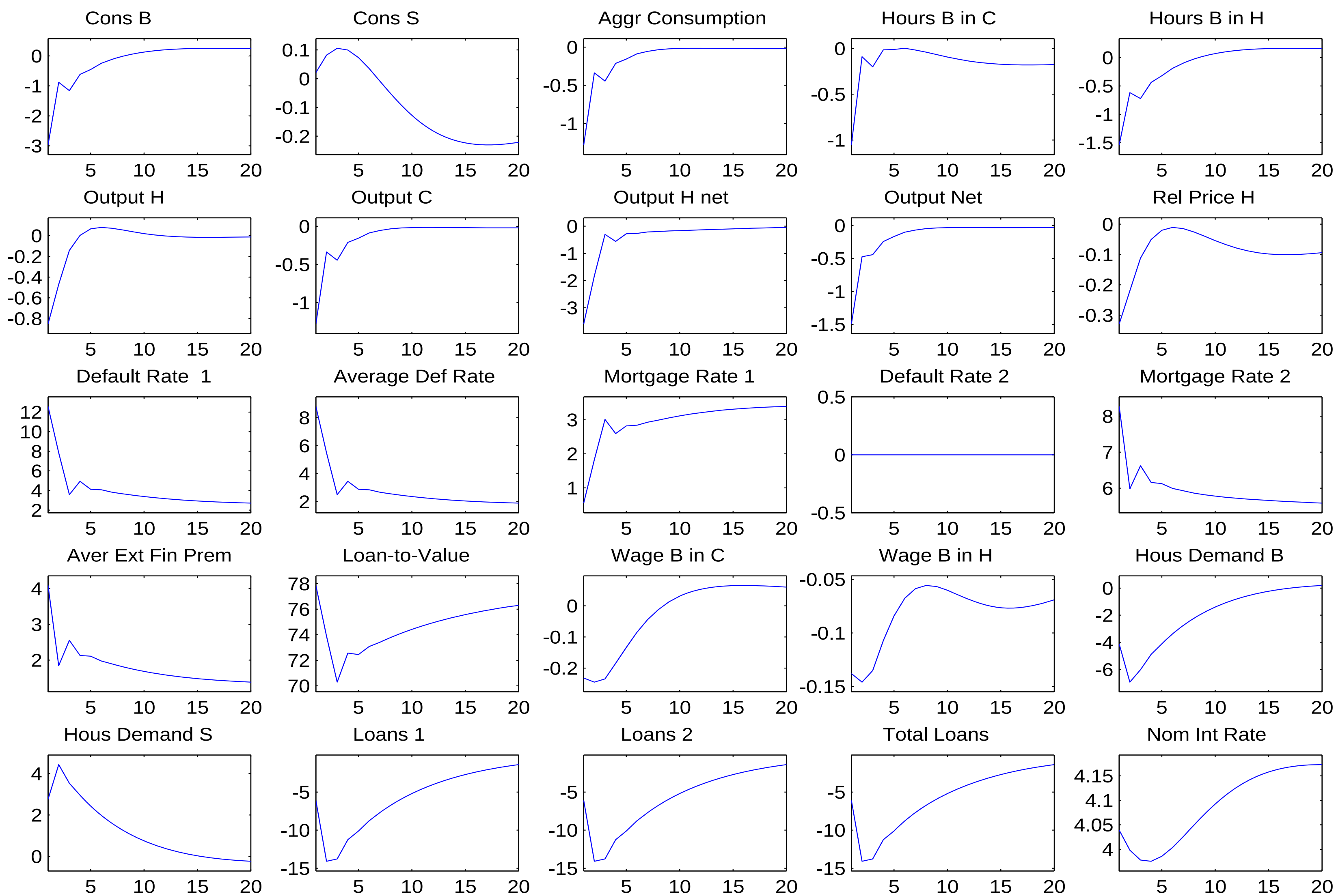
This analysis suggests that non-traditional mortgage products that reduce initial mortgage repayments, such as negative-amortization or interest-rate-only mortgages, increase aggregate housing demand, housing prices and leverage.

5 Amplification

We analyze the dynamic response of our model to an unanticipated increase in $\sigma_{\omega,t}$, the standard deviation of the distribution of idiosyncratic housing investment risk ω_1 and ω_2. This increase in risk means to capture the situation in which loans are made on the basis of an expected distribution for idiosyncratic risk, but the actual distribution turns out to be characterized by a higher standard deviation. Hence, the riskiness of mortgages changes over time and these changes are persistent. A mean-preserving increase in $\sigma_{\omega,t}$ implies an increase in the skewness of the distribution of $\omega_{j,t}$. Since the log-normal distribution does not take negative values, the lower tail of the distribution becomes thicker. Thus, for the same value of $\bar{\omega}_{j,t}$, a higher standard deviation implies a higher cumulative distribution function and therefore a higher default rate on mortgages.

Figure 3 shows the impulse responses of our model to a 50% increase in σ_{ω}. Default and interest rates are annual and in percentage points. The delinquency rate on U.S. real estate loans reached 10.44% in the first quarter of 2010. The size of the shock is chosen so as to raise the average default rate on impact to around 10%. In this example we set $x = 0.7$. Hence, equilibrium default on two-period loans is equal to zero. The shift in the distribution of $\omega_{1,t}$ and $\omega_{2,t}$ increase the rate of default on one-period loans while default on two-period loans remains equal to zero. Financial conditions of Borrowers worsen significantly. First, Borrower household members who default experience a housing loss while those who repay face on average higher adjustable mortgage rates and external finance premia. In fact, the adjustable mortgage rate on two-period loans increases to compensate Savers for the two-period loans that will not be repaid because of default on the first period installment and its increase more than compensates the reduction in R_{Z1}. Second, the LTV ratio falls, thereby limiting the capacity to borrow out of the existing housing stock. Third, Borrowers experience an increase in real debt via the Fisher effect stemming from deflation (not shown in the graph). Fourth, wages fall in both sectors, which in turn trigger a reduction in hours and wage income. As a result, Borrowers de-leverage and reduce their consumption of non-durable goods as well as their housing demand. The fall in housing demand by Borrowers causes housing prices to fall. Savers are consumption smoothers and respond to lower house prices and lower interest rates by raising non-durable consumption and housing demand.

Wages fall in both sectors due to reduced demand and in spite of wage rigidity. Wage differential arise in equilibrium both across groups and across sectors because of imperfect substitutability of hours both on the supply and the demand side. The housing sector experiences a contraction driven by Borrowers' house downsizing, no matter whether output in the H sector is measured gross or net of monitoring costs. The C sector also experiences a sharp recession



Note: Default rate, external finance premium, mortgage interest rate and nominal interest rate are annual and in percentage points. The loan-to-value and leverage ratios are in percentage points. B = Borrower; S = Saver

Figure 3: Impulse Responses to a 50% Increase in σ_ω: $x = 0.7$

stemming from Borrowers' reduction in consumption. Total output in the economy falls.

Figure 4 compares the responses of our model to a housing risk shock for three different values of $x : 0.99, 0.5, 0.01$. With $x = 0.99$ the mortgage is repaid almost completely in the first period. We will refer to this case as high early amortization. $x = 0.5$ represents the case of equal amortization among the two periods; $x = 0.01$ is the case where the mortgage is almost entirely repaid at the end of its term. We will refer to this case as low early amortization. Figure 4 shows that reducing x and therefore reducing early amortization generates amplification. Moving from $x = 0.99$ to $x = 0.01$ triples the fall in net output, from -1% to -3%; the housing sector experiences a much deeper and sharper recession with low early amortization.

Once again Borrower's incentives depend on whether default on the second installment is zero or positive in equilibrium. Consider first the case where $x = 0.5$ and $x = 0.99$. When a sizable fraction of the mortgage is repaid early on, it is extremely unlikely to experience negative equity in the house in the second period so that default on the second installment is zero. An unanticipated housing risk shock raises default on the first installment of the mortgage, which in turn raises Borrower losses due to confiscation of the housing stock connected to the defaulted loans by lenders. This negative wealth effect is higher when $x = 0.5$ because of a steady-state effect. In fact, the steady-state housing stock and leverage of Borrowers is about 5% higher when $x = 0.5$ relative to the case where $x = 0.99$. Following a risk shock, Borrowers much cut both consumption, housing demand and total loans by more in the economy with higher leverage. The deeper fall in the demand of non-durable goods reduces real wages more and explains the larger drop in housing prices. These first-round effects generate further contraction and amplification. Lower housing prices exacerbate default on one-period loans because more Borrower members experience negative equity in the house, thereby further worsening the financial conditions of Borrowers and the economic downturn.

Consider now the impulse responses to a housing risk shock when $x = 0.01$. The default rate on two-period loans is positive in equilibrium with low early amortization. In this case an additional factor, a stronger negative wealth effect, contributes to generate amplification. Borrowers prefer to default on the second installment of the mortgage so as to keep the housing stock for an additional period. As a result, the default rate on one-period loans responds by less with low early amortization relative to the case where $x = 0.5$ or $x = 0.99$. This has two consequences. First, a lower default rate on the first installment leaves the Borrower household

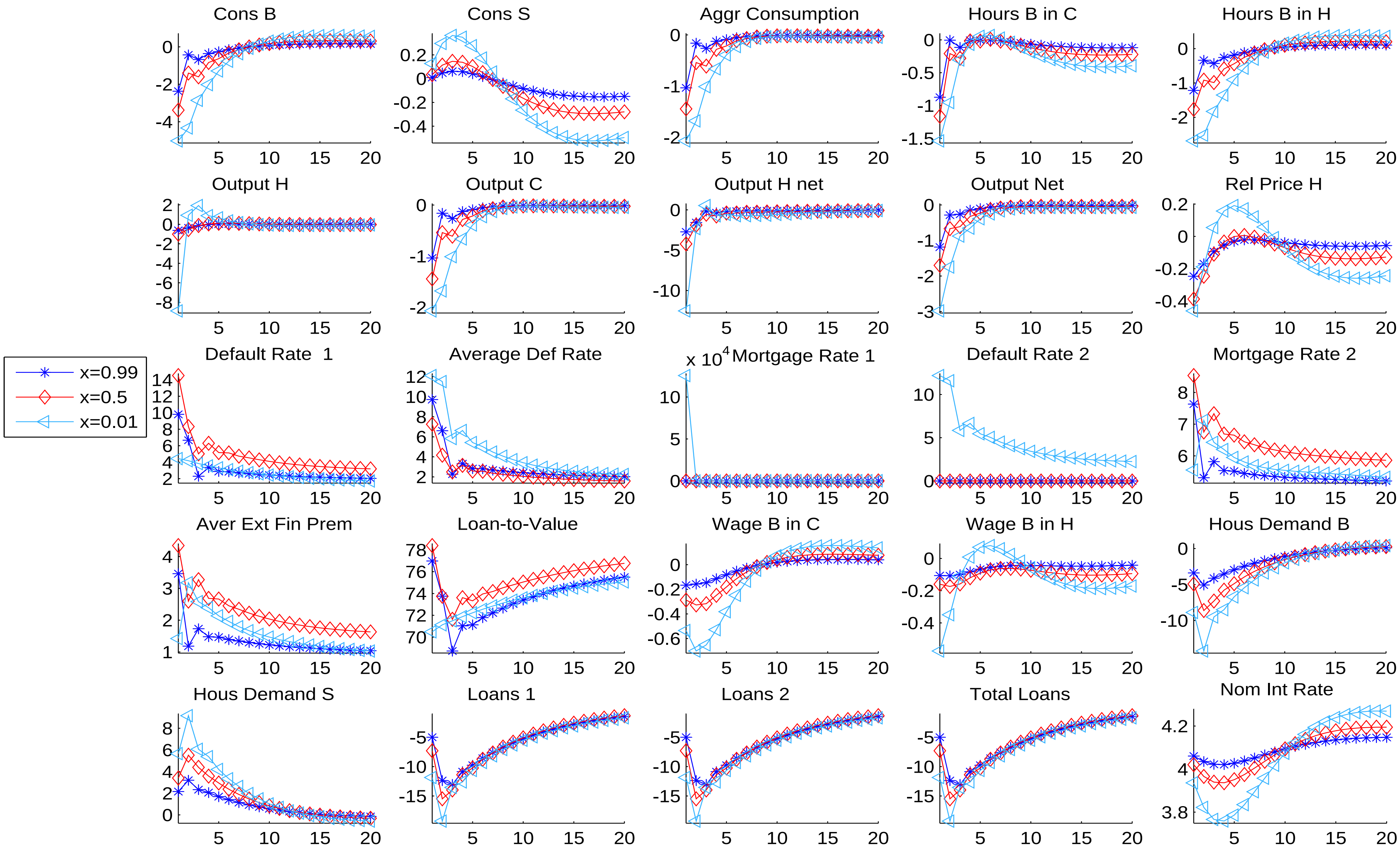


Figure 4: Impulse Responses to a 50% Increase in σ_ω: A Comparison

with a larger housing stock and reduces his demand for new houses,[5] which in turn generates a stronger contraction in the real wage and output in the housing sector. Second, a lower default rate on the first installment of the mortgage reduces the LTV ratio, which instead increases for the other values of x. A lower LTV ratio further erodes Borrower's capacity to borrow and consume non-durable goods. The deeper contraction in the non-durable sector further reduces the real wage in C.

6 Optimal Amortization

So far the parameter x has been treated as an exogenously given parameter. Borrowers could increase or decrease their total mortgages keeping the ratio of one- and two-period loans as constant. Here we relax this assumption and allow Borrowers to choose optimally $L_{1,t+1}$ and $L_{2,t+1}$. In terms of the model, Borrowers now have separate first-order conditions for each of these two variables.

For our benchmark calibration Borrowers choose $x^{opt} = 0.0227$. In words, the one-period loan represents 2.27% of the total mortgage and early amortization is very low. Positive default rates arise both on first- and second-period installments. Hence, if free to choose Borrowers prefer low early or even negative amortization – in fact, $x = 0.0227$ implies negative amortization in our model. Not surprisingly, Borrowers prefer to have small first-period installments and to repay most of the loan at the end of the term. This allows them to postpone default on the mortgage to the the second installment and take advantage of the housing stock for an additional period. Low early amortization schedules imply low first-period repayments and thereby low sunk costs. Interestingly, the Borrower does not choose $x = 0$, namely he does not choose to eliminate first-period payments altogether.

Figure 5 shows the responses of our model to the same housing risk shock analyzed in the previous section. In line with the findings of the previous section, low values of x come hand in hand with greater amplification. The main difference with respect to figure 4 lies in the behavior of one-period loans $L_{1,t+1}$. By cutting more aggressively one-period loans, the Borrower is able to better smooth his housing demand.

[5] To see why this is the case recall that $H^T_{t+1} = H_{t+1} + (1-\delta)(1-G(\bar{\omega}_t))H_t$.

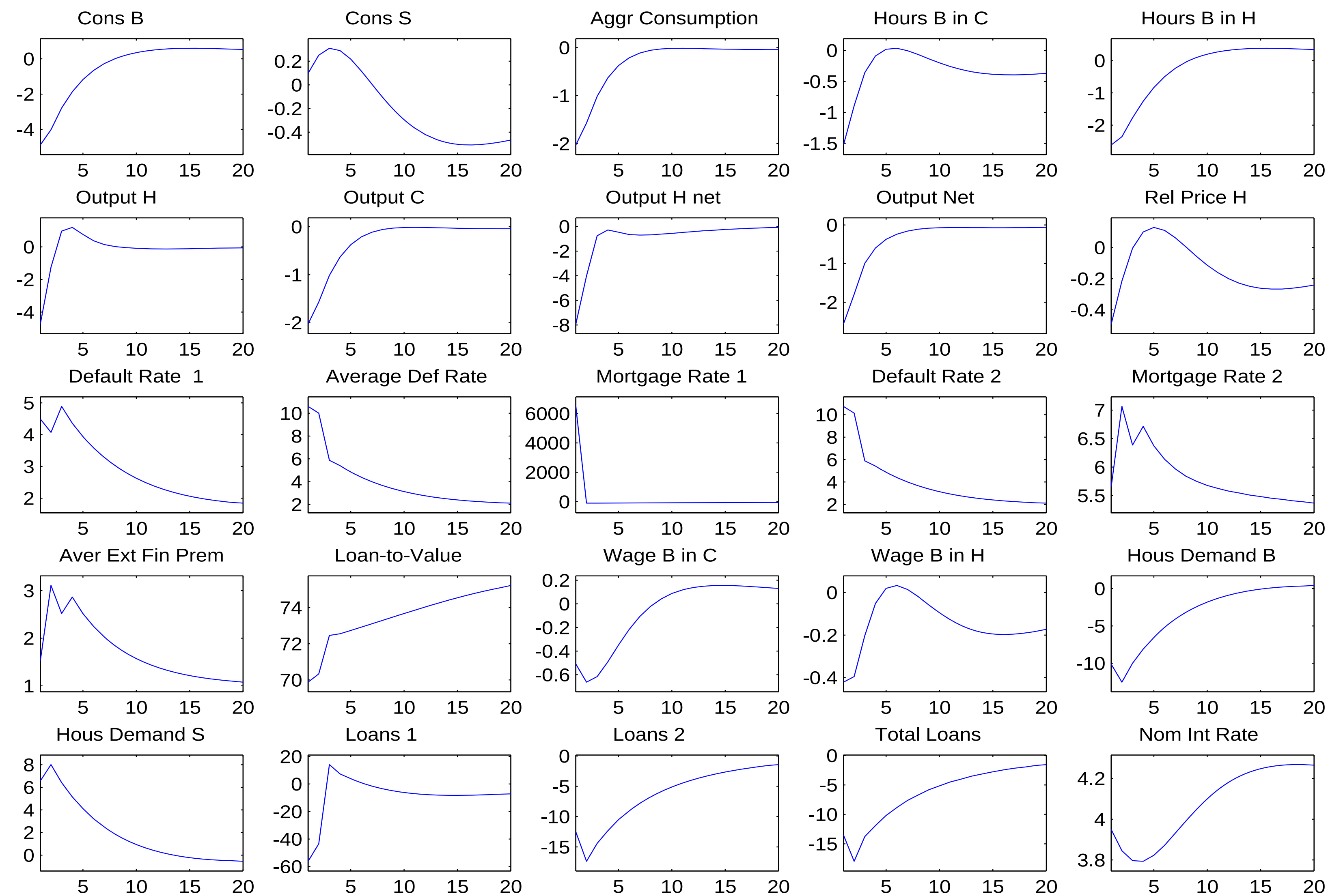


Figure 5: Impulse Responses to a 50% Increase in σ_ω: x chosen optimally

7 Conclusions

We analyze the role of mortgage amortization in the transmission of a housing risk shock that we model as an unexpected increase in the standard deviation of the distribution of idiosyncratic housing investment risk.

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A Model Equations

A.1 Borrowers

$$U_{C,t} - \lambda_{BC,t} = 0, \tag{40}$$

$$U_{H,t} - \lambda_{BC,t} p_{H,t} + \beta(1-\delta) E_t \left\{ U_{H,t+1} + (1-\mu) G_{t+1}(\bar{\omega}_{1,t+1}) p_{H,t+1} \lambda_{BC,t+1} + \lambda_{PC,t+1} p_{H,t+1} \right. \tag{41}$$

$$\times \left[\Gamma_{t+1}(\bar{\omega}_{1,t+1}) - \mu G_{t+1}(\bar{\omega}_{1,t+1}) \right] \} + \beta^2 (1-\delta)^2 E_t p_{H,t+2} \left\{ \lambda_{BC,t+2} [1 - G_{t+1}(\bar{\omega}_{1,t+1})] \right.$$

$$\left. [1 - \mu G_{t+1}(\bar{\omega}_{2,t+2})] + \lambda_{PC,t+2} [\Gamma_{t+2}(\bar{\omega}_{2,t+2}) - \mu G_{t+2}(\bar{\omega}_{2,t+2})] \right\} = 0,$$

$$U_{N_j,t} + \lambda_{BC,t} \frac{w_{j,t}}{\mu_{j,t}} = 0, \quad j \in \{C, H\}, \tag{42}$$

$$\lambda_{BC,t} - \beta(1 + R_{L1,t}) x E_t \left[\frac{\lambda_{PC1,t+1} - \lambda_{BC,t+1}}{\pi_{C,t+1}} \right] + \beta^2 (1 + R_{L2,t})(1-x) \left[\frac{\lambda_{PC2,t+2} + \lambda_{BC,t+2}}{\pi_{C,t+1} \pi_{C,t+2}} \right] = 0, \tag{43}$$

$$-\frac{U_{H,t+1}}{\beta(1-\delta)} + E_t p_{H,t+1} \left\{ \lambda_{BC,t+1}(1-\mu) + \lambda_{PC1,t+1} \left[\frac{\Gamma'_{t+1}(\bar{\omega}_{1,t+1})}{G'_{t+1}(\bar{\omega}_{1,t+1})} - \mu \right] \right\} + E_t p_{H,t+2} \beta (1-\delta) \tag{44}$$

$$\left\{ \lambda_{BC,t+2} [1 - \mu G_{t+2}(\bar{\omega}_{2,t+2})] + \left[\pi_{C,t+2} \frac{Q_{t+1,t+2}}{\beta} \lambda_{PC1,t+1} \right] [\Gamma_{t+2}(\bar{\omega}_{2,t+2}) - \mu G_{t+2}(\bar{\omega}_{2,t+2})] \right\} = 0$$

$$-\lambda_{BC,t+2} \mu G'_{t+2}(\bar{\omega}_{2,t+2}) + [\lambda_{PC2,t+2} - \lambda_{PC1,t+1} \pi_{C,t+2} Q_{t+1,t+2}] \left[\Gamma'_{t+2}(\bar{\omega}_{2,t+2}) - \mu G'_{t+2}(\bar{\omega}_{2,t+2}) \right] = 0, \tag{45}$$

where $\lambda_{BC,t}$ is the Lagrangian multiplier on Borrowers budget constraint (13), $\lambda_{PC1,t+1}$ is the Lagrangian multiplier on the participation constraint (15), $\lambda_{PC2,t+2}$ is the Lagrangian multiplier on the participation constraint (14), $\mu_{j,t}, j = C, H$ is the Lagrangian multiplier on the labor market constraint (18) in sector j. Notice that the first-order conditions with respect to $\bar{\omega}_{1,t+1}$ and $\bar{\omega}_{2,t+2}$ are state-by-state and not in expected terms.

To find the first-order conditions relative to $w^j_{C,t}, w^j_{H,t}$, we rewrite the relevant part of the Lagrangian associated with the optimization problem of Borrowers as follows

$$\mathcal{L}^w_t = E_t \sum_{s=0}^{\infty} (\varrho\beta)^s \lambda_{BC,t+s} \left\{ \left[N^d_{C,t+s} w^j_{C,t} \left(\frac{w^j_{C,t}}{w_{C,t+s}} \right)^{-\eta_w} + N^d_{H,t+s} w^j_{H,t} \left(\frac{w^j_{H,t}}{w_{H,t+s}} \right)^{-\eta_w} \right] + \right. \tag{46}$$

$$\left. \frac{w_{C,t+s}}{\mu^w_{C,t+s}} \left[N_{C,t+s} - N^d_{C,t+s} \left(\frac{w^j_{C,t}}{w_{C,t+s}} \right)^{-\eta_w} \right] + \frac{w_{H,t+s}}{\mu^w_{H,t+s}} \left[N_{H,t+s} - N^d_{H,t+s} \left(\frac{w^j_{H,t}}{w_{H,t+s}} \right)^{-\eta_w} \right] \right\}.$$

The first-order condition relative to $w^j_{C,t}$ is

$$E_t \sum_{s=0}^{\infty} (\varrho\beta)^s \lambda_{BC,t+s} N^d_{C,t+s} \left(\frac{w^j_{C,t}}{w_{C,t+s}} \right)^{-\eta_w} \prod_{z=0}^{s} \pi^{\eta_w}_{C,t+z} \left[-\left(\frac{\eta_w - 1}{\eta_w} \right) \frac{w^j_{C,t}}{\prod_{z=0}^{s} \pi_{C,t+z}} + \frac{w_{C,t+s}}{\mu^w_{C,t+s}} \right] = 0,$$

and the first-order condition relative to $w^j_{H,t}$ is

$$E_t \sum_{s=0}^{\infty} (\varrho\beta)^s \lambda_{BC,t+s} N^d_{H,t+s} \left(\frac{w^j_{H,t}}{w_{H,t+s}} \right)^{-\eta_w} \prod_{z=0}^{s} \pi^{\eta_w}_{C,t+z} \left[-\left(\frac{\eta_w - 1}{\eta_w} \right) \frac{w^j_{H,t}}{\prod_{z=0}^{s} \pi_{C,t+z}} + \frac{w_{H,t+s}}{\mu^w_{H,t+s}} \right] = 0.$$

To write these equations in a recursive manner we define

$$f_{1z,t} = \frac{\eta_w - 1}{\eta_w} w^j_{z,t} E_t \sum_{s=0}^{\infty} \lambda_{BC,t+s} N^d_{z,t+s} \left(\frac{w^j_{z,t}}{w_{z,t+s}} \right)^{-\eta_w} \prod_{k=0}^{s} \pi^{\eta_w - 1}_{C,t+k}, \quad z = C, H,$$

$$f_{2z,t} = -(w^j_{z,t})^{-\eta_w} E_t \sum_{s=0}^{\infty} \lambda_{BC,t+s} N^d_{z,t+s} w^{\eta_w}_{z,t+s} \frac{w_{z,t+s}}{\mu_{z,t+s}} \prod_{k=0}^{s} \pi^{\eta_w}_{C,t+k}, \quad z = C, H.$$

Making use of (42) and simplifying, the wage-setting equations simplify to

$$f_{1z,t} = \frac{\eta_w - 1}{\eta_w} w^j_{z,t} N^d_{z,t} \left(\frac{w^j_{z,t}}{w_{z,t}} \right)^{-\eta_w} + \varrho\beta E_t \pi^{\eta_w - 1}_{C,t+1} \left(\frac{w^j_{z,t+1}}{w^j_{z,t}} \right)^{\eta_w - 1} f_{1z,t+1}, \quad z = C, H, \tag{47}$$

$$f_{2z,t} = -U_{N_z,t} \left(\frac{w^j_{z,t}}{w_{z,t}} \right)^{-\eta_w} N^d_{z,t} + \varrho\beta E_t \pi^{\eta_w}_{C,t+1} \left(\frac{w^j_{z,t+1}}{w^j_{z,t}} \right)^{\eta_w} f_{2z,t+1}, \quad z = C, H, \tag{48}$$

$$f_{1z,t} = f_{2z,t}, \quad z = C, H. \tag{49}$$

The Borrowers' stochastic discount factor between period t and $t + k$ is given by

$$Q_{t,t+k} \equiv \beta^k \frac{P_{C,t}[\lambda_{BC,t+1} + \lambda_{PC1,t+1}]}{\lambda_{BC,t} P_{C,t+k}}. \tag{50}$$

A.2 Savers

$$U_{\widetilde{C},t} - \widetilde{\lambda}_{BC,t} = 0, \tag{51}$$

$$U_{\widetilde{H},t+1} - \widetilde{\lambda}_{BC,t} p_{H,t} + \gamma(1-\delta) E_t \left[\widetilde{\lambda}_{BC,t+1} p_{H,t+1} \right] = 0, \tag{52}$$

$$U_{\widetilde{N}_j,t} + \widetilde{\lambda}_{BC,t}\widetilde{w}_{j,t} = 0, \qquad j \in \{C, H\}, \tag{53}$$

$$-\widetilde{\lambda}_{BC,t} + \gamma(1 + R_{L1,t})E_t\left[\frac{\widetilde{\lambda}_{BC,t+1}}{\pi_{C,t+1}}\right] = 0, \tag{54}$$

$$-\widetilde{\lambda}_{BC,t} + \gamma^2(1 + R_{L2,t})E_t\left[\frac{\widetilde{\lambda}_{BC,t+2}}{\pi_{C,t+1}\pi_{C,t+2}}\right] = 0, \tag{55}$$

$$-p_{A_l,t}\widetilde{\lambda}_{BC,t} + E_t\left[(p_{A_l,t+1} + r_{A_l,t+1})\widetilde{\lambda}_{BC,t+1}\right] = 0, \tag{56}$$

where $\widetilde{\lambda}_{BC,t}$ is the Lagrangian multiplier on Savers budget constraint (20). The first-order conditions relative to $\widetilde{w}^j_{C,t}, \widetilde{w}^j_{H,t}$ are given by

$$\widetilde{f}_{1z,t} = \frac{\eta_w - 1}{\eta_w}\widetilde{w}^j_{z,t}\widetilde{N}^d_{z,t}\left(\frac{\widetilde{w}^j_{z,t}}{\widetilde{w}_{z,t}}\right)^{-\eta_w} + \widetilde{\varrho}\gamma E_t\pi^{\eta_w-1}_{C,t+1}\left(\frac{\widetilde{w}^j_{z,t+1}}{\widetilde{w}^j_{z,t}}\right)^{\eta_w-1}\widetilde{f}_{1z,t+1}, \qquad z = C, H, \tag{57}$$

$$\widetilde{f}_{2z,t} = -\widetilde{U}_{\widetilde{N}_z,t}\left(\frac{\widetilde{w}^j_{z,t}}{\widetilde{w}_{z,t}}\right)^{-\eta_w}\widetilde{N}^d_{z,t} + \widetilde{\varrho}\gamma E_t\pi^{\eta_w}_{C,t+1}\left(\frac{\widetilde{w}^j_{z,t+1}}{\widetilde{w}^j_{z,t}}\right)^{\eta_w}\widetilde{f}_{2z,t+1}, \qquad z = C, H, \tag{58}$$

$$\widetilde{f}_{1z,t} = \widetilde{f}_{2z,t}, \qquad z = C, H. \tag{59}$$

A.3 Intermediate Firms in the Non-Durable Sector

The maximization problem for firm i is given by

$$E_t\left\{\sum_{k=0}^{\infty}\theta^k_C\Lambda_{t,t+k}\left[P^*_{C,t}(i)Y_{C,t+k|t}(i) - W_{C,t+k}N_{C,t+k|t}(i) - \widetilde{W}_{C,t+k}\widetilde{N}_{C,t+k|t}(i)\right.\right.$$
$$\left.\left.+mc_{C,t+k|t}(i)P_{C,t+k}\left[A_{C,t+k}\left[\zeta^{\frac{1}{\varsigma}}N_{C,t+k|t}(i)^{\frac{\varsigma-1}{\varsigma}} + (1-\zeta)^{\frac{1}{\varsigma}}\widetilde{N}_{C,t+k|t}(i)^{\frac{\varsigma-1}{\varsigma}}\right]^{\frac{\varsigma}{\varsigma-1}} - Y_{C,t+k|t}(i)\right]\right]\right\} \tag{60}$$

where $Y_{C,t+k|t}(i)$ denotes output in period $t+k$ for a firm i that last changed its price in period t. A similar interpretation applies to $N_{C,t+k|t}(i)$ and $\widetilde{N}_{C,t+k|t}(i)$. $mc_{C,t+k|t}(i)$ is the real marginal cost of a firm i that last changed its price in period t.

In (60) the demand and the stochastic discount factor are respectively given by

$$Y_{C,t+k|t}(i) = \left(\frac{P^*_{C,t}(i)}{P_{C,t+k}}\right)^{-\varepsilon_C}Y_{C,t+k}, \qquad \Lambda_{t,t+k} \equiv \frac{P_{C,t}}{P_{C,t+k}}\frac{\gamma^k\widetilde{\lambda}_{BC,t+k}}{\widetilde{\lambda}_{BC,t}}.$$

The first-order conditions relative to $N_{C,t+k|t}(i)$ and $\widetilde{N}_{C,t+k|t}(i)$ are

$$-W_{C,t+k} + mc_{C,t+k|t}(i)P_{C,t+k}A_{C,t+k}^{1-\frac{1}{\varsigma}}Y_{C,t+k|t}(i)^{\frac{1}{\varsigma}}\zeta^{\frac{1}{\varsigma}}N_{C,t+k|t}(i)^{-\frac{1}{\varsigma}} = 0, \tag{61}$$

$$-\widetilde{W}_{C,t+k} + mc_{C,t+k|t}(i)P_{C,t+k}A_{C,t+k}^{1-\frac{1}{\varsigma}}Y_{C,t+k|t}(i)^{\frac{1}{\varsigma}}(1-\zeta)^{\frac{1}{\varsigma}}\widetilde{N}_{C,t+k|t}(i)^{-\frac{1}{\varsigma}} = 0, \tag{62}$$

which state that the nominal marginal cost equals the ratio of the nominal wage to the marginal product of each type of labor input. By rearranging (61) and (62) we obtain:

$$mc_{C,t+k|t}(i) = \frac{1}{A_{C,t+k}P_{C,t+k}}\left[\zeta W_{C,t+k}^{1-\varsigma} + (1-\zeta)\widetilde{W}_{C,t+k}^{1-\varsigma}\right]^{\frac{1}{1-\varsigma}}. \tag{63}$$

According to (63), $mc_{C,t+k|t}(i) = mc_{C,t+k}$. Marginal costs are equal across firms because wages are the same across all firms.

The first-order condition relative to the price is given by

$$E_t\left\{\sum_{k=0}^{\infty}\theta_C^k\Lambda_{t,t+k}\left[\left(\frac{P_{C,t}^*(i)}{P_{C,t+k}}\right)^{(-\varepsilon_C-1)}Y_{C,t+k}\left(\frac{P_{C,t}^*(i)}{P_{C,t+k}} - \frac{\varepsilon_C}{\varepsilon_C-1}mc_{C,t+k}\right)\right]\right\} = 0. \tag{64}$$

Finally, it can be shown[6] that, under Calvo price setting, the optimal price set by re-optimizing firms is linked to the aggregate price behavior by the following condition:

$$\left(\frac{P_{C,t}^*}{P_{C,t}}\right)^{(1-\varepsilon_C)} = \frac{1-\theta_C\pi_{C,t}^{\varepsilon_C-1}}{1-\theta_C}, \tag{65}$$

where $\pi_{C,t}$ denotes gross inflation in sector C.

A.4 Intermediate Firms in the Housing Sector

The first-order conditions with respect to $N_{H,t}(i)$, $\widetilde{N}_{H,t}(i)$ and $A_{l,t}$ are

$$-W_{H,t} + mc_{H,t}(i)P_{H,t}\kappa Y_{H,t}(i)\left[\zeta^{\frac{1}{\varsigma}}N_{H,t}(i)^{\frac{\varsigma-1}{\varsigma}} + (1-\zeta)^{\frac{1}{\varsigma}}\widetilde{N}_{H,t}(i)^{\frac{\varsigma-1}{\varsigma}}\right]^{-1}\zeta^{\frac{1}{\varsigma}}N_{H,t}(i)^{-\frac{1}{\varsigma}} = 0, \tag{66}$$

$$-\widetilde{W}_{H,t} + mc_{H,t}(i)P_{H,t}\kappa Y_{H,t}(i)\left[\zeta^{\frac{1}{\varsigma}}N_{H,t}(i)^{\frac{\varsigma-1}{\varsigma}} + (1-\zeta)^{\frac{1}{\varsigma}}\widetilde{N}_{H,t}(i)^{\frac{\varsigma-1}{\varsigma}}\right]^{-1}(1-\zeta)^{\frac{1}{\varsigma}}\widetilde{N}_{H,t}(i)^{-\frac{1}{\varsigma}} = 0, \tag{67}$$

[6] For a formal proof see for instance Woodford (2003).

$$-R_{A_l,t} + mc_{H,t}(i)P_{H,t}(1-\kappa)Y_{H,t}(i)A_{l,t}^{-1} = 0, \tag{68}$$

By using (66), (67) and (68) we obtain:

$$mc_{H,t}(i) = \frac{1}{A_{H,t}P_{H,t}\kappa^{\kappa}(1-\kappa)^{(1-\kappa)}} R_{A_l,t}^{1-\kappa} \left[\zeta W_{H,t}^{1-\varsigma} + (1-\zeta)\widetilde{W}_{H,t}^{1-\varsigma}\right]^{\frac{\kappa}{1-\varsigma}}. \tag{69}$$

Condition (69) states that the marginal cost of producing one more unit in the H sector is equal across firms and depends on Dixit-Stiglitz aggregator of the input prices, i.e. the land rental rate, Borrower and Saver wages.

The first-order condition relative to the price is given by

$$\begin{aligned} &\frac{P_{H,t}^{*}(i)}{P_{H,t}} - \frac{\varepsilon_H}{\varepsilon_H - 1} mc_{H,t} - \frac{\varepsilon_H}{\varepsilon_H - 1} g'\left(\left(\frac{P_{H,t}^{*}(i)}{P_{H,t}}\right)^{-\varepsilon} Y_{H,t} - \left(\frac{P_{H,t-1}^{*}(i)}{P_{H,t-1}}\right)^{-\varepsilon} Y_{H,t-1}\right) \\ &+ \frac{\varepsilon_H}{\varepsilon_H - 1} E_t\left\{\Lambda_{t+1,t}\pi_{H,t} g'\left(\left(\frac{P_{H,t+1}^{*}(i)}{P_{H,t+1}}\right)^{-\varepsilon} Y_{H,t+1} - \left(\frac{P_{H,t}^{*}(i)}{P_{H,t}}\right)^{-\varepsilon} Y_{H,t}\right)\right\} = 0. \end{aligned} \tag{70}$$

According to condition (70), in the absence of adjustment costs firms set prices as a mark-up over the their marginal cost.