Financial Frictions and the Demand for Money*

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Abstract

We develop a continuous time model in which money and bond holdings are consistent with individual decisions and aggregate variables such as production and interest rates. Agents are infinitely-lived, have constant-elasticity preferences, and receive a fraction of their income in money. Each agent solves a Baumol-Tobin money management problem. Markets are segmented because financial frictions make agents trade bonds for money at different times. The timing of the financial transactions is endogenous. We show the implications of the model about the optimal quantity of money, the welfare cost of inflation, and different ways of financing the government. The government consumption multiplier is larger when the timing of the transactions is endogenous. The multiplier is substantially bigger if public consumption is financed with seigniorage.

JEL Codes: E3, E4, E5.

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INTRODUCTION

Typically in the standard macro models, the dates at which agents can readjust their portfolios are exogenous. Here we study if that assumption is important when studying the cost of inflation and the effects of government consumption. Andre Silva (forthcoming) shows that the welfare costs of inflation are non trivial when lump sum taxation is available, the welfare cost of a ten percent inflation instead of a zero percent inflation increases from 0.01 percent of income, in a fixed periods setting, to one percent in an optimal periods setting. Here we study if this result is robust to the elimination of lump sum taxation. Additionally, we also study the effects of a permanent government consumption shock in the two settings.

THE MODEL

The model is a general equilibrium Baumol-Tobin model, similar to the one described in Alvarez et al. (2009), but with the timing of the financial transactions being an endogenous variable. Time is continuous and denoted by $t \in [0, \infty)$. At any moment there are markets for assets, the good and labor. There are two assets, money and nominal bonds. The markets for assets and the market for the good are physically separated in this economy.

There is an unit mass of households that are infinitely lived and have preferences over consumption and leisure. Households have two financial accounts: a brokerage account in which they hold bonds and a bank account in which they hold money. We assume that readjustments in the brokerage account have a fixed cost. As only money can be used to buy the good, households need to maintain, until they make a transfer of funds, an inventory of money in their bank account large enough to pay for their flow of consumption expenditures.

Firms are perfect competitors and hire labor to produce the good. There is a government that must finance government expenditures with either income taxes or seigniorage.

Firms

At date t, with $t \in [0, \infty)$, the firms combine the labor supplied by each household i, $h_t(i)$ with $i \in [0, 1]$, to produce the good of date t. The production function is linear in the amount of labor used:

$$y_t = A \int_0^1 h_t(i) di,$$

where A is a technological parameter.

Each hour of labor costs firms $W_t(1 + \tau)$, where W_t is the nominal wage received by the worker and τ is the labor tax. Per each unit of the good sold firms receive the price P_t. As firms are perfect competitors, the real wage received by the households is

$$w_t \equiv \frac{W_t}{P_t} = \frac{A}{(1+\tau)}. \tag{1}$$

Since the real wage is time invariant, we drop the subscript t from w_t.

Households

There is a continuum of infinitely lived households with measure one. Each household sells hours of labor h_t, to the firms and receives labor income $W_t h_t$, which goes to the brokerage account. The funds deposited in the brokerage account cannot be used to buy goods but receive nominal interest r. The money in the bank account can be used to buy goods. The household chooses optimally when to transfer funds between accounts. The transfer of funds has a real fixed cost γ.

Household i, decides consumption $c_t(i)$, labor supply $h_t(i)$, the dates when transfers to the bank are made $T_j(i)$, $j = 1, 2, ...$, money holdings in the bank account $M_t(i)$, and bond holdings in the brokerage account $B_t(i)$. Let $B_{T_j^-(i)}(i)$ and $M_{T_j^-(i)}(i)$ represent bonds and money holdings just before $t = T_j(i)$, and let $B_{T_j^+(i)}(i)$ and $M_{T_j^+(i)}(i)$ represent bonds and money holdings just after $t = T_j(i)$. Formally, $B_{T_j^-(i)}(i) \equiv \lim_{t \to T_j(i), t<T_j(i)} B_t(i)$ and $B^+_{T_j(i)}(i) \equiv \lim_{t \to T_j(i), t>T_j(i)} B_t(i)$. The definitions of $M_{T_j^-(i)}(i)$ and $M_{T_j^-(i)}(i)$ are similar.

Household i, has an initial endowment of wealth $\mathbb{W}_0(i)$ which is divided exclusively between money $M_0(i)$ in the checking account and $B_0(i)$ in the brokerage account, i.e. $\mathbb{W}_0(i) = M_0(i) + B_0(i)$. Define an holding period as the interval between any two consecutive transfer times, i.e. $[T_j(i), T_{j+1}(i))$, for $j = 1, 2, ...$ The first time household i adjusts its portfolio of bonds is $T_1(i)$ and its first holding period is $[T_1(i), T_2(i)$.

As we will be concentrating on the steady state equilibrium, we assume, without loss of generality, that the distribution of the initial endowments of wealth among the households is such that the fraction of households that choose to readjust their portfolio of bonds is the same at any moment and, the duration of the holding periods, N, is the same across households.

The wealth level of household i, $\mathbb{W}_s(i)$ at date s is given by the sum of its assets holdings, at the last date the household readjusted its portfolio, plus the accumulated labor and interest income minus the accumulated consumption expenditure. Let $T_0(i) \equiv 0$. Formally, $\mathbb{W}_s(i)$ for $s \in (T_j(i), T_{j+1}(i)]$, $j = 0, 1, 2, ...$, is given by the expression

$$\begin{aligned}\mathbb{W}_s(i) &\equiv M^+_{T_j(i)}(i) + e^{(s-T_j(i))r} B^+_{T_j(i)}(i) - \int_{T_j(i)}^{s} P_t c_t(i)\, dt \\ &+ \int_{T_j(i)}^{s} e^{(t-T_j(i))r} P_t w_t h_t(i)\, dt, \end{aligned} \tag{2}$$

At each date $T_j(i)$, for $j = 1, 2, ...$, household i chooses to readjust its portfolio. The restriction it faces is that the portfolio chosen plus the real cost of readjusting must be smaller or equal to the current wealth,

$$M^+_{T_j(i)}(i) + B^+_{T_j(i)}(i) + P_{T_j(i)}\gamma \leq \mathbb{W}_{T_j(i)}(i), \text{ for } j = 1, 2, ... \tag{3}$$

Additionally the household faces a cash in advance constraint,

$$\int_{T_j(i)}^{s} P_t c_t(i)\, dt \leq M^+_{T_j(i)}(i), \text{ for } s \in [T_j(i), T_{j+1}(i)), \text{ for } j = 0, 1, 2, ... \tag{4}$$

Using the equations involving $\mathbb{W}_s$, (2) and (3), we can write the budget constraint of holding period j, for $j = 0, 1, 2, ...$, for household i as

$$\begin{aligned}M^+_{T_{j+1}(i)}(i) + B^+_{T_{j+1}(i)}(i) + P_{T_{j+1}(i)}\gamma &\leq M^+_{T_j(i)}(i) + B^+_{T_j(i)} e^{(T_{j+1}(i)-T_j(i))r} - \int_{T_j(i)}^{T_{j+1}(i)} P_t c_t(i)\, dt \\ &+ \int_{T_j(i)}^{T_{j+1}(i)} P_t e^{(T_{j+1}(i)-t)r} w h_t(i)\, dt,\end{aligned}$$

Let $Z_{T_{j+1}(i)} \equiv e^{-(T_{j+1}(i)-T_j(i))r}$ denote the price of a bond at moment $T_j(i)$ that pays 1 dollar at moment $T_{j+1}(i)$. Define $Q_{T_{j+1}(i)} \equiv Z_{T_1(i)}...Z_{T_{j+1}(i)} = e^{-T_{j+1}r}$ as the price at moment 0 of a bond that pays 1 dollar at $T_{j+1}(i)$. If we multiply the budget constraint of holding period j, for all $j = 0, 1, 2, ..., k$, by $Q_{T_{j+1}(i)}$ and add up all of them we obtain

$$\begin{aligned}&\sum_{j=1}^{k} Q_{T_j(i)}\left(1 - Z_{T_{j+1}(i)}\right) M^+_{T_j(i)}(i) + Q_{T_{k+1}(i)}(B^+_{T_{k+1}(i)}(i) + M^+_{T_{k+1}(i)}(i)) \\ \leq\ & -\sum_{j=0}^{k} Q_{T_{j+1}(i)}\left(\int_{T_j(i)}^{T_{j+1}(i)} P_t c_t(i)\, dt + P_{T_{j+1}(i)}\gamma\right) \\ & +\sum_{j=0}^{k} Q_{T_{j+1}(i)}\left(\int_{T_j(i)}^{T_{j+1}(i)} P_t e^{(T_{j+1}(i)-t)r} w_t h_t(i)\, dt\right) + B^+_0(i) + Q_{T_1(i)} M^+_0(i).\end{aligned}$$

Using the definition of $Q_{T_j(i)}$ and the fact that at the optimum $\lim_{T_k(i)\to\infty} Q_{T_k(i)}(B^+_{T_k(i)}(i) + M^+_{T_k(i)}(i)) = 0$ we get

$$\begin{aligned}
&\sum_{j=1}^{\infty} Q_{T_j}\left[M^+_{T_j}(i) + P_{T_j}\gamma\right] + V_0(i) \\
\leq \; &\sum_{j=0}^{k} \int_{T_j(i)}^{T_{j+1}(i)} P_t e^{-rt} w_t h_t(i)\, dt + B^+_0(i) + Q_{T_1(i)} M^+_0(i) \,. \qquad (5)
\end{aligned}$$

where $V_0(i) = Q_{T_1} \int_0^{T_1(i)} P_t c_t dt$. Expression (5) is the intertemporal budget constraint of household i.

Household i has an intertemporal utility function

$$\sum_{j=0}^{\infty} \int_{T_j(i)}^{T_{j+1}(i)} e^{-\rho t} u(c_t(i), h_t(i))\, dt. \qquad (6)$$

We take the momentary utility function to be a Greenwood-Hercowitz-Huffman, GHH, utility function: $u(c_t(i), h_t(i)) = \frac{1}{1-1/\eta}\left(c_t(i) - \frac{(h_t(i))^{1+\chi}}{1+\chi}\right)^{1-1/\eta}$. The household problem is to choose the vector $\left\{c_t(i), h_t(i), M_{T^+_j(i)}(i), T_j(i)\right\}$ that maximizes the intertemporal utility function (6) subject to intertemporal budget constraint (5) and the cash in advance constraint that we rewrite here again, as (7)

$$\int_{T_j(i)}^{T_{j+1}(i)} P_t c_t(i)\, dt \leq M^+_{T_j(i)}(i)\,. \qquad (7)$$

Let λ be the lagrange multiplier of (5) and $\mu_{T_j(i)}$ the lagrange multiplier of (7). Let $t \in [T_j(i), T_{j+1}(i))$ for all $j = 1, 2, ...$The first order condition with respect to $c_t(i)$ is

$$e^{-\rho t}\left(c_t(i) - \frac{(h_t(i))^{1+\chi}}{1+\chi}\right)^{-1/\eta} = \mu_{T_j(i)} P_t, \qquad (8)$$

with respect to $h_t(i)$ is

$$e^{-\rho t}\left(c_t(i) - \frac{(h_t(i))^{1+\chi}}{1+\chi}\right)^{-1/\eta} (h_t(i))^{\chi} = \lambda e^{-tr} P_t w_t.$$

with respect to $M^+_{T_j(i)}(i)$

$$\lambda Q_{T_j(i)} = \mu_{T_j(i)}, \qquad (9)$$

and with respect to $T_j(i)$ is

$$\begin{aligned}
&e^{-\rho T_j} u\left(c^-_{T_j(i)}, l^-_{T_j(i)}\right) - e^{-\rho T_j} u\left(c^+_{T_j(i)}, l^+_{T_j(i)}\right) + \lambda[-\dot{Q}_{T_j(i)} M^+_{T_j}(i) - Q_{T_j} \dot{M}^+_{T_j}(i) \\
&- \gamma\left(P_{T_j}\dot{Q}_{T_j} + Q_{T_j}\dot{P}_{T_j}\right) + P_{T_j(i)} e^{-rT_j(i)} w_{T_j(i)} h^-_{T_j(i)}(i) - P_{T_j(i)} e^{-rT_j(i)} w_{T_j(i)} h^+_{T_j(i)}(i)] \\
&+ \mu_{T_j}\left(\dot{M}^+_{T_j}(i) + P_{T_j} c^+_{T_j}\right) + \mu_{T_{j-1}}\left(-P_{T_j} c^-_{T_j}\right) \\
&= 0.
\end{aligned} \tag{10}$$

Conditions, (8)-(9), imply

$$(h_t(i))^{\chi} = \frac{\lambda Q_t P_t w_t}{\lambda Q_{T_j(i)} P_t} = w e^{-r(t-T_j(i))} \equiv w^*_t(i). \tag{11}$$

This equation equates the marginal rate of substitution between leisure and consumption, the left hand side, to the adjusted real wage, w^*_t. As expected, since preferences are GHH, the supply of labor is only a function of the adjusted real wage. The adjusted real wage is a function of the real wage, the nominal interest rate and the distance to the previous transfer time. Even if the size of the holding period, $T_{j+1}(i) - T_j(i)$, is the same for all households the labor supply will decrease within the holding period, as the relevant real wage, $w^*_t(i)$, decreases within the holding period at a constant rate: $w^*_n = we^{-rn}$ for $n = [0, N]$.

Government

The government is continuously in the asset markets exchanging bonds for money. However, to derive the intertemporal government budget constraint it is convenient to assume a fictitious discrete timing economy, with intervals of dimension δ, which later we make arbitrarily small.

The government issues non state-contingent debt, B_t and money M_t, makes consumption expenditures g, and taxes labor income at rate τ. At any moment t, total public debt, D_t, can take two forms: the government can choose to divide it between M_t and B_t.

$$M_t + B_t = D_t$$

The financial responsibilities the government at time $t+\delta$, for small δ, are equal to $M_t + B_t e^{\delta r} - P_t g + \tau P_t w h_t$. Thus, the time $t+\delta$ budget constraint of the government can be written as

$$M_{t+\delta} + B_{t+\delta} \leq M_t + B_t e^{\delta r} - P_t g + \tau P_t w h_t.$$

Similarly for the $t+2\delta$ budget constraint of the government,

$$M_{t+2\delta}+B_{t+2\delta}=M_{t+\delta}+B_{t+\delta}e^{\delta r}-P_{t+\delta}g+\tau P_{t+\delta}wh_{t+\delta}.$$

After multiplying the $t+\delta$ budget constraint by $e^{-\delta r}$, the time $t+2\delta$ budget constraint $e^{-2\delta r}$, etc..., and adding up all of them we get

$$\begin{aligned} & \sum_{s=0}^{k}\left(e^{-(s+1)\delta r}-e^{-(s+1)\delta r}\right)M_{t+(s+1)\delta}+e^{-k\delta r}\left(M_{t+k\delta}+B_{t+k\delta}\right) \\ = & \sum_{s=0}^{k}e^{-(s+1)\delta r}P_{t+s\delta}\left(\tau wh_{t+s\delta}-g\right)+B_t+e^{-\delta r}M_t. \end{aligned}$$

Since $e^{-k\delta r}\left(M_{t+k\delta}+B_{t+k\delta}\right)\longrightarrow 0$ as $k\longrightarrow\infty$, then

$$\sum_{s=0}^{\infty}\left(e^{-(s+1)\delta r}-e^{-(s+2)\delta r}\right)M_{t+(s+1)\delta}=\sum_{s=0}^{\infty}e^{-(s+1)\delta r}P_{t+s\delta}\left(w\tau h_{t+s\delta}-g\right)+B_t+e^{-\delta r}M_t.$$

As $\delta\longrightarrow 0$

$$\begin{aligned} B_t+M_t &= r\int_t^\infty e^{-r(s-t)}M_s ds+\int_t^\infty e^{-r(s-t)}\tau P_s wh_s ds \\ &\quad -\int_t^\infty e^{-r(s-t)}P_s g ds. \end{aligned}$$

After dividing by P_t both sides of the equation one gets the intertemporal budget constraint of the government,

$$\begin{aligned} b_0+m_0 &= r\int_0^\infty e^{(\pi-r)s}m_s ds+\int_0^\infty e^{(\pi-r)s}\tau wh_s ds \\ &\quad -\int_0^\infty e^{(\pi-r)s}g ds. \end{aligned} \tag{12}$$

Clearing conditions

In equilibrium, all markets clear. Labor market clearing was already assumed to save on notation. The money demand is equal to the supply of money:

$$\int_0^1 M_t\left(i\right)di=M_t. \tag{13}$$

The demand for bonds by each household is equal to the total supply:

$$\int_0^1 B_t(i)\, di = B_t. \tag{14}$$

The clearing of the goods markets implies that aggregate private consumption plus public consumption plus financial services equals production:

$$g + \int_0^1 c_t(i) di + \frac{\gamma}{N} = y_t. \tag{15}$$

SOLVING FOR THE EQUILIBRIUM

A competitive equilibrium is a sequence of policies, allocations and prices such that: (i) the private agents (firms and households) solve their problems given the sequences of policies and prices, (ii) the budget constraints of the government are satisfied and (iii) markets clear

We are interested in studying the steady state equilibria of this economy, as such we assume that the initial distribution of bonds and money among the households is such that the economy is in the steady state. The equilibrium steady state has the properties that all holding periods, have the same duration, N, and that all households behave similarly during their holding periods. Thus, all households readjust their portfolio in the same way, being equal the fraction of households that readjust their portfolio at any moment in this interval. Thus, household $i \in [0, 1]$, that initially adjusts its portfolio at date $n(i) \in [0, N)$, will also readjust its portfolio at dates $n(i) + jN$ for $j = 1, 2, ...$

Consumption

We start by computing the consumption of household i. First we compute the equilibrium value of an artificial variable, $\mathfrak{c}_t$, given the value of this variable we get the equilibrium values of consumption $c_t(i)$ and of hours $h_t(i)$. From (8), (9) and (11) we get

$$\mathfrak{c}_t(i) \equiv \left(c_t(i) - \frac{(w_t^*(i))^{\frac{1+\chi}{\chi}}}{1+\chi} \right) = \frac{e^{-\eta r \left(t - T_{j(i)}\right)}}{[\lambda P_0]^\eta}, \tag{16}$$

where we used the fact that the nominal interest rate, r, in the steady state is

$$r = \rho + \pi.$$

It is clear, from (16), that in the interval $t \in [T_j(i), T_{j+1}(i))$, $\mathfrak{c}_t$ decreases at rate ηr. Let $\mathfrak{c}_0$ be the value of $\mathfrak{c}_t$ at the beginning of the holding period, then

$$\mathfrak{c}_t(i) = \mathfrak{c}_0 e^{-\eta r\left(t - T_{j(i)}\right)} \text{ for } t \in [T_j(i), T_{j+1}(i)). \tag{17}$$

The equilibrium condition in the good market can be rewritten as

$$g + \int_0^1 \mathfrak{c}_t(i) di + \int_0^1 \frac{(w_t^*(i))^{\frac{1+\chi}{\chi}}}{1+\chi} + \frac{\gamma}{N} = A \int_0^1 h(i)\, di.$$

We can solve for $\mathfrak{c}_0$ by replacing $\mathfrak{c}_t(i)$, given by (17), in the equation above,

$$\mathfrak{c}_0 = \frac{\eta r N}{(1 - e^{-\eta r N})} \left(\frac{A w^{\frac{1}{\chi}}}{N} \frac{1 - e^{-\frac{rN}{\chi}}}{\frac{r}{\chi}} - g - \frac{w^{\frac{1+\chi}{\chi}}}{N(1+\chi)} \frac{1 - e^{-rN\frac{1+\chi}{\chi}}}{r\left(\frac{1+\chi}{\chi}\right)} - \frac{\gamma}{N} \right). \tag{18}$$

Given $\mathfrak{c}_0$ we obtain $\mathfrak{c}_t(i)$ from (17). Given w and $w_t^*(i)$ we obtain the equilibrium values for $c_t(i)$ from 16 and $h_t(i)$ from 11.

Holding Period

Here we compute the size of the holding period, N. The derivation of the expression for N is straightforward but cumbersome, as such the details are in the Appendix. After using conditions 8, 9, 10, and replacing μ_{T_j} with $\lambda Q_{T_j(i)}$, one can obtain

$$\begin{aligned} & rN\mathfrak{c}_0 \left(\frac{e^{(\pi - \eta r)N} - 1}{(\pi - \eta r) N} - \frac{e^{-(\eta-1) rN} - 1}{-(\eta - 1) rN} \right) + \gamma (r - \pi) \\ & + rN \left(\frac{w^{\frac{1+\chi}{\chi}}}{1+\chi} \right) \left(\frac{e^{\left(\pi - r\frac{1+\chi}{\chi}\right)N} - 1}{\left(\pi - r\frac{1+\chi}{\chi}\right) N} - \chi \frac{1 - e^{-\frac{rN}{\chi}}}{rN} \right) \\ = \; & 0 \end{aligned} \tag{19}$$

Money Demand

At any time t there will be households that after the initial period, are in their $j+1$ holding period, while others are in their j holding period. For a household i that is in its $j+1$ holding period, its real money demand is

$$M_t(i) = \int_t^{T_{j+2}(i)} P_0 e^{\pi z} c_z(i)\, dz$$

while the money demand for a household that is in its j holding period at time t is

$$M_t(i) = \int_t^{T_{j+1}(i)} P_0 e^{\pi z} c_z(i)\, dz$$

It can be shown, see the appendix, that the aggregate real money demand, $\frac{M_t}{P_t} = \frac{1}{P_t}\int_0^1 M_t(i)\, di$, can be rewritten as

$$\Rightarrow \frac{M_t}{P_t} = \frac{\mathfrak{c}_0}{N(\eta r-\pi)}\left[\frac{1-e^{-r\eta N}}{r\eta} - \frac{e^{-(\eta r-\pi)N}\left(1-e^{-\pi N}\right)}{\pi}\right] + \frac{P_0 w^{\frac{1+\chi}{\chi}} e^{\pi t}}{N(1+\chi)\left(r\frac{1+\chi}{\chi}-\pi\right)}\left[\frac{1-e^{-r\frac{1+\chi}{\chi}N}}{r\frac{1+\chi}{\chi}} - \frac{e^{-r\frac{1+\chi}{\chi}N}\left(e^{\pi N}-1\right)}{\pi}\right]. \tag{20}$$

Equilibrium

Given $\{g, r\}$ the steady state equilibrium conditions for $\{w, \mathfrak{c}_0, N, \tau, m\}$ solve the system of equations (11), (12), (18), (19) and (20). The remaining equilibrium variables, $c_t(i)$ and $h_t(i)$ can be obtained from:

$$\mathfrak{c}_t(i) = \mathfrak{c}_0 e^{-\eta r\left(t-T_{j(i)}\right)}, \text{ for } t \in [T_j(i), T_{j+1}(i))$$

$$h_t(i) = w e^{-\frac{r}{\chi}(t-T_j(i))}, \text{ for } t \in [T_j(i), T_{j+1}(i))$$

and

$$\left(c_t(i) - \frac{(h_t(i))^{1+\chi}}{1+\chi}\right) = \mathfrak{c}_t(i).$$

The following results are derived in the appendix. The first says that the equilibrium holding period increases if the transfer cost increases. And the second result says that the equilibrium holding period decreases if the nominal interest rate increases. Both of them are quite intuitive.

Results 1: $\frac{\partial N}{\partial \gamma} > 0$
Proof: See Appendix

Result 2: $\frac{\partial N}{\partial r} < 0$.
Proof: See Appendix

RAMSEY PROBLEM

Here we study the Ramsey problem of this economy. Clearly, if lump-sum taxation was available setting the nominal interest rate equal to zero would be the first best. With distortionary taxation, the answer is less trivial. We investigate which is the best allocation associated with a steady state equilibrium. The Ramsey problem is

$$\max_{\{c_t(i),h(i),m,\tau,\pi\}} U^T \equiv \int_0^1 \frac{1}{1-1/\eta}\left(c_t\left(i\right) - \frac{\left(h_t\left(i\right)\right)^{1+\chi}}{1+\chi}\right)^{1-1/\eta} di$$

subject to five constraints: (11), (12), (18), (19) and (20). The objective function of this problem can be rewritten as

$$U^T = \left(\mathfrak{c}_0\right)^{1-1/\eta}\left[\frac{e^{(1-\eta)rN}-1}{N\left(1-\eta\right)r}\right].$$

It is optimal for the government to make the initial total public debt equal to zero. So we assume, without loss of generality, that $b_0 + m_0 = 0$. Thus, in the steady state, public consumption is equal to the inflation tax plus the income tax,

$$rm + \tau w \int_0^1 h\left(i\right) di = g. \tag{21}$$

Result 3: The Friedman rule applies, Friedman (1969).

Proof: (sketch) If $r = 0$ then all households equate their MRS between themselves and to the real wage w. The only remaining distortion is between the MRT and the MRS due to the tax, since $w = \frac{A}{(1+\tau)}$.

Welfare Cost of Inflation

Let $\bar{r}$ be the lower interest rate and ask by how much would consumers need to be compensated to be as well as after the interest rate increase to r. Let $U^T(\bar{r}, \overline{g}, \Delta) = \left(\mathfrak{c}_0\right)^{1-1/\eta}\left[\frac{e^{(1-\eta)rN}-1}{N(1-\eta)r}\right]$, where $\mathfrak{c}_0$ and N are the equilibrium values for the economy when the nominal interest rate and government expenditures are $\bar{r}$ and $\overline{g}$ respectively, and there is an exogenous transfer to each household of an extra flow of real income equal to Δ. The income compensation so that agents are indifferent between $\bar{r}$ and r is defined as $U^T\left(\bar{r}, \overline{g}, 0\right) = U^T\left(r, \overline{g}, \Delta\right)$. In the economy with variables $\left(r, \overline{g}, \Delta\right)$ the market clearing condition is

$$\overline{g} + \int_0^1 c_t(i) di + \frac{\gamma}{N} = A \int_0^1 h\left(i\right) di + \Delta.$$

The equation for consumption becomes

$$\mathfrak{c}_0 = \frac{\eta r N}{\left(1 - e^{-\eta r N}\right)} \left(\frac{A w^{\frac{1}{\chi}}}{N} \frac{1 - e^{-\frac{rN}{\chi}}}{\frac{r}{\chi}} + \Delta - \overline{g} - \frac{w^{\frac{1+\chi}{\chi}}}{N(1+\chi)} \frac{1 - e^{-rN\frac{1+\chi}{\chi}}}{r\left(\frac{1+\chi}{\chi}\right)} - \frac{\gamma}{N} \right)$$

Of all the equilibrium equations, the equation above is the only one that changes. To the previous system of equilibrium allocations we add up one more variable, Δ, and one more equation

$$\frac{(\mathfrak{c}_0)^{1-1/\eta}}{1-1/\eta} \left[\frac{e^{(1-\eta)rN} - 1}{N(1-\eta) r} \right] = U^T(r, \overline{g}, \Delta)$$

As we have as many equations as unknowns we can solve the system of equations. But, first we parameterize the economy.

Parameterization

Labor supply elasticity: $\frac{1}{\chi}$, is set to 0.5. The highest value found in the literature is 1.6; lower values are validated by micro studies. Degree of risk aversion: $1/\eta$, is set to 2. Usually the estimates for η, the elasticity of intertemporal substitution, are above 0.1 and below 10. Government consumption set equal to 20% of the output and the real interest rate to 3%. The cost of the transfer is set equal to the output produced during a quarter of a day. Using this parameterization we compute the welfare cost of increasing the inflation rate from 0% to 10%. The optimal value of N when inflation is 0% is about 3 months and 2 weeks and when inflation is 10% is 1 month and 3 weeks. The welfare cost of inflation, when N is endogenous is 0.38% of the output. The welfare cost of inflation, when N is fixed at 3 months and 2 weeks is 0.0049%.

GOVERNMENT CONSUMPTION MULTIPLIER

In this section we consider the KPR preferences and physical capital. The King-Plosser-Rebelo, KPR, utility function is

$$u(c_t(i), h_t(i)) = \frac{\left[c_t(i)(1 - h_t(i))^{\alpha}\right]^{1-1/\eta}}{1 - 1/\eta}.$$

Now we also assume that production requires physical capital in addition to labor. It is given by

$$y_t = A K_t^{\theta} L_t^{1-\theta}.$$

We want to compute the effect over total consumption and output of an increase in government consumption financed by either i) an increase in the wage tax or ii) an increase in inflation?

Parameterization

We chose standard values for the parameters: $\alpha = 0.5$; $\eta = 2$; $\theta = 0.3$. Depreciation of capital was set equal to 10%. Government consumption was set equal to 20% of the output. The real interest rate was set equal to 3% and the inflation rate to 2%. The transfer cost, γ, was set equal to the output produced during half of a day. In equilibrium $N = 3$ weeks for a nominal interest rate of 5% and the tax rate to 29%. First we study the case when the increase in public consumption is financed by the income tax. When we fix N at its optimal level for G equal to 20% of the output, a 10% increase in government consumption leads to an increase of 0.05% in the output, while private consumption drops 4.3%. When N is endogenous, the 10% increase in government consumption leads to an increase of 0.25% in the output. Private consumption drops more, 4.8%, the holding period and real money holdings decrease. Capital, labor and taxes increase.

The second case considered was the case when the increase in public consumption is financed by seigniorage. With N fixed at the optimal level for G equal to 20% of output, a 10% increase in government consumption leads to a 0.58% increase in the output. The equilibrium inflation increases from 2% to 42% and consumption decreases 3.4%. With N endogenous, a 10% increase in government consumption leads to a 9.7% increase in the output. The equilibrium inflation increases from 2% to 19% and consumption decreases 21%.

CONCLUSIONS

The consideration of endogenous timing for the transactions increases the welfare costs of inflation even when lump sum taxation is not available.

The government consumption multiplier is larger when the timing of the transactions is endogenous. The multiplier is much bigger if public consumption is financed with seigniorage.

APPENDIX

To be completed.

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