Financial Frictions and Interest Rate Shocks*

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Abstract

We study the effects of interest rate shocks in a model with financial frictions. The shocks are unexpected increases in the nominal interest rate. Financial frictions are introduced as a cost to transfer money between bond and goods markets. Financial frictions imply market segmentation because only part of the agents participates in open market operations at each time. Moreover, market segmentation is endogenous because agents decide the moment to participate. We compare the predictions of the model with fixed and endogenous segmentation. Endogenous segmentation reproduces the following two empirical facts: the decrease in the stock of money just after the shocks, and the gradual decrease in the real quantity of money after an increase in the interest rate.

JEL classification: E3, E4, E5.

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1. Introduction

We obtain a slow response of prices and money and a decrease in the quantity of money after interest rate shocks. The shocks are modeled as unexpected increases in the nominal interest rate. The key aspect of the model is endogenous market segmentation: the number of agents that participate in open market operations changes with the interest rate. A slow response of prices and money and a decrease in the quantity of money after interest rate shocks is found for example in Cochrane (1994), King and Watson (1996), Leeper et al. (1996), Christiano et al. (1999), Bernanke et al. (2005), and Uhlig (2005). We offer a model to explain these facts.

We study two shocks: a permanent and a temporary increase of one percentage point of the nominal interest rate. These shocks simulate increases in the federal funds rate induced by the monetary authority. The main contribution is to obtain the transitions of prices and money with the following two characteristics: (1) In the short run, slow responses of prices and money and a decrease in the quantity of money. (2) In the long run, money growth rate equal to the inflation rate and, for the permanent increase in the interest rate, a decrease in the real quantity of money. These two characteristics agree with the empirical evidence on the short and long run behavior of prices and money. Market segmentation has been used to explain the slow response of prices and money after shocks. The advantage of endogenous segmentation, instead of fixed segmentation, is to reproduce the decrease in the quantity of money.

An economy shows market segmentation when only a part of the agents trades in the asset market at each time. Only those agents in the asset market participate in open market operations. As it happens in practice, moreover, agents do not receive money from the monetary authority through transfers. The monetary authority changes the supply of money through open market operations, trading with those agents in the asset market. Grossman and Weiss (1983) and Rotemberg (1984) proposed the first models of market segmentation, further developed, among others, by Fuerst (1992), Alvarez and Atkeson (1997), and Occhino (2004).

The novelty is to make market segmentation endogenous: the time to trade in the asset market is a choice. When the interest rate increases, the agents anticipate their trades and so the number of agents in open market operations increases. This change implies a stronger decrease in the stock of money and a better match with the data.

Prices are flexible. There are no assumptions on sticky wages, staggering price setting, or sticky prices. To make clear the role of fixed and endogenous segmentation, we compare the transitions with the two forms of segmentation. We find that the slow response of prices is a result of market segmentation and that the decrease in the quantity of money is a result of endogenous segmentation.

The friction is a transfer cost. Agents trade bonds for money infrequently because there is a cost to transfer money from the asset market, where agents trade bonds for money, to the goods market, where agents trade goods. As in the Baumol (1952) and Tobin (1956) model of the demand for money, agents decrease the interval between transfers when the interest rate increases. By decreasing the interval between transfers, they decrease money holdings. Comparing with fixed segmentation, the decrease in money holdings after an interest rate increase is stronger.

The change in the transfer times happens gradually. As Friedman (1969, p. 13) writes when describing the effects of changes in monetary policy, "it takes time for people to catch on to what is happening. (...) [T]hey delay the adjustment of actual to desired balances." In the model, the delay happens because agents change gradually their transfer times: those with less money adjust faster than those with more money. The delay does not happen in the model for lack of information, all agents know about the change in policy. The delay happens because agents prefer to postpone the payment of the transfer. Empirically, Christiano et al. (1996), and Vissing-Jorgensen

(2002) find evidence that households take time to adjust their portfolios after a shock.

This paper is most related to Alvarez et al. (2002) and Alvarez et al. (2009). As in Alvarez et al. (2002), the agents pay a fixed cost to make transfers. However, Alvarez et al. (2002) assume that the cash constraint binds in every period and, as a result, velocity is constant. In Alvarez et al. (2009), the cash constraint does not bind in every period because the agents maintain some of their balances for the future. The heterogeneity of money holdings complicates the problem when there are short run fluctuations, however, and so Alvarez et al. (2009) simplify the model by fixing the transfer times. The present paper can be understood as an extension of Alvarez et al. (2002) by letting agents maintain their balances over longer periods, or of Alvarez et al. (2009) by endogenizing the transfer times.

We endogenize the transfer times but simplify the economy in a different way: we study the transition given the new interest rate paths and remove all other shocks to the economy. The monetary authority unexpectedly announces the new interest rate paths from an initial steady state. An advantage of removing these shocks is to focus on the effects of segmentation.

This paper is also related to Khan and Thomas (2010). We let agents decide the interval between transfers while agents in Khan and Thomas decide whether to make transfers according to a random draw of the transfer cost. Another difference is that Khan and Thomas study changes in the money growth rate while we study the effects on prices and money after changes in the nominal interest rate.[1]

We use the model of Silva (forthcoming), which combines the general equilibrium Baumol-Tobin models in Jovanovic (1982) and Romer (1986) and the market seg-

[1] Other papers that study the effects of shocks with different versions of the Baumol-Tobin model are Romer (1987), Fusselman and Grossman (1989), Heathcote (1998), and Chiu (2007). These models have the transfer cost in the utility function. Here, the transfer cost is in goods, which is important for the convergence. Romer keeps the real interest rate constant. Heathcote has a discrete-time version of Romer. Chiu finds that the effects of segmentation disappear with large shocks. Here, the effects of segmentation are always present.

mentation models in Grossman and Weiss (1983), Rotemberg (1983) and Grossman (1987). From the Baumol-Tobin model, the time to trade bonds for money is a choice. From the market segmentation model, the monetary authority can only change the stock of money through open market operations (there are no helicopter drops of money) and only a fraction of the agents are in the asset market at each time. We add to the model the possibility of interest rate shocks and obtain the transition after the shocks.

The advantage of the model is the variation of the demand for money caused by changes in the nominal interest rate. Cash-in-advance models or models with fixed segmentation imply small variation in velocity (Hodrick et al. 1991). Here, the financial frictions imply that the agents change their behavior according to the nominal interest rate more strongly. Silva (forthcoming) focus on the steady state to calculate the welfare cost of inflation for each steady state. Here, we focus on the transition after a change in the nominal interest rate. The fluctuations of the demand for money after the shocks imply the real effects of the nominal interest rate shocks.

2. The Model

There is a continuum of agents with measure one. They are indexed by n. Each agent has a brokerage account and a bank account, as in Alvarez et al. (2009). The brokerage account contains bonds and the bank account contains money. Time is continuous to avoid integer constraints on the time to make transfers between the brokerage account and the bank account, $t \geq 0$. At $t = 0$, each agent has $M_0(n)$ in money in the bank account and $B_0(n)$ in bonds in the brokerage account.

Money is used in the brokerage account and in the bank account. Only the money in the bank account can be used to buy goods. On the other hand, bond trades can only be done with the money in the brokerage account. Money in the brokerage account cannot be used to buy goods and does not receive interest. So, any money sent to the brokerage account is immediately converted into bonds. The agents have to make a transfer from the brokerage account to the bank account to buy goods.

A transfer of money between the brokerage account and the bank account costs Γ goods. The transfer cost is the crucial assumption of the paper. It implies a nondegenerate distribution of money holdings across agents and a slow response of prices after monetary shocks. Let $M(t,n)$ denote money holdings of agent n at time t available in the bank account to buy goods. Let $T_j(n)$, $j = 1, 2, ...$, denote the times of each transfer of agent n. The time interval $[T_j(n), T_{j+1}(n))$ defines a holding period. Just before $T_j(n)$, agent n has money holdings $M^-(T_j(n), n) \equiv \lim_{t \to T_j, t<T_j} M(t,n)$. In the same way, agent n has $M^+(T_j(n), n) \equiv \lim_{t \to T_j, t>T_j} M(t,n)$ just after a transfer. Let $P(t)$ denote the price level at t. In order to start the jth holding period with $M^+(T_j(n))$ in the bank account, the agent has sell $M^+(T_j(n)) - M^+(T_j(n)) + P(T_j)\Gamma$ dollars of bonds, pay the transfer cost $P(T_j)\Gamma$ with the resources of the brokerage account, and transfer $M^+(T_j(n)) - M^+(T_j(n))$ to the bank account.

The consumption good is produced by firms. The firms have linear production function and produce Y units of goods with one unit of labor. Production is constant to isolate the effects of market segmentation and to simplify the computation of the transition. At time t, the firms sell their production at the goods market for $P(t)Y$, send the proceeds to their brokerage account, and convert them into bonds.

An agent is composed of a shopper, a trader, and a worker, as in Lucas (1990) (and as it is common in models of market segmentation). The shopper uses the money holdings $M(t,n)$ to buy goods from firms. The trader manages the bond trades of the brokerage account and the transfers between account. The worker supplies one unit of labor inelastically to the firms. Given the linear production function, the workers receive $P(t)Y$ for their work and the firms make zero profits. To pay their workers, the firms make transfers of $P(t)Y$ from their brokerage account to the brokerage account the workers. Therefore, the workers receive their income in interest-bearing bonds, but have to pay a transfer cost to transform this income into money to buy goods. This setup is implicit in the Baumol-Tobin model (Karni 1973).[2]

The agents choose consumption $c(t,n)$, money $M(t,n)$, and the transfer times $T_j(n)$, $j = 1, 2, ...$ Let the bond price at time zero be given by $Q(t)$, with $Q(0) = 1$. The nominal interest rate at time t is then given by $r(t) = -d \log Q(t)/dt$. During a holding period, bonds in the brokerage account follow $\dot{B}(t,n) = r(t) B(t,n) + P(t) Y$, where $\dot{x}(t) \equiv dx(t)/dt$. At the time of the transfer T_j, the quantity of bonds and money must satisfy

$$M^+(T_j(n)) + B^+(T_j(n)) + P(T_j(n)) \Gamma = M^-(T_j(n)) + B^-(T_j(n)), \; j = 1, 2, ..., \quad (1)$$

where B^+ and B^- denote bonds just after and just before the transfer. Substituting recursively the value for bond holdings, together with the condition $\lim_{t\to\infty} Q(t) B(t,n) = 0$ yields the present value budget constraint

$$\sum_{j=1}^{\infty} Q(T_j(n)) \left[M^+(T_j(n), n) + P(T_j(n)) \Gamma \right] \leq \sum_{j=1}^{\infty} Q(T_1(n)) M^-(T_j(n), n) + W_0(n), \quad (2)$$

where $W_0(n) = B_0(n) + \int_0^\infty Q(t) P(t) Y dt$.

The maximization problem of agent n is then

$$\max_{c,M,T_j} \sum_{j=0}^{\infty} \int_{T_j(n)}^{T_{j+1}(n)} e^{-\rho t} u(c(t,n)) \, dt \quad (3)$$

[2] In Silva (forthcoming), the firms send the fraction $1 - a$ of the sales proceeds to the brokerage account and send the fraction a directly to the workers in money. These flows of money affect the transition after shocks beyond the segmentation effect caused by the transfers times $T_j(n)$. We set $a = 0$ here to concentrate on the effects of market segmentation caused by the decision on the transfer times.

subject to (2) and

$$\dot{M}(t,n) = -P(t)\,c(t,n)\,,\ t \neq T_1(n)\,,T_2(n)\,,...,\tag{4}$$

$M(t,n) \geq 0$, $T_{j+1}(n) \geq T_j(n)$, given $M_0(n) \geq 0$, where $\rho > 0$ is the rate of intertemporal discounting. We use logarithmic utility $u(c(t,n)) = \log c(t,n)$ to simplify the characterization of the transition. At $t = T_1(n)\,,T_2(n)\,,...$, constraint (4) is replaced by $\dot{M}(T_j(n)\,,n)^+ = -P(t)\,c^+(T_j(n)\,,n)$, where $\dot{M}(T_j(n)\,,n)^+$ is the right derivative of $M(t,s)$ at $t = T_j(n)$ and $c^+(T_j(n)\,,n)$ is consumption just after the transfer. $T_0(n) \equiv 0$ to simplify the exposition, there is not a transfer at $t = 0$, unless the agent decides to set $T_1(n) = 0$.

Equation (4) is a cash-in-advance constraint. It states that it is only possible to buy goods with money. In this deterministic setting, the agents will choose $M^+(T_j(n))$ so that $M^-(T_{j+1}(n)) = 0$. If $M^-(T_{j+1}(n)) > 0$, the agent would have lost income from interest during the holding period $[T_j, T_{j+1})$. It would always be optimal to decrease $M^+(T_{j+1}(n))$ and then finish the holding period with $M^-(T_{j+1}(n)) = 0$. Only $M^-(T_1(n))$ can be positive, as M_0 is given rather than being a choice. In particular, an interest shock may surprise an agent with too much money. The agent then may anticipates transfer, even having $M^-(T_1(n)) > 0$ at that time. After $j = 1$, the agents will always set $M^-(T_{j+1}(n)) = 0$. Therefore, equation (4) implies that

$$M^+(T_j(n)) = \int_{T_j(n)}^{T_{j+1}(n)} P(t)\,c(t,n)\,dt.\tag{5}$$

Let $\Gamma = \gamma Y$ to make the budget constraint (2) linear in income. With this, consumption and the demand for money will be linear in income. The consumption-income ratio will be independent of a particular level of income. Moreover, the elasticity of the demand for money with respect to income will be equal to one, which agrees with the evidence in Lucas (2000) and in others.

The government controls the quantity of money through open market operations. As a result, the initial quantity of bonds is equal to the present value of all changes in the quantity of money. The government budget constraint is then given by $B_0^G = \int_0^\infty Q(t)\,\dot{M}(t)\,dt$, where B_0^G is the aggregate quantity of bonds and $M(t)$ is the aggregate money supply.[3]

The market clearing conditions for money and bonds are $\int M(t,n)\,dF(n) = M(t)$ and $\int B_0(t,n)\,dF(n) = B_0^G$, where F is a given distribution of agents. For the market clearing condition for goods, we have to take into account the transfer cost and the number of transfers. The number of transfers varies if the interest rate changes. It increases when the interest rate increases, for example. This is so because the agents trade bonds more frequently to decrease their money holdings. Let $A(t,\delta) \equiv \{n : T_j(n) \in [t, t+\delta]\}$ denote the set of agents that make a transfer during $[t, t+\delta]$. The number of goods used on average during $[t, t+\delta]$ to pay the transfer cost is then given by $\int_{A(t,\delta)} \frac{1}{\delta}\Gamma dF(s)$. The market clearing condition for goods is given by $\int c(t,n)\,dF(n) + \lim_{\delta\to 0}\int_{A(t,\delta)} \frac{1}{\delta}\Gamma dF(n) = Y$.

An equilibrium is defined as prices $P(t)$, $Q(t)$, allocations $c(t,n)$, $M(t,n)$, $B(t,n)$, transfer times $T_j(n)$, $j = 1, 2, ...$, and a distribution of agents F such that (i) $c(t,n)$, $M(t,n)$, $B(t,n)$ and $T_j(n)$ solve the maximization problem of each agent given $P(t)$ and $Q(t)$ for all $t \geq 0$ and n in the support of F; (ii) the government budget constraint holds; and (iii) the market clearing conditions for money, bonds, and goods hold.

To solve the agent's maximization problem, substitute $M^-(T_{j+1}(n)) = 0$ and the expression of $M^+(T_j(n))$ given by (5) in the constraint (2). The first order condition

[3] Alternatively, this equation states that the government budget constraint is such that the government collects seigniorage and redistributes it to agents as initial bonds, $B_0^G = \int_0^\infty Q(t)\,P(t)\,\frac{\dot{M}(t)}{P(t)}dt$

with respect to consumption then implies

$$P(t)\, c(t,n)\, \lambda(n)\, Q(T_j(n)) = e^{-\rho t},\ t \in (T_j(n), T_{j+1}(n)),\ j = 1, 2, ..., \tag{6}$$

where $\lambda(n)$ is the Lagrange multiplier of (2). The first order condition with respect to $T_j(n)$, $j \geq 2$, implies

$$\gamma Y[r(T_j(n)) - \pi(T_j(n))] + r(T_j(n)) \int_{T_j(n)}^{T_{j+1}(n)} \frac{P(t)c(t,n)}{P(T_j(n))} dt = c^+(T_j(n)) \log \frac{c^+(T_j(n))}{c^-(T_j(n))}, \tag{7}$$

where $c^-(T_j(n), n)$ denote consumption just before $T_j(n)$ and $\pi(t)$ denote inflation at time t.

The left hand side of (7) is the marginal gain of delaying the transfer and the right hand side is the marginal loss. The marginal gain is given by postponing the transfer and decreasing real balances from $T_j(n)$ to $T_{j+1}(n)$. The marginal loss is the effect in utility of changing the length of holding periods $T_{j-1}(n)$, $T_j(n)$ and $T_j(n)$, $T_{j+1}(n)$. Denote the interval between transfers by $N_j(n) = T_j(n) - T_{j-1}(n)$, $j = 1, 2, ...$ So, $T_j(n) = \sum_{s=1}^{j} N_s(n)$. In a standard cash-in-advance model, the difference $N_j(n) = T_j(n) - T_{j-1}(n)$ is constant, equal to one quarter for example. Here, N_j is obtained in equilibrium.

We model the economy so that an interest rate shock hits the economy in the steady state. We need then to find the distribution of money holdings in the steady state. When a shock hits the economy, the values of $M_0(n)$ are equal to the distribution of money holdings in the steady state. The agents then solve their maximization problems given $M_0(n)$.

A steady state is defined as an equilibrium in which the interest rate is constant and the inflation rate is constant. Moreover, to make the agents homogeneous in their characteristics, we define the steady state as an equilibrium in which the agents have the same pattern of consumption within holding periods. In particular, they start a holding period with the same level of consumption $c^+ (T_j (n), n) = c_0$, to be found in equilibrium. The only difference between agents in the steady is their position in the holding period $[T_j (n), T_{j-1} (n))$. As stated in Silva (forthcoming), these properties of the steady state imply that all agents have the same size of holding periods, $N_j (n) = N$, and that the distribution of agents over their position in the holding period is uniform. We can then index the agents by $n \in [0, N)$, with a uniform distribution $F (n)$ over $[0, N)$.

Substituting $r (t) = r$ and $\pi (t) = \pi$ in (6), implies $\frac{\dot{c}(t,n)}{c(t,n)} = -\rho - \pi$ and so individual consumption can be written as $c (t, n) = c_0 e^{(r-\pi-\rho)t} e^{-r(t-T_j)}$, taking the largest j such that $t \in [T_j (n), T_{j+1} (n))$. Aggregating the expressions of $c (t, n)$ implies that aggregate consumption is given by $C (t) = c_0 e^{(r-\pi-\rho)t} \frac{1-e^{rN}}{rN}$. As consumption is constant in the steady state, the nominal interest rate and inflation in the steady state are such that $r = \rho + \pi$.

As a result, the individual behavior in this economy is different from the aggregate behavior. In equilibrium, $r = \rho + \pi$ implies $\frac{\dot{c}(t,n)}{c(t,n)} = -r$. Therefore, although $C (t)$ is constant in equilibrium, consumption decreases faster if the nominal interest rate increases. When a shock hits the economy, the individual reactions on consumption have little impact on aggregate consumption. As a result, the price level reacts slowly to an increase in the nominal interest rate. The real interest rate will then increase after a nominal change. This segmentation effect is also present in Grossman and Weiss (1983) and more recently in Alvarez at al. (2009). In addition to this effect, the changes in the transfer times after the shock imply a decrease in the quantity of money.

The contrast between individual and aggregate variables is an implication of the frictions that each agent in this economy faces. Because it is optimal to make infrequent transfers of money to the bank account, each agent adopts (S, s) policies for consumption and money holding. These policies show variation of consumption and money over time. But aggregate consumption and money are smooth over time. For a recent survey on (S, s) models, see Caplin and Leahy (2010).

To characterize the value of c_0, use the market clearing condition for consumption. It implies $\frac{1}{N}\int_0^N \hat{c}_0(r,N)\, e^{-rx}dx + \frac{\gamma}{N} = 1$, where $\hat{c}$ is the consumption-income ratio. Therefore, $\hat{c}_0(r,N) = (1-\gamma/N)\left(\frac{1-e^{-rN}}{rN}\right)^{-1}$.

The value of the optimal interval between transfers N is obtained with the first order conditions for $T_j(n)$ and $c(t,n)$. Silva (2011) shows that N is given by the positive root of

$$\hat{c}_0(r,N)\, rN\left(1-\frac{1-e^{-\rho N}}{\rho N}\right) = \rho\gamma \tag{8}$$

where $\hat{c}_0(N) = (1-\gamma/N)\frac{rN}{1-e^{-rN}}$. N exists and is unique for all positive r, ρ and γ. N increases with the transfer cost and decreases with the interest rate.

The initial money and bond holdings $M_0(n)$ and $B_0(n)$ to imply a steady state equilibrium are such that agent n consumes at the steady state rate of consumption, and makes transfers at $t = n$, $n+N$, $n+2N$ and so on. We have

$$M_0(n) = P_0 Y n e^{r(n-N)}\hat{c}_0(r,N)\frac{1-e^{-\rho n}}{\rho n}, \tag{9}$$

and $B_0(n)$ is such that $W_0(n) = P_0 Y[\hat{c}_0(N)\frac{1-e^{-\rho N}}{\rho} + \gamma\frac{e^{-\rho n}}{1-e^{-\rho N}}]$, where N is given by (8) and $n \in [0, N)$. $W_0(n)$ is equal to the present value of the transfer fees plus the present value of the transfer amounts, $B_0(n)$ is found by $B_0(n) = W_0(n) - \int_0^\infty Q(t)\, P(t)\, Y dt$. $M_0(n)$ is increasing in n: agents with more initial balances make the first transfer later. We obtain the steady state by assigning $M_0(n)$ and $B_0(n)$ to the agents for a given nominal interest rate r.

The stock of money is obtained by summing individual money holdings. The stock of money at time zero is then $M_0 = \int_0^N \frac{1}{N} M_0(n)\, dn$. The stock of money in the steady state grows at the same rate of inflation, $r-\rho$. So, in the steady state,

$M(t) = M_0 e^{(r-\rho)t}$. The money-income ratio $m(r,\gamma,\rho) \equiv \frac{M}{PY}$, the measure for the real stock of money on which we focus, is constant through time in the steady state for given r,γ, and ρ. Substituting $M(t)$ and $P(t)$ implies

$$m(r) = \frac{\hat{c}_0(r,N)\, e^{-rN(r)}}{\rho} \left[\frac{e^{rN} - 1}{rN(r)} - \frac{e^{(r-\rho)N(r)} - 1}{(r-\rho)\, N(r)} \right], \tag{10}$$

where $\hat{c}_0(r,N) = \left(1 - \frac{\gamma}{N}\right)\left(\frac{1-e^{-rN}}{rN}\right)^{-1}$ and $N(r)$ given by (8).

The elasticity of $m(r)$ with respect to the interest rate is $-1/2$ and the semi-elasticity is -12.5. We interpret these elasticities as long-run interest rate elasticities, as r stands for a value in the steady state. The value for the semi-elasticity is in accordance with the long-run elasticities found in Guerron-Quintana (2009). Lucas (2000) concludes that a money-income ratio with elasticity of $-1/2$ best fits the data. In fact, the expression in (10) is close to a Baumol-Tobin money demand $m(r) = Ar^{-1/2}$. Alvarez and Lippi (2009) also find values for the elasticity close to $-1/2$.

The effects of endogenous segmentation appear through the effects on N. With fixed segmentation, the agents cannot change the value of N. For this reason, the interest-elasticity of the money-income ratio is close to zero when N is fixed (in fact, slightly positive). This result is not particular to this version of the Baumol-Tobin model. Romer (1986) shows that the real stock of money increases with the interest rate if the transfer interval is unresponsive to the interest rate. The main mechanism to adjust money holdings is to change N. With endogenous segmentation, the changes in N imply a higher sensitivity of money holdings to changes in the interest rate.

To calibrate the model, we have to set the parameters ρ and γ. Set $\rho = 3\%$ per year, so that $r = 3\%$ per year implies zero inflation. We set γ by matching the money-income ratio in the data. We use a similar dataset as in Lucas (2000). In particular, we use M1 for the monetary aggregate and commercial paper rate for the interest rate. We choose γ such that m passes through the mean of the money-income ratio and interest rate in the data, $m = 0.257$ and $r = 3.6\%$. This implies $\gamma = 1.79$. With $r = 4\%$ (inflation of 1% per year), $N = 181$ days or 2 transfers per year from high-yielding assets to money. These are not ATM withdrawals. Transfers in the model convert high-yielding bonds to money whereas ATM withdrawals convert bank deposits into cash but do not change the stock of money.

In accordance with other models of market segmentation, the model requires a large transfer cost and a large interval between transfers (Edmond and Weill 2008). Alvarez et al. (2009), for example, have calibrations with N equal to 24 and to 36 months. Khan and Thomas (2010) have average N from 1.2 to 2.4 years and maximum N from 1.5 to 2.5 years. (Khan and Thomas have average and maximum transfer intervals because the transfer cost is stochastic.) The transfer period of 6 months obtained here is smaller mainly because we use M1 and Alvarez et al. and Khan and Thomas use M2.[4]

The large transfer intervals reflect the large money holdings and the low frequency of trades found in the data. The 0.257 money-income ratio implies that each person in the U.S. holds about 9,000 dollars at each time, with per capita income of 35,000 dollars in 2000. Moreover, the ratio of cash to assets of U.S. firms more than doubled from 1980 to 2004 (Bates et al. 2009), which is relevant as U.S. firms in 2000 hold more than 62% of M1 (Bover and Watson 2005). Finally, a large fraction of households trades assets with high yields less than once a year (Vissing-Jorgensen 2002).

These observations are surprising given the technological innovations in financial services of the last decades. The objective of this paper, however, is not to explain the large cash holdings of households and firms (for this, see for example Bates et al. 2009 and Alvarez and Lippi 2009). We take as given the fact that households

[4] Alvarez et al. and Khan and Thomas also assume that agents receive 60 percent of their income directly in the bank account. This assumption increases my calibrated N to 16 months when $r = 4\%$.

and firms adjust their demands for money to maintain cash during a holding period. Taking into account this behavior, we calculate the effects of shocks to the nominal interest rate.

3. Interest Rate Shocks

We study two shocks, a permanent shock and a temporary shock. In the permanent shock, the nominal interest rate increases from 3% to 4% per year and stays at 4% per year. In the temporary shock, the nominal interest rate increases to 4% and gradually returns to 3% per year.

We change the interest rate rather than the money supply to be closer to the policy of central banks and to simplify the analysis. Central banks usually track the interest rate in their daily operations rather than the quantity of money (Woodford 2003). Implicitly, they assume that the quantity of money changes according to the interest rate. The path of the nominal interest rate in the model is the counterpart of a relevant overnight interest rate, such as the federal funds rate. Woodford (2003), and Nautz and Schmidt (2009), for example, discuss how a central bank is able to affect the dynamics of the federal fund rate. Other papers that study monetary policy shocks by setting the nominal interest rate and assuming that the money supply changes to satisfy the market clearing condition are Grossman (1987), Christiano et al. (2005) and Alvarez et al. (2009).

We also decrease the dimensionality of the problem by setting the nominal interest rate. Instead of working with two equilibrium prices, $P(t)$ and $r(t)$, and two market clearing conditions, for goods and money, we only have to find $P(t)$, with the market clearing condition for goods. With $P(t)$ and the given interest rate, we obtain the money supply through the market clearing for money. The problem would be intractable if we had to find $P(t)$ and $r(t)$ with endogenous segmentation and two market clearing conditions.

When we set an exogenous interest rate path, it is well known that the price level path is indeterminate unless we set an additional nominal variable (Sargent and Wallace 1975). Alvarez et al. (2009), for example, keep the price level at $t = 0$ constant on impact to determine the price level path. The solution here is to introduce contingent bonds. If the probability of a change in the interest rate path is small then the Lagrange multiplier λ is close to its value before the shock. The additional variable that we set, therefore, is λ, calculated in equilibrium for the initial steady state.[5]

Suppose therefore the possibility of two states for the interest rate path and of bonds contingent on the states. In state 1, the nominal interest rate is r_1 in all periods. In state 2, the nominal interest rate has the path $r(t)$. The realization of the state happens at $t = 0$ and the probability of state 2 is small. The nominal interest rate is constant in state 1 and the probability of the shock is small to make the economy behave as if it were in the initial steady state and the interest rate changed unexpectedly.

Each agent now maximizes the expected value of utility weighted by the probabilities of each state. As money is not contingent on the states, agent n uses $M_0(n)$ from $t = 0$ to the first transfer $T_1(n)$. We then have two budget constraints from $t = 0$ to $T_1(n)$, one for each price level path in each state. After $T_1(n)$, on the other hand, we have only one budget constraint because agents use contingent bonds to transfer resources between states. (The analytical statement of the problem is in the appendix.)

The initial cross section of money is close to the one in the initial steady state, as the probability of the shock is small. To obtain optimal consumption and transfer times, therefore, first calculate money and bond holdings such that the economy is in

[5] Grossman (1987) also specifies a Lagrange multiplier to study interest rate shocks. However, he writes the price response as a function of the multiplier and assumes different values for the multiplier. We show that λ is the same for all agents in the initial steady state and use the equilibrium λ. See Adão et al. (2011) for the question of determinacy of interest-rate rules.

the steady state under r_1. Then, calculate optimal consumption and transfer times for the transition given the interest rate path $r(t)$ and initial money holdings $M_0(n)$.

Using this procedure, consumption at time t and transfer times $T_j(n) \equiv N_1(n) + ... + N_j(n)$ for each agent n are such that

$$c(t,n) = \frac{e^{-\rho t}}{\lambda P(t) Q(T_j(n))},\ t \in (T_j(n), T_{j+1}(n)),\ j \geq 1, \tag{11}$$

$$[R(T_j(n)) - R(T_{j-1}(n))] - \frac{\gamma Y [r(T_j(n)) - \pi(T_j(n))]}{c^+(T_j(n))} = r(T_j(n)) \frac{1 - e^{-\rho N_{j+1}(n)}}{\rho},\ j \geq 2, \tag{12}$$

where $R(t) \equiv -\int_0^t r(s)\,ds$, $c^+(T_j(n)) = e^{-\rho T_j(n)} / [\lambda Q(T_j(n)) P(T_j(n))]$ is consumption just after the jth transfer, and λ is the Lagrange multiplier of the budget constraint after $T_1(n)$. Contingent bonds play their role here. With contingent bonds, λ is the same across states. With a small probability of the shock, λ is close to its value in the first steady (the steps to obtain λ are in the appendix). Once the value of λ in the first steady state is substituted in (11), the value of $c(t,n)$ depends only on one unknown variable, $P(t)$. Substituting $c(t,n)$ for each consumer in the market clearing condition, we have then an equation to determine $P(t)$.

Equation (12) shows how N_j relates to N_{j+1} during the transition. The term with γ appears because the transfer cost is paid in goods. It relates the price level with the decision of transfer times. This term disappears with transfer cost in the utility function.

Different shocks are described by different paths of the interest rate. For the permanent shock, the monetary authority sets $r(t) = 4\%$ per year. For the temporary shock, the monetary authority sets $r(t) = 4\%$ per year at time zero and then reduces it toward 3% per year at a constant rate. The process is $r(t) = r_1 + (r_2 - r_1) e^{-\eta t}$, where $r_1 = 3\%$, $r_2 = 4\%$ and η is the persistency of the shock. We set η to approximate $r(t)$ to the response of a shock similar the response shown in Christiano et al. (1999) and Uhlig (2005), as in Alvarez et al. (2009). For the time in days,

$\eta = (-12 \log 0.87)/365$.

We assume that the economy is initially in equilibrium with a constant nominal interest rate equal to 3% per year. This implies zero inflation before the shock as $\rho = 3\%$ per year. The initial steady state is arbitrary. We choose an initial steady state with zero inflation to facilitate the comparison of the economy before and after the shocks. An initial state with positive inflation does not change conclusions.[6]

We proceed numerically to obtain the equilibrium price level and the other equilibrium values. For each agent, we have a system of equations in the form $h(N_j, N_{j+1}) = 0$. We assume that each agent chooses $N_{J+1} = N'$ after the Jth transfer, where N' is the steady state interval under the new interest rate, to have a finite system. J is large, equal to 40, to approximate the solution. We then have a system of J equations and J unknowns N_1, ..., N_J for each agent. (See the appendix for a detailed description of the algorithm.) The model period is one day.

Proceed as follows to find the price level during the transition. (i) Start with a guess for the price level during the transition. (ii) Calculate the transfer times and consumption for each agent. (iii) Check the market clearing condition. (iv) If the difference between demand and supply is smaller than a preestablished value for every t, stop. If not, change $P(t)$ and repeat steps (i)-(iii).

The results of the simulations for the permanent and the temporary shocks are in figures (1) and (2).[7] The liquidity effect is common to both shocks. In the short run, the price level responds slowly, and money and real balances decrease. After six months, there is an overshooting in the price level for both the permanent and the temporary shocks, followed by dampened oscillations toward the new steady state.

[6] Unless the initial inflation is very high (much higher than the inflation rates between zero and 10% observed for the U.S.). With high inflation, N would approach zero and the economy would be close to an economy without market segmentation.

[7] The results of the daily values show oscillations with decreasing amplitude. In order to focus on the main results of the simulations, figures (1) and (2) show the annual means of money and of the money-income ratio.

The dynamics of money shows the liquidity effect (the initial decrease in the quantity of money) and the later adjustment of money to the new steady state (constant growth for the permanent shock, or convergence to the new quantity of money for the temporary shock).

As the money-income ratio is the inverse of velocity, velocity increases just after an increase of the interest rate. In standard cash-in-advance models, such as Cooley and Hansen (1989), velocity is constant, equal to one. Hodrick et al. (1991) change the standard cash-in-advance model in various ways to allow variation in velocity. They find that, even with the changes, velocity is close to constant. Here, endogenous segmentation implies substantial movements in velocity.

Figures (1) and (2) include the case with fixed segmentation for comparison. Using the same model as above, we let agents optimize between transfers but fix the transfer times. We fix the transfer interval to its value in the first steady state. The two economies behave in the same way in the first steady state. They are different after the shocks. With $r = 3\%$, both economies have $N = 209$ days. With the permanent shock to $r = 4\%$, the agents in the economy with endogenous segmentation gradually decrease N to 181 days. The agents maintain the initial N in the economy with fixed segmentation. With the temporary shock, the agents in the economy with endogenous segmentation temporarily decrease N to 195 days on average. Later, they return to the initial value of 209 days as r returns to 3%. In the simulations, we discretize the interval $[0, N)$ in units of 0.10 days, and say that agent n makes a transfer at day t if $t \leq T_j(n) < t + 1$. With the permanent shock, the average number of transfers per day increases from 10 to 11.6.

The economy with fixed segmentation maintains the number of transfers per day during the transition. We can obtain analytical formulas for the transition with fixed segmentation. The model with fixed segmentation is similar to Alvarez et al. (2009) and to Grossman and Weiss (1983). The difference is that the model is in continuous time and that we removed the short-run variations in the interest rate. Both fixed and endogenous segmentation show a slow adjustment of prices toward the new steady state. A cash-in-advance model would show an instantaneous adjustment. The difference with endogenous segmentation is on the response of money. Because agents anticipate their transfers, money decreases sharply after an increase in the interest rate with endogenous segmentation.

Permanent shock

For the permanent shock, money decreases about 11% during the first two years. The price level increases at a rate lower than its long-run growth rate for the first six months. Real money decreases slowly toward its new steady state. After one year, it decreases about 12% from its initial level and is 2% higher than the new steady state level.

The behavior of money after the shock is compatible with the estimations in Christiano et al. (1999): money decreases for two quarters after a contractionary monetary policy. The model period is one day, as mentioned above. So, the decrease in the quantity of money lasts much beyond the model period.

In the long run, the real stock of money decreases, as the interest rate is higher. Moreover, prices and money grow at the inflation rate, 1% per year, equal to the difference between nominal and real interest rates. The Fisher effect prevails in the long run: the increase in inflation compensates the increase in the nominal interest rate. After the initial stickiness in the price level, there is a sharp overshooting as agents initially synchronize the timing of their response to the shock. This effect on prices is common to both shocks, we analyze it in more detail below.

With endogenous segmentation, the real stock of money decreases after a permanent increase in the nominal interest rate. In accordance with this result, Meltzer (1963) and Lucas (1988), for example, find a negative long-run relation between interest rates and real balances. With fixed segmentation, long-run real balances are approximately constant (they increase 0.1%). This result is mentioned by Romer (1986). Grossman (1987) also mentions that the interest elasticity is close to zero with fixed transfer periods. The key to decrease money holdings is to decrease the interval between transfers. With fixed segmentation, agents change their consumption within holding periods, but this is not enough to decrease real balances. The steady states with fixed and endogenous segmentation are similar if the transfer intervals are the same. But the transition after the shock is very different.

Temporary shock

For the temporary interest-rate increase, nominal and real balances decrease 5% during the first year. The price level increases toward its new steady state level in the long run, 0.6% higher than its initial value. It does not jump to a higher value, as we would have in a standard cash-in-advance model with constant output. Note that this is the response of an increase in the interest rate, not a temporary contraction of the money supply. Grossman and Weiss (1983) study the response after money supply shocks. Grossman (1987) and Alvarez et al. (2009), with fixed segmentation, also study increases in the interest rate and find a long-run increase in the price level.

As the nominal interest rate returns to its initial value, equal to the real interest rate, the inflation rate returns to zero. In the long run, prices and money are constant and real balances return to their initial level. The stock of money is higher than its initial level because there is inflation during the transition.

With the gradual decrease in the interest rate, agents will eventually hold more real balances. As real balances return to their initial level, there is an increase in the stock of money to offset the increase in prices. Prices and money both increase 0.6% in the long run. In the long run, the effects of the price level are the same as in a standard cash-in-advance model.[8] The effects of market segmentation appear in the

[8] The price level increases because output is constant and because the nominal interest rate is always higher than the real interest rate. Output is constant also to isolate the effects of market segmentation, and to facilitate the comparison of the model with the benchmark cash-in-advance

short run.

The price level falls for both shocks and returns to its initial level only after around 30 days. During the first quarter, the price level is approximately constant: the difference between the geometric mean of the price level during the first quarter and the initial price level is only 0.01% for the permanent shock and 0.02% for the temporary shock. A researcher with access to the average price level during this period would probably conclude that the price level is sticky after the shocks.

The reason for the effects in the short run and the dampened oscillations is the different behavior of agents according to their initial balances. The transfer cost makes agents economize in the use of money to avoid making a transfer too soon. Agents with little balances make a transfer sooner. They consume at a faster rate because they decrease the interval between transfers to reflect the higher interest rate. Initially, the number of agents that have made a transfer after the shock is small. Consequently, prices do not change instantaneously with the change in the nominal interest rate.

After about six months, two groups of agents with different consumption patterns meet. The first group is composed of agents who had little balances and were about to make a transfer when the shock hit the economy. They are now making their second transfer. The second group is composed of those who had substantial money holdings at the time of the shock and have not made a transfer since that time. When the two groups meet, there is a fast increase in the price level because the agents in the first group consume at a faster rate and are now making the second transfer. The new steady state interval between transfers for 4% interest rate is 181 days, as stated above, approximately six months. The temporary fluctuations in the price level, at intervals of six months, reflect these large groups of households synchronizing the timing of their response to the shock.

model without further frictions.

Prices become smooth and we have convergence because agents pay the transfer cost in goods. The synchronization of transfers is temporary because prices increase in these dates. Agents change their transfer times to periods in which prices are lower. This behavior eventually makes the number of transfers per day constant and the economy converges to the new steady state. With transfer cost in utility, the price level disappears from the first order conditions and, in contrast, prices and money do not converge. The redistribution of transfers, however, is slow. The economy experiences changes in the price level five years after the shocks. A different calibration would make the transition faster, but the qualitative aspects do not change. If we consider only the period after 1980, with lower money-income ratio, N decreases to 170 days under $r = 3\%$ and to 147 days under 4%. The oscillations would occur at intervals of five months, decaying over time.

Although the short and long run implications of the model are compatible with the empirical evidence, the fluctuations in the price level, temporarily higher than the steady state changes, are not. The model focus on only one mechanism of convergence, the change in the transfer times, and abstracts from several other elements of the actual economies. We can understand the role of endogenous segmentation by comparing of the model with fixed and endogenous N. It is beyond the objectives of this paper to predict all price and money movements with this single modification. We would need to add other elements such as different types of agents, endogenous production, and other kinds of shocks.

Friedman (1969) studies the effects of a once-and-for-all change in the quantity of money and of a continuous increase in the quantity of money. We relate the two exercises with the temporary and permanent increase in the interest rate. The reason is that the final effect of a temporary increase in the interest rate is a once-and-for-all increase in the quantity of money. And the final effect of a permanent increase is a continuous increase in the quantity of money. As mentioned in the introduction, Friedman states that "it takes time for people to catch on to what is happening." The present model gives an analytical explanation for the effects of the shocks. The advantage is that now we can quantify the effects and give a meaning to what we understand by the short and long run. The short run stands for six months, as most effects occur in the first six months, and the long run stands for the behavior after two years, as prices and money are close to their new steady state values.

4. Conclusions

The main contribution of this paper is to show that money adjusts faster to interest rate shocks with endogenous segmentation. Alvarez et al. (2009) show that a model with fixed market segmentation reproduces the slow reaction of prices to changes in monetary policy. We advance to show that endogenous segmentation predicts sharper effects on the quantity of money after changes in the interest rate.

The implications of the model agree with the empirical evidence on the short and long run behavior of prices and money. The price level slowly adjusts to the new steady state. Nominal and real balances decrease after the shocks and slowly adapt to the shocks.

Endogenous segmentation changes results in important ways. First, the quantity of money decreases just after the shocks. Second, real balances decrease with a permanent interest rate increase. Endogenous segmentation helps to explain how a monetary authority affects the stock of money by changing the interest rate.

Appendix

First order conditions and Lagrange multipliers

The first order conditions of (1)-(4) with respect to $c\,(t,n)$ imply $P\,(t)\,c\,(t,n) = \frac{e^{-\rho t}}{\lambda(n)Q(T_j)}$, $t \in (T_j, T_{j+1})$, $P(T_j)c^{+}(T_j,n) = \frac{e^{-\rho T_j}}{\lambda(n)Q(T_j)}$, and $P(T_{j+1})c^{-}(T_{j+1},n) = \frac{e^{-\rho T_{j+1}}}{\lambda(n)Q(T_j)}$.

For $T_j(n)$, $j = 2, 3...$, the first order conditions imply $\frac{e^{-\rho T_j}}{\lambda(n)P(T_j)Q(T_j)} \log \frac{c^+(T_j,n)}{c^-(T_j,n)} = r(T_j) \int_{T_j}^{T_{j+1}} \frac{P(t)c(t,n)}{P(T_j)} dt + c^+(T_j, n) - \frac{Q(T_{j-1})}{Q(T_j)} c^-(T_j, n) + \gamma Y[r(T_j) - \pi(T_j)]$. For $T_1(n)$, $\frac{e^{-\rho T_1}}{\lambda(n)P(T_1)Q(T_1)} \log \frac{c^+(T_1,n)}{c^-(T_1,n)} = r(T_1) \int_{T_1}^{T_2} \frac{P(t)c(t,n)}{P(T_1)} dt - \frac{r(T_1)M^-(T_1)}{P(T_1)} + c^+(T_1, n) - \frac{\mu(n)c^-(T_1,n)}{\lambda(n)Q(T_1)} + \gamma Y[r(T_1) - \pi(T_1)]$. Using the budget constraint and the first order conditions for consumption, we have $\lambda(n) = \frac{e^{-\rho T_1}}{\rho}[W_0(n) + Q(T_1) M^-(T_1) - \gamma Y \sum_{j=1}^{\infty} Q(T_j) P(T_j)]^{-1}$. The value of λ in the steady state is the same for all agents, $\lambda(n) = \frac{1}{P_0 Y \hat{c}_0}$. The first order conditions for consumption for $0 \leq t < T_1(n)$ are analogous to the conditions for $t \geq T_1(n)$. In particular, $P(t) c(t,n) \mu(n) = e^{-\rho t}$ for $t \in (0, T_1(n))$. With the budget constraint for $0 \leq t < T_1(n)$, we obtain $\mu(n) = \frac{1}{M_0(n) - M^-(T_1)} \frac{1 - e^{-\rho T_1(n)}}{\rho}$.

Maximization problem for the transition

The maximization problem of each agent is

$$\max \quad \theta \sum_{j=0}^{\infty} \int_{T_j(n;1)}^{T_{j+1}(n;1)} e^{-\rho t} u(c(t,n;1))\, dt + (1-\theta) \sum_{j=0}^{\infty} \int_{T_j(n;2)}^{T_{j+1}(n;2)} e^{-\rho t} u(c(t,n;2))\, dt$$

subject to $\sum_{s=1,2} \sum_{j=1}^{\infty} Q(T_j(n;s)) [\int_{T_j(n;s)}^{T_{j+1}(n;s)} P(t;s) c(t,n;s)\, dt + P(T_j(n;s))\gamma Y] \leq \sum_{s=1,2} Q(T_1(n;s)) M^-(n;s) + W_0(n)$, and $\int_0^{T_1(n;s)} P(t;s) c(t,n;s)\, dt + M^-(n;s) = M_0(n)$, where $W_0(n) \equiv B_0(n) + \sum_{s=1,2} \int_0^{\infty} Q(t;s) P(t;s) Y dt$.

The first order conditions with respect to $c(t,n)$ and $T_j(n)$ in the state 2 imply, for $j \geq 2$, $c^+(T_j, n) [R(T_j(n)) - R(T_{j-1}(n))] - \gamma Y[r(T_j(n)) - \pi(T_j(n))] = r(T_j(n)) \int_{T_j(n)}^{T_{j+1}(n)} \frac{P(t)c(t,n)}{P(T_j(n))} dt$, where $c^+(T_j, n) = [\lambda e^{\rho T_j(n)} Q(T_j(n)) P(T_j(n))]^{-1}$. For $T_1(n)$, the first order conditions imply $c^+(T_1, n) R(T_1(n)) - \gamma Y[r(T_1(n)) - \pi(T_1(n))] - \log \frac{\lambda}{\mu(n)} + \frac{r(T_1(n))M^-(n)}{P(T_1(n))} = r(T_1(n)) \int_{T_1(n)}^{T_2(n)} \frac{P(t)c(t,n)}{P(T_1(n))} dt$.

Algorithm for the transition with endogenous segmentation

The objective is to find the equilibrium price level $P(t)$ from time zero and on. The economy is initially in the steady state with nominal interest rate $r_1 = 3\%$ per year.

We need first to describe the economy in the initial steady state. With the values of

r_1, γ and ρ, we find N in the initial steady state. Production Y and the money supply before the shock M_0^S are normalized to 1. The price level before the shock is then given by $P_0 = M_0^S/m$. The initial money holdings $M_0(n)$ of each agent $n \in [0, N)$ to make the economy start in the steady state are given by (9).

With the announcement of the new policy $r(t)$, agents choose $c(t,n)$ and $T_j(n)$. The optimal transfer times $T_j(n)$ for agent n are given by the system of equations

$$R(T_1) - \lambda \frac{Q(T_1) P(T_1) [r(T_1) - \pi(T_1)]}{e^{-\rho T_1}} \gamma Y + \log \frac{\mu(n)}{\lambda} + r(T_1) \lambda \frac{e^{-R(T_1)} M^-(T_1)}{e^{-\rho T_1}} = r(T_1) \frac{1 - e^{-\rho N_2}}{\rho}, \tag{13}$$

$$[R(T_j) - R(T_{j-1})] - \lambda \frac{Q(T_j) P(T_j) [r(T_j) - \pi(T_j)]}{e^{-\rho T_j}} \gamma Y = r(T_j) \frac{1 - e^{-\rho N_{j+1}}}{\rho}, \tag{14}$$

for $j \geq 2$. We have $\lambda = \frac{1}{P_0 c_0}$. If $M^-(T_1) = 0$ then $\mu(n) = \frac{1 - e^{-\rho T_1(n)}}{\rho M_0(n)}$ and equation (13) simplifies to

$$R(T_1) - \frac{e^{-R(T_1)}}{e^{-\rho T_1}} \gamma Y \frac{P(T_1)}{P_0 c_0} [r(T_1) - \pi(T_1)] + \log \frac{\mu(n)}{\lambda} = r(T_1) \frac{1 - e^{-\rho N_2}}{\rho},$$

and we have to check the condition $\mu(n) > Q(T_1) \lambda$ in this case. If $M^-(T_1) > 0$ then $\frac{\mu(n)}{\lambda} = Q(T_1)$. Given the optimal transfer times, we find optimal consumption for agent n by $c(t,n) = \frac{e^{-\rho t}}{\lambda P(t) Q(T_j)}$, $c^+(T_j, n) = \frac{e^{-\rho T_j}}{\lambda P(T_j) Q(T_j)}$, $c^-(T_{j+1}, n) = \frac{e^{-\rho T_{j+1}}}{\lambda P(T_{j+1}) Q(T_j)}$, for $T_j \leq t \leq T_{j+1}$, $j \geq 1$, and $c(t,n) = \frac{e^{-\rho t}}{\mu P(t)}$, $c^+(0,n) = \frac{1}{\mu P(0)^+}$, $c^-(T_1, n) = \frac{e^{-\rho T_1}}{\mu P(T_1)}$, for $0 \leq t \leq T_1(n)$. We have $Q(t) = e^{-R(t)}$, $R(t) = \int_0^t r(s)\, ds$. For the permanent shock, $r(t) = r_2$ and $R(t) = r_2 t$. For the temporary shock, $r(t) = r_1 + (r_2 - r_1) e^{-\eta t}$ and $R(t) = r_1 t - \frac{r_2 - r_1}{\eta} e^{-\eta t} + \frac{r_2 - r_1}{\eta}$.

To obtain a finite system with (13) and (14), assume that agents choose the new steady state interval N' after a long period under the new interest rate. That is, $N_{J+1} = N'$ for a large J. We have then a system of J equations in N_1, ..., N_J for each agent. We can solve this system for a given price path $P(t)$. In the simulations,

$J = 40$ which implies $N_{41} = N'$ in about 20 years after the shock.

In order to solve the system for each n, the interval $[0, N)$ is discretized as $\{n_1, n_2, ..., n_{\max}\}$ where $n_1 = 0$ and $n_{\max}$ is smaller than N but sufficiently close. In the simulations, the number of agents is such that $n_{i+1} - n_i$ is equal to 0.10 day. This implies $2,094$ agents for the parameters used.

We update the price path with the market clearing condition for goods. We have to sum consumption at time t for each agent and total resources used for transfers at time t to find aggregate demand. In equilibrium, we have,

$$\frac{1}{n_{\max}} \sum_{n} c\left(t, n; P\right) + \frac{1}{n_{\max}} \gamma Y \times \textit{Number of Transfers}\left(t; P\right) = Y,$$

for each time t, where P stands for the path of the price level. The number of transfers at t is calculated summing the agents with $T_j\left(n\right)$ such that $t \leq T_j\left(n\right) < t + 1$, that is, the unit of time is one day. Several agents make a transfer at each day. The left-hand side is divided by $n_{\max}$ because the density of agents is uniform over $[0, N)$. If demand is higher than supply at time t then increase $P\left(t\right)$ and recalculate the optimal transfer intervals for the new price. If demand is lower than supply at time t, decrease $P\left(t\right)$.

For the demand for money. Given individual spending $P\left(t\right) c\left(t, n\right)$ implied by the first order conditions, individual money demand is $M\left(t, n\right) = e^{R(T_j)} \frac{e^{-\rho t} - e^{-\rho T_{j+1}}}{\lambda \rho}$, for $T_j \leq t < T_{j+1}$, $j \geq 1$, and $M\left(t, n\right) = \frac{e^{-\rho t} - e^{-\rho T_1}}{\mu(n) \rho}$ for $0 \leq t < T_1$. With the values of T_j, we obtain individual money demand for each agent. We then aggregate over agents to find aggregate money demand at time t.

The initial guess for the price path is $P\left(t\right) = P_0 e^{\pi t}$ where π is inflation in the new steady state. Several other simulations were done with different numbers of intervals (different J's), number of agents, transfer costs, and initial guesses for prices. These changes do not affect the qualitative behavior of the price level or of the other equilibrium variables.

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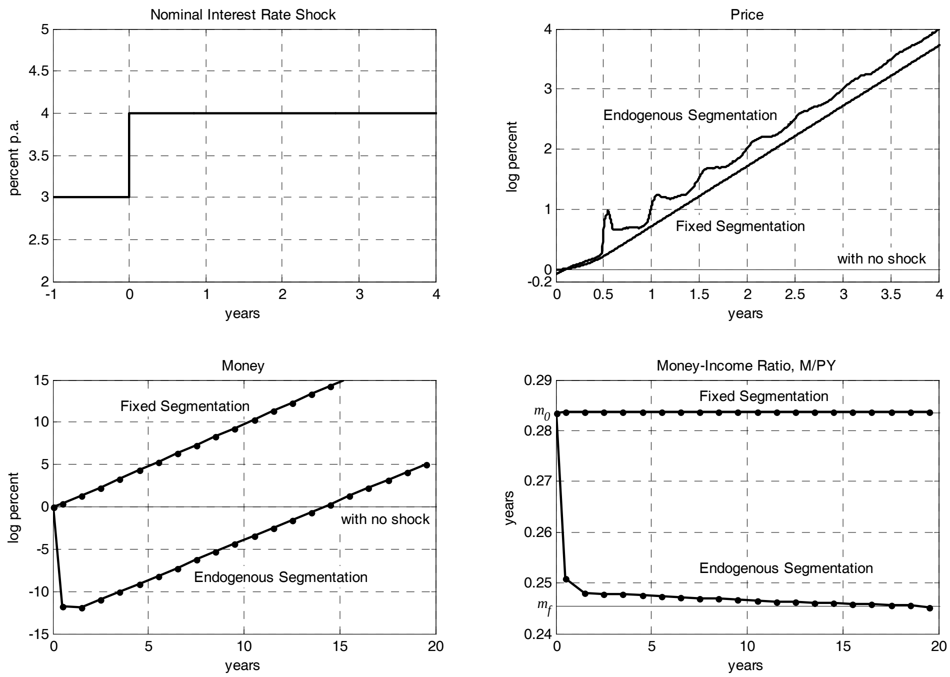

FIG. 1. Permanent interest rate shock from 3 to 4% per year. Log percent from the values before the shock (without the shock, the paths of money and prices are constant, shown in the straight lines). m_0 and m_f: steady states before and after the shock. Annual means of money and money-income ratio, the first point is the value before the shock. The dynamics of the price level continues in a similar way as shown for the remaining years. We present the price level for the first four years for clarity. Model period: one day. With endogenous segmentation, prices respond slowly, overshoot and then increase at a constant rate; money decreases after the shock (there is a liquidity effect) and later increases at the inflation rate; the money-income ratio gradually decreases to the new steady state.

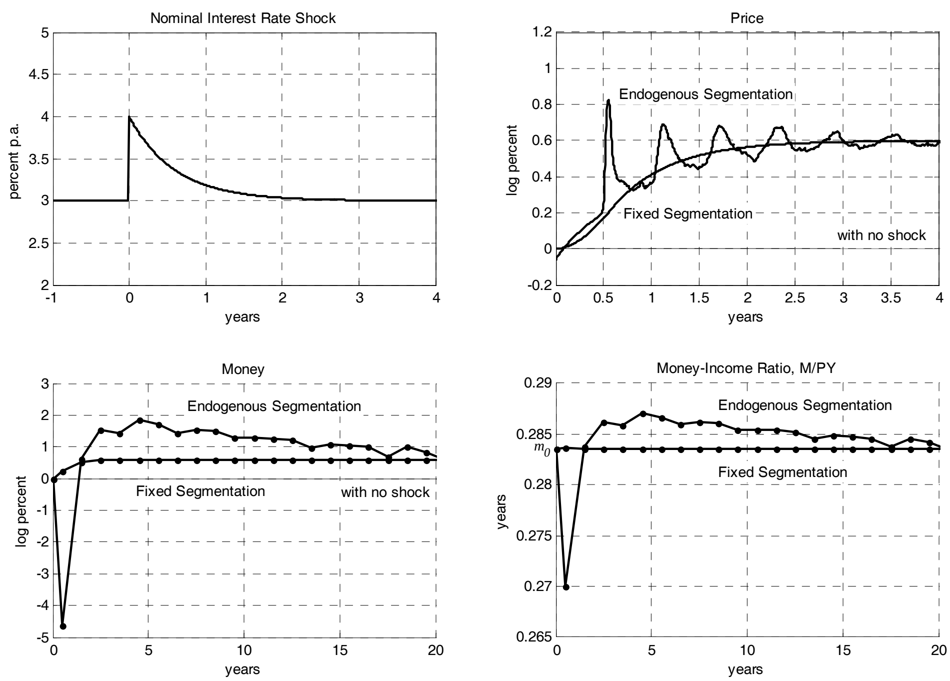

FIG. 2. Effects of a temporary interest rate shock from 3 to 4% per year. See figure (1) for definitions. Model period: one day. With endogenous segmentation, prices respond slowly, overshoot, and then converge to the new steady state; money decreases just after the shock and later converges to the new steady state, this implies constant M/P in the long run; the money-income ratio initially decreases and later returns to its previous value as inflation and the interest rate return to their previous values.